NATURAL SCIENCES TRIPOS Part III

Tuesday 18 January 2011 2:00pm to 4:00pm

EXPERIMENTAL AND THEORETICAL PHYSICS (4)

Particle Physics

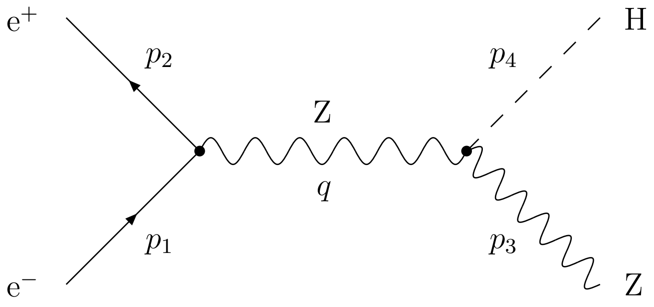
Answer **two** *questions only. The approximate number of marks allotted to each part of a question is indicated in the right hand margin where appropriate. The paper contains FIVE sides including this one and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS
20 page answer book
Rough workpad
Metric graph paper

SPECIAL REQUIREMENTS
Mathematical formulae handbook
Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1 At a future e^+e^- linear collider, a low mass Higgs boson would be produced via the Higgsstrahlung process shown below.



The respective vertex factors for the e^+e^-Z and ZZH interactions, are

$$-ig_Z\gamma^\mu\tfrac{1}{2}(v_e - a_e\gamma^5) \quad \text{and} \quad -ig_{ZZH}$$

and the corresponding matrix element is

$$M_{fi} = \overline{v}(p_2)g_Z\gamma^\mu\tfrac{1}{2}(v_e - a_e\gamma^5)u(p_1)\frac{g_{\mu\nu}}{s-m_Z^2}g_{ZZH}\epsilon_\lambda^\nu(p_3),$$

where ϵ_λ is the polarisation of the final state Z.

a) Taking $m_Z = 91\,\text{GeV}$ and $m_H = 120\,\text{GeV}$, find the energy and momentum of the final state Z for a centre-of-mass energy of $\sqrt{s} = 250\,\text{GeV}$. [4]

b) Show that the e^+e^-Z vertex factor can be written as

$$-ig_Z\gamma^\mu\left[c_R\tfrac{1}{2}(1+\gamma^5) + c_L\tfrac{1}{2}(1-\gamma^5)\right] \ ,$$

and find an expression for $(c_R^2 + c_L^2)$ in terms of v_e^2 and a_e^2. [3]

c) Hence show that there are only two combinations of electron/positron spins which give non-zero matrix elements, and that these are:

$$M_R = g_Z g_{ZZH}\frac{g_{\mu\nu}}{s-m_Z^2}c_R j_R^\mu \epsilon_\lambda^\nu(p_3), \quad \text{and} \quad M_L = g_Z g_{ZZH}\frac{g_{\mu\nu}}{s-m_Z^2}c_L j_L^\mu \epsilon_\lambda^\nu(p_3),$$

where $j_R^\mu = \overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)$ and $j_L^\mu = \overline{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)$. [6]

d) Taking the z-axis be along the direction of the initial state electron, it can be shown that $j_R^\mu = \sqrt{s}(0,-1,+i,0)$ and $j_L^\mu = \sqrt{s}(0,-1,-i,0)$. Show that the spin-averaged matrix element is

$$\langle|M^2|\rangle = ks(c_L^2 + c_R^2)\left(2 + \frac{p_Z^2}{m_Z^2}\sin^2\theta\right),$$

where θ and $p_{\rm Z}$ are the polar angle and magnitude of the momentum of the final state Z, respectively. Give an expression for the factor k in terms $g_{\rm Z}$ and $g_{\rm ZZH}$. [8]

e) In the Standard Model, $g_{\rm Z} \times g_{\rm ZZH} = 8m_{\rm Z}^3 G_{\rm F}/\sqrt{2}$. Using this relation, show that the total Higgsstrahlung cross section is

$$\sigma = A\frac{G_{\rm F}^2 m_{\rm Z}^4}{s}(a_e^2 + v_e^2)\left(1 - \frac{m_{\rm Z}^2}{s}\right)^{-2}\beta_{if}\left(\frac{12m_{\rm Z}^2}{s} + \beta_{if}^2\right),$$

where β_{if} is the ratio of the final to initial state momenta in the centre-of-mass frame, $\beta_{if} = |p_f^*|/|p_i^*|$, and A is a numerical factor which should be determined. [6]

What is the Higgsstrahlung cross section (in nb) for $m_{\rm H} = 120\,{\rm GeV}$ and a centre-of-mass collision energy of $250\,{\rm GeV}$? [3]

