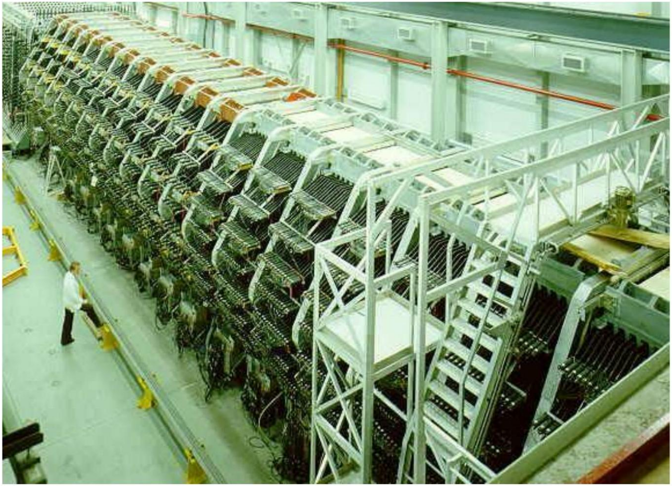
Dr C.G. Lester, 2023



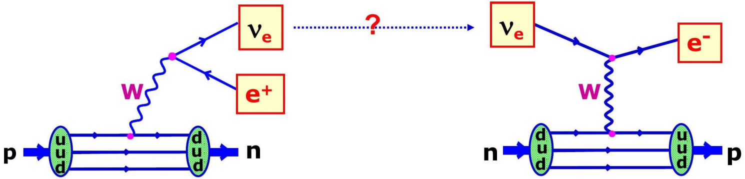
H10: Leptonic Weak Interactons and Neutrino Deep Inelastic Scattering

Aside : Neutrino Flavours

- Recent experiments (see Handout 11) imply neutrinos have mass (albeit very small).
- The textbook neutrino states, ν_e, ν_μ, ν_τ, are not the fundamental particles; these are ν_1, ν_2, ν_3.
- Concepts like 'electron number' conservation are now known not to hold.
- So what are ν_e, ν_μ, ν_τ ?
- Never directly observe neutrinos — can only detect them by their weak interactions. Hence by definition ν_e is the neutrino state produced along with an electron. Similarly, charged current weak interactions of the state ν_e produce an electron.

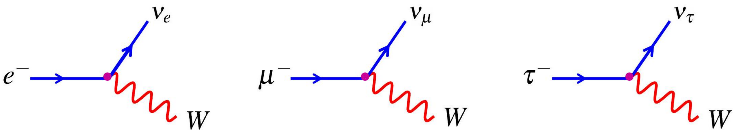
$$\boxed{\{\nu_e, \nu_\mu, \nu_\tau\} = \text{weak eignestates}}$$

- Unless dealing with very large distances: the neutrino produced with a positron will interact to produce an electron. For the discussion of the weak interaction continue to use ν_e, ν_μ, ν_τ as if they were the fundamental particle states.

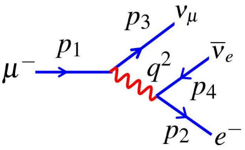


Muon Decay and Lepton Universality

- The leptonic charged current ($W^{\pm}$) interaction vertices are:



- Consider muon decay:



- It is straight-forward to write down the matrix element

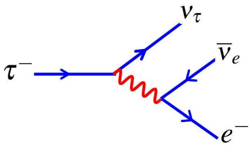
$$M_{fi} = \frac{g_W^{(e)} g_W^{(\mu)}}{8m_W^2}[\bar{u}(p_3)\gamma^{\mu}(1-\gamma^5)u(p_1)]g_{\mu\nu}[\bar{u}(p_2)\gamma^{\nu}(1-\gamma^5)v(p_4)].$$

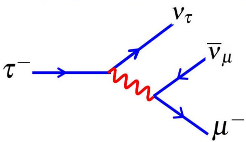
- Note: for lepton decay $q^2 \ll m_W^2$ so propagator is a constant (Fermi theory limit!).
- Its evaluation and subsequent treatment of a three-body decay is rather tricky (and not particularly interesting). Here will simply quote the result (over page):

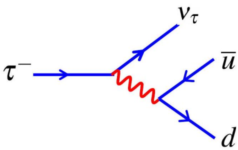
- The muon to electron rate is

$$\Gamma(\mu \to e\nu\nu) = \frac{G_F^e G_F^\mu m_\mu^5}{192\pi^3} = \frac{1}{\tau_\mu} \quad \text{with} \quad G_F = \frac{g_W^2}{4\sqrt{2}m_W^2}.$$

- Similarly for tau to electron $\Gamma(\tau \to e\nu\nu) = \frac{G_F^e G_F^\tau m_\tau^5}{192\pi^3}$ however, the tau can decay to a number of final states:







$Br(\tau \to e\nu\nu) = 0.1784(5)$ $Br(\tau \to \mu\nu\nu) = 0.1736(5)$

- Recall total width (total transition rate) is the sum of the partial widths

$$\Gamma = \sum_i \Gamma_i = \frac{1}{\tau}$$

- Can relate partial decay width to total decay width and therefore lifetime:

$$\Gamma(\tau \to e\nu\nu) = \Gamma_\tau Br(\tau \to e\nu\nu) = Br(\tau \to e\nu\nu)/\tau_\tau$$

- Therefore predict $\tau_\mu = \frac{192\pi^3}{G_F^e G_F^\mu m_\mu^5}$ $\tau_\tau = \frac{192\pi^3}{G_F^e G_F^\tau m_\tau^5} Br(\tau \to e\nu\nu).$

