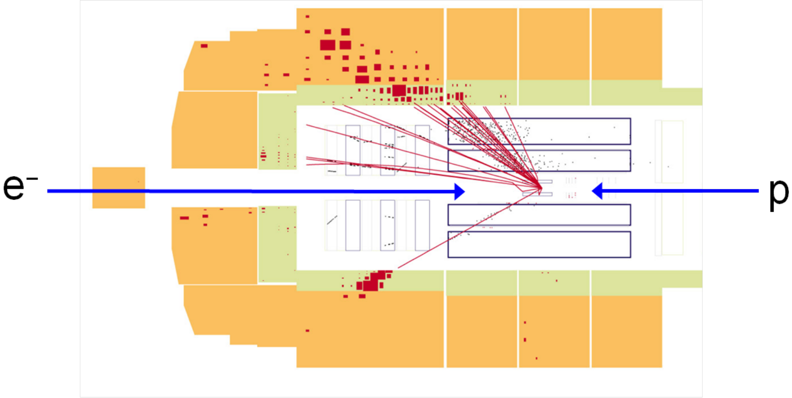
Dr C.G. Lester, 2023



H6: Deep Inelastic Scattering

Electron-Proton Elastic Scattering at High Q^2 $(= -q^2)$

- At high Q^2 the Rosenbluth expression for elastic scattering becomes

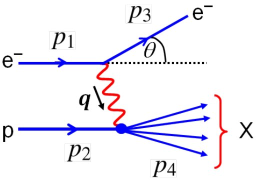
$$\left(\frac{d\sigma}{d\Omega}\right)_{elastic} = \frac{\alpha^2}{4E_1^2 \sin^4\theta/2}\frac{E_3}{E_1}\left(\frac{q^2}{2M^2}G_M^2 \sin^2\frac{\theta}{2}\right) \qquad \text{with} \qquad \tau = -\frac{q^2}{4M^2} \gg 1$$

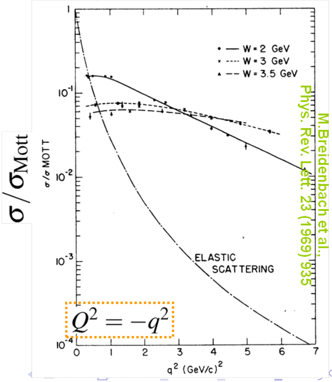
- From $e^- p$ elastic scattering, the proton magnetic form factor is

$$G_M(q^2) \approx \frac{1}{(1+q^2/(0.71\,\mathrm{GeV}^2))^2} \qquad \text{so} \qquad G_M(q^2) \propto q^{-4} \text{at high } q^2$$

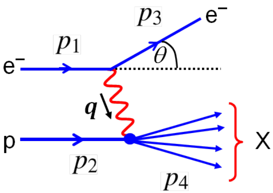
$$\Longrightarrow \quad \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{elastic}} \propto q^{-6}.$$

- Due to the finite proton size, elastic scattering at high Q^2 is unlikely and inelastic reactions where the proton breaks up dominate.





Kinematic Variables for Inelastic Scattering



- For inelastic scattering the mass M_X of the final state hadronic system X is larger than the proton mass, M, since X contains at least one baryon.
- Therefore: $M_X \geq M$ with equality for elastic and inequality for inelastic scattering.

- For inelastic scattering we will introduce four new kinematic variables:

$$Q^2 \equiv -q^2, \qquad x \equiv \frac{Q^2}{2p_2 \cdot q}, \qquad y \equiv \frac{p_2 \cdot q}{p_2 \cdot p_1} \qquad \text{and} \qquad \nu \equiv \frac{p_2 \cdot q}{M}.$$

('Bjorken x') ('Bjorken y')

Properties of x and Q^2:

1. $Q^2 \geq 0$
   (since $Q^2 = -(p_1 - p_3)^2 = 2p_1 \cdot p_3 = 2(|p_1||p_3| - \vec{p}_1 \cdot \vec{p}_3) \geq 0$),
2. $2p_2 \cdot q \geq Q^2$ (with equality for elastic and inequality for inelastic scattering)
   (since: $2p_2 \cdot q = (p_2 + q)^2 - p_2^2 - q^2 = p_4^2 - p_2^2 + Q^2 = M_X^2 - M^2 + Q^2 \geq Q^2$),
3. and so $0 < x < 1$ for inelastic and $x = 1$ for elastic scattering.

(In many text books W is often used in place of M_X.)

Properties of Bjorken y:

1. In the lab frame $p_1 = (E_1, 0, 0, E_1)$, $p_2 = (M, 0, 0, 0)$ and $q = (E_1 - E_3, \vec{p}_1 - \vec{p}_3)$ so

$$y \equiv \frac{p_2 \cdot q}{p_2 \cdot p_1} = \frac{M(E_1 - E_3)}{ME_1} = 1 - \frac{E_3}{E_1}$$

   and so y is the fractional energy loss of the incoming particle in the lab frame.

