Particle Physics

Michaelmas Term 2011
**Prof Mark Thomson**
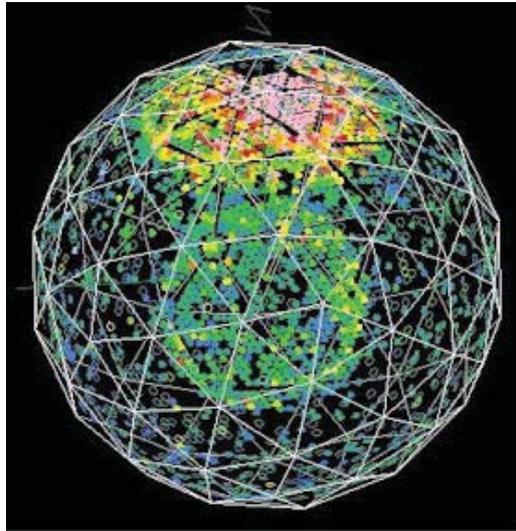
Handout 9 : The Weak Interaction and V-A

Parity

★ **The parity operator performs spatial inversion through the origin:**

$$\psi'(\vec{x},t) = \hat{P}\psi(\vec{x},t) = \psi(-\vec{x},t)$$

- **applying** $\hat{P}$ **twice:** $\quad \hat{P}\hat{P}\psi(\vec{x},t) = \hat{P}\psi(-\vec{x},t) = \psi(\vec{x},t)$

  so $\qquad \hat{P}\hat{P} = I \qquad \longrightarrow \qquad \hat{P}^{-1} = \hat{P}$

- **To preserve the normalisation of the wave-function**

$$\langle\psi|\psi\rangle = \langle\psi'|\psi'\rangle = \langle\psi|\hat{P}^\dagger\hat{P}|\psi\rangle$$

$$\hat{P}^\dagger\hat{P} = I \qquad \longrightarrow \qquad \hat{P} \quad \text{Unitary}$$

- **But since** $\hat{P}\hat{P} = I \qquad \hat{P} = \hat{P}^\dagger \qquad \longrightarrow \qquad \hat{P} \quad \text{Hermitian}$

  which implies Parity is an observable quantity. If the interaction Hamiltonian commutes with $\hat{P}$ **, parity is an observable conserved quantity**

- **If** $\psi(\vec{x},t)$ **is an eigenfunction of the parity operator with eigenvalue** P

$$\hat{P}\psi(\vec{x},t) = P\psi(\vec{x},t) \qquad \longrightarrow \qquad \hat{P}\hat{P}\psi(\vec{x},t) = P\hat{P}\psi(\vec{x},t) = P^2\psi(\vec{x},t)$$

  since $\hat{P}\hat{P} = I \qquad P^2 = 1$

  ➡ **Parity has eigenvalues** $P = \pm1$

★ **QED and QCD are invariant under parity**

★ **Experimentally observe that Weak Interactions do not conserve parity**

Intrinsic Parities of fundamental particles:

Spin-1 Bosons

- From Gauge Field Theory can show that the gauge bosons have $P = -1$

$$P_\gamma = P_g = P_{W^+} = P_{W^-} = P_Z = -1$$

Spin-½ Fermions

- From the Dirac equation showed (handout 2):

    Spin ½ particles have opposite parity to spin ½ anti-particles

- Conventional choice: spin ½ particles have $P = +1$

$$P_{e^-} = P_{\mu^-} = P_{\tau^-} = P_\nu = P_q = +1$$

and anti-particles have opposite parity, i.e.

$$P_{e^+} = P_{\mu^+} = P_{\tau^+} = P_{\bar{\nu}} = P_{\bar{q}} = -1$$

★ For Dirac spinors it was shown (handout 2) that the parity operator is:

$$\hat{P} = \gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Parity Conservation in QED and QCD

- Consider the QED process $e^- q \rightarrow e^- q$
- The Feynman rules for QED give:

$$-iM = [\bar{u}_e(p_3) ie\gamma^\mu u_e(p_1)] \frac{-ig_{\mu\nu}}{q^2} [\bar{u}_q(p_4) ie\gamma^\nu u_q(p_2)]$$

