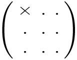
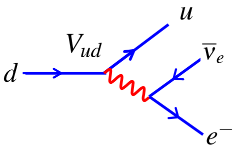
Appendix XVIII: Determination of the CKM Matrix

- The experimental determination of the CKM matrix elements comes mainly from measurements of leptonic decays (the leptonic part is well understood).
- It is easy to produce/observe meson decays, however theoretical uncertainties associated with the decays of bound states often limits the precision
- Contrast this with the measurements of the PMNS matrix, where there are few theoretical uncertainties and the experimental difficulties in dealing with neutrinos limits the precision.

❶ **$|V_{ud}|$** **from nuclear beta decay**





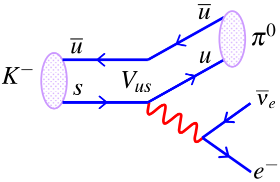
Super-allowed $0^+ \to 0^+$ beta decays are relatively free from theoretical uncertainties

$$\Gamma \propto |V_{ud}|^2$$

$$|V_{ud}| = 0.97377 \pm 0.00027 \qquad (\approx \cos\theta_c)$$

❷ **$|V_{us}|$** **from semi-leptonic kaon decays**

$$\begin{pmatrix} \cdot & \times & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$



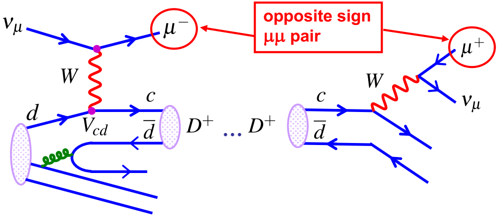
$$\Gamma \propto |V_{us}|^2$$

$$|V_{us}| = 0.2257 \pm 0.0021 \qquad (\approx \sin\theta_c)$$

❸ **$|V_{cd}|$** **from neutrino scattering** $\nu_\mu + N \rightarrow \mu^+\mu^- X$

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \times & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

Look for opposite charge di-muon events in ν_μ scattering from production and decay of a $D^+(c\bar{d})$ meson



$$\text{Rate} \propto |V_{cd}|^2 \text{Br}(D^+ \rightarrow X\mu^+\nu_\mu)$$

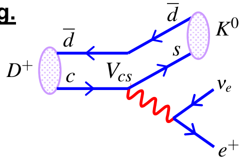
Measured in various collider experiments

$$\Rightarrow \quad |V_{cd}| = 0.230 \pm 0.011$$

❹ **$|V_{cs}|$** **from semi-leptonic charmed meson decays**

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \times & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

e.g.



$$\Gamma \propto |V_{cs}|^2$$

•**Precision limited by theoretical uncertainties**

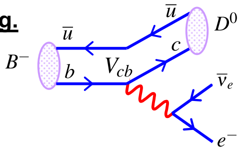
$$|V_{cs}| = 0.957 \pm 0.017 \pm 0.093$$

experimental error — theory uncertainty

❺ **$|V_{cb}|$** **from semi-leptonic B hadron decays**

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \end{pmatrix}$$

e.g.



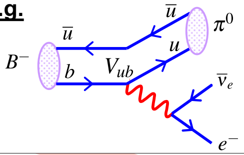
$$\Gamma \propto |V_{cb}|^2$$

$$|V_{cb}| = 0.0416 \pm 0.0006$$

❻ **$|V_{ub}|$** **from semi-leptonic B hadron decays**

$$\begin{pmatrix} \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

e.g.



$$\Gamma \propto |V_{ub}|^2$$

$$|V_{ub}| = 0.0043 \pm 0.0003$$

Appendix XIX: Particle–AntiParticle Mixing

-The wave-function for a single particle with lifetime $\tau = 1/\Gamma$ evolves with time as:

$$\psi(t) = Ne^{-\Gamma t/2}e^{-iMt}$$

which gives the appropriate exponential decay of

$$\langle \psi(t) \mid \psi(t) \rangle = \langle \psi(0) \mid \psi(0) \rangle e^{-t/\tau}$$

