

# Flavour Physics (of quarks)

## Part 3: Measuring the CKM parameters

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**Cavendish HEP Graduate Lectures**

February 2024

## Lecture 1: Flavour in the SM

- ▶ Flavour in the SM
- ▶ Quark Model History
- ▶ The CKM matrix

## Lecture 2: Mixing and $CP$ violation

- ▶ Neutral Meson Mixing (no CPV)
- ▶  $B$ -meson production and experiments
- ▶  $CP$  violation

## Lecture 3: Measuring the CKM parameters (Today)

- ▶ Measuring CKM elements and phases
- ▶ Global CKM fits
- ▶  $CPT$  and  $T$ -reversal
- ▶ Dipole moments

## Lecture 4: Flavour Changing Neutral Currents

- ▶ Effective Theories
- ▶ New Physics in  $B$  mixing
- ▶ New Physics in rare  $b \rightarrow s$  processes
- ▶ Lepton Flavour Violation

## 1. Recap

## Recap

- ▶ Last time we discussed neutral meson mixing and all three types of  $CPV$
- ▶ Saw the “master” equations for neutral meson decays which are characterised by

$$\lambda_f = \frac{q}{p} \frac{\bar{A}_f}{A_f}$$

- ▶  $CPV$  in decay (the only type possible for a charged initial state) [ $|\bar{A}_f/A_f| \neq 0$ ]
  - ▶  $CPV$  in mixing [ $|q/p| \neq 1$ ]
  - ▶  $CPV$  in the interference between mixing and decay [ $|\arg(\lambda_f)| \neq 0$ ]
- ▶ We got two important expressions which we will see again today
1. The direct (time-integrated)  $CP$  asymmetry arising when we have two amplitudes with different strong ( $\delta$ ) and weak ( $\phi$ ) phases and magnitude ratio ( $r$ ):

$$\mathcal{A}_{CP} = \frac{2r \sin(\delta) \sin(\phi)}{1 + r^2 + 2r \cos(\delta) \cos(\phi)} \quad (1)$$

2. The general time-dependent  $CP$  asymmetry for a neutral meson to a  $CP$ -eigenstate

$$\mathcal{A}_{CP}(t) = \frac{C_f \cos(\Delta mt) - S_f \sin(\Delta mt)}{\cosh(\frac{1}{2} \Delta \Gamma t) + D_f \sinh(\frac{1}{2} \Delta \Gamma t)} \quad (2)$$

where

$$C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \quad D_f = \frac{2\mathcal{R}e(\lambda_f)}{1 + |\lambda_f|^2}, \quad S_f = \frac{2\mathcal{I}m(\lambda_f)}{1 + |\lambda_f|^2} \quad (3)$$

# Recap

- ▶ Recall the CKM matrix which governs quark weak transitions

## CKM exhibits a clear hierarchy

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim \begin{pmatrix} 1 & 0.2 & 0.004 \\ 0.2 & 1 & 0.04 \\ 0.008 & 0.04 & 1 \end{pmatrix}$$

experimentally  
determined values

## Commonly represented in the Wolfenstein parametrisation

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

4  $\mathcal{O}(1)$  real parameters ( $A, \lambda, \rho, \eta$ )

- ▶ Wolfenstein parameterisation ensures that

$$\bar{\rho} + i\bar{\eta} = -(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*) \quad (4)$$

is phase convention independent and CKM matrix written in  $(A, \lambda, \bar{\rho}, \bar{\eta})$  is unitary to all orders in  $\lambda$

$$\bar{\rho} = \rho(1 - \lambda^2/2 + \dots) \quad \text{and} \quad \bar{\eta} = \eta(1 - \lambda^2/2 + \dots) \quad (5)$$

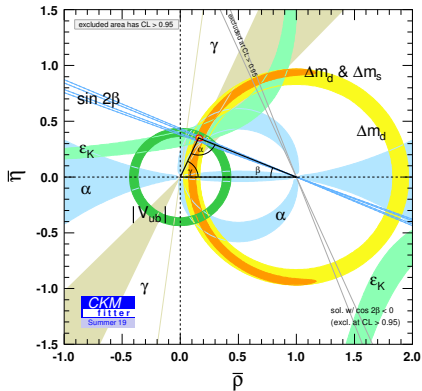
- ▶ The amount of  $CP$  violation in the SM is equivalent to asking how big is  $\eta$  relative to  $\rho$ .
- ▶ There are many experimental observables (9 element magnitudes and 4 phases) we can measure to over-constrain the CKM picture.

# CKM Unitarity Triangles

- ▶ Unitarity gives 6 constraints for off-diagonals represented as triangles in  $(\bar{\rho}, \bar{\eta})$  space
  - ▶ See [CKMfitter](#) or [UTfit](#) for the latest updates [1,2]

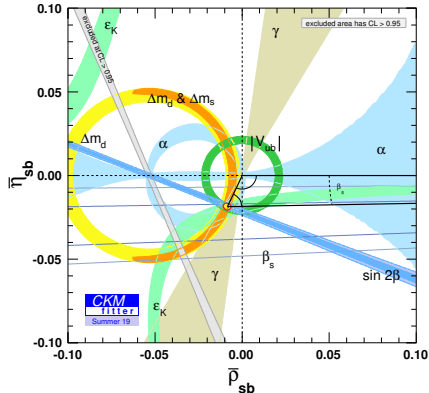
## The $(B^0)$ Unitarity Triangle

$$\bar{\rho}_{(db)} + i\bar{\eta}_{(db)} = -(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*)$$



## The $B_s^0$ Unitarity Triangle

$$\bar{\rho}_{sb} + i\bar{\eta}_{sb} = -(V_{us}V_{ub}^*)/(V_{cs}V_{cb}^*)$$



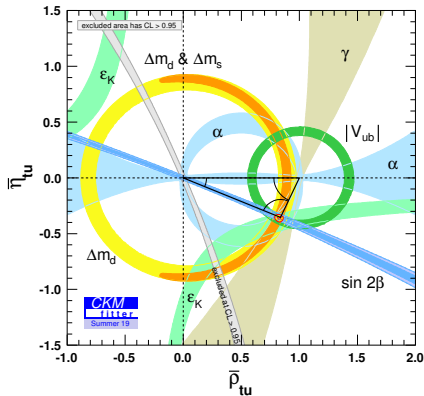


# CKM Unitarity Triangles

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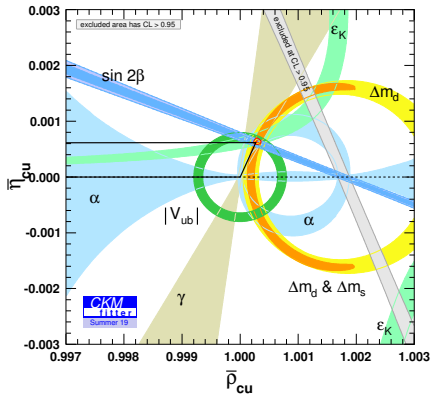
## The $tu$ Unitarity Triangle

$$\bar{\rho}_{(tu)} + i\bar{\eta}_{(tu)} = -(V_{td}V_{ud}^*)/(V_{ts}V_{us}^*)$$



## The $D$ Unitarity Triangle

$$\bar{\rho}_{cu} + i\bar{\eta}_{cu} = -(V_{cd}V_{ud}^*)/(V_{cs}V_{us}^*)$$



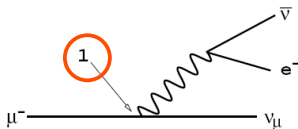
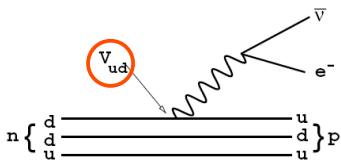
### 2. Measuring CKM matrix element magnitudes

# Measuring CKM matrix elements

## Measuring $V_{ud}$

- ▶ Compare rates of neutron,  $n^0$ , and muon,  $\mu^-$ , decays (see [PDG write-up](#)) [3]
- ▶ The ratio is proportional to  $|V_{ud}|^2$
- ▶  $|V_{ud}| = 0.947417 \pm 0.00021$
- ▶  $|V_{ud}| \approx 1$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



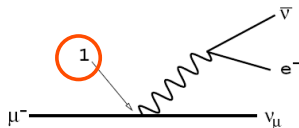
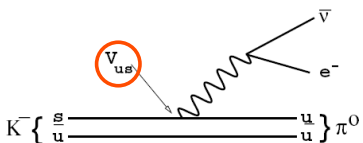
$$\frac{d\Gamma(n \rightarrow pe^- \bar{\nu}_e)}{dx_p} = \frac{G_F^2 m_n^2}{192\pi^2} |V_{ud}|^2 f(q^2)^2 \left( x_p^2 - 4 \frac{m_p^2}{m_n^2} \right)^{3/2}, \quad \text{where } x_p = \frac{2E_p}{m_n}$$

# Measuring CKM matrix elements

## Measuring $V_{us}$

- ▶ Compare rates of kaon,  $K^-$ , and muon,  $\mu^-$ , decays (see [PDG write-up](#)) [4]
- ▶ The ratio is proportional to  $|V_{us}|^2$
- ▶  $|V_{us}| = 0.2248 \pm 0.0006$
- ▶  $|V_{us}| \approx \sin(\theta_C) \approx \lambda$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



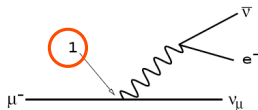
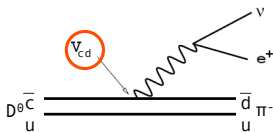
$$\frac{d\Gamma(\bar{K}^0 \rightarrow \pi^+ e^- \bar{\nu}_e)}{dx_\pi} = \frac{G_F^2 m_K^2}{192\pi^2} |V_{us}|^2 f(q^2)^2 \left(x_\pi^2 - 4 \frac{m_\pi^2}{m_K^2}\right)^{3/2}, \quad \text{where } x_\pi = \frac{2E_\pi}{m_K}$$

# Measuring CKM matrix elements

## Measuring $V_{cd}$ and $V_{cs}$

- ▶ Early measurements used neutrino DIS
- ▶ Now use semi-leptonic charm decays,  $D^0 \rightarrow \pi^- \ell^+ \nu_\ell$  ( $V_{cd}$ ) and  $D^0 \rightarrow K^- \ell^+ \nu_\ell$  ( $V_{cs}$ ) (see [PDG write-up](#)) [4]
- ▶  $|V_{cd}| = 0.220 \pm 0.005$
- ▶  $|V_{cs}| = 0.995 \pm 0.016$
- ▶  $|V_{cd}| \approx \sin(\theta_C) \approx \lambda$
- ▶  $|V_{cs}| \approx 1$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



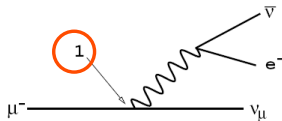
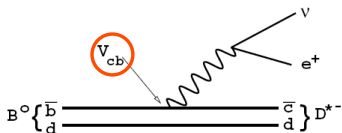
$$\frac{d\Gamma(\bar{D}^0 \rightarrow \pi^0 e^- \bar{\nu}_e)}{dx_\pi} = \frac{G_F^2 m_D^2}{192\pi^2} |V_{cd}|^2 f(q^2)^2 \left( x_\pi^2 - 4 \frac{m_\pi^2}{m_D^2} \right)^{3/2}, \quad \text{where } x_\pi = \frac{2E_\pi}{m_D}$$

