Solar Powered Hot Air Balloons

Candidate number: 8291T Supervisor: Dr Chris Lester

Abstract

Solar Balloons are heated by the sun, through a dark skin absorbing the suns light and heat. A theoretical model for these balloons was constructed, backed up with measured properties of various materials, and compared with the performance of real balloons under artificial light conditions. It is concluded that lifts of 1.5 Nm⁻³ would be easily attainable in a country such as Britain, meaning that an 80kg person could be lifted by a 550m³ balloon, which compares favourably with the sizes of regular hot air balloons. More complex balloons incorporating materials with different properties can increase the internal temperature and therefore the lift even more.

Notation

P(0): atmospheric pressure (taken to be 101 kPa)

- M: average molecular mass of air (approximately 29 g mol⁻¹)
- g: acceleration due to gravity (taken to be 9.81 ms⁻²)
- R: ideal gas constant (8.31 J K⁻¹ mol⁻¹)

V: balloon volume

- T_{in}: average internal temperature
- $T_{\mbox{\scriptsize out}}$: ambient air temperature
- L: lift achieved
- α: absorption coefficient
- β : emission coefficient between balloon/absorber and atmosphere
- γ: emission coefficient between absorber and balloon
- I: power/unit area arriving at the balloon
- A: balloon surface area

1 Introduction

Solar powered hot air balloons are conceptually very simple – a black balloon is heated by absorbing the sunlight shining on it, this heats the air inside, which becomes less dense than the air outside, and experiences an upwards lift force. These balloons can be used for weather observation or recreation, and it has even been suggested that they can be used to harness solar power¹.

There are many factors that affect the performance of such a balloon, so some important parameters are investigated and a simple theoretical model to describe the lift achieved by such a balloon is developed and validated with lab data. Section 2 develops a basic theoretical framework for the performance of such a balloon, based on heating and cooling processes, and how these affect the lift of a balloon. Section 3 details the methods involved in determining parameters relating to the materials and balloons involved, and the results of these experiments are covered in section 4. In section 5, the main results are detailed and compared with predictions made by the model. Section 6 is a discussion of all the methods and results, and section 7 lists some final conclusions.

2 Theoretical model

2.1 Lift

The lift achieved by a hot air balloon will obviously depend on temperature and size, however the way in which these affect lift are not necessarily obvious. It can be calculated fairly simply by integrating the pressure difference over the surface of a balloon with a given temperature distribution (noting that the pressures at the bottom, which is open, are equal). The problem is that this pressure difference will depend on the internal temperature distribution, which will be determined by internal convection currents driven by a hot surface which is being heated by the sun – modelling of these processes is very complicated. Here a simple empirical model will be used Appendix 1 shows the calculations of the lift generated by a cylindrical balloon with two different temperature differences – uniform temperature and a linear increase with height (discovered in section 5 to be roughly the distribution inside real balloons). However, comparing the two results for the sort of temperatures encountered when using solar balloons, there is very little difference (~1%) between the values for the linear temperature case and those for the uniform temperature where the same average temperature is used. To further simplify, the result from the uniform temperature case can easily be approximated for small balloons to the result we will use for the lift

$$L \approx P(0) \frac{mgV}{R} \left(\frac{1}{T_{out}} - \frac{1}{T_{in}}\right)$$
(2.1)

The values for P(0), m, and g, are all taken as standard values, ignoring any variations from altitude, time of year, or weather conditions. The value for m is given by the value for dry air – in real conditions there is always some water vapour present which reduces the average molecular mass, so high humidity will lead to a reduction in lift. These variations are not very large (they will be smaller than the errors due to approximations already made) so they will be ignored.

The constants can be combined to give

$$L \approx 3457 \times V(\frac{1}{T_{out}} - \frac{1}{T_{in}})$$
(2.2)

It should be noted that this is the total lift achieved by the balloon – the balloon mass should be subtracted to give the "spare" lift that could be used to carry a payload. The balloon mass should be a factor when designing a balloon, from the materials chosen to any internal structure such as support tape.

