Particle Physics

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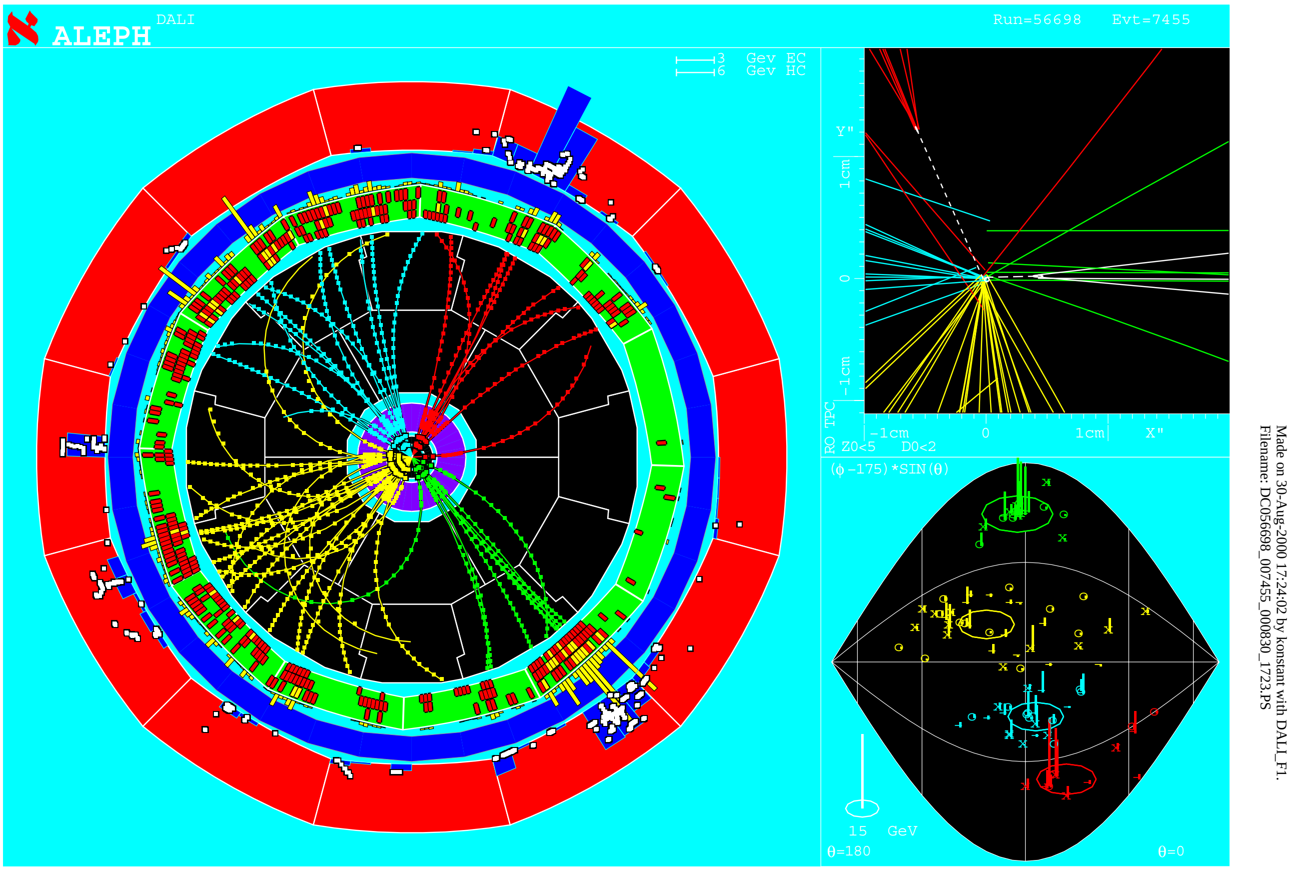

Part II, Lent Term 2004
HANDOUT I

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see : http://www.hep.phy.cam.ac.uk/∼thomson/particles/

Course Synopsis

Overview:

❶ Introduction to the Standard Model

❷ Quantum Electrodynamics QED

❸ Strong Interaction QCD

❹ The Quark Model

❺ Relativistic Quantum Mechanics

❻ Weak Interaction

❼ Electroweak Unification

❽ Beyond the Standard Model

Recommended Reading:

➠ *Particle Physics* : Martin B.R. and G. Shaw (2nd Edition, 1997)

➠ *Introduction to High Energy Physics* : D.H.Perkins (4th Edition, 2000)

Introduction to the Standard Model

Particle Physics is the study of

★ **MATTER:** the fundamental constituents that make up the universe - the elementary particles

★ **FORCE:** the basic forces in nature i.e. the interactions between the elementary particles

Try to categorize the PARTICLES and FORCES in as simple and fundamental manner as possible.

Current understanding embodied in the STANDARD MODEL

★ Explains all current experimental observations.

★ Forces described by particle exchange.

★ It is not the ultimate theory - many mysteries.

The Briefest History of Particle Physics

the Greek View

★ *c*. 400 B.C : Democritus : concept of matter comprised of indivisible "atoms".

★ "Fundamental Elements" : air, earth, water, fire

Newton's Definition

★ 1704 : matter comprised of "primitive particles ... incomparably harder than any porous Bodies compounded of them, even so very hard, as never to wear out or break in pieces."

★ A good definition - e.g. kinetic theory of gases.

CHEMISTRY

★ Fundamental particles : "elements"

★ Patterns 1869 Mendeleev's Periodic Table ➟ sub-structure

★ Explained by atomic shell model

ATOMIC PHYSICS

★ Bohr Model

★ Fundamental particles : electrons orbiting the atomic nucleus

NUCLEAR PHYSICS

★ Patterns in nuclear structure - e.g. magic numbers in the shell model ➠ sub-structure

★ Fundamental particles :

proton,neutron,electron,neutrino

★ Fundamental forces :

ELECTROMAGNETIC : atomic structure

STRONG: nuclear binding

WEAK: β-decay $n \rightarrow p e^- \nu_e$

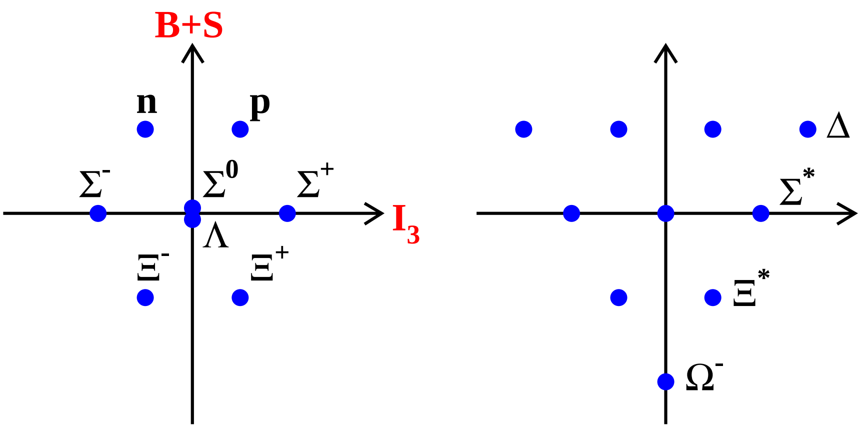
★ Nuclear physics is complicated : not dealing with fundamental particles/forces

1960s PARTICLE PHYSICS

★ Fundamental particles ? : far too many !

