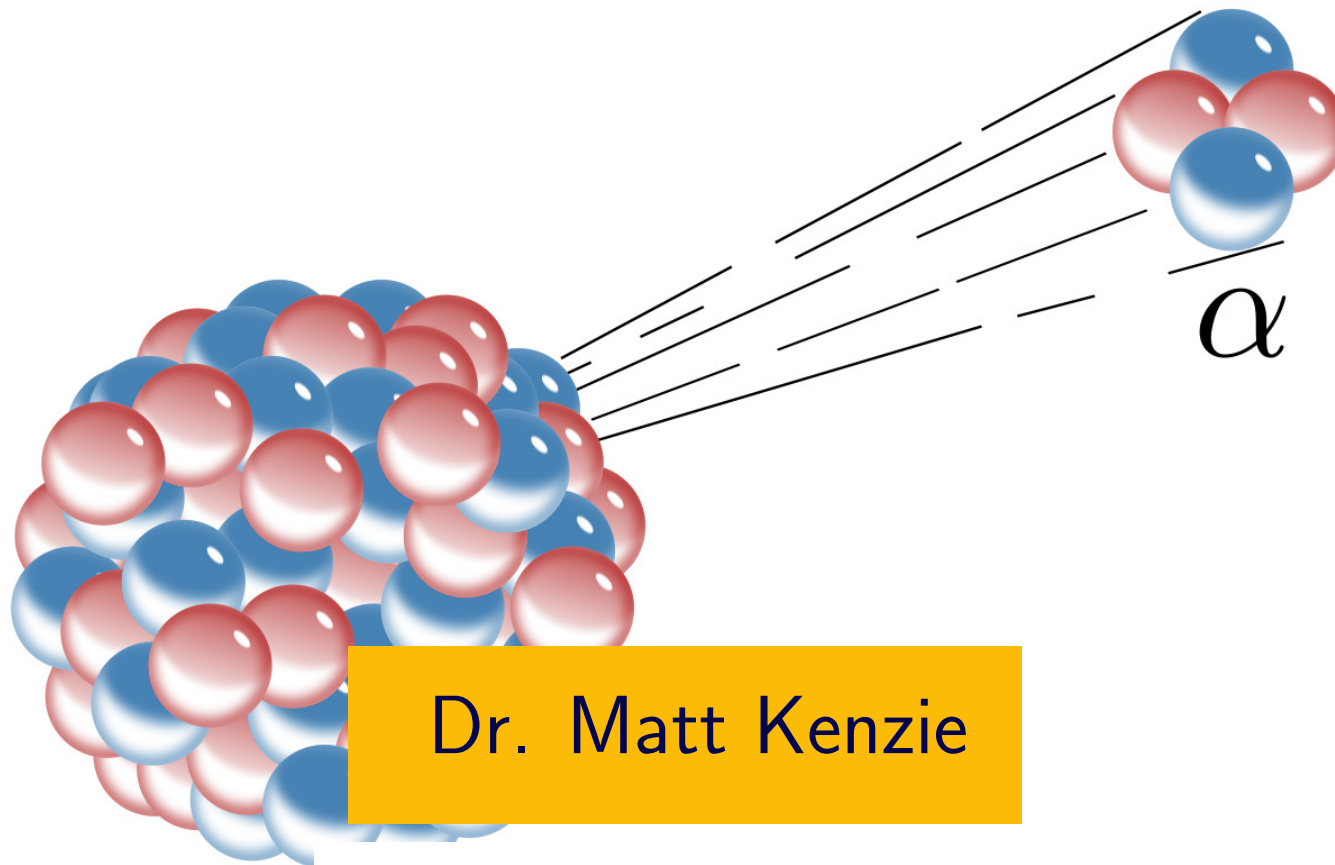


15. Nuclear Decay

Particle and Nuclear Physics



Dr. Matt Kenzie



UNIVERSITY OF
CAMBRIDGE

In this section...

- Radioactive decays
- Radioactive dating
- α decay
- β decay
- γ decay

Radioactivity

Natural radioactivity: three main types α, β, γ , and in a few cases, spontaneous fission.

α decay ${}^4_2\text{He}$ nucleus emitted.



For decay to occur, energy must be released $Q > 0$

$$Q = m_{\text{X}} - m_{\text{Y}} - m_{\text{He}} = B_{\text{Y}} + B_{\text{He}} - B_{\text{X}}$$

β decay emission of electron e^- or positron e^+

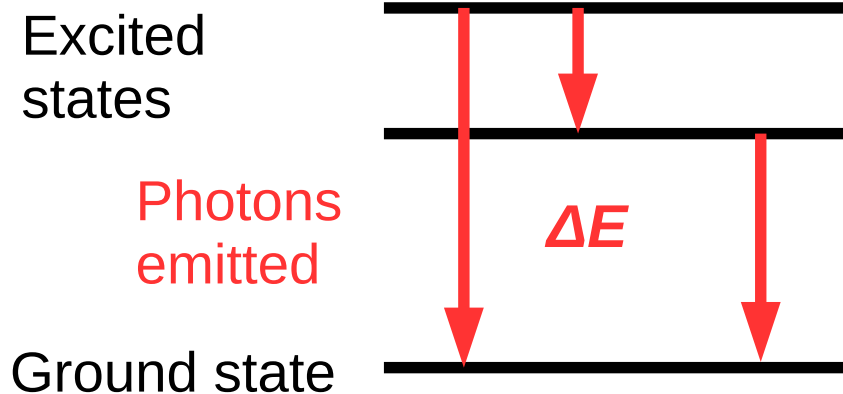


n.b. of these processes, only $n \rightarrow pe\nu$ can occur outside a nucleus.

Radioactivity

γ decay

Nuclei in excited states can decay by emission of a photon γ . Often follows α or β decay.



	ΔE	λ
Atom	$\sim 10 \text{ eV}$	$\sim 10^{-7} \text{ m}$ optical
	$\sim 10 \text{ keV}$	$\sim 10^{-10} \text{ m}$ X-ray
Nucleus	$\sim \text{MeV}$	$\sim 10^{-12} \text{ m}$ γ -ray

A variant of γ decay is **Internal Conversion**:

- an excited nucleus loses energy by emitting a virtual photon,
- the photon is absorbed by an atomic e^- , which is then ejected
- n.b. not β decay, as nucleus composition is unchanged (e^- not from nucleus)

Natural Radioactivity

The **half-life**, $\tau_{1/2}$, is the time over which 50% of the nuclei decay

$$\tau_{1/2} = \frac{\ln 2}{\lambda} = 0.693\tau$$

λ Transition rate

τ Average lifetime

Some $\tau_{1/2}$ values may be long compared to the age of the Earth.

Series Name	Type	Final Nucleus (stable)	Longest-lived Nucleus	$\tau_{1/2}$ (years)
Thorium	4n	^{208}Pb	^{232}Th	1.41×10^{10}
Neptunium	4n+1	^{209}Bi	^{237}Np	2.14×10^6
Uranium	4n+2	^{206}Pb	^{238}U	4.47×10^9
Actinium	4n+3	^{207}Pb	^{235}U	7.04×10^8

n is an integer

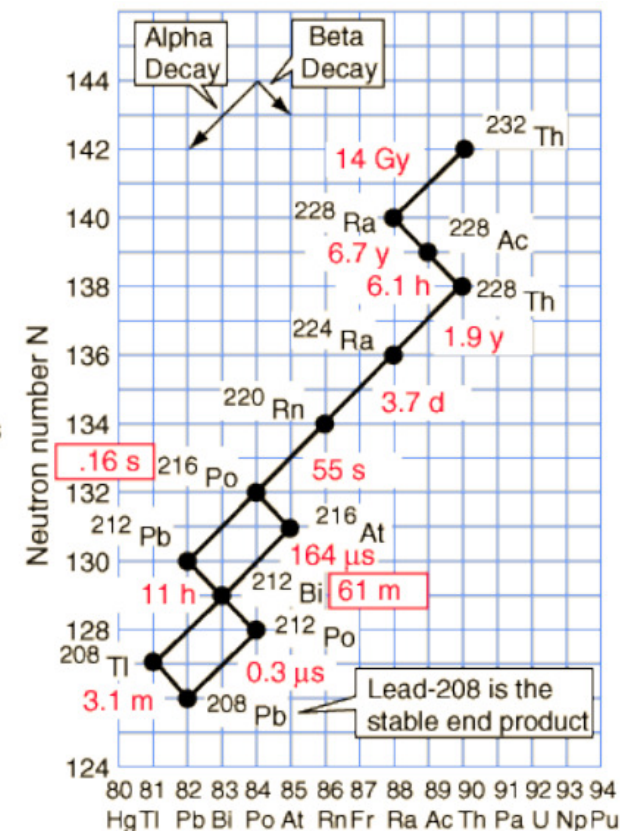
4n series

The Thorium-232 Decay Series

- ☐ ^{235}U Series
- ☒ ^{232}Th Series
- ☐ ^{238}U Series
- ☐ ^{237}Np Series

The four natural radioactive series

Boxed values for half-life are for multiple decay paths



Radioactive Dating

Geological Dating

Can use β^- decay to age the Earth, ${}^{87}\text{Rb} \rightarrow {}^{87}\text{Sr}$ ($\tau_{1/2} = 4.8 \times 10^{10}$ years)
 N_1 N_2
 ${}^{87}\text{Sr}$ is stable $\rightarrow \lambda_2 = 0$

So in this case, we have (using expressions from Chapter 2)

$$N_2(t) = N_1(0) [1 - e^{-\lambda_1 t}] + N_2(0) = N_1(t) [e^{\lambda_1 t} - 1] + N_2(0)$$

Assume we know λ_1 , and can measure $N_1(t)$ and $N_2(t)$ e.g. chemically.
But we don't know $N_2(0)$.