-The wave-function satisfies the time-dependent wave equation:

$$\hat{H}|\psi(t)\rangle = \left(M - \frac{1}{2}i\Gamma\right)|\psi(t)\rangle = i\frac{\partial}{\partial t}|\psi(t)\rangle$$

-For a bound state such as a K^0 the mass term includes the "mass" from the weak interaction "potential" $\hat{H}_{\text{weak}}$

$$M = m_{K^0} + \left\langle K^0 \middle| \hat{H}_{\text{weak}} \middle| K^0 \right\rangle + \sum_j \frac{\left| \left\langle K^0 \middle| \hat{H}_{\text{weak}} \middle| j \right\rangle \right|^2}{m_{K^0} - E_j} \leftarrow$$

Sum over intermediate states j

The third term is the 2nd order term in the perturbation expansion corresponding to box diagrams resulting in $K^0 \rightarrow K^0$

Not examinable

- The total decay rate is the sum over all possible decays $K^0 \to f$

$$\Gamma = 2\pi \sum_f \left| \left\langle f \left| \hat{H}_{\text{weak}} \right| K^0 \right\rangle \right|^2 \rho_F \longleftarrow \text{ Density of final states}$$

⋆ Because there are also diagrams which allow $K^0 \leftrightarrow \bar{K}^0$ mixing need to consider the time evolution of a mixed stated

$$\psi(t) = a(t)K^0 + b(t)\bar{K}^0$$

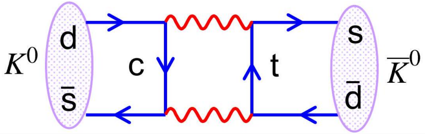
⋆ The time dependent wave-equation of (A1) becomes

$$\begin{pmatrix} M_{11} - \frac{1}{2}i\Gamma_{11} & M_{12} - \frac{1}{2}i\Gamma_{12} \\ M_{21} - \frac{1}{2}i\Gamma_{21} & M_{22} - \frac{1}{2}i\Gamma_{22} \end{pmatrix} \begin{pmatrix} \left|K^0(t)\right\rangle \\ \left|\bar{K}^0(t)\right\rangle \end{pmatrix} = i\frac{\partial}{\partial t} \begin{pmatrix} \left|K^0(t)\right\rangle \\ \left|\bar{K}^0(t)\right\rangle \end{pmatrix}$$

the diagonal terms are as before, and the off-diagonal terms are due to mixing.

$$M_{11} = m_{K^0} + \left\langle K^0 \left| \hat{H}_{\text{weak}} \right| K^0 \right\rangle + \sum_n \frac{\left| \left\langle K^0 \left| \hat{H}_{\text{weak}} \right| K^0 \right\rangle \right|^2}{m_{K^0} - E_n}$$

$$M_{12} = \sum_j \frac{\langle K^0 | \hat{H}_{\text{weak}} | j \rangle^* \langle j | \hat{H}_{\text{weak}} | \overline{K}^0 \rangle}{m_{K^0} - E_j}$$



-The off-diagonal decay terms include the effects of interference between decays to a common final state

$$\Gamma_{12} = 2\pi \sum_f \left\langle f \left| \hat{H}_{\text{weak}} \right| K^0 \right\rangle^* \left\langle f \left| \hat{H}_{\text{weak}} \right| \bar{K}^0 \right\rangle \rho_F$$

-In terms of the time dependent coefficients for the kaon states, (A3) becomes

$$\left[\mathbf{M} - i\frac{1}{2}\Gamma \right] \begin{pmatrix} a \\ b \end{pmatrix} = i\frac{\partial}{\partial t} \begin{pmatrix} a \\ b \end{pmatrix}$$

where the Hamiltonian can be written:

$$\mathbf{H} = \mathbf{M} - i\frac{1}{2}\Gamma = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} - \frac{1}{2} \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix}$$

-Both the mass and decay matrices represent observable quantities and are Hermitian

$$M_{11} = M_{11}^*, \quad M_{22} = M_{22}^*, \quad M_{12} = M_{21}^*$$
$$\Gamma_{11} = \Gamma_{11}^*, \quad \Gamma_{22} = \Gamma_{22}^*, \quad \Gamma_{12} = \Gamma_{21}^*$$