# Measuring CKM matrix elements

## Measuring $V_{cb}$

- ▶ Compare rates of  $B^0 \rightarrow D^{*-} \ell^+ \nu_\ell$  and muon decays (see [PDG write-up](#)) [4]
- ▶ Ratio is proportional to  $|V_{cb}|^2$
- ▶  $|V_{cb}| = 0.0405 \pm 0.0013$
- ▶  $|V_{cd}| \approx \sin^2(\theta_C) \approx \lambda^2$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

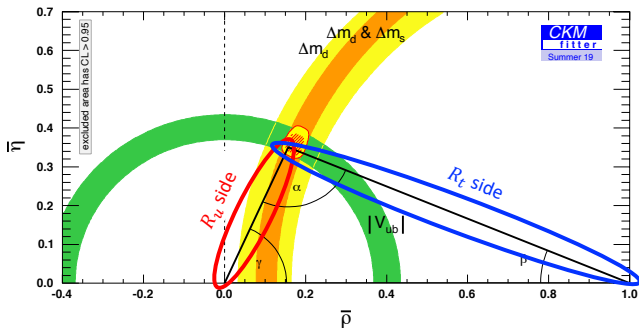


$$\frac{d\Gamma(b \rightarrow u_\alpha \ell^- \bar{\nu}_\ell)}{dx} = \frac{G_F^2 m_b^5}{192\pi^2} |V_{\alpha b}|^2 \left( 2x^2 \left( \frac{1-x-\xi}{1-x} \right)^2 \left( 3 - 2x + \xi + \frac{2\xi}{1-x} \right) \right)$$

$$\text{where } \alpha = u, c, \quad \xi = \frac{m_\alpha^2}{m_b^2}, \quad x = \frac{2E_\ell}{m_b}$$

# Measuring CKM matrix elements

- ▶ The sides of the ( $B^0$ ) unitarity triangle are constrained by
  - ▶ The ratio  $V_{ub}/V_{cb}$  for the left side (known sometimes as  $R_u$ )
  - ▶ The ratio  $\Delta m_d/\Delta m_s$  for the right side (known sometimes as  $R_t$ )
- ▶ Sometimes called “UT constraints from  $CP$ -conserving quantities



- ▶ Notice that using the CKM paradigm we can observe  $CPV$  without  $CP$ -violating quantities

# Measurements of $V_{ub}$

- ▶ There are three ways to determine  $V_{ub}$ 
  1. “Inclusive” decays of  $b \rightarrow u\ell^-\bar{\nu}_\ell$ 
    - ▶ Of course there are no bare quarks so we are really looking at a sum of exclusive decays of the form  $B_{(s)}^{0(-)} \rightarrow \pi^{0(-)}\ell^-\bar{\nu}_\ell X$
  2. “Exclusive” decays e.g.  $\bar{B}^0 \rightarrow \pi^+\ell^-\bar{\nu}_\ell$
  3. Leptonic “annihilation” decays e.g.  $B^+ \rightarrow \ell^+\nu_\ell$
- ▶ These each come with various requirements on theory (form factors) and the results have historically been rather inconsistent
  - ▶ This is typical in flavour physics
  - ▶ Is the discrepancy a theory issue, an experimental issue or New Physics (or some combination)?

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- ▶ Measure the sum of all processes containing  $b \rightarrow u\ell^-\bar{\nu}_\ell$

- ▶ Just think about what this means and how hard this is to achieve

From [HFLAV 2021 write-up](#) [5]

- ▶ Experimentally this is incredibly challenging due to backgrounds from the dominant  $b \rightarrow c$  semileptonic decays

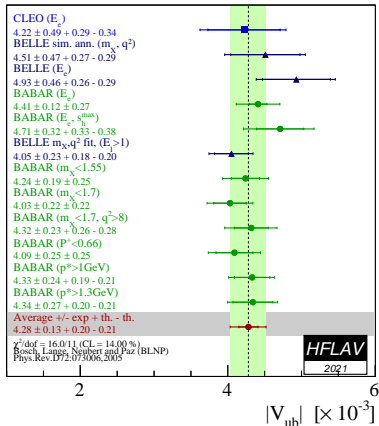
- ▶ These backgrounds are reduced by either

- ▶ Cutting on the mass of the  $X_u$  system or
  - ▶ Cutting on the lepton energy (use the end-point to reject  $X_c$ )

- ▶ Essential to have a hermetic detector (need to resolve the neutral) so can only be done at Belle and BaBar

- ▶ It is the mass or end-point cuts which then introduce large theory uncertainties

- ▶ Need to estimate how much of the  $X_u$  phase space is being removed by these cuts

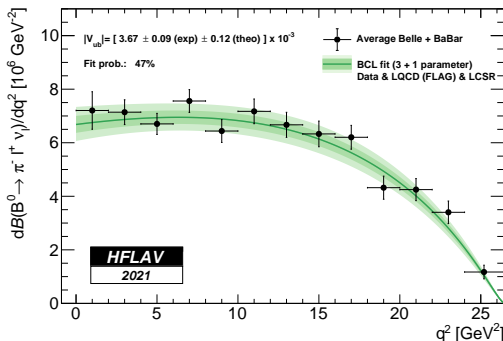


- ▶ Determined by fitting the decay rate seen by BaBar and Belle in e.g.  $B^0 \rightarrow \pi^- \ell^+ \nu_\ell$

$$\frac{d\Gamma}{dq^2} = |V_{ub}|^2 \frac{G_F^2}{192\pi^3 m_B^3} \lambda(m_B, m_\pi, q^2)^{3/2} |f_+(q^2)|^2$$

- ▶ Much more straightforward experimentally but more challenging for the theory
  - ▶ Have a dependence on form-factors,  $f_+(q^2)$ , for the  $B \rightarrow \pi$  transition
  - ▶ Use Lattice QCD calculations

From [HFLAV 2021 write-up](#) [5]

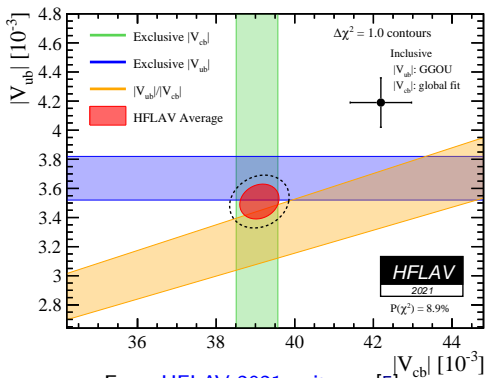


# Measurements of $V_{ub}$ and $V_{cb}$

- ▶ LHCb has also pioneered an approach with the  $\Lambda_b^0 \rightarrow p\mu^-\bar{\nu}_\mu$  decay
- ▶ Take the ratio with  $\Lambda_b^0 \rightarrow \Lambda_c^+\mu^-\bar{\nu}_\mu$  to get  $|V_{ub}|/|V_{cb}|$
- ▶ Requires the form factor ratio,  $R_{FF}$ , from the Lattice

$$\frac{\mathcal{B}(\Lambda_b^0 \rightarrow p\mu^-\bar{\nu}_\mu)}{\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+\mu^-\bar{\nu}_\mu)} = \frac{|V_{ub}|^2}{|V_{cb}|^2} R_{FF}$$

- ▶ The global average exhibits a considerable tension between inclusive and exclusive



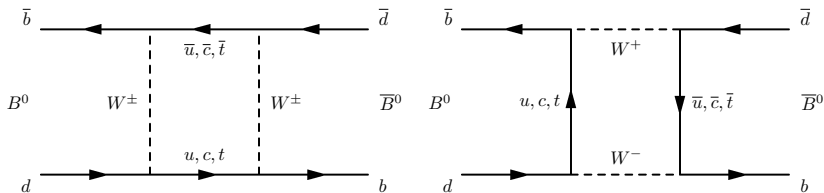
# Measurements of $V_{td}$ and $V_{ts}$

- ▶ There is no top decay but can obtain indirect measurements from the loops which appear in  $B^0$  and  $B_s^0$  mixing

- ▶  $|V_{ts}| = 0.0082 \pm 0.0006$

- ▶  $|V_{td}| = 0.0400 \pm 0.0027$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ \color{red}V_{td} & \color{red}V_{ts} & V_{tb} \end{pmatrix}$$



- ▶ Ratio of frequencies for  $B^0$  and  $B_s^0$ :

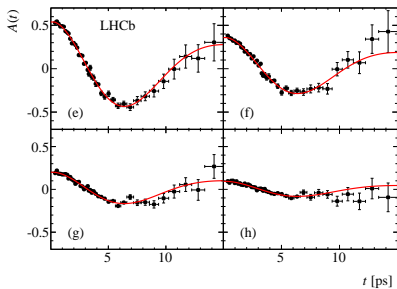
$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s^0}}{m_{B^0}} \frac{f_{B_s^0}^2}{f_{B^0}^2} \frac{B_{B_s^0}^2}{B_{B^0}} \frac{|V_{ts}|^2}{|V_{td}|^2} = \frac{m_{B_s^0}}{m_{B^0}} \xi^2 \frac{|V_{ts}|^2}{|V_{td}|^2} \quad (6)$$

# Measurements of the $R_t$ side

- ▶  $B^0$  and  $B_s^0$  oscillation frequencies (which we use to get constraints on  $V_{td}$  and  $V_{ts}$ ) measured at LEP, Tevatron,  $B$ -factories and LHCb
- ▶ Most precise measurements now come from LHCb

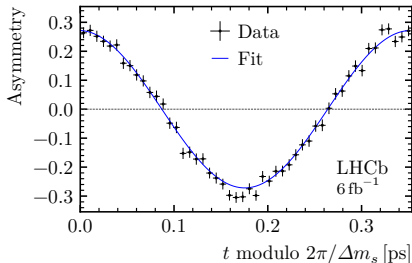
$\Delta m_d$  from  $B^0 \rightarrow D^{(*)-} \mu^+ \nu_\mu X$

LHCb - [EPJ C 76 (2016) 412] [6]



$\Delta m_s$  from  $B_s^0 \rightarrow D_s^- \pi^+$

LHCb - [Nature Phys. 18 (2022) 1] [7]



## Measuring CKM matrix elements

- ▶ These measurements have all been for the **magnitudes** of the CKM elements
  - ▶ Developed over a long period of time using several experiments

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

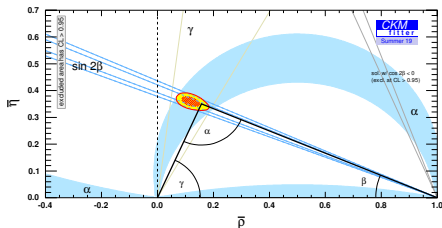
$$\lambda \approx \sin(\theta_C) = \sin(\theta_{12}) \approx 0.22$$

- ▶ These give no information on the phase(s)
  - ▶ Let's now consider measurements of this imaginary part
  - ▶ To find the imaginary part we need *CPV*