2. In the C.o.M. Frame (neglecting the electron and proton masses) $p_1 = (E, 0, 0, E)$, $p_2 = (E, 0, 0, -E)$ and $p_3 = (E, E\sin\theta^*, 0, E\cos\theta^*)$ so for $E \gg M$:

$$y = \frac{1}{2}(1 - \cos\theta^*).$$

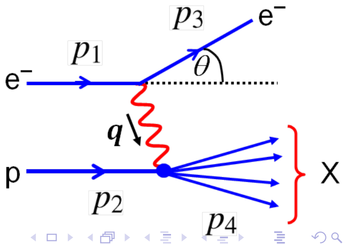
3. $0 < y < 1$ follows from either of the last two properties.

Properties of ν:

1. In the Lab Frame:

$$\nu \equiv \frac{p_2 . q}{M} = \frac{M(E_1 - E_3)}{M} = E_1 - E_3$$

   so ν is the energy lost by the incoming particle in the lab frame.



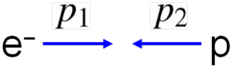
Relationships between Kinematic Variables for inelastic collisions

- We saw in lecture 1 that elastic collisions **need only two variables** to describe their kinematic differences, as all four external particles have fixed masses.
  (These two variables are usually chosen to be Mandelstam s together with one of t or u).)
- However, in inelastic collisions the proton breaks up, and so we need an extra variable to describe the variability in the mass of p_4:
  inelastic collisions **require three variables** to describe their kinematic differences.

Thus, at fixed s, at most two of

$$Q^2 \equiv -q^2, \quad x \equiv \frac{Q^2}{2p_2.q}, \quad y \equiv \frac{p_2.q}{p_2.p_1} \quad \text{and} \quad \nu \equiv \frac{p_2.q}{M}$$

are independent! There must exist many relationships among them.



- E.g. using $s = (p_1 + p_2)^2 = m_e^2 + M^2 + 2p_1 \cdot p_2 \approx 2p_1 \cdot p_2 + M^2$ we can show (when m_e is negligible) that:

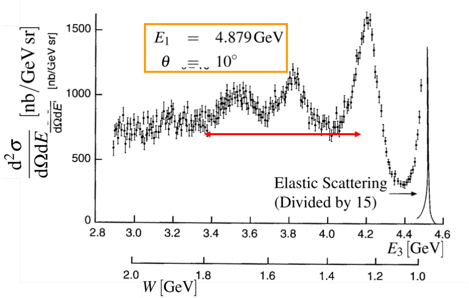
$$x = \frac{Q^2}{2M\nu} \quad \text{and} \quad y = \frac{2M}{s - M^2}\nu$$

so

$$xy = \frac{Q^2}{s - M^2} \quad \text{and} \quad Q^2 = (s - M^2)xy.$$

Elastic vs Inelastic vs Deep Inelastic Scattering

Example: Scattering of 4.879 GeV electrons from protons at rest



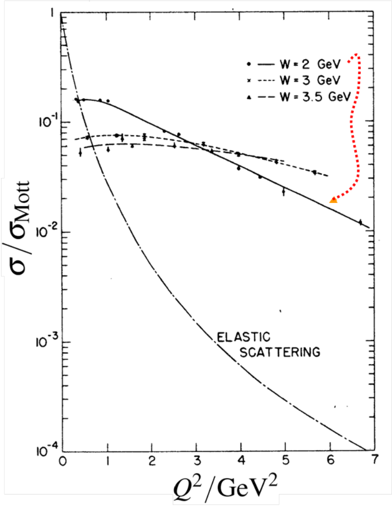
- Place detector at 10° to beam and measure the energies of scattered e^-.
- Kinematics fully determined from the electron energy and angle !
- e.g. for this energy and angle : the invariant mass of the final state hadronic system $W^2 = M_X^2 = 10.06 - 2.03E_3$. (Try and show this!)

- **Elastic Scattering**: proton remains intact ($W = M$).
- **Inelastic Scattering** (red arrow on plot) produces 'excited states' of proton, e.g. $\Delta^+(1232)$, $W = M_\Delta$.
- **Deep Inelastic Scattering** is where the proton breaks up completely resulting in a many particle final state (far L.H.S. of plot where $W \gg M$).

Deep Inelastic Scattering = Large W = Total proton breakup.

Inelastic Cross Sections

M.Breidenbach et al.,
Phys. Rev. Lett. 23 (1969) 935



- Repeat experiments at different angles/beam energies and determine Q^2 dependence of elastic and inelastic cross-sections.
- Elastic scattering falls of rapidly with due to the proton not being point-like (i.e. form factors).
- Inelastic scattering cross sections only weakly dependent on Q^2.
- Deep Inelastic scattering cross sections almost independent of Q^2.
  i.e. "Form factor" $\rightarrow 1$.

There appears to be Scattering from point-like objects within the proton!

Elastic → Inelastic Scattering

Elastic scattering (Handout 5). Recall:

- Only one independent variable desscribes final state. In Lab. frame express differential cross section in terms of the electron scattering angle (Rosenbluth formula)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4 \theta/2} \frac{E_3}{E_1} \left(\frac{G_E^2 + \tau G_M^2}{(1+\tau)} \cos^2 \frac{\theta}{2} + 2\tau G_M^2 \sin^2 \frac{\theta}{2} \right) \qquad (\text{with } \tau = \frac{Q^2}{4M^2}).$$

  Note: here the energy of the scattered electron E_3 is completely determined by the angle θ.

