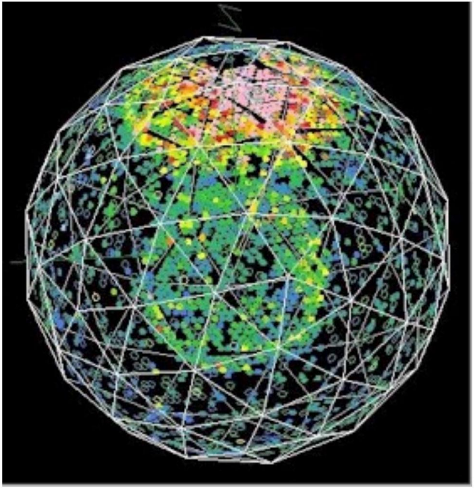
Dr C.G. Lester, 2023



H9: The Weak Interaction and $V - A$

Parity (again/reminder)

- **The parity operator performs spatial inversion through the origin:**

$$\psi'(\vec{x}, t) = \hat{P}\psi(\vec{x}, t) = \psi(-\vec{x}, t).$$

Doing this **twice** gets us back to the start, so $\hat{P}^{-1} = \hat{P}$.

- To preserve the normalisation of the wave-function

$$\langle \psi \mid \psi \rangle = \langle \psi' | \psi' \rangle = \left\langle \psi \middle| \hat{P}^\dagger \hat{P} \middle| \psi \right\rangle$$

so $\hat{P}^\dagger \hat{P} = I$ and thus $\hat{P}$ is **unitary**, i.e. $\hat{P}^{-1} = \hat{P}^\dagger$.

- The last two bullet points taken together mean that $\hat{P}^\dagger = \hat{P}$, i.e. that $\hat{P}$ is Hermitian, which implies that **parity is an observable quantity**. Furthermore, if the interaction Hamiltonian commutes with $\hat{P}$, then **parity is a conserved quantity**.
- If $\psi(\vec{x}, t)$ is an eigenfunction of the parity operator with eigenvalue P $\hat{P}\hat{P} = I$ forces observable parity values (eigenvalues) to be ± 1.
$[(\hat{P}\vec{x} = \lambda\vec{x}) \implies (\hat{P}\hat{P}\vec{x} = \lambda^2\vec{x}) \implies (\vec{x} = \lambda^2\vec{x}) \implies (\lambda = \pm 1).]$

- We will see later (page 355) that QED and QCD are invariant under parity, but that
- **the weak interactions does not conserve parity**.

Intrinsic Parities of fundamental particles

Spin-1 Bosons

From Gauge Field Theory can show that the gauge bosons have $P = -1$

$$P_\gamma = P_g = P_{W^+} = P_{W^-} = P_Z = -1.$$

Spin-$\frac{1}{2}$ Fermions

From the Dirac equation showed (Handout 2):
Spin-$\frac{1}{2}$ particles have opposite parity to spin-$\frac{1}{2}$ anti-particles. Conventional choice is:

Particles:	$P_{e^-} = P_{\mu^-} = P_{\tau^-} = P_\nu = P_q = +1,$
Anti-particles:	$P_{e^+} = P_{\mu^+} = P_{\tau^+} = P_{\bar{\nu}} = P_{\bar{q}} = -1.$

For Dirac spinors it was shown (in Handout 2) that the parity operator is:

$$\hat{P} = \gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

Parity transformation for adjoint spinors

The transformation we already know for ordinary spinors is:

$$u \xrightarrow{\hat{P}} u' \equiv \hat{P} u = \gamma^0 u. \qquad (123)$$

The transformation for adjoint spinors must therefore be:

$$\bar{u} \xrightarrow{\hat{P}} \bar{u}' \equiv \overline{\hat{P} u} = \bar{u}\hat{P} = \bar{u}\gamma^0. \qquad (124)$$