- Which can be expressed in terms of the electron and quark 4-vector currents:

$$M = -\frac{e^2}{q^2} g_{\mu\nu} j_e^\mu j_q^\nu = -\frac{e^2}{q^2} j_e \cdot j_q$$

with $j_e = \bar{u}_e(p_3)\gamma^\mu u_e(p_1)$ and $j_q = \bar{u}_q(p_4)\gamma^\mu u_q(p_2)$
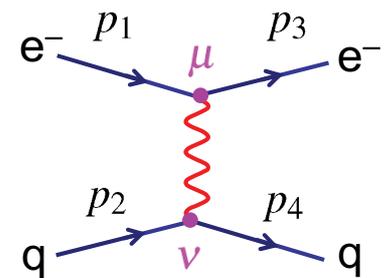
★ Consider the what happen to the matrix element under the parity transformation

- Spinors transform as $\boxed{u \xrightarrow{\hat{P}} \hat{P}u = \gamma^0 u}$

- Adjoint spinors transform as

$$\bar{u} = u^\dagger \gamma^0 \xrightarrow{\hat{P}} (\hat{P}u)^\dagger \gamma^0 = u^\dagger \gamma^{0\dagger}\gamma^0 = u^\dagger \gamma^0 \gamma^0 = \bar{u}\gamma^0$$

$$\boxed{\bar{u} \xrightarrow{\hat{P}} \bar{u}\gamma^0}$$

- Hence $j_e = \bar{u}_e(p_3)\gamma^\mu u_e(p_1) \xrightarrow{\hat{P}} \bar{u}_e(p_3)\gamma^0\gamma^\mu\gamma^0 u_e(p_1)$

★ **Consider the components of the four-vector current**

$\boxed{0:}$ $j_e^0 \xrightarrow{\hat{P}} \bar{u}\gamma^0\gamma^0\gamma^0 u = \bar{u}\gamma^0 u = j_e^0$ since $\gamma^0\gamma^0 = 1$

$\boxed{k=1,2,3:}$ $j_e^k \xrightarrow{\hat{P}} \bar{u}\gamma^0\gamma^k\gamma^0 u = -\bar{u}\gamma^k\gamma^0\gamma^0 u = -\bar{u}\gamma^k u = -j_e^k$ since $\gamma^0\gamma^k = -\gamma^k\gamma^0$

- **The time-like component remains unchanged and the space-like components change sign**

- **Similarly** $j_q^0 \xrightarrow{\hat{P}} j_q^0$ $j_q^k \xrightarrow{\hat{P}} -j_q^k$ $k = 1, 2, 3$

★ **Consequently the four-vector scalar product**

$j_e \cdot j_q = j_e^0 j_q^0 - j_e^k j_q^k \xrightarrow{\hat{P}} j_e^0 j_q^0 - (-j_e^k)(-j_q^k) = j_e \cdot j_q$ $k = 1, 3$

$\boxed{\begin{array}{c} \text{or} \quad j^\mu \xrightarrow{\hat{P}} j_\mu \\[4pt] j^\mu \cdot j^\nu \xrightarrow{\hat{P}} j_\mu \cdot j_\nu \\[4pt] \xrightarrow{\hat{P}} j^\mu \cdot j^\nu \end{array}}$

$\boxed{\textbf{QED Matrix Elements are Parity Invariant}}$

➡ $\boxed{\textbf{Parity Conserved in QED}}$

★ **The QCD vertex has the same form and, thus,**

$\boxed{\textbf{Parity Conserved in QCD}}$

Parity Violation in β-Decay

★ **The parity operator** $\hat{P}$ **corresponds to a discrete transformation** $x \to -x, \, etc.$

★ **Under the parity transformation:**

Vectors change sign
$$\begin{cases} \vec{r} \xrightarrow{\hat{P}} -\vec{r} \\[4pt] \vec{p} \xrightarrow{\hat{P}} -\vec{p} \quad (p_x = \frac{\partial}{\partial x}, \, etc.) \end{cases}$$

Axial-Vectors unchanged
$$\begin{cases} \vec{L} \xrightarrow{\hat{P}} \vec{L} \quad (\vec{L} = \vec{r} \wedge \vec{p}) \\[4pt] \vec{\mu} \xrightarrow{\hat{P}} \vec{\mu} \quad (\vec{\mu} \propto \vec{L}) \end{cases}$$

$\boxed{\begin{array}{l}\textbf{Note B is an} \\ \textbf{axial vector} \\ \mathrm{d}\vec{B} \propto \vec{J} \wedge \vec{r}\, \mathrm{d}^3\vec{r}\end{array}}$

