Symmetry → Discrete ✗

Symmetry → Continuous → Matrix Rep (n×n) → Generators → Mystery "Casimir" operator "G^2":

$\hookrightarrow G^2 = \sum_i G_i G_i$ → Is Hermitian by construction so G^2 is OBSERVABLE

Matrix Rep → U

Generators → G_i

U → $\text{Det}\,U = 1$

U → $U^{-1} = U^{\dagger}$ (UNITARY)

U, G_i → $U = \mathbb{1} + i\varepsilon_i G_i$

$\text{Det}\,U = 1$ and $U^{-1} = U^{\dagger}$ → $SU(n)$

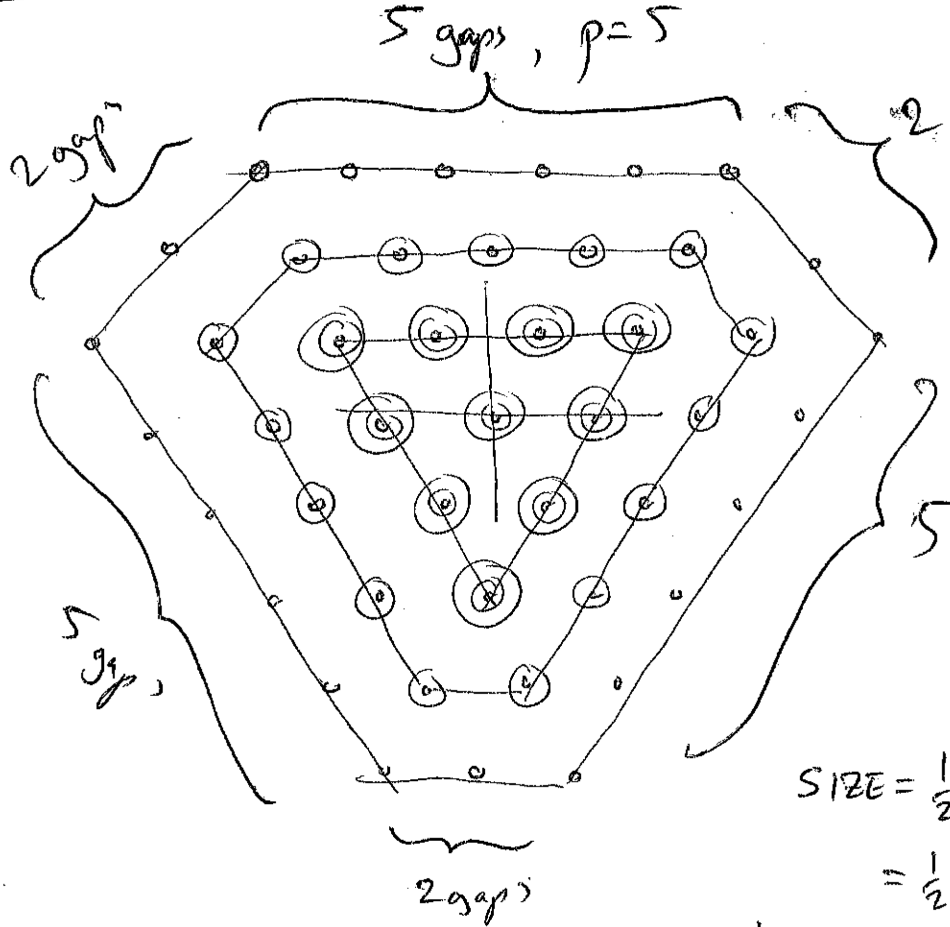
$SU(n) \longrightarrow G_i = G_i^{\dagger} \rightarrow G_i$ Hermitian and TRACELESS → G_i OBSERVABLE.

G_i OBSERVABLE → Mysteriously $[G^2, G_i] = 0$ ⟹ G^2 & G_i simultaneously observable

NOT ALL OF G_i are simultaneously observable ⟺ $[G_i, G_j] \neq 0$ "not always"