Solution is to normalise to another (stable) isotope – ${}^{86}\text{Sr}$ – for which number is $N_0(t) = N_0(0)$.

$$\frac{N_2(t)}{N_0} = \frac{N_1(t)}{N_0} [e^{\lambda_1 t} - 1] + \frac{N_2(0)}{N_0}$$

Method: plot $N_2(t)/N_0$ vs $N_1(t)/N_0$ for lots of minerals.

Gradient gives $[e^{\lambda_1 t} - 1]$ and hence t .

Intercept = $N_2(0)/N_0$, which should be the same for all minerals (determined by chemistry of formation).

Radioactive Dating

Dating the Earth

$$\frac{N_2(t)}{N_0} = \frac{N_1(t)}{N_0} [e^{\lambda_1 t} - 1] + \frac{N_2(0)}{N_0}$$

Method: plot $N_2(t)/N_0$ vs $N_1(t)/N_0$ for lots of minerals.

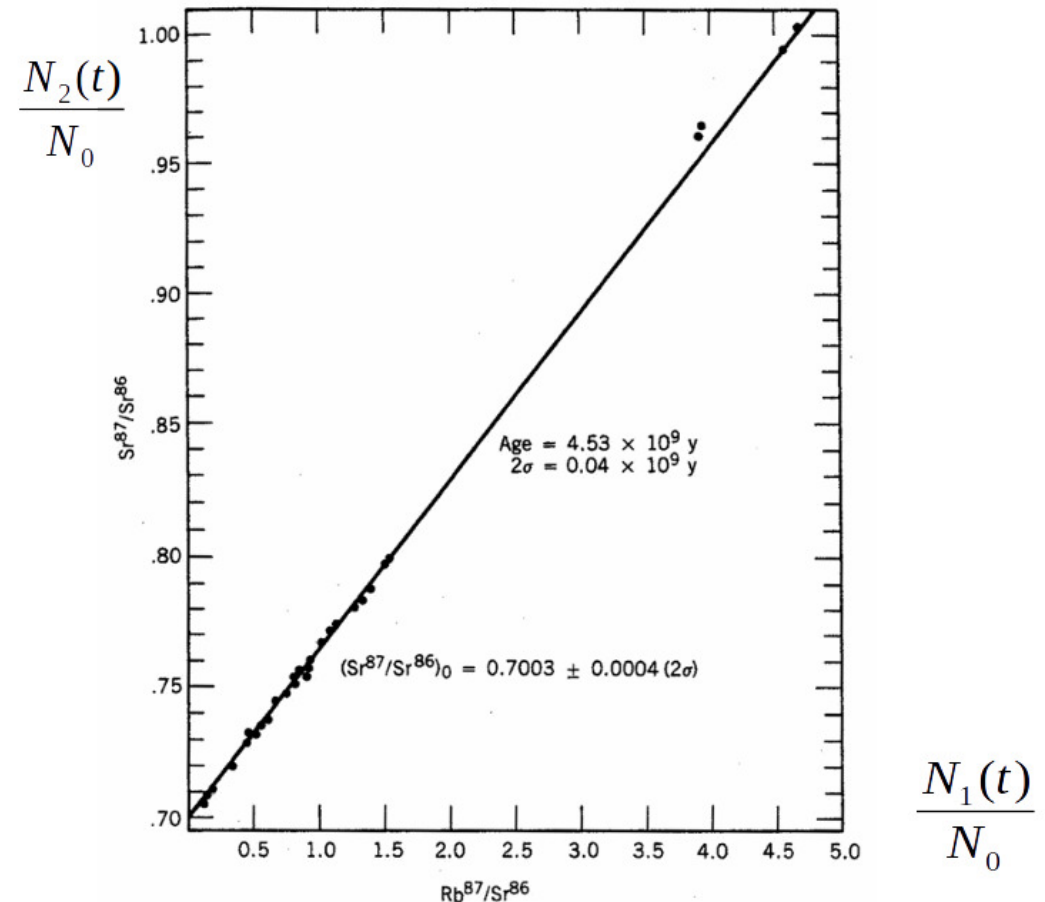
Gradient gives $[e^{\lambda_1 t} - 1]$ and hence t .

Intercept = $N_2(0)/N_0$, which should be the same for all minerals (determined by chemistry of formation).

Using minerals from the Earth, Moon and meteorites.

Intercept gives $N_2(0)/N_0 = 0.70$

Slope gives the age of the Earth = 4.5×10^9 yrs

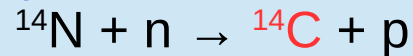


Radioactive Dating

Radio-Carbon Dating

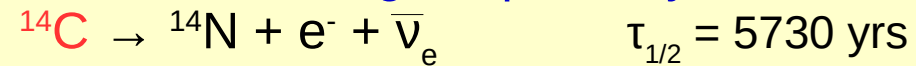
For recent organic matter, use ^{14}C dating

Continuously formed in the upper atmosphere at approx. constant rate.



Atmospheric carbon continuously exchanged with living organisms.
Equilibrium: 1 atom of ^{14}C to every 10^{12} atoms of other carbon isotopes
(98.9% ^{12}C , 1.1% ^{13}C)

Undergoes β^- decay



No more ^{14}C intake for dead organisms.

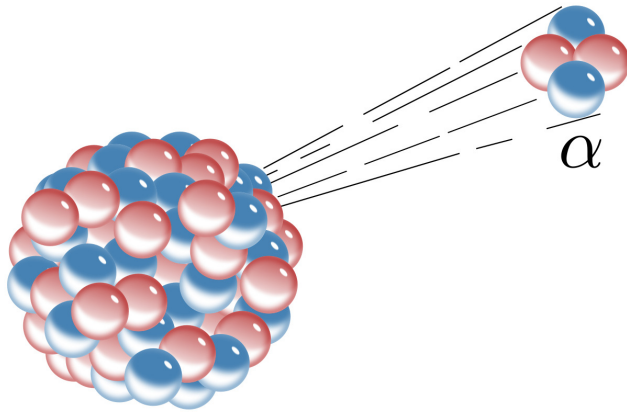
Fresh organic material
~11 decays/minute/gram of carbon.

Measure the **specific activity** of material to obtain age, i.e. number of decays per second per unit mass

Complications for the future!

Burning of fossil fuels increases ^{12}C in atmosphere,
Nuclear bomb testing (adds ^{14}C to atmosphere)

α Decay



- α decay is due to the emission of a ${}^4_2\text{He}$ nucleus.
- ${}^4_2\text{He}$ is **doubly magic** and very **tightly bound**.
- α decay is energetically favourable for almost all with $A \geq 190$ and for many $A \geq 150$.

Why α rather than any other nucleus?

