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PART II PARTICLES EXAMPLES CLASS OUTLINE ANSWERS

(2) Number of Neutrinos

(b) Due to radiation from incoming beams the centre-of-mass energy of the e^+e^- collisions is reduced. Consider $(E, \tilde{\text{p}})$ for the case where the photon is radiated from the e^- (neglect m_e and assume e^- direction unchanged):

$$\begin{aligned}
e^- &: (E - E_\gamma, 0, 0, E - E_\gamma) \\
e^+ &: (E, 0, 0, E) \\
\text{Invariant mass} \quad m^2_{e^+e^-} = s' &= E^2 - \tilde{\text{p}}^2 \\
s' &= (E + E - E_\gamma)^2 - (E - E_\gamma + E)^2 \\
&= 4E^2 - 4EE_\gamma \\
&= s - 2E_\gamma\sqrt{s}
\end{aligned}$$

(d) Number of interactions observed in a collider experiment N:

$$N = \sigma\mathcal{L}$$

where $\mathcal{L}$ is intergrated luminosity of the colliding beams. It emcompasses many things *e.g.* beam current (number of e^+e^-), beam size (number density), whether the beams completely overlap, *etc.*

Here $\mathcal{L} = 22.5\ \text{pb}^{-1}$ which means that for a cross section of 1 pb expect 22.5 interactions. Observe 208 single photon events of which expect 20 are attributed to come from background processes. Therefore, the number of genuine observed $\nu\bar{\nu}\gamma$ events is $208 - 20 = 188$. Since detection efficiency is 66%, the best estimate of number of interactions is $\nu\bar{\nu}\gamma$ is 188/0.66 :

$$\begin{aligned}
\sigma &= \frac{N - N_B}{\epsilon\mathcal{L}} \\
&= \frac{208 - 20}{0.66 \times 22.5}\ \text{pb} \\
&= 12.7\ \text{pb}
\end{aligned}$$

(e) Mean measured photon energy is 2.85 GeV:

$$\begin{aligned}
\langle E_\gamma \rangle &= 2.85\ GeV \\
\Rightarrow \quad \text{mean} \ s' &= \sqrt{s} - 2\langle E_\gamma \rangle \sqrt{s} \\
&= 91.3^2 - 2 \times 2.85 \times 91.3 \\
\sqrt{s'} &= 88.4\ \text{GeV}
\end{aligned}$$

(f) Breit-Wigner expression for cross section refers to the total $\sigma_{\nu\bar{\nu}}$ cross section. Only observe the fraction where a photon of energy greater than 1.75 GeV has been radiated (12.7 pb). This constitues 1/180 of the total cross section, therefore:

$$\begin{aligned}
\sigma_{\nu\bar{\nu}} &= 180 \times 12.7\ \text{pb} \\
&= 2.29\ \text{nb}
\end{aligned}$$

Using Breit-Wigner formula for cross section and noting that we are below the peak of the Z^0 resonance:

$$\begin{aligned}\sigma_{\nu\overline{\nu}} &= \frac{3\pi\Gamma_{ee}\Gamma_{\nu\overline{\nu}}}{s'[(\sqrt{s'}-M_{Z^0})^2-\Gamma_Z{}^2/4]} \\ &= \frac{3\pi\times 0.084\Gamma_{\nu\overline{\nu}}}{7815[(88.4-91.187)^2-2.49^2/4]} \\ &= 1.09\times 10^{-5}\Gamma_{\nu\overline{\nu}}\text{GeV}^{-2}.\end{aligned}$$

(Note $\Gamma_{\nu\overline{\nu}}$ is measured in GeV). Need to convert from natural units to S.I. units *i.e.* pb. Recall 1 barn $= 10^{-28}$ m and putting back missing factors of $\hbar, c$

$$\begin{aligned}[\sigma]=[L]^2 &= [E^{-2}][\hbar]^m[c]^n \\ &= [E^{-2}][E]^m[T]^m[L]^n[T]^{-n} \\ \Rightarrow n=m &= 2 \\ \therefore \quad 1.09\times 10^{-5}\times(\Gamma_{\nu\overline{\nu}}/\text{GeV})\ \text{GeV}^{-2} &= 1.09\times 10^{-5}\hbar^2c^2\text{GeV}^{-2}(\Gamma_{\nu\overline{\nu}}/\text{GeV})\ m^2 \\ &= (\Gamma_{\nu\overline{\nu}}/\text{GeV})\times 4.31\times 10^{-37}\ m^2 \\ &= (\Gamma_{\nu\overline{\nu}}/\text{GeV})\times 4.31\ \text{nb}\end{aligned}$$

