For completeness, the same table includes the positions of the unobservable near- and far-edges. Note that although the position of the high-edge is just the higher of $m^{\max}_{l_{\text{near}}q}$ and $m^{\max}_{l_{\text{far}}q}$, the position of the low-edge has a more interesting form. This asymmetry arises because it is not always kinematically possible for the invariant mass of the lepton-quark pair coming from the lower of the near- and far-distribution to approach $\min\left[m^{\max}_{l_{\text{near}}q}, m^{\max}_{l_{\text{far}}q}\right]$ arbitrarily closely, while simultaneously requiring that this invariant mass is less than that of the other lepton-quark pair.

5.7.6 Dislepton production, $M_{T2}^{\max}(\chi)$, ΔM and the $\tilde{l}\bar{\tilde{l}}$ edge

One of the novel parts of this analysis concerns the introduction of ΔM, a variable designed to better constrain the slepton-LSP mass difference.

It is easiest to describe ΔM in terms of a related variable, $M_{T2}(\chi)$ proposed by the author (in collaboration) in [28].

$M_{T2}(\chi)$ may be used to study events where:

- the event contains two identical two-body decays (i.e. $X \to Y a_1 a_2$ followed by $a_1 \to b_1 c_1$ and $a_2 \to b_2 c_2$),
- $m_A = m_{a_1} \equiv m_{a_2}$ is unknown,
- $m_C = m_{c_1} \equiv m_{c_2}$ is unknown,
- $m_B = m_{b_1} \equiv m_{b_2}$ is known,
- particles of type c are undetectable,
- the longitudinal momentum of the incoming particles is also unknown, and
- Y does not contain any unobservable particles such as neutrinos.

In such cases, the variable provides a kinematic constraint on the masses of particles of type a and type c. In this analysis, $M_{T2}(\chi)$ will be applied to dislepton events of the form $pp \to Y\tilde{l}_R^{\pm}\tilde{l}_R^{\mp} \to Yl^{\pm}l^{\mp}\tilde{\chi}_1^0\tilde{\chi}_1^0$ (Figure 5.4) using the assignment $a = \tilde{l}$, $b = e/\mu$ and $c = \tilde{\chi}_1^0$. These events are used to constrain $m_{\tilde{l}}$ and $m_{\tilde{\chi}_1^0}$ in the following way:

Suppose, for the moment, that the mass of the neutralino were known, and denoted by $m_{\tilde{\chi}}$ (the numbers in $\tilde{\chi}_1^0$ have been dropped to avoid confusion with subscripts indicating which decay the neutralino came from). Then, if it were possible to observe the momenta of each neutralino, and correctly identify its associated lepton, it would be possible to compute the invariant mass of either slepton using the exact relation:

$$m_{\tilde{l}}^2 = m_l^2 + m_{\tilde{\chi}}^2 + 2(E_T^l E_T^{\tilde{\chi}} \cosh(\Delta y) - \mathbf{p}_T^l \cdot \mathbf{p}_T^{\tilde{\chi}}) \tag{5.9}$$

where $E_T^l = \sqrt{{\mathbf{p}_T^l}^2 + m_l^2}$, $E_T^{\tilde{\chi}} = \sqrt{{\mathbf{p}_T^{\tilde{\chi}}}^2 + m_{\tilde{\chi}}^2}$ and Δy is the difference between the rapidities (see Appendix C) of the lepton and the neutralino. This definition includes the lepton masses for completeness, although these are neglected in all computations. Of course, observing the neutralinos is impossible at ATLAS, so the difference in rapidities, Δy, will certainly not be known. Nevertheless, using the fact that $\cosh \Delta y \geq 1$, it is possible to make the reduced claim:

$$m_{\tilde{l}}^2 \geq m_T^2(\mathbf{p}_T^l, \mathbf{p}_T^{\tilde{\chi}}, m_{\tilde{\chi}}) \equiv m_l^2 + m_{\tilde{\chi}}^2 + 2(E_T^l E_T^{\tilde{\chi}} - \mathbf{p}_T^l \cdot \mathbf{p}_T^{\tilde{\chi}}), \tag{5.10}$$