$p, n, \pi^{\pm}, \pi^0, \Sigma^{\pm}, \Lambda, \eta, \eta', K^{\pm}, K^0, \rho,$
$\omega, \Omega^-, \phi, a_1, a_2, f_1, f_2, J/\psi, \Delta, \ldots$

★ Again Patterns emerged:



★ sub-structure - explained by QUARK model : u,d,c,s

TODAY

★ Simple/Elegant description of the fundamental particles/forces

★ These lectures will describe our current understanding and most recent experimental results

Modern Particle Physics

Our current theory is embodied in the

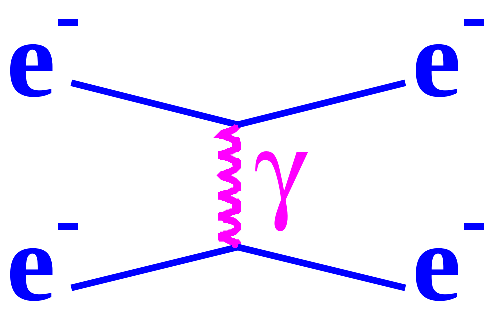
Standard Model

which accurately describes all data.

MATTER: made of spin-$\frac{1}{2}$ **FERMIONS** of which there are two types.

- ★ **LEPTONS:** e.g. e^-, ν_e
- ★ **QUARKS:** e.g. up quark and down quark
  - proton - (u,u,d)
- ★ **+ ANTIMATTER:** e.g. positron e^+,
  - anti-proton - $(\bar{u},\bar{u},\bar{d})$

FORCES: forces between quarks and leptons mediated by the exchange of spin-1 bosons - the **GAUGE BOSONS**.



Electromagnetic	Photon	γ
Strong	Gluon	g
Weak	W and Z	$W^{\pm}, Z^0$
Gravity	Graviton	G

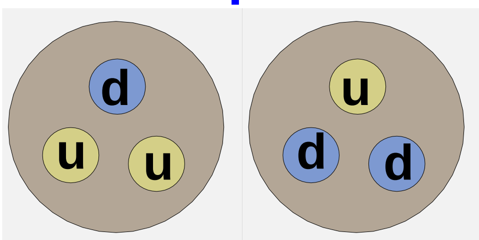
Matter: 1^{st} Generation

Almost all phenomena you will have encountered can be described by the interactions of FOUR spin-half particles : "the First Generation"

particle	symbol	type	charge
Electron	e^-	lepton	-1
Neutrino	ν_e	lepton	0
Up Quark	u	quark	+2/3
Down Quark	d	quark	-1/3

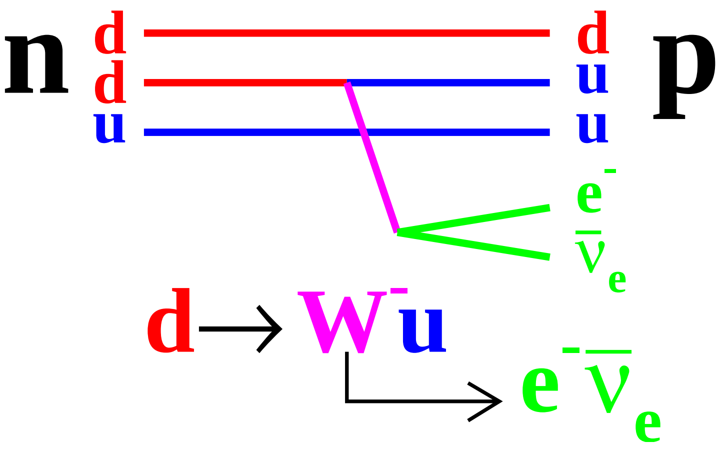
The proton and the neutron are the lowest energy states of a combination of three quarks:

- ★ Proton = (uud)
- ★ Neutron = (udd)



e.g. beta-decay viewed in the quark picture

$$n \rightarrow p + e^- + \overline{\nu}_e$$
$$d \rightarrow u + e^- + \overline{\nu}_e$$



GENERATIONS

★ Nature is not quite that simple.

★ There are 3 GENERATIONS of fundamental fermions.

First Generation		Second Generation		Third Generation	
Electron	e^-	Muon	μ^-	Tau	τ^-
Electron Neutrino	ν_e	Muon Neutrino	ν_μ	Tau Neutrino	ν_τ
Up Quark	u	Charm Quark	c	Top Quark	t
Down Quark	d	Strange Quark	s	Bottom Quark	b

★ Each generation e.g. (μ^-, ν_μ, c, s) is an exact copy of (e^-, ν_e, u, d).

★ The only difference is the mass of the particles - with first generation the lightest and third generation heaviest.

★ Clear symmetry - origin of 3 generations is NOT UNDERSTOOD

The LEPTONS

PARTICLES which do not interact via the STRONG interaction - "colour charge" = 0.

- ★ spin 1/2 fermions
- ★ 6 distinct FLAVOURS of leptons
- ★ 3 charged leptons : e^-,μ^-,τ^-

Muon (μ^-) - heavier version of the electron ($m_\mu/m_e \approx 207$)

- ★ 3 neutral leptons : neutrinos

Gen.	flavour		q	Approx. Mass
1^{st}	Electron	e^-	-1	0.511 MeV/c^2
1^{st}	Electron neutrino	ν_e	0	massless ?
2^{nd}	Muon	μ^-	-1	105.7 MeV/c^2
2^{nd}	Muon neutrino	ν_μ	0	massless ?
3^{rd}	Tau	τ^-	-1	1777.0 MeV/c^2
3^{rd}	Tau neutrino	ν_τ	0	massless ?

- ★ + antimatter partners, e^+, $\overline{\nu}_e$

Neutrinos

stable and (almost ?) massless:

ν_e Mass < 3 eV$/c^2$

ν_μ Mass < 0.19 MeV$/c^2$

ν_τ Mass < 18.2 MeV$/c^2$

★ **Charged leptons only experience the ELECTROMAGNETIC and WEAK and forces.**

★ **Neutrinos only experience the WEAK force.**

The Quarks

- ★ spin 1/2 fermions
- ★ fractional charge
- ★ 6 distinct FLAVOURS of quarks

Generation	flavour		charge	Approx. Mass
1^{st}	down	d	-1/3	0.35 GeV/c^2
1^{st}	up	u	+2/3	0.35 GeV/c^2
2^{nd}	strange	s	-1/3	0.5 GeV/c^2
2^{nd}	charm	c	+2/3	1.5 GeV/c^2
3^{rd}	bottom	b	-1/3	4.5 GeV/c^2
3^{rd}	top	t	+2/3	175 GeV/c^2

Mass quoted in units of GeV/c^2. To be compared with $M_{\rm proton}$ = 0.938 GeV/c^2.

- ★ Quarks come in 3 "COLOURS"

  "RED", "GREEN", "BLUE"

COLOUR is a label for the charge of the strong interaction. Unlike the electric charge of an electron ($-e$), the strong charge comes in three "orthogonal colours" RGB.

- ★ quarks confined within *HADRONS*

  e.g. p $\equiv$ (uud), $\pi^+ \equiv (u\bar{d})$

- ★ Quarks experience the ALL forces: ELECTROMAGNETIC, STRONG and WEAK (and of course gravity).

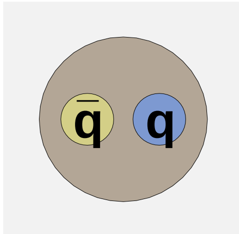
HADRONS

★ single free QUARKS are NEVER observed

★ quarks always CONFINED in HADRONS
i.e. ONLY see bound states of $(\mathrm{q\overline{q}})$ or (qqq).

