NATURAL SCIENCES TRIPOS Part III

Tuesday 16 January 2007 9:00am to 10:30am

EXPERIMENTAL AND THEORETICAL PHYSICS (4)
Particle Physics

Answer **two** *questions only. The approximate number of marks allotted to each part of a question is indicated in the right hand margin where appropriate. The paper contains FIVE sides including this one and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
20 Page Answer Book	Mathematical Formulae Handbook
Rough Work Pad	
Metric Graph Paper	

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

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1 In the Standard Model, and using standard notation, the vector and axial-vector couplings between the Z boson and the fermions are given by $c_{\mathrm{V}} = I_{\mathrm{W}}^3 - 2Q\sin^2\theta_{\mathrm{W}}$ and $c_{\mathrm{A}} = I_{\mathrm{W}}^3$, where I_{W}^3 is the third component of weak isospin, Q is the electromagnetic charge and θ_{W} is the weak mixing angle. Use the left- and right-handed chiral projection operators, $P_{\mathrm{L}} = \frac{1}{2}(1-\gamma^5)$ and $P_{\mathrm{R}} = \frac{1}{2}(1+\gamma^5)$, to express the Z interaction vertex in terms of the couplings to left and right-handed fermions, c_{L} and c_{R}, and show that

$$c_{\mathrm{L}} = \frac{1}{2}(c_{\mathrm{V}} + c_{\mathrm{A}}), \quad c_{\mathrm{R}} = \frac{1}{2}(c_{\mathrm{V}} - c_{\mathrm{A}}).$$ [4]

Using $\sin^2\theta_{\mathrm{W}} = 0.23$ determine the values of c_{L} and c_{R} for both an electron and for a neutrino. [2]

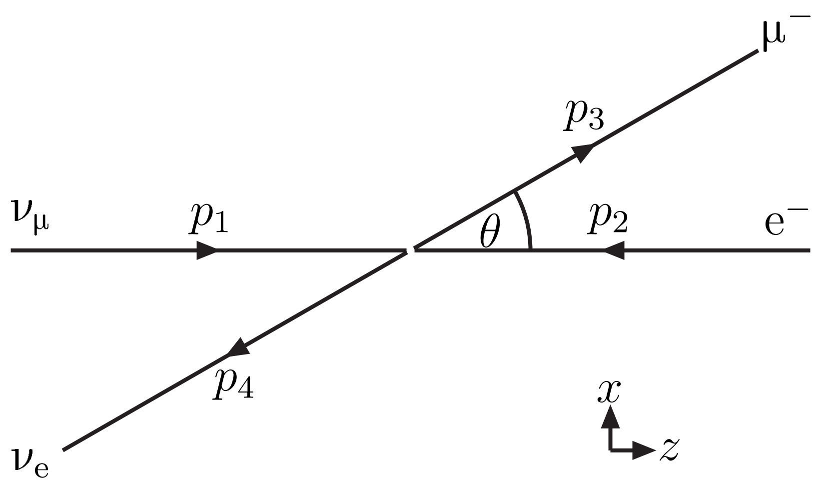
In the centre-of-mass frame, the spin-averaged matrix element for the neutral current process $\nu_\mu e^- \to \nu_\mu e^-$ is

$$\langle |M_{fi}|^2 \rangle = \frac{1}{2}(|M_{\mathrm{LL}}|^2 + |M_{\mathrm{LR}}|^2)$$

where

$$M_{\mathrm{LL}} = c_{\mathrm{L}}^{(\nu)} c_{\mathrm{L}}^{(\mathrm{e})} \frac{g_{\mathrm{Z}}^2 s}{m_{\mathrm{Z}}^2} \qquad M_{\mathrm{LR}} = c_{\mathrm{L}}^{(\nu)} c_{\mathrm{R}}^{(\mathrm{e})} \frac{g_{\mathrm{Z}}^2 s}{m_{\mathrm{Z}}^2} \frac{1}{2}(1+\cos\theta),$$

and s is the centre-of-mass energy squared and θ is the neutrino scattering angle. Explain the angular dependences of the two matrix elements and explain why $M_{\mathrm{LR}} = 0$ for the interactions of the $\mathrm{W}^{\pm}$. [4]



Draw the Feynman diagram for the charged current weak interaction $\nu_\mu e^- \to \mu^- \nu_e$ (shown above). Starting from the Feynman rules, show that the corresponding matrix element can be written $M = M_{\mathrm{LL}}$ where

$$M_{\mathrm{LL}} = \frac{g_{\mathrm{W}}^2}{2m_{\mathrm{W}}^2} j_{(\nu_\mu)} \cdot j_{(\mathrm{e})}$$

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and $j^{\mu}_{(\nu_\mu)} = \overline{u}_\downarrow(p_3)\gamma^\mu u_\downarrow(p_1)$, and $j^{\nu}_{(e)} = \overline{u}_\downarrow(p_4)\gamma^\nu u_\downarrow(p_2)$. Neglect the fermion masses and take $E_\nu \ll m_W$. [4]

Write down the spinors involved in the interaction and show that

$$M_{LL} = \frac{g_W^2 s}{m_W^2}.$$ [8]

Draw the Feynman diagrams for the process $\nu_e e^- \to \nu_e e^-$ and, by using the above expressions for the neutral current and charged current matrix elements, show that

$$\frac{\sigma(\nu_\mu e^- \to \nu_\mu e^-)}{\sigma(\nu_e e^- \to \nu_e e^-)} \approx \frac{1}{6}.$$ [8]

