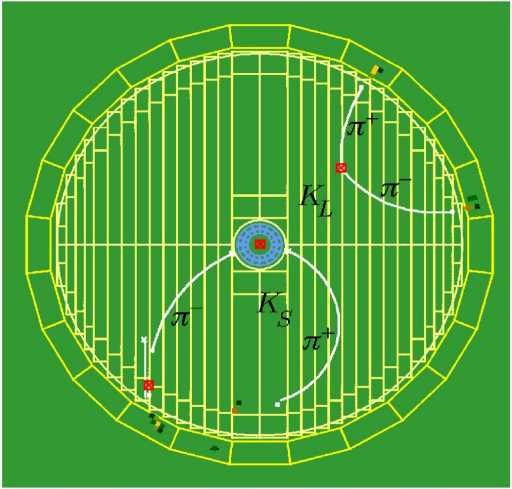
Dr C.G. Lester, 2023



H12: The CKM Matrix and CP Violation

CP Violation in the Early Universe

- Today the universe is matter dominated. There is no evidence for anti-galaxies, *etc.* The matter/anti-matter asymmetry is estimated to be

$$\xi = \frac{n_B - n_{\bar{B}}}{n_\gamma} \approx \frac{n_B}{n_\gamma} \approx 10^{-9}$$

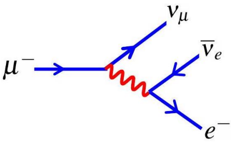
  i.e. for every baryon in the universe today there are 10^9 photons and no anti-baryons.
- One possible explanation is that for every 10^9 anti-baryons in the early universe there were $10^9 + 1$ baryons, and that these annihilated to 1 baryon $+ \sim 10^9$ photons + no anti-baryons.
- To generate such an asymmetry from a symmetric precursor state, **three conditions** must be met (Sakharov, 1967) [7]:
  1. **Baryon number violation**: i.e. $n_B - n_{\bar{B}}$ is not constant,
  2. **C and CP violation**: if CP is conserved for a reaction which generates a net number of baryons over anti-baryons there would be a CP conjugate reaction generating a net number of anti-baryons.
  3. **Departure from thermal equilibrium**: In thermal equilibrium any baryon number violating process will be balanced by the inverse reaction.
- [Aside: Your lecturer is not entirely convinced by the arguments that the early universe should be symmetric, but he is in a minority!]

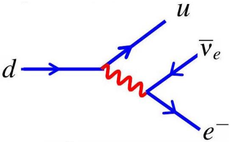
We would like to know if the Standard Model of particle physics provides enough CP violation to generate the observed matter antimatter asymmetry of the unvierse from a symmetric starting point.

- There are two places in the SM where CP violation enters: complex phases in the **PMNS matrix** (neutrinos) and the **CKM matrix** (quarks).
- To date CP violation has been confirmed only in the quark sector.
- We will approach quark-sector oscillations it in two stages first without and then with CP violation.
- We will see many features in common with neutrino oscillations — except that the oscillating particles (mesons) will have finite rather than infinte lifetimes (neutrinos).

The Weak Interaction of Quarks

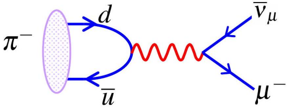
- A slightly different values of G_F is measured in μ decay and nuclear β decay:

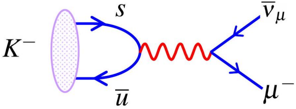




$G_F^\mu = (1.16632 \pm 0.00002) \times 10^{-5}\ \mathrm{GeV}^{-2}$ $\quad$ $G_F^\beta = (1.136 \pm 0.003) \times 10^{-5}\ \mathrm{GeV}^{-2}$

- In addition, certain hadronic decay modes are observed to be suppressed, e.g. compare $K^- \to \mu^- \bar{\nu}_\mu$ and $\pi^- \to \mu^- \bar{\nu}_\mu$. Kaon decay rate suppressed factor 20 compared to the expectation assuming a universal weak interaction for quarks.



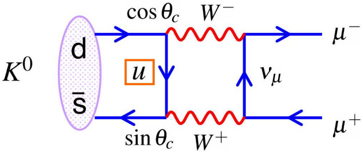


- Both observations explained by **Cabibbo hypothesis** (1963) [8]: weak eigenstates are different from mass eigenstates, i.e. weak interactions of quarks have same strength as for leptons but a u-quark couples to a linear combination of s and d.

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

GIM Mechanism

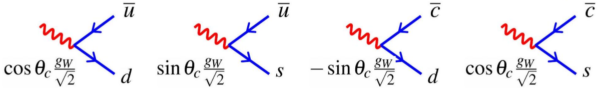
- In the weak interaction have couplings between both *ud* and *us* which implies that neutral mesons can decay via box diagrams, e.g.



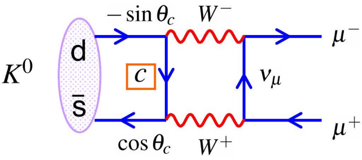
$$M_1 \propto g_W^4 \cos\theta_c \sin\theta_c.$$

Historically, the observed branching was much smaller than predicted.

- Led Glashow, Illiopoulos and Maiani to postulate existence of an extra quark - before discovery of charm quark in 1974. Weak interaction couplings become:



- Gives another box diagram for $K^0 \to \mu^+\mu^-$ with $M_2 \propto -g_W^4 \cos\theta_c \sin\theta_c$:

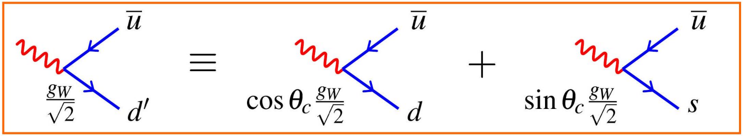


- Same final state so sum amplitudes:

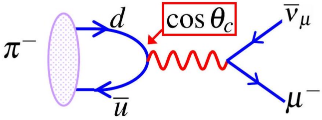
$$|M|^2 = |M_1 + M_2|^2 \approx 0.$$

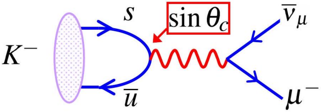
- Cancellation not exact because $m_u \neq m_c$.

- i.e. weak interaction couples different generations of quarks:



- Can explain the observations on the previous pages with $\theta_c = 13.1°$.
  Kaon decay is suppressed by a factor of $\tan^2\theta_c \approx 0.05$ relative to pion decay.

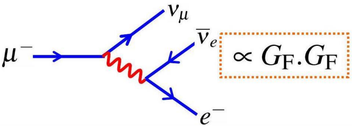


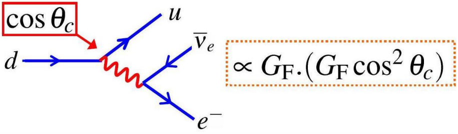


$\Gamma\left(\pi^- \to \mu^- \bar{\nu}_\mu\right) \propto |M|^2 \propto \cos^2\theta_c$

$\Gamma\left(K^- \to \mu^- \bar{\nu}_\mu\right) \propto |M|^2 \propto \sin^2\theta_c$

- Hence expect $G_{\mathrm{F}}^{\beta} = G_{\mathrm{F}}^{\mu}\cos\theta_c$



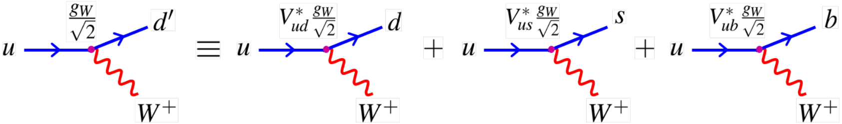


Cabibbo–Kobayashi–Maskawa (CKM) Matrix

- Extend ideas to three quark flavours (analogue of three flavour neutrino treatment)

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Weak eigenstates, CKM Matrix , Mass Eigenstates
(Cabibbo, Kobayashi, Maskawa)
e.g. Weak eigenstate d' is produced in weak decay of an up quark:

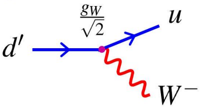


- The CKM matrix elements V_{ij} are complex constants.
- The CKM matrix is unitary.
- The V_{ij} are not predicted by the SM — have to be determined from experiment.

Depending on the order of the interaction $u \to d$ or $d \to u$, the CKM matrix enters as either V_{ud} or V_{ud}^*.

[Spoiler: the happy quark index on V always comes first, and no conjugation is needed if a walk backwards up the fermion line also encounters the happy quark first. See below!]

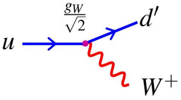
- For $d' \to u$ the weak current is:



$$j_{d'u} = \bar{u}\left[-i\frac{g_W}{\sqrt{2}}\gamma^\mu\frac{1}{2}\left(1-\gamma^5\right)\right]d'$$

giving the $d \to u$ weak current: $\boxed{j_{du} = \bar{u}\left[-i\frac{g_W}{\sqrt{2}}\gamma^\mu\frac{1}{2}\left(1-\gamma^5\right)\right]V_{ud}\,d}$.