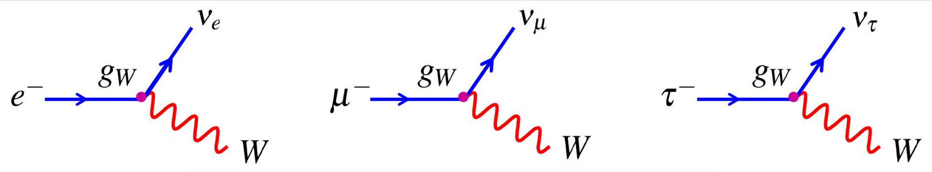
- All these quantities are precisely measured: $\left\{ \begin{array}{l} m_\mu = 0.1056583692(94)\,\mathrm{GeV} \\ m_\tau = 1.77699(28)\,\mathrm{GeV} \\ \tau_\tau = 0.2906(10) \times 10^{-12}\,\mathrm{s} \\ \tau_\mu = 2.19703(4) \times 10^{-6}\,\mathrm{s} \\ Br(\tau \to e\nu\nu) = 0.1784(5) \end{array} \right\}$ so

$$\frac{G_F^\tau}{G_F^\mu} = \frac{m_\mu^5 \tau_\mu}{m_\tau^5 \tau_\tau} Br(\tau \to e\nu\nu) = 1.0024 \pm 0.0033.$$

- Similarly by comparing $Br(\tau \to \mu\nu\nu)$ and $Br(\tau \to e\nu\nu)$

$$\frac{G_F^e}{G_F^\mu} = 1.000 \pm 0.004.$$

The above demonstrates the weak charged current is the same for all leptonic vertices. This is referred to as 'Charged Current Lepton Universality'.



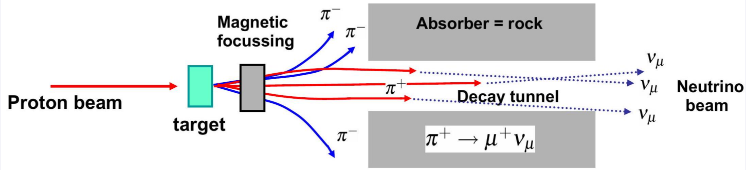
Neutrino Scattering

In Handout 6 considered electron-proton Deep Inelastic Scattering where a virtual photon is used to probe nucleon structure.

- Can also consider the weak interaction equivalent: Neutrino Deep Inelastic Scattering where a virtual W-boson probes the structure of nucleons. **This provides additional information about parton structure functions.**
- provides a good example of calculations of weak interaction cross sections.

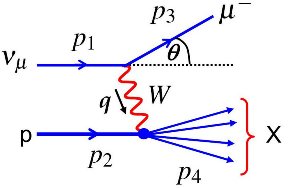
Neutrino beams are needed. To make them:

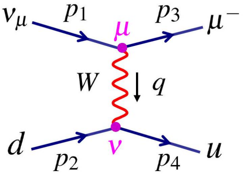
- Smash high energy protons into a fixed target to make hadrons.
- Focus positive pions/kaons.
- Allow them to decay $\pi^+ \to \mu^+\nu_\mu + K^+ \to \mu^+ \nu_\mu (BR \approx 64\%)$;
- this gives a beam of "collimated" ν_μ.
- Focus negative pions/kaons to give beam of $\bar{\nu}_\mu$.



Neutrino-Quark Scattering

- For ν_μ-proton Deep Inelastic Scattering the underlying process is $\nu_\mu d \to \mu^- u$





- In the limit $q^2 \ll m_W^2$ the W-boson propagator is $\approx ig_{\mu\nu}/m_W^2$. so the Feynman rules give:

$$-iM_{fi} = \left[-i\frac{g_W}{\sqrt{2}}\bar{u}(p_3)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)u(p_1)\right] \frac{ig_{\mu\nu}}{m_W^2} \left[-i\frac{g_W}{\sqrt{2}}\bar{u}(p_4)\tfrac{1}{2}\gamma^\nu(1-\gamma^5)u(p_2)\right]$$

so

$$\boxed{M_{fi} = \frac{g_W^2}{2m_W^2}g_{\mu\nu}\left[\bar{u}(p_3)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)u(p_1)\right]\left[\bar{u}(p_4)\tfrac{1}{2}\gamma^\nu(1-\gamma^5)u(p_2)\right]}.$$

- Evaluate the matrix element in the extreme relativistic limit where the muon and quark masses can be neglected.

- In this limit the helicity states are equivalent to the chiral states. Furthermore

$$\frac{1}{2}(1-\gamma^5)u_\uparrow(p_1) = 0 \quad \text{and} \quad \frac{1}{2}(1-\gamma^5)u_\downarrow(p_1) = u_\downarrow(p_1)$$

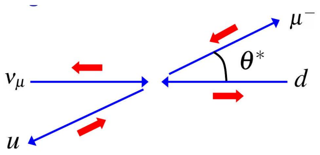
so $M_{fi} = 0$ for $u_\uparrow(p_1)$ and $u_\uparrow(p_2)$.

- Since the weak interaction 'conserves the helicity', the only helicity combination where the matrix element is non-zero is

$$M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} \left[\bar{u}_\downarrow(p_3)\gamma^\mu u_\downarrow(p_1)\right]\left[\bar{u}_\downarrow(p_4)\gamma^\nu u_\downarrow(p_2)\right].$$

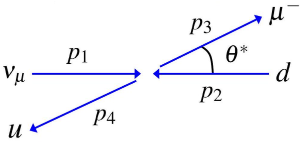
(We could have written this down straight away as in the ultra-relativistic limit only LH helicity particle states participate in the weak interaction.)

- We will next consider this scattering in the C.o.M frame:



Evaluation of Neutrino-Quark Scattering Matrix Element

- Go through the calculation in gory detail (fortunately only one helicity combination)
- In the $\nu_\mu d$ CMS frame, neglecting particle masses:



$$p_1 = (E, 0, 0, E),$$
$$p_2 = (E, 0, 0, -E)$$
$$p_3 = (E, E\sin\theta^*, 0, E\cos\theta^*)$$
$$p_4 = (E, -E\sin\theta^*, 0, -E\cos\theta^*)$$

- Dealing with LH helicity particle spinors. From Handout 2 (page 106), for a massless particle travelling in direction θ, ϕ:

$$u_\downarrow = \sqrt{E}\begin{pmatrix} -s \\ ce^{i\phi} \\ s \\ -ce^{i\phi} \end{pmatrix} \qquad c = \cos\frac{\theta}{2}; \quad s = \sin\frac{\theta}{2}$$

- Here $(\theta_1, \phi_1) = (0, 0)$; $(\theta_2, \phi_2) = (\pi, 0)$; $(\theta_3, \phi_3) = (\theta^*, 0)$; $(\theta_4, \phi_4) = (\pi - \theta^*, \pi)$ giving:

$$u_\downarrow(p_1) = \sqrt{E}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}; \; u_\downarrow(p_2) = \sqrt{E}\begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}; \; u_\downarrow(p_3) = \sqrt{E}\begin{pmatrix} -s \\ c \\ s \\ -c \end{pmatrix}; \; u_\downarrow(p_4) = \sqrt{E}\begin{pmatrix} -c \\ -s \\ c \\ s \end{pmatrix}.$$

Neutrino-Quark scattering (cont.)