Measured value of 2.29 nb corresponds to

$$\begin{aligned}\Gamma_{\nu\overline{\nu}}/\text{GeV}) &= 2.29/4.31 = 0.53 \\ \Gamma_{\nu\overline{\nu}} &= 0.53\ \text{GeV}\end{aligned}$$

From lecture notes, $\Gamma_{\nu\overline{\nu}} = 0.167 N_{\nu\overline{\nu}}$ GeV giving:

$$\begin{aligned}N_{\nu\overline{\nu}} &= 0.53/0.167 \\ &= 3.18\pm 0.22\end{aligned}$$

where the error is from the statistical uncertainty associated with the 208 observed events *i.e.* $\sqrt{N}$.

(3) The Charmed Baryons

(a) Quantum Numbers for $(cdd), (cuu), (css)$ baryons

Treat quarks as identical particles labelled by flavour giving the form of the the baryon wave-function

$$\psi_{baryon} = \psi_{colour}\psi_{space}\psi_{flavour}\psi_{spin}$$

which must be ANTI-SYMMETRIC under interchange of any two quarks. For all baryons ψ_{colour} is ANTI-SYMMETRIC *i.e.* the colourless singlet state. For ground state (L=0) baryons the relative orbital angular momentum between all quarks is zero and ψ_{space} is SYMMETRIC. Therefore $\psi_{flavour}\psi_{spin}$ is SYMMETRIC under interchange of any two quarks.

$(cdd), (cuu), (css)$ baryons Consider the light quark pair in *e.g.* (cdd). Flavour wave-function is SYMMETRIC ($\psi_{flavour} = dd$), therefore spin wave-function must be SYMMETRIC, *i.e.* must have total spin $S_{qq} = 1$.

$$J(\text{Baryon}) = S_c + S_{qq} = \frac{1}{2}+1$$

i.e. $(cdd), (cuu), (css)$ baryons can exist in spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ states.

$\underline{\boldsymbol{(cdu), (cds), (cus)}}$ **baryons** Consider the light quark pair in *e.g.* (cud) still requiring $\psi_{flavour}\psi_{spin}$ to be SYMMETRIC Flavour wave-function can be:

$$\begin{array}{ll} \text{symmetric} & \frac{1}{\sqrt{2}}(ud+du) \\ \text{or anti}-\text{symmetric} & \frac{1}{\sqrt{2}}(ud-du) \end{array}$$

Since there is no spin-$\frac{3}{2}$ anti-symmetric spin state then cannot make a spin-$\frac{3}{2}$ state from the anti-symmetric flavour combination. No suce restriction for spin-$\frac{1}{2}$ since both symmetric and anti-symmetric spin states exist, In total there are 6 spin-$\frac{1}{2}$ and 3 spin-$\frac{3}{2}$ states.

Intrinsic parities $= P_1P_2P_3(-1)^L(-1)^{L'} = +1$ since parities of fermions are defined to be positive.

(b) *$J=\frac{3}{2}$ Baryon Masses*

- $J = \vec{S}_1+\vec{S}_2+\vec{S}_3 = \frac{3}{2}$. If $\vec{S}_{ij}=0$ then $J = \mathbf{\frac{1}{2}}+\mathbf{0} = \frac{1}{2}$. Therefore any pair of quarks in a spin $\frac{3}{2}$ baryon must have total spin 1.

- $$\begin{aligned} \vec{S}_{ij} &= \vec{S}_i+\vec{S}_j \\ \vec{S}_{ij}^2 &= \vec{S}_i^2+\vec{S}_j^2+2\vec{S}_i\cdot\vec{S}_j \\ \vec{S}_i\cdot\vec{S}_j &= \frac{1}{2}[1(1+1)-2\frac{1}{2}(1+\frac{1}{2})] \\ &= \frac{1}{4} \end{aligned}$$

  for any pair of quarks in a spin-$\frac{3}{2}$ baryon.