2.2 Heating

The air inside a solar powered balloon is heated by conduction/convection from the portion of the balloons surface which is being irradiated by the sun. The balloon loses heat by a combination of radiation and convection. The model used will treat the balloon as being made up of 2 distinct parts – an absorber and a separate balloon. The absorber is the area of the balloon surface facing the sun, which absorbs heat at a certain rate, and loses it through radiation and convection. The absorber transfers heat

to the rest of the balloon (mainly to the air inside by conduction, and spread throughout the volume of the balloon via convection). The remainder of the balloon loses heat to the environment by the same mechanisms as the absorber. We will assume that the absorber is at a uniform temperature, T_2 and the rest of the balloon is at a uniform temperature T_{in} . The environment is at temperature T_{out} and we will only analyse the steady state case assuming no wind-forced convection, etc. In this model the lift can be described by equation 2.2.

In section 4 (Fig 4.1) we determine an empirical formula for the cooling of a piece of material, at temperature T, to be

$$\frac{dT}{dt} \propto (T - T_{out}) \tag{2.3}$$

We assume that this will apply to the heat loss from both the absorber and the balloon, and the energy transfer between the absorber and the balloon (though this rate will have a different constant as it will not occur through radiation but only through conduction and convection). We also assume that the rate at which energy is absorbed is independent of temperature, and so will be simply a fraction of the incident intensity. Within this framework, the following equations describe the energy transfer for the absorber and the balloon in equilibrium

$$Q_{1} = \frac{\alpha IA}{2} - \beta (T_{2} - T_{out}) \frac{A}{2} - \gamma (T_{2} - T_{in}) \frac{A}{2} = 0$$
(2.4)

$$Q_2 = \gamma (T_2 - T_{in}) \frac{A}{2} - \beta (T_{in} - T_{out}) \frac{A}{2} = 0$$
(2.5)

The implicit assumption here is that the sun is shining from the side. In practice this is not going to be the case; the sun will be shining at an angle to the horizontal. However considerations for an actual flight, such as wind conditions, mean that it is likely balloons would be used in the morning or evening when the sun is low. For a high sun, the balloon may well react differently as the convection currents inside will not occur in the same manner. This will not be studied here, although it is likely there will not be a very large difference experimentally. Under the assumed conditions the area factor cancels, and the internal temperature should not depend on balloon size.



The values for α and β will be taken from section 4 where the properties of materials are measured. There is, however, no reason to assume that $\gamma = \beta$, moreover there is reason to assume they are not equal as radiation will play almost no part in transferring heat to the air in the balloon whereas it is highly likely to be the dominant mechanism in losing heat to the surroundings. The simulation of convective heat transfer is complex, so γ will be determined from measurements on a balloon in combination with the measured values of α and β .

2.3 Sun power

The intensity of sunlight above the atmosphere has been measured to be 1360 Wm⁻², however the amount of that power that reaches a balloon near the surface depends on the absorption by the atmosphere, which in turn depends on the angle of the sun and the weather conditions. For simplicity, the sky can be assumed to be clear – this is when the maximum intensity is available. The usual assumptions are that the intensity falls off exponentially, and the depth of atmosphere traversed by the suns radiation is expressed as an air mass number, m = cosec θ (θ being measured from the horizon such that 90° is noon at the equator). Fig 2.2 shows how the intensity varies with the suns angle using these assumptions. The absorption coefficient was calculated assuming that the peak intensity at noon is 1000 Wm⁻². This corresponds very well with real measurements that have been made².

The materials used are assumed to absorb mainly in the visible wavelengths (as this is where the peak solar intensity is, and the materials chosen will all be black, which means that they strongly absorb visible wavelengths). The maximum power available will therefore not be the total incident power, but the luminous power. The luminous efficiency of the bulbs used is roughly 3% ³, and the luminous efficiency of the sun is roughly 12% (see appendix 2). This gives a way of directly comparing the results obtained using artificial light with those expected for sunlight.



Fig 2.2 – computed variation of solar intensity with angle.

3 Method

3.1 Material Properties

In order to test the heat absorption and loss properties of the materials, samples were placed at 1.2 m from a set of lights and the temperature was recorded as a function of time. Then the lights were turned off and again the temperature was recorded. The heating and cooling rates (dT/dt) were fitted to a linear distribution and the difference was taken to give the pure heating rate. The number of lights was varied, and different samples were used. The different samples were mainly bin liner materials of different grades, with one being taken from a "solar airship" toy.

