Symmetry → Discrete ×

Symmetry → Continuous → Matrix Rep $(n \times n)$ → Generators → Mystery "Casimir" operator "G^2":

$\hookrightarrow G^2 = \sum_i G_i G_i$ → Is Hermitian by construction so G^2 is OBSERVABLE

Matrix Rep → U

$U \to \mathrm{Det}\, U = 1$

$U \to U^{-1} = U^{+}$ (UNITARY)

$\mathrm{Det}\, U = 1$, $U^{-1} = U^{+}$ → $SU(n)$

Generators → G_i → $U = \mathbb{1} + i\epsilon_i G_i$

$SU(n)$ → $G_i = G_i^{+}$ → G_i Hermitian → G_i OBSERVABLE.

Mysteriously $[G^2, G_i] = 0$ ⟹ G^2 & G_i simultaneously observable

NOT ALL OF G_i are simultaneously observable ⟺ $[G_i, G_j] \neq 0$ "not always"

Example - Symmetry

1-particle basis: $|u\rangle = \begin{pmatrix}1\\0\end{pmatrix}$, $|d\rangle = \begin{pmatrix}0\\1\end{pmatrix}$

2-particle basis: $|uu\rangle, |ud\rangle, |du\rangle, |dd\rangle$

or "$|u\rangle|u\rangle$"

eg: 1 confused particle:

$\frac{1}{\sqrt{2}}|u\rangle + \frac{i}{\sqrt{2}}|d\rangle = \begin{pmatrix}1/\sqrt{2}\\ i/\sqrt{2}\end{pmatrix}$

2 non-confused particles:

$|uu\rangle \equiv |u\rangle|u\rangle \equiv \begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}$

Generators — DIAGONAL?

$G_1 = \text{"}T_1\text{"} = \frac{\sigma_1}{2}$ ×

$G_2 = \text{"}T_2\text{"} = \frac{\sigma_2}{2}$ ×

$\boxed{G_3 = \text{"}T_3\text{"} = \frac{\sigma_3}{2}}$ ✓

DIAGONAL!

Casimir op

$\boxed{G^2 = \text{"}T^2\text{"} = T_1^2 + T_2^2 + T_3^2}$ ← DIAGONAL

$\left(= \frac{3}{4}\mathbb{1}_{2\times 2} \text{ when acting on single particle states}\right)$

$\boxed{G^2 \;\&\; G_3 = \text{maximal simultaneously diagonalisable/observable set of ops.}}$

Better basis

Label states by e-vals under:

T^2, T_3

T^2 ↕ Thing preserved by ladder ops = "which multiplet we are in"

T_3 ↔ ISOSPIN (x-co-ord in multiplet)

$\left[\{T^{\pm} = T_1 \pm iT_2\} = 2 \text{ ladder ops change } x\text{-co-ord} \quad \xleftrightarrow{T^{\pm}}\right]$

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1 particle basis: $|u\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix}$ $|d\rangle = \begin{pmatrix}0\\1\\0\end{pmatrix}$ $|s\rangle = \begin{pmatrix}0\\0\\1\end{pmatrix}$

2 particle basis $|uu\rangle, |ud\rangle, |us\rangle, |du\rangle, |dd\rangle, \ldots$ (9 of them)

e.g.

1 confused particle:

$\frac{1}{\sqrt{2}}|u\rangle + \frac{i}{\sqrt{2}}|d\rangle = \begin{pmatrix}1/\sqrt{2}\\ i/\sqrt{2}\\ 0\end{pmatrix}$

2 non-confused particles.

$|us\rangle = |u\rangle|s\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}$

↑ IN BOTH CASES ABOVE:

$|\psi\rangle \to |\psi\rangle' = U|\psi\rangle$ assumed to be symmetry of them

DIAGONAL?

$G_1 = \text{"}\lambda_1\text{"}/2$ ×

$G_2 = \text{"}\lambda_2\text{"}/2$ ×

$\boxed{G_3 = \text{"}\lambda_3\text{"}/2}$ ✓

$G_4 = \text{"}\lambda_4\text{"}/2$ ×

$G_5 = \text{"}\lambda_5\text{"}/2$ ×

$G_6 = \text{"}\lambda_6\text{"}/2$ ×

$G_7 = \text{"}\lambda_7\text{"}/2$ ×

$\boxed{G_8 = \text{"}\lambda_8\text{"}/2}$ ✓

$\boxed{G^2 = G_1^2 + G_2^2 + \ldots + G_8^2}$ ← DIAGONAL

$\left(= \frac{16}{3}\mathbb{1}_{3\times 3} \text{ when acting on single particle states}\right)$

$\boxed{G^2 \;\&\; G_3 \;\&\; G_8 = \text{maximal simultaneously diagonalisable/observable set of ops}}$

Label states by e-vals under: ⊛

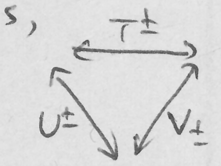
G^2, G_3, G_8

G^2 ← Thing preserved by ladder ops = "which multiplet we are in".