- We could express the differential cross section in terms of a single variable Q^2. (Q13 on examples sheet.)

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[\frac{G_E^2 + \tau G_M^2}{(1+\tau)} \left(1 - y - \frac{M^2 y^2}{Q^2} \right) + \frac{1}{2} y^2 G_M^2 \right]$$

  which can be written as

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[f_2(Q^2) \left(1 - y - \frac{M^2 y^2}{Q^2} \right) + \frac{1}{2} y^2 f_1(Q^2) \right].$$

Inelastic scattering:

- For Deep Inelastic Scattering (DIS) have two independent variables for final sate. Therefore need a double differential cross section for DIS ... as shown on next page.

Deep Inelastic Scattering

- It can be shown that the most general Lorentz Invariant expression for $e^- p \to e^- X$ inelastic scattering (via a single exchanged photon is):

$$\frac{d^2\sigma}{dx\, dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1 - y - \frac{M^2y^2}{Q^2}\right)\frac{F_2(x, Q^2)}{x} + y^2 F_1(x, Q^2)\right] \quad \boxed{\text{INELASTIC}} \tag{112}$$

c.f. $$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1 - y - \frac{M^2y^2}{Q^2}\right) f_2(Q^2) + \frac{1}{2}y^2 f_1(Q^2)\right] \quad \boxed{\text{ELASTIC}}$$

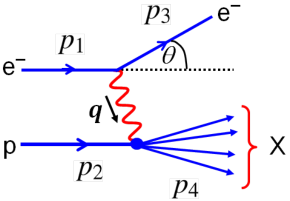
  We will soon see how this connects to the quark model of the proton.

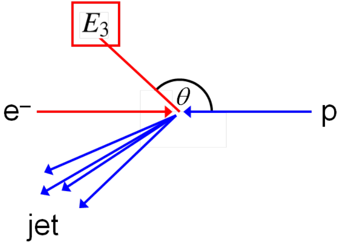
- NOTE: The **form factors** $f_1(Q^2)$ and $f_2(Q^2)$ have been replaced by the **structure functions** $F_1(x, Q^2)$ and $F_2(x, Q^2)$ which are functions of x and Q^2: they cannot be interpreted as the Fourier transforms of the charge and magnetic moment distributions. We shall soon see that they describe the **momentum distribution** of the quarks within the proton.

- In the limit of high energy (or more correctly $Q^2 \gg M^2y^2$) equation (112) becomes:

$$\boxed{\frac{d^2\sigma}{dx\, dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1 - y)\frac{F_2(x, Q^2)}{x} + y^2 F_1(x, Q^2)\right]}. \tag{113}$$

- In the Lab. frame it is convenient to express the cross section in terms of the angle, θ, and energy, E_3, of the scattered electron – which is experimentally well measured.





$$Q^2 = 4E_1E_3\sin^2\theta/2; \quad x = \frac{Q^2}{2M(E_1 - E_3)}; \quad y = 1 - \frac{E_3}{E_1}; \quad \nu = E_1 - E_3$$

- In the Lab. frame, Equation (113) becomes: (see examples sheet Q13)

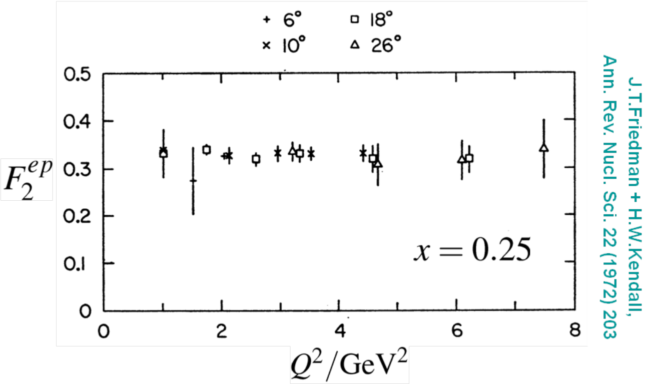
$$\frac{d^2\sigma}{dE_3\,d\Omega} = \frac{\alpha^2}{4E_1^2\sin^4(\theta/2)}\left[\frac{1}{\nu}F_2(x, Q^2)\cos^2\left(\frac{\theta}{2}\right) + \frac{2}{M}F_1(x, Q^2)\sin^2\left(\frac{\theta}{2}\right)\right] \quad (114)$$

Electromagnetic Structure Function ; Pure Magnetic Structure Function

Measuring the Structure Functions

- To determine $F_1(x, Q^2)$ and $F_2(x, Q^2)$ for a given x and Q^2 need measurements of the differential cross section at several different scattering angles and incoming electron beam energies (see Q13 on examples sheet)

  Example: electron-proton scattering F_2 vs. Q^2 at fixed x:



- Experimentally it is observed that both F_1 and F_2 are (almost) independent of Q^2!