Consider energy release (Q) in various possible decays of ${}^{232}\text{U}$

	n	p	${}^2\text{H}$	${}^3\text{H}$	${}^3\text{He}$	${}^4\text{He}$	${}^5\text{He}$	${}^6\text{Li}$	${}^7\text{Li}$
Q/MeV	-7.26	-6.12	-10.70	-10.24	-9.92	+5.41	-2.59	-3.79	-1.94

α is easy to form inside a nucleus $2p \uparrow\downarrow + 2n \uparrow\downarrow$

(though the extent to which α particles really exist inside a nucleus is still debatable)

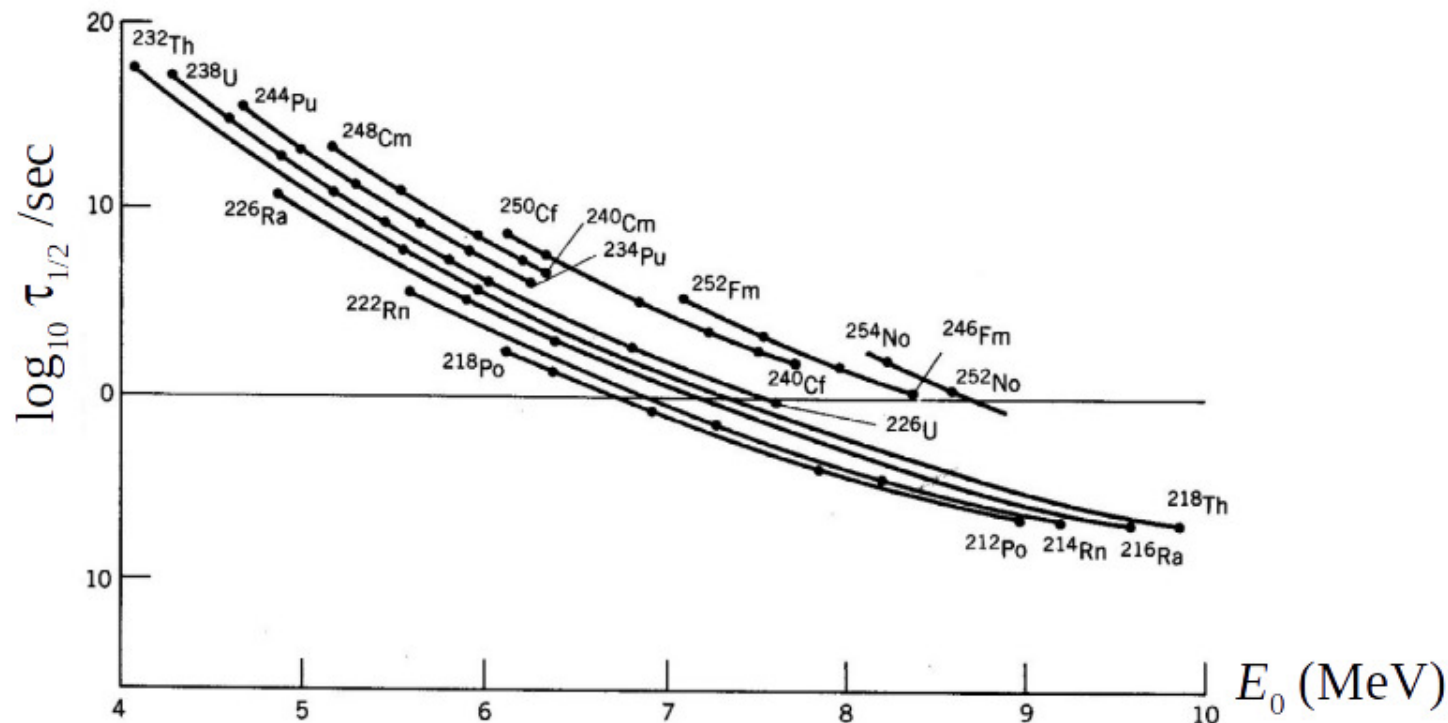
α Decay Dependence of $\tau_{1/2}$ on E_0

(Geiger and Nuttall 1911)

A **very** striking feature of α decay is the strong dependence of lifetime on E_0

Example ^{232}Th $E_0 = 4.08 \text{ MeV}$ $\tau_{1/2} = 1.4 \times 10^{10} \text{ yrs}$
 ^{218}Th $E_0 = 9.85 \text{ MeV}$ $\tau_{1/2} = 1.0 \times 10^{-7} \text{ s}$

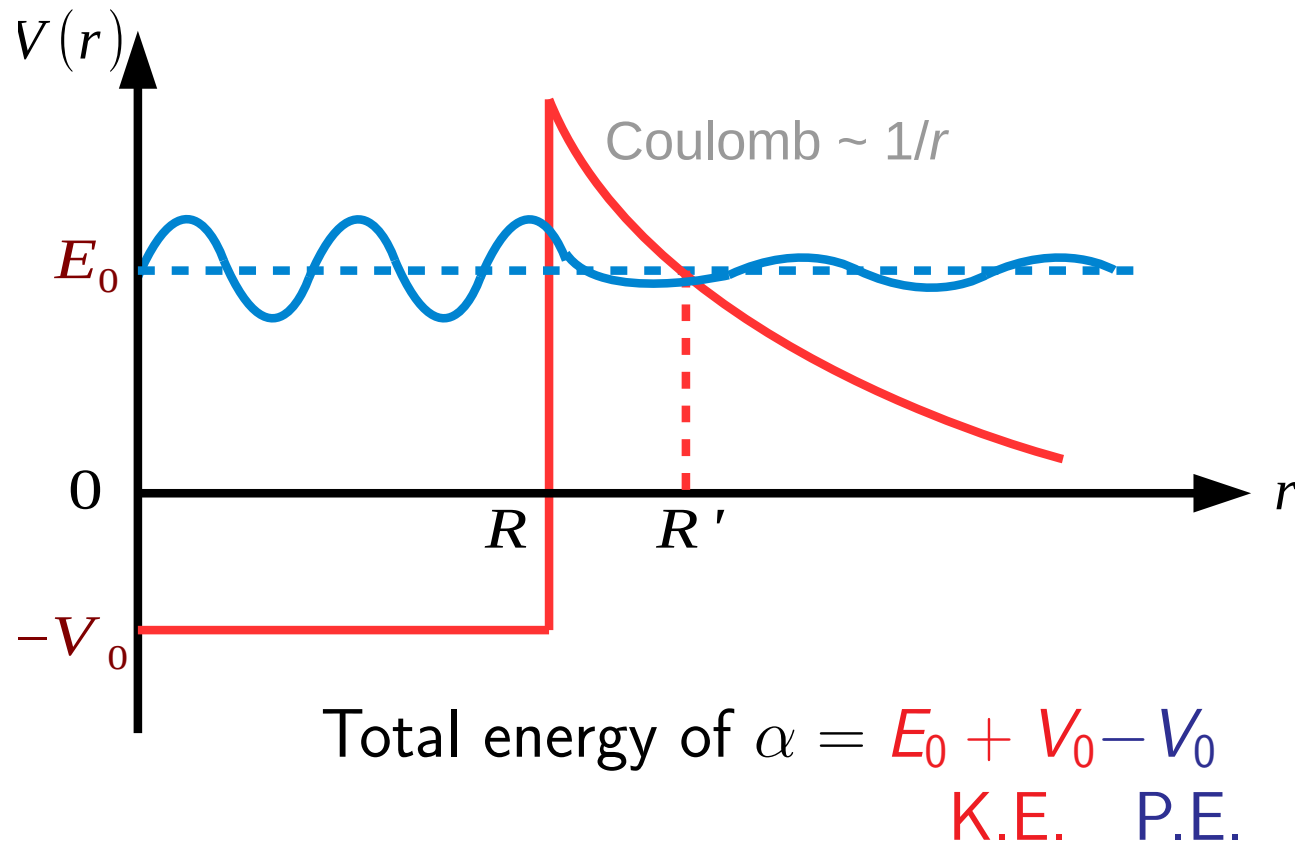
A factor of ~ 2.5 in $E_0 \Rightarrow$ factor 10^{24} in $\tau_{1/2}$!



e.g. even N , even Z nuclei for a given Z see smooth trend ($\tau_{1/2}$ increases as Z does)

α Decay *Quantum Mechanical Tunnelling*

The nuclear potential for the α particle due to the daughter nucleus includes a **Coulomb barrier** which inhibits the decay.



Classically, α particle cannot enter or escape from nucleus.
Quantum mechanically, α particle can penetrate the Coulomb barrier
 $\Rightarrow$ **Quantum Mechanical Tunnelling**

α Decay *Simple Theory* (Gamow, Gurney, Condon 1928)

Assume α exists inside the nucleus and hits the barrier.

$$\alpha \text{ decay rate, } \lambda = f P$$

f = escape trial frequency, P = probability of tunnelling through barrier

$$\text{semi - classically, } f \sim v/2R$$

v = velocity of a particle inside nucleus, given by: $v^2 = (2E_\alpha/m_\alpha)$
and R = radius of nucleus

Typical values: $V_0 \sim 35$ MeV, $E_0 \sim 5$ MeV $\Rightarrow E_\alpha = 40$ MeV inside nucleus

$$f \sim \frac{v}{2R} = \frac{1}{2R} \sqrt{\frac{2E_\alpha}{m_\alpha}} \sim 10^{22} \text{ s}^{-1} \quad \begin{array}{l} m_\alpha = 3.7 \text{ GeV} \\ R \sim 2.1 \text{ fm} \end{array}$$

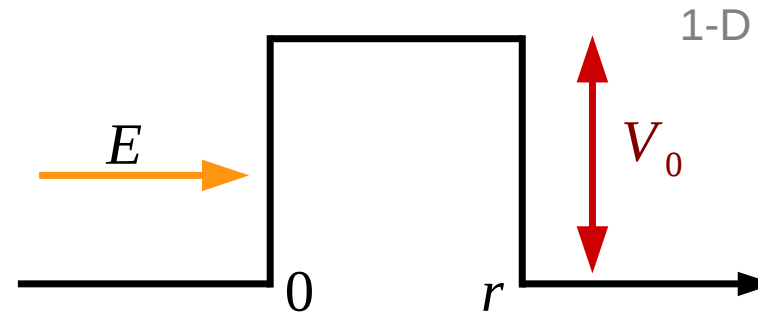
Obtain tunnelling probability, P , by solving Schrödinger equation in three regions and using boundary conditions.