-Furthermore, if CPT is conserved then the masses and decay rates of the $\bar{K}^0$ and K^0 are identical:

$$M_{11} = M_{22} = M; \quad \Gamma_{11} = \Gamma_{22} = \Gamma$$

-Hence the time evolution of the system can be written:

$$\begin{pmatrix} M - \frac{1}{2}i\Gamma & M_{12} - \frac{1}{2}i\Gamma_{12} \\ M_{12}^* - \frac{1}{2}i\Gamma_{12}^* & M - \frac{1}{2}i\Gamma \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = i\frac{\partial}{\partial t} \begin{pmatrix} a \\ b \end{pmatrix}$$

- To solve the coupled differential equations for $a(t)$ and $b(t)$, first find the eigenstates of the Hamiltonian (the K_L and K_S) and then transform into this basis. The eigenvalue equation is:

$$\begin{pmatrix} M - \frac{1}{2}i\Gamma & M_{12} - \frac{1}{2}i\Gamma_{12} \\ M_{12}^* - \frac{1}{2}i\Gamma_{12}^* & M - \frac{1}{2}i\Gamma \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \lambda \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

-Which has non-trivial solutions for

$$|\mathbf{H} - \lambda I| = 0$$

$$\Rightarrow \left(M - \frac{1}{2}i\Gamma - \lambda\right)^2 - \left(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*\right)\left(M_{12} - \frac{1}{2}i\Gamma_{12}\right) = 0$$

with eigenvalues

$$\lambda = M - \frac{1}{2}i\Gamma \pm \sqrt{\left(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*\right)\left(M_{12} - \frac{1}{2}i\Gamma_{12}\right)}$$

Not examinable

-The eigenstates can be obtained by substituting back into (A5)

$$\left(M - \tfrac{1}{2}i\Gamma\right)x_1 + \left(M_{12} - \tfrac{1}{2}i\Gamma_{12}\right) = \left(M - \tfrac{1}{2}i\Gamma \pm \sqrt{\left(M_{12}^* - \tfrac{1}{2}i\Gamma_{12}^*\right)\left(M_{12} - \tfrac{1}{2}i\Gamma_{12}\right)}\right)x_1$$

$$\Rightarrow \quad \frac{x_2}{x_1} = \pm\sqrt{\frac{M_{12}^* - \frac{1}{2}i\Gamma_{12}^*}{M_{12} - \frac{1}{2}i\Gamma_{12}}}$$

⋆ Define

$$\eta = \sqrt{\frac{M_{12}^* - \frac{1}{2}i\Gamma_{12}^*}{M_{12} - \frac{1}{2}i\Gamma_{12}}}$$

- Hence the normalised eigenstates are

$$|K_\pm\rangle = \frac{1}{\sqrt{1+|\eta|^2}}\begin{pmatrix} 1 \\ \pm\eta \end{pmatrix} = \frac{1}{\sqrt{1+|\eta|^2}}\left(\left|K^0\right\rangle \pm \eta\left|\bar{K}^0\right\rangle\right)$$

⋆ Note, in the limit where M_{12}, Γ_{12} are real, the eigenstates correspond to the CP eigenstates K_1 and K_2. Hence we can identify the general eigenstates as as the long and short lived neutral kaons:

$$|K_L\rangle = \frac{1}{\sqrt{1+|\eta|^2}}\left(\left|K^0\right\rangle + \eta\left|\bar{K}^0\right\rangle\right) \quad |K_S\rangle = \frac{1}{\sqrt{1+|\eta|^2}}\left(\left|K^0\right\rangle - \eta\left|\bar{K}^0\right\rangle\right)$$