- $$\begin{aligned} M(baryon) &= m_1+m_2+m_3+A\left(\frac{\vec{S}_1\cdot\vec{S}_2}{m_1m_2}+\frac{\vec{S}_1\cdot\vec{S}_3}{m_1m_3}+\frac{\vec{S}_2\cdot\vec{S}_3}{m_2m_3}\right) \\ M(J=\frac{3}{2}) &= m_1+m_2+m_3+A\left(\frac{\vec{S}_1\cdot\vec{S}_2}{m_1m_2}+\frac{\vec{S}_1\cdot\vec{S}_3}{m_1m_3}+\frac{\vec{S}_2\cdot\vec{S}_3}{m_2m_3}\right) \\ M(J=\frac{3}{2}) &= m_1+m_2+m_3+\frac{A}{4}\left(\frac{1}{m_1m_2}+\frac{1}{m_1m_3}+\frac{1}{m_2m_3}\right) \end{aligned}$$

(c) *$J=\frac{1}{2}$ Baryon Masses*

- $$\begin{aligned} \vec{S} &= \vec{S}_1+\vec{S}_2+\vec{S}_3 \\ \vec{S}_{ij}^2 &= \vec{S}_1^2+\vec{S}_2^2+\vec{S}_3^2+2(\vec{S}_1\cdot\vec{S}_2+\vec{S}_1\cdot\vec{S}_3+\vec{S}_2\cdot\vec{S}_3) \\ \vec{S}_1\cdot\vec{S}_2+\vec{S}_1\cdot\vec{S}_3+\vec{S}_2\cdot\vec{S}_3 &= \frac{1}{2}(\vec{S}^2-\vec{S}_1^2-\vec{S}_1^2-\vec{S}_1^2) \\ &= \frac{1}{2}\left(\frac{3}{4}-3\times\frac{3}{4}\right) \\ \vec{S}_1\cdot\vec{S}_2+\vec{S}_1\cdot\vec{S}_3+\vec{S}_2\cdot\vec{S}_3 &= -\frac{3}{4} \qquad (1) \end{aligned}$$

- For $\Sigma_c^0, \Sigma_c^{++}$ and Ω_c^0 (cdd,cuu,css) have $S_{qq} = 1$ from part (a).
- Λ_c^+ and Σ_c^+, both (cud), are the $S_{qq} = 0$ and $S_{qq} = 1$ (cud) states from part (a). Identify the $S_{qq} = 1$ state with Σ_c^+ - forming a *triplet* of states with the $\Sigma_c^0, \Sigma_c^{++}$. The Λ_c^+ is then identified as the $S_{qq} = 0$ (cud) state.
- For Λ_c^+ $S_{qq} = 0$:

$$\begin{aligned} 0 &= \vec{S}_u(\vec{S}_u + 1) + \vec{S}_d(\vec{S}_d + 1) + 2\vec{S}_d \cdot \vec{S}_u \\ \vec{S}_d \cdot \vec{S} &= -\frac{3}{4} \end{aligned}$$

Using Equation 1):

$$\begin{aligned} \vec{S}_d \cdot \vec{S}_u + \vec{S}_d \cdot \vec{S}_c + \vec{S}_u \cdot \vec{S}_c &= -\frac{3}{4} \\ \Rightarrow \vec{S}_d \cdot \vec{S}_c + \vec{S}_u \cdot \vec{S}_c &= 0 \end{aligned}$$

giving

$$M(\Lambda_c^+) = m_c + 2m_u - A \cdot \frac{3}{4m_u^2}$$

- $\Sigma_c^0, \Sigma_c^{++}$ and Ω_c^0 (cdd, cuu, css) have $S_{qq} = 1$ which implies $\vec{S}_{q_1}\vec{S}_{q_2} = -\frac{3}{4}$. Inserting in Equation 1 gives:

$$\vec{S}_c \cdot \vec{S}_{q_1} + \vec{S}_c \cdot \vec{S}_{q_2} = -1$$

Giving:

$$\begin{aligned} M(\Sigma_c) &= m_c + 2m_u + A\left(\frac{1}{4m_u^2} - \frac{1}{m_c m_u}\right) \\ M(\Omega_c^0) &= m_c + 2m_s + A\left(\frac{1}{4m_s^2} - \frac{1}{m_c m_s}\right) \end{aligned}$$