or even the stronger claim:

$$m_{\tilde{l}}^2 \geq \max\{m_T^2(\mathbf{p}_T^{l_1}, \mathbf{p}_T^{\tilde{\chi}_1}, m_{\tilde{\chi}}), m_T^2(\mathbf{p}_T^{l_2}, \mathbf{p}_T^{\tilde{\chi}_2}, m_{\tilde{\chi}})\} \tag{5.11}$$

using both of the decays. But neither $m_T^2(\mathbf{p}_T^l, \mathbf{p}_T^{\tilde{\chi}}, m_{\tilde{\chi}})$ nor the right hand side of (5.11) is an observable quantity, as each relies upon knowledge of the transverse components of the momenta of each neutralino. Under the prevailing assumptions, the only thing known about the neutralinos is the *sum* of their transverse momenta, given by the missing momentum of the event:

$$\not{\mathbf{p}}_T = \mathbf{p}_T^{\tilde{\chi}_1} + \mathbf{p}_T^{\tilde{\chi}_2}. \tag{5.12}$$

Although the form of the splitting (5.12) is not known, it is possible to turn the right hand side of (5.11) into a quantity $M_{T2}^2(m_{\tilde{\chi}})$ which *is* observable, by trying all possible splittings. This motivates the definition:

$$M_{T2}^2(m_{\tilde{\chi}}) \equiv \min_{\not{\mathbf{p}}_1 + \not{\mathbf{p}}_2 = \not{\mathbf{p}}_T} \left[\max\left\{ m_T^2(\mathbf{p}_T^{l_1}, \not{\mathbf{p}}_1, m_{\tilde{\chi}}), m_T^2(\mathbf{p}_T^{l_2}, \not{\mathbf{p}}_2, m_{\tilde{\chi}}) \right\} \right], \tag{5.13}$$

which must therefore satisfy

$$m_{\tilde{l}}^2 \geq M_{T2}^2(m_{\tilde{\chi}}). \tag{5.14}$$

In [28], the authors note that it *is* kinematically possible for equality to be reached in (5.14), and that this occurs when $\Delta y_1 = \Delta y_2 = 0$ and $\mathbf{v}_T^{l_1} - \mathbf{v}_T^{\tilde{\chi}_1}$ is parallel (note: not anti-parallel) to $\mathbf{v}_T^{l_2} - \mathbf{v}_T^{\tilde{\chi}_2}$, where $\mathbf{v}_T = \mathbf{p}_T / E_T$. Thus, by construction, $M_{T2}(m_{\tilde{\chi}})$ has the property that, for a large enough number of signal events in a perfect detector,

$$M_{T2}^{\max}(m_{\tilde{\chi}_1^0}) \equiv \max_{\text{events}} \left[M_{T2}(m_{\tilde{\chi}_1^0}) \right] = m_{\tilde{l}}, \tag{5.15}$$

which is equivalent to saying that the endpoint of the $M_{T2}(m_{\tilde{\chi}})$ distribution occurs at the slepton mass.

Note that up to this point it has been necessary to assume knowledge of the neutralino mass, $m_{\tilde{\chi}} \equiv m_{\tilde{\chi}_1^0}$, which is a little unfair, given that it is one of the quantities which the analysis seeks to determine. Formally, then, the M_{T2} distribution is not a single distribution, but a family of distributions: one for each possible value of the neutralino mass. For example; suppose it is proposed that the neutralino mass is equal to "χ" (not necessarily close to the correct value). This would motivate plotting the distribution of $M_{T2}(\chi)$, generated from dislepton events under the hypothesis that the neutralino mass may be taken to be χ rather than $m_{\tilde{\chi}}$ in definition (5.13). Having plotted this distribution, the location $M_{T2}^{\max}(\chi)$ of its endpoint could be ascertained, and the interpretation of $M_{T2}^{\max}(\chi)$ would be "the slepton mass favoured by the data given the assumed neutralino mass".