Above result follows from invariance of $\bar{u}u$ given $(\hat{P})^2 = 1$, or (if you prefer) because:

$$\begin{aligned}
\bar{u}' &\equiv \overline{\hat{P} u} && \text{(by definition of } u'\text{)} \\
&= (\hat{P} u)^\dagger \gamma^0 && \text{(by definition pf adjoint operation)} \\
&= (\gamma^0 u)^\dagger \hat{P} && \text{(since } \hat{P} = \gamma^0\text{)} \\
&= u^\dagger \gamma^{0\dagger} \hat{P} && \text{(by } (AB)^\dagger = B^\dagger A^\dagger\text{)} \\
&= u^\dagger \gamma^0 \hat{P} && \text{(since } \gamma^0 \text{ is Hermitiarn)} \\
&= \bar{u}\hat{P} && \text{(by definition of } \bar{u}\text{).}
\end{aligned}$$

Parity transformations for currents $j^\mu = \bar{\phi}\gamma^\mu\psi$:

Appendix V already proved that all currents transform as Lorentz four-vectors. However, that proof relied on Lorentz transformations being continuously connected to the identity.

Parity transformations are not continuously connected to the identity, so we need to **consider how currents transform under parity separately**.

From (123) and (124) we deduce the following transformations under Parity:

$$j^\mu = \bar{\phi}\gamma^\mu\psi \xrightarrow{\hat{P}} (j')^\mu = (\bar{\phi}\gamma^0)\gamma^\mu(\gamma^0\psi) = \bar{\phi}(\gamma^0\gamma^\mu\gamma^0)\psi$$

and so

$$\textbf{time:} \quad j^0 \longrightarrow \bar{\phi}\gamma^0\gamma^0\gamma^0\psi = \bar{\phi}\gamma^0(\gamma^0\gamma^0)\psi = \bar{\phi}\gamma^0\psi = j^0, \quad \text{and}$$

$$\textbf{space:} \quad j^i \longrightarrow \bar{\phi}\gamma^0\gamma^i\gamma^0\psi = -\bar{\phi}\gamma^i(\gamma^0\gamma^0)\psi = -\bar{\phi}\gamma^i\psi = -j^i.$$

Thus, even **under parity currents still behave like bona fide four-vectors**: the time part is unchanged and the space part changes sign:

$$\boxed{\left(j^0, +\vec{j}\right) \xrightarrow{\hat{P}} \left(j^0, -\vec{j}\right)} \tag{125}$$

thus **scalar products between currents** (or any four-vectors) **are invariant under parity**:

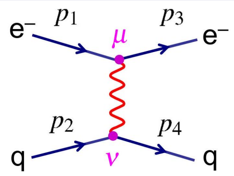
$$j_e \cdot j_q = j_e^0 j_q^0 - \vec{j}_e \cdot \vec{j}_q \xrightarrow{\hat{P}} j_e^0 j_q^0 - \left(-\vec{j}_e\right) \cdot \left(-\vec{j}_q\right) = j_e \cdot j_q. \tag{126}$$

Parity Conservation in QED and QCD

Consider the QED process $\mathrm{e^- q \to e^- q}$:

The Feynman rules give:

$$-iM \propto \left[\bar{u}_e\left(p_3\right) ie\gamma^{\mu} u_e\left(p_1\right)\right] \frac{-ig_{\mu\nu}}{q^2} \left[\bar{u}_q\left(p_4\right) ie\gamma^{\nu} u_q\left(p_2\right)\right]$$



which can be re-cast in terms of the electron and quark currents as

$$M \propto -\frac{e^2}{q^2} g_{\mu\nu} j_e^{\mu} j_q^{\nu} = -\frac{e^2}{q^2} j_e \cdot j_q.$$

with: $\boxed{j_e^{\mu} = \bar{u}_e\left(p_3\right) \gamma^{\mu} u_e\left(p_1\right)}$ and $\boxed{j_q^{\mu} = \bar{u}_q\left(p_4\right) \gamma^{\mu} u_q\left(p_2\right)}$.