- For $u \to d'$ the weak current is:



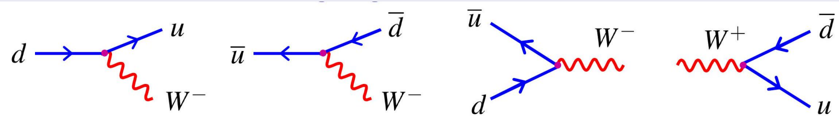
$$j_{ud'} = \bar{d}'\left[-i\frac{g_W}{\sqrt{2}}\gamma^\mu\frac{1}{2}\left(1-\gamma^5\right)\right]u.$$

This time $\bar{d}' = d'^\dagger\gamma^0 \to (V_{ud}\,d)^\dagger\,\gamma^0 = V_{ud}^* d^\dagger\gamma^0 = V_{ud}^*\bar{d}$

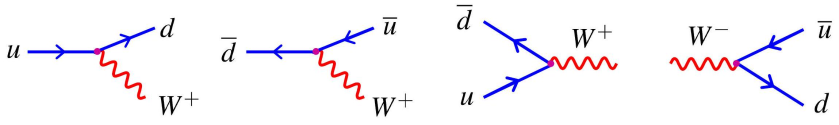
giving the $u \to d$ weak current: $\boxed{j_{ud} = \bar{d}\,V_{ud}^*\left[-i\frac{g_W}{\sqrt{2}}\gamma^\mu\frac{1}{2}\left(1-\gamma^5\right)\right]u}$.

Examples showing use of what was explained on the last slide:

$-i\frac{g_W}{\sqrt{2}}V_{ud}\gamma^{\mu}\frac{1}{2}\left(1-\gamma^5\right)$ is the vertex factor the following diagrams:



$-i\frac{g_W}{\sqrt{2}}V^*_{ud}\gamma^{\mu}\frac{1}{2}\left(1-\gamma^5\right)$ is the vertex factor for the following diagrams:



CKM summary

- Experimentally (see Appendix XVIII) determine:

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.97373 & 0.2243 & 0.0038 \\ 0.221 & 0.975 & 0.041 \\ 0.009 & 0.042 & 1.01 \end{pmatrix}.$$

- Assuming unitarity of CKM matrix, e.g. $|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1$, get (2022 PDG):

$$|V_{\rm CKM}| = \begin{pmatrix} 0.97435 \pm 0.00016 & 0.22500 \pm 0.00067 & 0.00369 \pm 0.00011 \\ 0.22486 \pm 0.00067 & 0.97349 \pm 0.00016 & 0.04182^{+0.00085}_{-0.00074} \\ 0.00857^{+0.00020}_{-0.00018} & 0.04110^{+0.00083}_{-0.00072} & 0.999118^{+0.000031}_{-0.000036} \end{pmatrix}$$

which is **fairly diagonal — very different from PMNS!**

Notes:

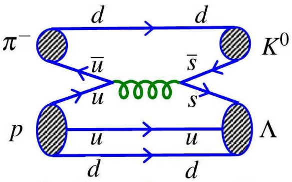
- Weak Charged Current (i.e. $W^{\pm}$) is only SM interaction that changes flavour.
- As off-diagonal elements are small, flavour changes are 'discouraged'.
- Weak interactions are largest between quarks of the same generation.
- Coupling between first and third generation quarks is very small!
- Just as for the PMNS matrix — the CKM matrix allows CP violation in the SM.

The Neutral Kaon System

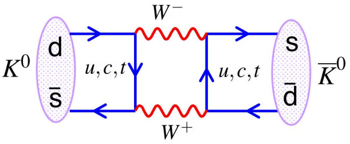
- Neutral Kaons are produced in strong interaction, e.g.:

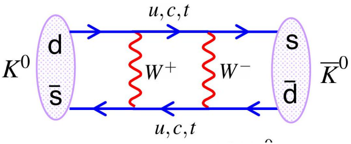
$$\pi^-(d\bar{u}) + p(uud) \to \Lambda(uds) + K^0(d\bar{s})$$
$$\pi^+(u\bar{d}) + p(uud) \to K^+(u\bar{s}) + \bar{K}^0(s\bar{d}) + p(uud)$$

but decay via the weak interaction.



- The Weak Interaction also allows mixing of neutral kaons via "**box diagrams**"





- This allows transitions between the strong eigenstates states $K^0, \bar{K}^0$.
- Consequently, the neutral kaons propagate as eigenstates of the overall strong + weak interaction (Appendix XIX); i.e. as linear combinations of $K^0, \bar{K}^0$

These neutral kaon mass states are called the "K-short", K_S, and the "K-long', K_L. These states have approximately the same mass, $m(K_S) \approx m(K_L) \approx 498\ \mathrm{MeV}$, but have very different lifetimes: $\tau(K_S) = 0.9 \times 10^{-10}\ \mathrm{s}$ versus $\tau(K_L) = 0.5 \times 10^{-7}\ \mathrm{s}$.

CP Eigenstates

The K_S and K_L are closely related to eigenstates of the combined charge conjugation and parity operators: CP.

- The strong eigenstates $K^0(d\bar{s})$ and $\bar{K}^0(s\bar{d})$ have $J^P = 0^-$ and

$$\hat{P}\left|K^0\right\rangle = -\left|K^0\right\rangle, \quad \hat{P}\left|\overline{K^0}\right\rangle = -\left|\bar{K}^0\right\rangle.$$

- The charge conjugation operator changes particle into anti-particle and vice versa:

$$\hat{C}\left|K^0\right\rangle = \hat{C}|d\bar{s}\rangle = +|s\bar{d}\rangle = \left|\bar{K}^0\right\rangle. \quad \text{Likewise:} \quad \hat{C}\left|\bar{K}^0\right\rangle = \left|K^0\right\rangle.$$

(The plus sign is purely conventional. We could have used a minus sign with no physical consequences.)

- Consequently:

$$\hat{C}\hat{P}\left|K^0\right\rangle = -\left|\bar{K}^0\right\rangle \quad \hat{C}\hat{P}\left|\bar{K}^0\right\rangle = -\left|K^0\right\rangle$$

i.e. neither K^0 or $\bar{K}^0$ are eigenstates of CP.

- Form CP eigenstates $|K_1\rangle$ and $|K_2\rangle$ from linear combs. of $\left|K^0\right\rangle$ and $\left|\bar{K}^0\right\rangle$ as follows:

$$\boxed{\begin{aligned} |K_1\rangle &= \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle - \left|\bar{K}^0\right\rangle\right) \\ |K_2\rangle &= \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle + \left|\bar{K}^0\right\rangle\right) \end{aligned}} \quad \text{so that} \quad \boxed{\begin{aligned} \hat{C}\hat{P}\,|K_1\rangle &= +\,|K_1\rangle \\ \hat{C}\hat{P}\,|K_2\rangle &= -\,|K_2\rangle \end{aligned}}.$$

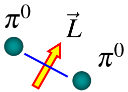
Neutral Kaon decays

Neutral kaons often decay to two or three pions.

This is because:

- Pions are the lightest hadrons.
- The kaon masses are approximately 498 MeV and the pion masses are approximately 140 MeV, and so there is (just!) room to make three pions.

Neutral Kaon decays to Two Pions



Decays to Two Pions:

★ $K^0 \to \pi^0\pi^0 \qquad J^P: \quad 0^- \to 0^- + 0^-$

- **Conservation of angular momentum** ➡ $\vec{L} = 0$

$$\Rightarrow \hat{P}(\pi^0\pi^0) = -1.-1.(-1)^L = +1$$

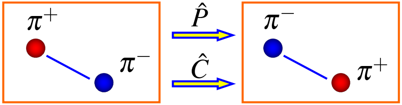
- **The** $\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$ **is an eigenstate of** $\hat{C}$

$$C(\pi^0\pi^0) = C\pi^0.C\pi^0 = +1.+1 = +1$$

$$\Rightarrow \boxed{CP(\pi^0\pi^0) = +1}$$

★ $K^0 \to \pi^+\pi^-$ **as before** $\hat{P}(\pi^+\pi^-) = +1$

★ **Here the C and P operations have the identical effect**



Hence the combined effect of $\hat{C}\hat{P}$ **is to leave the system unchanged**

$$\boxed{\hat{C}\hat{P}(\pi^+\pi^-) = +1}$$

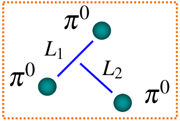
Conclusion:

Neutral kaon decays to two pions occur in CP-even (i.e. $CP = +1$) eigenstates.

Neutral Kaon decays to Three Pions

Decays to Three Pions:

☆ $K^0 \to \pi^0\pi^0\pi^0$ $\qquad J^P: \quad 0^- \to 0^- + 0^- + 0^-$



•**Conservation of angular momentum:**

Remember L is magnitude of angular momentum vector

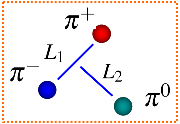
$$L_1 \oplus L_2 = 0 \;\Rightarrow\; L_1 = L_2$$

$$P(\pi^0\pi^0\pi^0) = -1.-1.-1.(-1)^{L_1}.(-1)^{L_2} = -1$$

$$C(\pi^0\pi^0\pi^0) = +1.+1.+1$$

$$\Rightarrow \quad CP(\pi^0\pi^0\pi^0) = -1$$

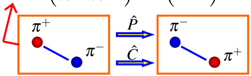
☆ $K^0 \to \pi^+\pi^-\pi^0$



•**Again** $L_1 = L_2$

$$P(\pi^+\pi^-\pi^0) = -1.-1.-1.(-1)^{L_1}.(-1)^{L_2} = -1$$

$$C(\pi^+\pi^-\pi^0) = +1.C(\pi^+\pi^-) = P(\pi^+\pi^-) = (-1)^{L_1}$$



Hence: $\quad CP(\pi^+\pi^-\pi^0) = -1.(-1)^{L_1}$

•**The small amount of energy available in the decay,** $m(K) - 3m(\pi) \approx 70\,\text{MeV}$ **means that the L>0 decays are strongly suppressed by the angular momentum barrier effects (recall QM tunnelling in alpha decay)**

Conclusion:

Neutral kaon decays to three pions **mostly** occur in *CP*-odd (i.e. $CP = -1$) eigenstates.

If CP were conserved in the weak decays of neutral kaons ...