★ **1957:** C.S.Wu et al. studied beta decay of polarized cobalt-60 nuclei:

$$^{60}\text{Co} \to\, ^{60}Ni^* + e^- + \bar{\nu}_e$$
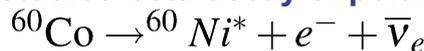
★ **Observed electrons emitted preferentially in direction opposite to applied field**
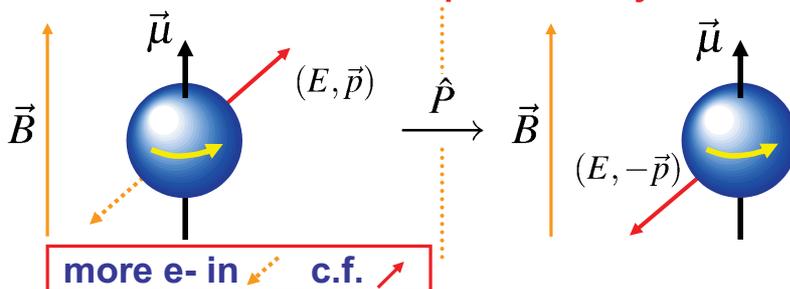
$\boxed{\begin{array}{l}\textbf{If parity were conserved:} \\ \textbf{expect equal rate for} \\ \textbf{producing } \mathbf{e^-} \textbf{ in directions} \\ \textbf{along and opposite to the} \\ \textbf{nuclear spin.}\end{array}}$

$\boxed{\text{more e- in} \quad \nearrow \quad \text{c.f.} \quad \nearrow}$

$\boxed{\begin{array}{l} ★\textbf{Conclude parity is violated in WEAK INTERACTION} \\ \quad\Rightarrow \textbf{ that the WEAK interaction vertex is NOT of the form } \bar{u}_e \gamma^\mu u_\nu \end{array}}$

Bilinear Covariants

★ **The requirement of Lorentz invariance of the matrix element severely restricts the form of the interaction vertex. QED and QCD are "VECTOR" interactions:**

$$j^\mu = \overline{\psi}\gamma^\mu \phi$$

★ **This combination transforms as a 4-vector (Handout 2 appendix V)**

★ **In general, there are only 5 possible combinations of two spinors and the gamma matrices that form Lorentz invariant currents, called "bilinear covariants":**

Type	Form	Components	"Boson Spin"
♦ SCALAR	$\overline{\psi}\phi$	1	0
♦ PSEUDOSCALAR	$\overline{\psi}\gamma^5\phi$	1	0
♦ VECTOR	$\overline{\psi}\gamma^\mu\phi$	4	1
♦ AXIAL VECTOR	$\overline{\psi}\gamma^\mu\gamma^5\phi$	4	1
♦ TENSOR	$\overline{\psi}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)\phi$	6	2

★ **Note that in total the sixteen components correspond to the 16 elements of a general 4x4 matrix: "decomposition into Lorentz invariant combinations"**

★ **In QED the factor $g_{\mu\nu}$ arose from the sum over polarization states of the virtual photon (2 transverse + 1 longitudinal, 1 scalar) = (2J+1) + 1**

★ **Associate SCALAR and PSEUDOSCALAR interactions with the exchange of a SPIN-0 boson, etc. – no spin degrees of freedom**


# V-A Structure of the Weak Interaction

★ **The most general form for the interaction between a fermion and a boson is a linear combination of bilinear covariants**

★ **For an interaction corresponding to the exchange of a spin-1 particle the most general form is a linear combination of VECTOR and AXIAL-VECTOR**

★ **The form for WEAK interaction is determined from experiment to be VECTOR – AXIAL-VECTOR (V – A)**
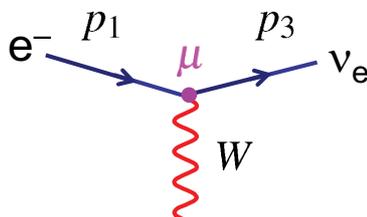
$$j^\mu \propto \overline{u}_{\nu_e}(\gamma^\mu - \gamma^\mu\gamma^5)u_e$$