Example · Symmetry	Generators	DIAGONAL?	Casimir op	Better basis
1-particle basis: $\lvert u\rangle = \begin{pmatrix}1\\0\end{pmatrix}$, $\lvert d\rangle = \begin{pmatrix}0\\1\end{pmatrix}$ 2-particle basis: $\lvert uu\rangle, \lvert ud\rangle, \lvert du\rangle, \lvert dd\rangle$ or "$\lvert u\rangle\lvert u\rangle$" eg: 1 confused particle: $\frac{1}{\sqrt2}\lvert u\rangle + \frac{i}{\sqrt2}\lvert d\rangle = \begin{pmatrix}1/\sqrt2\\ i/\sqrt2\end{pmatrix}$ 2 non-confused particles: $\lvert uu\rangle \equiv \lvert u\rangle\lvert u\rangle \equiv \begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}$	$G_1 = $ "T_1" $= \frac{\sigma_1}{2}$ $G_2 = $ "T_2" $= \frac{\sigma_2}{2}$ $\boxed{G_3 = \text{"}T_3\text{"} = \frac{\sigma_3}{2}}$ DIAGONAL!	✗ ✗ ✓	$\boxed{G^2 = \text{"}T^2\text{"} = T_1^2 + T_2^2 + T_3^2}$ DIAGONAL $\left(= \frac{3}{4}\mathbb{1}_{2\times2}\right.$ when acting on single particle states$\left.\right)$	Label states by e-vals under: T^2, T_3 T^2: Thing preserved by ladder ops = "which multiplet we are in" T_3: THIRD CPT ISOSPIN (x-co-ord in multiplet) $\left[\{T^{\pm} = T_1 \pm iT_2\} = 2 \text{ ladder ops change } x\text{-co-ord} \quad \xleftrightarrow{T^{\pm}}\right]$
$\boxed{G^2 \text{ \& } G_3 = \text{maximal simultaneously diagonalisable/observable set of ops.}}$				
1 particle basis: $\lvert u\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix}$ $\lvert d\rangle = \begin{pmatrix}0\\1\\0\end{pmatrix}$ $\lvert s\rangle = \begin{pmatrix}0\\0\\1\end{pmatrix}$ 2 particle basis $\lvert uu\rangle, \lvert ud\rangle, \lvert us\rangle, \lvert du\rangle, \lvert dd\rangle, \ldots$ (9 of them) e.g. 1 confused particle: $\frac{1}{\sqrt2}\lvert u\rangle + \frac{i}{\sqrt2}\lvert d\rangle = \begin{pmatrix}1/\sqrt2\\ i/\sqrt2\\ 0\end{pmatrix}$ 2 non-confused particles. $\lvert us\rangle = \lvert u\rangle\lvert s\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}$	$G_1 = $ "$\lambda_1/2$" $G_2 = $ "$\lambda_2/2$" $\boxed{G_3 = \lambda_3/2}$ $G_4 = $ "λ_4"$/2$ $G_5 = $ "λ_5"$/2$ $G_6 = $ "λ_6"$/2$ $G_7 = $ "λ_7"$/2$ $\boxed{G_8 = \text{"}\lambda_8\text{"}/2}$	✗ ✗ ✓ ✗ ✗ ✗ ✗ ✓	$\boxed{G^2 = G_1^2 + G_2^2 + \ldots + G_8^2}$ DIAGONAL $\left(= \frac{16}{3}\mathbb{1}_{3\times3}\right.$ when acting on single particle states$\left.\right)$	Label states by e-vals under: ✱ G^2, G_3, G_8 G^2: Thing preserved by ladder ops = "which multiplet we are in". G_3: ISOSPIN = (x-co-ord in multiplet) G_8: Hypercharge (up to factors of $\frac{1}{2}$ or $\frac{1}{\sqrt2}$) ∥ (y-co-ord in multiplet) $\left[\begin{matrix}T^{\pm} = G_1 \pm iG_2\\ V^{\pm} = G_4 \pm iG_5\\ U^{\pm} = G_6 \pm iG_7\end{matrix}\right] = 6$ ladder ops, change x & y co-ords $\quad T^{\pm}, U^{\pm}, V^{\pm}$
↑ IN BOTH CASES ABOVE: $\lvert\psi\rangle \rightarrow \lvert\psi\rangle' = U\lvert\psi\rangle$ assumed to be symmetry of theory	$\boxed{G^2 \text{ \& } G_3 \text{ \& } G_8 = \text{maximal simultaneously diagonalisable/observable set of ops}}$			

SU(3) General (p,q) multiplet:



$= \text{"}(5,2)\text{"}$

$\boxed{81}$ dots.

$$\text{SIZE} = \frac{1}{2}(p+1)(q+1)(p+q+2)$$
$$= \frac{1}{2}(\text{dots})(\text{dots})(\text{dots} + \text{dots})$$

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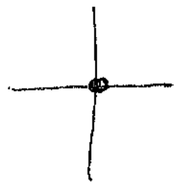
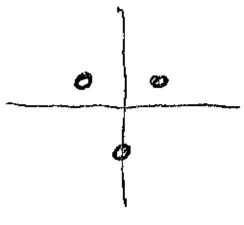
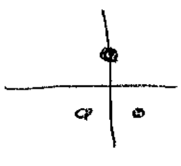
examples:



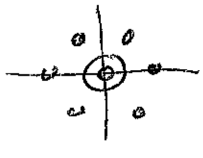
$= \text{"}(0,0)\text{"}$ $\boxed{1}$ dots



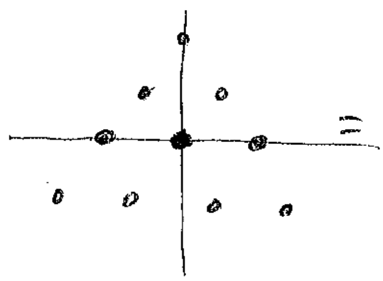
$= \text{"}(1,0)\text{"}$ $\boxed{3}$ dots



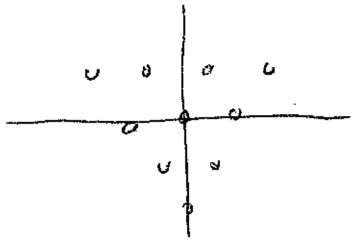
$= \text{"}(0,1)\text{"}$ $\boxed{3}$ dots ("$\bar{3}$")



$= \text{"}(1,1)\text{"}$ $\boxed{8}$ dots



$= \text{"}(0,3)\text{"}$ $\boxed{10}$ dots ("$\overline{10}$")



$= \text{"}(3,0)\text{"}$ $\boxed{10}$ dots ("10")

Trivia: (non examinable)

$(2,1)$ & $(4,0)$ both $\Rightarrow \boxed{15}$

$(13,0)$ & $(6,2)$ both $\Rightarrow \boxed{105}$

$(9,1)$, $(14,0)$, $(5,3)$ $\Rightarrow \boxed{120}$