Thus the overall result obtained from a full set of $M_{T2}(\chi)$ distributions, is a function $M_{T2}^{\max}(\chi)$, describing the most favoured slepton mass as a function of neutralino mass χ.

In Section D.7 of the Appendix it is shown that the function $M_{T2}^{\max}(\chi)$ is in fact dependent on only one unknown parameter $p > 0$; namely the modulus of the three-momentum of the lepton (or the neutralino) in the rest frame of the slepton. In general p is given by

$$p^2 = \frac{m_{\tilde{l}}^4 + m_{\tilde{\chi}_1^0}^4 + m_l^4 - 2(m_{\tilde{l}}^2 m_{\tilde{\chi}_1^0}^2 + m_{\tilde{l}}^2 m_l^2 + m_{\tilde{\chi}_1^0}^2 m_l^2)}{4 m_{\tilde{l}}^2}. \tag{5.16}$$

In Section D.7 the general form of the dependence of $M_{T2}^{\max}(\chi)$ upon p is shown to be:

$$M_{T2}^{\max}(\chi) = \sqrt{m_l^2 + p^2} + \sqrt{\chi^2 + p^2}. \tag{5.17}$$

Significantly, (5.17) may be inverted for arbitrary χ yielding:

$$p^2 = \frac{\tilde{L}^4 + \chi^4 + m_l^4 - 2(\tilde{L}^2\chi^2 + \tilde{L}^2 m_l^2 + \chi^2 m_l^2)}{4\tilde{L}^2} = \text{"independent of } \chi\text{"}, \tag{5.18}$$

where $M_{T2}^{\max}(\chi)$ has been abbreviated to $\tilde{L}$. The reader is encouraged to compare (5.18) with (5.16). The result (5.18) is important for two reasons. Firstly it tells us that we no longer need to worry about choosing a particular value of χ in the region of $m_{\tilde{\chi}_1^0}$, and secondly it shows us that p from (5.16) is the quantity which $M_{T2}^{\max}(\chi)$ is *really* constraining. In fact, the most elegant way of extracting p from $M_{T2}(\chi)$ measurements actually *exploits* the freedom to choose an arbitrary value of χ. Paradoxically, the best choice for χ turns out to be neither 0 nor $m_{\tilde{\chi}_1^0}$ (which one might have expected from the original definition of χ) but is in fact m_l. Using this value one may define the quantity $\Delta M \geq 0$ by

$$(\Delta M)^2 \equiv \frac{1}{4}\left(M_{T2}(m_l)\right)^2 - m_l^2 \tag{5.19}$$

and thereby obtain from (5.17) the significant result that:

$$\Delta M^{\max} = p = p(m_{\tilde{l}}, m_{\tilde{\chi}_1^0}, m_l). \tag{5.20}$$

As a consequence of this important result, it is distributions of ΔM which are chosen to be plotted to extract model independent information from dislepton events.[h] The result (5.20) shows us that the location of the ΔM edge (i.e. $\Delta M^{\max}$) is a direct measumenet of p given by (5.16).

Thus far, the only approximation which has been made was the presumption that transverse slepton boosts have a negligible effect on the prediction for $\Delta M^{\max}$ given in (5.20). Within this thesis, however, there is additionally a motivation for neglecting the lepton masses:

In order to maximise the size of the signal, the analysis presented in this thesis will seek both e^+e^- and $\mu^+\mu^-$ events. The presence of the lepton mass in (5.20) (or equivalently in the definition of p) suggests that one ought to analyse events with different