V – A

★ **Can this account for parity violation?**

★ **First consider parity transformation of a pure AXIAL-VECTOR current**

$$j_A = \overline{\psi}\gamma^\mu\gamma^5\phi \qquad \text{with} \qquad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3; \quad \gamma^5\gamma^0 = -\gamma^0\gamma^5$$

$$j_A = \overline{\psi}\gamma^\mu\gamma^5\phi \xrightarrow{\hat{P}} \overline{\psi}\gamma^0\gamma^\mu\gamma^5\gamma^0\phi = -\overline{\psi}\gamma^0\gamma^\mu\gamma^0\gamma^5\phi$$

$$j_A^0 = \xrightarrow{\hat{P}} -\overline{\psi}\gamma^0\gamma^0\gamma^0\gamma^5\phi = -\overline{\psi}\gamma^0\gamma^5\phi = -j_A^0$$

$$j_A^k = \xrightarrow{\hat{P}} -\overline{\psi}\gamma^0\gamma^k\gamma^0\gamma^5\phi = +\overline{\psi}\gamma^k\gamma^5\phi = +j_A^k \quad k = 1,2,3 \qquad \boxed{\text{or} \quad j_A^\mu \xrightarrow{\hat{P}} -j_{A\mu}}$$

- **The space-like components remain unchanged and the time-like components change sign (the opposite to the parity properties of a vector-current)**

$$\boxed{ j_A^0 \xrightarrow{\hat{P}} -j_A^0; \quad j_A^k \xrightarrow{\hat{P}} +j_A^k; } \quad \boxed{ j_V^0 \xrightarrow{\hat{P}} +j_V^0; \quad j_V^k \xrightarrow{\hat{P}} -j_V^k }$$

- **Now consider the matrix elements**

$$M \propto g_{\mu\nu} j_1^\mu j_2^\nu = j_1^0 j_2^0 - \sum_{k=1,3} j_1^k j_2^k$$

- **For the combination of a two axial-vector currents**

$$j_{A1} \cdot j_{A2} \xrightarrow{\hat{P}} (-j_1^0)(-j_2^0) - \sum_{k=1,3} (j_1^k)(j_2^k) = j_{A1} \cdot j_{A2}$$

- **Consequently parity is conserved for both a pure vector and pure axial-vector interactions**

- **However the combination of a vector current and an axial vector current**

$$j_{V1} \cdot j_{A2} \xrightarrow{\hat{P}} (j_1^0)(-j_2^0) - \sum_{k=1,3} (-j_1^k)(j_2^k) = -j_{V1} \cdot j_{A2}$$

changes sign under parity – can give parity violation !

★ **Now consider a general linear combination of VECTOR and AXIAL-VECTOR (note this is relevant for the Z-boson vertex)**

$$j_1 = \overline{\phi}_1 (g_V \gamma^\mu + g_A \gamma^\mu \gamma^5) \psi_1 = g_V j_1^V + g_A j_1^A$$

$$\frac{g_{\mu\nu}}{q^2 - m^2}$$

$$j_2 = \overline{\phi}_2 (g_V \gamma^\mu + g_A \gamma^\mu \gamma^5) \psi_2 = g_V j_2^V + g_A j_2^A$$

$$M_{fi} \propto j_1 \cdot j_2 = g_V^2 j_1^V \cdot j_2^V + g_A^2 j_1^A \cdot j_2^A + g_V g_A (j_1^V \cdot j_2^A + j_1^A \cdot j_2^V)$$

- **Consider the parity transformation of this scalar product**

$$j_1 \cdot j_2 \xrightarrow{\hat{P}} g_V^2 j_1^V \cdot j_2^V + g_A^2 j_1^A \cdot j_2^A - g_V g_A (j_1^V \cdot j_2^A + j_1^A \cdot j_2^V)$$

- **If either g_A or g_V is zero, Parity is conserved, i.e. parity conserved in a pure VECTOR or pure AXIAL-VECTOR interaction**

- **Relative strength of parity violating part $\propto \dfrac{g_V g_A}{g_V^2 + g_A^2}$**

Maximal Parity Violation for V-A (or V+A)

Chiral Structure of QED (Reminder)

★ **Recall (Handout 4) introduced CHIRAL projections operators**

$$P_R = \tfrac{1}{2}(1 + \gamma^5); \qquad P_L = \tfrac{1}{2}(1 - \gamma^5)$$

project out **chiral** right- and left- handed states

★ **In the ultra-relativistic limit, chiral states correspond to helicity states**

★ **Any spinor can be expressed as:**

$$\psi = \tfrac{1}{2}(1 + \gamma^5)\psi + \tfrac{1}{2}(1 - \gamma^5)\psi = P_R\psi + P_L\psi = \psi_R + \psi_L$$