★ HADRONS = { MESONS,BARYONS }

MESONS = $\mathrm{q\overline{q}}$



A meson is a bound state of a QUARK and an ANTI-QUARK

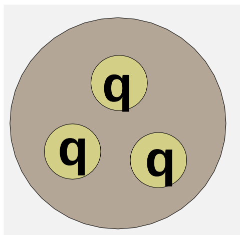
- All have INTEGER spin 0,1,2,...
- e.g. $\pi^+ \equiv \mathrm{u\overline{d}}$

  charge, Q_{π^+} = $Q_u + Q_{\overline{d}}$ = $\frac{2}{3} + \frac{1}{3} = +1$

  π^+ is the ground state (L=0) of $\mathrm{u\overline{d}}$

  there are other states, e.g. ρ^+, ...

BARYONS=qqq



- All have half-INTEGER spin $\frac{1}{2}, \frac{3}{2}$,...
  e.g. $\mathrm{p} \equiv (\mathrm{uud})$

  e.g. $\mathrm{n} \equiv (\mathrm{udd})$
- Plus ANTI-BARYONS= $\mathrm{\overline{q}\,\overline{q}\,\overline{q}}$
  e.g. anti-proton $\bar{\mathrm{p}} \equiv (\bar{\mathrm{u}}\bar{\mathrm{u}}\bar{\mathrm{d}})$

  e.g. anti-neutron $\bar{\mathrm{n}} \equiv (\bar{\mathrm{u}}\bar{\mathrm{d}}\bar{\mathrm{d}})$

★ Composite $\Rightarrow$ relatively complicated

Forces

Consider ELECTROMAGNETISM and scattering of electrons from a proton:

Classical Picture

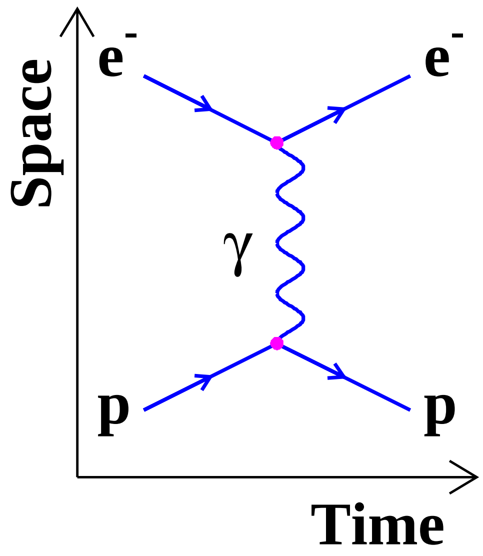
Electrons scatter in the static potential of the proton:

$$V(r) \propto -\frac{1}{r}$$

NEWTON : "...that one body can act upon another at a distance, through a vacuum, without the mediation of anything else,..., is to me a great absurdity"

Modern Picture

Particles interact via the exchange of particles GAUGE BOSONS. The PHOTON is the gauge bosons of electromagnetic force.

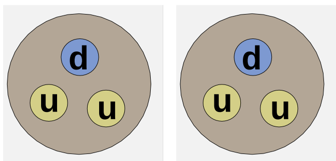


Early next week we'll learn how to calculate Quantum Mechanical amplitudes for scattering via Gauge Boson Exchange.

All (known) particle interactions can be explained by 4 fundamental forces:

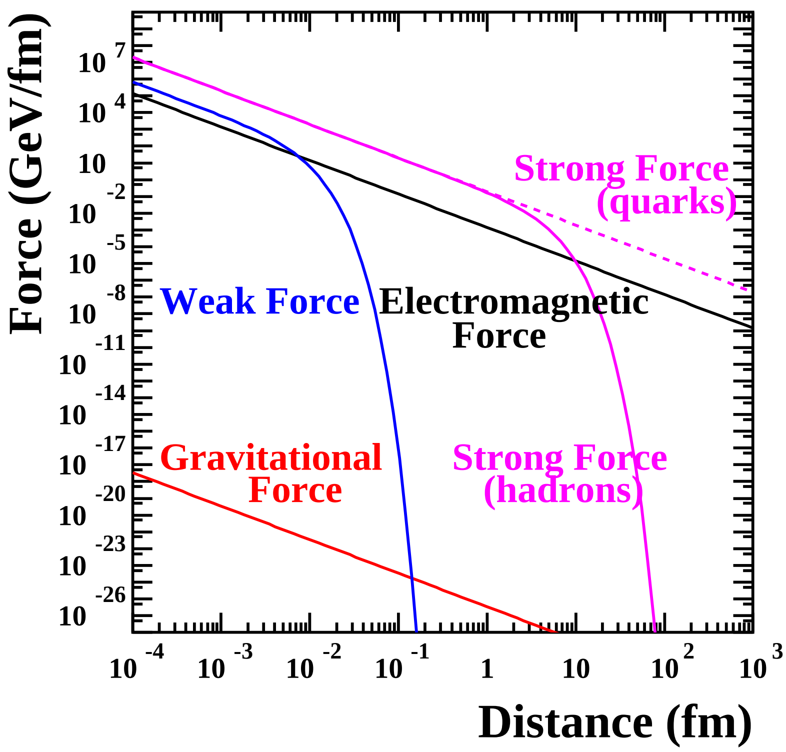
Electromagnetic, Strong, Weak, Gravity

Relative strengths of the forces between two protons just in contact (10^{-15} m):



Strong	**1**
Electromagnetic	10^{-2}
Weak	10^{-7}
Gravity	10^{-39}

At very small distances (high energies) - UNIFICATION



The Gauge Bosons

- ★ GAUGE BOSONS mediate the fundamental forces
- ★ Spin-1 particles (i.e. VECTOR BOSONS)
- ★ The manner in which the GAUGE BOSONS interact with the LEPTONS and QUARKS determines the nature of the fundamental forces.

Force	Boson	Mass (GeV/c^2)	Range (m)
Electromagnetic	Photon	massless	∞
Weak	$W^{\pm}, Z$	80/90	10^{-17}
Strong	Gluon	massless	$\infty/10^{-15}$

Practical Particle Physics

OR How do we study the particles and forces ?

★ **Static Properties:**

Mass

Spin, Parity : J^P e.g. 1^-

Magnetic Moments (see Quark Model handout)

★ **Particle Decays :**

Allowed/Forbidden decays $\longrightarrow$ conservation laws

Particle lifetimes

Force	Typical Lifetime
STRONG	10^{-23} s
E-M	10^{-20} s
WEAK	10^{-8} s

★ **Accelerator physics - particle scattering:**

Direct production of new massive particles in

Matter/Antimatter ANNIHILATION

Study of particle interaction cross sections

FORCE	Typical Cross section
STRONG	10 mb
E-M	10^{-2} mb
WEAK	10^{-13} mb

Scattering and Annihilation in Quantum Electrodynamics are the main topics of the next two lectures. Particle decay will be covered later in the course.