α Decay Simple Theory (Gamow, Gurney, Condon 1928)

Transmission probability (1D square barrier):

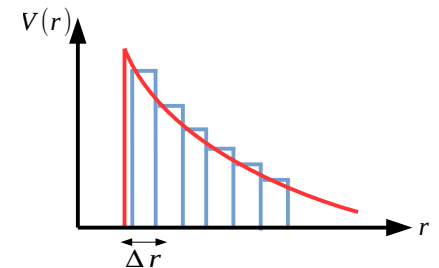
$$P = \left[1 + \frac{V_0^2}{4(V_0 - E)E} \sinh^2 ka \right]^{-1}$$

$$\frac{\hbar^2 k^2}{2m} = V_0 - E \quad m = \text{reduced mass}$$



For $ka \gg 1$, P is dominated by the exp. decay within barrier $\Rightarrow P \sim e^{-2ka}$.

Coulomb potential, $V \propto 1/r$, and thus k varies with r .
Divide into rectangular pieces and multiply together exponentials, i.e. sum exponents.



Probability to tunnel through Coulomb barrier

$$P = \prod_i e^{-2k_i \Delta R} = e^{-2G} \quad k = \frac{[2m_\alpha(V(r) - E_0)]^{1/2}}{\hbar}$$

The **Gamow Factor** $G = \int_R^{R'} \frac{[2m_\alpha(V(r) - E_0)]^{1/2}}{\hbar} dr = \int_R^{R'} k(r) dr$

α Decay Simple Theory (Gamow, Gurney, Condon 1928)

For $r > R$,
$$V(r) = \frac{Z_\alpha Z' e^2}{4\pi\epsilon_0 r} = \frac{B}{r} \quad Z' = Z - Z_\alpha \quad (Z_\alpha = 2)$$

α -particle escapes at $r = R'$, $V(R') = E_0 \Rightarrow R' = B/E_0$

$$\therefore G = \int_R^{R'} \left(\frac{2m_\alpha}{\hbar^2} \right)^{1/2} \left[\frac{B}{r} - E_0 \right]^{1/2} dr = \left(\frac{2m_\alpha B}{\hbar^2} \right)^{1/2} \int_R^{R'} \left[\frac{1}{r} - \frac{1}{R'} \right]^{1/2} dr$$

See Appendix H

$$G = \left(\frac{2m_\alpha}{E_0} \right)^{1/2} \frac{B}{\hbar} \left[\cos^{-1} \left(\frac{R}{R'} \right)^{1/2} - \left\{ \left(1 - \frac{R}{R'} \right) \left(\frac{R}{R'} \right) \right\}^{1/2} \right]$$

To perform integration, substitute $r = R' \cos^2 \theta$

In most practical cases $R \ll R'$, so term in [...] $\sim \pi/2$

$$G \sim \left(\frac{2m_\alpha}{E_0} \right)^{1/2} \frac{B\pi}{\hbar 2} \quad B = \frac{Z_\alpha Z' e^2}{4\pi\epsilon_0}$$

e.g. typical values: $Z = 90$, $E_0 \sim 6$ MeV $\Rightarrow R' \sim 40$ fm $\gg R$

$$G \sim Z' \left(\frac{3.9 \text{ MeV}}{E_0} \right)^{1/2}$$

α Decay Simple Theory (Gamow, Gurney, Condon 1928)

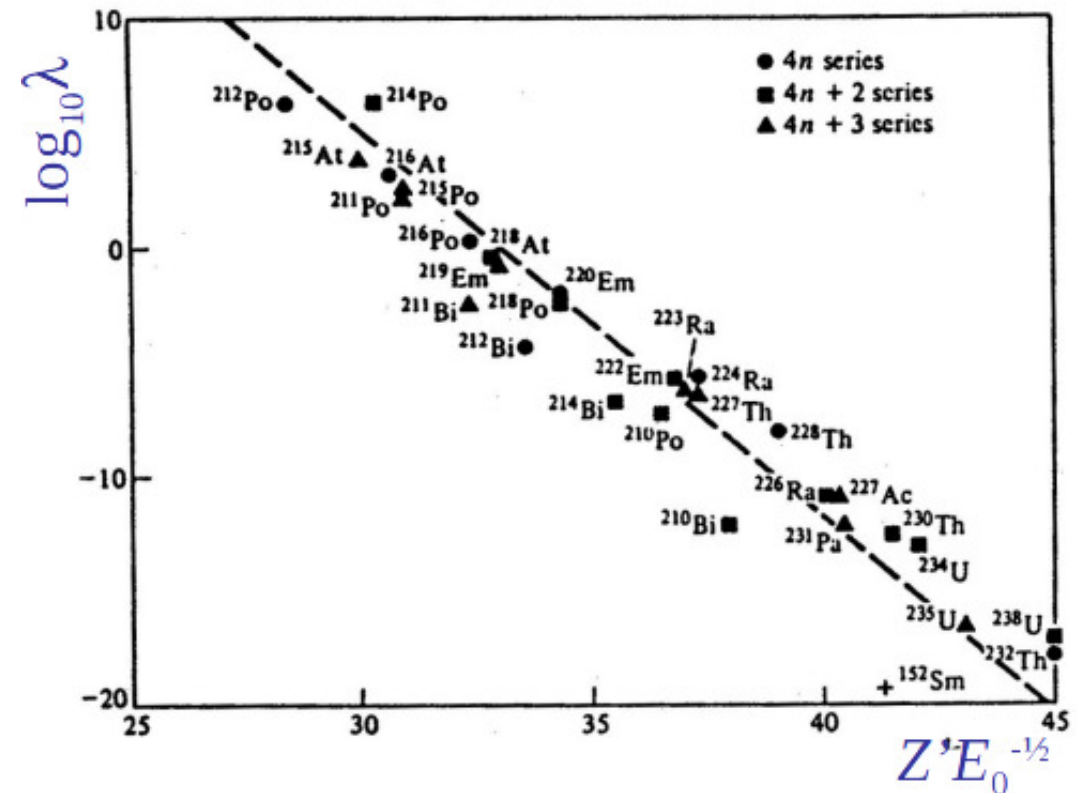
Lifetime $\tau = \frac{1}{\lambda} = \frac{1}{fP} \sim \frac{2R}{v} e^{2G}$

$$\Rightarrow \ln \tau \sim 2G + \ln \frac{2R}{v}$$

$$\ln \lambda \sim -\frac{Z'}{E_0^{1/2}} + \text{constant}$$

Geiger-Nuttall Law

Not perfect, but provides an explanation of the dominant trend of the data



Simple tunnelling model accounts for

- strong dependence of $\tau_{1/2}$ on E_0
- $\tau_{1/2}$ increases with Z
- disfavoured decay to heavier fragments e.g. ^{12}C

$$G \propto m^{1/2} \quad \text{and} \quad G \propto \text{charge of fragment}$$

α Decay *Simple Theory* (Gamow, Gurney, Condon 1928)

Deficiencies/complications with simple tunnelling model:

- Assumed existence of a single α particle in nucleus and have taken no account of probability of formation.
- Assumed “semi-classical” approach to estimate escape trial frequency, $f \sim v/2R$, and make absolute prediction of decay rate.
- If α is emitted with some angular momentum, L , the radial wave equation must include a centrifugal barrier term in Schrödinger equation

$$V' = \frac{L(L+1)\hbar^2}{2\mu r^2}$$

L = relative a.m. of α and daughter nucleus

μ = reduced mass

which raises the barrier and suppresses emission of α in high L states.

α Decay Selection rules

Nuclear Shell Model: α has $J^P = 0^+$

Angular momentum

e.g. $X \rightarrow Y + \alpha$

Conserve J : $J_X = J_Y \oplus J_\alpha = J_Y \oplus L_\alpha$

L_α can take values from $J_X + J_Y$ to $|J_X - J_Y|$

Parity

Parity is conserved in α decay (strong force).