- we would expect decays to pions to occur from states of definite CP (i.e. K_1 and K_2):

$$|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle) \qquad \hat{C}\hat{P}|K_1\rangle = +|K_1\rangle \qquad K_1 \to \pi\pi \qquad \text{CP EVEN}$$
$$|K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle) \qquad \hat{C}\hat{P}|K_2\rangle = -|K_2\rangle \qquad K_2 \to \pi\pi\pi \qquad \text{CP ODD};$$

- we would expect lifetimes of CP-eigenstates to be very different given that:
  energy available in **two** pion decay is $m_K - 2m_\pi \approx 220$ MeV and
  energy available in **three** pion decay is $m_K - 3m_\pi \approx 80$ MeV;
- and we would expect decays to two pions to be more rapid than decays to three pions due to increased phase space.

This is exactly what is observed:

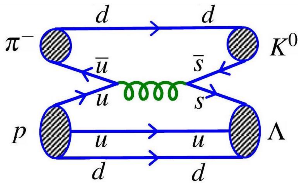
A short-lived state **"K-short"** which decays to (mainly) to two pions and a long-lived state **"K-long"** which decays (only) to three pions.
Therefore: in the absence of CP-violation we can identify:

$$|K_S\rangle = |K_1\rangle \equiv \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right) \quad \text{with decays:} \quad K_S \to \pi\pi, \quad \text{and}$$
$$|K_L\rangle = |K_2\rangle \equiv \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right) \quad \text{with decays:} \quad K_L \to \pi\pi\pi.$$

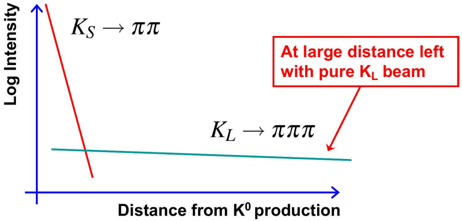
Neutral Kaon Decays to pions

- Consider the decays of a beam of K^0. The decays to pions occur in states of definite CP.
- If CP is conserved in the decay, need to express K^0 in terms of K_S and K_L:

$$|K_0\rangle = \frac{1}{\sqrt{2}}(|K_S\rangle + |K_L\rangle)\,.$$



- Hence from the point of view of decays to pions, a K^0 beam is a linear combination of CP eigenstates containing a rapidly decaying CP-even component and a long-lived CP-odd component.
- Therefore, expect to see predominantly two-pion decays near start of beam and predominantly three pion decays further downstream:



- Suppose we have a beam of pure K^0 at time $t = 0$: $|\psi(t=0)\rangle = \frac{1}{\sqrt{2}}\left(|K_S\rangle + |K_L\rangle\right)$.
- Put in the time dependence: $|K_S(t)\rangle = |K_S\rangle\, e^{-im_S t - \Gamma_S t/2}$
  with m_S being the mass of the K-short and $\Gamma_S = 1/\tau_S$ being its decay rate.

Aside: The term $e^{-\Gamma_S t/2}$ ensures the K_S probability density decays exponentially, i.e.:

$$|\psi_S|^2 = \langle K_S(t) \mid K_S(t)\rangle = e^{-\Gamma_S t} = e^{-t/\tau_S}.$$

- Hence the wave-function evolves as:

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left[|K_S\rangle\, e^{-\left(im_S + \frac{\Gamma_S}{2}\right)t} + |K_L\rangle\, e^{-\left(im_L + \frac{\Gamma_L}{2}\right)t}\right].$$

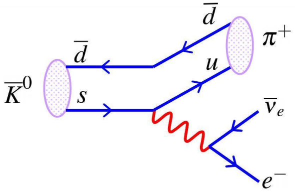
- Compressing by defining $\theta_S(t) = e^{-(im_S + \Gamma_S/2)t}$ and $\theta_L(t) = e^{-(im_L + \Gamma_L/2)t}$ we have:

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(\theta_S(t)\,|K_S\rangle + \theta_L(t)\,|K_L\rangle\right).$$

- As anticipated, the decay rate to two pions for a state which was produced as K^0 is:

$$\Gamma\left(K^0_{t=0} \to \pi\pi\right) \propto |\langle K_S \mid \psi(t)\rangle|^2 \propto |\theta_S(t)|^2 = e^{-\Gamma_S t} = e^{-t/\tau_S}.$$

Neutral Kaons decays to Leptons



- Neutral Kaons can also decays to Leptons:

$$\begin{array}{ll} \bar{K}^0 \to \pi^+ e^- \bar{\nu}_e & \bar{K}^0 \to \pi^+ \mu^- \bar{\nu}_\mu \\ K^0 \to \pi^- e^+ \nu_e & K^0 \to \pi^- \mu^+ \nu_\mu \end{array}.$$

- Note: the final states are not CP eigenstates which is why we express these decays in terms of $K^0, \bar{K}^0$.
- Neutral kaons propagate as combined eigenstates of weak + strong interaction i.e. the K_S, K_L.
- The **main** decay modes/branching fractions are:

$$\begin{array}{lll} K_S & \to \pi^+\pi^- & BR = 69.2\% \\ & \to \pi^0\pi^0 & BR = 30.7\% \\ & \to \pi^- e^+ \nu_e & BR = 0.03\% \\ & \to \pi^+ e^- \bar{\nu}_e & BR = 0.03\% \\ & \to \pi^- \mu^+ \nu_\mu & BR = 0.02\% \\ & \to \pi^+ \mu^- \bar{\nu}_\mu & BR = 0.02\% \end{array} \qquad \begin{array}{lll} K_L & \to \pi^+\pi^-\pi^0 & BR = 12.6\% \\ & \to \pi^0\pi^0\pi^0 & BR = 19.6\% \\ & \to \pi^- e^+ \nu_e & BR = 20.2\% \\ & \to \pi^+ e^- \bar{\nu}_e & BR = 20.2\% \\ & \to \pi^- \mu^+ \nu_\mu & BR = 13.5\% \\ & \to \pi^+ \mu^- \bar{\nu}_\mu & BR = 13.5\% \end{array}.$$

- Leptonic decays are more likely for the K-long because the three pion decay modes have a lower decay rate than the two pion modes of the K-short.

Strangeness Oscillations (neglecting CP violation)

The "semi-leptonic" decay rate to $\pi^- e^+ \nu_e$ occurs from the K^0 state. Hence to calculate the expected decay rate, need to know the K^0 component of the wave-function. For example, for a beam which was initially K^0 we have:

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}} \left(\theta_S(t) \, |K_S\rangle + \theta_L(t) \, |K_L\rangle \right). \tag{175}$$

Writing K_S and K_L in terms of K^0 and $\bar{K}^0$:

$$\begin{aligned} |\psi(t)\rangle &= \frac{1}{2} \left[\theta_S(t) \left(\left| K^0 \right\rangle - \left| \bar{K}^0 \right\rangle \right) + \theta_L(t) \left(\left| K^0 \right\rangle + \left| \bar{K}^0 \right\rangle \right) \right] \\ &= \frac{1}{2} (\theta_S + \theta_L) \left| K^0 \right\rangle + \frac{1}{2} (\theta_L - \theta_S) \left| \bar{K}^0 \right\rangle . \end{aligned}$$

Because $\theta_S(t) \neq \theta_L(t)$ a state that was initially a K^0 evolves with time into a mixture of K^0 and $\bar{K}^0$ — "strangeness oscillations".

The K^0 intensity (i.e. K^0 fraction), whose form we will improve over pages, is thus:

$$\Gamma \left(K^0_{t=0} \to K^0 \right) = \left| \left\langle K^0 \mid \psi(t) \right\rangle \right|^2 = \frac{1}{4} |\theta_S + \theta_L|^2 . \tag{176}$$

Similarly:

$$\Gamma \left(K^0_{t=0} \to \bar{K}^0 \right) = \left| \left\langle \bar{K}^0 \mid \psi(t) \right\rangle \right|^2 = \frac{1}{4} |\theta_S - \theta_L|^2 . \tag{177}$$

- Using the identity $|z_1 \pm z_2|^2 = |z_1|^2 + |z_2|^2 \pm 2\,\mathrm{Re}\,(z_1 z_2^*)$ we find:

$$
\begin{aligned}
|\theta_S \pm \theta_L|^2 &= \left| e^{-\left(im_S + \frac{1}{2}\Gamma_S\right)t} \pm e^{-\left(im_L + \frac{1}{2}\Gamma_L\right)t} \right|^2 \\
&= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2\,\mathrm{Re}\left\{ e^{-im_S t} e^{-\frac{1}{2}\Gamma_S t} \cdot e^{+im_L t} e^{-\frac{1}{2}\Gamma_L t} \right\} \\
&= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\,\mathrm{Re}\left\{ e^{-i(m_S - m_L)t} \right\} \\
&= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t} \cos\left(m_S - m_L\right)t \\
&= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t} \cos \Delta m t.
\end{aligned}
$$

Thus we see oscillations between neutral kaon states

The frequency is given by the mass splitting $\Delta m = m\left(K_L\right) - m\left(K_S\right)$.

This is reminiscent of neutrino oscillations! Only this time we have decaying states!

Thus, from (176) and (177) get form of kaon fractions that is nicer than on last page:

$$\Gamma\left(K^0_{t=0} \to K^0\right) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2} \cos \Delta m t\right] \tag{178}$$

$$\Gamma\left(K^0_{t=0} \to \bar{K}^0\right) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2} \cos \Delta m t\right]. \tag{179}$$

Observed strangeness oscillations in the Kaon System

Experimentally it is measured/found that:

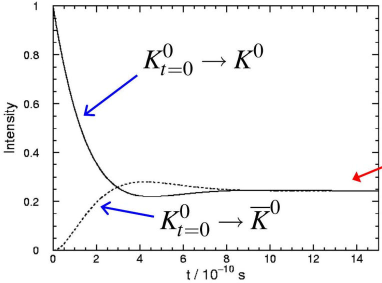
$\tau(K_S) = 0.9 \times 10^{-10}\ \mathrm{s}$, $\tau(K_L) = 0.5 \times 10^{-7}\ \mathrm{s}$ and

$\Delta m = (3.506 \pm 0.006) \times 10^{-15}\ \mathrm{GeV}$

i.e. the K-long mass is greater than the K-short by one part in 10^{16}.