We saw in (125) and (126) that such currents are *bona fide* four-vectors and that their dot products (and thus M above) are therefore invariant under parity.
The QCD vertex has the same spinor structure as that of QED, and the result is also valid for higher order matrix elements in QED and QCD, therefore:

QED and QCD conserve parity, and the predictions of these theories do not change when and experiment is parity-inverted (i.e. when it is mirrored).

Vectors vs Axial-Vectors

Under a parity transformation vectors and axial-vectors transform differently:

$$\text{Vectors change sign}\begin{cases}\vec{r} \xrightarrow{\hat{P}} -\vec{r} & \\ \vec{p} \xrightarrow{\hat{P}} -\vec{p} & \qquad \left(p_x = \frac{\partial}{\partial x},\ \text{etc.}\right)\end{cases}$$

$$\text{Axial-Vectors are unchanged}\begin{cases}\vec{L} \xrightarrow{\hat{P}} +\vec{L} & \qquad (\vec{L} = \vec{r} \wedge \vec{p}) \\ \vec{\mu} \xrightarrow{\hat{P}} +\vec{\mu} & \qquad (\vec{\mu} \propto \vec{L})\end{cases}$$

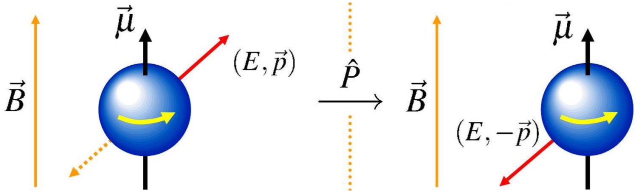
Parity Violation in β-Decay

- In 1957 C.S.Wu et al. studied beta decay of polarized cobalt-60 nuclei:

$$^{60}\mathrm{Co} \to {}^{60}Ni^{*} + e^{-} + \bar{\nu}_e .$$

  and observed that **electrons were emitted preferentially in direction opposite to applied B-field** !

- Having such a preference (*irrespective of whether it favours 'opposite to' or 'in the same direction as'* $\vec{B}$) is incompatible with parity being a symmetry of nature because parity transforms vector $\vec{p}$ and axial-vector $\vec{B}$ differently:



- If parity were a symmetry of nature we would expect equal rate for producing e^{-}in directions along and opposite to the nuclear spin.

Conclusion: parity is somehow violated by the weak interaction.

$\Longrightarrow$ The weak interaction vertex is not of the form $\bar{u}_e \gamma^{\mu} u_{\nu}$!

Bilinear Covariants

- The requirement of Lorentz invariance of the matrix element severely restricts the form of the interaction vertex.
- QED and QCD are called **vector interactions** because their currents:

$$j^\mu = \bar{\psi}\gamma^\mu\phi$$

transform as **non-axial four-vectors** (Appendix V and (125)).

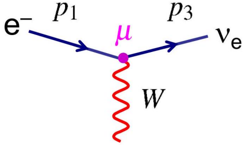
- In general, there are only 5 possible combinations of two spinors and the gamma matrices that form Lorentz covariant currents. They are the so-called "bilinear covariants":

Type	Form	Components	Boson Spin
Scalar	$\bar{\psi}\phi$	1	0
Pseudoscalar	$\bar{\psi}\gamma^5\phi$	1	0
Vector	$\bar{\psi}\gamma^\mu\phi$	4	1
Axial Vector	$\bar{\psi}\gamma^\mu\gamma^5\phi$	4	1
Tensor	$\bar{\psi}\left(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu\right)\phi$	6	2

- Note that in total the sixteen components correspond to the 16 elements of a general 4x4 matrix: "decomposition into Lorentz covariant combinations".