3.2 Balloon Construction

In conventional hot air balloons the material used is normally some sort of nylon, which is sewn together to form the balloon envelope. The polyethylene sheets used here would not respond well to being sewn together, and would probably need some sort of leak-sealing, so two different methods of joining the plastic were investigated – sticky tape and "welded" seams. The weights of 3 different types of tape are listed in table 4.1, showing that even the lightest tape will add a weight of 0.6g for every metre. Assuming that strips no more than 1m wide are used to create the balloon, this adds an extra 5-10% of the weight of the balloon, whereas if the seams are melted together an overlap of only 1-2cm is needed – the seams will then constitute less than 5% of the weight of the balloon. If done well the welded seams were also found to be as strong and air tight as taped seams, although tape was used to reinforce some corner areas where stresses would build up (more of a problem with the balloon design).

Таре	width / mm	mass/length
masking tape	19	1.75 g/m
parcel tape	48	1.75 g/m
selotape	18	0.6 g/m

Fig 3.1 – Table showing mass per unit length of various tapes

An iron was used to form the seams – the edge was briefly touched against the overlapping edges, causing them to melt just enough to fuse together. Care was taken not to allow the iron to melt straight through, although a few small holes were patched with tape. Some seams would not fuse together correctly due to the iron not being hot enough and would come apart, requiring further work to repair them.

On a larger balloon, the strength would be taken into account. In this case it would make sense to reinforce seams with tape, sacrificing lift to the extra weight. The forming of seams with an iron can also be a time-consuming process, so for construction of a large balloon it would be necessary to investigate other methods or instruments that would be able to form seams much faster.

3.3 Temperature Distribution

In order to measure the temperature distribution inside the balloons, a thermocouple was held at various known positions over the height of the balloon whilst the temperature was measured. The thermocouple was simply attached to a metre rule, and all the measurements were taken at least 4 minutes after any adjustments had been made to ensure the system could reach equilibrium.

The thermocouple was kept on the opposite side of the metre rule to the lights, as it was discovered that the measurements were about 2°C higher if it was facing lights.

This is thought to be due to direct heating of the metre rule by unabsorbed light, which causes a local "hot-spot" around the thermocouple. This shouldn't occur if the thermocouple is on the side facing away from the lights. Obviously this does suggest that the measurement process could be affecting the internal temperature of the balloon, however the effect should be small relative to the heat absorption and loss by the balloon skin.

The lights were placed roughly 1m away from the closest surface. This is closer than in the measurements of absorption coefficients because the parts of the balloon further round the sides will be further away and receive a lower intensity, so the average intensity should be almost the same.

The surface temperature distribution was measured much more crudely. Owing to the inability to move a thermocouple easily over the surface, the temperature was simply measured at a point roughly central on the side facing the light source, and a similar point on the other side using a thermocouple taped in place temporarily.

3.4 Lift

As none of the balloons were large enough to achieve more lift than needed to support their own weight, they were hung from the ceiling by a spring. The spring was used to determine the overall downwards force exerted by the balloon, which gave values for the lift obtained by each balloon. This system was periodically recalibrated.

4 Preliminary Results

4.1 Lights

The lights used were 500W halogen lights, which had built in reflectors. It was estimated that most of the emitted light fell over an area of approximately 7 m^2 (roughly circular with diameter 3m) at the distance of 1.2 m (the point at which the samples were placed). Assuming 3% luminous efficiency ³ this gives an intensity at 1.2m of 2.14 Wm⁻² of visible light.

4.2 Material Properties

As shown in fig 4.1, the heating and cooling rates can easily be approximated to straight lines. This is only because they are over a small temperature range, however as there is little interest in using solar powered hot air balloons much outside of this temperature range the linear approximation will be used. The absorption coefficient was obtained by taking the difference between the two lines to give the rate at which heat is absorbed.



Fig 4.1 – example graphs of heating and cooling rates for two tests – one using 2 lights and one using 4 lights. Both tests were on the solar balloon material.

The samples were all assumed to be made out of polyethylene (the material from which bin liners are usually made), though it was not possible to confirm this for certain. The value for the heat capacity of polyethylene was taken to be 1000 J kg⁻¹K⁻¹, although this is only really a guide – the heat capacity cancels in the calculations for balloon temperatures so the results will be unaffected.