Bjorken Scaling and the Callan-Gross Relation

- The near (see later) independence of the structure functions on Q^2 is known as **Bjorken Scaling**, i.e.

$$F_1(x, Q^2) \to F_1(x) \quad \text{and} \quad F_2(x, Q^2) \to F_2(x).$$

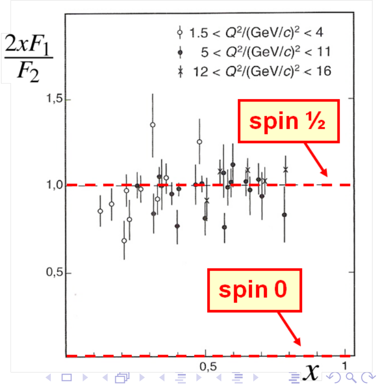
- It is **strongly suggestive of scattering from point-like constituents** within the proton

- It is also observed that $F_1(x)$ and $F_2(x)$ are not independent but satisfy the **Callan-Gross relation**

$$F_2(x) = 2xF_1(x).$$

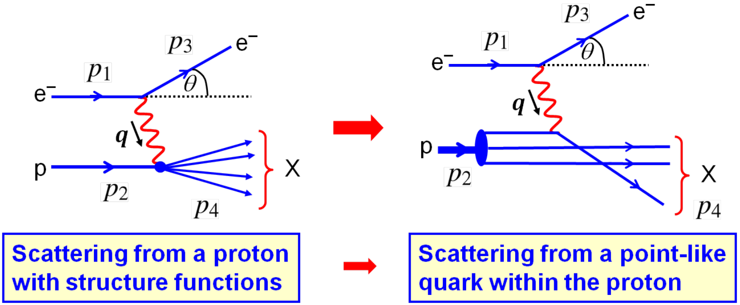
- As we shall soon see this is exactly what is expected for scattering from spin-half quarks.

- **Note:** if quarks were spin zero particles we would expect the purely magnetic structure function to be zero, i.e. $F_1(x) = 0$.



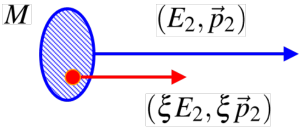
The Quark-Parton Model

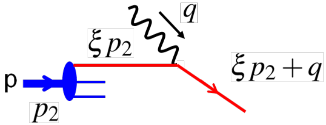
- Before quarks and gluons were generally accepted Feynman proposed that the proton was made up of point-like constituents "partons".
- Both **Bjorken Scaling** and the **Callan-Gross relationship** can be explained by assuming that Deep Inelastic Scattering is dominated by the scattering of a single virtual photon from point-like spin-half constituents of the proton.



- How do these two pictures of the interaction relate to each other?

- In the parton model the basic interaction is ELASTIC scattering from a "quasi-free" spin-$\frac{1}{2}$ quark in the proton, i.e. treat the quark as a free particle!
- The parton model is most easily formulated in a frame where the proton has very high energy, often referred to as the "infinite momentum frame", where we can neglect the proton mass, and $p_2 = (E_2, 0, 0, E_2)$.
- In this frame can also neglect the mass of the quark and any momentum transverse to the direction of the proton.
- Let the quark carry a fraction ξ of the proton's four-momentum. $(\xi^2 p_2^2 = m_q^2 \approx 0)$





- After the interaction the struck quark's four-momentum is $\xi p_2 + q$ and it is still a quark, therefore: $m_q^2 = (\xi p_2 + q)^2 = (\xi p_2)^2 + 2\xi p_2 \cdot q + q^2 = m_q^2 + 2\xi p_2 \cdot q + q^2$ and so comparing the first and last expressions we see that $\xi = -q^2/(2p_2 \cdot q)$ and thus

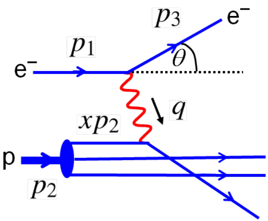
$$\xi = \frac{Q^2}{2p_2.q} = x$$

...

therefore Bjorken x can be identified as the fraction of the proton momentum carried by the struck quark in the infinite momentum frame.

- In terms of the proton momentum

$$s = (p_1 + p_2)^2 \simeq 2p_1.p_2 \quad y = \frac{p_2.q}{p_2.p_1} \quad x = \frac{Q^2}{2p_2.q}$$



- But for the underlying quark interaction:

$$s^q = (p_1 + xp_2)^2 = 2xp_1.p_2 = xs$$

$$y_q = \frac{p_q.q}{p_q.p_1} = \frac{xp_2.q}{xp_2.p_1} = y \qquad \text{and} \qquad x_q = 1.$$

(i.e. quark coll. is elastic: quark does not break up).

- Previously derived the Lorentz Invariant cross section for $e^-\mu^- \to e^-\mu^-$ elastic scattering in the ultra-relativistic limit (Handout 4 + Q10 on examples sheet). Now apply this to $e^-q \to e^-q$ assuming quark charge is e_q (e.g. $e_u = +2/3$):

$$\frac{d\sigma}{dq^2} = \frac{2\pi\alpha^2 e_q^2}{q^4}\left[1 + \left(1 + \frac{q^2}{s_q}\right)^2\right]$$