Not examinable

Substituting these states back into (A2):

$$
\begin{aligned}
|\psi(t)\rangle &= a(t)\left|K^0\right\rangle + b(t)\left|\bar{K}^0\right\rangle \\
&= \sqrt{1+|\eta|^2}\left[\frac{a(t)}{2}(K_L+K_S) + \frac{b(t)}{2\eta}(K_L-K_S)\right] \\
&= \sqrt{1+|\eta|^2}\left[\left(\frac{a(t)}{2}+\frac{b(t)}{2\eta}\right)K_L + \left(\frac{a(t)}{2}-\frac{b(t)}{2\eta}\right)K_S\right] \\
&= \frac{\sqrt{1+|\eta|^2}}{2}\left[a_L(t)K_L + a_S(t)K_S\right]
\end{aligned}
$$

with

$$
a_L(t) \equiv a(t) + \frac{b(t)}{\eta} \quad a_S(t) \equiv a(t) - \frac{b(t)}{\eta}
$$

- Now consider the time evolution of $a_L(t)$

$$
i\frac{\partial a_L}{\partial t} = i\frac{\partial a}{\partial t} + \frac{i}{\eta}\frac{\partial b}{\partial t}
$$

⋆ Which can be evaluated using (A4) for the time evolution of $a(t)$ and $b(t)$:

$$i\frac{\partial a_L}{\partial t} = \left[\left(M - \frac{1}{2}i\Gamma_{12}\right)a + \left(M_{12} - \frac{1}{2}i\Gamma_{12}\right)b\right] + \frac{1}{\eta}\left[\left(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*\right)a + \left(M - \frac{1}{2}i\Gamma\right)b\right]$$

$$= \left(M - \frac{1}{2}i\Gamma\right)\left(a + \frac{b}{\eta}\right) + \left(M_{12} - \frac{1}{2}i\Gamma_{12}\right)b + \frac{1}{\eta}\left(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*\right)a$$

$$= \left(M - \frac{1}{2}i\Gamma\right)a_L + \left(M_{12} - \frac{1}{2}i\Gamma_{12}\right)b + \left(\sqrt{\left(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*\right)\left(M_{12} - \frac{1}{2}i\Gamma_{12}\right)}\right)a$$

$$= \left(M - \frac{1}{2}i\Gamma\right)a_L + \left(\sqrt{\left(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*\right)\left(M_{12} - \frac{1}{2}i\Gamma_{12}\right)}\right)\left(a + \frac{b}{\eta}\right)$$

$$= \left(M - \frac{1}{2}i\Gamma\right)a_L + \left(\sqrt{\left(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*\right)\left(M_{12} - \frac{1}{2}i\Gamma_{12}\right)}\right)a_L$$

$$= \left(m_L - \frac{1}{2}i\Gamma_L\right)a_L$$

⋆ Hence:

$$i\frac{\partial a_L}{\partial t} = \left(m_L - \frac{1}{2}i\Gamma_L\right)a_L$$

with $m_L = M + \mathrm{Re}\left\{\sqrt{\left(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*\right)\left(M_{12} - \frac{1}{2}i\Gamma_{12}\right)}\right\}$

⋆ Following the same procedure obtain:

$$i\frac{\partial a_S}{\partial t} = \left(m_S - \frac{1}{2}i\Gamma_S\right) a_S$$

with $m_S = M - \Re\left\{\sqrt{\left(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*\right)\left(M_{12} - \frac{1}{2}i\Gamma_{12}\right)}\right\}$

and $\Gamma_S = \Gamma + 2\Im\left\{\sqrt{\left(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*\right)\left(M_{12} - \frac{1}{2}i\Gamma_{12}\right)}\right\}$

⋆ In matrix notation we have

⋆ Solving we obtain

$$\begin{pmatrix} M_L - \frac{1}{2}i\Gamma_L & 0 \\ 0 & M_S - \frac{1}{2}i\Gamma_S \end{pmatrix}\begin{pmatrix} a_L \\ a_S \end{pmatrix} = i\frac{\partial}{\partial t}\begin{pmatrix} a_L \\ a_S \end{pmatrix}$$

$$a_L(t) \propto e^{-im_L t - \Gamma_L t/2} \quad a_S(t) \propto e^{-im_S t - \Gamma_S t/2}$$