• **The QED vertex** $\overline{\psi}\gamma^\mu\phi$ **in terms of chiral states:**

$$\overline{\psi}\gamma^\mu\phi = \overline{\psi}_R\gamma^\mu\phi_R + \overline{\psi}_R\gamma^\mu\phi_L + \overline{\psi}_L\gamma^\mu\phi_R + \overline{\psi}_L\gamma^\mu\phi_L$$

conserves chirality, e.g.

$$
\begin{aligned}
\overline{\psi}_R\gamma^\mu\phi_L &= \tfrac{1}{2}\psi^\dagger(1+\gamma^5)\gamma^0\gamma^\mu\tfrac{1}{2}(1-\gamma^5)\phi \\
&= \tfrac{1}{4}\psi^\dagger\gamma^0(1-\gamma^5)\gamma^\mu(1-\gamma^5)\phi \\
&= \tfrac{1}{4}\overline{\psi}\gamma^\mu(1+\gamma^5)(1-\gamma^5)\phi = 0
\end{aligned}
$$
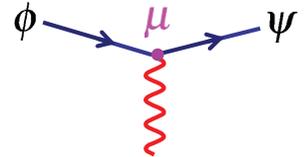
★ **In the ultra-relativistic limit only two helicity combinations are non-zero**
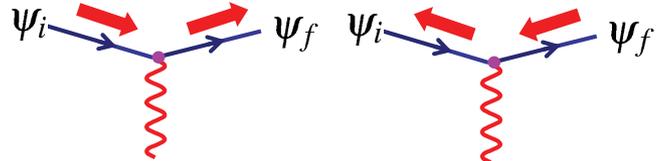
Helicity Structure of the WEAK Interaction

★ **The charged current (W±) weak vertex is:**

$$\frac{-ig_w}{\sqrt{2}}\frac{1}{2}\gamma^\mu(1-\gamma^5)$$
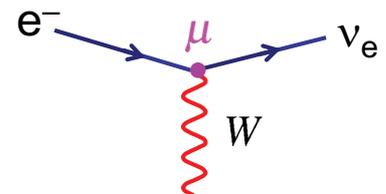
★ **Since** $\tfrac{1}{2}(1-\gamma^5)$ **projects out left-handed chiral particle states:**

$$\overline{\psi}\tfrac{1}{2}\gamma^\mu(1-\gamma^5)\phi = \overline{\psi}\gamma^\mu\phi_L \qquad \text{(question 16)}$$

★ **Writing** $\overline{\psi} = \overline{\psi}_R + \overline{\psi}_L$ **and from discussion of QED,** $\overline{\psi}_R\gamma^\mu\phi_L = 0$ **gives**

$$\overline{\psi}\tfrac{1}{2}\gamma^\mu(1-\gamma^5)\phi = \overline{\psi}_L\gamma^\mu\phi_L$$

> **Only the left-handed chiral components of particle spinors and right-handed chiral components of anti-particle spinors participate in charged current weak interactions**

★ **At very high energy** $(E \gg m)$ **, the left-handed chiral components are helicity eigenstates :**

$$\tfrac{1}{2}(1-\gamma^5)u \implies$$

LEFT-HANDED PARTICLES
   Helicity = -1

$$\tfrac{1}{2}(1-\gamma^5)v \implies$$

RIGHT-HANDED ANTI-PARTICLES
   Helicity = +1

In the ultra-relativistic limit only **left-handed particles** and **right-handed antiparticles** participate in charged current weak interactions

e.g. In the relativistic limit, the only possible electron – neutrino interactions are:
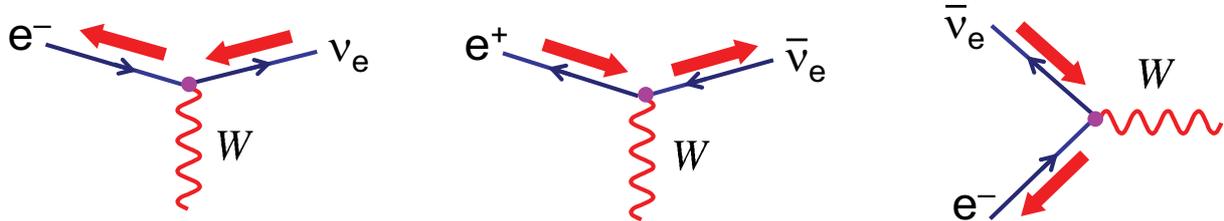
★ **The helicity dependence of the weak interaction** ⟷ **parity violation**