- To calculate

$$M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} \left[\bar{u}_\downarrow(p_3)\gamma^\mu u_\downarrow(p_1)\right] \left[\bar{u}_\downarrow(p_4)\gamma^\nu u_\downarrow(p_2)\right]$$

need twice to evaluate terms of the form

$$\begin{aligned}
\overline{\psi}\gamma^0\phi &= \psi^\dagger\gamma^0\gamma^0\phi = \psi_1^*\phi_1 + \psi_2^*\phi_2 + \psi_3^*\phi_3 + \psi_4^*\phi_4, \\
\overline{\psi}\gamma^1\phi &= \psi^\dagger\gamma^0\gamma^1\phi = \psi_1^*\phi_4 + \psi_2^*\phi_3 + \psi_3^*\phi_2 + \psi_4^*\phi_1, \\
\overline{\psi}\gamma^2\phi &= \psi^\dagger\gamma^0\gamma^2\phi = -i(\psi_1^*\phi_4 - \psi_2^*\phi_3 + \psi_3^*\phi_2 - \psi_4^*\phi_1), \\
\overline{\psi}\gamma^3\phi &= \psi^\dagger\gamma^0\gamma^3\phi = \psi_1^*\phi_3 - \psi_2^*\phi_4 + \psi_3^*\phi_1 - \psi_4^*\phi_2.
\end{aligned}$$

- Using

$$u_\downarrow(p_1) = \sqrt{E}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix};\ u_\downarrow(p_2) = \sqrt{E}\begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix};\ u_\downarrow(p_3) = \sqrt{E}\begin{pmatrix} -s \\ c \\ s \\ -c \end{pmatrix};\ u_\downarrow(p_4) = \sqrt{E}\begin{pmatrix} -c \\ -s \\ c \\ s \end{pmatrix}$$

we get

$$\boxed{\begin{aligned} \bar{u}_\downarrow(p_3)\gamma^\mu u_\downarrow(p_1) &= 2E(c, s, -is, c) \\ \bar{u}_\downarrow(p_4)\gamma^\nu u_\downarrow(p_2) &= 2E(c, -s, -is, -c) \end{aligned}}$$

$$\Longrightarrow \boxed{M_{fi} = \frac{g_W^2}{2m_W^2} 4E^2(c^2 + s^2 + s^2 + c^2) = \frac{g_W^2 \hat{s}}{m_W^2}}$$

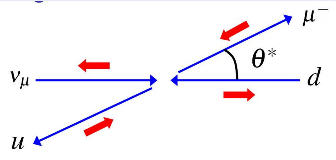
where $\hat{s} = (2E)^2$.

Neutrino-Quark scattering (cont.)

Note that the Matrix Element is isotropic:

$$M_{fi} = \frac{g_W^2}{m_W^2}\hat{s}.$$

We could have anticipated this since the helicity combination (spins anti-parallel) has $S_z = 0$ so expect no preferred polar angle.



- As before need to sum over all possible spin states and average over all possible initial state spin states. Here only one possible spin combination ($LL \to LL$) and only two possible initial state combinations (the neutrino is always produced in a LH helicity state).

$$\left\langle |M_{fi}|^2 \right\rangle = \frac{1}{2}\left|\frac{g_W^2}{m_W^2}\hat{s}\right|^2$$

  The factor of a half arises because half of the time the quark will be in a RH states and won't participate in the charged current Weak interaction.
- From Handout 1, in the extreme relativistic limit, the cross section for any $2 \to 2$ body scattering process is

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2\hat{s}}\langle |M_{fi}|^2\rangle.$$

Neutrino-Quark scattering (conclusion).

- Therefore:

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2 \hat{s}} \langle |M_{fi}|^2 \rangle = \frac{1}{64\pi \hat{s}} \frac{1}{2} \left(\frac{g_W^2 \hat{s}}{m_W^2} \right)^2 = \left(\frac{g_W^2}{8\sqrt{2}\pi m_W^2} \right)^2 \hat{s}.$$

- Using $\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8 m_W^2}$ the above simplifies to

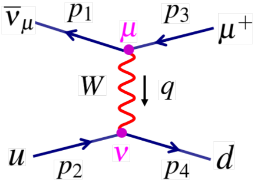
$$\boxed{\frac{d\sigma}{d\Omega^*} = \frac{G_F^2}{4\pi^2} \hat{s}} \tag{127}$$

and integrating this isotropic distribution over $d\Omega^*$ gives

$$\boxed{\sigma_{\nu q} = \frac{G_F^2 \hat{s}}{\pi}} \tag{128}$$

- Since the cross section is a (longitudinally) Lorentz invariant, (128) is also the cross section for scattering in the lab frame.

Antineutrino-Quark Scattering



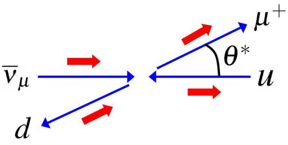
- In the ultra-relativistic limit, the charged-current interaction matrix element is:

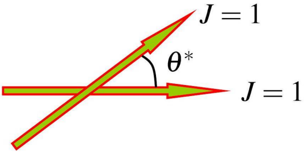
$$M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} \Big[\bar{v}(p_1)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)v(p_3)\Big] \Big[\bar{u}(p_4)\gamma^\nu \tfrac{1}{2}(1-\gamma^5)u(p$$

- In the extreme relativistic limit only LH Helicity particles and RH Helicity anti-particles participate in the charged current weak interaction:

$$M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} [\bar{v}_\uparrow(p_1)\gamma^\mu v_\uparrow(p_3)] [\bar{u}_\downarrow(p_4)\gamma^\nu u_\downarrow(p_2)]$$

In terms of the particle spins it can be seen that the interaction occurs in a total angular momentum $J = 1$ state:

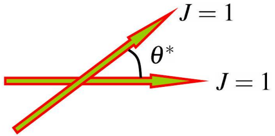




Antineutrino-Quark Scattering (cont.)

