

# Flavour Physics Problem Sheet Solutions

Cambridge HEP Graduate Lectures

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## Lecture 1

1. Estimate the ratio of the  $p + p \rightarrow d + \pi^+$  and  $p + n \rightarrow d + \pi^0$  cross sections (see <http://hep.physics.wayne.edu/harr/courses/7060/w03/lecture36.htm>)

- a) Have to figure out the isospin state of the deuteron. It is a bound state of  $p$  and  $n$ . Think of all possible bound states of  $p = |1/2, +1/2\rangle$  and  $n = |1/2, -1/2\rangle$  and you get

$$|1, 1\rangle = |pp\rangle \quad (1)$$

$$|1, 0\rangle = \frac{|pn\rangle + |np\rangle}{\sqrt{2}} \quad (2)$$

$$|1, -1\rangle = |nn\rangle \quad (3)$$

$$|0, 0\rangle = \frac{|pn\rangle - |np\rangle}{\sqrt{2}} \quad (4)$$

- b) The first three of these are in an isospin triplet, the last one an isospin singlet. There are no  $pp$  or  $nn$  states found in nature so turns out the deuteron is the isospin singlet,  $|0, 0\rangle$ .
  - c) Now we look at the isospin structure of the two decays.  $pp \rightarrow d\pi^+$  has initial state which can only be pure  $I = 1$ , whereas  $pn \rightarrow d\pi^0$  has an initial state that can be (in an admixture of)  $I = 0$  and  $I = 1$ . The final state in both cases is  $I = 1$ .
  - d) Thus we expect the relative cross section to be approx half.
2. What is the quark content of the ground state spin-0 mesons ( $K^\pm$ ,  $\bar{K}^0$ ,  $\pi^\pm$ ,  $\pi^0$ ,  $\eta$ ,  $\eta'$ ) and the ground state spin-1/2 baryons ( $n$ ,  $p$ ,  $\Sigma^\pm$ ,  $\Sigma^0$ ,  $\Lambda^0$ ,  $\Xi^0$ ,  $\Xi^-$ )
  3. What are the equivalent excited spin-1 and spin-3/2 states?
  4. Show that the CKM matrix has  $N(N-1)/2$  real parameters and  $(N-1)(N-2)/2$  phases for the case of  $N$  fermion generations.

5. Derive the Wolfenstein parameterisation
  - a) First define expansion parameter  $\lambda := V_{us}$  and derive  $c_{13} \approx 1$
  - b) Next write down the standard parameterisation of the CKM-matrix expressing  $s_{12}$  and  $c_{12}$  in terms of  $\lambda$  (include terms up to  $\mathcal{O}(\lambda^3)$ )
  - c) The experimental data suggested that  $1 \approx V_{ud} > V_{us} > V_{cb} > V_{ub}$ . Apply the ansatz  $V_{cb} := A\lambda^2$  and  $V_{ub} := A\lambda^3(\rho - i\eta)$  and express the CKM matrix in terms of  $\lambda$  up to  $\mathcal{O}(\lambda^3)$
6. Show that CKM matrix is unitary if written in the Wolfenstein parameterisation including terms up to  $\mathcal{O}(\lambda^4)$
7. Why is it that down-type neutral mesons contain the anti-quark species but the up-type species contain the quark? In other words why is the state  $(\bar{s}, d)$  (with an anti- $s$ -quark) labelled as  $K^0$  and the state  $(s, \bar{d})$  (with an  $s$ -quark) labelled as  $\bar{K}^0$  whereas the state  $(c, \bar{u})$  is labelled as  $D^0$  and the state  $(\bar{c}, u)$  labelled as  $\bar{D}^0$ ?

**SOLUTION:** It's essentially because of the relative charge difference between down quarks and up quarks and the relationship between charge and quantum numbers. The convention we use is that the sign of the flavour quantum numbers agree with the sign of the electric charge carried by the quarks of the corresponding flavour. Hence an  $s$  has strangeness quantum number,  $S = -1$ , whereas a  $c$  has charmness quantum number  $C = 1$ . We know from the Gell-Mann-Nishijima formula that electric charge is

$$Q = I_3 + Y/2, \quad (5)$$

where  $I_3$  is the third component of isospin and  $Y$  is the hypercharge, itself defined by

$$Y = B + S + C + B' + T', \quad (6)$$

in other words the sum of Baryon number,  $B$ , strangeness,  $S$ , charmness,  $C$ , bottomness  $B'$  and topness  $T'$ .

## Lecture 2

1. Why is it easier for theorists to make predictions for so-called “inclusive” processes, such as  $b \rightarrow c\bar{c}s$ ? Why is it easier for experimentalists to measure so-called “exclusive” modes in which all final states hadrons are identified, such as  $B^0 \rightarrow D^+D^-$ ? How could an experimentalist make an “inclusive” measurement? How could a theorist make an “exclusive” mode prediction?
- 2.
- 3.
4. Derivation

5. The argument of the trigonometric functions in the mixing equations is  $\Delta Mt/2$  thus an oscillation occurs where

$$\frac{\Delta mt}{2} = \pi \implies t = \frac{2\pi}{\Delta m} \quad (7)$$

Thus the length is

$$x = vt' = \beta\gamma ct = \beta\gamma c \frac{2\pi}{\Delta m} \quad (8)$$