- Ξ_c^0, Ξ_c^+ (cds, cus). The strong interaction couples to colour and does not distinguish between different flavour quarks. In the limit where $m_u, m_s \ll m_c$ we would expect $\vec{S}_c \cdot \vec{S}_u = \vec{S}_c \cdot \vec{S}_s$.
- Ξ_c^0, Ξ_c^+ (cds, cus). For the $S_{qq} = 0$ states (from discussion of Λ_c^+)

$$\begin{aligned} \vec{S}_c \cdot \vec{S}_u + \vec{S}_c \cdot \vec{S}_s &= 0 \\ \text{giving} \quad \vec{S}_c \cdot \vec{S}_u &= \vec{S}_c \cdot \vec{S}_s = 0 \end{aligned}$$

For the $S_{qq} = 1$ states (see discussion of Λ_c^+) then

$$\begin{aligned} \vec{S}_c \cdot \vec{S}_u + \vec{S}_c \cdot \vec{S}_s &= -1 \\ \text{whichheregives} \vec{S}_c \cdot \vec{S}_u &= \vec{S}_c \cdot \vec{S}_s == \frac{1}{2} \end{aligned}$$

$$\begin{aligned} M(\Xi_c;\, S_{qq} = 0) &= m_c + m_s + m_u - A \cdot \frac{3}{4m_s m_u} \\ M(\Xi_c;\, S_{qq} = 1) &= m_c + m_s + m_u + A\left(\frac{1}{4m_s m_u} - \frac{1}{2m_c m_s} - \frac{1}{2m_c m_u}\right). \end{aligned}$$

		$J=\frac{1}{2}$		$J=\frac{3}{2}$	
		predicted	observed	predicted	observed
Λ_c^+		2281	2285	$---$	$---$
$\Sigma_c^0,\ \Sigma_c^+,\ \Sigma_c^{++}$		2438	2453	2502	2530
$\Xi_c^0,\ \Xi_c^+$	$S_{qq}=0$	2505	2470/2465	$---$	$---$
$\Xi_c^0,\ \Xi_c^+$	$S_{qq}=1$	2604	?	2658	?
Ω_c^0		2775	2710	2818	?

(d) *Comparison with Experiment*

$$m_u = m_d = 363\,\text{MeV}\,; \qquad m_s = 538\,\text{MeV}\,; \qquad m_c = 1705\,\text{MeV}\,; \qquad A/m_u^2 = 200\,\text{MeV}$$

(4) Experimental Determination of the Spin of the π^+ Meson

(a) In cms frame $E^{*2} - p^{*2} = E^2_{cms}$ since $p = 0$ (starred quantities refer to cms system). Since $(E^2 - p^2)$ is invariant.

$$E^2_{cms} \;=\; E^2_{lab} - p_{lab}$$

For process $a + b(\text{at rest}) \to c + d$

$$\begin{aligned} E^2_{cms} &= (E_1 + E_2)^2 - p_1^2 \\ &= E_1^2 + E_2^2 + 2E_1E_2 - p_1^2 \\ &= m_1^2 + p_1^2 + m_2^2 + 2(m_1 + T_1)m_2 - p_1^2 \\ \Rightarrow \quad E^2_{cms} &= [m_1^2 + m_2^2 + 2(m_1 + T_1)m_2]^{\frac{1}{2}} \end{aligned}$$

(b) cross section is transition rate divided by flux:

$$\begin{aligned} \sigma &= \frac{\Gamma_{if}}{v_i} \\ \therefore \quad \frac{\sigma(A)}{\sigma(B)} &= \frac{\Gamma_{if}^{(A)}}{\Gamma_{if}^{(B)}} \frac{v_i^{(B)}}{v_i^{(A)}} \\ &= \frac{|M_A|^2 (dn/dE)_A}{|M_B|^2 (dn/dE)_B} \frac{v_i^{(B)}}{v_i^{(A)}} \end{aligned}$$