[h]It should be noted that montecarlo studies suggest that the evaluation of $M_{T2}(\chi)$ at values of χ different than $m_{\tilde{\chi}_1^0}$ can introduce a slight dependence of the edge position upon the maximum transverse boost attained by the sleptons. For example, it has been observed that in a situation where *both* sleptons are able to be emitted collinearly and with transverse momenta of 20 GeV (or 100 GeV or 1000 GeV), then the value of $\Delta M^{\max}$ predicted by (5.20) for heavy (left) sleptons at S5 would underestimate th true value by about 2% (or 8% or 14%). As expected, transverse boosts are observed to have no effect on $M_{T2}^{\max}(m_{\tilde{\chi}_1^0})$. The precise reason for this effect is still under investigation, but within this analysis the effect may safely be ignored for two reasons. Firstly, dislepton production is threshold dominated, and so sleptons typically have small transverse momenta. Secondly, and more importantly, the cut limiting jet activity to "at most one with $p_T < 40$ GeV" puts a strong constraint on the momentum available for the sleptons to recoil against. This restricts the washing out of the edge to a 2% effect, which is far less than the order 10% error on the edge position due to statistics. In situations where the effect cannot be neglected, systematic error could be minimised by evaluating $M_{T2}(\chi)$ at the best avaliable guess for $m_{\tilde{\chi}_1^0}$.

leptons differently. Note, however, that in cases where the $\Delta M^{\max}$ is observed to lie at a mass much in excess of m_l, one may immediately deduce that $m_l \ll p$. The assumption $m_l \ll p$, together with definition (5.16), may be used to show that $m_l \ll m_{\tilde{l}} - m_{\tilde{\chi}_1^0}$, which itself permits p to be expanded in terms of $\epsilon = m_l/(m_{\tilde{l}} - m_{\tilde{\chi}_1^0})$ as follows:

$$\Delta M^{\max} = p = p_{\text{approx}} \left\{ 1 - \frac{m_{\tilde{l}}^2 + m_{\tilde{\chi}_1^0}^2}{(m_{\tilde{l}} + m_{\tilde{\chi}_1^0})^2}\epsilon^2 - \frac{2m_{\tilde{l}}^2 m_{\tilde{\chi}_1^0}^2}{(m_{\tilde{l}} + m_{\tilde{\chi}_1^0})^4}\epsilon^4 + O(\epsilon^6) \right\} \tag{5.21}$$

where

$$p_{\text{approx}} \equiv \frac{m_{\tilde{l}}^2 - m_{\tilde{\chi}_1^0}^2}{2m_{\tilde{l}}}. \tag{5.22}$$

This shows that when $\Delta M^{\max} \gg m_l$, then one may take $\Delta M^{\max}$ to be really just a measurement of p_{approx} given by (5.22).

The ΔM distributions arising from two test models examined in this thesis do in fact happen to satisfy $\Delta M^{\max} \gg \max[m_e, m_\mu]$ at the 100 GeV $\gg$ 100 MeV level, and so within this thesis $\Delta M^{\max}$ is taken to be equivalent to p_{approx} as defined in (5.22). This means that within this part of the analysis it is safe to take the leptons to be massless and to analyse both the e^+e^- and the $\mu^+\mu^-$ events together.

To summarise the analysis of the dislepton edge:

- The distributions which will be plotted and whose edges will be identified, will be distributions of ΔM.

- The observation that $\Delta M^{\max} \gg m_\mu$ for the models under study permits us to: neglect lepton masses, analyse both e^+e^- and $\mu^+\mu^-$ events together, and interpret $\Delta M^{\max}$ as a measurement of $p_{\text{approx}} = (m_{\tilde{l}}^2 - m_{\tilde{\chi}_1^0}^2)/(2m_{\tilde{l}})$.

- Were the analysis to be performed at another point where $\Delta M^{\max}$ was much less than (or order) m_l, then the analysis could still proceed, but it would no longer be possible to treat $\Delta M^{\max}$ as a measurement of p_{approx}. $\Delta M^{\max}$ would instead constrain p. Beacuse p depends on m_l, one would have to consider e^+e^- events separately from $\mu^+\mu^-$ events.

The ΔM edge can thus be included in the analysis to provide an additional constraint on the slepton and neutralino masses. The name 'ΔM' is chosen because, to first order, $\Delta M^{\max} \approx p_{\mathrm{approx}}$ is well approximated by the slepton/neutralino mass difference provided that the slepton is not *much* heavier than the neutralino.