### 3. Measuring CKM matrix angles

# Measuring CKM matrix phases

Amplitude	Rel. magnitude	phase
$b \rightarrow c$	Dominant	0
$b \rightarrow u$	Supressed	$\gamma$
$t \rightarrow d$	Time-dependent	$2\beta$
$t \rightarrow s$	Time-dependent	$-2\beta_s$



- ▶  $\gamma$  in interference between  $b \rightarrow u$  and  $b \rightarrow c$  transitions
- ▶  $\beta$  in interference between  $B^0$  mixing and decay
- ▶  $\beta_s \approx \phi_s$  in interference between  $B_s^0$  mixing and decay
- ▶  $\alpha$  arises in the interference between different  $b \rightarrow u$  transitions

$$V_{CKM} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{-i\beta_s} & |V_{tb}| \end{pmatrix} + \mathcal{O}(\lambda^5)$$

- ▶ Arises in the interference between  $B^0 \rightarrow f_{CP}$  and  $B^0 \rightarrow \bar{B}^0 \rightarrow f_{CP}$
- ▶ The **golden mode** is  $B^0 \rightarrow J/\psi K_S^0$  because the master equations (see Lecture 2) simplify considerably

1. For a  $B^0$  we have no (or at least negligible)  $CPV$  in mixing

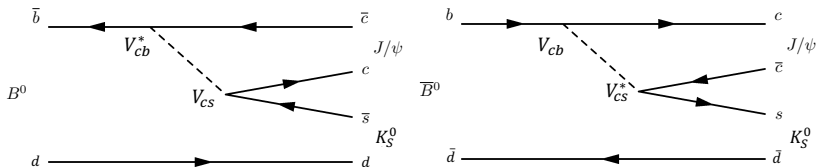
$$\left| \frac{q}{p} \right| \approx 1$$

2. For the  $J/\psi K_S^0$  we have a  $CP$ -even final state so  $f = \bar{f}$  therefore

$$\lambda_f \equiv \frac{q}{p} \frac{\bar{A}_f}{A_f} = \frac{q}{p} \frac{\bar{A}_{\bar{f}}}{A_{\bar{f}}} \equiv \lambda_{\bar{f}}$$

3. The  $B^0$  and  $\bar{B}^0$  amplitudes to  $f$  are (almost) identical

$$|A_f| = |\bar{A}_f|$$



- ▶ Recall from the master equations (Lecture 2) that

$$C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \quad D_f = \frac{2\mathcal{R}e(\lambda_f)}{1 + |\lambda_f|^2}, \quad S_f = \frac{2\mathcal{I}m(\lambda_f)}{1 + |\lambda_f|^2}$$

- ▶ Giving a time-dependent asymmetry of

$$\mathcal{A}_{CP}(t) = \frac{\Gamma_{X^0 \rightarrow f}(t) - \Gamma_{\bar{X}^0 \rightarrow f}(t)}{\Gamma_{X^0 \rightarrow f}(t) + \Gamma_{\bar{X}^0 \rightarrow f}(t)} = \frac{C_f \cos(\Delta mt) - S_f \sin(\Delta mt)}{\cosh(\frac{1}{2}\Delta\Gamma t) + D_f \sinh(\frac{1}{2}\Delta\Gamma t)} \quad (7)$$

- ▶ In the case of  $B^0 \rightarrow J/\psi K_S^0$  this hugely simplifies as  $|\lambda_f| = 1$  and  $\Delta\Gamma = 0$  so that

$$\mathcal{A}_{CP}(t) = -\mathcal{I}m(\lambda_f) \sin(\Delta mt) \quad (8)$$

- ▶ Looking into more detail at what  $\lambda_f$  is in the case of  $B^0 \rightarrow J/\psi K_S^0$

$$\lambda_{J/\psi K_S^0} = \left(\frac{q}{p}\right)_{B^0} \frac{\bar{A}_{J/\psi K_S^0}}{A_{J/\psi K_S^0}} = \left(\frac{q}{p}\right)_{B^0} \frac{\bar{A}_{J/\psi K^0}}{A_{J/\psi K^0}} \left(\frac{p}{q}\right)_{K^0} \quad (9)$$

$$= - \underbrace{\left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\right)}_{B^0 \text{ mixing}} \underbrace{\left(\frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}}\right)}_{B^0 \rightarrow J/\psi K^0} \underbrace{\left(\frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}}\right)}_{K^0 \text{ mixing}} \quad (10)$$

$$= -e^{-2i\beta} \quad (11)$$

it's a useful exercise to show this using the equations from Lecture 2

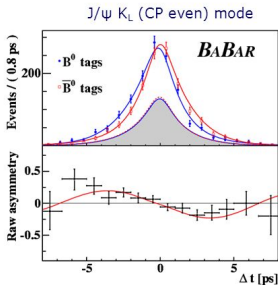
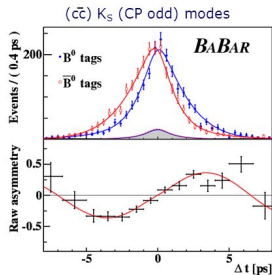
- ▶ So that the time-dependent asymmetry is

$$A_{CP}(t) = \pm \sin(2\beta) \sin(\Delta mt) \quad (12)$$

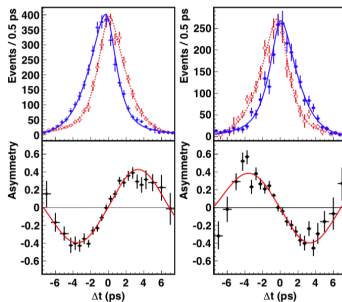
the  $\pm$  is for  $CP$ -even (e.g.  $J/\psi K_L^0$ ) or  $CP$ -odd (e.g.  $J/\psi K_S^0$ ) final states

- ▶ A theoretically and experimentally clean signature
- ▶ Also has a relatively large branching fraction,  $O(10^{-4})$

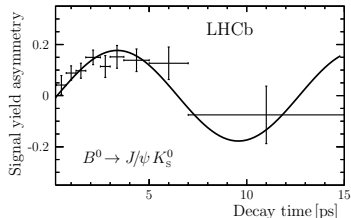
## BaBar [8]



## Belle [9]

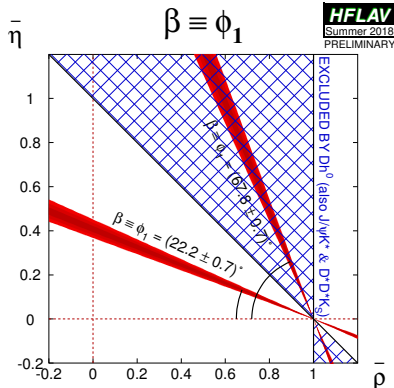
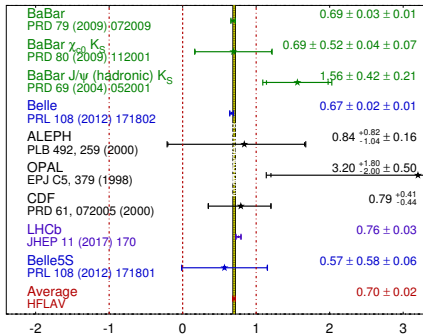


## LHCb [10]



Plots from [HFLAV 2021 write-up](#) [5]

$$\sin(2\beta) \equiv \sin(2\phi_1) \quad \text{HFLAV Moriond 2018 PRELIMINARY}$$



$$\sin(2\beta) = 0.699 \pm 0.017$$

$$\beta = (22.2 \pm 0.7)^\circ$$

- ▶ The  $B_s^0$  analogue of  $\beta$  (recall the squeezed  $B_s^0$  unitarity triangle)
- ▶ Use  $B_s^0 \rightarrow J/\psi\phi$  which is a spectator quark  $d \leftrightarrow s$  switch for  $B^0 \rightarrow J/\psi K_S^0$ 
  - ▶ There are four main differences:

	$B^0 \rightarrow J/\psi K_S^0$	$B_s^0 \rightarrow J/\psi\phi$
1. CKM element	$V_{td}$	$V_{ts}$
2. $\Delta\Gamma$	$\sim 0$	$\sim 0.1$
3. Final state (spin)	$K^0 : s = 0$	$\phi : s = 1$
4. Final state ( $K$ )	$K^0$ mixing	-

- ▶ Recall from the master equations the time-dependent  $CP$  asymmetry

$$\mathcal{A}_{CP}(t) = \frac{\Gamma_{X^0 \rightarrow f}(t) - \Gamma_{\bar{X}^0 \rightarrow f}(t)}{\Gamma_{X^0 \rightarrow f}(t) + \Gamma_{\bar{X}^0 \rightarrow f}(t)} = \frac{2C_f \cos(\Delta mt) - 2S_f \sin(\Delta mt)}{2 \cosh(\frac{1}{2}\Delta\Gamma t) + 2 \sinh(\frac{1}{2}\Delta\Gamma t)} \quad (13)$$

- ▶ We still have one dominant amplitude so  $A_f \approx A_{\bar{f}} \implies |\lambda_f| \approx 1 \implies C_f \approx 0$  so

$$\mathcal{A}_{CP}(t) = \frac{-\mathcal{I}m(\lambda_{J/\psi\phi}) \sin(\Delta mt)}{\cosh(\frac{1}{2}\Delta\Gamma t) + \mathcal{R}e(\lambda_{J/\psi\phi}) \sinh(\frac{1}{2}\Delta\Gamma t)} \quad (14)$$

- ▶ Looking into more detail at what  $\lambda_f$  is in the case of  $B_s^0 \rightarrow J/\psi\phi$

$$\lambda_{J/\psi\phi} = \left(\frac{q}{p}\right)_{B_s^0} \left( \frac{\eta_{J/\psi\phi} \bar{A}_{J/\psi\phi}}{A_{J/\psi\phi}} \right) \quad (15)$$

$$= (-1)^l \left( \frac{V_{tb}^* V_{ts}}{V_{tb} V_{ts}^*} \right) \left( \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \right) \quad (16)$$

$$= (-1)^l e^{-2i\beta_s} \quad (17)$$

$\eta$  represents the  $CP$ -eigenvalue

- ▶ Because we have two vectors in the final state there are three amplitudes to consider (as opposed to the one amplitude for  $B^0 \rightarrow J/\psi K_S^0$ )

$$A_{\parallel} \quad (\uparrow\uparrow) \quad l = 2$$

$$A_{\perp} \quad (\uparrow\rightarrow) \quad l = 1$$

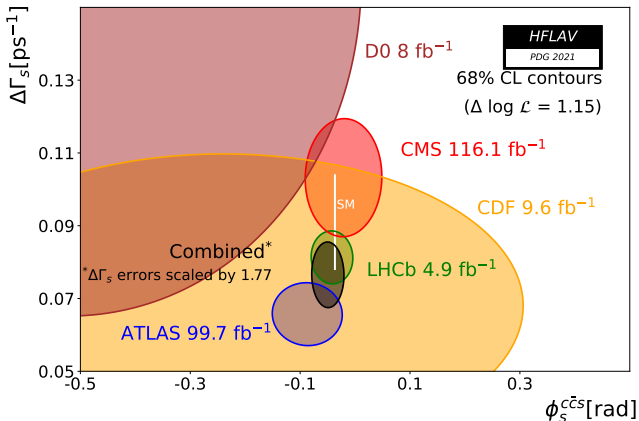
$$A_0 \quad (\uparrow\downarrow) \quad l = 0$$

