

Re-examine invariant mass: $M \rightarrow a b$

$$\begin{aligned} M^2 &= \left(\sqrt{m_a^2 + a_x^2 + a_y^2 + a_z^2} + \sqrt{m_b^2 + b_x^2 + b_y^2 + b_z^2} \right)^2 \\ &\quad - (a_x + b_x)^2 - (a_y + b_y)^2 - (a_z + b_z)^2 \\ &= (E_a + E_b)^2 - (a_x + b_x)^2 - (a_y + b_y)^2 - (a_z + b_z)^2 \\ &= m_a^2 + m_b^2 + 2(E_a E_b - a_x b_x - a_y b_y - a_z b_z) \end{aligned}$$

$$= m_a^2 + m_b^2 + 2(e_a e_b \cosh(\Delta\eta) - a_x b_x - a_y b_y)$$

where

$$\begin{aligned} e_a &= \sqrt{m_a^2 + a_x^2 + a_y^2} & \text{and} & & \eta_a &= \frac{1}{2} \ln((E_a + a_z)/(E_a - a_z)) \\ e_b &= \sqrt{m_b^2 + a_b^2 + a_b^2} & & & \eta_b &= \frac{1}{2} \ln((E_b + b_z)/(E_b - b_z)) \\ & & & & \Delta\eta &= \eta_a - \eta_b \end{aligned}$$