'$V - A$'-structure of the Weak Interaction

- The most general form for the interaction between a fermion and a boson is a linear combination of bilinear covariants.
- For an interaction corresponding to the exchange of a spin-1 boson the most general form is a linear combination of VECTOR AND AXIAL-VECTOR currents.
- Experimentally, the weak interaction's current is determined VECTOR minus AXIAL VECTOR — called '$V - A$' for short:



$$\boxed{j^\mu \propto \bar{u}_{\nu_e}(\gamma^\mu - \gamma^\mu\gamma^5)u_e \quad \mathbf{V - A}}$$

⋆ Can this account for parity violation?

⋆ First consider parity transformation of a pure AXIAL-VECTOR current

$$j_A = \bar{\psi}\gamma^\mu\gamma^5\phi \quad \text{with} \quad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3; \quad \gamma^5\gamma^0 = -\gamma^0\gamma^5$$

$$j_A = \bar{\psi}\gamma^\mu\gamma^5\phi \xrightarrow{\hat{P}} \bar{\psi}\gamma^0\gamma^\mu\gamma^5\gamma^0\phi = -\bar{\psi}\gamma^0\gamma^\mu\gamma^0\gamma^5\phi$$

$$j_A^0 = \xrightarrow{\hat{P}} -\bar{\psi}\gamma^0\gamma^0\gamma^0\gamma^5\phi = -\bar{\psi}\gamma^0\gamma^5\phi = -j_A^0$$

$$j_A^k = \xrightarrow{\hat{P}} -\bar{\psi}\gamma^0\gamma^k\gamma^0\gamma^5\phi = +\bar{\psi}\gamma^k\gamma^5\phi = +j_A^k \quad k = 1, 2, 3.$$

- The space-like components remain unchanged and the time-like components change sign (the opposite to the parity properties of a vector-current):

$$\boxed{\text{Vector}: \quad \left(j^0, \vec{j}\right) \xrightarrow{\hat{P}} \left(j^0, -\vec{j}\right)} \quad \text{versus} \quad \boxed{\text{Axial}: \quad \left(j^0, \vec{j}\right) \xrightarrow{\hat{P}} \left(-j^0, \vec{j}\right)}$$

- Now consider the matrix element:

$$M \propto g_{\mu\nu} j_1^{\mu} j_2^{\nu} = j_1^0 j_2^0 - \vec{j}_1 \cdot \vec{j}_2.$$

- An M containing a **combination of a two axial-vector currents is invariant under parity** because:

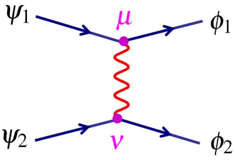
$$j_{A1} \cdot j_{A2} \xrightarrow{\hat{P}} \left(\left(-j_{A1}^0\right)\left(-j_{A2}^0\right) - \left(\vec{j}_{A1}\right) \cdot \left(\vec{j}_{A2}\right)\right) = j_{A1} \cdot j_{A2}$$

- Consequently parity is conserved for both a **pure vector** and **pure axial-vector** interactions.
- However, a scalar product between a **vector** current **and** an **axial vector** current:

$$j_{V1} \cdot j_{A2} \xrightarrow{\hat{P}} \left(\left(j_{V1}^0\right)\left(-j_{A2}^0\right) - \left(-\vec{j}_{V1}\right) \cdot \left(\vec{j}_{A2}\right)\right) = -j_{V1} \cdot j_{A2}$$

changes sign under parity. This can **interfere** with parity invariant terms in an M having both, to give parity violating cross sections!