NATURAL UNITS

SI UNITS: kg m s

$$[M]\,[L]\,[T]$$

- ★ For everyday physics SI units are a natural choice : M(Widdecombe)$\sim$ 250 kg
- ★ Not so good for particle physics:
  $M_{\text{proton}} \sim 10^{-27}$ kg
- ★ use a different basis - NATURAL UNITS
- ★ based on language of particle physics, *i.e.*
  Quantum Mechanics and Relativity
    unit of action in QM $\hbar$ (Js)
    velocity of light c (ms^{-2})
- ★ Unit of energy : GeV = 10^9 eV= 1.6×10^{-10} J

Natural Units

GeV ħ c

Units become

Energy	GeV	Time	(GeV/$\hbar$)$^{-1}$
Momentum	GeV/c	Length	(GeV/$\hbar$c)$^{-1}$
Mass	GeV/c^2	Area	(GeV/$\hbar$c)$^{-2}$

Simplify (!) by choosing

$$\boxed{\hbar = c = 1}$$

All quantities expressed in powers of GeV

Energy	GeV	Time	GeV^{-1}
Momentum	GeV	Length	GeV^{-1}
Mass	GeV	Area	GeV^{-2}

★ Convert back to S.I. units by reintroducing 'missing' factors of $\hbar$ and c

EXAMPLE: $\text{Area} = 1\ \text{GeV}^{-2}$

$$
\begin{aligned}
[\mathrm{L}]^2 &= [\mathrm{E}]^{-2}[\hbar]^n[c]^m \\
[\mathrm{L}]^2 &= [\mathrm{E}]^{-2}[\mathrm{E}]^n[\mathrm{T}]^n[\mathrm{L}]^m[\mathrm{T}]^{-m} \\
\therefore n = 2 \quad &\text{and} \quad m = 2 \\
\rightarrow \text{Area(S.I.)} &= 1\ \text{GeV}^{-2} \times \hbar^2 c^2 \\
&= 3.8 \times 10^{-32}\ m^2 \\
&= 0.38 \times 10\ mb
\end{aligned}
$$

$$\boxed{\text{Heaviside-Lorentz units } \epsilon_0 = \mu_0 = \hbar = c = 1}$$

Fine structure constant α:

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137}$$

becomes $$\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}$$

Relativistic Kinematics

In Special Relativity $(t,\tilde{\mathrm{x}})$ and $(E,\tilde{\mathrm{p}})$ transform from frame-to-frame, BUT

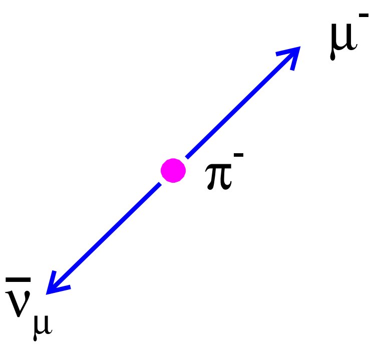
$$d^2 = t^2\text{-}x^2\text{-}y^2\text{-}z^2$$
$$m^2c^4 = E^2 - \tilde{\mathrm{p}}^2c^2$$

are CONSTANT (invariant interval, invariant mass)

Using natural units:

$$\boxed{m^2 = E^2 - \tilde{\mathrm{p}}^2}$$

EXAMPLE: $\pi^- \rightarrow \mu^-\overline{\nu}_\mu$ **at rest.**



Conservation of Energy

$$E_\pi = E_\mu + E_\nu$$

Conservation of Momentum

$$0 = \vec{p}_\mu + \vec{p}_\nu$$

(assume $m_\nu = 0$)

$$E_\pi = m_\pi, \quad E_\mu^2 = p_\mu^2 + m_\mu^2, \quad E_\nu = |p_\nu|$$

$$\begin{aligned} E_\pi &= E_\mu + E_\nu \\ \Rightarrow \quad m_\pi &= E_\mu + p_\mu \\ \Rightarrow \quad (m_\pi - E_\mu)^2 &= p_\mu^2 \\ \text{but} \quad E_\mu^2 - m_\mu^2 &= p_\mu^2 \end{aligned}$$

$$\therefore \quad E_\mu = \frac{m_\pi^2 + m_\mu^2}{2m_\pi} = \frac{(140\ \mathrm{MeV})^2 + (106\ \mathrm{MeV})^2}{2(140\ \mathrm{MeV})}$$
$$= 110\ \mathrm{MeV}$$
$$|p_\mu| = |p_\nu| = 30\ \mathrm{MeV}/c$$

(see Question 1 on the problem sheet)

Colliders and $\sqrt{s}$

Consider the collision of two particles:

$$\xrightarrow{\qquad\qquad} \qquad\qquad \xleftarrow{\qquad\qquad}$$

$$\mathbf{p}_1^\mu(E_1,\bar{\mathbf{p}}_1) \qquad \mathbf{p}_2^\mu(E_2,\bar{\mathbf{p}}_2)$$

The invariant quantity $s = (p_1^\mu + p_2^\mu)(p_{1_\mu} + p_{2_\mu})$

$$\begin{aligned} s &= (p_1^\mu p_{1_\mu} + p_2^\mu p_{2_\mu} + 2p_1^\mu p_{2_\mu}) \\ &= E_1^2 - \tilde{\mathrm{p}}_1^2 + E_2^2 - \tilde{\mathrm{p}}_2^2 + 2(E_1E_2 - \tilde{\mathrm{p}}_1.\tilde{\mathrm{p}}_2) \\ &= m_1^2 + m_2^2 + 2(E_1E_2 - \tilde{\mathrm{p}}_1.\tilde{\mathrm{p}}_2) \\ &= m_1^2 + m_2^2 + 2(E_1E_2 - |\tilde{\mathrm{p}}_1||\tilde{\mathrm{p}}_2|\cos\theta) \end{aligned}$$

$\sqrt{s}$ is the energy in the zero momentum frame.

It is the amount of energy available to interaction *e.g.* the maximum energy/mass of a particle produced in matter-antimatter annihilation.

Fixed Target Collision

$$\xrightarrow{\hspace{2cm}} \quad \bullet$$
$$p_1^\mu(E,\bar{p}) \qquad p_2^\mu(m_2,0)$$

$$s = m_1^2 + m_2^2 + 2E_1 m_2$$

for $E_1 \gg m_1, m_2$ $\quad s = 2E_1 m_2$

C.O.M. Energy $\sqrt{s} = \sqrt{2E_1 m_2}$

e.g. 100 GeV proton hitting a proton at rest:

$$\sqrt{s} = \sqrt{2E_p m_p} \approx \sqrt{2.100.1}$$
$$\approx 14 \text{ GeV}$$

Collider Experiment

$$\xrightarrow{\hspace{2cm}} \qquad \xleftarrow{\hspace{2cm}}$$
$$p_1^\mu(E,\bar{p}) \qquad p_2^\mu(E,-\bar{p})$$

Now consider two protons colliding head-on.

$$s = m_1^2 + m_2^2 + 2(E_1 E_2 - |\tilde{p}_1||\tilde{p}_2|\cos\theta)$$

If $E \gg m_1, m_2$ **then** $|\tilde{p}| = E$ **and**

$$s = 2(E^2 - E^2\cos\theta)$$
$$s = 4E^2$$
$$\sqrt{s} = 2E$$

e.g. 100 GeV proton colliding with a 100 GeV proton :

$$\sqrt{s} = 2.100 = 200 \text{ GeV}$$

In a fixed target experiment most of the proton's energy is wasted - providing momentum to the C.O.M system rather than being available for the interaction.

(NOTE: UNITS G = Giga =10^9, M = Mega =10^6)

Summary

FERMIONS : spin $\frac{1}{2}$

leptons **Charge**

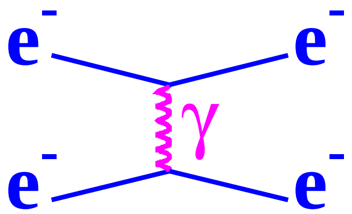
$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \quad \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \quad \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix} \qquad \begin{matrix} -1 \\ 0 \end{matrix}$$

quarks e.g. proton (uud)

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix} \qquad \begin{matrix} +\frac{2}{3} \\ -\frac{1}{3} \end{matrix}$$