- ▶ Thus the time-dependent asymmetry becomes

$$\mathcal{A}_{CP}(t) = \frac{-\eta \sin(2\beta_s) \sin(\Delta mt)}{\cosh(\frac{1}{2}\Delta\Gamma t) + \eta \cos(2\beta_s) \sinh(\frac{1}{2}\Delta\Gamma t)} \quad (18)$$

at least it does for each polarisation amplitude independently (the interference between the amplitudes is slightly more complicated)

Plot from [HFLAV 2021 write-up](#) [5]



$$\phi_s^{c\bar{c}s} = -0.050 \pm 0.019$$

► But watch out for polluting penguin contributions!

$$\phi_s = -2\beta_s + \delta\phi^{\text{SM}} + \phi^{\text{NP}} \tag{19}$$

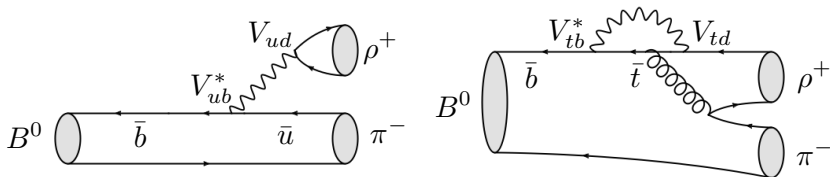
# CKM angle $\alpha$

- ▶ Following a similar logic to that of  $B^0 \rightarrow J/\psi K_S^0$  for  $\beta$  one finds that  $\alpha$  arises in the time-dependent asymmetry for modes containing a  $b \rightarrow u\bar{u}d$  transition
  - ▶ For example  $B^0 \rightarrow \pi^+\pi^-$  or  $B^0 \rightarrow \rho^+\rho^-$
- ▶ Recalling the master equations with  $\Delta\Gamma = 0$
- ▶ Nominally we should have  $C_f = 0$  and  $S_f = \sin(2\alpha)$  to give

$$\mathcal{A}_{CP}(t) = \pm \sin(2\alpha) \sin(\Delta mt) \quad (20)$$

exactly equivalent to the extraction of  $\beta$

- ▶ However, in this case there is a **non-negligible contribution from penguin decays** of  $b \rightarrow d\bar{u}u$ 
  - ▶ Similar in magnitude to the  $b \rightarrow u\bar{u}q$  transition but has a different weak phase
  - ▶ Therefore  $C \neq 0$  and  $S \neq \pm \sin(2\alpha)$
  - ▶ How do we deal with the penguin contamination?



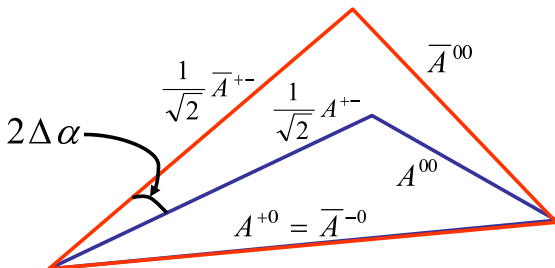
- ▶ The contributions from the penguin amplitudes can be accounted for using an “isospin analysis”

- ▶ Relate the amplitudes for isospin partners

$$A^{+-} \text{ for } B^0 \rightarrow \pi^+\pi^-, \quad A^{+0} \text{ for } B^+ \rightarrow \pi^+\pi^0, \quad A^{00} \text{ for } B^0 \rightarrow \pi^0\pi^0, \quad (21)$$

- ▶ There is no penguin contribution to  $A^{+0}$  and  $\bar{A}^{-0}$  because  $\pi^\pm\pi^0$  is a pure isospin-2 state and the QCD-penguin ( $\Delta I = 1/2$ ) only contributes to the isospin-0 final states
- ▶ Obtain isospin triangle relations

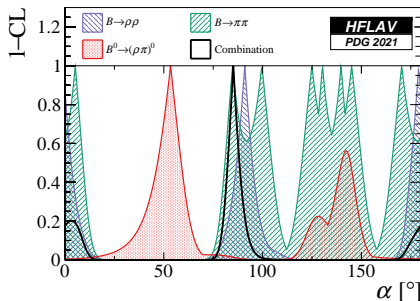
$$A^{+0} = \frac{1}{\sqrt{2}}A^{+-} + A^{00}, \quad \text{and} \quad \bar{A}^{-0} = \frac{1}{\sqrt{2}}\bar{A}^{+-} + \bar{A}^{00} \quad (22)$$



## Add in the related $B \rightarrow \rho\rho$ modes

- ▶ These are vectors (not scalars like the  $\pi$ s) so do not have a fixed  $CP$ -eigenvalue
- ▶ However it is found that these decays are almost entirely longitudinally polarised (so approximately  $CP$ -even)
- ▶ Much easier to reconstruct, have a much higher branching fraction and have much smaller penguin contributions (triangles are flatted) so have better sensitivity and reduced ambiguities

Plot from [HFLAV 2021 write-up](#) [5]



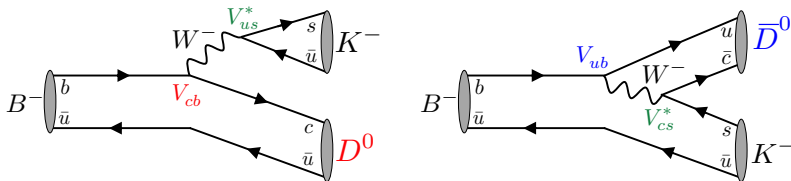
## Add the $B \rightarrow \rho\pi$ system

- ▶ Get a pentagonal (rather than triangular) isospin relation
- ▶ The relative amplitudes of  $\rho^+\pi^-$ ,  $\rho^-\pi^+$  and  $\rho^0\pi^0$  can all be determined from Dalitz analysis of  $B^0 \rightarrow \pi^+\pi^-\pi^0$

# CKM angle $\gamma$

- ▶  $\gamma$  is the phase between  $V_{ub}^*V_{ud}$  and  $V_{cb}^*V_{cd}$ 
  - ▶ Require interference between  $b \rightarrow cW$  and  $b \rightarrow uW$  to access it
  - ▶ No dependence on CKM elements involving the top
  - ▶ Can be measured using tree level  $B$  decays
- ▶ The “textbook” case is  $B^\pm \rightarrow \bar{D}^0 K^\pm$ :
  - ▶ Transitions themselves have different final states ( $D^0$  and  $\bar{D}^0$ )
  - ▶ Interference occurs when  $D^0$  and  $\bar{D}^0$  decay to the same final state  $f$

Reconstruct the  $D^0/\bar{D}^0$  in a final state accessible to both to achieve interference

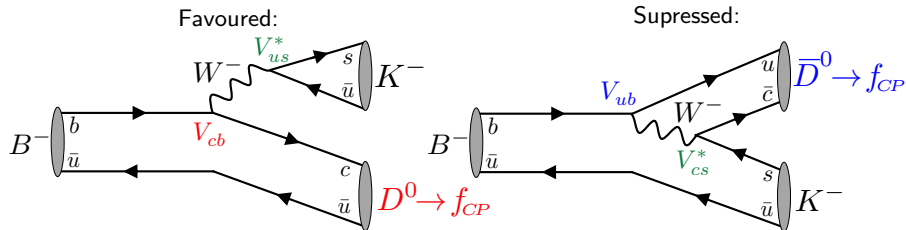


- ▶ The crucial feature of these (and similar) decays is that the  $D^0$  can be reconstructed in several different final states [all have same weak phase  $\gamma$ ]

# $\gamma$ with $D^0 \rightarrow CP$ eigenstates (GLW)

Consider only  $D$  decays to  $CP$  eigenstates,  $f_{CP}$  (Gronau-London-Wyler [11, 12])

- **Favoured:**  $b \rightarrow c$  with strong phase  $\delta_F$  and weak phase  $\phi_F$
- **Supressed:**  $b \rightarrow u$  with strong phase  $\delta_S$  and weak phase  $\phi_S$



Subsequent amplitude to final state  $f_{CP}$  is:

$$B^- : A_f = |F|e^{i(\delta_F - \phi_F)} + |S|e^{i(\delta_S - \phi_S)} \quad (23)$$

$$B^+ : \bar{A}_f = |F|e^{i(\delta_F + \phi_F)} + |S|e^{i(\delta_S + \phi_S)} \quad (24)$$

because strong phases ( $\delta$ ) don't change sign under  $CP$  while weak phases ( $\phi$ ) do

## $\gamma$ with $D^0 \rightarrow CP$ eigenstates (GLW)

- ▶ Can define the sum and difference of rates with  $B^+$  and  $B^-$

### Rate difference and sum

$$|\bar{A}_{\bar{f}}|^2 - |A_f|^2 = 2|F||S| \sin(\delta_F - \delta_S) \sin(\phi_F - \phi_S) \quad (25)$$

$$|\bar{A}_{\bar{f}}|^2 + |A_f|^2 = |F|^2 + |S|^2 + 2|F||S| \cos(\delta_F - \delta_S) \cos(\phi_F - \phi_S) \quad (26)$$

- ▶ Choose  $r_B = \frac{|S|}{|F|}$  (so that  $r < 1$ ) and use strong phase difference  $\delta_B = \delta_F - \delta_S$
- ▶  $\gamma$  is the weak phase difference  $\phi_F - \phi_S$
- ▶ Subsequently have two **experimental observables** which are

### GLW $CP$ asymmetry

$$\mathcal{A}_{CP} = \frac{\pm 2r_B \sin(\delta_B) \sin(\gamma)}{1 + r_B^2 \pm 2r_B \cos(\delta_B) \cos(\gamma)}$$

### GLW total rate

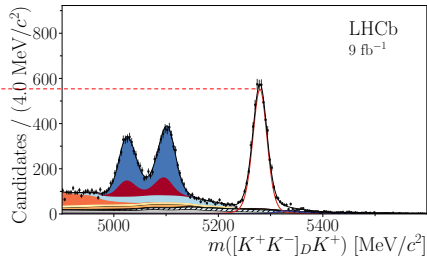
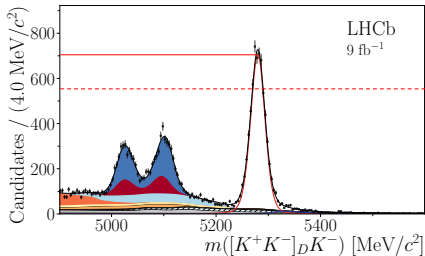
$$\mathcal{R}_{CP} = 1 + r_B^2 \pm 2r_B \cos(\delta_B) \cos(\gamma)$$

- ▶ The  $+(-)$  sign corresponds to  $CP$ -even (-odd) final states
- ▶ Note that  $r_B$  and  $\delta_B$  (ratio and strong phase difference of favoured and suppressed modes) are different for each  $B$  decay
- ▶ **The value of  $\gamma$  is shared by all such decays**

## GLW observables

$$A_{CP} = \frac{\pm 2r_B \sin(\delta_B) \sin(\gamma)}{1 + r_B^2 \pm 2r_B \cos(\delta_B) \cos(\gamma)} \quad (25)$$