In practice the ΔM edge can be distorted by the finite resolution of the detector or missing energy from soft underlying events. Additionally, standard model (SM) backgrounds provide constraints on minimum detectable slepton/neutralino mass differences (see Section 5.15) **XXX FIX REFERENCE!! XXX**. However the most important factor to control is the ability to correctly identify which production processes are contributing to an observed $\Delta M(\chi)$ edge – particularly since it is possible to have multiple edges in the *non* standard model contributions to $\Delta M(\chi)$ distributions. At S5, for example, where both the right- and left-sleptons are lighter than at O1 (see Table 5.1), both $\tilde{l}^+_R\tilde{l}^-_R$ and $\tilde{l}^+_L\tilde{l}^-_L$ events pass the cuts, and two edges are generated. The edge coming from the (lighter) right-slepton falls well under the SM background and so is not measurable, while the (heavier) left-slepton still has a cross section for pair-production high enough to let it form a good edge of its own at $\Delta M(m_{\tilde{\chi}^0_1}) = m_{\tilde{l}_L} - m_{\tilde{\chi}^0_1}$. It is important that this edge is not mistaken for one produced by $\tilde{l}_R$, so methods are needed to dismiss it. Discussion of these issues are delayed until Section **??** XXX FIX REFERENCE!! XXX.

5.7.7 Notes concerning interpretation of endpoint positions

Were all squarks to have the same mass, then (with perfect detector resolution and with infinite statistics) the endpoints of the edges listed above would be found at the positions listed above and summarised in Table 5.2. Note that these positions depend that $p < q$ is the condition for $m^{\max}_{l_{\rm near}q}$ to be added to the list of possible candidates for $m^{\max}_{lq({\rm low})}$. Note however, that the condition controlling its inclusion is precisely the condition which causes it to be added under "Case 1" anyway. Therefore "Case 2" adds no new information and may be ignored.

Case 3

There is only one unconstrained local maximum for $m_{l_{\rm far}q}$, and it occurs when the quark is pointing along $l_{\rm near}$ and back-to-back with $l_{\rm far}$ in the rest frame of the slepton. Note that, in contrast with "Case 2" all the momenta are now fixed. Moreover, this configuration has fixed $m_{l_{\rm near}q} = 0$, which means that $m_{l_{\rm far}q}$ in this state is most certainly not smaller than $m_{l_{\rm near}q}$. Thus "Case 3" generates no masses at all.

Summary

Putting all the results together, the following result is obtained:

$$(m^{\max}_{lq({\rm low})})^2 = \begin{cases} (m^{\max}_{l_{\rm near}q})^2 & \text{when} \quad p \leq q \\ \frac{(c-b)(a-z)}{2a-z} & \text{when} \quad p \geq q. \end{cases} \tag{D.36}$$

D.7 $\tilde{l}\bar{\tilde{l}}$ edge

It was mentioned in Section 5.7.6 that the condition for a event to yield a value of $M_{T2}(\chi)$ close to the maximum is that $\Delta y_1 = \Delta y_2 = 0$ and $\mathbf{v}_T^{l_1} - \mathbf{v}_T^{\tilde{\chi}_1}$ is parallel (note: not anti-parallel) to $\mathbf{v}_T^{l_2} - \mathbf{v}_T^{\tilde{\chi}_2}$. The definitions of Δy_i and $\mathbf{v}_T^X$ may be found by referring back to Section 5.7.6. To establish the value of $M_{T2}^{\max}(\chi)$ it is thus sufficient to calculate the value of $M_{T2}(\chi)$ in an event satisfying the condition. The simplest such event contains two sleptons, at rest in the frame of the detector, and with each decaying into a lepton travelling along the negative x-axis and a neutralino along the positive x-axis. In the frame of the sleptons, both the neutralino and the lepton will have a three momentum $\mathbf{p}$ with modulu p satisfying

$$p^2 = \frac{m_{\tilde{l}}^4 + m_{\tilde{\chi}_1^0}^4 + m_l^4 - 2(m_{\tilde{l}}^2 m_{\tilde{\chi}_1^0}^2 + m_l^2 m_{\tilde{\chi}_1^0}^2 + m_{\tilde{l}}^2 m_l^2)}{4m_{\tilde{l}}^2}. \tag{D.37}$$