+ anti-particles

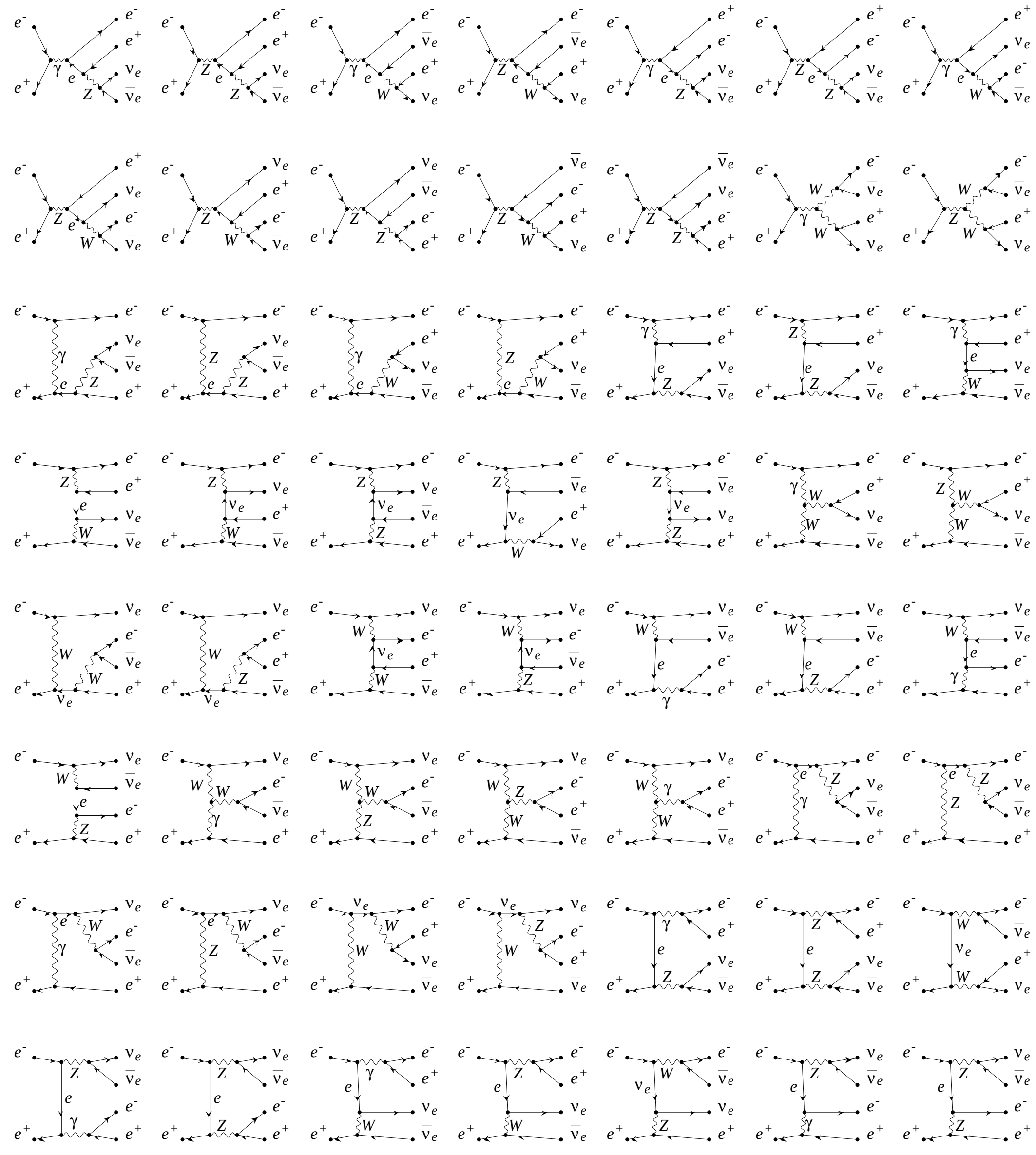
Fermion interactions = exchange of spin 1 bosons



BOSONS : spin 1

		Mass	Force
Photon	γ	0	Electromagnetic
W-boson	$W^{\pm}$	91.2 GeV	Weak (CC)
Z-boson	Z^0	80.3 GeV	Weak (NC)
Gluon	g	0	Strong (QCD)

Theoretical Framework



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(Lowest order Feynman diagrams for $e^+e^- \rightarrow e^+e^-\nu_e\overline{\nu}_e$)

Theory : Road-map

		Macroscopic	Microscopic
Slow		Classical mechanics	Quantum mechanics
Fast $v \to c$		Special Relativity	Quantum Field Theory

To describe the fundamental interactions of particles we need a theory of Relativistic Quantum Mechanics.

OUTLINE:

- ★ Klein-Gordon Equation
  - Anti-Matter
  - Yukawa Potential
- ★ Scattering in Quantum Mechanics
  - Born approximation (revision)
- ★ Quantum Electrodynamics
  - 2^{nd} Order Perturbation Theory
  - Feynman diagrams
- ★ Quantum Chromodynamics
  - the theory of the STRONG interaction
- ★ Dirac Equation
  - the relativistic theory of spin-1/2 particles
- ★ Weak interaction
- ★ Electro-weak Unification

The Klein-Gordon Equation

Schrödinger Equation for a free particle can be written as

$$\hat{\mathrm{E}}\psi = \frac{\hat{\mathrm{p}}^2}{2m}\psi,$$

with energy and momentum operators:

$$\hat{\mathrm{E}} = i\hbar\frac{\partial}{\partial t} \quad , \quad \hat{\mathrm{p}} = -i\hbar\nabla$$

giving $(\hbar = c = 1)$**:**

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2\mathrm{m}}\nabla^2\psi$$

which has plane wave solutions ($\tilde{\mathrm{p}} = \hbar\tilde{\mathrm{k}}$, $E = \hbar\omega$**)**

$$\psi(\tilde{\mathrm{r}}, \mathrm{t}) = N\mathrm{e}^{-\mathrm{i}(\mathrm{Et} - \tilde{\mathrm{p}}.\tilde{\mathrm{r}})}$$

Schrödinger Equation:

- ★ **1st Order in time derivative (**$\partial/\partial t$**)**
- ★ **2nd Order in space derivatives (**∇^2**)**
- ★ $\therefore$ **Not Lorentz Invariant !!!!**

Schrödinger Equation cannot be used to describe the physics of relativistic particles. We need a relativistic version of Quantum Mechanics. Our first attempt is the Klein-Gordon equation.

From Special Relativity (nat. units $c = 1$):

$$E^2 = p^2 + m^2$$

from Quantum Mechanics ($\hbar = 1$):

$$\hat{\mathrm{E}} = i\,\frac{\partial}{\partial \mathrm{t}} \quad , \quad \hat{\mathrm{p}} = -i\nabla$$

Combine to give the Klein-Gordon Equation:

$$\begin{aligned} -\frac{\partial^2\psi}{\partial t^2} &= -\nabla^2\psi + m^2\psi \\ \frac{\partial^2\psi}{\partial t^2} &= (\nabla^2 - m^2)\psi \end{aligned}$$

Second order in both space and time derivatives - by construction Lorentz invariant.

Plane wave solutions $\mathrm{e}^{-\mathrm{i}(\mathrm{Et}-\tilde{\mathrm{p}}.\tilde{\mathrm{r}})}$ solutions give

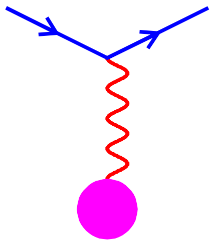
$$\begin{aligned} E^2 &= p^2 + m^2 \\ E &= \pm\sqrt{|p|^2 + m^2} \end{aligned}$$

ALLOWS NEGATIVE ENERGY SOLUTIONS

Anti-Matter

Static Solutions of K-G Equation

Consider a test particle near a massive source of bosons which mediate a force. Solutions of the K-G equation interpreted as either boson wave-function or as the potential



The bosons satisfy the Klein-Gordon Equation:

$$-\frac{\partial^2\psi}{\partial t^2} = -\nabla^2\psi + m^2\psi$$

If we now consider the static case: K-G becomes

$$\nabla^2\psi = m^2\psi$$

a solutions is

$$\psi(\tilde{\mathrm{r}}) = -\frac{g^2}{4\pi r}e^{-mr}$$

where the constant g gives the strength of the related force and m is the mass of the bosons.

$$\boxed{V(\tilde{\mathrm{r}}) = -\frac{g^2}{4\pi r}e^{-mr}}$$

the Yukawa Potential - originally proposed by Yukawa as the form of the nuclear potential.