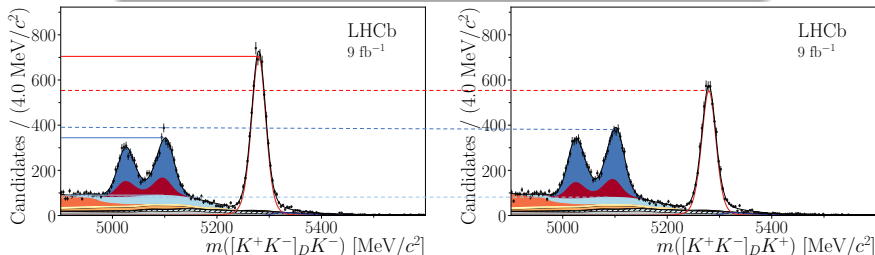
$$R_{CP} = 1 + r_B^2 \pm 2r_B \cos(\delta_B) \cos(\gamma) \quad (26)$$



## GLW observables

$$A_{CP} = \frac{\pm 2r_B(2F^+ + 1) \sin(\delta_B) \sin(\gamma)}{1 + r_B^2 \pm 2r_B(2F^+ + 1) \cos(\delta_B) \cos(\gamma)} \quad (25)$$

$$R_{CP} = 1 + r_B^2 \pm 2r_B(2F^+ + 1) \cos(\delta_B) \cos(\gamma) \quad (26)$$

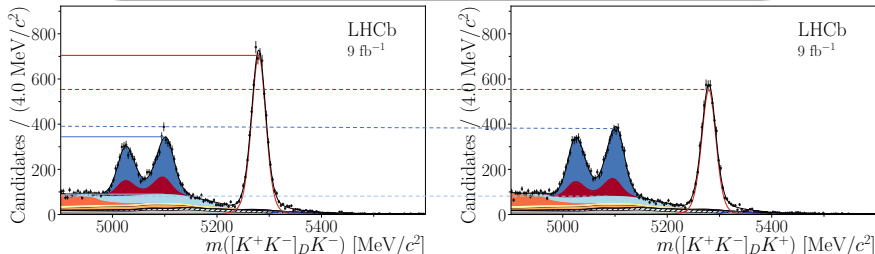


- ▶ LHCb has recently extracted GLW observables from partially reconstructed  $B^- \rightarrow D^{*0} K^-$  in the same fit [13]

## GLW observables

$$A_{CP} = \frac{\pm 2r_B(2F^+ + 1) \sin(\delta_B) \sin(\gamma)}{1 + r_B^2 \pm 2r_B(2F^+ + 1) \cos(\delta_B) \cos(\gamma)} \quad (25)$$

$$R_{CP} = 1 + r_B^2 \pm 2r_B(2F^+ + 1) \cos(\delta_B) \cos(\gamma) \quad (26)$$

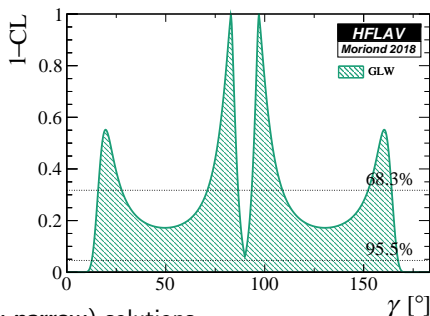


- ▶ LHCb has recently extracted GLW observables from partially reconstructed  $B^- \rightarrow D^{*0} K^-$  in the same fit [13]
- ▶ Can extend to quasi- $CP$ -eigenstates ( $D^0 \rightarrow KK\pi^0$ ) if fraction of  $CP$  content,  $F^+$ , is known

## GLW observables

$$A_{CP} = \frac{\pm 2r_B(2F^+ + 1) \sin(\delta_B) \sin(\gamma)}{1 + r_B^2 \pm 2r_B(2F^+ + 1) \cos(\delta_B) \cos(\gamma)} \quad (25)$$

$$R_{CP} = 1 + r_B^2 \pm 2r_B(2F^+ + 1) \cos(\delta_B) \cos(\gamma) \quad (26)$$



- ▶ Multiple (**but very narrow**) solutions
- ▶ Require knowledge of  $F^+$  from charm friends

## $\gamma$ with CF and DCS decays (ADS)

Proposed by Atwood, Dunietz and Soni [14, 15]

- ▶ A 2-body  $D$  decay to final state  $f$  accessible to both  $D^0$  and  $\bar{D}^0$  can be
  - ▶ Cabibbo-favoured (CF) -  $D^0 \rightarrow \pi^- K^+$
  - ▶ Doubly-Cabibbo-suppressed (DCS) -  $\bar{D}^0 \rightarrow \pi^- K^+$
- ▶ Introduces 2 new hadronic parameters:
  - ▶  $r_D$  - ratio of magnitudes for  $D^0$  and  $\bar{D}^0$  decay to  $f$
  - ▶  $\delta_D$  - relative phase for  $D^0$  and  $\bar{D}^0$  decay to  $f$
- ▶ Gives a modified asymmetry and rate definition

### ADS asymmetry

$$\mathcal{A}_{ADS} = \frac{2r_D r_B \sin(\delta_B + \delta_D) \sin(\gamma)}{r_D^2 + r_B^2 + 2r_B r_D \cos(\delta_B + \delta_D) \cos(\gamma)}$$

### ADS ratio

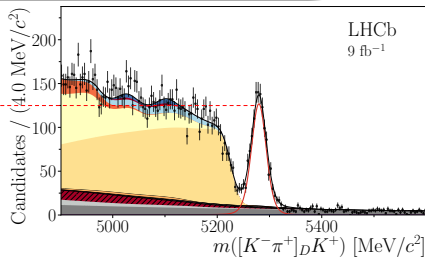
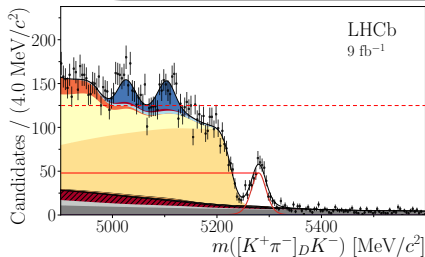
$$\mathcal{R}_{ADS} = \frac{|\bar{A}_f|^2 + |A_f|^2}{|A_f|^2 + |\bar{A}_f|^2} = r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D) \cos(\gamma)$$

- ▶ Hadronic parameters  $r_D$  and  $\delta_D$  can be independently determined

## ADS observables

$$A_{ADS} = \frac{2r_B r_D \sin(\delta_B + \delta_D) \sin(\gamma)}{r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D) \cos(\gamma)} \quad (25)$$

$$R_{ADS} = r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D) \cos(\gamma) \quad (26)$$

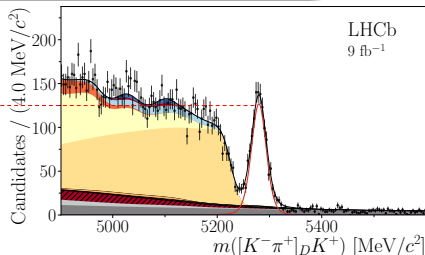
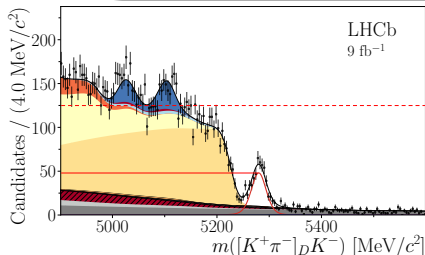


- **Much harder to extract partially reconstructed observables because of  $B_s^0 \rightarrow D^{(*)0} K^+ \pi^-$  backgrounds.**

## ADS observables

$$A_{ADS} = \frac{2r_B r_D \kappa_D \sin(\delta_B + \delta_D) \sin(\gamma)}{r_B^2 + r_D^2 + 2r_B r_D \kappa_D \cos(\delta_B + \delta_D) \cos(\gamma)} \quad (25)$$

$$R_{ADS} = r_B^2 + r_D^2 + 2r_B r_D \kappa_D \cos(\delta_B + \delta_D) \cos(\gamma) \quad (26)$$

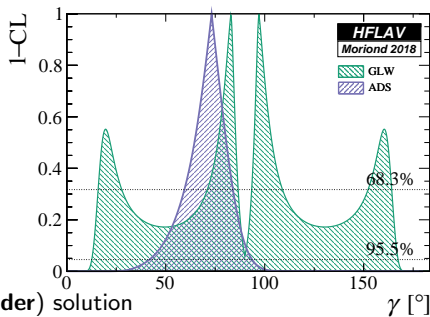


- ▶ **Much harder to extract partially reconstructed observables because of  $B_s^0 \rightarrow D^{(*)0} K^+ \pi^-$  backgrounds.**
- ▶ Can extend to multibody-DCS-decays ( $D^0 \rightarrow K \pi \pi^0$ ) if dilution from interference,  $\kappa_D$ , is known

## ADS observables

$$A_{ADS} = \frac{2r_B r_D \sin(\delta_B + \delta_D) \sin(\gamma)}{r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D) \cos(\gamma)} \quad (25)$$

$$R_{ADS} = r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D) \cos(\gamma) \quad (26)$$



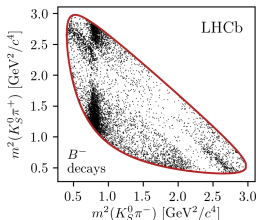
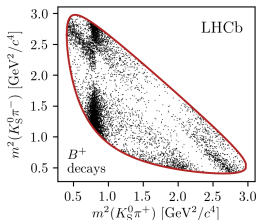
- ▶ A single (**yet broader**) solution
- ▶ Require knowledge of  $r_D$ ,  $\delta_D$ ,  $\kappa_D$  from charm friends

# $\gamma$ with 3-body self-conjugate states (BPGGSZ)

- ▶ Now get additional sensitivity over the 3-body phase space [16, 17, 18]
- ▶ Idea is to perform a GLW/ADS type analysis across the  $D$  decay phase space
- ▶ For example  $D^0 \rightarrow K_S^0 \pi^+ \pi^-$  has contributions from
  - ▶ Singly-Cabibbo-suppressed decay  $D^0 \rightarrow K_S^0 \rho^0$
  - ▶ Doubly-Cabibbo-suppressed decay  $D^0 \rightarrow K^{*+} \pi^-$
  - ▶ Interference between them enhances sensitivity and resolves ambiguities in  $\gamma$

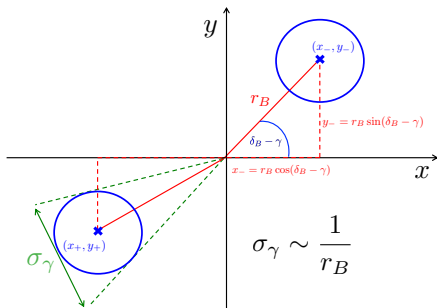
**BPGGSZ observables** (partial rate as function of Dalitz position)

$$d\Gamma_{B^\pm(\mathbf{x})} = A_{(\pm, \mp)}^2 + r_B^2 A_{(\mp, \pm)}^2 + 2A_{(\pm, \mp)} A_{(\mp, \pm)} \left[ r_B \cos(\delta_B \pm \gamma) \cos(\delta_{D(\pm, \mp)}) + r_B \sin(\delta_B \pm \gamma) \sin(\delta_{D(\pm, \mp)}) \right] \quad (27)$$



## BPGGSZ observables (partial rate as function of Dalitz position)

$$d\Gamma_{B^\pm(\mathbf{x})} = A_{(\pm, \mp)}^2 + r_B^2 A_{(\mp, \pm)}^2 + 2A_{(\pm, \mp)}A_{(\mp, \pm)} \left[ \underbrace{r_B \cos(\delta_B \pm \gamma)}_{x_\pm} \underbrace{\cos(\delta_{D(\pm, \mp)})}_{c_i} + \underbrace{r_B \sin(\delta_B \pm \gamma)}_{y_\pm} \underbrace{\sin(\delta_{D(\pm, \mp)})}_{s_i} \right] \quad (28)$$



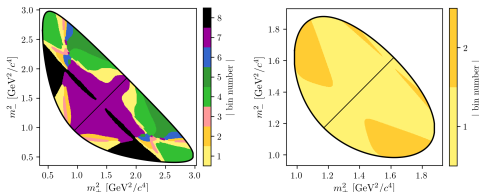
$$\sigma_\gamma \sim \frac{1}{r_B}$$

- ▶  $x_\pm + iy_\pm = r_B e^{i(\delta_B \pm \gamma)}$
- ▶ **Uncertainty on  $\gamma$  is inversely proportional to central value of hadronic unknown!!**
- ▶ Fluctuation in nuisance parameter = fluctuation in error on parameter of interest!