e.g. $\overline{\nu}_e + e^- \rightarrow W^-$

$\xrightarrow{\hat{P}}$

| RH anti-particle | LH particle | | RH particle | LH anti-particle |

Valid weak interaction | **Does not occur**
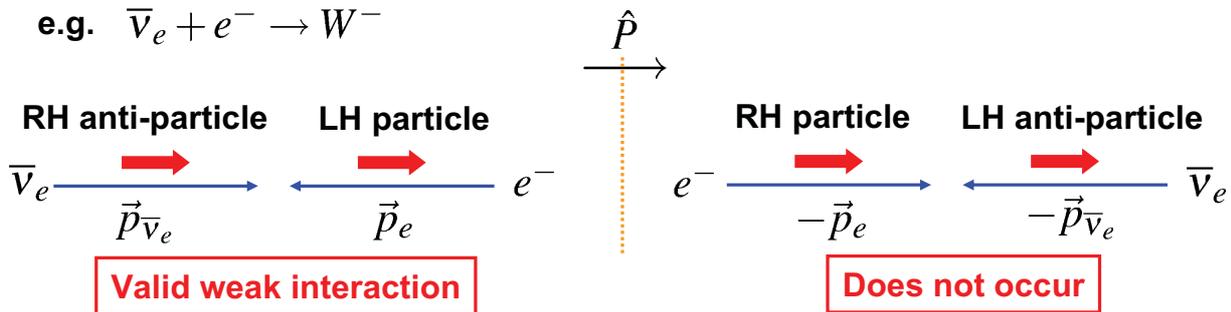
Helicity in Pion Decay

★The decays of charged pions provide a good demonstration of the role of helicity in the weak interaction
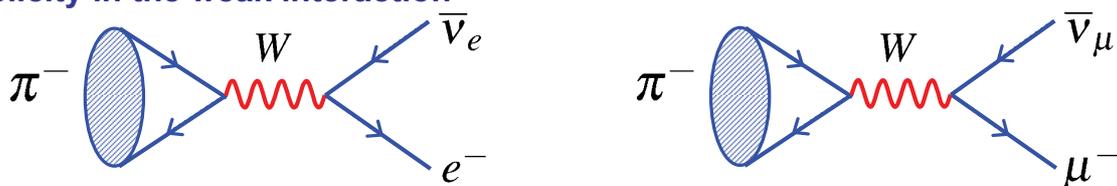
EXPERIMENTALLY: $\dfrac{\Gamma(\pi^- \rightarrow e^- \overline{\nu}_e)}{\Gamma(\pi^- \rightarrow \mu^- \overline{\nu}_\mu)} = 1.23 \times 10^{-4}$

• Might expect the decay to electrons to dominate – due to increased phase space…. The opposite happens, the electron decay is **helicity suppressed**

★Consider decay in pion rest frame.
   • Pion is spin zero: so the spins of the $\overline{\nu}$ and μ are opposite
   • Weak interaction only couples to **RH chiral** anti-particle states. Since neutrinos are (almost) massless, must be in **RH Helicity** state
   • Therefore, to conserve angular mom. muon is emitted in a **RH HELICITY** state

$$\overline{\nu}_\mu \quad \longleftarrow \quad \bullet \quad \longrightarrow \quad \mu^-$$

• But only **left-handed CHIRAL** particle states participate in weak interaction

★ **The general right-handed helicity solution to the Dirac equation is**

$$u_\uparrow = N \begin{pmatrix} c \\ e^{i\phi}s \\ \frac{|\vec{p}|}{E+m}c \\ \frac{|\vec{p}|}{E+m}e^{i\phi}s \end{pmatrix} \quad \text{with} \quad c = \cos\frac{\theta}{2} \quad \text{and} \quad s = \sin\frac{\theta}{2}$$

• **project out the left-handed chiral part of the wave-function using**

$$P_L = \frac{1}{2}(1 - \gamma^5) = \frac{1}{2}\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}$$

giving

$$P_L u_\uparrow = \frac{1}{2}N\left(1 - \frac{|\vec{p}|}{E+m}\right)\begin{pmatrix} c \\ e^{i\phi}s \\ -c \\ -e^{i\phi}s \end{pmatrix} = \frac{1}{2}N\left(1 - \frac{|\vec{p}|}{E+m}\right)u_L$$

In the limit $m \ll E$ this tends to zero

• **similarly**

$$P_R u_\uparrow = \frac{1}{2}N\left(1 + \frac{|\vec{p}|}{E+m}\right)\begin{pmatrix} c \\ e^{i\phi}s \\ c \\ e^{i\phi}s \end{pmatrix} = \frac{1}{2}N\left(1 + \frac{|\vec{p}|}{E+m}\right)u_R$$

