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# PART II PARTICLES EXAMPLES CLASS

1. **The Forces**

Give an account of the nature and properties of the strong, electromagnetic and weak interactions, illustrating your answer with Feynman diagrams involving relevant quarks, leptons and gauge bosons.

Draw possible Feynman diagrams for the following processes, and state what interactions are involved in each case:

(a) $e^+ + e^- \to W^+ + W^-$ (only two of the lowest order diagrams are required.)

(b) $\Delta^+(uud) \to n(udd) + \pi^+(u\bar{d})$;

(c) $\Sigma^+(uus) \to p(uud) + \gamma$.

[The quark compositions of the hadrons are shown.]

2. **The Number of Neutrino Species.**

The $\mathrm{Z}^0$ boson is produced in $e^+e^-$ collisions at centre-of-mass energies near the $\mathrm{Z}^0$ resonance and can decay into pairs of neutrinos with mass less than $M_Z/2$. The existence of a substantial $\mathrm{Z}^0$ "invisible width", $\Gamma_{\mathrm{inv}}$, can be inferred from the precision measurements of the production cross-sections of hadrons and charged leptons.

This *indirect* measurement of $\Gamma_{\mathrm{inv}}$ can be interpreted as a measurement of the number of light neutrino species, $N_\nu$, and the final results from the LEP collider at CERN give

$$N_\nu = 2.9841 \pm 0.0083.$$

A *direct* measurement of the invisible width can also be made from the observation of events in which either the initial state electron or positron radiates a photon,

$$e^+e^- \to \mathrm{Z}^0 \to \nu\bar{\nu}\gamma.$$

(a) Draw one of the lowest order Feynman diagrams representing this process.

(b) If the radiated photon has energy, $E_\gamma$, show that the centre-of-mass energy squared of the $e^+e^-$ collisions, $s'$, is given by

$$s' \approx s - 2\,E_\gamma\sqrt{s}$$

where $\sqrt{s} = 2E$ and $E$ is the energy of the electron and positron beams. You may assume that the deflection of the initial state electron due to the radiation of a photon is small.

(c) The cross-section for invisible particle production via $\mathrm{Z}^0$ exchange can be written as,

$$\sigma(s') = \frac{3\pi\Gamma_{ee}\Gamma_{\mathrm{inv}}}{s'\left[(\sqrt{s'} - M_Z)^2 + \Gamma_Z^2/4\right]}$$

where $\Gamma_{ee}$ is the $Z^0$ partial width to electrons and $\Gamma_Z$ is the total width. Describe the origin of the terms in this equation (without detailed calculation).

(d) In 1992, the OPAL experiment at LEP collected an integrated luminosity of 22.5 $\mathrm{pb}^{-1}$ at a centre-of-mass energy, $\sqrt{s} = 91.3$ GeV. They observed 208 events containing single photon with $E_\gamma > 1.75$ GeV. The photon detection efficiency is 66% and the estimated number of background events is 20. (Note expected number of interactions (events), $N$ is related to luminosity, $\mathcal{L}$, by $N = \mathcal{L}\sigma$).

Calculate the measured cross-section for single photon production.

(e) The observed photons in the single photon event sample have a mean energy of 2.85 GeV. Calculate the mean value of $s'$.

(f) The probability that an initial state electron radiates a photon with $E_\gamma > 1.75$ GeV has been calculated and is 1/180. Hence, calculate the invisible width assuming:

$$M_Z = 91.187\,\mathrm{GeV} \qquad \Gamma_\mathrm{Z} = 2.49\,\mathrm{GeV} \quad \text{and} \quad \Gamma_\mathrm{ee} = 0.084\,\mathrm{GeV}.$$

What number of neutrino species does this value correspond to and how does it compare to the number extracted from the *indirect* method ? (In the Standard Model the expected partial width to neutrinos is $\Gamma_{\nu\overline{\nu}} = 0.168\,\mathrm{GeV}$.

(g) Finally, draw a Feynman diagram ($e^+e^- \rightarrow \nu\overline{\nu}\gamma$) involving a single W boson that would contribute to the measured cross-section. Can you suggest an electromagnetic process that would be the dominant background source ? (This latter background gives a single observed photon but has no neutrinos in the final state).

3. **The Charmed Baryons**

In this example, the quark model predictions for the quantum numbers, masses and decay modes of the ground state baryons

$$\Lambda_c^+ \text{ (cud)}; \quad \Sigma_c^0, \Sigma_c^+, \Sigma_c^{++} \text{ (cdd,cud,cuu)}; \quad \Xi_c^0, \Xi_c^+ \text{ (cds,cus)}; \quad \Omega_c^0 \text{ (css)}$$

containing a single charm quark c combined with two light quarks u,d,s are investigated and compared with experiment.