⋆ Hence in terms of the $\mathrm{K_L}$ and $\mathrm{K_S}$ basis the states propagate as independent particles with definite masses and lifetimes (the mass eigenstates). The time evolution of the neutral kaon system can be written

$$|\psi(t)\rangle = A_L e^{-im_L t - \Gamma_L t/2}|K_L\rangle + A_S e^{-im_S t - \Gamma_S t/2}|K_S\rangle$$

where A_L and A_S are constants

Appendix XX: CP Violation : $\pi\pi$ decays

⋆ Consider the development of the $K^0 - \bar{K}^0$ system now including CP violation

⋆ Repeat previous derivation using

$$|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[|K_1\rangle + \varepsilon\,|K_2\rangle\right] \quad |K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[|K_2\rangle + \varepsilon\,|K_1\rangle\right]$$

-Writing the CP eigenstates in terms of $K^0, \bar{K}^0$

$$|K_L\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+|\varepsilon|^2}}\left[(1+\varepsilon)\,|K_0\rangle + (1-\varepsilon)\left|\bar{K}^0\right\rangle\right]$$

$$|K_S\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+|\varepsilon|^2}}\left[(1+\varepsilon)\,|K_0\rangle - (1-\varepsilon)\left|\bar{K}^0\right\rangle\right]$$

- Inverting these expressions obtain

$$\left|K^0\right\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2}}\frac{1}{1+\varepsilon}\left(|K_L\rangle + |K_S\rangle\right) \quad \left|\bar{K}^0\right\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2}}\frac{1}{1-\varepsilon}\left(|K_L\rangle - |K_S\rangle\right)\,|$$

-Hence a state that was produced as a K^0 evolves with time as:

$$|\psi(t)\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2}}\frac{1}{1+\varepsilon}\left(\theta_L(t)\,|K_L\rangle + \theta_S(t)\,|K_S\rangle\right)$$

where as before $\theta_S(t) = e^{-\left(im_S+\frac{\Gamma_S}{2}\right)t}$ and $\theta_L(t) = e^{-\left(im_L+\frac{\Gamma_L}{2}\right)t}$

-If we are considering the decay rate to $\pi\pi$ need to express the wave-function in terms of the CP eigenstates (remember we are neglecting CP violation in the decay)

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\frac{1}{1+\varepsilon}\left[\left(|K_2\rangle + \varepsilon|K_1\rangle\right)\theta_L(t) + \left(|K_1\rangle + \varepsilon|K_2\rangle\right)\theta_S(t)\right]$$

$$= \frac{1}{\sqrt{2}}\frac{1}{1+\varepsilon}\left[\left(\theta_S + \varepsilon\theta_L\right)|K_1\rangle + \left(\theta_L + \varepsilon\theta_S\right)|K_2\rangle\right]$$

CP Eigenstates

-Two pion decays occur with $\mathrm{CP} = +1$ and therefore arise from decay of the $\mathrm{CP} = +1$ kaon eigenstate, i.e. K_1

$$\Gamma\left(K^0_{t=0} \to \pi\pi\right) \propto |\langle K_1 \mid \psi(t)\rangle|^2 = \frac{1}{2}\left|\frac{1}{1+\varepsilon}\right|^2 |\theta_S + \varepsilon\theta_L|^2$$

- Since $|\varepsilon| \ll 1$

$$\left|\frac{1}{1+\varepsilon}\right|^2 = \frac{1}{(1+\varepsilon^*)(1+\varepsilon)} \approx \frac{1}{1+2\Re\{\varepsilon\}} \approx 1 - 2\Re\{\varepsilon\}$$

- Now evaluate the $|\theta_S + \varepsilon\theta_L|^2$ term again using

$$|z_1 \pm z_2|^2 = |z_1|^2 + |z_2|^2 \pm 2\Re\left(z_1 z_2^*\right)$$

$$|\theta_S + \varepsilon\theta_L|^2 = \left| e^{-im_S t - \frac{\Gamma_S}{2} t} + \varepsilon e^{-im_L t - \frac{\Gamma_L}{2} t} \right|^2$$

$$= e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} + 2\,\mathrm{Re}\left\{ e^{-im_S t - \frac{\Gamma_S}{2} t} \cdot \varepsilon^* e^{+im_L t - \frac{\Gamma_L}{2} t} \right\}$$