In the limit $m \ll E$, $\quad P_R u_\uparrow \to u_R$

★ **Hence** $\quad u_\uparrow = P_R u_\uparrow + P_L u_\uparrow = \frac{1}{2}\left(1 + \frac{|\vec{p}|}{E+m}\right)u_R + \frac{1}{2}\left(1 - \frac{|\vec{p}|}{E+m}\right)u_L$

RH Helicity   RH Chiral   LH Chiral

• **In the limit $E \gg m$, as expected, the RH chiral and helicity states are identical**
• **Although only LH chiral particles participate in the weak interaction the contribution from RH Helicity states is not necessarily zero !**

$$\overline{\nu}_\mu \quad \longleftarrow \quad \textcolor{red}{\bullet} \quad \longrightarrow \quad \mu^-$$

$m_\nu \approx 0$: RH Helicity ≡ RH Chiral

$m_\mu \neq 0$: RH Helicity has LH Chiral Component

★ **Expect matrix element to be proportional to LH chiral component of RH Helicity electron/muon spinor**

$$M_{fi} \propto \frac{1}{2}\left(1 - \frac{|\vec{p}|}{E+m}\right) = \frac{m_\mu}{m_\pi + m_\mu}$$

from the kinematics of pion decay at rest

★ **Hence because the electron mass is much smaller than the pion mass the decay** $\pi^- \to e^- \overline{\nu}_e$ **is heavily suppressed.**

Evidence for V-A

★ **The V-A nature of the charged current weak interaction vertex fits with experiment**

   EXAMPLE charged pion decay   (question 17)

- **Experimentally measure:** $\dfrac{\Gamma(\pi^- \to e^- \overline{\nu}_e)}{\Gamma(\pi^- \to \mu^- \overline{\nu}_\mu)} = (1.230 \pm 0.004) \times 10^{-4}$

- **Theoretical predictions** (depend on Lorentz Structure of the interaction)

  V-A $(\overline{\psi}\gamma^\mu(1-\gamma^5)\phi)$ **or V+A** $(\overline{\psi}\gamma^\mu(1+\gamma^5)\phi)$ ➡ $\dfrac{\Gamma(\pi^- \to e^- \overline{\nu}_e)}{\Gamma(\pi^- \to \mu^- \overline{\nu}_\mu)} \approx 1.3 \times 10^{-4}$

  Scalar $(\overline{\psi}\phi)$ **or Pseudo-Scalar** $(\overline{\psi}\gamma^5\phi)$ ➡ $\dfrac{\Gamma(\pi^- \to e^- \overline{\nu}_e)}{\Gamma(\pi^- \to \mu^- \overline{\nu}_\mu)} = 5.5$

  EXAMPLE muon decay

Measure electron energy and angular distributions relative to muon spin direction. Results expressed in terms of general S+P+V+A+T form in "Michel Parameters"
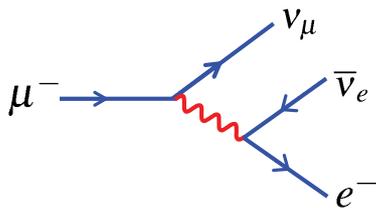
e.g. TWIST expt: 6x10⁹ μ decays
Phys. Rev. Lett. 95 (2005) 101805

$$\rho = 0.75080 \pm 0.00105$$

V-A Prediction: $\rho = 0.75$
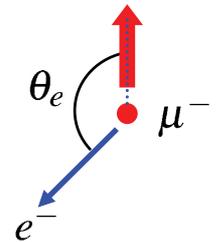
Weak Charged Current Propagator

★ **The charged-current Weak interaction is different from QED and QCD in that it is mediated by massive W-bosons (80.3 GeV)**

★ **This results in a more complicated form for the propagator:**
- **in handout 4 showed that for the exchange of a massive particle:**

    massless   **massive**

$$\frac{1}{q^2} \longrightarrow \frac{1}{q^2 - m^2}$$

- **In addition the sum over W boson polarization states modifies the numerator**

● **W-boson propagator**

  spin 1 $W^\pm$ $\dfrac{-i\left[g_{\mu\nu} - q_\mu q_\nu/m_W^2\right]}{q^2 - m_W^2}$
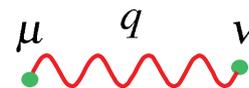