Consider a general linear combination of VECTOR and AXIAL-VECTOR (note this is relevant for the Z-boson vertex)



$$\left\{ \begin{array}{c} j_1 = \overline{\phi}_1 (g_V \gamma^\mu + g_A \gamma^\mu \gamma^5) \psi_1 = g_V j_1^V + g_A j_1^A \\ \dfrac{g_{\mu\nu}}{q^2 - m^2} \\ j_2 = \overline{\phi}_2 (g_V \gamma^\mu + g_A \gamma^\mu \gamma^5) \psi_2 = g_V j_2^V + g_A j_2^A \end{array} \right.$$

$$M_{fi} \propto j_1 \cdot j_2 = g_V^2 \, j_1^V \cdot j_2^V + g_A^2 \, j_1^A \cdot j_2^A + g_V g_A \left(j_1^V \cdot j_2^A + j_1^A \cdot j_2^V \right).$$

- Consider the parity transformation of this scalar product

$$j_1 \cdot j_2 \xrightarrow{\hat{P}} g_V^2 \, j_1^V \cdot j_2^V + g_A^2 \, j_1^A \cdot j_2^A - g_V g_A \left(j_1^V \cdot j_2^A + j_1^A \cdot j_2^V \right).$$

- If **either** g_A **or** g_V **is zero, parity is conserved**. (Theories with pure vector or pure axial-vector interactions have parity symmetric cross sections.)
- Relative strength of parity violating part is proportional to:

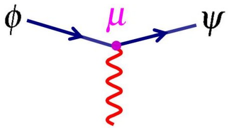
$$\frac{g_V g_A}{g_V^2 + g_A^2}.$$

Chiral Structure of QED (reminder of material from Handout 4)

- Chiral projections operators are: $P_R = \frac{1}{2}\left(1+\gamma^5\right)$; $\quad P_L = \frac{1}{2}\left(1-\gamma^5\right)$.
- In the ultra-relativistic limit, chiral states correspond to helicity states.

- Any spinor can be expressed as:

$$\psi \equiv \frac{1}{2}\left(1+\gamma^5\right)\psi + \frac{1}{2}\left(1-\gamma^5\right)\psi \equiv P_R\psi + P_L\psi \equiv \psi_R + \psi_L.$$

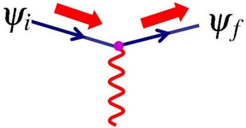


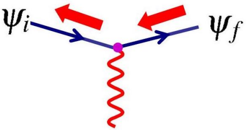
- The QED current $\bar{\psi}\gamma^\mu\phi$ could be written (see page 190) in terms of chiral states as:

$$\bar{\psi}\gamma^\mu\phi \equiv \bar{\psi}_R\gamma^\mu\phi_R + \bar{\psi}_L\gamma^\mu\phi_L.$$

since $\bar{\psi}_R\gamma^\mu\phi_L \equiv 0$ and $\bar{\psi}_L\gamma^\mu\phi_R \equiv 0$.

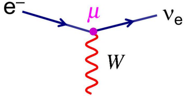
- A consequence of the above is that in the ultra-relativistic limit only two helicity combinations are non-zero:





Chiral and Helicity Structure of the Weak Interaction

⋆ The charged current $\left(W^{\pm}\right)$ weak vertex is:



$$\frac{-ig_W}{\sqrt{2}}\frac{1}{2}\gamma^\mu\left(1-\gamma^5\right).$$

⋆ Since $\frac{1}{2}\left(1-\gamma^5\right)$ projects out left-handed chiral particle states:

$$\bar{\psi}\frac{1}{2}\gamma^\mu\left(1-\gamma^5\right)\phi=\bar{\psi}\gamma^\mu\phi_L.$$

⋆ Writing $\bar{\psi}=\bar{\psi}_R+\bar{\psi}_L$ and from discussion of QED, $\bar{\psi}_R\gamma^\mu\phi_L=0$ gives

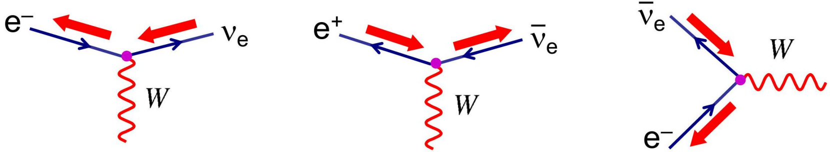
$$\bar{\psi}\frac{1}{2}\gamma^\mu\left(1-\gamma^5\right)\phi=\bar{\psi}_L\gamma^\mu\phi_L.$$

Only the left-handed chiral components of particle spinors and right-handed chiral components of anti-particle spinors participate in charged current weak interactions.