- Similarly to the neutrino-quark scattering calculation obtain:

$$\frac{d\sigma_{\bar{\nu}q}}{d\Omega^*} = \frac{d\sigma_{\nu q}}{d\Omega^*}\frac{1}{4}(1+\cos\theta^*)^2\hat{s}$$



- The factor $\frac{1}{4}(1+\cos\theta^*)^2$ can be understood in terms of the overlap of the initial and final angular momentum wave-functions.
- Integrating over solid angle:

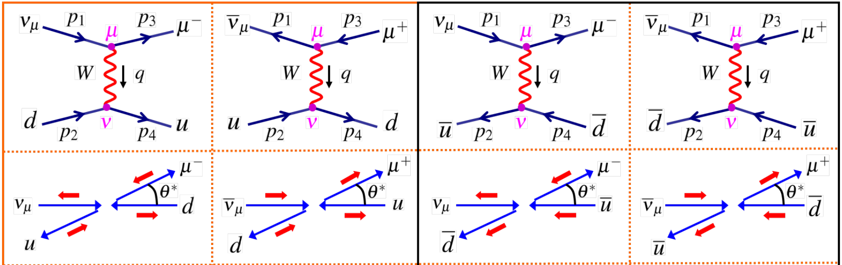
$$\int (1+\cos\theta^*)^2\, d\Omega^* = 2\pi \int_{-1}^{+1} (1+\cos\theta^*)^2\, d(\cos\theta^*) = \frac{16\pi}{3}$$

$$\Longrightarrow \qquad \boxed{\sigma_{\bar{\nu}q} = \frac{G_F^2 \hat{s}}{3\pi}}.$$

- This is a factor three smaller than the neutrino quark cross-section: $\boxed{\frac{\sigma_{\bar{\nu}q}}{\sigma_{\nu q}} = \frac{1}{3}}$.

(Anti)neutrino-(Anti)quark Scattering

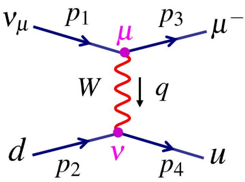
- Non-zero anti-quark component to the nucleon also consider scattering from q
- Cross-sections can be obtained immediately by comparing with quark scattering and remembering to only include LH particles and RH anti-particles



$S_z = 0$	$S_z = +1$	$S_z = -1$	$S_z = 0$
$\frac{d\sigma_{\nu q}}{d\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s}$	$\frac{d\sigma_{\bar{\nu} q}}{d\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2\hat{s}$	$\frac{d\sigma_{\nu \bar{q}}}{d\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2\hat{s}$	$\frac{d\sigma_{\bar{\nu} \bar{q}}}{d\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s}$
$\sigma_{\nu q} = \frac{G_F^2\hat{s}}{\pi}$	$\sigma_{\bar{\nu} q} = \frac{G_F^2\hat{s}}{3\pi}$	$\sigma_{\nu \bar{q}} = \frac{G_F^2\hat{s}}{3\pi}$	$\sigma_{\bar{\nu} \bar{q}} = \frac{G_F^2\hat{s}}{\pi}$

Differential Cross Section in form $d\sigma/dy$

- We derived the differential neutrino scattering cross sections in C.o.M frame. Let us convert them to Lorentz invariant form.



- As for DIS use Lorentz invariant $y \equiv \frac{p_2.q}{p_2.p_1}$.
- In relativistic limit y can be expressed in terms of the C.o.M. scattering angle: $y = \frac{1}{2}(1 - \cos\theta^*)$.
- In the lab. frame $y = 1 - \frac{E_3}{E_1}$.

- Convert from $\frac{d\sigma}{d\Omega^*} \to \frac{d\sigma}{dy}$ using

$$\frac{d\sigma}{dy} = \left|\frac{d\cos\theta^*}{dy}\right| \frac{d\sigma}{d\cos\theta^*} = \left|\frac{d\cos\theta^*}{dy}\right| 2\pi \frac{d\sigma}{d\Omega^*} = 4\pi \frac{d\sigma}{d\Omega^*}.$$

- But we already found (in (127)) that: $\frac{d\sigma}{d\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s}$ hence:

$$\boxed{\frac{d\sigma_{\nu q}}{dy} = \frac{d\sigma_{\bar{\nu}\bar{q}}}{dy} = \frac{G_F^2}{\pi}\hat{s}}.$$

Similarly

$$\frac{d\sigma_{\bar{\nu}q}}{d\Omega^*} = \frac{d\sigma_{\nu\bar{q}}}{d\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2\hat{s}$$

becomes

$$\frac{d\sigma_{\bar{\nu}q}}{d\Omega^*} = \frac{d\sigma_{\nu\bar{q}}}{d\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2\hat{s}$$

from $y = \frac{1}{2}(1-\cos\theta^*)$ $\rightarrow 1-y = \frac{1}{2}(1+\cos\theta^*)$ and hence

$$\boxed{\frac{d\sigma_{\bar{\nu}q}}{dy} = \frac{d\sigma_{\nu\bar{q}}}{dy} = \frac{G_F^2}{\pi}(1-y)^2\hat{s}}.$$

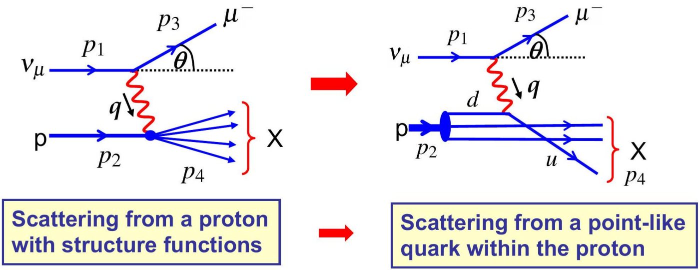
For comparison, the electro-magnetic $e^{\pm}q \rightarrow e^{\pm}q$ cross section is:

QED: $$\frac{d\sigma_{e^{\pm}q}}{dy} = \frac{2\pi\alpha^2}{q^4}e_q^2\left[1+(1-y)^2\right]\hat{s}$$

Weak: $$\frac{d\sigma_{\bar{\nu}q}}{dy} = \frac{d\sigma_{\nu\bar{q}}}{dy} = \frac{G_F^2}{\pi}(1-y)^2\hat{s}.$$

(Differences are in (i) Helicity Structure, and (ii) in interaction+propagator.)