★ **However in the limit where** q^2 **is small compared with** $m_W = 80.3\,\text{GeV}$ **the interaction takes a simpler form.**

● **W-boson propagator (** $q^2 \ll m_W^2$ **)**

$$\frac{ig_{\mu\nu}}{m_W^2}$$

       μ ν

- **The interaction appears point-like (i.e no q² dependence)**

Connection to Fermi Theory

★ **In 1934, before the discovery of parity violation, Fermi proposed, in analogy with QED, that the invariant matrix element for β-decay was of the form:**

$$M_{fi} = G_F \, g_{\mu\nu} [\overline{\psi}\gamma^\mu \psi][\overline{\psi}\gamma^\nu \psi]$$

where $G_F = 1.166 \times 10^{-5} \, \text{GeV}^{-2}$
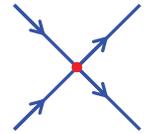
- **Note the absence of a propagator : i.e. this represents an interaction at a point**

★ **After the discovery of parity violation in 1957 this was modified to**

$$M_{fi} = \frac{G_F}{\sqrt{2}} \, g_{\mu\nu} [\overline{\psi}\gamma^\mu (1-\gamma^5)\psi][\overline{\psi}\gamma^\nu (1-\gamma^5)\psi]$$

(the factor of √2 was included so the numerical value of G_F did not need to be changed)

★ **Compare to the prediction for W-boson exchange**

$$M_{fi} = [\tfrac{g_W}{\sqrt{2}}\overline{\psi}\tfrac{1}{2}\gamma^\mu(1-\gamma^5)\psi]\frac{g_{\mu\nu} - q_\mu q_\nu/m_W^2}{q^2 - m_W^2}[\tfrac{g_W}{\sqrt{2}}\overline{\psi}\tfrac{1}{2}\gamma^\nu(1-\gamma^5)\psi]$$

which for $q^2 \ll m_W^2$ **becomes:**

$$M_{fi} = \frac{g_W^2}{8m_W^2} \, g_{\mu\nu} [\overline{\psi}\gamma^\mu (1-\gamma^5)\psi][\overline{\psi}\gamma^\nu (1-\gamma^5)\psi]$$

$$\boxed{\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2}}$$

Still usually use G_F to express strength of weak interaction as the is the quantity that is precisely determined in muon decay

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Strength of Weak Interaction

★ **Strength of weak interaction most precisely measured in muon decay**
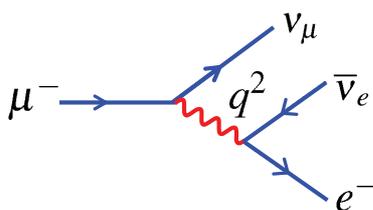
- **Here** $q^2 < m_\mu \; (0.106\,\text{GeV})$

- **To a very good approximation the W-boson propagator can be written**

$$\frac{-i[g_{\mu\nu} - q_\mu q_\nu/m_W^2]}{q^2 - m_W^2} \approx \frac{ig_{\mu\nu}}{m_W^2}$$

- **In muon decay measure** g_W^2/m_W^2

- **Muon decay** ➡ $G_F = 1.16639(1) \times 10^{-5}\,\text{GeV}^{-2}$

$$\boxed{\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2}}$$

★ **To obtain the intrinsic strength of weak interaction need to know mass of W-boson:** $m_W = 80.403 \pm 0.029\,\text{GeV}$ **(see handout 14)**

$$\alpha_W = \frac{g_W^2}{4\pi} = \frac{8m_W^2 G_F}{4\sqrt{2}\pi} = \frac{1}{30}$$

> **The intrinsic strength of the weak interaction is similar to, but greater than, the EM interaction ! It is the massive W-boson in the propagator which makes it appear weak. For** $q^2 \gg m_W^2$ **weak interactions are more likely than EM.**

Summary

★ **Weak interaction is of form Vector – Axial-vector (V-A)**

$$\frac{-ig_w}{\sqrt{2}}\frac{1}{2}\gamma^{\mu}\left(1-\gamma^5\right)$$

★ **Consequently only** left-handed chiral particle states **and** right-handed chiral anti-particle states **participate in the weak interaction**

➡ **MAXIMAL PARITY VIOLATION**

★ **Weak interaction also violates Charge Conjugation symmetry**

★ **At low** q^2 **weak interaction is only weak because of the large W-boson mass**

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2}$$

★ **Intrinsic strength of weak interaction is similar to that of QED**