- Using $-q^2 = Q^2 = (s_q - m^2)x_q y_q \implies \frac{q^2}{s_q} = -y_q = -y$

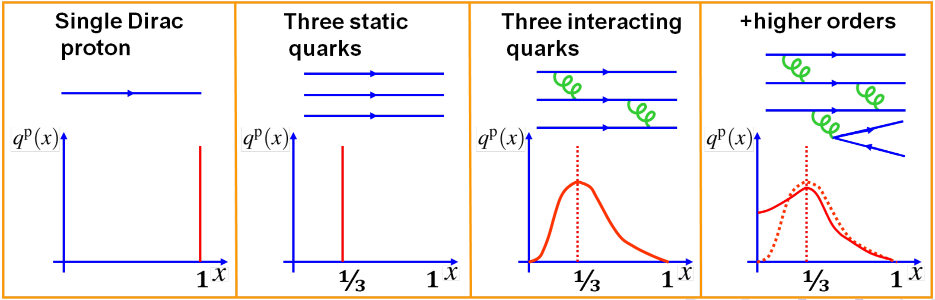
$$\frac{d\sigma}{dQ^2} = \frac{2\pi\alpha^2 e_q^2}{Q^4}\left[1 + (1-y)^2\right]$$

(where the last two expressions assume the massless limit $m = 0$).

Parton Distribution Functions

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2 e_q^2}{Q^4}\left[(1-y)+\frac{y^2}{2}\right] \qquad (115)$$

- This is the expression for the differential cross-section for elastic $e^- q$ scattering from a quark carrying a fraction x of the proton momentum.
- Now need to account for distribution of quark momenta within proton
- Introduce **parton distribution functions** such that $q^{\mathrm{p}}(x)dx$ is **the number of quarks of type** q **within a proton** p **with momenta between** $x \to x + dx$**.**
- Possible forms of the parton distribution functions:



- The cross section for scattering from a particular quark type within the proton, which in the range $x \to x + \mathrm{d}x$, is

$$\frac{d^2\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y) + \frac{y^2}{2}\right] \cdot e_q^2 q^{\mathrm{p}}(x)\mathrm{d}x$$

- Summing over all types of quarks within the proton gives the expression for the electron-proton scattering cross section

$$\frac{d^2\sigma^{\mathrm{ep}}}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y) + \frac{y^2}{2}\right] \sum_q e_q^2 q^{\mathrm{p}}(x) \qquad (116)$$

- Compare with the electron-proton scattering cross section in terms of structure functions (equation (113)):

$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2(x,Q^2)}{x} + y^2 F_1(x,Q^2)\right] \qquad (117)$$

- By comparing (116) and (117), obtain the parton model prediction for the structure functions in the general Lorentz invariant form for the differential cross section

$$\boxed{F_2^{\mathrm{p}}(x,Q^2) = 2xF_1^{\mathrm{p}}(x,Q^2) = x\sum_q e_q^2 q^{\mathrm{p}}(x)}$$

Can relate measured structure functions to the underlying quark distributions.

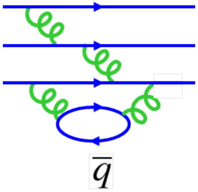
Predictions of The Parton Model

The parton model predicts:

- **Bjorken Scaling** $F_1(x, Q^2) \to F_1(x)$ $F_2(x, Q^2) \to F_2(x)$. This is due to scattering from point-like particles within the proton
- **Callan-Gross Relation** $F_2^{\mathrm{p}}(x) = x \sum_q e_q^2 q^{\mathrm{p}}(x)$. This is due to scattering from spin half Dirac particles where the magnetic moment is directly related to the charge; hence the 'electro-magnetic' and 'pure magnetic' terms are fixed with respect to each other.
- At present parton distributions cannot be calculated from QCD. (Can't use perturbation theory due to large coupling constant.)
- Measurements of the structure functions enable us to determine the parton distribution functions !

- For electron-proton scattering we have:

$$F_2^{\mathrm{p}}(x) = x \sum_q e_q^2 q^{\mathrm{p}}(x)$$



- Due to higher orders, the proton contains not only up and down quarks but also anti-up and anti-down quarks (will neglect the small contributions from heavier quarks).

- For electron-proton scattering have:

$$F_2^{\mathrm{ep}}(x) = x \sum_q e_q^2 q^{\mathrm{p}}(x) = x \left(\frac{4}{9} u^{\mathrm{p}}(x) + \frac{1}{9} d^{\mathrm{p}}(x) + \frac{4}{9} \overline{u}^{\mathrm{p}}(x) + \frac{1}{9} \overline{d}^{\mathrm{p}}(x) \right)$$

- For electron-neutron scattering have:

$$F_2^{\mathrm{en}}(x) = x \sum_q e_q^2 q^{\mathrm{n}}(x) = x \left(\frac{4}{9} u^{\mathrm{n}}(x) + \frac{1}{9} d^{\mathrm{n}}(x) + \frac{4}{9} \overline{u}^{\mathrm{n}}(x) + \frac{1}{9} \overline{d}^{\mathrm{n}}(x) \right)$$

- Now assume "isospin symmetry", i.e. that the neutron (ddu) is the same as a proton (uud) with up and down quarks interchanged, i.e.