Parton Model For Neutrino Deep Inelastic Scattering



- **Neutrino**-proton scattering can occur via scattering from a **down-quark** or from an **up-antiquark**.
- In the parton model, number of down quarks within the proton in the $x \rightarrow x + dx$ momentum fraction range is $d^p(x)\,dx$. Their contribution to the neutrino scattering cross-section is obtained by multiplying by the $\nu_\mu d \rightarrow \mu^- u$ cross-section derived previously

$$\frac{d\sigma^{\nu p}}{dy} = \frac{G_F^2}{\pi}\hat{s} d^p(x)\,dx$$

where $\hat{s}$ is the centre-of-mass energy of the $\nu_\mu d$.

- Similarly for the $\bar{u}$ contribution

$$\frac{d\sigma^{\nu p}}{dy} = \frac{G_F^2}{\pi}\hat{s}(1-y)^2\bar{u}^p(x)\,dx.$$

- The neutrino-**proton** scattering cross section is obtained by summing the two contributions and using $\hat{s} = xs$ to give:

$$\frac{d^2\sigma^{\nu p}}{dx\,dy} = \frac{G_F^2}{\pi}sx\left[d^p(x) + (1-y)^2\bar{u}^p(x)\right].$$

- The anti-neutrino **proton** differential cross section can be obtained in the same manner:

$$\frac{d^2\sigma^{\bar{\nu} p}}{dx\,dy} = \frac{GF^2}{\pi}sx\left[(1-y)^2u^p(x) + \bar{d}^p(x)\right].$$

- For (anti)neutrino-**neutron** scattering:

$$\frac{d^2\sigma^{\nu n}}{dx\,dy} = \frac{G_F^2}{\pi}sx\left[d^n(x) + (1-y)^2\bar{u}^n(x)\right]$$

$$\frac{d^2\sigma^{\bar{\nu} n}}{dx\,dy} = \frac{G_F^2}{\pi}sx\left[(1-y)^2u^n(x) + \bar{d}^n(x)\right].$$

- As before, define neutron distributions functions in terms of those of the proton
$u(x) \equiv u^{\mathrm{p}}(x) = d^{\mathrm{n}}(x); \qquad d(x) \equiv d^{\mathrm{p}}(x) = u^{\mathrm{n}}(x)$
$\overline{u}(x) \equiv \overline{u}^{\mathrm{p}}(x) = \overline{d}^{\mathrm{n}}(x); \qquad \overline{d}(x) \equiv \overline{d}^{\mathrm{p}}(x) = \overline{u}^{\mathrm{n}}(x)$

$$\frac{d^2\sigma^{\nu p}}{dx\,dy} = \frac{G_F^2}{\pi} sx \left[d(x) + (1-y)^2 \bar{u}(x)\right] \tag{129}$$

$$\frac{d^2\sigma^{\bar{\nu} p}}{dx\,dy} = \frac{G_F^2}{\pi} sx \left[(1-y)^2 u(x) + \bar{d}(x)\right] \tag{130}$$

$$\frac{d^2\sigma^{\nu n}}{dx\,dy} = \frac{G_F^2}{\pi} sx \left[u(x) + (1-y)^2 \bar{d}(x)\right] \tag{131}$$

$$\frac{d^2\sigma^{\bar{\nu} n}}{dx\,dy} = \frac{G_F^2}{\pi} sx \left[(1-y)^2 d(x) + \bar{u}(x)\right] \tag{132}$$

- Because neutrino cross sections are very small, need massive detectors. These are usually made of iron, hence, one experimentally measurea a combination of proton/neutron scattering cross sections.

- For an isoscalar target (i.e. equal numbers of protons and neutrons), the mean cross section per nucleon, N:

$$\frac{d^2\sigma^{\nu N}}{dx\,dy} = \frac{1}{2}\left(\frac{d^2\sigma^{\nu p}}{dx\,dy} + \frac{d^2\sigma^{\nu n}}{dx\,dy}\right)$$

$$\Longrightarrow \boxed{\frac{d^2\sigma^{\nu N}}{dx\,dy} = \frac{G_F^2}{2\pi} sx\left[u(x) + d(x) + (1-y)^2(\bar{u}(x) + \bar{d}(x))\right]}.$$

- Integrate over momentum fraction x:

$$\boxed{\frac{d\sigma^{\nu N}}{dy} = \frac{G_F^2}{2\pi} s\left[f_q + (1-y)^2 f_{\bar{q}})\right]} \tag{133}$$

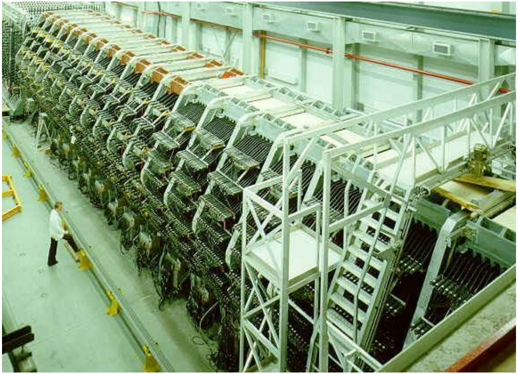
where f_q and $f_{\bar{q}}$ are the total momentum fractions carried by the quarks and by the anti-quarks within a nucleon:

$$f_q \equiv f_d + f_u = \int_0^1 x\,[u(x) + d(x)]\;dx; \qquad f_{\bar{q}} \equiv f_{\bar{d}} + f_{\bar{u}} = \int_0^1 x\left[\bar{u}(x) + \bar{d}(x)\right]\;dx.$$