Here $|M_A|^2 = |M_{pp\to\pi^+d}|^2 = |M_{\pi^+d\to pp}|^2 = |M_B|^2$ by the principle of detailed balance.

$$\therefore \quad \frac{\sigma(A)}{\sigma(B)} = \frac{(dn/dE)_A}{(dn/dE)_B} \frac{v_i^{(B)}}{v_i^{(A)}}$$

(c) Consider $a + b \to c + d$ in cms where $p_c = p_d = p^*$.

$$\begin{aligned}
E_{cms} &= E_c^* + E_d^* \\
E_{cms}^2 &= E_c^{*2} + E_d^{*2} + 2E_c^* E_d^* \\
&= m_c^2 + p^{*2} + m_d^2 + p^{*2} + 2E_c^* E_d^* \\
E_{cms}^2 - m_c^2 - m_d^2 - 2p^{*2} &= 2E_c^* E_d^* \\
(E_{cms}^2 - m_c^2 - m_d^2 - 2p^{*2})^2 &= 4E_c^{*2} E_d^{*2} \\
(E_{cms}^2 - m_c^2 - m_d^2)^2 + 4p^{*2} - 4p^{*2}(E_{cms}^2 - m_c^2 - m_d^2) &= 4(m_c^2 + p^{*2})(m_d^2 + p^{*2}) \\
p^{*2} &= \frac{(E_{cms}^2 - m_c^2 - m_d^2)^2 - 4m_c^2 m_d^2}{4E_{cms}^2}
\end{aligned}$$

(d) $g = (2S_c + 1)(2S_d + 1)$ spin states. If particles are identical then an extra factor of $\frac{1}{2}$ to avoid double counting when integrating over 4π solid angle.

(e)

$$\begin{aligned}
E_{cms} &= E_c^* + E_d^* \\
\frac{dE_{cms}}{dp^*} &= \frac{dE_c^*}{dp^*} + \frac{dE_d^*}{dp^*} \\
\text{but} \quad E^2 &= p^2 + m^2 \\
2EdE &= 2pdp \\
\frac{dE}{dp} &= \frac{p}{E} = \frac{\gamma m v}{\gamma m} = v \\
\therefore \quad \frac{dE_{cms}}{dp^*} &= |v_c^*| + |v_d^*| = v_f
\end{aligned}$$

(f) In cms frame

$$\begin{aligned}
\frac{\sigma(A)}{\sigma(B)} &= \frac{(dn/dE)_A}{(dn/dE)_B} \frac{v_i^{(B)}}{v_i^{(A)}} \\
&= \frac{(dn/dp^*)_A (dp^*/dE^*)_A}{(dn/dE)_B (dp^*/dE^*)_B} \frac{v_i^{(B)}}{v_i^{(A)}} \\
&= \frac{p_A^{*2} g_A v_f^(A)}{p_B^{*2} g_B v_f^(B)} \frac{v_i^{(B)}}{v_i^{(A)}}
\end{aligned}$$

Symmetric reactions at same cms energy $\Rightarrow v_f^(B) = v_i^(A)$ and $v_f^(A) = v_i^(B)$, giving

$$\begin{aligned}
\frac{\sigma(A)}{\sigma(B)} &= \frac{p_A^{*2} g_A}{p_B^{*2} g_B} \\
\text{but} \quad g_A &= (2S_\pi + 1)(2S_d + 1) \\
\text{and} \quad g_B &= \frac{1}{2}(2S_p + 1)(2S_p + 1) \\
\frac{\sigma(A)}{\sigma(B)} &= \frac{3}{2}(2S_\pi + 1)\frac{p_A^{*2}}{p_B^{*2}} \\
&= 0.064(2S_\pi + 1) \\
\therefore \quad 0.058 \pm 0.020 &= 0.064(2S_\pi + 1) \\
S_\pi &= -0.05 \pm 0.15
\end{aligned}$$