At very high energy ($E \gg m$), the left-handed chiral components are helicity eigenstates, so:

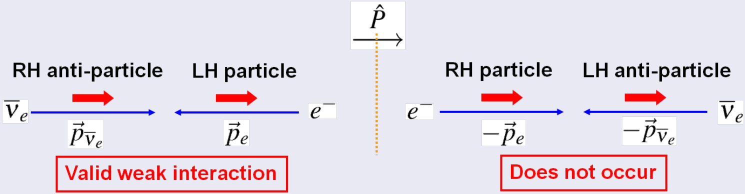
In the ultra-relativistic limit only left-handed particles and right-handed antiparticles participate in charged current weak interactions.

E.g.: in the relativistic limit, the only possible electron-neutrino interactions are:



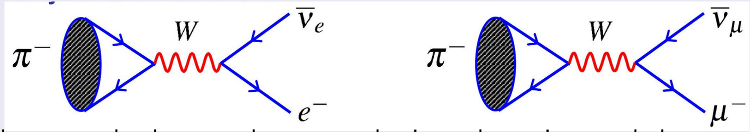
Example of parity violating consequence:

Annihilation of the form: e.g. $\bar{\nu}_e + e^- \to W^-$ is spin-dependent:



Helicity in Pion Decay

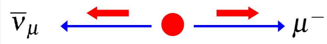
Pion decay demonstrates importance of helicity in the weak interaction.



- Might expect the decay to electrons to dominate due to increased phase space, but the opposite happens. The electron decay is helicity suppressed.
- Experimentally see something **very different**: $\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} = 1.23 \times 10^{-4}$.

Consider decay in pion rest frame.

- Pion is spin zero: so the spins of the $\bar{\nu}$ and μ are opposite.
- Weak interaction only couples to RH chiral anti-particle states. Since neutrinos are (almost) massless, neutrino must be in RH Helicity state.
- Therefore, to conserve angular mom. muon is emitted in a RH HELICITY state:



... but only left-handed CHIRAL particle states participate in weak interaction!

Helicity in pion decay (cont)

The general right-handed helicity solution to the Dirac equation is $u_\uparrow = N \begin{pmatrix} c \\ e^{i\phi} s \\ \frac{|\vec{p}|}{E+m} c \\ \frac{|\vec{p}|}{E+m} e^{i\phi} s \end{pmatrix}$.

Project out the left-handed chiral part of the wave-function using

$$P_L = \frac{1}{2}\left(1 - \gamma^5\right) = \frac{1}{2}\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}$$

giving

$$P_L u_\uparrow = \frac{1}{2} N \left(1 - \frac{|\vec{p}|}{E+m}\right) \begin{pmatrix} c \\ e^{i\phi} S \\ -c \\ -e^{i\phi} s \end{pmatrix} = \frac{1}{2}\left(1 - \frac{|\vec{p}|}{E+m}\right) u_L.$$

In the limit $m \ll E$ we see that $P_L u_\uparrow$ tends to zero. Similarly:

$$P_R u_\uparrow = \frac{1}{2} N \left(1 + \frac{|\vec{p}|}{E+m}\right) \begin{pmatrix} c \\ e^{i\phi} S \\ c \\ e^{i\phi} S \end{pmatrix} = \frac{1}{2}\left(1 + \frac{|\vec{p}|}{E+m}\right) u_R.$$

In the limit $m \ll E$, $\quad P_R u_\uparrow \to u_R$

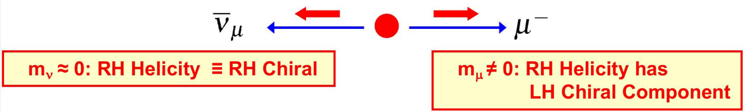
Helicity in pion decay (cont)

Hence

$$u_\uparrow = P_R u_\uparrow + P_L u_\uparrow = \frac{1}{2}\left(1 + \frac{|\vec{p}|}{E+m}\right) u_R + \frac{1}{2}\left(1 - \frac{|\vec{p}|}{E+m}\right) u_L.$$

In the limit $E \gg m$, as expected, the RH chiral and helicity states are identical.