- Similarly

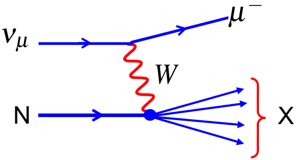
$$\boxed{\frac{d\sigma^{\bar{\nu} N}}{dy} = \frac{G_F^2}{2\pi} s\left[(1-y)^2 f_q + f_{\bar{q}})\right]}. \tag{134}$$

CDHS Experiment (CERN 1976-1984)

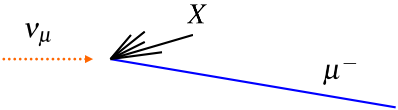


- 1250 tons
- Magnetized iron modules
- Separated by drift chambers

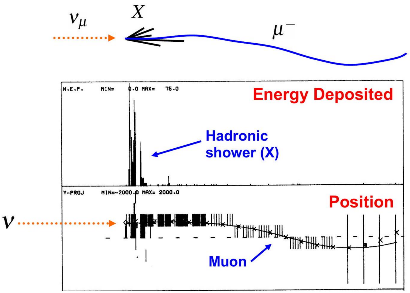
Study Neutrino Deep Inelastic Scattering



Experimental Signature:



CDHS (cont.)



- Measure energy of X, and call it E_X,
- Measure muon momentum from curvature in B-field. and call it E_μ.
- For each event can then determine neutrino energy and y:

$$E_\nu = E_X + E_\mu$$

and then

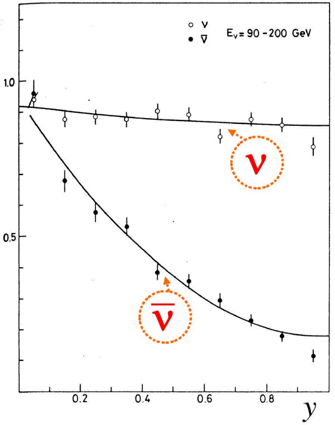
$$E_\mu = (1-y)E_\nu$$

implies

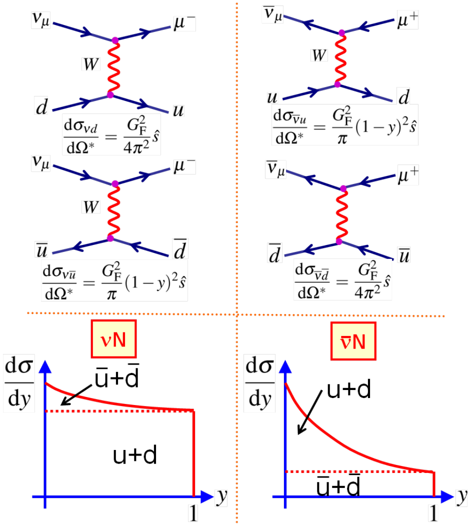
$$y = \left(1 - \frac{E_\mu}{E_\nu}\right).$$

Measured y Distribution

- CDHS measured y distribution
- Shapes can be understood in terms of (anti)neutrino – (anti)quark scattering



J. de Groot et al., Z.Phys. C1 (1979) 143



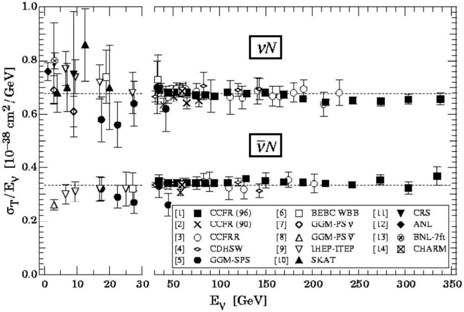
Measured Total Cross Sections

- Integrating the expressions for $\frac{d\sigma}{dy}$ (equations (133) and (134)):

$$\boxed{\sigma^{\nu N} = \frac{G_F^2 s}{2\pi}\left[f_q + \frac{1}{3}f_{\bar{q}}\right]} \quad \text{and} \quad \boxed{\sigma^{\bar{\nu} N} = \frac{G_F^2 s}{2\pi}\left[\frac{1}{3}f_q + f_{\bar{q}}\right]}.$$

- In lab frame $p^\mu(\nu) = (E_\nu, 0, 0, +E_\nu)$ and $p^\mu(p) = (m_p, 0, 0, 0)$ so $s = (E_\nu + m_p)^2 - E_\nu^2 = 2E_\nu m_p + m_p^2 \approx 2E_\nu m_p$ therefore **the Neutrino DIS cross section is approximately proportional to lab. frame neutrino energy!**

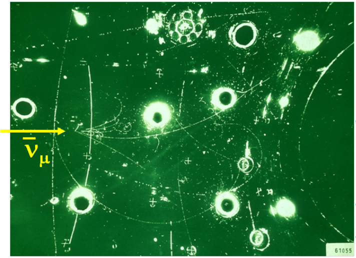
- Measure cross sections can be used to determine fraction of protons momentum carried by quarks, f_q, and fraction carried by anti-quarks, $f_{\bar{q}}$.
- Find: $f_q \approx 0.41$; $f_{\bar{q}} \approx 0.08$.
- $\sim 50\%$ of momentum carried by gluons (which don't interact with virtual W-boson).
- If no anti-quarks in nucleons expect $\frac{\sigma^{\nu N}}{\sigma^{\bar{\nu} N}} = 3$.
- Including anti-quarks $\frac{\sigma^{\nu N}}{\sigma^{\bar{\nu} N}} \approx 2$.



Weak Neutral Current

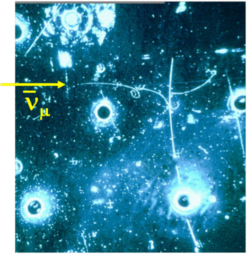
- Neutrinos also interact via the Neutral Current. First observed in the Gargamelle bubble chamber in 1973. Interaction of muon neutrinos produce a final state muon.