Material source	density (g m ⁻²)	absorption coefficient (α)	emission coefficient (β)
value black bin liner	12.7	0.518	0.157
garden heavy refuse	30.7	1.33	0.557
wheelie liner	18	0.748	0.344
solar balloon	13	0.647	0.249
value white bin liners	7.9	0.043	0.148

Fig 4.2 – Table showing measured properties of materials

The absorption of light by a thickness of a material can be characterised by a logarithmic law (in the same way as the attenuation of sunlight by the atmosphere). In this way it is simple to derive the power absorbed in terms of the thickness, x, and an attenuation coefficient, λ :

$$\alpha I = I(1 - e^{-\lambda x}) \tag{4.1}$$

Assuming the materials all have the same density, the relative thicknesses can be calculated, and using the obtained values for α , a plot of $-\ln(1-\alpha)$ against thickness should give a straight line through zero with the gradient equal to λ .



Fig 4.3 – Plot of $-\ln(1 - \alpha)$ against thickness, both axes arbitrary units.

Fig 4.3 clearly shows the three black bin liner materials forming a straight line, with the white bin liners having a much lower attenuation coefficient, and the solar balloon material slightly higher. This agrees with the suggestion that all the materials have the same density (kg m⁻³ rather than per area in this case) and the same heat capacity, with the only differences occurring in the colouring of the plastics. From fig 4.4 it is clear that the solar balloon is much blacker than the other materials, causing the higher absorption.



Fig 4.4 – photograph showing the colours of the solar balloon (left), and value black bin liner (right) materials. Both materials are roughly the same thickness, but the solar balloon absorbs more light due to its darker colour.

4.3 Balloon properties

Two balloons were initially made. Both were constructed using 6 octagonal "side" pieces (Fig 4.5) and a circular top cap. A replica of balloon 1 was constructed later because the original balloon had been slightly warped by the lights. The replica was created with slightly different seams but the same geometry. All the balloons were made from the "solar balloon" material because of its low mass and high absorption coefficient.



Balloon	calculated height/cm	calculated volume/m ³	calculated weight/g	measured weight/g
1	180	1.22	76	79
2	240	1	72	69.8

Fig 4.6 – table showing physical properties of the two balloons. Calculated values do not take into account any manufacturing flaws, bulging of the inflated balloon beyond the hexagonal shape or any weight added due to tape or overlap.

4.4 Temperature Distribution

The internal temperature distribution of the balloons tested was linear with height. Fig 4.7 shows the measured distribution for the two balloons with 3 lights and with 6 lights.

The model predicts that the average temperature will depend on the luminous flux, and material, and not the size of the balloon. However under 3 lights the balloons are both slightly collapsed and so the shapes are not uniform like they are assumed to be (and are when using a higher power). This could be the cause of the slight discrepancy between the two balloons temperatures in the case of using 3 lights, as the distance from the lights may well not be the same or the asymmetry they obtain when slightly collapsed could affect the absorption.



Fig 4.7 – vertical temperature distribution of balloons under horizontal illumination. Heights measured from mid-point of balloon.

4.5 Surface Temperature

The front (facing the lights) and back (facing away from the lights) surface temperatures were measured with a thermocouple on the surface, for the case with 6 lights on. These were used with the internal and external temperatures to form an estimate for γ as a function of β . From equation (2.5) we can see that

$$\gamma = \beta \frac{(T_{in} - T_{out})}{(T_2 - T_{in})}$$
(4.2)

T _{in} /∘C		T _{out} /°C		T₂/°C	•	γ/β	
	29		20	4	7	0.5	
T' - 10	N	σ	1 4 -				

Fig 4.8 –Measured temperatures giving a value for the ratio γ/β of 0.5.

5. Results

5.1 *Temperature*

Eliminating T₂ from equations (2.4) and (2.5) and taking $\gamma = \beta/2$, gives us an expression for the internal Temperature

$$T_{in} = T_{out} + \frac{\alpha I}{4\beta} \tag{5.1}$$

Assuming that, in the experiments, the whole balloon was evenly exposed to a luminous intensity of n x 2.14 Wm⁻², where n is the number of lights used. This gives us a temperature difference of 1.39n.