(a) *Quantum Numbers*

When the two light (u,d or s) quarks are of the same flavour (cdd,cuu,css baryons), show that the light quark pair must have total spin $S_{qq} = 1$ and hence that there should be just one spin $\frac{1}{2}$ and one spin $\frac{3}{2}$ baryon in each case. Show that, in all, 9 $J = \frac{1}{2}$ and 6 $J = \frac{3}{2}$ baryons containing a single charm quark are expected. Plot the positions of these baryons on diagrams with "isospin", on the horizontal axis and "strangeness", on the vertical axis. Draw separate diagrams for the $J = \frac{1}{2}$ and $J = \frac{3}{2}$ baryons. Recall isospin $= N_u - N_d + N_{\overline{d}} - N_{\overline{u}}$ and strangeness $= N_{\overline{s}} - N_s$ What are the intrinsic parities of these baryon states ?

(b) $J = \frac{3}{2}$ *Baryon Masses*

Explain why any pair of quarks in a spin $\frac{3}{2}$ baryon must have total spin 1. Hence show that $\vec{\mathrm{S}}_i \cdot \vec{\mathrm{S}}_j = \frac{1}{4}$ for any pair of quark spins $\vec{\mathrm{S}}_i$ and $\vec{\mathrm{S}}_j$, and that the quark model prediction for the spin $\frac{3}{2}$ baryon masses is

$$M(J=\tfrac{3}{2}) \;=\; m_1 + m_2 + m_3 + \frac{A}{4}\left(\frac{1}{m_1 m_2} + \frac{1}{m_1 m_3} + \frac{1}{m_2 m_3}\right)$$

where $A$ is a constant and $m_1$, $m_2$, $m_3$ are the quark masses.

(c) $J=\frac{1}{2}$ *Baryon Masses*

Show that, for the spin $\frac{1}{2}$ baryons, the quark spins $\vec{\mathrm{S}}_i$ satisfy the relation

$$\vec{\mathrm{S}}_1\cdot\vec{\mathrm{S}}_2 + \vec{\mathrm{S}}_1\cdot\vec{\mathrm{S}}_3 + \vec{\mathrm{S}}_2\cdot\vec{\mathrm{S}}_3 = -\tfrac{3}{4}.$$

The $\Sigma_c^+$ baryon has the same mass as the $\Sigma_c^0$ and $\Sigma_c^{++}$ Justtify the identification of spin states of the light quark pairs as $S_{qq} = 0$ for the $\Lambda_\mathrm{c}^+$ and $S_{qq} = 1$ for the $\Sigma_c$ and $\Omega_c^0$ baryons. Hence show that $\vec{\mathrm{S}}_u\cdot\vec{\mathrm{S}}_d = -\frac{3}{4}$ for the $\Lambda_\mathrm{c}^+$ baryon and that, assuming equal masses for the u and d quarks ($m_u = m_d$), the quark model predicts

$$M(\Lambda_\mathrm{c}^+) \;=\; \mathrm{m_c} + 2\mathrm{m_u} - \mathrm{A}\cdot\frac{3}{4\mathrm{m_u^2}}.$$

Similarly, show that the $J=\frac{1}{2}$ $\Sigma_c$ and $\Omega_c^0$ masses are predicted to be

$$\begin{aligned} M(\Sigma_c) &= m_c + 2m_u + A\left(\frac{1}{4m_u^2} - \frac{1}{m_c m_u}\right) \\ M(\Omega_c^0) &= m_c + 2m_s + A\left(\frac{1}{4m_s^2} - \frac{1}{m_c m_s}\right). \end{aligned}$$

Like the other baryons in the $J=\frac{1}{2}$ multiplet, the light quark pair in a spin $\frac{1}{2}$ $\Xi_c$ baryon is expected to be in a state of definite total spin: $S_{qq} = 0$ or $S_{qq} = 1$. [In practice, some mixing between the $S_{qq} = 0$ and $S_{qq} = 1$ states may occur, but this is expected to be small]. How would you expect the vector products $\vec{\mathrm{S}}_c\cdot\vec{\mathrm{S}}_1$ and $\vec{\mathrm{S}}_c\cdot\vec{\mathrm{S}}_2$ involving the charm quark spin $\vec{\mathrm{S}}_c$ and the light quark spins $\vec{\mathrm{S}}_1$, $\vec{\mathrm{S}}_2$ to be related ? Show that the $J=\frac{1}{2}$ $\Xi_c$ baryon masses are predicted to be

$$\begin{aligned} M(\Xi_c;\, S_{qq}=0) &= m_c + m_s + m_u - A\cdot\frac{3}{4m_s m_u} \\ M(\Xi_c;\, S_{qq}=1) &= m_c + m_s + m_u + A\left(\frac{1}{4m_s m_u} - \frac{1}{2m_c m_s} - \frac{1}{2m_c m_u}\right). \end{aligned}$$

