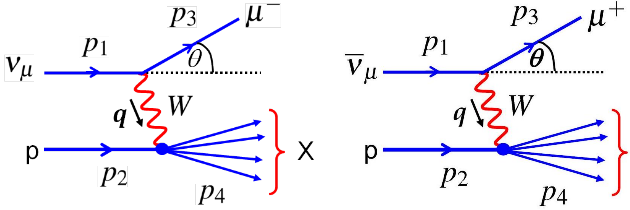
Appendix XVII: Deep-Inelastic Neutrino Scattering



- Two steps:
  - First write down most general cross section in terms of structure functions.
  - Then evaluate expressions in the quark-parton model.
- QED Revisited:
  - In the limit $s \gg M^2$the most general electro-magnetic deep-inelastic cross section (from single photon exchange) can be written (Eq. (113) of Handout 6) as

$$\frac{d^2\sigma_{e^\pm p}}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\,\frac{F_2(x,Q^2)}{x} + y^2 F_1(x,Q^2)\right].$$

  - For neutrino scattering typically measure the energy of the produced muon $E_\mu = E_\nu(1-y)$ and differential cross-sections expressed in terms of $dx\,dy$
  - $Q^2 = (s - M^2)xy \approx sxy \;\; \rightarrow$

$$\frac{d^2\sigma}{dx\,dy} = \left|\frac{dQ^2}{dy}\right| \frac{d^2\sigma}{dx\,dQ^2} = sx\frac{d^2\sigma}{dx\,dQ^2}.$$

- In the limit $s \gg M^2$ the general Electro-magnetic DIS cross section can be written

$$\frac{d^2\sigma^{e^\pm p}}{dx\,dy} = \frac{4\pi\alpha^2 s}{Q^4}\left[(1-y)\,F_2(x,Q^2) + y^2 x F_1(x,Q^2)\right]. \qquad (135)$$

- NOTE: This is the most general Lorentz Invariant parity conserving expression
- For neutrino DIS parity is violated and the general expression includes an additional term to allow for parity violation. New structure function: $F_3(x,Q^2)$:

$\nu_\mu p \to \mu^- X$

$$\frac{d^2\sigma^{\nu p}}{dx\,dy} = \frac{G_F^2 s}{2\pi}\left[(1-y)\,F_2^{\nu p}(x,Q^2) + y^2 x F_1^{\nu p}(x,Q^2) + y\left(1-\frac{y}{2}\right)xF_3^{\nu p}(x,Q^2)\right]$$

- For anti-neutrino scattering new structure function enters with opposite sign

$\bar{\nu}_\mu p \to \mu^+ X$

$$\frac{d^2\sigma^{\bar{\nu} p}}{dx\,dy} = \frac{G_F^2 s}{2\pi}\left[(1-y)\,F_2^{\bar{\nu} p}(x,Q^2) + y^2 x F_1^{\bar{\nu} p}(x,Q^2) - y\left(1-\frac{y}{2}\right)xF_3^{\bar{\nu} p}(x,Q^2)\right]$$

- Similarly for neutrino-neutron scattering

$\nu_\mu n \to \mu^- X$

$$\frac{d^2\sigma^{\nu n}}{dx\,dy} = \frac{G_F^2 s}{2\pi}\left[(1-y)\,F_2^{\nu n}(x,Q^2) + y^2 x F_1^{\nu n}(x,Q^2) + y\left(1-\frac{y}{2}\right)xF_3^{\nu n}(x,Q^2)\right]$$

$\bar{\nu}_\mu n \to \mu^+ X$

Not examinable

$$\frac{d^2\sigma^{\bar{\nu} n}}{dx\,dy} = \frac{G_F^2 s}{2\pi}\left[(1-y)\,F_2^{\bar{\nu} n}(x,Q^2) + y^2 x F_1^{\bar{\nu} n}(x,Q^2) - y\left(1-\frac{y}{2}\right)xF_3^{\bar{\nu} n}(x,Q^2)\right]$$

Neutrino Interaction Structure Functions

- In terms of the parton distribution functions we found (129):

$$\frac{d^2\sigma^{\nu p}}{dx\,dy} = \frac{G_F^2}{\pi} sx \left[d(x) + (1-y)^2 \bar{u}(x) \right]$$

- Compare coefficients of y with the general Lorentz Invariant form (135) and assume Bjorken scaling, i.e. $F(x, Q^2) \rightarrow F(x)$

$$\frac{d^2\sigma^{\nu p}}{dx\,dy} = \frac{G_F^2 s}{2\pi} \left[(1-y)\, F_2^{\nu p}(x) + y^2 x F_1^{\nu p}(x) + y\left(1 - \frac{y}{2}\right) x F_3^{\nu p}(x) \right]$$

- Re-writing (129):

$$\frac{d^2\sigma^{\nu p}}{dx\,dy} = \frac{G_F^2}{2\pi} s \left[2xd(x) + 2x\bar{u}(x) - 4xy\bar{u}(x) + 2xy^2\bar{u}(x) \right]$$

and equating powers of y

$$\begin{aligned} 2xd + 2x\bar{u} &= F_2 \\ -4x\bar{u} &= -F_2 + xF_3 \\ 2\bar{u} &= F_1 - xF_3/2 \end{aligned}$$

gives:

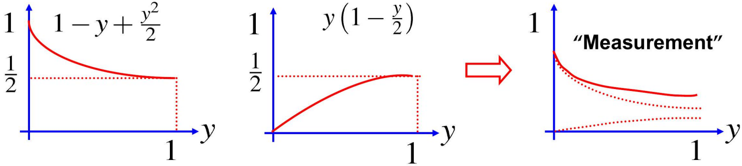
$$F_2^{\nu p} = 2xF_1^{\nu p} = 2x[d(x) + \bar{u}(x)]$$
$$xF_3^{\nu p} = 2x[d(x) - \bar{u}(x)].$$

Not examinable

- NOTE: again we get the Callan-Gross relation $F_2 = 2xF_1$.
- No surprise, underlying process is scattering from point-like spin-1/2 quarks

$$\frac{d^2\sigma^{\nu p}}{dx\,dy} = \frac{G_F^2 s}{2\pi}\left[\left(1 - y + \frac{y^2}{2}\right) F_2^{\nu p}(x) + y\left(1 - \frac{y}{2}\right) xF_3^{\nu p}(x)\right]$$

- Experimentally measure F_2 and F_3 from y distributions at fixed x
  - Different y dependencies (from different rest frame angular distributions) allow contributions from the two structure functions to be measured



  - Then use $F_2^{\nu p} = 2x[d(x) + \bar{u}(x)]$ and $F_3^{\nu p} = 2[d(x) - \bar{u}(x)]$ $\rightarrow d(x)$ and $\bar{u}(x)$ separately

- Neutrino experiments require large detectors (often iron) i.e. isoscalar target
  $F_2^{\nu N} = 2xF_1^{\nu N} = \frac{1}{2}\left(F_2^{\nu p} + F_2^{\nu n}\right) = x[u(x) + d(x) + \bar{u}(x) + \bar{d}(x)]$
  $xF_3^{\nu N} = \frac{1}{2}\left(xF_3^{\nu p} + xF_3^{\nu n}\right) = x[u(x) + d(x) - \bar{u}(x) - \bar{d}(x)]$
- For electron – nucleon scattering: $F_2^{ep} = 2xF_1^{ep} = x[\frac{4}{9}u(x) + \frac{1}{9}d(x) + \frac{4}{9}\bar{u}(x) + \frac{1}{9}\bar{d}(x)]$
  $F_2^{en} = 2xF_1^{en} = x[\frac{4}{9}d(x) + \frac{1}{9}u(x) + \frac{4}{9}\bar{d}(x) + \frac{1}{9}\bar{u}(x)]$ $F_2^{\nu N} = \frac{18}{5}F_2^{eN}$
- For an isoscalar target

$$F_2^{eN} = \frac{1}{2}\left(F_2^{ep} + F_2^{en}\right) = \frac{5}{18}x[u(x) + d(x) + \bar{u}(x) + \bar{d}(x)]$$

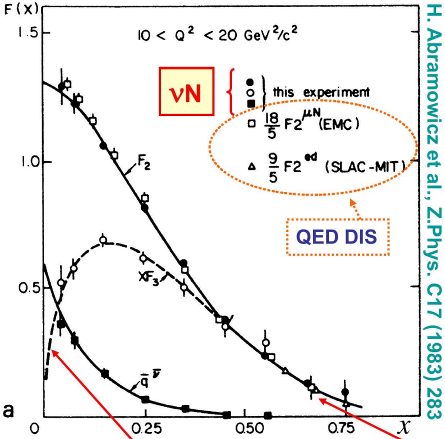
- Note that the factor $\frac{5}{18} = \frac{1}{2}\left(q_u^2 + q_d^2\right)$ and by comparing neutrino to electron scattering structure functions measure the sum of quark charges

Experiment: 0.29 ± 0.02

Measurements of F2(x) and F3(x)

Not examinable

- CHDS Experiment $\nu_\mu + \mathrm{Fe} \to \mu^- + X$



Sea dominates so expect xF_3 to go to zero as q(x) = q̄(x)

$$F_2^{\nu N} = x[u(x) + d(x) + \overline{u}(x) + \overline{d}(x)]$$

$$xF_3^{\nu N} = x[u(x) + d(x) - \overline{u}(x) - \overline{d}(x)]$$

$$\Rightarrow F_2^{\nu N} - xF_3^{\nu N} = 2x[\overline{u} + \overline{d}]$$

★ **Difference in neutrino structure functions measures anti-quark (sea) parton distribution functions**

Sea contribution goes to zero

Valence Contribution

- Separate parton density functions into sea and valence components

$$u(x) = u_V(x) + u_S(x) = u_V(x) + S(x)$$

$$d(x) = d_V(x) + d_S(x) = d_V(x) + S(x)$$

$$\bar{u}(x) = \bar{u}_S(x) = S(x)$$

$$\bar{d}(x) = \bar{d}_S(x) = S(x)$$

$$\rightarrow F_3^{\nu N} = [u(x) + d(x) - \bar{u}(x) - \bar{d}(x)] = u_V(x) + d_V(x) \ \rightarrow$$

$$\int_0^1 F_3^{\nu N}(x)\, dx = \int_0^1 (u_V(x) + d_V(x))\ dx = N_u^V + N_d^V$$

- Area under measured function gives a measurement of the total number of valence quarks in a nucleon! Expect

$$\int_0^1 F_3^{\nu N}(x)\, dx = 3$$

"Gross–Llewellyn-Smith sum rule" Experiment: 3.0 ± 0.2.

- Note: $F_2^{\bar{\nu}p} = F_2^{\nu n}$; $F_2^{\bar{\nu}n} = F_2^{\nu p}$; $F_3^{\bar{\nu}p} = F_3^{\nu n}$; $F_3^{\bar{\nu}n} = F_3^{\nu p}$ and anti-neutrino structure functions contain same pdf information.