Oscillation period is thus $T_{osc} = \dfrac{2\pi\hbar}{\Delta m} \approx 1.2 \times 10^{-9}\ \mathrm{s}$.

- The oscillation period is relatively long compared to the K_S lifetime. Consequently, we only observe a slight wiggle. (See below!)

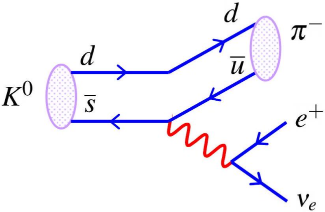


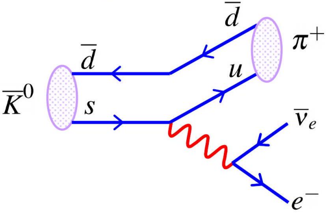
$$\Gamma(K^0_{t=0} \to K^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

$$\Gamma(K^0_{t=0} \to \overline{K}^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

After a few K_S lifetimes, left with a pure K_L beam which is half K^0 and half $\overline{K}^0$

⋆ Strangeness oscillations can be studied by looking at semi-leptonic decays





⋆ The charge of the observed pion (or lepton) tags the decay as from either a $\bar{K}^0$ or K^0 because

$$\begin{array}{l} K^0 \rightarrow \pi^- e^+ \nu_e \\ \bar{K}^0 \rightarrow \pi^+ e^- \bar{\nu}_e \end{array} \quad \text{but} \quad \begin{array}{l} \bar{K}^0 \not\rightarrow \pi^- e^+ \nu_e \\ K^0 \not\rightarrow \pi^+ e^- \bar{\nu}_e \end{array}$$

is not allowed (see Question 23).

⋆ So for an initial K^0 beam, observe the decays to both charge combinations:

$$K^0_{t=0} \longrightarrow K^0 \hookrightarrow \pi^- e^+ \nu_e \qquad\qquad K^0_{t=0} \longrightarrow \bar{K}^0 \hookrightarrow \pi^+ e^{-\bar{\nu}_e}$$

which provides a way of measuring strangeness oscillations.

The CPLEAR Experiment [9]

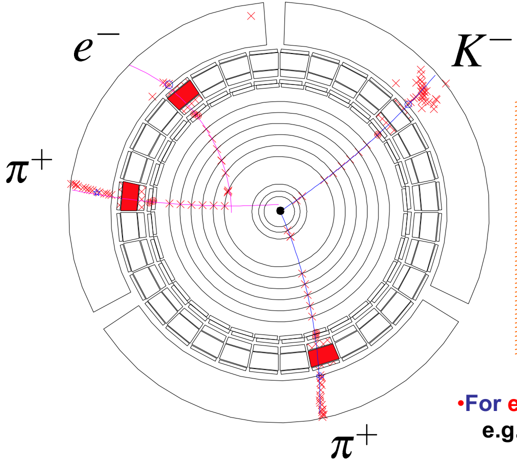
- CPLEAR used a low energy anti-proton beam at CERN (1990-1996).
- Produced neutral kaons in reactions like:

$$\bar{p}p \to K^-\pi^+K^0,$$
$$\bar{p}p \to K^+\pi^-\bar{K}^0,$$

- Low energy, so particles produced almost at rest.
- Observe production process and decay in the same detector.

- Charge of $K^\pm\pi^\mp$ in the production process tags the initial neutral kaon as either K^0 or $\bar{K}^0$.
- Charge of decay products tags the decay as either as being either K^0 or $\bar{K}^0$.
- Provides a direct probe of strangeness oscillations.

An example of a CPLEAR event



$K^-(s\bar{u})$
$K^0(d\bar{s})$
$\bar{K}^0(s\bar{d})$

Production:

$$\bar{p}p \longrightarrow K^-\pi^+ K^0$$

Mixing

Decay:

$$\bar{K}^0 \longrightarrow \pi^+ e^- \bar{\nu}_e$$

- **For each event know initial wave-function,** **e.g. here:** $|\psi(t=0)\rangle = |K^0\rangle$

Quantifying the asymmetry

Can measure decay rates as a function of time for all combinations:
e.g. $R^+ = \Gamma\left(K^0_{t=0} \to \pi^- e^+ \bar{\nu}_e\right) \propto \Gamma\left(K^0_{t=0} \to K^0\right)$.

From equations (178) and (179) and similar relations:

$$R_+ \equiv \Gamma\left(K^0_{t=0} \to \pi^- e^+ \nu_e\right) = N_{\pi e \nu} \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

$$R_- \equiv \Gamma\left(K^0_{t=0} \to \pi^+ e^- \bar{\nu}_e\right) = N_{\pi e \nu} \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

$$\bar{R}_- \equiv \Gamma\left(\bar{K}^0_{t=0} \to \pi^+ e^- \bar{\nu}_e\right) = N_{\pi e \nu} \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

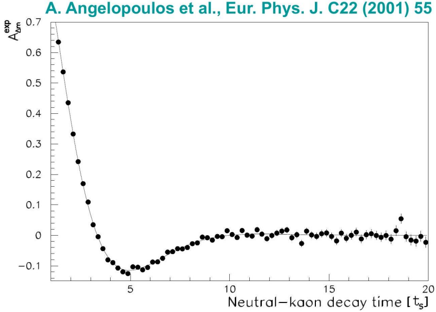
$$\bar{R}_+ \equiv \Gamma\left(\bar{K}^0_{t=0} \to \pi^- e^+ \nu_e\right) = N_{\pi e \nu} \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

where $N_{\pi e \nu}$ is some overall normalisation factor.

Express measurements as an "asymmetry" to remove dependence on $N_{\pi e \nu}$:

$$A_{\Delta m} = \frac{(R_+ + \bar{R}_-) - (R_- + \bar{R}_+)}{(R_+ + \bar{R}_-) + (R_- + \bar{R}_+)} = \frac{2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t}{e^{-\Gamma_S t} + e^{-\Gamma_L t}}.$$

CPLEAR's results:



- Points show the data.
- The line shows the theoretical prediction for the value of Δm most consistent with the CPLEAR data:

$$\Delta m = 3.485 \times 10^{-15} \text{ GeV}$$

using

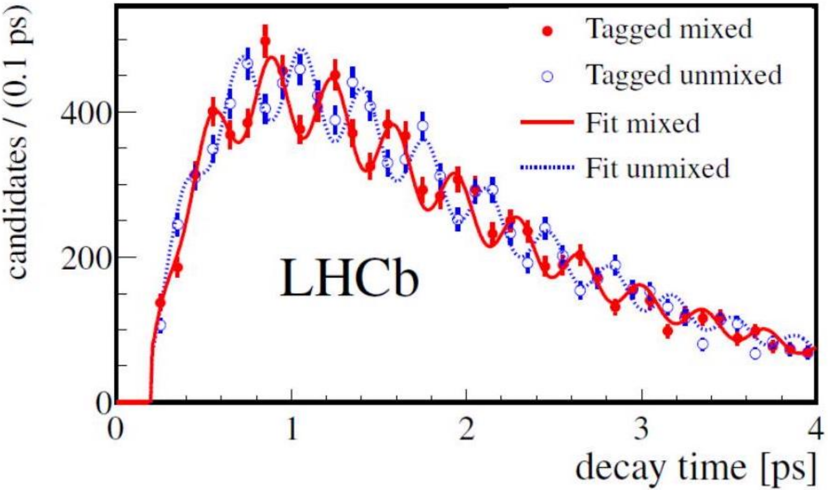
$$A_{\Delta m} = \frac{2e^{-(\Gamma_S+\Gamma_L)t/2} \cos \Delta mt}{e^{-\Gamma_S t} + e^{-\Gamma_L t}}.$$

- The sign of Δm is not determined here but is known from other experiments.

When the CPLEAR results are combined with experiments at FermiLab obtain:

$$\Delta m = m(K_L) - m(K_S) = (3.506 \pm 0.006) \times 10^{-15} \text{ GeV}.$$

In 2013 even nicer oscillations were seen in B-hadrons:



[If you are interested, look could up the B_s^0–$\bar{B}_s^0$ mixing evidence in arXiv:1304.4741 [10].]

CP-Violation in the Kaon System

- So far we have ignored CP-violation in the neutral kaon system.
- We identified the K-short as the CP-even state and the K-long as the CP-odd state:

$$|K_S\rangle = |K_1\rangle \equiv \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right) \quad \text{with decays:} \quad K_S \to \pi\pi \quad \boxed{\text{CP} = +1},$$
$$|K_L\rangle = |K_2\rangle \equiv \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right) \quad \text{with decays:} \quad K_L \to \pi\pi\pi \quad \boxed{\text{CP} = -1}.$$

- At a long distance from the production point a beam of neutral kaons will be 100% K-long (the K-short component will have decayed away). Hence, if CP is conserved, would expect to see only three-pion decays.

In 1964 Fitch & Cronin (joint Nobel prize) observed 45 $K_L \to \pi^+\pi^-$ decays in a sample of 22700 kaon decays a long distance from the production point.

This implies that weak interactions violate CP!

- CP is violated in hadronic weak interactions, but only at the level of 2 parts in 1000.