[*You may require the following information:*

i) $\gamma^{5\dagger} = \gamma^5;\quad \gamma^{0\dagger} = \gamma^0;\quad \gamma^5\gamma^\mu = -\gamma^\mu\gamma^5$

ii) For a massive boson with momentum $\boldsymbol{p} = (p\sin\theta, 0, p\cos\theta)$, a suitable choice of the three possible polarisation vectors is:

$$\epsilon_1^\nu = (0,0,1,0),\ \epsilon_2^\nu = (0,\cos\theta,0,-\sin\theta),\ \epsilon_L^\nu = \frac{1}{m_{\rm Z}}(p_{\rm Z}, E_{\rm Z}\sin\theta, 0, E_{\rm Z}\cos\theta)\ .$$

iii) In the centre-of-mass frame, the differential cross section for $a + b \to c + d$ is given by

$$\frac{{\rm d}\sigma}{{\rm d}\Omega} = \frac{1}{64\pi^2 s}\frac{p_f^*}{p_i^*}\langle|M^2|\rangle,$$

where p_i^* and p_f^* are the magnitudes of the initial and final state particle momenta.

iv) $G_{\rm F} = 1.166 \times 10^{-5}\,{\rm GeV}^{-2}$; $v_e = -0.04$; $a_e = -0.5$.]

(TURN OVER

2 In an extension to the Standard Model, a new fourth generation heavy charged lepton, L^-, is proposed with a mass of 1.6 TeV. In addition there would be a new heavy neutrino, ν_L with a mass of 500 GeV. The fourth generation leptons would have the same couplings to the W, Z and γ as the first three generations.

a) The L^- would decay via the weak interaction, e.g. $L^- \to \mathrm{e}^- \nu_L \overline{\nu}_e$. Draw the Feynman diagram for this decay and by considering the other possible decays of the W-boson, estimate the branching fraction for $L^- \to \nu_L + \text{hadrons}$. [6]

b) Heavy charged leptons could be produced in proton-proton collisions via the Drell-Yan process, i.e. the production of (L^+L^-) through the annihilation of a quark and anti-quark into a photon. Draw the Feynman diagram for this interaction and explain why the cross section is non-zero for proton-proton collisions. [3]

c) If the squared centre-of-mass energy of the proton-proton collision is s, show that the centre-of-mass energy squared of the $q\overline{q}$ system is $\hat{s} = x_1 x_2 s$, where x_1 and x_2 are the fractional momenta carried by the partons involved in the collision. [3]

d) The QED cross section for $q\overline{q} \to \ell^+\ell^-$, where ℓ is a charged lepton, is given by

$$\sigma = \frac{4\pi}{3}\frac{\alpha^2}{s} e_q^2 \times f(s, m_\ell),$$

where e_q is the quark charge (i.e. $e_u = +2/3$ and $e_d = -1/3$) and $f(s, m_\ell)$ is a kinematic factor which depends on the lepton mass. Assuming that the $\overline{\mathrm{u}}$ and $\overline{\mathrm{d}}$ parton distribution functions can be described by a single function, $S(x)$, and neglecting the strange quark contribution, show that the parton model prediction for the $\mathrm{pp} \to L^+L^-X$ differential cross section can be written

$$\frac{d^2\sigma}{\mathrm{d}x_1 \mathrm{d}x_2} = f(sx_1x_2, m_L)\frac{2\pi\alpha^2}{81 x_1 x_2 s}\{9u_V(x_1)S(x_2) + 9u_V(x_2)S(x_1) + 20S(x_1)S(x_2)\},$$

where $u_V(x)$ is the valence up-quark parton distribution function. Clearly state any assumptions you have made. [10]

e) Draw a diagram showing the region of x_1 versus x_2 which contributes to the cross section for $\mathrm{pp} \to L^+L^-X$ at the LHC operating at $\sqrt{s} = 7$ TeV. [2]

f) Explaining your reasoning, which of the terms, $20S(x_1)S(x_2)$ or $9u_V(x_1)S(x_2) + 9u_V(x_2)S(x_1)$, would you expect to dominate in this region? [2]

g) In the relevant regions of x, the parton distribution functions can be taken to have the approximate forms, $u_V(x) \approx ax^{-\lambda}$ and $S(x) \approx bx^{-\lambda}$. Taking $f(sx_1x_2, m_L) = 1$, and by performing the appropriate integration over x_1 and x_2, obtain an approximate expression for the Drell-Yan cross section for heavy lepton production in terms of $\alpha, a, b, \lambda, M_L$ and s. [4]

3 In the form of a list with bullet points, write brief notes on **three** of the following topics:

(a) *reactor* neutrino experiments and what has been learnt from them. [10]

(b) helicity and chirality in particle physics. [10]

(c) electron-Proton elastic scattering and the measurement of form factors. [10]

(d) electroweak unification and precision tests of the Standard Model at LEP. [10]

(e) SU(3) flavour symmetry and how it can be used to derive the proton wave-function. [10]

END OF PAPER