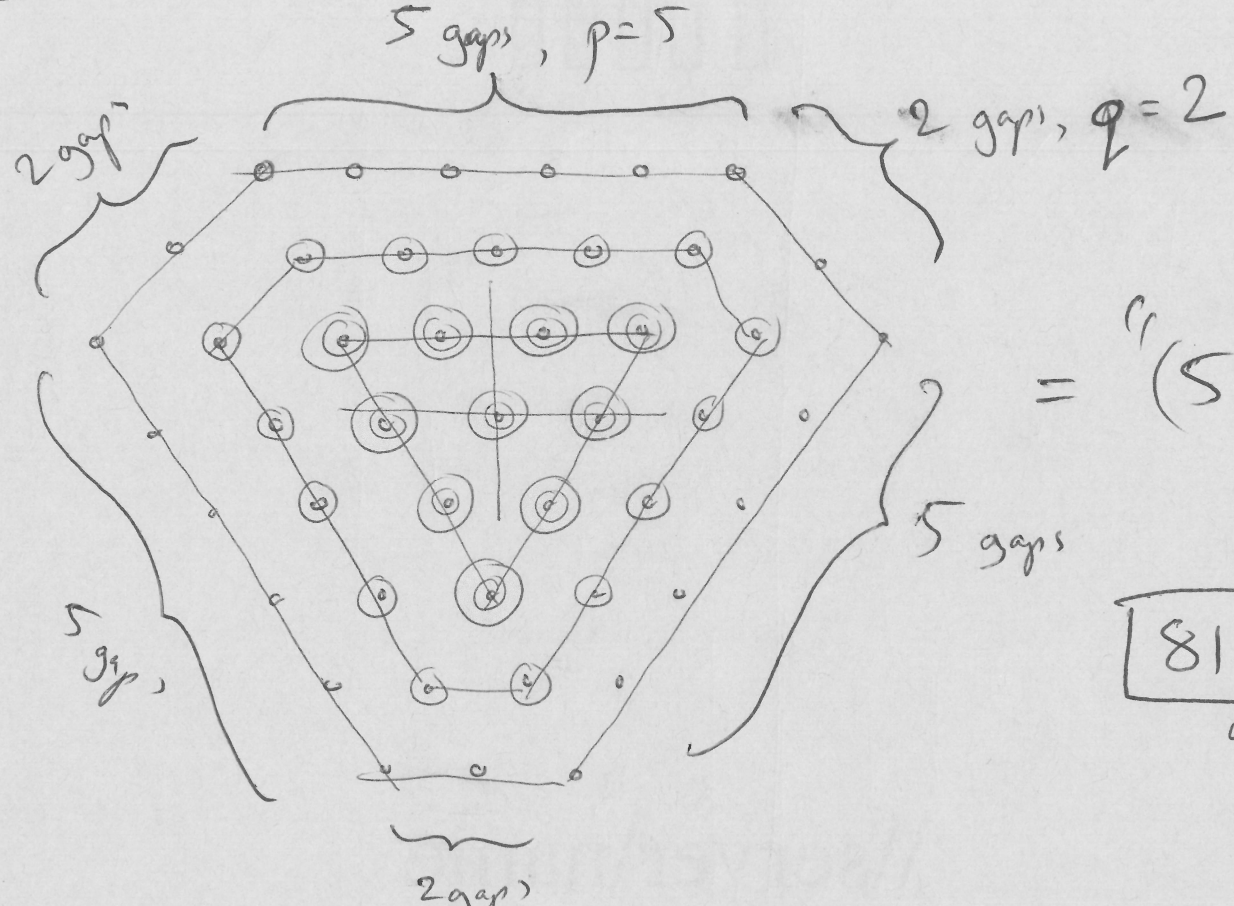
G_3 ← ISOSPIN = (x-co-ord in multiplet)

G_8 ← Hypercharge (up to factors of $\frac{1}{2}$ or $\frac{1}{\sqrt{2}}$) = (y-co-ord in multiplet)

$\left[\begin{cases} T^{\pm} = G_1 \pm iG_2 \\ V^{\pm} = G_4 \pm iG_5 \\ U^{\pm} = G_6 \pm iG_7 \end{cases} = 6 \text{ ladder ops, change } x \,\&\, y \text{ co-ords}\right.$



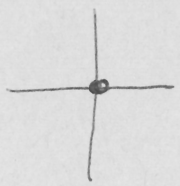
SU(3) General (p, q) multiplet :



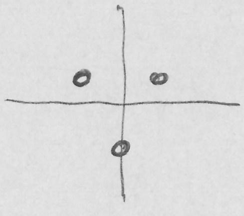
= "(5, 2)"

$\boxed{81}$ dots.

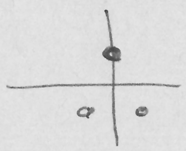
examples:



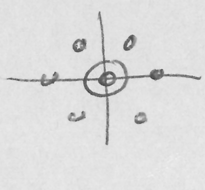
= "(0, 0)" $\boxed{1}$ dots



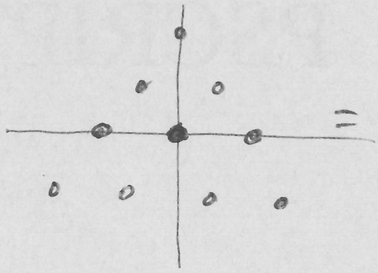
= "(1, 0)" $\boxed{3}$ dots



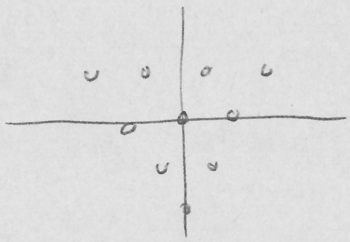
= "(0, 1)" $\boxed{3}$ dots ("$\bar{3}$")



= "(1, 1)" $\boxed{8}$ dots



= "(0, 3)" $\boxed{10}$ dots ("$\overline{10}$")



= "(3, 0)" $\boxed{10}$ dots ("10")

Trivia: (non examinable)

(2,1) & (4,0) both ⇒ $\boxed{15}$

(13,0) & (6,2) both ⇒ $\boxed{105}$

(9,1), (14,0), (5,3) ⇒ $\boxed{120}$