K_L to pion BRs:

K_L	$\to \pi^+\pi^-\pi^0$	$BR = 12.6\%$	$CP = -1$
K_L	$\to \pi^0\pi^0\pi^0$	$BR = 19.6\%$	$CP = -1$
K_L	$\to \pi^+\pi^-$	$BR = 0.20\%$	$CP = +1$
K_L	$\to \pi^0\pi^0$	$BR = 0.08\%$	$CP = +1$

There are two possible explanations of CP violation in the kaon system:

1. The K_S and K_L do not correspond exactly to the CP-eigenstates K_1 and K_2:

$$|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[|K_1\rangle + \varepsilon\,|K_2\rangle\right] \qquad |K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[|K_2\rangle + \varepsilon\,|K_1\rangle\right]$$

with $|\varepsilon| \sim 2\times 10^{-3}$.
In this case the observation of $K_L \to \pi\pi$ is accounted for by:

$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[|K_2\rangle + \varepsilon|K_1\rangle\right]$$

$|K_1\rangle \to \pi\pi$ **CP = +1**

$|K_2\rangle \to \pi\pi\pi$ **CP = -1**

2. and/or CP is violated in the decay

$$|K_L\rangle = |K_2\rangle$$ **CP = -1**

$\to \pi\pi\pi$ **CP = -1**

$\to \pi\pi$ **CP = +1**

This effect is parameterised by a parameter ϵ'.

- Experimentally both possibilities are found to contribute, but the first source dominates since $\epsilon'/\varepsilon = (1.7\pm 0.3)\times 10^{-3}$ (NA48(CERN) and KTeV (Fermilab)).
- The dominant mechanism is discussed in Appendix XX.

CP Violation in Semi-leptonic decays

- If observe a neutral kaon beam a long time after production (i.e. at large distances) it will consist of a pure K_L component:

$$|K_L\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+|\varepsilon|^2}}\left[(1+\varepsilon)|K_0\rangle + (1-\varepsilon)|\overline{K}^0\rangle\right]$$

($|K_0\rangle \to \pi^- e^+ \nu_e$; $|\overline{K}^0\rangle \to \pi^+ e^- \overline{\nu}_e$)

- Decays to $\pi^- e^+ \nu_e$ must come from the K^0 component, and decays to $\pi^+ e^- \bar{\nu}_e$ must come from the $\bar{K}^0$ component so:

$$\Gamma\left(K_L \to \pi^+ e^- \bar{\nu}_e\right) \propto \left|\left\langle \bar{K}^0 \mid K_L \right\rangle\right|^2 \propto |1-\varepsilon|^2 \approx 1 - 2\Re\{\varepsilon\},$$

$$\Gamma\left(K_L \to \pi^- e^+ \nu_e\right) \propto \left|\left\langle K^0 \mid K_L \right\rangle\right|^2 \propto |1+\varepsilon|^2 \approx 1 + 2\Re\{\varepsilon\}.$$

- This results in a small difference in decay rates:

  the K_L decay to $\pi^- e^+ \nu_e$ is 0.7% more likely than the K_L decay to $\pi^+ e^- \bar{\nu}_e$.

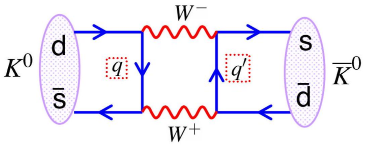
- This difference has been observed and thus provides the first direct evidence for an absolute difference between matter and anti-matter.

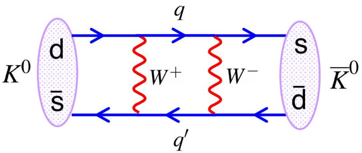
It also provides an unambiguous definition of matter which could, for example, be transmitted to aliens in a distant galaxy!

"The electrons in our atoms have the same charge as those emitted **least** often in the decays of the long-lived neutral kaon."

CP Violation and the CKM Matrix

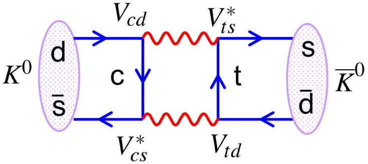
- Imaginary parts of the CKM matrix should be able to lead to *CP*-violation for the reasons already discussed when we considered the PMNS Matrix.
- Can we link $\Gamma\left(\bar{K}^0_{t=0} \to K^0\right) \neq \Gamma\left(K^0_{t=0} \to \bar{K}^0\right)$ to imaginary parts of the CKM matrix?
- Consider the box diagrams responsible for mixing, i.e.:





where $q = \{u, c, t\}$ and $q' = \{u, c, t\}$.

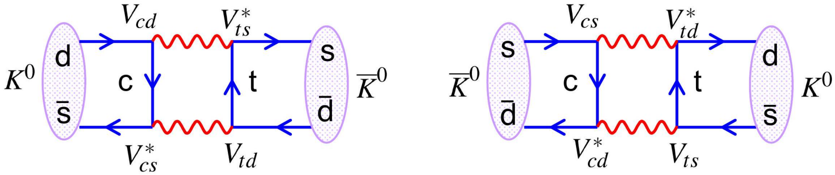
- Have to sum over all possible quark exchanges in the box. For simplicity consider just one diagram:



$$M_{fi} \propto A_{ct} V_{cd} V^*_{cs} V_{td} V^*_{ts}$$

A constant related to integrating over virtual momenta

- Compare the equivalent box diagrams for $K^0 \to \bar{K}^0$ and $\bar{K}^0 \to K^0$



$M_{fi} \propto A_{ct} V_{cd} V_{cs}^* V_{td} V_{ts}^*$ $\qquad$ $M'_{fi} \propto A_{ct} V_{cd}^* V_{cs} V_{td}^* V_{ts} = M_{fi}^*$

- Evidently $M_{fi} - M'_{fi} = 2\Im\{M_{fi}\}$ ... yet the imaginary part of the CKM Matrix generates CP-violation.
- Hence likely that the factors which drive CP-violation are related to $M_{fi} - M'_{fi}$.

$$|\varepsilon| \propto \Im\{M_{fi}\}\,.$$

[See also Thomson's "Modern Particle Physics", Chap 14.]

- In the kaon system we can show that:

$$|\varepsilon| \propto A_{ut} \cdot \Im\{V_{ud} V_{us}^* V_{td} V_{ts}^*\} + A_{ct} \cdot \Im\{V_{cd} V_{cs}^* V_{td} V_{ts}^*\} + A_{tt} \cdot \Im\{V_{td} V_{ts}^* V_{td} V_{ts}^*\}$$

(see Question 25).

Summary of quark sector *CP*-violation

The weak interactions of quarks are controlled by the CKM matrix.

Similar structure to the lepton sector, although unlike the PMNS matrix, the CKM matrix is nearly diagonal.

CP violation enters through via a complex phase in the CKM matrix for same reasons as PMNS matrix.

There is a great deal of experimental evidence for CP violation in the weak interactions of quarks

Some kind of *CP*-violation might explain matter-antimatter asymmetry in the Universe.

CP violation in the SM is not sufficient to explain the matter-antimatter asymmetry.

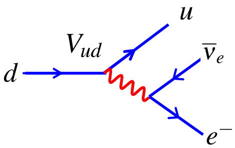
Either there is a different as-yet-undiscovered mechanism of *CP*-violation, **or** there was just more matter than antimatter to start with!

Appendix XVIII: Determination of the CKM Matrix

- The experimental determination of the CKM matrix elements comes mainly from measurements of leptonic decays (the leptonic part is well understood).
- It is easy to produce/observe meson decays, however theoretical uncertainties associated with the decays of bound states often limits the precision
- Contrast this with the measurements of the PMNS matrix, where there are few theoretical uncertainties and the experimental difficulties in dealing with neutrinos limits the precision.

❶ **$|V_{ud}|$** **from nuclear beta decay**

$$\begin{pmatrix} \times & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$



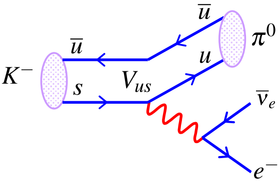
Super-allowed $0^+ \rightarrow 0^+$ beta decays are relatively free from theoretical uncertainties

$$\Gamma \propto |V_{ud}|^2$$

$$|V_{ud}| = 0.97377 \pm 0.00027 \qquad (\approx \cos\theta_c)$$

❷ **$|V_{us}|$** **from semi-leptonic kaon decays**

$$\begin{pmatrix} \cdot & \times & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$



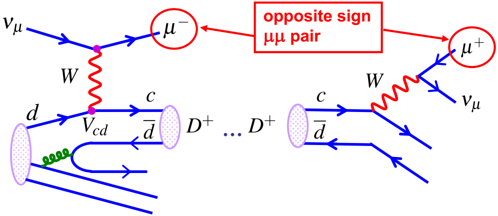
$$\Gamma \propto |V_{us}|^2$$

$$|V_{us}| = 0.2257 \pm 0.0021 \qquad (\approx \sin\theta_c)$$

❸ **$|V_{cd}|$** **from neutrino scattering** $\nu_\mu + N \to \mu^+\mu^- X$

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \times & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

Look for opposite charge di-muon events in ν_μ scattering from production and decay of a $D^+(c\bar{d})$ meson

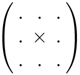


$$\text{Rate} \propto |V_{cd}|^2 \text{Br}(D^+ \to X\mu^+\nu_\mu)$$

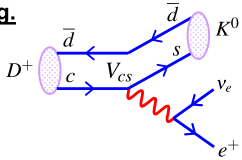
Measured in various collider experiments

$\Rightarrow$ $$|V_{cd}| = 0.230 \pm 0.011$$

❹ **$|V_{cs}|$** **from semi-leptonic charmed meson decays**



e.g.