# Model-independent BPGGSZ Analysis

- ▶ Consider both  $D \rightarrow K_S^0 \pi \pi$  and  $D \rightarrow K_S^0 K K$  decays
- ▶ Divide up the Dalitz space into  $2N$  symmetric bins chosen to optimise sensitivity to  $\gamma$

LHCb - [JHEP 02 (2021) 169] - [19]



Decay amplitude is a superposition of suppressed and favoured contributions

$$A_B(m_-^2, m_+^2) \propto A_D(m_-^2, m_+^2) + r_B e^{i(\delta_B - \gamma)} A_{\bar{D}}(m_-^2, m_+^2)$$

Expected number of  $B^+$  ( $B^-$ ) events in bin  $i$

$$N_{\pm i}^+ = h_{B^+} \left[ F_{\mp i} + (x_+^2 + y_+^2) F_{\pm i} + 2\sqrt{F_i F_{-i}} (x_+ c_{\pm i} - y_+ s_{\pm i}) \right]$$

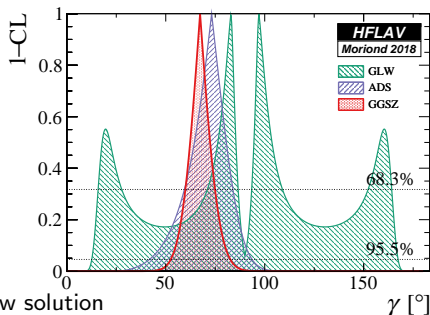
$$N_{\pm i}^- = h_{B^-} \left[ F_{\pm i} + (x_-^2 + y_-^2) F_{\mp i} + 2\sqrt{F_i F_{-i}} (x_- c_{\pm i} - y_- s_{\pm i}) \right]$$

- ▶  $N_{\pm i}^{\pm}$  - events in each bin
- ▶  $F_{\pm i}$  - from  $B \rightarrow D^{*\pm} \mu^{\mp} \nu_{\mu} X$
- ▶  $c_i, s_i$  - from CLEO-c (QC  $D^0 \bar{D}^0$ ) measurements
- ▶  $h_{B^{\pm}}$  - overall normalisation

Expected number of  $B^+$  ( $B^-$ ) events in bin  $i$

$$N_{\pm i}^+ = h_{B^+} \left[ F_{\mp i} + (x_+^2 + y_+^2) F_{\pm i} + 2\sqrt{F_i F_{-i}}(x_+ c_{\pm i} - y_+ s_{\pm i}) \right]$$

$$N_{\pm i}^- = h_{B^-} \left[ F_{\pm i} + (x_-^2 + y_-^2) F_{\mp i} + 2\sqrt{F_i F_{-i}}(x_- c_{\pm i} - y_- s_{\pm i}) \right]$$

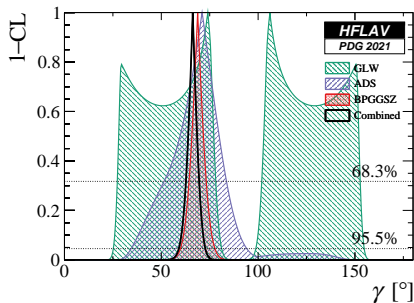
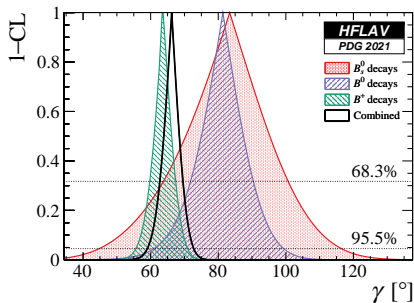


- ▶ A single and narrow solution
- ▶ Require knowledge of  $c_{\pm i}$  and  $s_{\pm i}$  from charm friends

# Combined constraints on $\gamma$

World Average (HFLAV) - [PDG 2021] [4]

$$\gamma = (66.2^{+3.4}_{-3.6})^\circ$$



**Indirect constraints are:**  $\gamma = (65.3^{+1.0}_{-2.5})^\circ$

Comparison between different methods gives robust cross check of systematics [?]

### 4. CKM constraints from kaon decays

- ▶ CPV first observed in  $2\pi$  decays of  $K_L^0$  mesons
  - ▶ Is this just mixing induced or is it direct CPV also (i.e. CPV in decay)?
  - ▶ For the CPV in kaon mixing we introduce the complex parameter  $\epsilon$  such that

$$|K_S^0\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}} (|K_1\rangle + \epsilon|K_2\rangle) \quad \text{and} \quad |K_L^0\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}} (|K_2\rangle + \epsilon|K_1\rangle) \quad (29)$$

- ▶ If CPV is only mixing induced then we expect  $K_L^0 : K_S^0$  amplitude ratios to be equivalent for neutral and charged final states (i.e.  $\eta_{00} = \eta_{+-}$ ) where

$$\eta_{00} = \frac{\mathcal{A}(K_L^0 \rightarrow \pi^0\pi^0)}{\mathcal{A}(K_S^0 \rightarrow \pi^0\pi^0)}, \quad \eta_{+-} = \frac{\mathcal{A}(K_L^0 \rightarrow \pi^+\pi^-)}{\mathcal{A}(K_S^0 \rightarrow \pi^+\pi^-)}. \quad (30)$$

- ▶ But we also see evidence for CPV in kaon decay (via semileptonic decays)

$$\delta \equiv \mathcal{A}_{CP}(K_L^0 \rightarrow \ell^+ \nu_\ell \pi^-) \quad (31)$$

- ▶ Can then summarise CPV in the kaon system using two parameters,  $(\epsilon, \epsilon')$  where

$$\eta_{00} = \epsilon - 2\epsilon' \quad (32)$$

$$\eta_{11} = \epsilon + \epsilon' \quad (33)$$

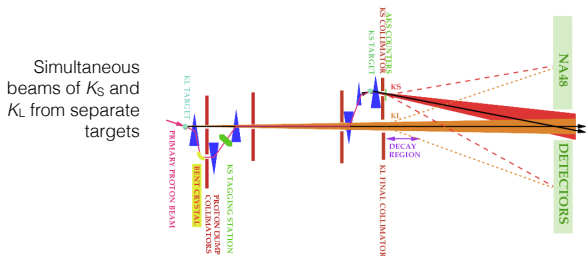
$$\delta = \frac{2\mathcal{R}e(\epsilon)}{1+|\epsilon|^2} \quad (34)$$

# NA48 experiment

- ▶ Established that  $\mathcal{R}e(\epsilon'/\epsilon) \neq 0$  by NA48 at CERN [?] and KTEV in Japan [?]
- ▶ NA48 is a fixed target experiment in CERN's North Area
- ▶ Measure the double ratio of  $\pi^0\pi^0$  and  $\pi^+\pi^-$  decays from  $K_L^0$  and  $K_S^0$

$$R = \frac{|\eta_{00}|^2}{|\eta_{+-}|^2} \approx 1 - 6\mathcal{R}e\left(\frac{\epsilon'}{\epsilon}\right) \quad (35)$$
$$= (13.7 \pm 2.5 \pm 1.8) \times 10^{-4}$$

- ▶ Now replaced by NA62 an even more sensitive kaon physics experiment looking for very rare kaon decays



### 5. Status of CKM matrix global fits

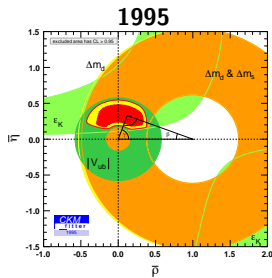
## Putting all the constraints together

- ▶ All of these separate measurements can be put together to over-constrain the CKM picture
- ▶ This is incredibly powerful because we can attack the  $(\rho, \eta)$  vertex of the unitarity triangle in several ways

### World Averages are performed by several groups

- ▶ CKMfitter (frequentist)
  - ▶ <http://ckmfitter.in2p3.fr/>
- ▶ UTFit (Bayesian)
  - ▶ <http://www.utfit.org/UTFit/>
- ▶ Heavy Flavour Averaging Group (HFLAV)
  - ▶ <https://hflav.web.cern.ch/>
- ▶ Particle Data Group (PDG)
  - ▶ <http://pdg.lbl.gov/>

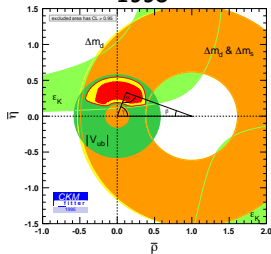
# The CKM fit



- ▶ Before the  $B$ -factories and LHC the CKM picture was not even established

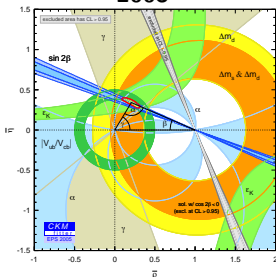
# The CKM fit

1995



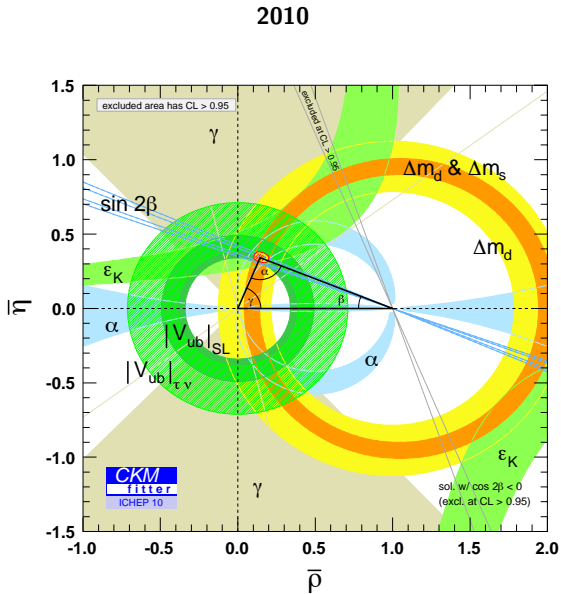
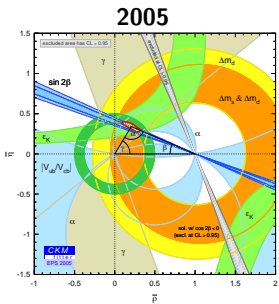
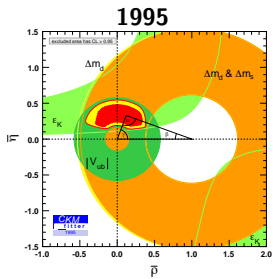
▶ With data from the Tevatron and  $B$ -factories the CKM picture is verified