-Writing $\varepsilon = |\varepsilon| e^{i\phi}$

$$|\theta_S + \varepsilon\theta_L|^2 = e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} + 2|\varepsilon| e^{-(\Gamma_S+\Gamma_L)t/2}\,\mathrm{Re}\left\{ e^{i(m_L - m_S)t - \phi} \right\}$$

$$= e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} + 2|\varepsilon| e^{-(\Gamma_S+\Gamma_L)t/2} \cos(\Delta m.t - \phi)$$

-Putting this together we obtain:

$$\Gamma\left(K^0_{t=0} \to \pi\pi\right) = \frac{1}{2}(1 - 2\,\mathrm{Re}\{\varepsilon\}) N_{\pi\pi} \left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} + 2|\varepsilon| e^{-(\Gamma_S+\Gamma_L)t/2} \cos(\Delta m.t - \phi) \right]$$

Short lifetime

component

$K_S \to \pi\pi$

CP violating long lifetime component KL¦p

Interference term

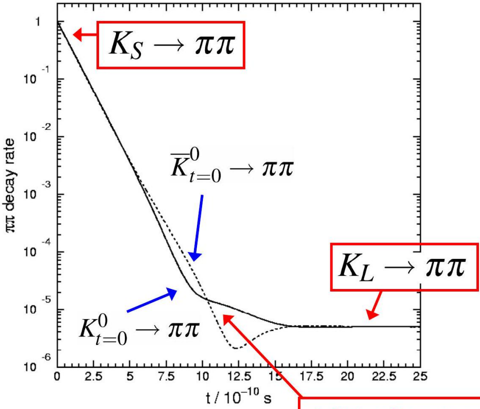
-In exactly the same manner obtain for a beam which was produced as $\bar{K}^0$

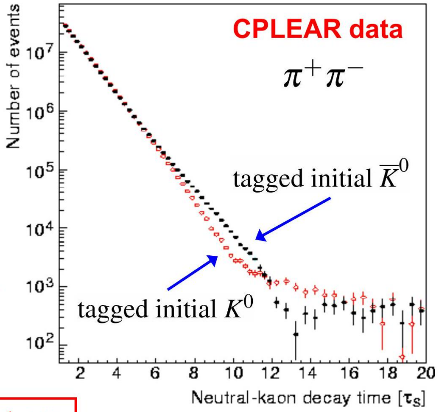
Not examinable

$$\Gamma\left(K^0_{t=0} \to \pi\pi\right) \to \frac{1}{2}(1 - 2\,\mathrm{Re}\{\varepsilon\})N_{\pi\pi} \cdot |\varepsilon|^2 e^{-\Gamma_L t}$$

i.e. CP violating $K_L \to \pi\pi$ decays

⋆ Since CPLEAR can identify whether a K^0 or $\bar{K}^0$ was produced, able to measure $\Gamma\left(K^0_{t=0} \to \pi\pi\right)$ and $\Gamma\left(\bar{K}^0_{t=0} \to \pi\pi\right)$





± interference term

⋆ The CPLEAR data shown previously can be used to measure $\varepsilon = |\varepsilon| e^{i\phi}$ -Define the asymmetry: $A_{+-} = \frac{\Gamma\left(\bar{K}^0_{t=0}\to\pi\pi\right)-\Gamma\left(K^0_{t=0}\to\pi\pi\right)}{\Gamma\left(\bar{K}^0_{t=0}\to\pi\pi\right)+\Gamma\left(K^0_{t=0}\to\pi\pi\right)}$