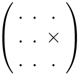
$$\Gamma \propto |V_{cs}|^2$$

•**Precision limited by theoretical uncertainties**

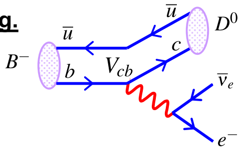
$$|V_{cs}| = 0.957 \pm 0.017 \pm 0.093$$

experimental error **theory uncertainty**

❺ **$|V_{cb}|$** **from semi-leptonic B hadron decays**



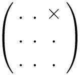
e.g.



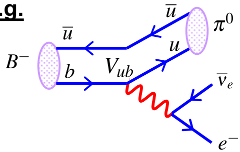
$$\Gamma \propto |V_{cb}|^2$$

$$|V_{cb}| = 0.0416 \pm 0.0006$$

❻ **$|V_{ub}|$** **from semi-leptonic B hadron decays**



e.g.



$$\Gamma \propto |V_{ub}|^2$$

$$|V_{ub}| = 0.0043 \pm 0.0003$$

Appendix XIX: Particle–AntiParticle Mixing

-The wave-function for a single particle with lifetime $\tau = 1/\Gamma$ evolves with time as:

$$\psi(t) = Ne^{-\Gamma t/2}e^{-iMt}$$

which gives the appropriate exponential decay of

$$\langle\psi(t) \mid \psi(t)\rangle = \langle\psi(0) \mid \psi(0)\rangle e^{-t/\tau}$$

-The wave-function satisfies the time-dependent wave equation:

$$\hat{H}|\psi(t)\rangle = \left(M - \frac{1}{2}i\Gamma\right)|\psi(t)\rangle = i\frac{\partial}{\partial t}|\psi(t)\rangle$$

-For a bound state such as a K^0 the mass term includes the "mass" from the weak interaction "potential" $\hat{H}_{\text{weak}}$

$$M = m_{K^0} + \left\langle K^0 \middle| \hat{H}_{\text{weak}} \middle| K^0 \right\rangle + \sum_j \frac{\left|\left\langle K^0 \middle| \hat{H}_{\text{weak}} \middle| j \right\rangle\right|^2}{m_{K^0} - E_j} \leftarrow$$

Sum over intermediate states j

The third term is the 2nd order term in the perturbation expansion corresponding to box diagrams resulting in $K^0 \to \bar{K}^0$

Not examinable

- The total decay rate is the sum over all possible decays $K^0 \rightarrow f$

$$\Gamma = 2\pi \sum_f \left| \left\langle f \left| \hat{H}_{\text{weak}} \right| K^0 \right\rangle \right|^2 \rho_F \longleftarrow \text{Density of final states}$$

$\star$ Because there are also diagrams which allow $K^0 \leftrightarrow \bar{K}^0$ mixing need to consider the time evolution of a mixed stated

$$\psi(t) = a(t)K^0 + b(t)\bar{K}^0$$

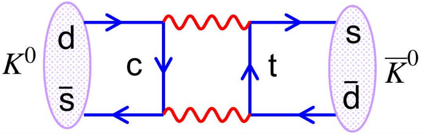
$\star$ The time dependent wave-equation of (A1) becomes

$$\begin{pmatrix} M_{11} - \frac{1}{2}i\Gamma_{11} & M_{12} - \frac{1}{2}i\Gamma_{12} \\ M_{21} - \frac{1}{2}i\Gamma_{21} & M_{22} - \frac{1}{2}i\Gamma_{22} \end{pmatrix} \begin{pmatrix} \left|K^0(t)\right\rangle \\ \left|\bar{K}^0(t)\right\rangle \end{pmatrix} = i\frac{\partial}{\partial t} \begin{pmatrix} \left|K^0(t)\right\rangle \\ \left|\bar{K}^0(t)\right\rangle \end{pmatrix}$$

the diagonal terms are as before, and the off-diagonal terms are due to mixing.

$$M_{11} = m_{K^0} + \left\langle K^0 \left| \hat{H}_{\text{weak}} \right| K^0 \right\rangle + \sum_n \frac{\left| \left\langle K^0 \left| \hat{H}_{\text{weak}} \right| K^0 \right\rangle \right|^2}{m_{K^0} - E_n}$$

$$M_{12} = \sum_j \frac{\langle K^0|\hat{H}_{\text{weak}}|j\rangle^* \langle j|\hat{H}_{\text{weak}}|\overline{K}^0\rangle}{m_{K^0} - E_j}$$



-The off-diagonal decay terms include the effects of interference between decays to a common final state

$$\Gamma_{12} = 2\pi \sum_f \left\langle f \left| \hat{H}_{\text{weak}} \right| K^0 \right\rangle^* \left\langle f \left| \hat{H}_{\text{weak}} \right| \bar{K}^0 \right\rangle \rho_F$$

-In terms of the time dependent coefficients for the kaon states, (A3) becomes

$$\left[\mathbf{M} - i\frac{1}{2}\Gamma\right] \begin{pmatrix} a \\ b \end{pmatrix} = i\frac{\partial}{\partial t} \begin{pmatrix} a \\ b \end{pmatrix}$$

where the Hamiltonian can be written:

$$\mathbf{H} = \mathbf{M} - i\frac{1}{2}\Gamma = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} - \frac{1}{2} \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix}$$

-Both the mass and decay matrices represent observable quantities and are Hermitian

$$M_{11} = M_{11}^*, \quad M_{22} = M_{22}^*, \quad M_{12} = M_{21}^*$$
$$\Gamma_{11} = \Gamma_{11}^*, \quad \Gamma_{22} = \Gamma_{22}^*, \quad \Gamma_{12} = \Gamma_{21}^*$$

-Furthermore, if CPT is conserved then the masses and decay rates of the $\bar{K}^0$ and K^0 are identical:

$$M_{11} = M_{22} = M; \quad \Gamma_{11} = \Gamma_{22} = \Gamma$$

Not examinable

-Hence the time evolution of the system can be written:

$$\begin{pmatrix} M - \frac{1}{2}i\Gamma & M_{12} - \frac{1}{2}i\Gamma_{12} \\ M_{12}^* - \frac{1}{2}i\Gamma_{12}^* & M - \frac{1}{2}i\Gamma \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = i\frac{\partial}{\partial t}\begin{pmatrix} a \\ b \end{pmatrix}$$

- To solve the coupled differential equations for $a(t)$ and $b(t)$, first find the eigenstates of the Hamiltonian (the K_L and K_S) and then transform into this basis. The eigenvalue equation is:

$$\begin{pmatrix} M - \frac{1}{2}i\Gamma & M_{12} - \frac{1}{2}i\Gamma_{12} \\ M_{12}^* - \frac{1}{2}i\Gamma_{12}^* & M - \frac{1}{2}i\Gamma \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \lambda \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

-Which has non-trivial solutions for

$$|\mathbf{H} - \lambda I| = 0$$

$$\Rightarrow \left(M - \frac{1}{2}i\Gamma - \lambda\right)^2 - \left(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*\right)\left(M_{12} - \frac{1}{2}i\Gamma_{12}\right) = 0$$

with eigenvalues

$$\lambda = M - \frac{1}{2}i\Gamma \pm \sqrt{\left(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*\right)\left(M_{12} - \frac{1}{2}i\Gamma_{12}\right)}$$

Not examinable

Not examinable

-The eigenstates can be obtained by substituting back into (A5)

$$\left(M - \tfrac{1}{2}i\Gamma\right)x_1 + \left(M_{12} - \tfrac{1}{2}i\Gamma_{12}\right) = \left(M - \tfrac{1}{2}i\Gamma \pm \sqrt{\left(M_{12}^* - \tfrac{1}{2}i\Gamma_{12}^*\right)\left(M_{12} - \tfrac{1}{2}i\Gamma_{12}\right)}\right)x_1$$

$$\Rightarrow \quad \frac{x_2}{x_1} = \pm\sqrt{\frac{M_{12}^* - \frac{1}{2}i\Gamma_{12}^*}{M_{12} - \frac{1}{2}i\Gamma_{12}}}$$

⋆ Define

$$\eta = \sqrt{\frac{M_{12}^* - \frac{1}{2}i\Gamma_{12}^*}{M_{12} - \frac{1}{2}i\Gamma_{12}}}$$

- Hence the normalised eigenstates are

$$|K_\pm\rangle = \frac{1}{\sqrt{1+|\eta|^2}}\begin{pmatrix}1\\ \pm\eta\end{pmatrix} = \frac{1}{\sqrt{1+|\eta|^2}}\left(\left|K^0\right\rangle \pm \eta\left|\bar{K}^0\right\rangle\right)$$

⋆ Note, in the limit where M_{12}, Γ_{12} are real, the eigenstates correspond to the CP eigenstates K_1 and K_2. Hence we can identify the general eigenstates as as the long and short lived neutral kaons:

$$|K_L\rangle = \frac{1}{\sqrt{1+|\eta|^2}}\left(\left|K^0\right\rangle + \eta\left|\bar{K}^0\right\rangle\right) \quad |K_S\rangle = \frac{1}{\sqrt{1+|\eta|^2}}\left(\left|K^0\right\rangle - \eta\left|\bar{K}^0\right\rangle\right)$$