$$d^{\mathrm{n}}(x) = u^{\mathrm{p}}(x) \qquad \text{and} \qquad u^{\mathrm{n}}(x) = d^{\mathrm{p}}(x)$$

and then define the neutron distributions functions in terms of those of the proton
$u(x) \equiv u^{\mathrm{p}}(x) = d^{\mathrm{n}}(x)$; $\qquad d(x) \equiv d^{\mathrm{p}}(x) = u^{\mathrm{n}}(x)$
$\overline{u}(x) \equiv \overline{u}^{\mathrm{p}}(x) = \overline{d}^{\mathrm{n}}(x)$; $\qquad \overline{d}(x) \equiv \overline{d}^{\mathrm{p}}(x) = \overline{u}^{\mathrm{n}}(x)$

giving:
$$F_2^{\mathrm{ep}}(x) = 2xF_1^{\mathrm{ep}}(x) = x \left(\frac{4}{9} u(x) + \frac{1}{9} d(x) + \frac{4}{9} \overline{u}(x) + \frac{1}{9} \overline{d}(x) \right) \qquad (118)$$

$$F_2^{\mathrm{en}}(x) = 2xF_1^{\mathrm{en}}(x) = x \left(\frac{4}{9} d(x) + \frac{1}{9} u(x) + \frac{4}{9} \overline{d}(x) + \frac{1}{9} \overline{u}(x) \right). \qquad (119)$$

- Integrating (118) and (119):

$$\int_0^1 F_2^{\text{en}}(x)dx = \int_0^1 x\left(\frac{4}{9}[d(x)+\overline{d}(x)] + \frac{1}{9}[u(x)+\overline{u}(x)]\right)dx = \frac{4}{9}f_d + \frac{1}{9}f_u \qquad \text{and}$$

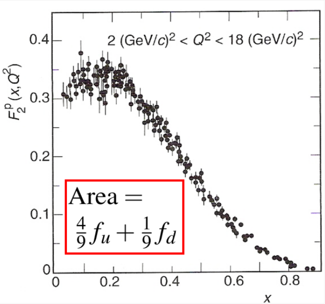
$$\int_0^1 F_2^{\text{ep}}(x)dx = \int_0^1 x\left(\frac{4}{9}[u(x)+\overline{u}(x)] + \frac{1}{9}[d(x)+\overline{d}(x)]\right)dx = \frac{4}{9}f_u + \frac{1}{9}f_d \qquad \text{where}$$

$$f_u = \int_0^1 [xu(x) + x\overline{u}(x)]dx$$

is the fraction of the proton momentum carried by the up and anti-up quarks.

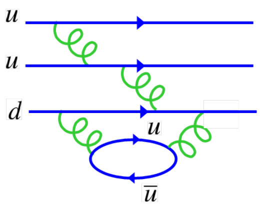
Experimentally:

$$\begin{matrix} \int F_2^{\text{ep}}(x)dx \approx 0.18 \\ \int F_2^{\text{en}}(x)dx \approx 0.12 \end{matrix} \implies \boxed{f_u \approx 0.36 \text{ and } f_d \approx 0.18}$$



- In the proton, as expected, the up quarks carry twice the momentum of the down quarks.
- The quarks carry just over 50% of the total proton momentum. The rest is carried by gluons (which being neutral doesn't contribute to electron-nucleon scattering).

Valence and Sea Quarks



- The proton is complex! its parton distribution functions include contributions from the "valence" quarks and the virtual quarks produced by gluons: the "sea".
- Resolving into valence and sea contributions:

$$u(x) = u_{\rm V}(x) + u_{\rm S}(x) \qquad \overline{d}(x) = \overline{d}_{\rm S}(x)$$
$$\overline{u}(x) = \overline{u}_{\rm S}(x) \qquad d(x) = d_{\rm V}(x) + d_{\rm S}(x)$$

- Proton contains two valence up quarks and one valence down quark so would expect:

$$\int_0^1 u_{\rm V}(x)dx = 2 \quad \text{and} \quad \int_0^1 d_{\rm V}(x)dx = 1$$

  ... but we have no a priori expectation for the total number of sea quarks!
- Sea quarks arise from gluon quark/anti-quark pair production and with $m_u = m_d$ it is reasonable to expect

$$u_{\rm S}(x) = d_{\rm S}(x) = \overline{u}_{\rm S}(x) = \overline{d}_{\rm S}(x) = S(x).$$

- With these relations (118) and (119) become:

$$F_2^{\rm ep}(x) = x\left(\frac{4}{9}u_{\rm V}(x) + \frac{1}{9}d_{\rm V}(x) + \frac{10}{9}S(x)\right) \;\&\; F_2^{\rm en}(x) = x\left(\frac{4}{9}d_{\rm V}(x) + \frac{1}{9}u_{\rm V}(x) + \frac{10}{9}S(x)\right)$$

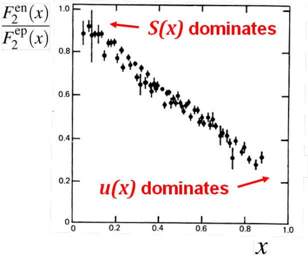
... giving the ratio

$$\frac{F_2^{\mathrm{en}}(x)}{F_2^{\mathrm{ep}}(x)} = \frac{4d_V(x) + u_V(x) + 10S(x)}{4u_V(x) + d_V(x) + 10S(x)}.$$

- The sea component arises from processes such as $g \to \overline{u}u$. Due to the $1/q^2$ dependence of the gluon propagator, much more likely to produce low energy gluons. Expect the sea to comprise of low energy q and $\overline{q}$.
- Therefore at low x expect the sea to dominate:

$$\frac{F_2^{\mathrm{en}}(x)}{F_2^{\mathrm{ep}}(x)} \to 1 \quad \text{as} \quad x \to 0.$$

Experiment agrees!