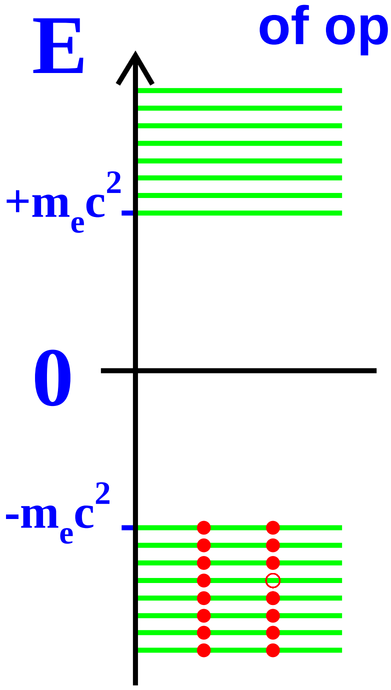
NOTE: for $m = 0$ we recover the Coulomb potential: $-\frac{\alpha}{r}$.

(the connection between the Yukawa Potential and a force mediated by the exchange of massive bosons will soon become apparent)

Anti-Particles

Negative energy solutions of the KG Equation ?

★ Dirac : vacuum corresponds to the state with all $E < 0$ states occupied by two electrons of opposite spin.



Pauli exclusion principle prevents $+$ve energy electron falling into $-$ve energy state

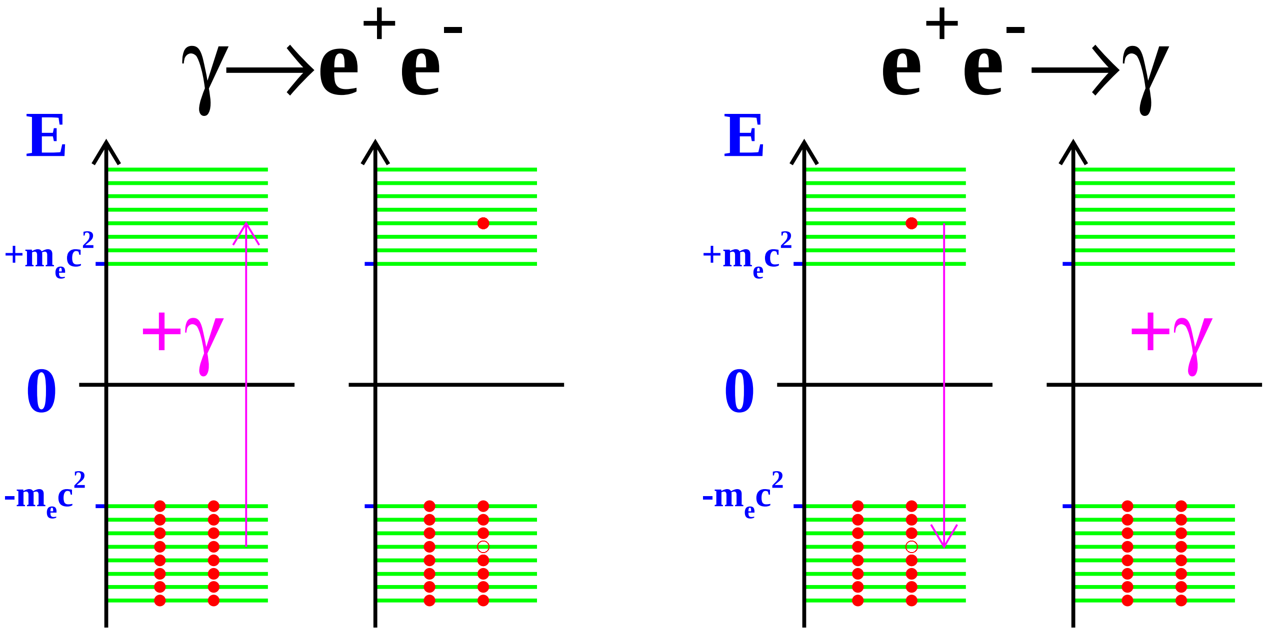
★ In this picture - a hole in the normally full set of $E < 0$ states corresponds to

Energy = $-E_{\text{hole}}$ *i.e.* $E_{e^+} > 0$

Charge = $-q_e{}^-$, *i.e.* $q_e{}^+ > 0$

A hole in the negative energy **particle** electron states corresponds to a **positively charged**, **positive energy** **anti-particle**

★ In the Dirac picture predict e^+e^- pair creation and e^+e^- annihilation.



★ 1931 : positron (e^+) was first observed.

Now favour Feynman interpretation:

★ $E < 0$ solutions represent negative energy particle states traveling backward in time.

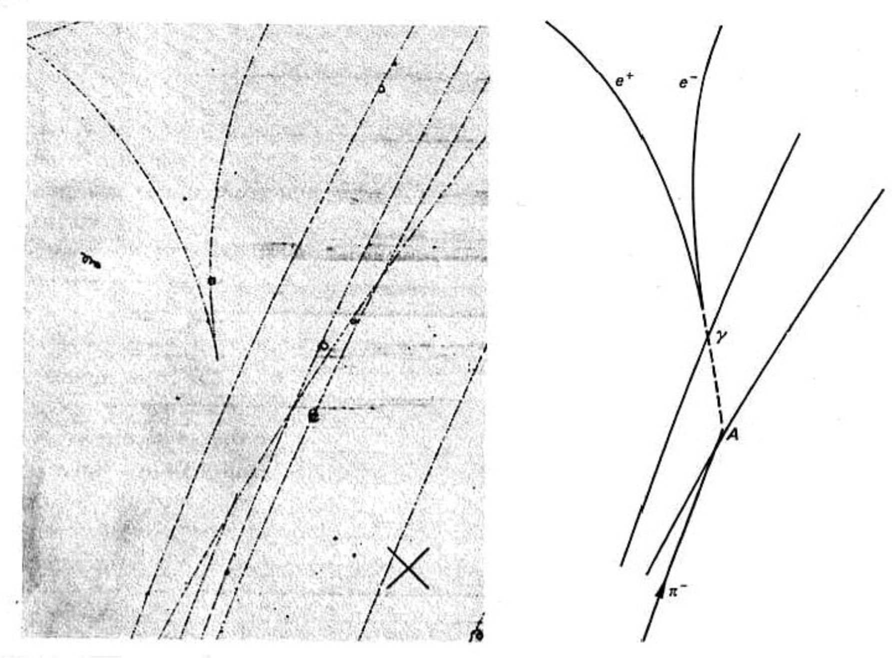
➡ Interpreted as positive energy anti-particles, of opposite charge, traveling forward in time.

★ Anti-particles have the same mass and equal but opposite charge.