Not examinable

Not examinable

Substituting these states back into (A2):

$$
\begin{aligned}
|\psi(t)\rangle &= a(t)\left|K^0\right\rangle + b(t)\left|\bar{K}^0\right\rangle \\
&= \sqrt{1+|\eta|^2}\left[\frac{a(t)}{2}(K_L + K_S) + \frac{b(t)}{2\eta}(K_L - K_S)\right] \\
&= \sqrt{1+|\eta|^2}\left[\left(\frac{a(t)}{2} + \frac{b(t)}{2\eta}\right)K_L + \left(\frac{a(t)}{2} - \frac{b(t)}{2\eta}\right)K_S\right] \\
&= \frac{\sqrt{1+|\eta|^2}}{2}\left[a_L(t)K_L + a_S(t)K_S\right]
\end{aligned}
$$

with

$$
a_L(t) \equiv a(t) + \frac{b(t)}{\eta} \quad a_S(t) \equiv a(t) - \frac{b(t)}{\eta}
$$

- Now consider the time evolution of $a_L(t)$

$$
i\frac{\partial a_L}{\partial t} = i\frac{\partial a}{\partial t} + \frac{i}{\eta}\frac{\partial b}{\partial t}
$$

⋆ Which can be evaluated using (A4) for the time evolution of $a(t)$ and $b(t)$:

Not examinable

$$
\begin{aligned}
i\frac{\partial a_L}{\partial t} &= \left[\left(M - \frac{1}{2}i\Gamma_{12}\right)a + \left(M_{12} - \frac{1}{2}i\Gamma_{12}\right)b\right] + \frac{1}{\eta}\left[\left(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*\right)a + \left(M - \frac{1}{2}i\Gamma\right)b\right] \\
&= \left(M - \frac{1}{2}i\Gamma\right)\left(a + \frac{b}{\eta}\right) + \left(M_{12} - \frac{1}{2}i\Gamma_{12}\right)b + \frac{1}{\eta}\left(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*\right)a \\
&= \left(M - \frac{1}{2}i\Gamma\right)a_L + \left(M_{12} - \frac{1}{2}i\Gamma_{12}\right)b + \left(\sqrt{\left(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*\right)\left(M_{12} - \frac{1}{2}i\Gamma_{12}\right)}\right)a \\
&= \left(M - \frac{1}{2}i\Gamma\right)a_L + \left(\sqrt{\left(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*\right)\left(M_{12} - \frac{1}{2}i\Gamma_{12}\right)}\right)\left(a + \frac{b}{\eta}\right) \\
&= \left(M - \frac{1}{2}i\Gamma\right)a_L + \left(\sqrt{\left(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*\right)\left(M_{12} - \frac{1}{2}i\Gamma_{12}\right)}\right)a_L \\
&= \left(m_L - \frac{1}{2}i\Gamma_L\right)a_L
\end{aligned}
$$

⋆ Hence:

$$
i\frac{\partial a_L}{\partial t} = \left(m_L - \frac{1}{2}i\Gamma_L\right)a_L
$$

with $m_L = M + \text{Re}\left\{\sqrt{\left(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*\right)\left(M_{12} - \frac{1}{2}i\Gamma_{12}\right)}\right\}$

Not examinable

Not examinable

⋆ Following the same procedure obtain:

$$i\frac{\partial a_S}{\partial t} = \left(m_S - \frac{1}{2}i\Gamma_S\right)a_S$$

with $m_S = M - \Re\left\{\sqrt{\left(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*\right)\left(M_{12} - \frac{1}{2}i\Gamma_{12}\right)}\right\}$

and $\Gamma_S = \Gamma + 2\Im\left\{\sqrt{\left(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*\right)\left(M_{12} - \frac{1}{2}i\Gamma_{12}\right)}\right\}$

⋆ In matrix notation we have

⋆ Solving we obtain

$$\begin{pmatrix} M_L - \frac{1}{2}i\Gamma_L & 0 \\ 0 & M_S - \frac{1}{2}i\Gamma_S \end{pmatrix}\begin{pmatrix} a_L \\ a_S \end{pmatrix} = i\frac{\partial}{\partial t}\begin{pmatrix} a_L \\ a_S \end{pmatrix}$$

$$a_L(t) \propto e^{-im_L t - \Gamma_L t/2} \quad a_S(t) \propto e^{-im_S t - \Gamma_S t/2}$$

⋆ Hence in terms of the $\mathrm{K_L}$ and $\mathrm{K_S}$ basis the states propagate as independent particles with definite masses and lifetimes (the mass eigenstates). The time evolution of the neutral kaon system can be written

$$|\psi(t)\rangle = A_L e^{-im_L t - \Gamma_L t/2}|K_L\rangle + A_S e^{-im_S t - \Gamma_S t/2}|K_S\rangle$$

where A_L and A_S are constants

Not examinable

Appendix XX: CP Violation : $\pi\pi$ decays

⋆ Consider the development of the $K^0 - \bar{K}^0$ system now including CP violation
⋆ Repeat previous derivation using

$$|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[|K_1\rangle + \varepsilon\,|K_2\rangle\right] \quad |K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[|K_2\rangle + \varepsilon\,|K_1\rangle\right]$$

-Writing the CP eigenstates in terms of $K^0, \bar{K}^0$

$$|K_L\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+|\varepsilon|^2}}\left[(1+\varepsilon)\,|K_0\rangle + (1-\varepsilon)\left|\bar{K}^0\right\rangle\right]$$

$$|K_S\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+|\varepsilon|^2}}\left[(1+\varepsilon)\,|K_0\rangle - (1-\varepsilon)\left|\bar{K}^0\right\rangle\right]$$

- Inverting these expressions obtain

$$\left|K^0\right\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2}}\frac{1}{1+\varepsilon}\left(|K_L\rangle + |K_S\rangle\right) \quad \left|\bar{K}^0\right\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2}}\frac{1}{1-\varepsilon}\left(|K_L\rangle - |K_S\rangle\right) |$$

-Hence a state that was produced as a K^0 evolves with time as:

$$|\psi(t)\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2}}\frac{1}{1+\varepsilon}\left(\theta_L(t)\,|K_L\rangle + \theta_S(t)\,|K_S\rangle\right)$$

where as before $\theta_S(t) = e^{-\left(im_S + \frac{\Gamma_S}{2}\right)t}$ and $\theta_L(t) = e^{-\left(im_L + \frac{\Gamma_L}{2}\right)t}$

-If we are considering the decay rate to $\pi\pi$ need to express the wave-function in terms of the CP eigenstates (remember we are neglecting CP violation in the decay)

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\frac{1}{1+\varepsilon}\left[\left(|K_2\rangle + \varepsilon|K_1\rangle\right)\theta_L(t) + \left(|K_1\rangle + \varepsilon|K_2\rangle\right)\theta_S(t)\right]$$

$$= \frac{1}{\sqrt{2}}\frac{1}{1+\varepsilon}\left[\left(\theta_S + \varepsilon\theta_L\right)|K_1\rangle + \left(\theta_L + \varepsilon\theta_S\right)|K_2\rangle\right]$$

CP Eigenstates

-Two pion decays occur with $\mathrm{CP} = +1$ and therefore arise from decay of the $\mathrm{CP} = +1$ kaon eigenstate, i.e. K_1

$$\Gamma\left(K^0_{t=0} \to \pi\pi\right) \propto |\langle K_1 \mid \psi(t)\rangle|^2 = \frac{1}{2}\left|\frac{1}{1+\varepsilon}\right|^2 |\theta_S + \varepsilon\theta_L|^2$$

- Since $|\varepsilon| \ll 1$

$$\left|\frac{1}{1+\varepsilon}\right|^2 = \frac{1}{(1+\varepsilon^*)(1+\varepsilon)} \approx \frac{1}{1+2\Re\{\varepsilon\}} \approx 1 - 2\Re\{\varepsilon\}$$

- Now evaluate the $|\theta_S + \varepsilon\theta_L|^2$ term again using

$$|z_1 \pm z_2|^2 = |z_1|^2 + |z_2|^2 \pm 2\Re\left(z_1 z_2^*\right)$$

Not examinable

$$|\theta_S + \varepsilon\theta_L|^2 = \left| e^{-im_S t - \frac{\Gamma_S}{2}t} + \varepsilon e^{-im_L t - \frac{\Gamma_L}{2}t} \right|^2$$

$$= e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} + 2\,\mathrm{Re}\left\{ e^{-im_S t - \frac{\Gamma_S}{2}t} \cdot \varepsilon^* e^{+im_L t - \frac{\Gamma_L}{2}t} \right\}$$

-Writing $\varepsilon = |\varepsilon| e^{i\phi}$

$$|\theta_S + \varepsilon\theta_L|^2 = e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} + 2|\varepsilon| e^{-(\Gamma_S+\Gamma_L)t/2}\,\mathrm{Re}\left\{ e^{i(m_L - m_S)t - \phi} \right\}$$

$$= e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} + 2|\varepsilon| e^{-(\Gamma_S+\Gamma_L)t/2} \cos(\Delta m.t - \phi)$$

-Putting this together we obtain:

$$\Gamma\left(K^0_{t=0} \to \pi\pi\right) = \frac{1}{2}(1 - 2\,\mathrm{Re}\{\varepsilon\}) N_{\pi\pi} \left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} + 2|\varepsilon| e^{-(\Gamma_S+\Gamma_L)t/2} \cos(\Delta m.t - \phi) \right]$$