Fig 5.1 – Plot showing the experimentally determined temperature differences and the temperature difference that the model predicts.

Clearly (fig 5.1) the model does not accurately predict the internal temperature of the balloon, however it is reasonably close considering the fairly large assumptions made in deriving the model. The discrepancies could also be due to heating of the system holding the thermocouple in place causing a systematic error in the measurements, as mentioned in section 3.3. There is also scope for large errors to occur when estimating the light intensity at the balloon surface, which is definitely not uniform as assumed. With this in mind the model does reasonably well. For temperatures or intensities well outside the range studied here it is unlikely the model will do so well.

5.2 Lift

The calculated volumes of the balloons are likely to be underestimates of the actual volumes when inflated, as there will be some "bulging" of the sides that were assumed to be flat. Also the manufacturing process was not particularly precise (especially the top piece which is challenging to attach without large overlaps), and this could have affected the volumes. New volumes can be calculated on the basis of the measured temperatures and lifts, with equation (2.2). These new volumes are both roughly 30% larger than the initial estimates, although for the cases where 3 lights are being used, the volumes are smaller as the balloons were not fully inflated. Fig 5.2 shows the data obtained for the lift/volume ratio of the balloons used at the two powers tested, as well as the predicted values for an ambient temperature of 5°C, 20°C, and 40°C. Clearly the predictions are less than the experimentally determined values, but this error comes from the model underestimating the balloons' internal temperatures.



Fig 5.2 – Lift as a function of incident luminous intensity. 120Wm⁻² corresponds to peak solar intensity (sun vertically above).

5.3 White tops

The top sections of both balloons were replaced with white bin liner material, to see if the reduction in radiative heat loss would increase the internal temperature, leading to more lift, whilst also reducing the balloons' weight.





Fig 5.3 – (a) differences between the temperature distributions of the balloon 1 with white and black tops. (b) the same measurements for balloon 2.

Fig 5.3 shows the temperature distributions obtained with a white top, compared with those for the same balloon with a black top. There does appear to be slightly larger temperature differences when using the white top, although for balloon 1 with three lights the order is reversed. Fig 5.4 shows the lift obtained in each case. The evidence does suggest that the white top results in increased temperatures, causing more lift.

test	balloon 1 (6 lights)	balloon 1 (3 lights)	balloon 2 (6 lights)	balloon 2 (3 lights)
black	0.477	0.137	0.425	0.138
white	0.5	0.182	0.432	0.16
% increase	4.8	32.8	1.6	15.9

Fig 5.4 – Lifts obtained (in Newtons) for various balloons, demonstrating the increased lift from replacing the top with white plastic. Clearly it had a larger effect on balloon 1 than balloon 2, and at lower powers.

5.4 Outdoor test

It was intended to perform an outdoor test in real sunlight to see how well the assumptions about sunlight held up. The test was intended to be performed either in the morning or evening to avoid windy conditions. Unfortunately a very light wind (3-4 knots) caused a much larger force sideways against the balloon than the internal air pressure would apply and so the balloon was unable to inflate without almost completely still air. On the one day there was little enough wind, the sun was behind a cloud, and so there was not enough intensity at the balloon to perform the intended measurements. A temperature reading was made at the top of the balloon, of 20.4°C, and the ambient air temperature was measured to be 13.8°C. The suns angle was roughly 30° (estimated by eye). It is difficult to relate this to the model as the balloon was not fully inflated and the suns intensity kept changing as the clouds moved relative to the suns position in the sky, however it is at least similar to the results obtained when using 3 lights in the lab tests. If the intensities are equivalent, then

taking into account luminous efficiencies this corresponds to an intensity of 53.5 Wm^{-2} , or about 10% of the suns full intensity at the time. This estimate seems low, but not unreasonable.