- At high x expect the sea contribution to be small

$$\frac{F_2^{\mathrm{en}}(x)}{F_2^{\mathrm{ep}}(x)} = \frac{4d_V(x) + u_V(x) + 10S(x)}{4u_V(x) + d_V(x) + 10S(x)}$$

Note: $u_V = 2d_V$ would give ratio $2/3$ as $x \to 1$

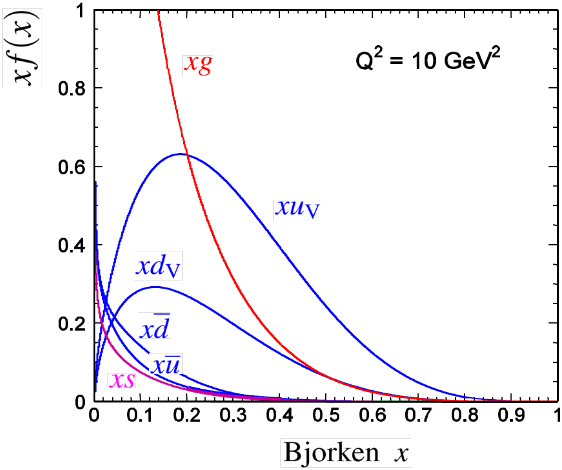
Experiment disagrees:

$$F_2^{\mathrm{en}}(x)/F_2^{\mathrm{ep}}(x) \to 1/4 \quad \text{i.e.} \quad d(x)/u(x) \to 0 \quad \text{as} \quad x \to 1.$$

This behaviour is not understood!

Parton Distribution Functions

- Ultimately the parton distribution functions are obtained from a fit to all experimental data including neutrino scattering (see Handout 10)
- Hadron-hadron collisions give information on gluon pdf, $g(x)$.



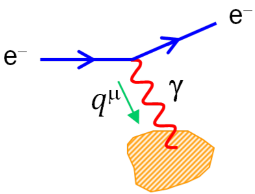
Note:

- Apart from at large x, $u_V(x) \approx 2d_V(x)$.
- For $x < 0.2$ gluons dominate.
- In fits to data assume $u_s(x) = \overline{u}(x)$
- $\overline{d}(x) > \overline{u}(x)$ not understood — exclusion principle?
- Small strange quark component $s(x)$.

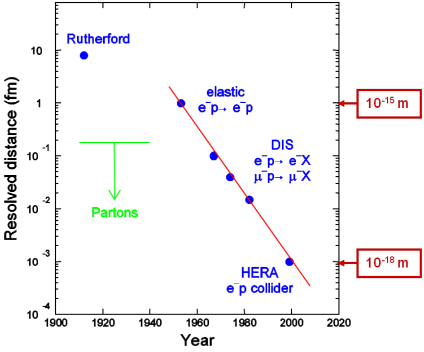
(Now try Question 12)

Scaling Violations

- In last 40 years, experiments have probed the proton with virtual photons of ever increasing energy
- Non-point like nature of the scattering becomes apparent when $\lambda_\gamma \sim$ size of scattering centre



$$\lambda_\gamma = \frac{h}{|\vec{q}|} \sim O\left(\frac{1}{|\vec{q}|/\mathrm{GeV}}\right)\ \mathrm{fm}.$$



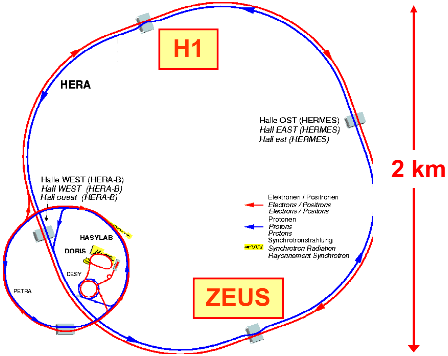
- Scattering from point-like quarks gives rise to Bjorken scaling: no Q^2 cross section dependence
- IF quarks were not point-like, at high Q^2 (when the wavelength of the virtual photon i$\sim$ size of quark) would observe rapid decrease in cross section with increasing Q^2.
- To search for quark sub-structure want to go to highest Q^2 ... see HERA collider (next slide).

The HERA $e^{\pm}$-proton Collider : 1991-2007

★ **DESY (Deutsches Elektronen-Synchroton) Laboratory, Hamburg, Germany**

$e^{\pm}$ **27.5 GeV** → ← **820 GeV** p

$\sqrt{s} = 300$ **GeV**

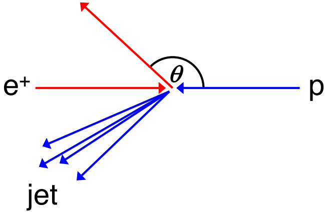


★ **Two large experiments : H1 and ZEUS**

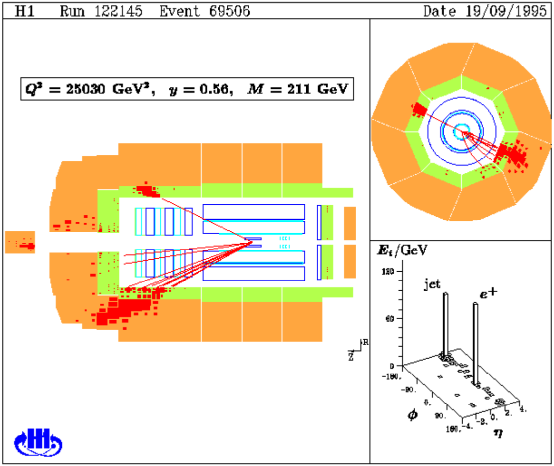
★ **Probe proton at very high Q^2 and very low x**