$\bar{\nu}_\mu + N \rightarrow \bar{\nu}_\mu + \text{hadrons}$



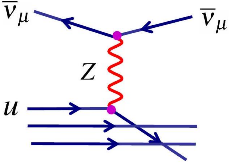
F.J. Hasert et al., Phys. Lett. 46B (1973) 138

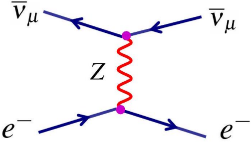
$\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$



F.J. Hasert et al., Phys. Lett. 46B (1973) 121

- Cannot be due to W exchange – this is first evidence for Z-boson!

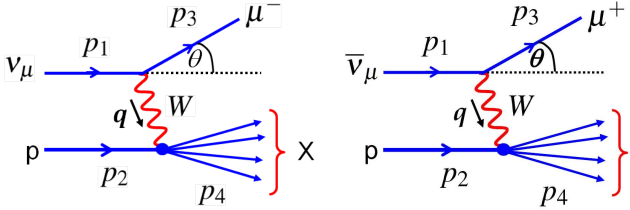




Summary

- Derived neutrino/anti-neutrino – quark/anti-quark weak charged current (CC) interaction cross sections.
- Neutrino-nucleon scattering allows us to measure anti-quark content of protons nad neutrons because:
  - ν couples to d and $\bar{u}$,
  - $\bar{\nu}$ couples to u and $\bar{d}$,
  - $\nu\bar{q}$ scattering is suppressed by factor $(1-y)^2$ compared with νq, and
  - $\bar{\nu}q$ scattering is suppressed by factor $(1-y)^2$ compared with $\bar{\nu}\bar{q}$.
- Further aspects of neutrino deep-inelastic scattering (expressed in general structure functions) are covered in Appendix XVII.
- Finally observe that neutrinos interact via weak neutral currents (NC).

Appendix XVII: Deep-Inelastic Neutrino Scattering



- Two steps:
  - First write down most general cross section in terms of structure functions.
  - Then evaluate expressions in the quark-parton model.
- QED Revisited:
  - In the limit $s \gg M^2$the most general electro-magnetic deep-inelastic cross section (from single photon exchange) can be written (Eq. (113) of Handout 6) as

$$\frac{d^2\sigma_{e^\pm p}}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\,\frac{F_2(x,Q^2)}{x} + y^2 F_1(x,Q^2)\right].$$

  - For neutrino scattering typically measure the energy of the produced muon $E_\mu = E_\nu(1-y)$ and differential cross-sections expressed in terms of $dx\,dy$
  - $Q^2 = (s-M^2)xy \approx sxy \ \rightarrow$

$$\frac{d^2\sigma}{dx\,dy} = \left|\frac{dQ^2}{dy}\right| \frac{d^2\sigma}{dx\,dQ^2} = sx\frac{d^2\sigma}{dx\,dQ^2}.$$

- In the limit $s \gg M^2$ the general Electro-magnetic DIS cross section can be written

$$\frac{d^2\sigma^{e^\pm p}}{dx\,dy} = \frac{4\pi\alpha^2 s}{Q^4}\left[(1-y)\,F_2(x,Q^2) + y^2 x F_1(x,Q^2)\right]. \tag{135}$$

- NOTE: This is the most general Lorentz Invariant parity conserving expression
- For neutrino DIS parity is violated and the general expression includes an additional term to allow for parity violation. New structure function: $F_3(x,Q^2)$:
  $\nu_\mu p \to \mu^- X$

$$\frac{d^2\sigma^{\nu p}}{dx\,dy} = \frac{G_F^2 s}{2\pi}\left[(1-y)\,F_2^{\nu p}(x,Q^2) + y^2 x F_1^{\nu p}(x,Q^2) + y\left(1-\frac{y}{2}\right)xF_3^{\nu p}(x,Q^2)\right]$$

- For anti-neutrino scattering new structure function enters with opposite sign
  $\bar{\nu}_\mu p \to \mu^+ X$

$$\frac{d^2\sigma^{\bar{\nu} p}}{dx\,dy} = \frac{G_F^2 s}{2\pi}\left[(1-y)\,F_2^{\bar{\nu} p}(x,Q^2) + y^2 x F_1^{\bar{\nu} p}(x,Q^2) - y\left(1-\frac{y}{2}\right)xF_3^{\bar{\nu} p}(x,Q^2)\right]$$

- Similarly for neutrino-neutron scattering
  $\nu_\mu n \to \mu^- X$

$$\frac{d^2\sigma^{\nu n}}{dx\,dy} = \frac{G_F^2 s}{2\pi}\left[(1-y)\,F_2^{\nu n}(x,Q^2) + y^2 x F_1^{\nu n}(x,Q^2) + y\left(1-\frac{y}{2}\right)xF_3^{\nu n}(x,Q^2)\right]$$

  $\bar{\nu}_\mu n \to \mu^+ X$

$$\frac{d^2\sigma^{\bar{\nu} n}}{dx\,dy} = \frac{G_F^2 s}{2\pi}\left[(1-y)\,F_2^{\bar{\nu} n}(x,Q^2) + y^2 x F_1^{\bar{\nu} n}(x,Q^2) - y\left(1-\frac{y}{2}\right)xF_3^{\bar{\nu} n}(x,Q^2)\right]$$

Not examinable

Neutrino Interaction Structure Functions

- In terms of the parton distribution functions we found (129):

$$\frac{d^2\sigma^{\nu p}}{dx\,dy} = \frac{G_F^2}{\pi} sx \left[d(x) + (1-y)^2 \bar{u}(x) \right]$$

- Compare coefficients of y with the general Lorentz Invariant form (135) and assume Bjorken scaling, i.e. $F(x, Q^2) \to F(x)$

$$\frac{d^2\sigma^{\nu p}}{dx\,dy} = \frac{G_F^2 s}{2\pi} \left[(1-y)\,F_2^{\nu p}(x) + y^2 x F_1^{\nu p}(x) + y\left(1 - \frac{y}{2}\right) x F_3^{\nu p}(x) \right]$$

- Re-writing (129):

$$\frac{d^2\sigma^{\nu p}}{dx\,dy} = \frac{G_F^2}{2\pi} s \left[2xd(x) + 2x\bar{u}(x) - 4xy\bar{u}(x) + 2xy^2\bar{u}(x) \right]$$

and equating powers of y

$$\begin{aligned} 2xd + 2x\bar{u} &= F_2 \\ -4x\bar{u} &= -F_2 + xF_3 \\ 2\bar{u} &= F_1 - xF_3/2 \end{aligned}$$