6 Discussion

6.1 Full-scale balloons

Extending the results obtained here to larger balloons in full sunlight, it is relatively easy to compare directly with regular hot air balloons and other solar powered hot air balloons. For example, early in the morning in springtime in England – the temperature may well be 10°C, with the sun at around 20-30° above the horizon. With these conditions on a cloudless, windless day, the lift obtained would be 1.5 Nm⁻³, requiring a 550m³ balloon to lift an 80kg person. A spherical balloon of this volume would have a radius of over 5m. A small hot air balloon carrying a basket, burner, and multiple people usually have volumes in excess of 2000m³, so the solar powered versions seem to compare favourably. Examples of solar powered hot air balloons made by members of the public agree well with the results here, even up to the size of carrying a person⁴.

6.2 Absorption

In deriving the absorption coefficients, we assumed that the plastics only absorbed over the visible spectrum. The appearance of an absorption coefficient greater than 1 for the heavy refuse bin liners could easily have arisen from absorption beyond this range, over the IR/UV spectra, for example. It could also be due to absorption of radiation that has been reflected from nearby surfaces, which is not taken into account in the model. The absorption of this reflected radiation, along with the absorption of scattered light from the sky could cause further deviations from the model. The large absorption coefficients are not a problem, as they are compensated for by using the luminous intensity, rather than the total intensity.

The transfer of heat from the absorbent surface into the balloon was not studied, and could behave quite differently to the way it was treated here. This could affect the results, especially at higher powers and temperature differences than seen in the tests performed.

The luminous efficiencies used here are meant to be a way of comparing the intensities of the two sources (sun, and halogen lights) at the relevant wavelengths. The luminous efficiency of the sun, however, is effectively altered by the atmosphere, as it absorbs and reflects large amounts of UV and other radiation, as well as scattering blue wavelengths much more than red wavelengths, so the direct radiation is deficient in these wavelengths

6.3 Balloons

The balloons used here were manufactured by hand, using an iron to fuse pieces of plastic together. To make larger balloons this process would take excessively long, and isn't very reliable. Having better temperature control of the iron, and a longer heated section, would reduce the production time. For larger balloons intended to carry loads, the strength of the balloon would have to be studied – often in normal hot air balloons there is some sort of load-bearing system that helps maintain the balloons

shape and reduce stresses on balloon seams. This would increase the weight quite substantially and so could only be used on larger balloons.

There are other ways of building a solar hot air balloon, for example using the greenhouse effect⁵ by having a clear skin and an inner absorber, which may be another internal balloon. This would tend to reduce losses but at the cost of added weight, and so is only suitable for reasonably large balloons. To treat this type of balloon with the type of basic analysis here would be difficult, and would be unlikely to work very well.

6.4 Experimental problems

There are a number of areas in which the experiment could have been improved given the resources. Firstly, it was not possible to measure the temperature distribution over the full height of the balloon, or laterally, and it was not possible to control or adjust the ambient lab temperature, so the tests were over a narrow range of temperatures, from 17°C to 22°C, depending on the temperature in the lab at that particular time. These restrictions meant it was not possible to test the model over a very broad range of conditions. The lack of temperature readings from near the top of the balloon means that it is not known whether the linear distribution holds true at the top, or whether the temperature does something else in the vicinity of the skin. It also makes it reasonably hard to compare the white-topped balloons with the black-topped balloons.

There were also errors introduced in having to replace blown 500W bulbs with 400W, energy efficient, bulbs. It is not clear how this affects the measurements, as they are designed to produce the same amount of visible light. However it is likely that less power will be absorbed by the balloon when using the energy efficient replacements, leading to lower temperatures and less lift. All of the direct comparisons between measurements in sections 4 and 5 used data from tests with equivalent sets of lights, as far as possible.

6.5 further work and improvements

There are a number of ways to improve on the results obtained, for example repeating the tests with more control over the temperature and light intensity, or with a wider range of balloons and conditions will provide a better test of the model. The model can be improved by treating the balloons as more than 2 different parts, for example taking a large number of small surface elements at different temperatures, and solving the heat transfer equations for the whole system numerically, or by an improved model of heat transfer, (conduction from the balloon walls, and convection currents) both within the balloon and over the outer surface. This would hopefully lead to a better understanding of how the shape of the balloon could affect the temperature. The model assumes a uniform internal temperature, however it was discovered that this is not the case. It is not clear whether the distribution found on small balloons would apply on much larger balloons, and how this might affect the validity of the model used.

7 Conclusions

1. A model for the performance of a simple solar powered hot air balloon was developed, treating the balloon as a two-part system consisting of an absorber and a balloon.

