Flavour Physics Problem Sheet

Cambridge HEP Graduate Lectures

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Lecture 1

1. Estimate the ratio of the $p + p \to d + \pi^+$ and $p + n \to d + \pi^0$ cross sections
   a) Note the deuteron is a bound state of a proton, p, and a neutron, n so think about all the possible isospin configurations of two nucleons.
   b) Identify which isospin state is that of the deuteron (using the observation that bound states of two protons and two neutrons are not found in nature)
   c) Now consider the total isospin in the intial and final states of the two reactions above
   d) Now you can approximate the relative cross section

2. What is the quark content of the ground state spin-0 mesons ($K^\pm$, $\overset{(-)}{K}{}^0$, $\pi^\pm$, π^0, η, η') and the ground state spin-1/2 baryons (n, p, $\Sigma^\pm$, Σ^0, Λ^0, Ξ^0, Ξ^-)

3. What are the equivalent excited spin-1 and spin-3/2 states?

4. Show that the CKM matrix has $N(N-1)/2$ real parameters and $(N-1)(N-2)/2$ phases for the case of N fermion generations.

5. Derive the Wolfenstein parameterisation
   a) First define expansion parameter $\lambda := V_{us}$ and derive $c_{13} \approx 1$
   b) Next write down the standard parameterisation of the CKM-matrix expressing s_{12} and c_{12} in terms of λ (include terms up to $\mathcal{O}(\lambda^3)$
   c) The experimental data suggested that $1 \approx V_{ud} > V_{us} > V_{cb} > V_{ub}$. Apply the ansatz $V_{cb} := A\lambda^2$ and $V_{ub} := A\lambda^3(\rho - i\eta)$ and express the CKM matrix in terms of λ up to $\mathcal{O}(\lambda^3)$

6. Show that CKM matrix is unitary if written in the Wolfenstein parameterisation including terms up to $\mathcal{O}(\lambda^4)$

7. Why is it that down-type neutral mesons contain the anti-quark species but the up-type species contain the quark? In other words why is the state $(\overline{s}, d)$ (with an anti-s-quark) labelled as K^0 and the state $(s, \overline{d})$ (with an s-quark) labelled as $\overline{K}^0$ whereas the state $(c, \overline{u})$ is labelled as D^0 and the state $(\overline{c}, u)$ labelled as $\overline{D}^0$?

Lecture 2

1. Why is it easier for theorists to make predictions for so-called "inclusive" processes, such as $b \to c\overline{c}s$? Why is it easier for experimentalists to measure so-called "exclusive" modes in which all final states hadrons are identificed, such as $B^0 \to D^+D^-$? How could an experimentalist make an "inclusive" measurement? How could a theorist make an "exclusive" mode prediction?

2. Draw the leading order Feynman diagrams for the following decays
   a) $B^+ \to \tau^+\nu_\tau$
   b) $B^+ \to \overline{D}^0\mu^+\nu_\mu$
   c) $B^- \to D^0K^-$ (via both the favoured V_{cb} and the suppressed V_{ub})
   d) $B_s^0 \to J/\psi\phi$ and $\overline{B}_s^0 \to J/\psi\phi$ (note that the J/ψ is a $c\overline{c}$ state and the ϕ is a $s\overline{s}$ state)
   e) $B^0 \to J/\psi K_S^0$ and $\overline{B}^0 \to J/\psi K_S^0$

3. Compare the Feynman diagrams for $\overline{B}^0 \to D^+\pi^-$ and $\overline{B}^0 \to \pi^+\pi^-$ and make a naive prediction of their ratio of branching fractions

$$\frac{\mathcal{B}(\overline{B}^0 \to D^+\pi^-)}{\mathcal{B}(\overline{B}^0 \to \pi^+\pi^-)} = ? \tag{1}$$