Orbital wavefunction has $P = (-1)^L$

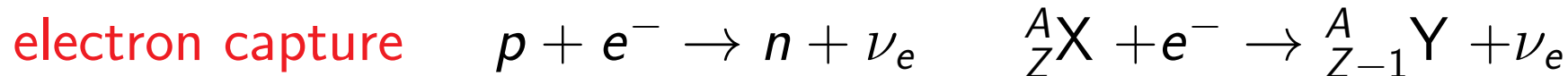
X, Y same parity $\Rightarrow L_\alpha$ must be even

X, Y opposite parity $\Rightarrow L_\alpha$ must be odd

e.g. if X, Y are both even-even nuclei in their ground states, shell model predicts both have $J^P = 0^+ \Rightarrow L_\alpha = 0$.

More generally, if X has $J^P = 0^+$, the states of Y which can be formed in α decay are $J^P = 0^+, 1^-, 2^+, 3^-, 4^+, \dots$

β Decay



- β decay is a weak interaction mediated by the W boson.
- Parity is violated in β decay.
- Responsible for Fermi postulating the existence of the neutrino.
- Kinematics: Decay is possible if energy release $E_0 > 0$

Nuclear Masses

$$\beta^- \quad E_0 = m_X - m_Y - m_e - m_\nu$$

$$\beta^+ \quad E_0 = m_X - m_Y - m_e - m_\nu$$

$$\text{e.c.} \quad E_0 = m_X - m_Y + m_e - m_\nu$$

Atomic Masses

$$E_0 = M_X - M_Y - m_\nu$$

$$E_0 = M_X - M_Y - 2m_e - m_\nu$$

$$E_0 = M_X - M_Y - m_\nu$$

(and note that $m_\nu \sim 0$)

using $M(A, Z) = m(A, Z) + Zm_e$

n.b. electron capture may be possible even if β^+ not allowed

β Decay Nuclear stability against β decay

Consider nuclear mass as a function of N and Z

$$m(A, Z) = Zm_p + (A - Z)m_n - a_V A + a_S A^{2/3} + \frac{a_C Z^2}{A^{1/3}} + a_A \frac{(N - Z)^2}{A} - \delta(A)$$

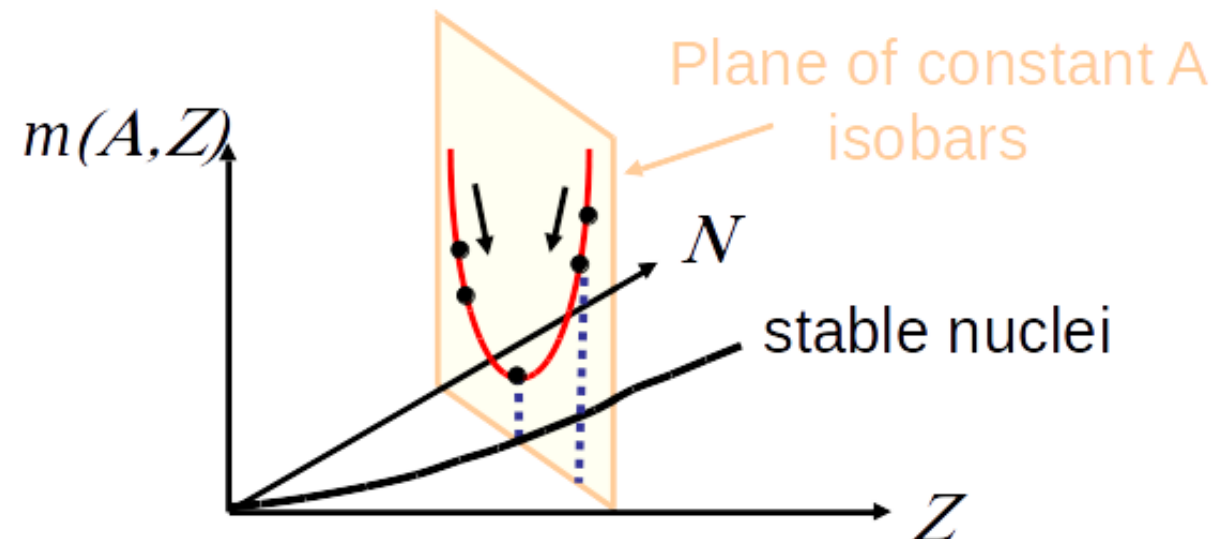
using SEMF

For β decay, A is constant,

but Z changes by ± 1 and $m(A, Z)$ is quadratic in Z

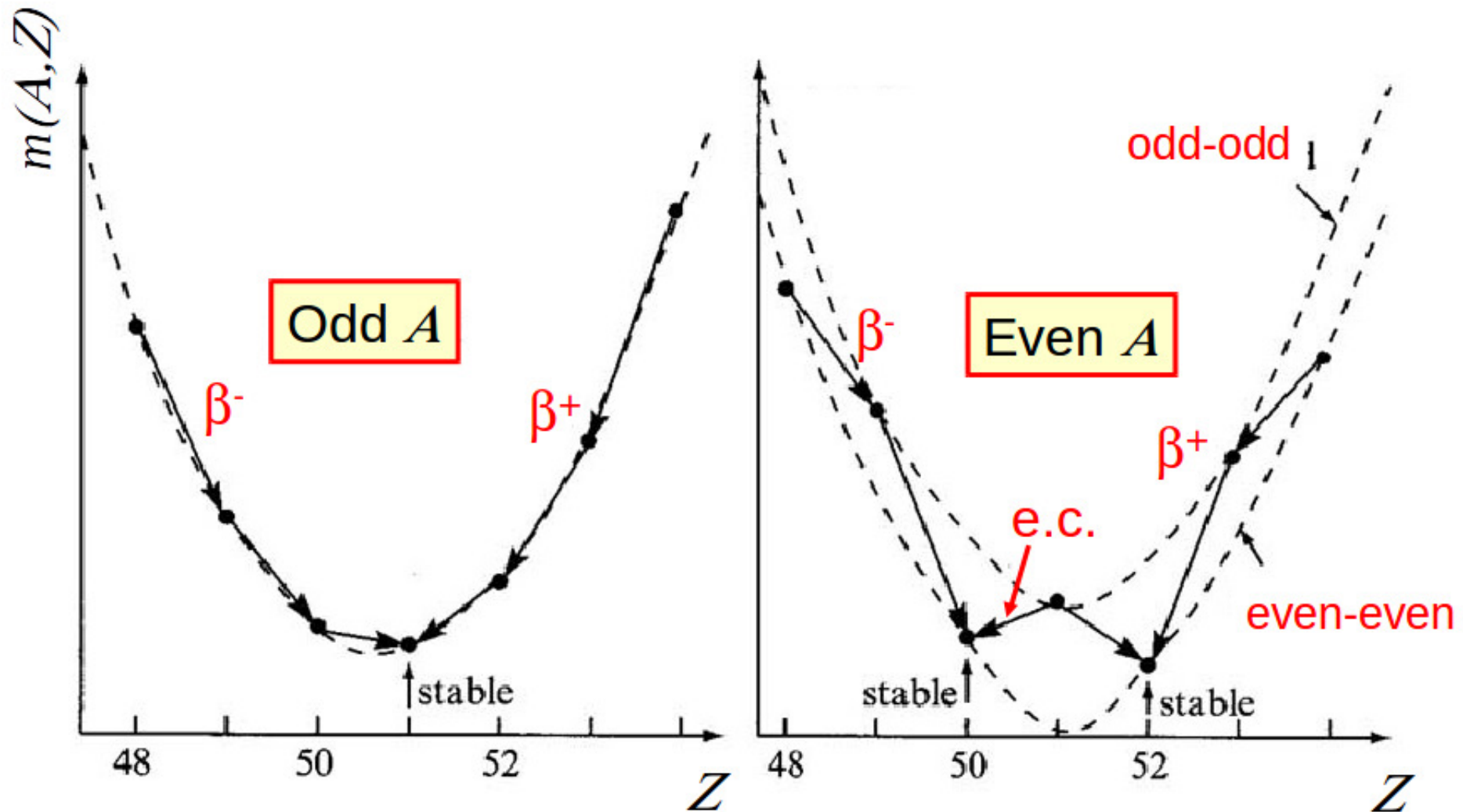
Most stable nuclide when

$$\left[\frac{\partial m(A, Z)}{\partial Z} \right]_A = 0$$



β Decay

Typical situation at constant A

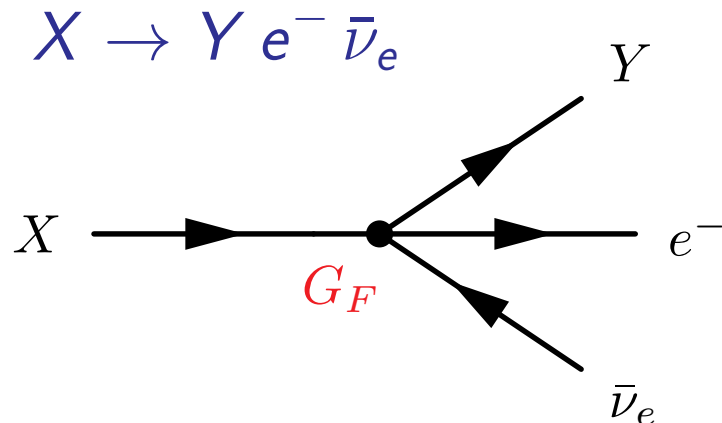


Usually only one isotope stable against β -decay; occasionally two.