Not much more than noting (for a plane wave)

$$e^{-iEt} = e^{-i(-E)(-t)}$$

(more on this next lecture)



Bubble chamber photograph of the process $\gamma \rightarrow e^+e^-$. Only charged particles are visible - i.e. only charged particles leave TRACKS

The full sequence of events is :

- ★ π^- **enters from bottom**
- ★ **Charge-exchange on a proton:**

$$\pi^- + p \rightarrow \pi^0 + n$$

- ★ π^0 **decays (lifetime 10^{-16} s) to two photons:**

$$\pi^0 \rightarrow \gamma\gamma$$

- ★ **Finally $\gamma \rightarrow e^+e^-$**

(the other photon is not observed within the region of the photograph)

Rates and Cross Sections

For reaction

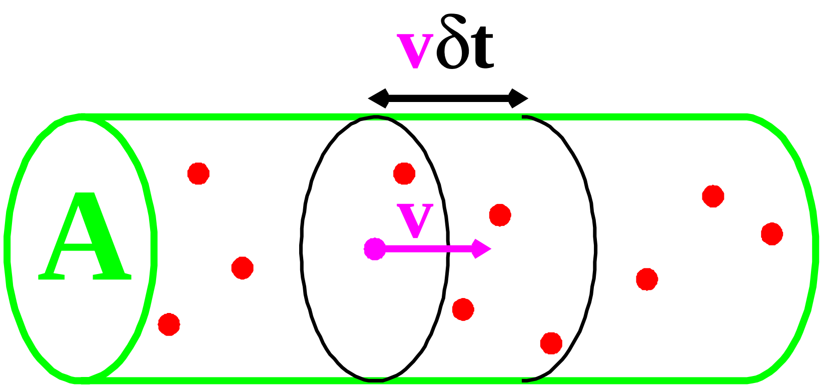
$$a + b \rightarrow c + d$$

The cross section, σ, is defined as the reaction rate per target particle, Γ, per unit incident flux, ϕ

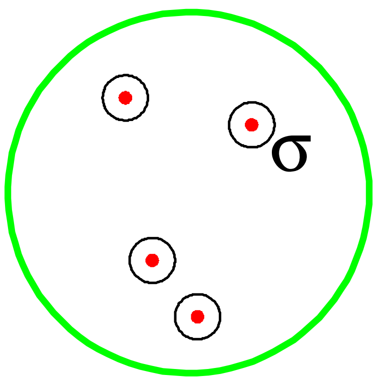
$$\Gamma = \phi\sigma$$

where Γ is given by Fermi's Golden Rule.

Example: Consider a single particle of type a traversing a beam (area A) of particles of type b of number density n_b.



In time δt traverses a region containing $v\delta t A n_b$ particles of type b.



Interaction probability defined as effective cross sectional area occupied by the $v\delta t A n_b$ particles of type b

$$\frac{v\delta t A n_b \sigma}{A} = v\delta t n_b \sigma$$

Therefore the reaction rate is $v n_b \sigma$.

(see Question 2 on the problem sheet)

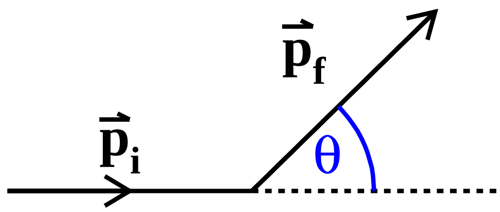
Scattering in Q.M.

REVISION (see Dr Ritchie's QMII transparencies 11.8-11.15)

NOTE: Natural Units used throughout $\hbar = c = 1$

$$\tilde{p} = \hbar\,\tilde{k} \rightarrow \tilde{p} = \tilde{k} \quad etc.$$

Consider a beam of particles scattering in Potential $V(r)$



★ Scattering rate characterized by the interaction cross section σ

$$\sigma = \frac{\text{number of particles scattered/unit time}}{\text{incident flux}}$$

Use FERMI'S GOLDEN RULE for Transition rate, Γ:

$$\Gamma \;=\; 2\pi|M|^2\rho(E_f)$$

where M is the Matrix Element and $\rho(E_f)$ = density of final states.

★ 1st Order Perturbation Theory using plane wave solutions of form $\psi = N\mathrm{e}^{-\mathrm{i}(\mathrm{E}t-\tilde{\mathrm{p}}.\tilde{\mathrm{r}})}$.

Require :

- wave-function normalization
- matrix element in perturbation theory
- expression for flux
- expression for density of states

Normalization: Normalize wave-functions to one particle in a box of side L

$$
\begin{aligned}
|\psi_i|^2 &= N^2 = 1/L^3 \\
N &= (1/L)^{\frac{3}{2}}
\end{aligned}
$$

Matrix Element: this contains the physics of the interaction

$$
\begin{aligned}
M &= \langle \psi_f | \hat{\mathrm{H}} | \psi_i \rangle \\
M &= \int \psi_f^* \hat{\mathrm{H}} \psi_i d^3\tilde{\mathrm{r}} \\
M &= \int N e^{-i\tilde{\mathrm{p}}_{\mathrm{f}}.\tilde{\mathrm{r}}} V(\tilde{\mathrm{r}}) N e^{i\tilde{\mathrm{p}}_{\mathrm{i}}.\tilde{\mathrm{r}}} d^3\tilde{\mathrm{r}} \\
M &= \frac{1}{L^3} \int e^{i\tilde{\mathrm{p}}.\tilde{\mathrm{r}}} V(\tilde{\mathrm{r}}) d^3\tilde{\mathrm{r}}
\end{aligned}
$$

where $\tilde{\mathrm{p}} = \tilde{\mathrm{p}}_{\mathrm{i}} - \tilde{\mathrm{p}}_{\mathrm{f}}$

Incident Flux: Consider a “target” of area A and a beam of particles traveling at $v = c$ towards the target. Any incident particle with in a volume cA will cross the target area every second. Flux = number of incident particles crossing unit area per second :

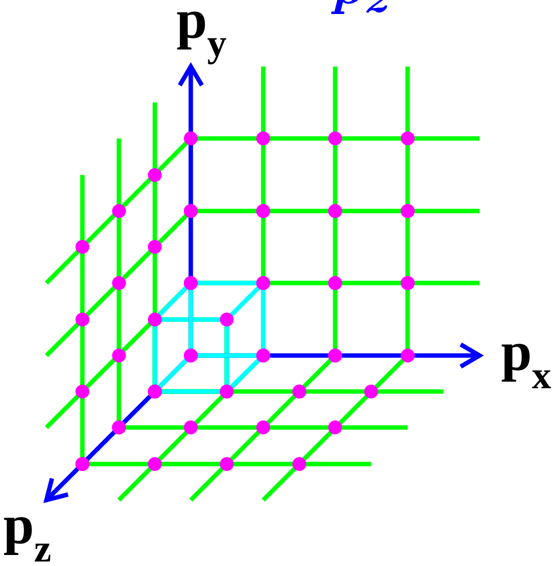
$$
\mathrm{flux} = \frac{cA}{A} n_i = c n_i
$$

where n_i is number density of incident particles = 1 per L^3

$$
\mathrm{flux} = c/L^3 = 1/L^3 \quad (c = 1)
$$

Density of states: for box of side L states are given by periodic boundary conditions:

$$\begin{aligned} k_x &= 2\pi n_x/L, \;\; etc. \\ \Rightarrow \qquad p_x &= 2\pi n_x/L \qquad (\hbar = 1) \\ p_y &= 2\pi n_y/L \\ p_z &= 2\pi n_z/L \end{aligned}$$



Volume of single state in momentum space:

$$\left(\frac{2\pi}{L}\right)^3$$

Number of final states between $p \to p + dp$:

$$\begin{aligned} dN &= p^2 dp d\Omega/(2\pi/L)^3 \\ \therefore \rho(p_f) &= dN/dp = p^2 d\Omega/(2\pi/L)^3 \end{aligned}$$

In almost all scattering process considered in these lectures the final state particles have $E \gg m$ and to a good approximation $E^2 = p^2 + m^2 \;\; \to E = p$.