 The internal temperature distribution and lift of small balloons was studied under artificial light conditions and compared with predictions from the theoretical model.
 The internal temperature was found to increase linearly with height, so the midpoint temperature was used. On larger balloons this could cause larger deviations from the predictions.

4. In sunlight a simple black balloon should be able to achieve 1.5 Nm⁻³ of lift.

5. Replacing the tops of balloons with white plastic increased the temperatures and lifts achieved. Further complexity with layered structures could also increase the lift achieved.

6. Further work could include a more extensive investigation, with control over lab temperature, a wider range of sizes and shapes of balloons, and outdoor tests to verify the assumptions made about sunlight.

7. For practical use the structural integrity of the balloons would need to be improved to cope with wind and support weight.

8 References

¹ Hot air balloon engine – Ian Edmonds, renewable energy, 34, 4, p1100-1105, 2009

² Principles of Environmental Physics, John L. Monteith, Edward Arnold Limited, 1973

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http://download.p4c.philips.com/l4bt/3/327366/plusline_small_double_ended_327366 __ffs_aen.pdf

⁴ <u>http://www.solar-balloons.com/videos.html</u>

⁵ <u>http://ballonsolaire.assoc.pagespro-orange.fr/en-historique2.htm</u>

Appendices

Appendix 1 – lift calculations

The lift of a cylindrical balloon is simple to calculate, as all the lift comes from the pressure difference at the top surface.

Using the ideal gas equation, and treating the air as horizontal slices of uniform density and thickness dz, the two following equations apply

$$P = \frac{\rho(z)}{m} RT$$

$$dP = \rho(z)gdz$$
(A.1)

Where P is the pressure, ρ is the air density, and other symbols are defined in the notation section above.

Equations (A.1) and (A.2) combine to give

$$\frac{dP}{P} = -\frac{mg}{RT(z)}dz \tag{A.3}$$

Integrating this for the relevant T(z) gives the pressure distribution, P(z). The atmospheric temperature is assumed to be constant, giving a pressure distribution of

$$P_{out} = P(0) \exp(\frac{-mgz}{RT_{out}})$$
(A.4)

For a cylindrical balloon, the lift is given by the area of the top surface multiplied by the pressure difference at the surface. Taking the height to be h, and the top surface area to be unity, then for an internal uniform temperature, T_{in} , the lift is given by

$$P(0)[\exp(\frac{-mgh}{RT_{out}}) - \exp(\frac{-mgh}{RT_{in}})]$$
(A.5)

The other case looked at is the linear distribution

$$T_z = T_1 + \frac{\Delta T}{h} z \tag{A.6}$$

And for this case, the pressure distribution is given by

$$P_{z} = P(0) \left[1 + \frac{z\Delta T}{hT_{1}} \right]^{\frac{-mgh}{R\Delta T}}$$
(A.7)

Binomially expanding the case for uniform temperature, and taking the terms linear in 1/T, leaves us with the usual lift equation (2.1).

Plotting the three possible equations (Fig A.1), with the uniform temperature cases having a temperature equal to $T_1 + \Delta T/2$, shows that there is very little difference between the approximations and the actual case, even for the extreme situation where there is a temperature difference of 20°C over a 1m tall balloon.



Fig A.1 – (a) plot of lift obtained for various internal temperatures. The linear distribution had a temperature difference of 20K between the top and bottom, which is a much higher gradient than was found in any balloon. (b) zoomed in version of a) for clarity

Appendix 2 – luminous intensity of the sun

The sun is assumed to be a black body at 5800K. Visible light is assumed to be in the range 400-700nm, or equivalently $4.29 \times 10^{14} - 7.5 \times 10^{14}$ hz. The power spectrum emitted by a black body is given by Plancks law

$$I(v,T) = \frac{2hv^{3}}{c^{2}} \frac{1}{e^{\frac{hv}{kT}} - 1}$$

And the luminous efficiency will be given by

$$\frac{\int_{v_1}^{v_2} I(v,T)}{\int_0^\infty I(v,T)}$$

Where v1 and v 2 are the frequencies given earlier. Using mathematica to numerically integrate this gives the value 0.116, or roughly 12%.