2005

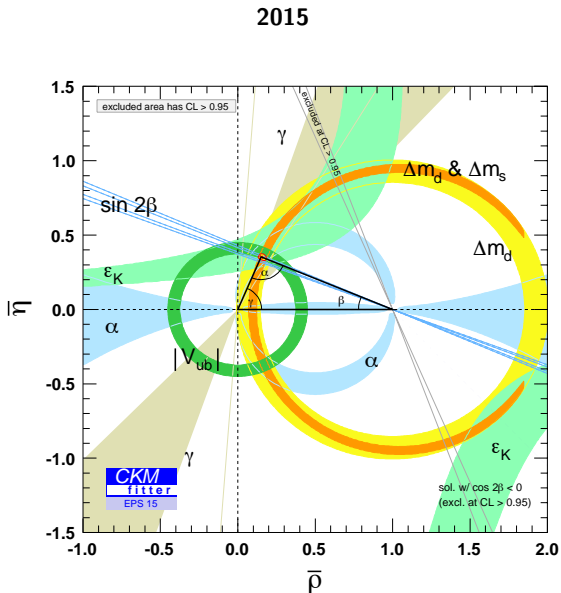
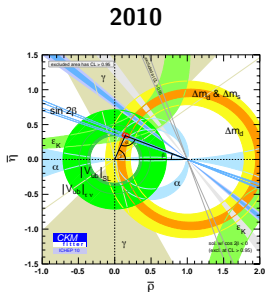
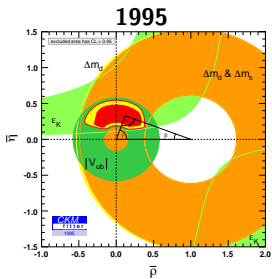


▶ When adding the LHC it now becomes a suite of precision physics measurements

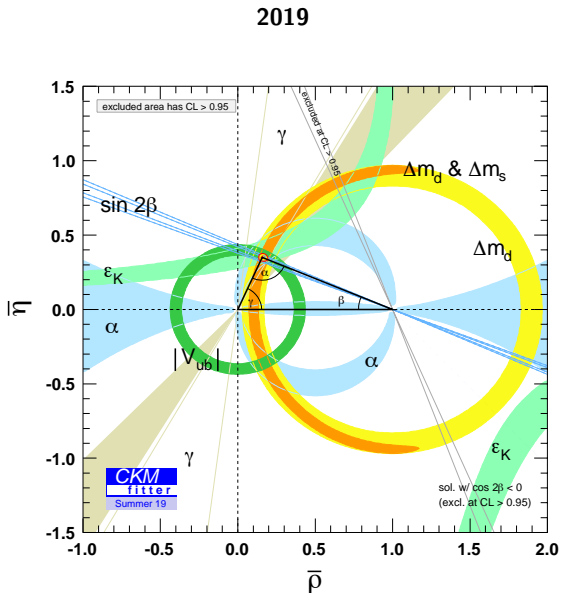
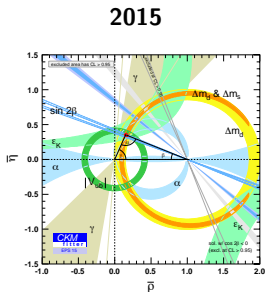
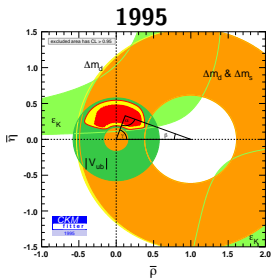
# The CKM fit



# The CKM fit

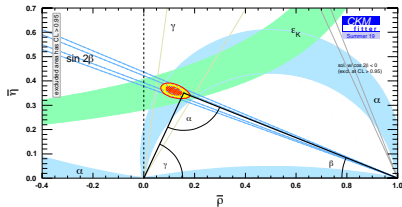
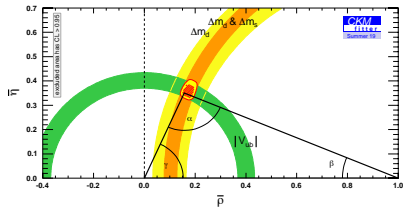


# The CKM fit

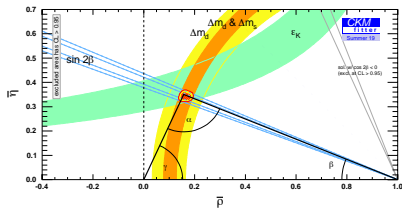
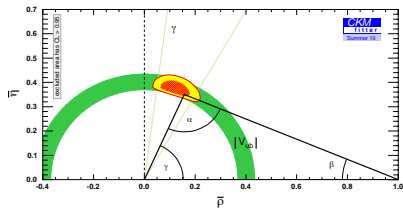


# The CKM fit

## Comparison between $CP$ -conserving (lengths of sides) and $CP$ -violating (angles)



## Comparison between tree-level ( $\gamma, V_{ub}$ ) and loop-level ( $\alpha, \beta, \Delta m, \epsilon$ )



## 6. CPT and T-reversal

- ▶ It is not possible to write a quantum field theory that is Lorentz invariant, with a Hermitian Hamiltonian  $H = H^\dagger$ , that violates the product of  $CPT$ 
  - ▶ *i.e.* one in which measurements are not invariant under position translations and Lorentz boosts of the system
- ▶ There are several important consequences that  $CPT$  invariance implies
  1. Mass and lifetime of particles and antiparticles are identical
  2. Quantum numbers of antiparticles are opposite those of particles
  3. Integer spin particles obey Bose-Einstein statistics and half-integer spin particles obey Fermi-Dirac statistics
- ▶ Time reversal symmetry translates  $t \rightarrow -t$ 
  - ▶ Obviously we can't test this experimentally (cannot run an experiment backwards in time)
  - ▶ However if  $CP$  is violated and the product  $CPT$  is conserved then  $T$  must also be violated

# $T$ violation in the $B$ system

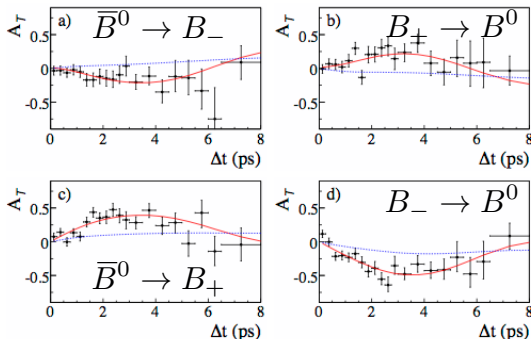
- ▶ This can actually be tested in the  $B$  system
- ▶ A generalisation of the  $\sin(2\beta)$  analysis
- ▶ Identify the flavour of the  $B$  by tagging the other  $B$  in the event and in addition separate the events by  $CP$ -odd ( $J/\psi K_S^0$ ) and  $CP$ -even ( $J/\psi K_L^0$ ) final states
- ▶ A  $T$  reversal violation would appear as a difference in the rates between

$$\bar{B}^0(t_1) \rightarrow B_-(t_2) \quad \text{and} \quad B_-(t_1) \rightarrow \bar{B}^0(t_2)$$

- ▶  $T$  violation has been observed by BaBar ([\[arXiv:1207.5832\]](https://arxiv.org/abs/1207.5832))

$$\Delta S_T^+ = -1.37 \pm 0.15$$

$$\Delta S_T^- = 1.17 \pm 0.21$$



## 7. Dipole Moments

# Magnetic dipole moments

- ▶ A “spinning” charge acts as a magnetic dipole with moment,  $\mu$ , which gives an energy shift to an externally applied magnetic field

$$\Delta E = -\vec{\mu} \cdot \vec{B} \quad (36)$$

- ▶ The prediction of  $g = 2$  (classically  $g = 1$ ) was a big success of the Dirac equation
- ▶ In an external field  $A^\mu$

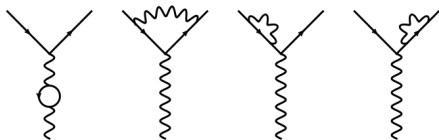
$$\left( \frac{1}{2m} (\vec{p} + e\vec{A}) + \frac{e}{2m} \vec{\sigma} \cdot \vec{B} - eA^0 \right) \psi = E\psi \quad (37)$$

- ▶ The magnetic dipole moment  $\mu$  is given by

$$\vec{\mu} = -\frac{e}{2m} \vec{\sigma} = -g \frac{\mu_B}{\hbar} \vec{S} \quad (38)$$


- ▶ Receives corrections from higher order processes (e.g. at order  $\alpha^2$ )

$$g = 2 + \frac{\alpha}{2\pi} + \mathcal{O}(\alpha^2)$$

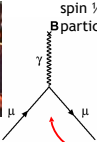


# Anomalous magnetic moment


Dirac



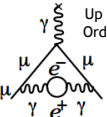
Charged, spin 1/2 B particle



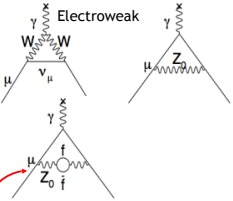
12672 diagrams




Up to 10<sup>th</sup> Order QED



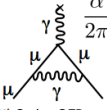
Electroweak



Schwinger

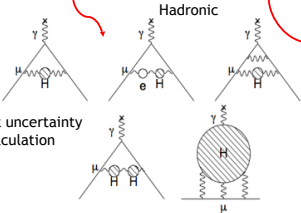


$\frac{\alpha}{2\pi} = 0.00232$



1<sup>st</sup> Order QED

Hadronic



Dominant uncertainty in calculation

**$g_\mu = 2.002\ 331\ 841\ 78(126)$**

Slide from Becky Chislett (via Tom Blake)

## Anomalous magnetic moments

- ▶  $(g - 2)_e$  is a powerful precision test of QED

$$(g - 2)_e = (1159.652186 \pm 0.000004) \times 10^{-6}$$

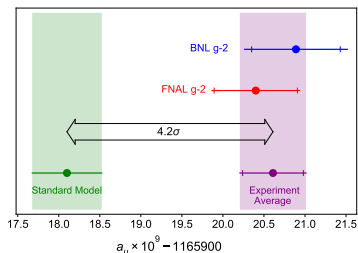
- ▶  $(g - 2)_\mu$  receives important Weak and QCD contributions. The latest experimental value from Brookhaven E821 and Fermilab  $g - 2$  experiments

$$(g - 2)_\mu = (116591810 \pm 43) \times 10^{-11} \text{ (Theory)}$$

$$(g - 2)_\mu = (116592061 \pm 41) \times 10^{-11} \text{ (Experiment)}$$

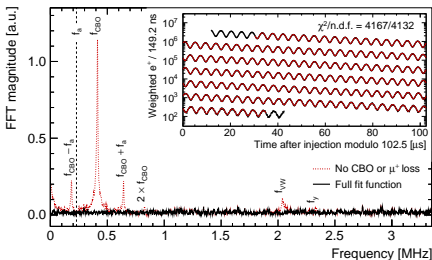
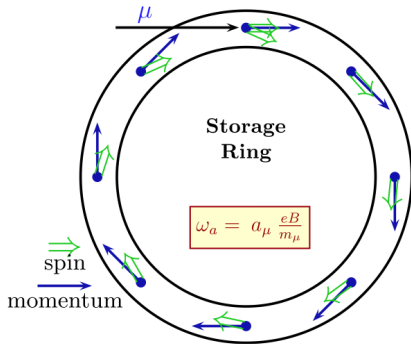
from [arXiv:2104.03281] is  $4.2\sigma$  from the SM expectation [arXiv:2006.04822]

- ▶ Is this a hint of a NP contribution to  $(g - 2)_\mu$  (review in [arXiv:0902.3360])?