Although only LH chiral particles participate in the weak interaction, the contribution from RH Helicity states is not necessarily zero:



Expect matrix element to be proportional to LH chiral component of RH Helicity electron/muon spinor so (using special case of Ex.Sheet Q3):

$$M_{fi} \propto \frac{1}{2}\left(1 - \frac{|\vec{p}|}{E+m}\right) = \frac{m_\mu}{m_\pi + m_\mu}.$$

Hence because the electron mass is much smaller than the pion and muon masses the decay $\pi^- \to e^- \bar{\nu}_e$ is heavily suppressed relative to $\pi^- \to \mu^- \bar{\nu}_\mu$.

Examples of evidence for V-A

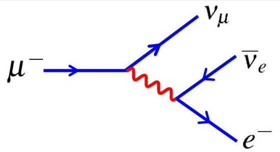
Charged pion decay (Ex.Sheet Q17)

Experimentally measure: $\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} = (1.230 \pm 0.004) \times 10^{-4}$.

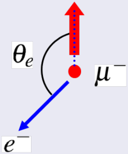
Theoretical predictions depend on Lorentz Structure of the interaction:

- **V-A and V+A** $(\bar{\psi}\gamma^\mu(1 \mp \gamma^5)\phi)$ predict . . . . $\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} \approx 1.3 \times 10^{-4}$.
- **Scalar** $(\bar{\psi}\phi)$ **and Pseudo-Scalar** $(\bar{\psi}\gamma^5\phi)$ predict . . . . $\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} \approx 5.5$.

Muon decay



Measure electron energy and angular distributions relative to muon spin direction. Results expressed in terms of general S+P+V+A+T form in 'Michel Parameters'.

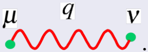


Measurement of TWIST expt $(6 \times 10^9 \mu$ decays,
Phys. Rev. Lett. 95 (2005) 101805) was . . . . $\rho = 0.75080 \pm 0.00105$.
V-A prediction: . . . . $\rho = 0.75$.

Weak Charged Current Propagator

The charged-current weak interaction is different from QED and QCD in that it is mediated by massive W-bosons ($M_W \approx 80.3\ \mathrm{GeV}$). The W propagator is thus different:

- As seen in Handout 4 denominator changes: $\frac{1}{q^2} \longrightarrow \frac{1}{q^2 - m^2}$.
- In addition the sum over **W** boson polarization states modifies the numerator.

Resulting W-**boson propagator** is: $\dfrac{-i\left[g_{\mu\nu} - q_\mu q_\nu / m_W^2\right]}{q^2 - m_W^2}$ .

However in the limit where q^2 is small compared with $m_W = 80.3\ \mathrm{GeV}$ the interaction takes a simpler form:

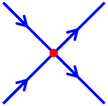
W-boson propagator in the limit $\left(q^2 \ll m_W^2\right)$:

$$\frac{i g_{\mu\nu}}{m_W^2}.$$

The interaction appears point-like (i.e. no q^2 dependence).

Connection to Fermi Theory

In 1934, before the discovery of parity violation, Fermi proposed, in analogy with QED, that the invariant matrix element for β-decay was of the form:



$$M_{fi} = G_F \cdot g_{\mu\nu} \left[\bar{\psi}\gamma^{\mu}\psi\right] \left[\bar{\psi}\gamma^{\nu}\psi\right]$$

where $G_F = 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$.