Typically two even-even nuclides are stable against β -decay; almost no odd-odd ones (pairing term).

Fermi Theory of β -decay

In nuclear decay, weak interaction taken to be a **4-fermion contact interaction**:



No “propagator” – absorb the effect of the exchanged W boson into an effective coupling strength given by the **Fermi constant**
 $G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$.

Use Fermi’s Golden Rule to get the transition rate $\Gamma = 2\pi |M_{\text{fi}}|^2 \rho(E_f)$

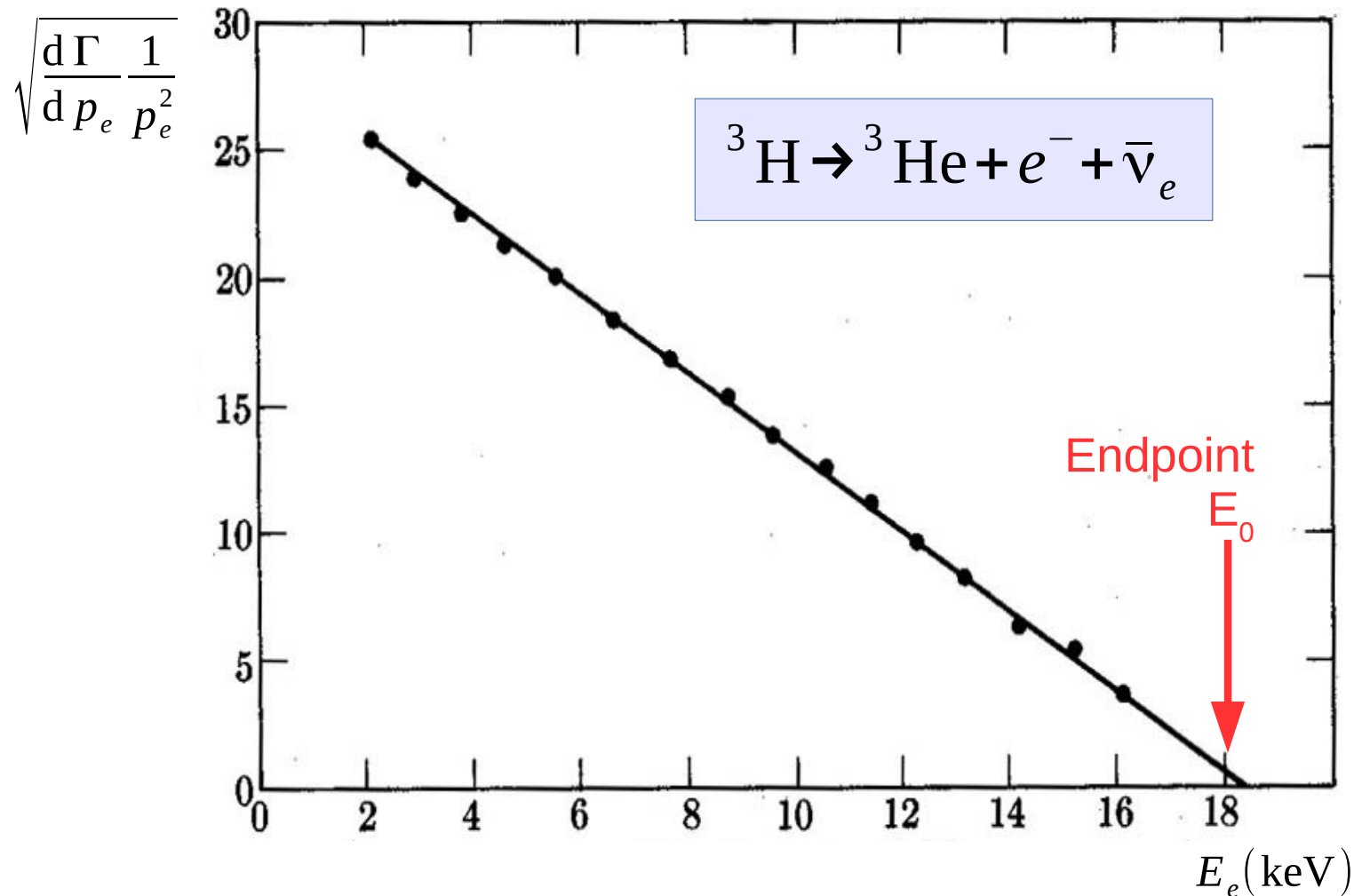
where M_{fi} is the matrix element and $\rho(E_f) = \frac{dN}{dE_f}$ is the density of final states.

$$\Gamma = \frac{G_F^2 |M_{\text{nuclear}}|^2}{2\pi^3} \int_0^{E_0} (E_0 - E_e)^2 E_e^2 dE_e$$

Total decay rate given by Sargent’s Rule, $\Gamma \propto E_0^5$

Fermi Theory of β -decay

β decay spectrum described by $\sqrt{\frac{d\Gamma}{dp_e} \frac{1}{p_e^2}} \propto (E_0 - E_e)$ **Kurie Plot**



Fermi Theory of β -decay

BUT, the momentum of the electron is modified by the Coulomb interaction as it moves away from the nucleus (different for e^- and e^+).

⇒ Multiply spectrum by **Fermi function** $F(Z_Y, E_e)$

$$\Gamma = \frac{G_F^2 |M_{\text{nuclear}}|^2}{2\pi^3} \int_0^{E_0} (E_0 - E_e)^2 E_e^2 F(Z_Y, E_e) dE_e$$

All the information about the nuclear wavefunctions is contained in the matrix element. Values for the complicated **Fermi Integral** are tabulated.

$$f(Z_Y, E_0) = \frac{1}{m_e^5} \int_0^{E_0} (E_0 - E_e)^2 E_e^2 F(Z_Y, E_e) dE_e$$

Mean lifetime $\tau = 1/\Gamma$, half-life $\tau_{1/2} = \frac{\ln 2}{\Gamma}$

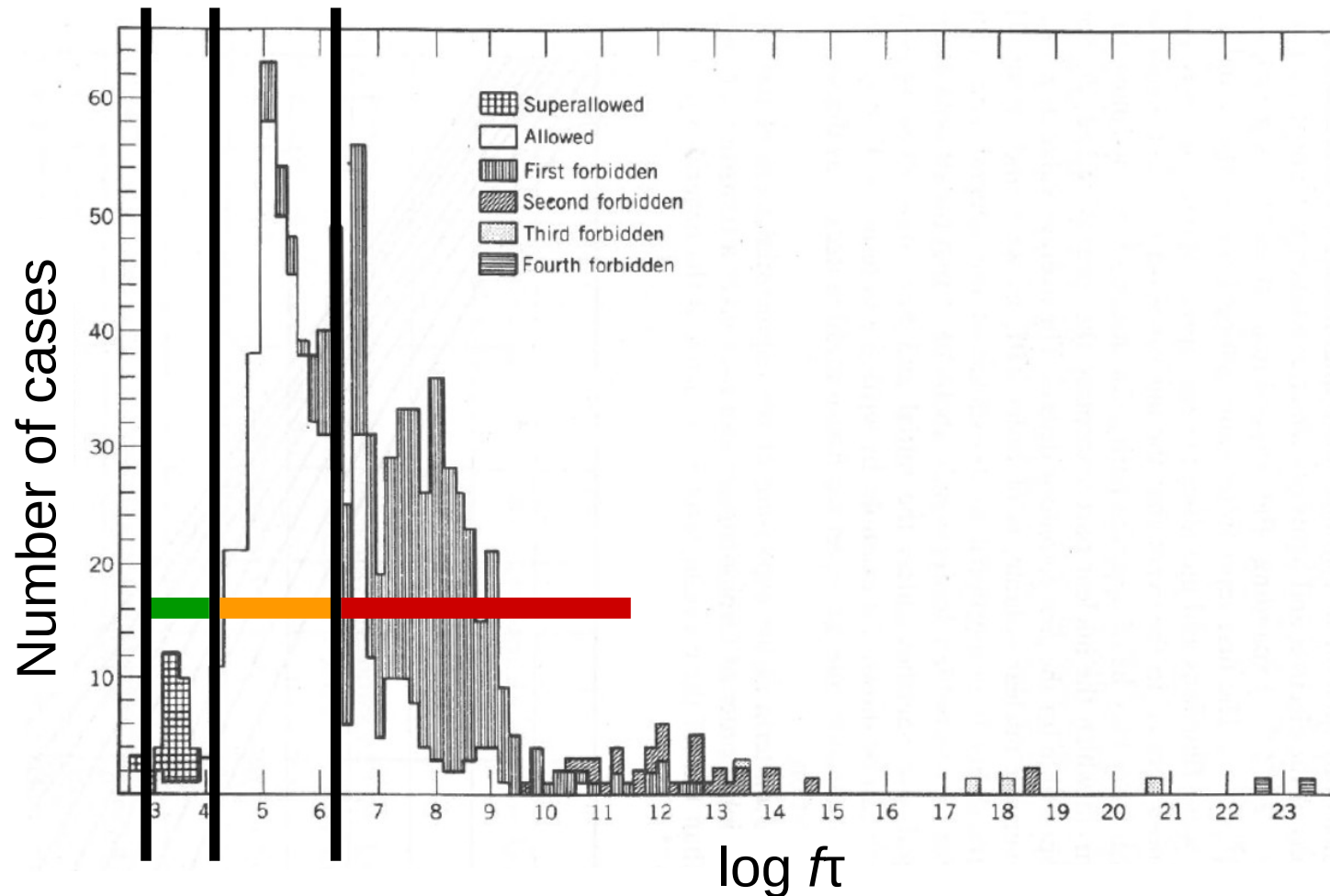
$$f \tau_{1/2} = \ln 2 \frac{2\pi^3}{m_e^5 G_F^2 |M_{\text{nuclear}}|^2}$$