4. Derive the equations for the time-evolution of a pure flavour state of a neutral meson at time $t = 0$
   a) First show that when diagonisling the neutral meson mixing matrix from the flavour basis into the mass basis that the following relations hold

$$\Delta M^2 - \frac{1}{4}\Delta\Gamma^2 = 4|M_{12}|^2 - |\Gamma_{12}|^2 \tag{2}$$

$$\Delta M \cdot \Delta\Gamma = -4\Re(M_{12}\Gamma_{12}^*) \tag{3}$$

$$\frac{q}{p} = -\frac{\Delta M + \frac{i}{2}\Delta\Gamma}{2M_{12} - \Gamma_{12}} \tag{4}$$

b) Solve these for the mass and decay rate difference to give

$$2\Delta M^2 = \sqrt{(4|M_{12}|^2 - |\Gamma_{12}|^2)^2 + 16|M_{12}|^2|\Gamma_{12}|^2\cos^2\phi_{12}} + 4|M_{12}|^2 - |\Gamma_{12}|^2 \tag{5}$$

$$\frac{1}{2}\Delta\Gamma^2 = \sqrt{(4|M_{12}|^2 - |\Gamma_{12}|^2)^2 + 16|M_{12}|^2|\Gamma_{12}|^2\cos^2\phi_{12}} - 4|M_{12}|^2 + |\Gamma_{12}|^2 \tag{6}$$

with $\phi_{12} = \arg(-M_{12}/\Gamma_{12})$

c) Now use the time-dependent Schrodinger equation to show how the flavour states evolve with time, *i.e.* that

$$\begin{aligned} |B^0(t)\rangle &= g_+(t)|B^0\rangle + \frac{q}{p}g_-(t)|\overline{B}^0\rangle \\ |\overline{B}^0(t)\rangle &= g_+(t)|\overline{B}^0\rangle + \frac{p}{q}g_-(t)|B^0\rangle \end{aligned} \tag{7}$$

with

$$\begin{aligned} g_+(t) &= e^{-iMt}e^{-\Gamma t/2}\left[\ \ \cosh\left(\frac{\Delta\Gamma t}{4}\right)\cos\left(\frac{\Delta m t}{2}\right) - i\sinh\left(\frac{\Delta\Gamma t}{4}\right)\sin\left(\frac{\Delta m t}{2}\right)\right] \\ g_-(t) &= e^{-iMt}e^{-\Gamma t/2}\left[-\sinh\left(\frac{\Delta\Gamma t}{4}\right)\cos\left(\frac{\Delta m t}{2}\right) + i\cosh\left(\frac{\Delta\Gamma t}{4}\right)\sin\left(\frac{\Delta m t}{2}\right)\right] \end{aligned} \tag{8}$$

where $M = (M_L + M_H)/2$ and $\Gamma = (\Gamma_L + \Gamma_H)/2$.

d) An alternative (perhaps easier) approach is to write the flavour states in terms of the mass states

$$|B^0\rangle = \frac{1}{2p}\left(|B^0_H\rangle + |B^0_L\rangle\right) \tag{9}$$

$$|\overline{B}^0\rangle = \frac{1}{2q}\left(|B^0_H\rangle - |B^0_L\rangle\right) \tag{10}$$

and then propagate the flavour states in time given the straightforward propagation of the mass states

$$|B^0_H(t)\rangle = e^{-iM_H t}e^{-\Gamma_H t/2}|B^0_H\rangle \tag{11}$$

$$|B^0_L(t)\rangle = e^{-iM_L t}e^{-\Gamma_L t/2}|B^0_L\rangle \tag{12}$$

5. Show that the probability to oscillate (+) or not (−) is given by

$$|g_\pm(t)|^2 = \frac{e^{-\Gamma t}}{2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) \pm \cos(\Delta m t)\right] \tag{13}$$

6. Write down an expression for the oscillation length of a neutral meson and estimate its value for K^0, D^0, B^0, B^0_s at LHCb

7. Explain the experimental vs theoretical mixing plot
8. Derive the equation for the time-integrated CP asymmetry for two amplitudes with different strong and weak phases
9. Derive the “master equations” for neutral meson decay rates