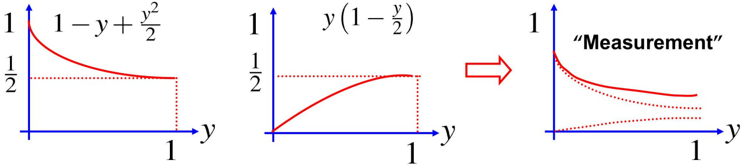
gives:

$$F_2^{\nu p} = 2xF_1^{\nu p} = 2x[d(x) + \bar{u}(x)]$$
$$xF_3^{\nu p} = 2x[d(x) - \bar{u}(x)].$$

- NOTE: again we get the Callan-Gross relation $F_2 = 2xF_1$.
- No surprise, underlying process is scattering from point-like spin-1/2 quarks

$$\frac{d^2\sigma^{\nu p}}{dx\,dy} = \frac{G_F^2 s}{2\pi}\left[\left(1 - y + \frac{y^2}{2}\right) F_2^{\nu p}(x) + y\left(1 - \frac{y}{2}\right) xF_3^{\nu p}(x)\right]$$

- Experimentally measure F_2 and F_3 from y distributions at fixed x
  - Different y dependencies (from different rest frame angular distributions) allow contributions from the two structure functions to be measured



  - Then use $F_2^{\nu p} = 2x[d(x) + \bar{u}(x)]$ and $F_3^{\nu p} = 2[d(x) - \bar{u}(x)]$ $\rightarrow d(x)$ and $\bar{u}(x)$ separately

Not examinable

- Neutrino experiments require large detectors (often iron) i.e. isoscalar target
  $F_2^{\nu N} = 2xF_1^{\nu N} = \frac{1}{2}\left(F_2^{\nu p} + F_2^{\nu n}\right) = x[u(x) + d(x) + \bar{u}(x) + \bar{d}(x)]$
  $xF_3^{\nu N} = \frac{1}{2}\left(xF_3^{\nu p} + xF_3^{\nu n}\right) = x[u(x) + d(x) - \bar{u}(x) - \bar{d}(x)]$
- For electron – nucleon scattering: $F_2^{ep} = 2xF_1^{ep} = x[\frac{4}{9}u(x) + \frac{1}{9}d(x) + \frac{4}{9}\bar{u}(x) + \frac{1}{9}\bar{d}(x)]$
  $F_2^{en} = 2xF_1^{en} = x[\frac{4}{9}d(x) + \frac{1}{9}u(x) + \frac{4}{9}\bar{d}(x) + \frac{1}{9}\bar{u}(x)]$ $F_2^{\nu N} = \frac{18}{5}F_2^{eN}$
- For an isoscalar target

$$F_2^{eN} = \frac{1}{2}\left(F_2^{ep} + F_2^{en}\right) = \frac{5}{18}x[u(x) + d(x) + \bar{u}(x) + \bar{d}(x)]$$

- Note that the factor $\frac{5}{18} = \frac{1}{2}\left(q_u^2 + q_d^2\right)$ and by comparing neutrino to electron scattering structure functions measure the sum of quark charges

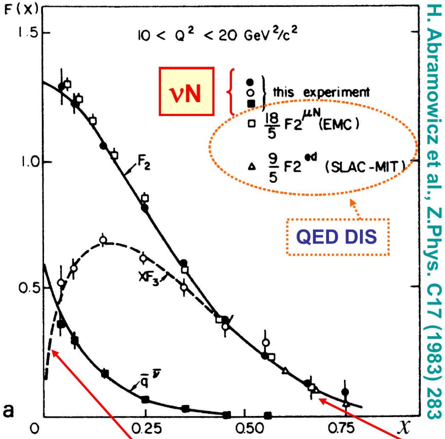
Experiment: 0.29 ± 0.02

Not examinable

Measurements of F2(x) and F3(x)

Not examinable

- CHDS Experiment $\nu_\mu + \mathrm{Fe} \rightarrow \mu^- + X$



Sea dominates so expect xF_3 to go to zero as q(x) = $\bar{q}$(x)

$$F_2^{\nu N} = x[u(x) + d(x) + \overline{u}(x) + \overline{d}(x)]$$
$$xF_3^{\nu N} = x[u(x) + d(x) - \overline{u}(x) - \overline{d}(x)]$$

$\Rightarrow$ $F_2^{\nu N} - xF_3^{\nu N} = 2x[\overline{u} + \overline{d}]$

★ **Difference in neutrino structure functions measures anti-quark (sea) parton distribution functions**

Sea contribution goes to zero

Valence Contribution

- Separate parton density functions into sea and valence components

$$u(x) = u_V(x) + u_S(x) = u_V(x) + S(x)$$

$$d(x) = d_V(x) + d_S(x) = d_V(x) + S(x)$$

$$\bar{u}(x) = \bar{u}_S(x) = S(x)$$

$$\bar{d}(x) = \bar{d}_S(x) = S(x)$$

$$\rightarrow F_3^{\nu N} = [u(x) + d(x) - \bar{u}(x) - \bar{d}(x)] = u_V(x) + d_V(x) \ \rightarrow$$

$$\int_0^1 F_3^{\nu N}(x)\, dx = \int_0^1 (u_V(x) + d_V(x))\ dx = N_u^V + N_d^V$$

- Area under measured function gives a measurement of the total number of valence quarks in a nucleon! Expect

$$\int_0^1 F_3^{\nu N}(x)\, dx = 3$$

"Gross–Llewellyn-Smith sum rule" Experiment: 3.0 ± 0.2.

- Note: $F_2^{\bar{\nu}p} = F_2^{\nu n}$; $F_2^{\bar{\nu}n} = F_2^{\nu p}$; $F_3^{\bar{\nu}p} = F_3^{\nu n}$; $F_3^{\bar{\nu}n} = F_3^{\nu p}$ and anti-neutrino structure functions contain same pdf information.