Example of a High Q^2 Event in the H1 Experiment at HERA

✶ **Event kinematics determined from electron angle and energy**



✶ **Also measure hadronic system (although not as precisely) – gives some redundancy**



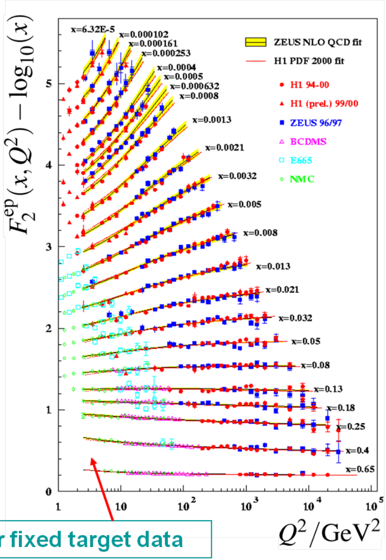
$F_2(x, Q^2)$ Results

- No evidence of rapid decrease of cross section at highest Q^2

$$\Longrightarrow \boxed{R_{\mathrm{quark}} < 10^{-18}\,\mathrm{m}}.$$

- For $x > 0.05$, only weak dependence of F_2 on Q^2 : consistent with the expectation from the quark-parton model.
- Observe clear scaling violations, particularly at low x:

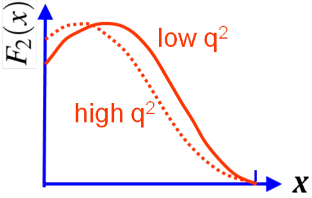
$$F_2(x, Q^2) \neq F_2(x).$$

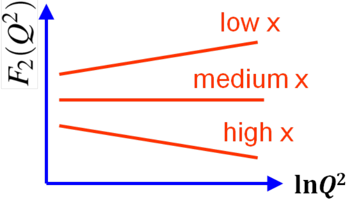


Origin of Scaling Violations

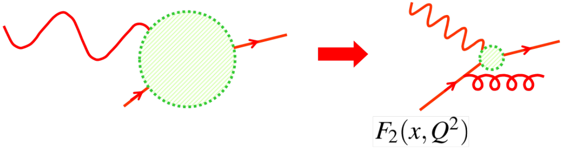
Not examinable

- Observe "small" deviations from exact Bjorken scaling $F_2(x) \to F_2(x, Q^2)$
- At high Q^2 observe more low x quarks.





- "Explanation": at high Q^2 (shorter wave-length) resolve finer structure: i.e. reveal quark is sharing momentum with gluons. At higher Q^2 expect to "see" more low x quarks.
- QCD cannot predict the x dependence of $F_2(x, Q^2)$



Not examinable

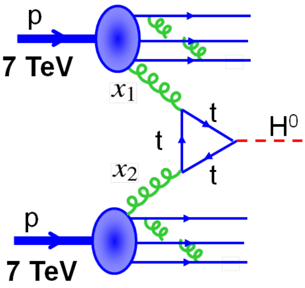
But QCD can predict the Q^2 dependence of $F_2(x, Q^2)$.

Proton-Proton Collisions at the LHC

- Measurements of structure functions not only provide a powerful test of QCD, the parton distribution functions are essential for the calculation of cross sections at pp and pp collider.

Example: Higgs production at the Large Hadron Collider LHC (2009-)

- The LHC collides up to $7\,TeV$ protons with $7\,TeV$ protons, however underlying collisions are between partons.



- Higgs production the LHC dominated by “gluon-gluon fusion”
- Cross section depends on gluon PDFs

$$\sigma(pp \to HX) \sim \int_0^1 \int_0^1 g(x_1) g(x_2) \sigma(gg \to H)\, dx_1\, dx_2.$$

- Uncertainty in gluon PDFs lead to a $\pm 5\%$ uncertainty in Higgs production cross section.
- Prior to HERA data uncertainty was $\pm 25\%$.

Summary

- At very high electron energies $\lambda \ll r_p$: the proton appears to be a sea of quarks and gluons.
- Deep Inelastic Scattering = Elastic scattering from the quasi-free constituent quarks.
  - **Bjorken Scaling:** $F_1(x, Q^2) \rightarrow F_1(x)$ (point-like scattering).
  - **Callan-Gross:** $F_2(x) = 2xF_1(x)$ (scattering from spin-$\frac{1}{2}$).
- Describe scattering in terms of parton distribution functions $u(x), d(x), \ldots$ which describe momentum distribution inside a nucleon.
- The proton is much more complex than just *uud* — sea of anti-quarks/gluons.
- Quarks carry only 50% of the protons momentum — the rest is due to low energy gluons.
- We will come back to this topic when we discuss neutrino scattering ...