-Using expressions on page 443

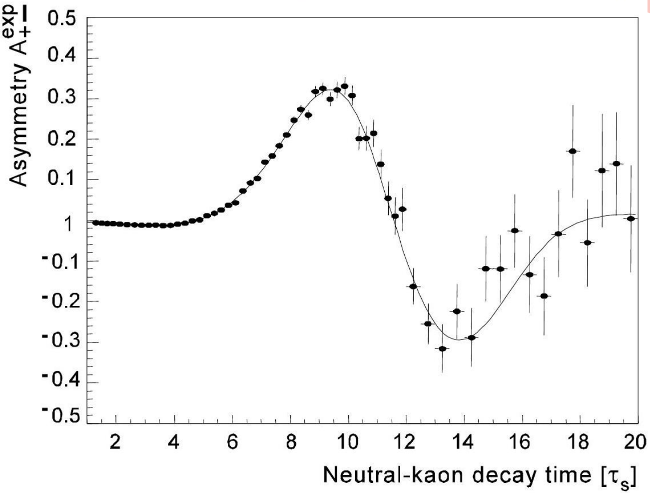
$$A_{+-} = \frac{4\Re\{\varepsilon\}\left[e^{-\Gamma_S t}+|\varepsilon|^2 e^{-\Gamma_L t}\right]-4|\varepsilon|e^{-(\Gamma_L+\Gamma_S)t/2}\cos(\Delta m.t-\phi)}{2\left[e^{-\Gamma_S t}+|\varepsilon|^2 e^{-\Gamma_L t}\right]-\underbrace{8\Re\{\varepsilon\}|\varepsilon|e^{-(\Gamma_L+\Gamma_S)t/2}\cos(\Delta m.t-\phi)}_{\propto |\varepsilon|\Re\{\varepsilon\}\text{ i.e. two small quantities can safely be neglected}}}$$

$$A_{+-} \approx \frac{2\Re\{\varepsilon\}\left[e^{-\Gamma_S t}+|\varepsilon|^2 e^{-\Gamma_L t}\right]-2|\varepsilon|e^{-(\Gamma_L+\Gamma_S)t/2}\cos(\Delta m.t-\phi)}{e^{-\Gamma_S t}+|\varepsilon|^2 e^{-\Gamma_L t}}$$

$$= 2\Re\{\varepsilon\} - \frac{2|\varepsilon|e^{-(\Gamma_L+\Gamma_S)t/2}\cos(\Delta m.t-\phi)}{e^{-\Gamma_S t}+|\varepsilon|^2 e^{-\Gamma_L t}}$$

$$= 2\Re\{\varepsilon\} - \frac{2|\varepsilon|e^{(\Gamma_S-\Gamma_L)t/2}\cos(\Delta m.t-\phi)}{1+|\varepsilon|^2 e^{(\Gamma_S-\Gamma_L)t}}$$

Not examinable



Best fit to the data:

$$|\varepsilon| = (2.264 \pm 0.035) \times 10^{-3}$$
$$\phi = (43.19 \pm 0.73)^\circ$$

Appendix XXI: CP Violation via Mixing

- A full description of the SM origin of CP violation in the kaon system is beyond the level of this course, nevertheless, the relation to the box diagrams is illustrated below

⋆ The K-long and K-short wave-functions depend on η

$$|K_L\rangle = \frac{1}{\sqrt{1+|\eta|^2}}\left(\left|K^0\right\rangle + \eta\left|\bar{K}^0\right\rangle\right) \, ||K_S\Big\rangle = \frac{1}{\sqrt{1+|\eta|^2}}\left(\left|K^0\right\rangle - \eta\left|\bar{K}^0\right\rangle\right)$$

$$\text{with} \quad \eta = \sqrt{\frac{M_{12}^* - \frac{1}{2}i\Gamma_{12}^*}{M_{12} - \frac{1}{2}i\Gamma_{12}}}$$

⋆ If $M_{12}^* = M_{12}$; $\Gamma_{12}^* = \Gamma_{12}$ then the K-long and K-short correspond to the CP eigenstates K_1 and K_2

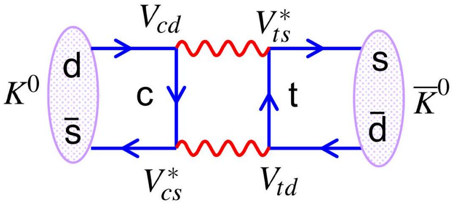
-CP violation is therefore associated with imaginary off-diagonal mass and decay elements for the neutral kaon system

-Experimentally, CP violation is small and $\eta \approx 1$

-Define: $\varepsilon = \frac{1-\eta}{1+\eta} \quad \Rightarrow \quad \eta = \frac{1-\varepsilon}{1+\varepsilon}$

- Consider the mixing term M_{12} which arises from the sum over all possible intermediate states in the mixing box diagrams

e.g.