(d) *Comparison with Experiment*

Evaluate the charmed baryon masses predicted by the quark model assuming:

$$m_u = m_d = 363\,\mathrm{MeV}\,; \quad m_s = 538\,\mathrm{MeV}\,; \quad m_c = 1705\,\mathrm{MeV}\,; \quad A/m_u^2 = 200\,\mathrm{MeV}$$

and enter the predicted masses (in MeV) into a table such as the following:

| | $J=\frac{1}{2}$ | | $J=\frac{3}{2}$ | |
|---|---|---|---|---|
| | predicted | observed | predicted | observed |
| $\Lambda_c^+$ | | | $---$ | $---$ |
| $\Sigma_c^0$, $\Sigma_c^+$, $\Sigma_c^{++}$ | | | | |
| $\Xi_c^0$, $\Xi_c^+$ $\quad S_{qq}=0$ | | | $---$ | $---$ |
| $\Xi_c^0$, $\Xi_c^+$ $\quad S_{qq}=1$ | | | | |
| $\Omega_c^0$ | | | | |

The following charmed baryons have been observed to date:

$$X^+(2285),\ X^0(2452),\ X^{++}(2453),\ X^+(2454),\ X^+(2465),\ X^0(2470),\ X^{++}(2530),\ X^0(2710)$$

where the numbers in parentheses are the masses in MeV. Suggest the most likely explanation for the observed states, and complete the table as far as possible.

(e) *Baryon Decays*

Explain why the $\Lambda_c^+$ can not decay via the strong or electromagnetic interactions, but must decay weakly. Give examples of allowed $\Lambda_c^+$ decays into final states containing (i) a strange meson and (ii) a strange baryon. Draw leading order Feynman diagrams corresponding to these examples.

The $J=\frac{1}{2}$ $\Sigma_c$ baryons decay via $\Sigma_c \to \Lambda_c^+ + \pi$ with branching ratios of almost 100%. Are these decays strong, weak or electromagnetic ? Draw a leading order Feynman diagram corresponding to the decay $\Sigma_c^+ \to \Lambda_c^+ + \pi^0$.

What would you expect to be the dominant decay mode of the $J=\frac{3}{2}$ $\Omega_c^0(2818)$ baryon ?

## 4. Experimental Determination of the Spin of the $\pi^+$ Meson

The spin of the $\pi^+$ meson was established in the early 1950's using the following cross section measurements:

$$\sigma(pp \to \pi^+ d) = 0.18 \pm 0.06 \text{ mb} \qquad (A)$$

$$\sigma(\pi^+ d \to pp) = 3.10 \pm 0.33 \text{ mb} \qquad (B)$$

the laboratory kinetic energies of the incident p and $\pi^+$ being 340 MeV and 29 MeV respectively. The rest masses of the $\pi^+$, p and d are 139.6, 938.3 and 1875.6 MeV/c$^2$. The rates $R$ for these reactions are given by Fermi's Golden Rule:

$$R = \frac{2\pi}{\hbar}|M|^2\frac{dn}{dE}$$

where $M$ is the quantum mechanical matrix element and $dn/dE$ is the density of final states. The principle of detailed balance says that:

$$|M_{i\to f}|^2 = |M_{f\to i}|^2 \qquad (C)$$

Thus the rates (and hence cross sections) from (A) and (B) will be related and information on the pion spin can be obtained.

(a) Equation (C) is true only if the centre of mass energies are the same for the two reactions. Show that

$$E_{cms} \simeq 2040 \text{ MeV} \quad \text{for (A)}$$
$$E_{cms} \simeq 2042 \text{ MeV} \quad \text{for (B)}$$

This is close enough to use detailed balance.
[Hint: $E^2_{cms} = E^2_{tot} - p^2_{tot}$ where the RHS being Lorentz invariant can be evaluated in any frame.]

(b) Show that

$$\frac{\sigma(A)}{\sigma(B)} = \frac{(dn/dE)_A}{(dn/dE)_B} \cdot \frac{(v_i)_B}{(v_i)_A}$$

where $(v_i)_A$ and $(v_i)_B$ are the velocities of approach of the incident particles for reactions (A) and (B).

(c) Show that the cms momenta $p^*$ of the final state particles are:

$$p^* \simeq 83 \text{ MeV} \quad \text{for (A)}$$
$$p^* \simeq 402 \text{ MeV} \quad \text{for (B)}$$

(d) For any two body reaction a+b→c+d considered in the cms frame, the density of final states is given by:

$$\frac{dn}{dp^*} = \frac{4\pi V p^{*2}}{h^3}$$

where the system is confined to a volume V and the particles are assumed to be spinless. Show that if c and d have spins $S_c$ and $S_d$ an extra factor

$$g = (2S_c + 1)(2S_d + 1)$$

is required. What extra modification is required if c and d are identical ?

(e) Show that

$$\frac{dE}{dp^*} = v_f$$

where E is the total cms energy and $v_f$ the velocity of separation of c and d.

(f) Hence show that:

$$\frac{\sigma(A)}{\sigma(B)} = \frac{3}{2}(2S_\pi + 1)\frac{p^{*2}_A}{p^{*2}_B}$$

and finally:

$$S_\pi = -0.05 \pm 0.15$$

(you may assume p and d have spins $\frac{1}{2}$ and 1).

**For hints see `http://www.hep.phy.cam.ac.uk/~thomson/particles/`**

**Answers will appear after the class**