In the limit where particle masses can be neglected the helicity eigenstates are

$$u_\uparrow = \sqrt{E}\begin{pmatrix} \cos(\theta/2) \\ e^{i\phi}\sin(\theta/2) \\ \cos(\theta/2) \\ e^{i\phi}\sin(\theta/2) \end{pmatrix}, \quad u_\downarrow = \sqrt{E}\begin{pmatrix} -\sin(\theta/2) \\ e^{i\phi}\cos(\theta/2) \\ \sin(\theta/2) \\ -e^{i\phi}\cos(\theta/2) \end{pmatrix}.$$

The four-vector current $j^\mu = \overline{\psi}\gamma^\mu\phi$ has components:

$$\begin{aligned}
j^0 = \overline{\psi}\gamma^0\phi &= \psi_1^*\phi_1 + \psi_2^*\phi_2 + \psi_3^*\phi_3 + \psi_4^*\phi_4 \\
j^1 = \overline{\psi}\gamma^1\phi &= \psi_1^*\phi_4 + \psi_2^*\phi_3 + \psi_3^*\phi_2 + \psi_4^*\phi_1 \\
j^2 = \overline{\psi}\gamma^2\phi &= -i\psi_1^*\phi_4 + i\psi_2^*\phi_3 - i\psi_3^*\phi_2 + i\psi_4^*\phi_1 \\
j^3 = \overline{\psi}\gamma^3\phi &= \psi_1^*\phi_3 - \psi_2^*\phi_4 + \psi_3^*\phi_1 - \psi_4^*\phi_2
\end{aligned}$$

The vertex factors governing the interaction of a fermion with a $W^\pm$ or Z boson are $-i(g_W/\sqrt{2})\cdot\gamma^\mu\frac{1}{2}(1-\gamma^5)$ and $-i(g_Z/2)\cdot\gamma^\mu(c_V - c_A\gamma^5)$, respectively. The constants g_W and g_Z are related via the expression $g_W = g_Z\cos\theta_W$, and the $W^\pm$ and Z masses via the expression $m_W = m_Z\cos\theta_W$.

2 In the deep inelastic scattering of an electron with four-momentum p_1 from a proton with four-momentum p_2, the following Lorentz invariant variables can be defined:

$$Q^2 \equiv -q^2; \quad x \equiv \frac{Q^2}{2p_2.q}; \quad y \equiv \frac{p_2.q}{p_2.p_1},$$

where $q = p_1 - p_3$ and p_3 is the four-momentum of the scattered electron.

Neglecting the mass of the electron:

(a) show that $Q^2 > 0$; [3]

(b) by considering the invariant mass of the hadronic system or otherwise, show that $0 < x \leq 1$ and comment on the significance of interactions with $x = 1$; [3]

(c) working in the laboratory frame, show that $0 < y < 1$; [2]

(d) when the proton mass can be neglected, show that $y = \frac{1}{2}(1 - \cos\theta^*)$, where θ^* is the scattering angle of e^- in the centre-of-mass frame. [2]

(e) in the parton model, show that x can be interpreted as the fraction of the proton's momentum carried by the struck quark. [3]

The Drell–Yan process is the production of leptons pairs ($\ell^+\ell^-$) in hadron-hadron collisions through the annihilation of a quark and anti-quark into a photon. Draw the Feynman diagram for this interaction and explain why the cross section is non-zero for proton-proton collisions. [2]

The cross section for $q\overline{q}$ annihilation is

$$\sigma = \frac{4\pi}{3}\frac{\alpha^2}{\widehat{s}}e_q^2,$$

where e_q is the quark charge (i.e. $e_u = +2/3$ and $e_d = -1/3$) and $\widehat{s}$ is the quark-anti-quark energy in the centre-of-mass frame. Using this expression, and neglecting the strange quark contribution, show that the parton model prediction for the $\mathrm{pp} \to \mu^+\mu^-\mathrm{X}$ differential cross section can be written

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x_1\mathrm{d}x_2} = \frac{4\pi\alpha^2}{91x_1x_2s}\{4[u(x_1)\overline{u}(x_2) + u(x_2)\overline{u}(x_1)] + d(x_1)\overline{d}(x_2) + d(x_2)\overline{d}(x_1)\}$$

In this expression x_1 and x_2 are the fractional momenta carried by the partons involved in the collistion, s is the centre of mass energy of the proton-proton collision, and $u(x)$, $d(x)$, $\overline{u}(x)$ and $\overline{d}(x)$ are the up and down quark/anti-quark parton distribution functions. [9]

Suggest how the Drell–Yan process can provide information on parton distribution functions. [2]

Drell–Yan production of muon pairs has been studied in pion collisions with carbon (an equal number of protons and neutrons). If the invariant mass of the observed $\mu^+\mu^-$ system is Q^2, explain, in the context of the parton model, why the ratio

$$\frac{\sigma(\pi^+\mathrm{C} \to \mu^+\mu^-\mathrm{X})}{\sigma(\pi^-\mathrm{C} \to \mu^+\mu^-\mathrm{X})}$$

is approximately unity for small Q^2 and tends to $\frac{1}{4}$ as Q^2 approachs s. [4]

3 Write brief notes on **three** of the following:

(a) the role of QCD in the quark model of hadrons; [10]

(b) CP violation in the decays of neutral kaons to pions; [10]

(c) reactor-based neutrino oscillation experiments; [10]

(d) experimental tests of the electroweak theory. [10]

[*Answers in the form of a logically ordered bullet-pointed list are acceptable. Diagrams and simple calculations should be included where appropriate.*]

END OF PAPER