Note the absence of a propagator: this represents an interaction at a point!

After the discovery of parity violation in 1957 this was modified to

$$\boxed{M_{fi} = \frac{G_F}{\sqrt{2}} g_{\mu\nu} \left[\bar{\psi}\gamma^{\mu}\left(1-\gamma^5\right)\psi\right] \left[\bar{\psi}\gamma^{\nu}\left(1-\gamma^5\right)\psi\right]}$$

(the factor of $\sqrt{2}$ was included so the numerical value of G_F did not need to be changed).

Compare to the prediction for W-boson exchange:

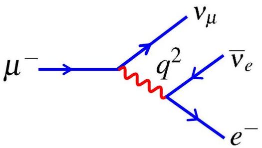
$$M_{fi} = \left[\frac{g_W}{\sqrt{2}}\bar{\psi}\frac{1}{2}\gamma^{\mu}\left(1-\gamma^5\right)\psi\right] \frac{g_{\mu\nu} - q_\mu q_\nu / m_W^2}{q^2 - m_W^2} \left[\frac{g_W}{\sqrt{2}}\bar{\psi}\frac{1}{2}\gamma^{\nu}\left(1-\gamma^5\right)\psi\right]$$

which for $q^2 \ll m_W^2$ becomes: $\boxed{M_{fi} = \frac{g_W^2}{8m_W^2} g_{\mu\nu} \left[\bar{\psi}\gamma^{\mu}\left(1-\gamma^5\right)\psi\right] \left[\bar{\psi}\gamma^{\nu}\left(1-\gamma^5\right)\psi\right]}$

and so consistency requires $\boxed{\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2}}$.

Still often use G_F to express strength of weak interaction as the is the quantity that is precisely determined in muon decay.

Strength of Weak Interaction as measured in muon decay



- Here $q^2 < m_\mu \sim 0.106\ \mathrm{GeV}$.
- To a very good approximation the W-boson propagator can be written: $\dfrac{-i\left[g_{\mu\nu} - q_\mu q_\nu / m_W^2\right]}{q^2 - m_W^2} \approx \dfrac{i g_{\mu\nu}}{m_W^2}$.
- In muon decay measure g_W^2/m_W^2.
- Convert to G_F measurement using $\boxed{\dfrac{G_\mathrm{F}}{\sqrt{2}} = \dfrac{g_W^2}{8 m_W^2}}$.
- Muon decay measurements find $G_F = 1.16639(1) \times 10^{-5}\ \mathrm{GeV}^{-2}$
- To obtain the intrinsic strength of weak interaction need to know mass of W-boson: $m_W = 80.403 \pm 0.029 \mathrm{GeV}$

$$\Rightarrow \quad \alpha_W = \frac{g_W^2}{4\pi} = \frac{8 m_W^2 G_\mathrm{F}}{4\sqrt{2}\pi} \approx \frac{1}{30}.$$

The intrinsic strength of the weak interaction is similar to, but greater than, the EM interaction! It is the massive W-boson in the propagator which makes it appear weak. For $q^2 \gg m_W^2$ weak interactions are more likely than EM.

Summary

- Weak interaction vertex has 'vector minus axial-vector' parts and so is termed a '$V - A$'-interaction:

$$(\text{weak vertex factor}) = \frac{-ig_W}{\sqrt{2}}\frac{1}{2}\gamma^\mu\left(1-\gamma^5\right).$$

- Consequently only left-handed chiral particle states and right-handed chiral anti-particle states participate in the weak interaction.
- $V - A$ is therefore also a form of 'Maximal Parity Violation'.
- The Weak interaction also violates charge conjugation symmetry.
- At low q^2 the weak interaction is only weak because of the large W-boson mass:

$$\frac{G_\mathrm{F}}{\sqrt{2}} = \frac{g_W^2}{8m_W^2}.$$

- Intrinsic strength of weak interaction is similar to that of QED.