Comparative half-life

this is rather useful because it depends only on the nuclear matrix element

Fermi Theory of β -decay

Comparative half-lives



In rough terms, decays with

$\log f\tau_{1/2} \sim 3 - 4$ known as **super-allowed**

$\sim 4 - 7$ known as **allowed**

≥ 6 known as **forbidden** (i.e. suppressed, small M_{if})

Fermi Theory of β -decay

Selection Rules

Fermi theory

$$M_{fi} = G_F \int \psi_p^* e^{-i(\vec{p}_e + \vec{p}_\nu) \cdot \vec{r}} \psi_n d^3\vec{r}$$

e, ν wavefunctions

Allowed Transitions $\log_{10} f \tau_{1/2} \sim 4 - 7$

Angular momentum of $e\nu$ pair relative to nucleus, $L = 0$.

Equivalent to: $e^{-i(\vec{p}_e + \vec{p}_\nu) \cdot \vec{r}} \sim 1$

Superaligned Transitions $\log_{10} f \tau_{1/2} \sim 3 - 4$

subset of Allowed transitions: often **mirror nuclei** in which p and n have approximately the same wavefunction

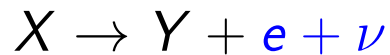
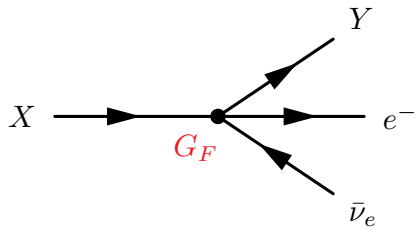
$$M_{\text{nuclear}} \sim \int \psi_p^* \psi_n d^3\vec{r} \sim 1$$

e, ν both have spin $1/2 \Rightarrow$ **Total spin of $e\nu$ system can be $S_{e\nu} = 0$ or 1 .**

There are **two** types of **allowed/superaligned** transitions depending on the relative spin states of the emitted e and ν ...

Fermi Theory of β -decay *Selection Rules*

For allowed/superaligned transitions, $L_{e\nu} = 0$



$$J_X = J_Y \oplus S_{e\nu} \oplus L_{e\nu}$$

e.g. $n \rightarrow pe^- \bar{\nu}_e$
 4 spin states of $e\nu$
 (3 G-T, 1 Fermi)

$S_{e\nu} = 0$ Fermi transitions

$$n \uparrow \rightarrow p \uparrow + \frac{1}{\sqrt{2}} [(e^- \uparrow \bar{\nu}_e \downarrow) - (e^- \downarrow \bar{\nu}_e \uparrow)] \quad \Delta J = 0$$

$$S_{e\nu} = 0, m_s = 0 \quad J_X = J_Y$$

$S_{e\nu} = 1$ Gamow-Teller transitions

$$n \uparrow \rightarrow p \uparrow + \frac{1}{\sqrt{2}} [(e^- \uparrow \bar{\nu}_e \downarrow) + (e^- \downarrow \bar{\nu}_e \uparrow)] \quad \Delta J = 0$$

$$S_{e\nu} = 1, m_s = 0 \quad 0 \rightarrow 0 \text{ forbidden} \quad J_X = J_Y$$

$$n \uparrow \rightarrow p \downarrow + e^- \uparrow + \bar{\nu}_e \uparrow \quad \Delta J = \pm 1$$

$$S_{e\nu} = 1, m_s = \pm 1 \quad J_X = J_Y \pm 1$$

No change in angular momentum of the $e\nu$ pair relative to the nucleus, $L_{e\nu} = 0$
 $\Rightarrow$ Parity of nucleus unchanged

Fermi Theory of β -decay

Selection Rules

Forbidden Transitions $\log_{10} f \tau_{1/2} \geq 6$

Angular momentum of $e\nu$ pair relative to nucleus, $L_{e\nu} > 0$.

$$e^{-i(\vec{p}_e + \vec{p}_\nu) \cdot \vec{r}} = 1 - i(\vec{p}_e + \vec{p}_\nu) \cdot \vec{r} + \frac{1}{2} [(\vec{p}_e + \vec{p}_\nu) \cdot \vec{r}]^2 - \dots$$

$L =$	0	1	2
$P = (-1)^L =$	even	odd	even
	Allowed	1 st forbidden	2 nd forbidden

Transition probabilities for $L > 0$ are small $\Rightarrow$ **forbidden transitions** (really means “suppressed”).

Forbidden transitions are only competitive if an allowed transition cannot occur (selection rules). Then the lowest permitted order of “forbiddenness” will dominate.

In general, n^{th} **forbidden** $\Rightarrow$ $e\nu$ system carries orbital angular momentum $L = n$, and $S_{e\nu} = 0$ (Fermi) or 1 (G-T). Parity change if L is odd.

Fermi Theory of β -decay

Selection Rules

Examples

$$^{34}\text{Cl}(0^+) \rightarrow ^{34}\text{S}(0^+)$$

$$^{14}\text{C}(0^+) \rightarrow ^{14}\text{N}(1^+)$$

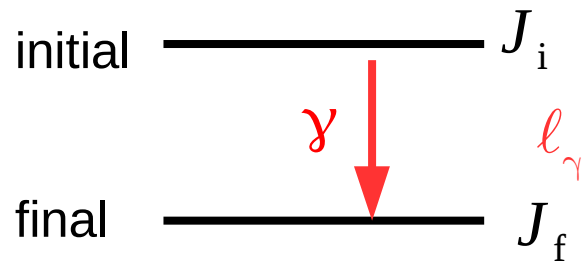
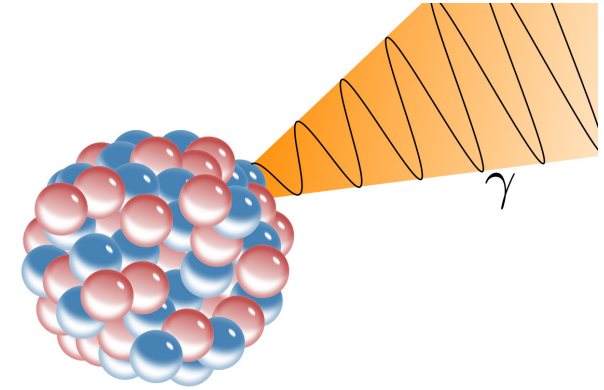
$$n(1/2^+) \rightarrow p(1/2^+)$$

$$^{39}\text{Ar}(7/2^-) \rightarrow ^{39}\text{K}(3/2^+)$$

$$^{87}\text{Rb}(3/2^-) \rightarrow ^{87}\text{Sr}(9/2^+)$$

γ Decay

Emission of γ -rays (EM radiation) occurs when a nucleus is created in an excited state (e.g. following α , β decay or collision).



The photon carries away net angular momentum L_γ when a proton in the nucleus makes a transition from its initial a.m. state J_i to its final a.m. state J_f .

$$\vec{J}_i = \vec{L}_\gamma \oplus \vec{J}_f \quad \text{and} \quad |\vec{J}_i - \vec{J}_f| \leq L_\gamma \leq |\vec{J}_i + \vec{J}_f|$$

The photon carries $J^P = 1^- \Rightarrow L_\gamma \geq 1$.