Short lifetime

component

$K_S \to \pi\pi$

CP violating long lifetime component KL¦p

Interference term

-In exactly the same manner obtain for a beam which was produced as $\bar{K}^0$

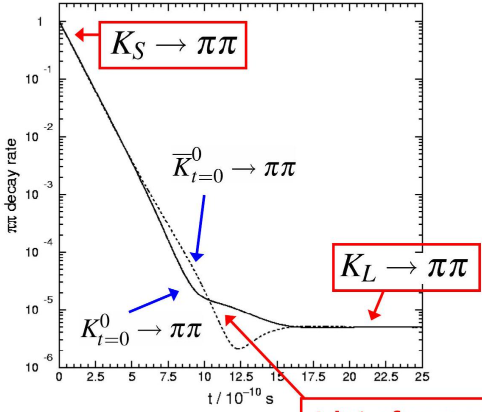
Not examinable

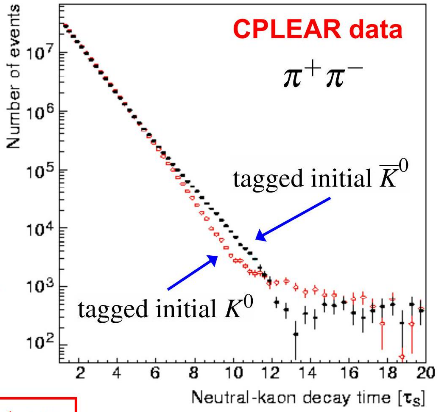
Not examinable

$$\Gamma\left(K^0_{t=0} \to \pi\pi\right) \to \frac{1}{2}(1 - 2\,\mathrm{Re}\{\varepsilon\})N_{\pi\pi} \cdot |\varepsilon|^2 e^{-\Gamma_L t}$$

i.e. CP violating $K_L \to \pi\pi$ decays

⋆ Since CPLEAR can identify whether a K^0 or $\bar{K}^0$ was produced, able to measure $\Gamma\left(K^0_{t=0} \to \pi\pi\right)$ and $\Gamma\left(\bar{K}^0_{t=0} \to \pi\pi\right)$





± interference term

$\star$ The CPLEAR data shown previously can be used to measure $\varepsilon = |\varepsilon| e^{i\phi}$ -Define the asymmetry: $A_{+-} = \frac{\Gamma(\bar{K}^0_{t=0} \to \pi\pi) - \Gamma(K^0_{t=0} \to \pi\pi)}{\Gamma(\bar{K}^0_{t=0} \to \pi\pi) + \Gamma(K^0_{t=0} \to \pi\pi)}$

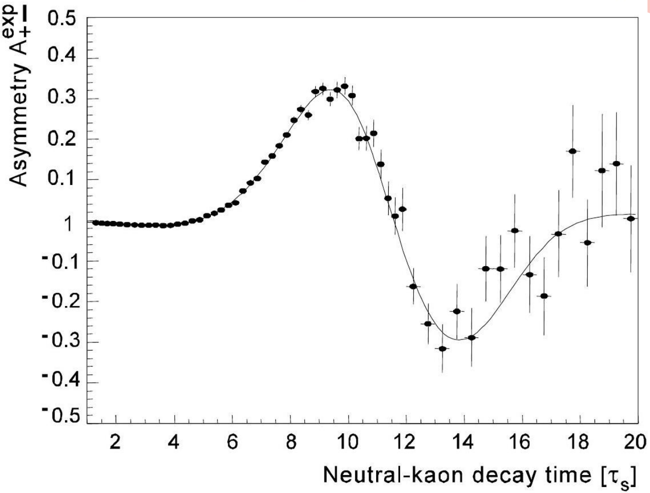
-Using expressions on page 443

$$A_{+-} = \frac{4\Re\{\varepsilon\}\left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t}\right] - 4|\varepsilon| e^{-(\Gamma_L + \Gamma_S)t/2}\cos(\Delta m.t - \phi)}{2\left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t}\right] - \underbrace{8\Re\{\varepsilon\}|\varepsilon| e^{-(\Gamma_L + \Gamma_S)t/2}\cos(\Delta m.t - \phi)}_{\propto |\varepsilon|\Re\{\varepsilon\} \text{ i.e. two small quantities can safely be neglected}}}$$

$$A_{+-} \approx \frac{2\Re\{\varepsilon\}\left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t}\right] - 2|\varepsilon| e^{-(\Gamma_L + \Gamma_S)t/2}\cos(\Delta m.t - \phi)}{e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t}}$$

$$= 2\Re\{\varepsilon\} - \frac{2|\varepsilon| e^{-(\Gamma_L + \Gamma_S)t/2}\cos(\Delta m.t - \phi)}{e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t}}$$

$$= 2\Re\{\varepsilon\} - \frac{2|\varepsilon| e^{(\Gamma_S - \Gamma_L)t/2}\cos(\Delta m.t - \phi)}{1 + |\varepsilon|^2 e^{(\Gamma_S - \Gamma_L)t}}$$



Best fit to the data:

$$|\varepsilon| = (2.264 \pm 0.035) \times 10^{-3}$$
$$\phi = (43.19 \pm 0.73)^{\circ}$$

Appendix XXI: CP Violation via Mixing

- A full description of the SM origin of CP violation in the kaon system is beyond the level of this course, nevertheless, the relation to the box diagrams is illustrated below

⋆ The K-long and K-short wave-functions depend on η

$$|K_L\rangle = \frac{1}{\sqrt{1+|\eta|^2}}\left(\left|K^0\right\rangle + \eta\left|\bar{K}^0\right\rangle\right) \,||K_S\bigg\rangle = \frac{1}{\sqrt{1+|\eta|^2}}\left(\left|K^0\right\rangle - \eta\left|\bar{K}^0\right\rangle\right)$$

$$\text{with} \quad \eta = \sqrt{\frac{M_{12}^* - \frac{1}{2}i\Gamma_{12}^*}{M_{12} - \frac{1}{2}i\Gamma_{12}}}$$

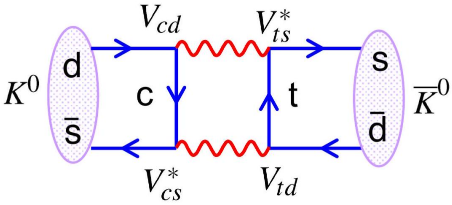
⋆ If $M_{12}^* = M_{12}$; $\Gamma_{12}^* = \Gamma_{12}$ then the K-long and K-short correspond to the CP eigenstates K_1 and K_2

-CP violation is therefore associated with imaginary off-diagonal mass and decay elements for the neutral kaon system

-Experimentally, CP violation is small and $\eta \approx 1$

-Define: $\varepsilon = \frac{1-\eta}{1+\eta} \quad \Rightarrow \quad \eta = \frac{1-\varepsilon}{1+\varepsilon}$

- Consider the mixing term M_{12} which arises from the sum over all possible intermediate states in the mixing box diagrams
e.g.



- In the Standard Model, CP violation is associated

with the imaginary components of the CKM matrix, and it can be shown that mixing leads to CP violation with

$$|\varepsilon| \propto \Im\{M_{12}\}$$

-The differences in masses of the mass eigenstates can be shown to be:

$$\Delta m_K = m_{K_L} - m_{K_S} \approx \sum_{q,q'} \frac{G_F^2}{3\pi^2} f_K^2 m_K \left| V_{qd} V_{qs}^* V_{q'd} V_{q's}^* \right| m_q m_{q'}$$

where q and q' are the quarks in the loops and f_K is a constant

- In terms of the small parameter ε

$$|K_L\rangle = \frac{1}{2\sqrt{1+|\varepsilon|^2}}\left[(1+\varepsilon)\left|K^0\right\rangle + (1-\varepsilon)\left|\bar{K}^0\right\rangle\right]$$
$$|K_S\rangle = \frac{1}{2\sqrt{1+|\varepsilon|^2}}\left[(1-\varepsilon)\left|K^0\right\rangle + (1+\varepsilon)\left|\bar{K}^0\right\rangle\right]$$

- If epsilon is non-zero we have CP violation in the neutral kaon system

Writing $\eta = \sqrt{\frac{M_{12}^* - \frac{1}{2}i\Gamma_{12}^*}{M_{12} - \frac{1}{2}i\Gamma_{12}}} = \sqrt{\frac{z^*}{z}}$ and $z = ae^{i\phi}$

gives $\eta = e^{-i\phi}$

- From which we can find an expression for ε

$$\varepsilon \cdot \varepsilon^* = \frac{1-e^{-i\phi}}{1+e^{-i\phi}} \cdot \frac{1-e^{+i\phi}}{1+e^{i\phi}} = \frac{2-\cos\phi}{2+\cos\phi} = \tan^2\frac{\phi}{2}$$
$$|\varepsilon| = \left|\tan\frac{\phi}{2}\right|$$

Experimentally we know ε is small, hence ϕ is small

$$|\varepsilon| \approx \frac{1}{2}\phi = \frac{1}{2}\arg z \approx \frac{1}{2}\frac{\Im\left\{M_{12} - \frac{1}{2}i\Gamma_{12}\right\}}{\left|M_{12} - \frac{1}{2}i\Gamma_{12}\right|}$$

Appendix XXII: Time Reversal Violation

Not examinable

-Previously in equations (178) and (179) we obtained expressions for strangeness oscillations in the absence of CP violation, e.g.:

$$\Gamma\left(K^0_{t=0} \to K^0\right) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

-This analysis can be extended to include the effects of CP violation to give the following rates (see Question 24):

$$\Gamma\left(K^0_{t=0} \to K^0\right) \propto \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

$$\Gamma\left(\bar{K}^0_{t=0} \to \bar{K}^0\right) \propto \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

$$\Gamma\left(\bar{K}^0_{t=0} \to K^0\right) \propto \frac{1}{4}(1 + 4\,\mathrm{Re}\{\varepsilon\})\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

$$\Gamma\left(K^0_{t=0} \to \bar{K}^0\right) \propto \frac{1}{4}(1 - 4\,\mathrm{Re}\{\varepsilon\})\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

⋆ Including the effects of CP violation find that

$$\Gamma\left(\bar{K}^0_{t=0} \to K^0\right) \neq \Gamma\left(K^0_{t=0} \to \bar{K}^0\right) \qquad \text{Violation of time reversal symmetry !}$$

Not examinable

- No surprise, as CPT is conserved, CP violation implies T violation

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