# The $g - 2$ experiment

- ▶ Experiment at Fermilab aiming for  $\sim 0.1 - 0.2\text{ppm}$  precision
- ▶ The anomalous magnetic moment causes the spin to precess at a different rate to the momentum vector
- ▶ Can use this precession to precisely measure  $g - 2$

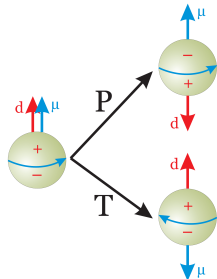


# Electric dipole moments

- ▶ Classically, EDMs are a measure of the spatial separation of positive and negative charges in a particle
  - ▶ A finite EDM can only exist if the charge centres do not coincide
- ▶ EDMs can also be measured for fundamental particles (electron, muon, neutron *etc.*)
  - ▶ Can interpret this as a measure of the “sphericity” of the particle
- ▶ This is tested using the Zeeman effect
  - ▶ Look for a shift in energy levels under an external electrical field (analogous to the magnetic moment)

$$\Delta E = -\vec{d} \cdot \vec{E} \quad (39)$$

- ▶ A non zero EDM would violate  $T$  and  $P$  symmetries
  - ▶ Under  $T$  reversal, the MDM would change direction but the EDM would remain unchanged
  - ▶ Under  $P$ , the EDM would change direction but the MDM remains unchanged
- ▶ Violation of  $P$  and  $T$  implies  $CP$  violation



# Electric dipole moments

- ▶ Electron EDM:
  - ▶  $d_e < 8.7 \times 10^{-29}$  [[arXiv:1310.7534](#)]
- ▶ Muon EDM:
  - ▶  $d_e < 1.9 \times 10^{-19}$  [[arXiv:0811.1207](#)]
- ▶ Neutron EDM:
  - ▶  $d_e < 3.0 \times 10^{-26}$  [[arXiv:hep-ex/0602020](#)]
- ▶ Probing incredibly small charge separation distances!

## Strong $CP$ problem

- ▶ The complicated nature of the QCD vacuum should give rise to a term in the Lagrangian like

$$\mathcal{L}_\theta = \theta \frac{\alpha_s}{8\pi} F_\alpha^{\mu\nu} \tilde{F}_{\alpha,\mu\nu} \quad (40)$$

- ▶ This is both  $P$  and  $T$ -violating but  $C$ -conserving (hence  $CP$ -violating)
- ▶ This terms would also contribute to the neutron dipole moment, but experimentally we know this is very small

$$d_n \sim e \cdot \theta \cdot m_q / M_N^2 \implies \theta \leq 10^{-9} \quad (41)$$

- ▶ This incredibly small size of the  $\theta$  parameter is (another) massive fine tuning problem (the so-called “strong  $CP$  problem”)
- ▶ What mechanism forces  $\theta$  to be so small?

- ▶ The Peccei-Quin solution to the strong  $CP$  problem is to introduce a  $U(1)$  symmetry that removes the strong  $CP$  problem by dynamically making  $\theta$  small
- ▶ Spontaneous breaking of this symmetry is associated with a pseudo-Nambu-Goldstone boson (in analogy with the Higgs mechanism), [the axion](#)
- ▶ The axion can be a light particle that couples very weakly to known SM particles
- ▶ There are a large number of searches for axions produced in particle colliders (direct searches)
- ▶ Can also be detected by the presence of axions converting into photons in the presence of a strong magnetic field (e.g. the CAST experiment at CERN)

### 8. The strong $CP$ problem

## Strong $CP$ problem

- ▶ The complicated nature of the QCD vacuum should give rise to a term in the Lagrangian like

$$\mathcal{L}_\theta = \theta \frac{\alpha_s}{8\pi} F_\alpha^{\mu\nu} \tilde{F}_{\alpha,\mu\nu} \quad (42)$$

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## 9. Recap

In this lecture we have covered

- ▶ Recap of the CKM matrix and unitarity triangles
- ▶ Measurements of the CKM matrix element magnitudes
  - ▶ In particular the sides of the unitarity triangle
  - ▶ The tension between inclusive and exclusive measurements of  $V_{ub}$
- ▶ Measurements of the CKM matrix angles
  - ▶ The angles  $\alpha$ ,  $\beta$ ,  $\gamma$  and  $\phi_s$
- ▶  $CP$  violation in the kaon system
- ▶ Global constraints on the CKM matrix and unitarity triangle(s)
- ▶  $T$  violation and  $CPT$
- ▶ Electric and magnetic dipole moments

End of Lecture 3

- [1] CKMfitter group, J. Charles *et al.*, *Current status of the standard model CKM fit and constraints on  $\Delta F = 2$  new physics*, *Phys. Rev.* **D91** (2015) 073007, [arXiv:1501.05013](https://arxiv.org/abs/1501.05013), updated results and plots available at <http://ckmfitter.in2p3.fr/>.
- [2] UTfit collaboration, M. Bona *et al.*, *The unitarity triangle fit in the standard model and hadronic parameters from lattice QCD: A reappraisal after the measurements of  $\Delta m_s$  and  $BR(B \rightarrow \tau\nu_\tau)$* , *JHEP* **10** (2006) 081, [arXiv:hep-ph/0606167](https://arxiv.org/abs/hep-ph/0606167), updated results and plots available at <http://www.utfit.org/>.
- [3] J. C. Hardy and I. S. Towner, *Superallowed  $0^+ \rightarrow 0^+$  nuclear  $\beta$  decays: 2020 critical survey, with implications for  $V_{ud}$  and CKM unitarity*, *Phys. Rev. C* **102** (2020) 045501.
- [4] Particle Data Group, R. L. Workman *et al.*, *Review of Particle Physics*, *PTEP* **2022** (2022) 083C01.
- [5] HFLAV, Y. S. Amhis *et al.*, *Averages of  $b$ -hadron,  $c$ -hadron, and  $\tau$ -lepton properties as of 2021*, [arXiv:2206.07501](https://arxiv.org/abs/2206.07501).

- [6] LHCb, R. Aaij *et al.*, *A precise measurement of the  $B^0$  meson oscillation frequency*, *Eur. Phys. J. C* **76** (2016) 412, [arXiv:1604.03475](#).
- [7] LHCb, R. Aaij *et al.*, *Precise determination of the  $B_s^0-\bar{B}_s^0$  oscillation frequency*, *Nature Phys.* **18** (2022) 1, [arXiv:2104.04421](#).
- [8] BaBar, B. Aubert *et al.*, *Measurement of Time-Dependent CP Asymmetry in  $B^0 \rightarrow c$  anti- $c$   $K^{(*)}0$  Decays*, *Phys. Rev. D* **79** (2009) 072009, [arXiv:0902.1708](#).
- [9] Belle, I. Adachi *et al.*, *Precise measurement of the CP violation parameter  $\sin 2\phi_1$  in  $B^0 \rightarrow (c\bar{c})K^0$  decays*, *Phys. Rev. Lett.* **108** (2012) 171802, [arXiv:1201.4643](#).
- [10] LHCb, R. Aaij *et al.*, *Measurement of CP violation in  $B^0 \rightarrow J/\psi K_S^0$  and  $B^0 \rightarrow \psi(2S)K_S^0$  decays*, *JHEP* **11** (2017) 170, [arXiv:1709.03944](#).
- [11] M. Gronau and D. London, *How to determine all the angles of the unitarity triangle from  $B(d)0 \rightarrow D K(s)$  and  $B(s)0 \rightarrow D0$* , *Phys. Lett. B* **253** (1991) 483.
- [12] M. Gronau and D. Wyler, *On determining a weak phase from CP asymmetries in charged B decays*, *Phys. Lett. B* **265** (1991) 172.

- [13] LHCb, R. Aaij *et al.*, *Measurement of CP observables in  $B^\pm \rightarrow D^{(*)}K^\pm$  and  $B^\pm \rightarrow D^{(*)}\pi^\pm$  decays using two-body D final states*, *JHEP* **04** (2021) 081, [arXiv:2012.09903](#).
- [14] D. Atwood, I. Dunietz, and A. Soni, *Enhanced CP violation with  $B \rightarrow K D0$  (anti- $D0$ ) modes and extraction of the CKM angle  $\gamma$* , *Phys. Rev. Lett.* **78** (1997) 3257, [arXiv:hep-ph/9612433](#).
- [15] D. Atwood, I. Dunietz, and A. Soni, *Improved Methods for Observing CP Violation in  $B^\pm \rightarrow KD$  and Measuring the CKM Phase  $\gamma$* , *Phys. Rev. D* **63** (2001) 036005, [arXiv:hep-ph/0008090](#).
- [16] A. Bondar, *Proceedings of BINP special analysis meeting on Dalitz analysis*, 2002.
- [17] A. Giri, Y. Grossman, A. Soffer, and J. Zupan, *Determining  $\gamma$  using  $B_{+-} \rightarrow DK_{+-}$  with multibody D decays*, *Phys. Rev. D* **68** (2003) 054018, [arXiv:hep-ph/0303187](#).

## References IV

- [18] Belle, A. Poluektov *et al.*, *Measurement of  $\phi(3)$  with Dalitz plot analysis of  $B^{+-} \rightarrow D^{**(*)} K_{+-}$  decay*, *Phys. Rev. D* **70** (2004) 072003, [arXiv:hep-ex/0406067](#).
- [19] LHCb, R. Aaij *et al.*, *Measurement of the CKM angle  $\gamma$  in  $B^{\pm} \rightarrow DK^{\pm}$  and  $B^{\pm} \rightarrow D\pi^{\pm}$  decays with  $D \rightarrow K_S^0 h^+ h^-$* , *JHEP* **02** (2021) 169, [arXiv:2010.08483](#).
- [20] R. D. Peccei and H. R. Quinn, *CP Conservation in the Presence of Instantons*, *Phys. Rev. Lett.* **38** (1977) 1440.