$\Rightarrow$ Single γ emission is **forbidden** for a transition between two $J = 0$ states.

($0 \rightarrow 0$ transitions can only occur via internal conversion (emitting an electron) or via the emission of more than one γ .)

γ Decay

Radiative transitions in nuclei are generally the same as for atoms, except

Atom $E_\gamma \sim \text{eV}$; $\lambda \sim 10^8 \text{ fm} \sim 10^3 \times r_{\text{atom}}$; $\Gamma \sim 10^9 \text{ s}^{-1}$

Only dipole transitions are important.

Nuclei $E_\gamma \sim \text{MeV}$; $\lambda \sim 10^2 \text{ fm} \sim 25 \times r_{\text{nuc}}$; $\Gamma \sim 10^{16} \text{ s}^{-1}$

Collective motion of many protons lead to higher transition rates.

$\Rightarrow$ Higher order transitions are also important.

Two types of transitions:

Electric (E) transitions arise from an oscillating charge which causes an oscillation in the external electric field.

Magnetic (M) transitions arise from a varying current or magnetic moment which sets up a varying magnetic field.

Obtain transition probabilities using Fermi's Golden Rule

$$\Gamma = 2\pi |M_{if}|^2 \rho(E_f)$$

γ Decay *Electric Dipole Transitions (E1) $L = 1$*

Insert dipole matrix element into FGR

$$\Gamma_{i \rightarrow f} = \frac{\omega^3}{3\pi\epsilon_0 c^3 \hbar} | \langle \psi_f | e\vec{r} | \psi_i \rangle |^2$$

see Adv. Quantum Physics; after averaging over initial and summing over final states

Order of magnitude estimate of this rate,

$$| \langle \psi_f | e\vec{r} | \psi_i \rangle |^2 \sim |eR|^2 \Rightarrow \Gamma \sim \frac{4}{3} \alpha E_\gamma^3 R^2$$

R = radius of nucleus,
 $\alpha = \frac{e^2}{4\pi\epsilon_0 c \hbar}$, $E_\gamma = \hbar\omega$, $\hbar = c = 1$.

e.g. $E_\gamma = 1 \text{ MeV}$, $R = 5 \text{ fm}$ ($\hbar c = 197 \text{ MeVfm}$, $\hbar = 6.6 \times 10^{-22} \text{ MeVs}$)

$$\Gamma(E1) = 0.24 \text{ MeV}^3 \text{fm}^2 = \frac{0.24}{(197)^2 \times 6.6 \times 10^{-22}} \text{ s}^{-1} = 10^{16} \text{ s}^{-1} \quad (\text{c.f. atoms } \Gamma \sim 10^9 \text{ s}^{-1})$$

As nuclear wavefunctions have definite parity, the matrix element can only be non-zero if the initial and final states have opposite parity.

$$e\vec{r} \xrightarrow{\hat{P}} -e\vec{r} \quad \text{ODD}$$

E1 transition $\Rightarrow$ parity change of nucleus

γ Decay *Magnetic Dipole Transitions (M1) $L = 1$*

Magnetic dipole matrix element $|\langle \psi_f | \mu \vec{\sigma} | \psi_i \rangle|^2$

μ = magnetic moment, $\vec{\sigma}$ = Pauli spin matrices

Typically $\langle \mu \sigma \rangle \sim \frac{e\hbar}{2m_p} = \mu_N$ Nuclear magneton

For a proton $\frac{\hbar}{m_p} \sim 0.2 \text{ fm} \sim \frac{R}{25}$ for $R = 5 \text{ fm}$

Compare to E1 transition rate $\frac{\Gamma(M1)}{\Gamma(E1)} = \left(\frac{e\hbar}{2m_p} \right)^2 \frac{1}{(eR)^2} = 10^{-3}$

Magnetic moment transforms the same way as angular momentum

$$e\vec{r} \times \vec{p} \xrightarrow{\hat{P}} e(-\vec{r}) \times (-\vec{p}) = e\vec{r} \times \vec{p} \quad \text{EVEN}$$

M1 transition $\Rightarrow$ no parity change of nucleus

γ Decay Higher Order Transitions (EL, ML , where $L > 1$)

If the initial and final nuclear states differ by more than 1 unit of angular momentum

$\Rightarrow$ higher multipole radiation

The perturbing Hamiltonian is a function of electric and magnetic fields and hence of the vector potential $\langle \psi_f | H'(\vec{A}) | \psi_i \rangle$

$\vec{A}$ for a photon is taken to have the form of a plane wave

$$\vec{A} e^{i\vec{p} \cdot \vec{r}} = 1 - i\vec{p} \cdot \vec{r} + \frac{1}{2}(\vec{p} \cdot \vec{r})^2 + \dots \frac{(-i\vec{p} \cdot \vec{r})^n}{n!}$$

	Dipole	Quadrupole	Octupole
$L =$	1	2	3
	E1, M1	E2, M2	E3, M3

Each successive term in the expansion of $\vec{A}$ is reduced from the previous one by a factor of roughly $\vec{p} \cdot \vec{r}$.

e.g. Compare E1 to E2 for $p \sim 1$ MeV, $R \sim 5$ fm
 $\Rightarrow pR \sim 5$ MeVfm ~ 0.025 , $|pR|^2 \sim 10^{-3}$

$$\frac{\Gamma(E2)}{\Gamma(E1)} \sim 10^{-3} \sim \frac{\Gamma(M1)}{\Gamma(E1)}$$

The matrix element for E2 transitions $\sim r^2$ i.e. even under a parity transformation.

γ Decay Transitions

In general, EL transitions $\text{Parity} = (-1)^L$
 ML transitions $\text{Parity} = (-1)^{L+1}$

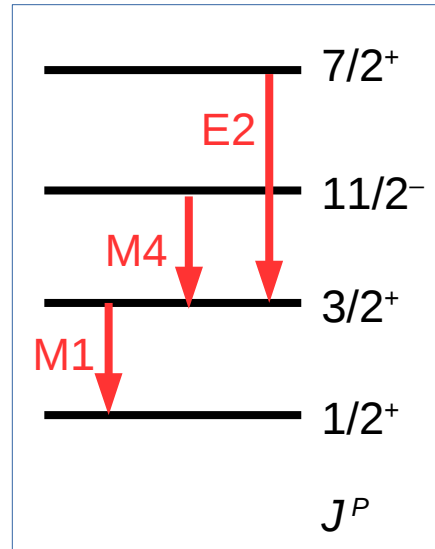
	Rate	1	10^{-3}	10^{-6}	10^{-9} ...
		E1	E2	E3	E4 ...
			M1	M2	M3 ...
Parity change		✓	✗	✓	✗
J^P of γ E:		1^-	2^+	3^-	4^+
M:			1^+	2^-	3^+

In general, a decay will proceed dominantly by the lowest order (i.e. fastest) process permitted by angular momentum and parity.

e.g. if a process has $\Delta J = 2$, no parity change, it will go by the E2, even though M3, E4 are also allowed.

γ Decay Transitions

e.g. $^{117}_{50}\text{Sn}$



3/2⁺ → 1/2⁺ **M1** (E2 also allowed)

11/2⁻ → 3/2⁺ **M4**
More likely than 11/2⁻ → 1/2⁺ (E5)

7/2⁺ → 3/2⁺ **E2**
 M2 } less likely 7/2⁺ → 11/2⁻
 M3 } 7/2⁺ → 1/2⁺

Information about the nature of transitions (based on rates and angular distributions) is very useful in inferring the J^P values of states.

Please note: this discussion of rates is fairly naïve. More complete formulae can be found in textbooks.

Also collective effects may be important if

- many nucleons participate in transitions,
- nucleus has a large electric quadrupole moment, Q , → rotational excited states enhance E2 transitions.

Summary

- Radioactive decays and dating.
- α -decay Strong dependence on E , Z
Tunnelling model (Gamow) – Geiger-Nuttall law $\ln \tau_{1/2} \sim \frac{Z'}{E_0^{1/2}} + \text{const.}$
- β -decay β^+ , β^- , electron capture; energetics, stability
Fermi theory – 4-fermion interaction plus 3-body phase space.
$$\Gamma = \frac{G_F^2 |M_{\text{nuclear}}|^2}{2\pi^3} \int_0^{E_0} (E_0 - E_e)^2 p_e^2 dp_e$$

Electron energy spectrum; Kurie plot.
Comparative half-lives.
Selection rules; Fermi, Gamow-Teller; allowed, forbidden.
- γ -decay Dipole, quadrupole; electric, magnetic transitions.
Selection rules.

Problem Sheet: q.37-41

Up next... Section 16: Fission and Fusion