$$\begin{aligned} \rho(E) &= \frac{dN}{dE} = \frac{dN}{dp}\frac{dp}{dE} \\ &= E^2 d\Omega/(2\pi/L)^3 \end{aligned}$$

$$\boxed{\rho(E) = \frac{E^2 d\Omega}{(2\pi)^3} L^3}$$

★ **Putting all the separate bits together:**

$$d\sigma = \frac{1}{\text{flux}} 2\pi |M|^2 \rho(E_f)$$

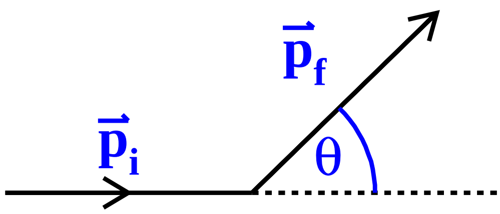
$$= L^3 2\pi \left| \frac{1}{L^3} \int e^{i\tilde{\mathrm{p}}.\tilde{\mathrm{r}}} V(r) d^3\tilde{\mathrm{r}} \right|^2 E^2 \left(\frac{L}{2\pi}\right)^3 d\Omega$$

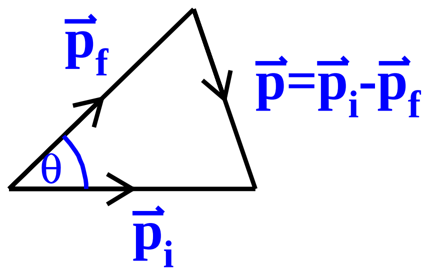
$$\frac{d\sigma}{d\Omega} = \frac{E^2}{(2\pi)^2} \left| \int e^{i\tilde{\mathrm{p}}.\tilde{\mathrm{r}}} V(r) d^3\tilde{\mathrm{r}} \right|^2$$

The normalization cancels and, in the limit where the incident particles have $v \approx c$ and the out-going particles have $E \gg m \rightarrow E_f = p_f$, arrive at a simple expression. Apply to the elastic scattering of a particle from in a Yukawa potential.

Scattering from Yukawa Potential

$$V(r) = -\frac{g^2}{4\pi}\frac{e^{-mr}}{r}$$





$$M_{fi} = \int e^{i\tilde{\mathrm{p}}.\tilde{\mathrm{r}}} V(r) d^3\tilde{\mathrm{r}}$$

$$M_{fi} = -\frac{g^2}{4\pi}\int_0^\infty\int_0^\pi\int_0^{2\pi} e^{i|\tilde{\mathrm{p}}|r\cos\theta'}\frac{e^{-mr}}{r} r^2 \sin\theta' dr d\theta' d\phi$$

Where for the purposes of the integration, the z-axis is been defined to lie in the direction of $\tilde{\mathrm{p}}$ and θ' is the polar angle with respect to this axis.

$$M_{fi} = -\frac{g^2}{4\pi}\int_0^{\infty}\int_0^{\pi}\int_0^{2\pi} e^{i|\tilde{\mathrm{p}}|r\cos\theta'}\frac{e^{-mr}}{r}r^2\sin\theta' d\theta' dr d\phi$$

Integrate over $d\phi$ and set $y = \cos\theta'$.

$$\begin{aligned}
M_{fi} &= -\frac{g^2}{2}\int_0^{\infty}\int_{-1}^{+1} r e^{i|\tilde{\mathrm{p}}|ry} e^{-mr} dr dy \\
&= -\frac{g^2}{2i|\tilde{\mathrm{p}}|}\int_0^{\infty}(e^{+i|\tilde{\mathrm{p}}|r} - e^{-i|\tilde{\mathrm{p}}|r})e^{-mr} dr \\
&= -\frac{g^2}{2i|\tilde{\mathrm{p}}|}\int_0^{\infty} e^{(i|\tilde{\mathrm{p}}|-m)r} - e^{-(i|\tilde{\mathrm{p}}|+m)r} dr \\
&= \frac{g^2}{2i|\tilde{\mathrm{p}}|}\left[\frac{1}{(i|\tilde{\mathrm{p}}| - m)} + \frac{1}{(i|\tilde{\mathrm{p}}| + m)}\right] \\
&= \frac{g^2}{2i|\tilde{\mathrm{p}}|}\left[\frac{2i|\tilde{\mathrm{p}}|}{(-|\tilde{\mathrm{p}}|^2 - m^2)}\right]
\end{aligned}$$

$$\boxed{M_{fi} = -\frac{g^2}{(m^2+|\vec{\mathrm{p}}|^2)}}$$

giving
$$\frac{d\sigma}{d\Omega} = \frac{E^2}{(2\pi)^2}\frac{g^4}{(m^2 + |\tilde{\mathrm{p}}|^2)^2}$$

★ Scattering in the Yukawa potential introduces a term $(m^2 + |\tilde{\mathrm{p}}|^2)$ in the denominator of the matrix element - this is known a the propagator.

Rutherford Scattering

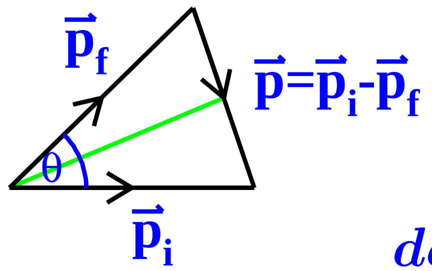
Let $m \to 0$ and replace $g^2 \to e^2 = 4\pi\alpha$

$$V(r) = -\frac{g^2}{4\pi}\frac{e^{-mr}}{r}$$

gives Coulomb potential: $$V(r) = -\alpha/r$$

Hence for elastic scattering in the Coulomb potential:

$$\frac{d\sigma}{d\Omega} = \frac{E^2}{(2\pi)^2}\frac{16\pi^2\alpha^2}{|\tilde{\mathrm{p}}|^4}$$

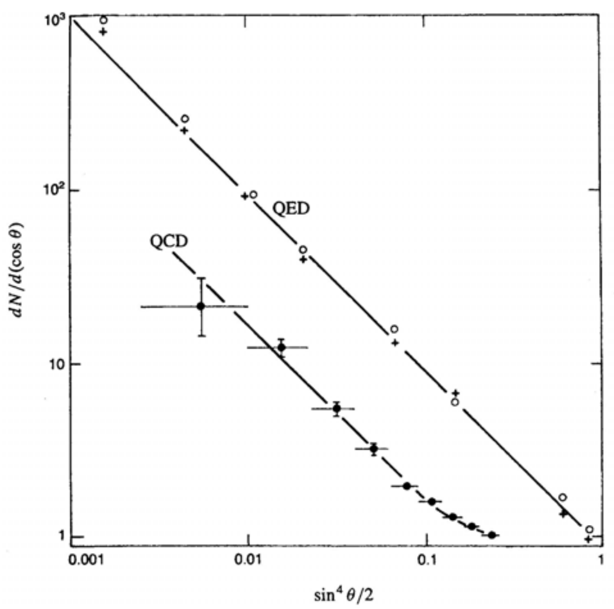


$$|\tilde{\mathrm{p}}| = 2|\tilde{\mathrm{p}}_i|\sin\frac{\theta}{2}$$

$$|\tilde{\mathrm{p}}| = 2E\sin\frac{\theta}{2}$$

$$\frac{d\sigma}{d\Omega} = \frac{E^2}{(2\pi)^2}\frac{16\pi^2\alpha^2}{16E^4\sin^4\frac{\theta}{2}}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Rutherford}} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}$$



e.g. The upper points are the Gieger and Marsden data (1911) for the elastic scattering of α particles as they traverse thin gold and silver foils. The scattering rate, plotted versus $\sin^4\frac{\theta}{2}$, follows the Rutherford formula. (note, plotted as log vs. log)