- In the Standard Model, CP violation is associated

with the imaginary components of the CKM matrix, and it can be shown that mixing leads to *CP* violation with

$$|\varepsilon| \propto \Im\{M_{12}\}$$

-The differences in masses of the mass eigenstates can be shown to be:

$$\Delta m_K = m_{K_L} - m_{K_S} \approx \sum_{q,q'} \frac{G_F^2}{3\pi^2} f_K^2 m_K \left| V_{qd} V_{qs}^* V_{q'd} V_{q's}^* \right| m_q m_{q'}$$

where q and q' are the quarks in the loops and f_K is a constant

- In terms of the small parameter ε

$$|K_L\rangle = \frac{1}{2\sqrt{1+|\varepsilon|^2}}\left[(1+\varepsilon)\left|K^0\right\rangle + (1-\varepsilon)\left|\bar{K}^0\right\rangle\right]$$
$$|K_S\rangle = \frac{1}{2\sqrt{1+|\varepsilon|^2}}\left[(1-\varepsilon)\left|K^0\right\rangle + (1+\varepsilon)\left|\bar{K}^0\right\rangle\right]$$

- If epsilon is non-zero we have CP violation in the neutral kaon system

Writing $\eta = \sqrt{\frac{M_{12}^* - \frac{1}{2}i\Gamma_{12}^*}{M_{12} - \frac{1}{2}i\Gamma_{12}}} = \sqrt{\frac{z^*}{z}}$ and $z = ae^{i\phi}$

gives $\eta = e^{-i\phi}$

- From which we can find an expression for ε

$$\varepsilon \cdot \varepsilon^* = \frac{1-e^{-i\phi}}{1+e^{-i\phi}} \cdot \frac{1-e^{+i\phi}}{1+e^{i\phi}} = \frac{2-\cos\phi}{2+\cos\phi} = \tan^2\frac{\phi}{2}$$
$$|\varepsilon| = \left|\tan\frac{\phi}{2}\right|$$

Experimentally we know ε is small, hence ϕ is small

$$|\varepsilon| \approx \frac{1}{2}\phi = \frac{1}{2}\arg z \approx \frac{1}{2}\frac{\Im\left\{M_{12} - \frac{1}{2}i\Gamma_{12}\right\}}{\left|M_{12} - \frac{1}{2}i\Gamma_{12}\right|}$$

Appendix XXII: Time Reversal Violation

-Previously in equations (166) and (167) we obtained expressions for strangeness oscillations in the absence of CP violation, e.g.:

$$\Gamma\left(K^0_{t=0} \to K^0\right) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

-This analysis can be extended to include the effects of CP violation to give the following rates (see Question 24):

$$\Gamma\left(K^0_{t=0} \to K^0\right) \propto \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

$$\Gamma\left(\bar{K}^0_{t=0} \to \bar{K}^0\right) \propto \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

$$\Gamma\left(\bar{K}^0_{t=0} \to K^0\right) \propto \frac{1}{4}(1 + 4\,\mathrm{Re}\{\varepsilon\})\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

$$\Gamma\left(K^0_{t=0} \to \bar{K}^0\right) \propto \frac{1}{4}(1 - 4\,\mathrm{Re}\{\varepsilon\})\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m t\right]$$

$\star$ Including the effects of CP violation find that

$$\Gamma\left(\bar{K}^0_{t=0} \to K^0\right) \neq \Gamma\left(K^0_{t=0} \to \bar{K}^0\right)$$ Violation of time reversal symmetry !

- No surprise, as CPT is conserved, CP violation implies T violation