In order to calculate $M_{T2}(\chi)$ in accordance with Definition (5.13) we need to evaluate

$$M_{T2}^2(\chi) \equiv \min_{\not{\mathbf{p}}_1 + \not{\mathbf{p}}_2 = \not{\mathbf{p}}_T} \left[\max\left\{m_T^2(\mathbf{p}_T^{l_1}, \not{\mathbf{p}}_1, \chi), m_T^2(\mathbf{p}_T^{l_2}, \not{\mathbf{p}}_2, \chi)\right\}\right]. \tag{D.38}$$

For the particular configuration described above we can say that

$$\begin{aligned} m_T^2(\mathbf{p}_T^{l_1}, \not{\mathbf{p}}_1, \chi) &= \chi^2 + m_l^2 + 2\left(\sqrt{m_l^2 + p^2}\sqrt{\chi^2 + (p+\delta)^2 + q^2} + p^2 + p\delta\right) &\text{(D.39)} \\ m_T^2(\mathbf{p}_T^{l_2}, \not{\mathbf{p}}_2, \chi) &= \chi^2 + m_l^2 + 2\left(\sqrt{m_l^2 + p^2}\sqrt{\chi^2 + (p-\delta)^2 + q^2} + p^2 - p\delta\right) &\text{(D.40)} \end{aligned}$$

where $\not{\mathbf{p}}_1$ and $\not{\mathbf{p}}_2$ have been parametrised by the two real numbers δ and q according to

$$\not{\mathbf{p}}_1 = \begin{pmatrix} p+\delta \\ q \end{pmatrix} \qquad \text{and} \qquad \not{\mathbf{p}}_2 = \begin{pmatrix} p-\delta \\ -q \end{pmatrix} \tag{D.41}$$

in order to ensure that $\not{\mathbf{p}}_1 + \not{\mathbf{p}}_2 = \not{\mathbf{p}}_T$ is automatically satisfied. Note that we may choose both $q \geq 0$ and $\delta \geq 0$ without loss of generality, and having done so we may then deduce that the right-hand-side of (D.39) is always greater than or equal to the right-hand-side of (D.40). With this fact in mind, the two values of m_T may be substituted into (D.38) yielding

$$M_{T2}^2(\chi) = \min_{q\geq 0, \delta \geq 0} \left[\chi^2 + m_l^2 + 2\left(\sqrt{m_l^2 + p^2}\sqrt{\chi^2 + (p+\delta)^2 + q^2} + p^2 + p\delta\right)\right]. \tag{D.42}$$

Minimisation over q is trivial and is achieved by setting $q = 0$. The derivative with respect to δ of the argument of the "max" in (D.42) may also be evaluated, and can be shown to be positive. The minimisation with respect to δ is thus achieved by setting δ also equal to zero. We now have all that is required to determine $M_{T2}(\chi)$ for this particular configuration, which by design provides us with the general formula for $(M_{T2}^{\max}(\chi))^2$:

$$(M_{T2}^{\max}(\chi))^2 = \chi^2 + m_l^2 + 2p^2 + 2\sqrt{m_l^2 + p^2}\sqrt{\chi^2 + p^2}, \tag{D.43}$$

from which we observe that

$$M_{T2}^{\max}(\chi) = \sqrt{m_l^2 + p^2} + \sqrt{\chi^2 + p^2}. \tag{D.44}$$

Notice that if χ is set equal to the real neutralino mass, $m_{\tilde{\chi}_1^0}$, then we recover the expected result $M_{T2}^{\max}(m_{\tilde{\chi}_1^0}) = E_l + E_{\tilde{\chi}_1^0} = E_{\tilde{l}} = m_{\tilde{l}}$. In passing I remark

D.8 NOT A HIGH PRIORITY BUT ...

XXX If desired, at this point I could include a description of a general edge finding process for finding $m_{abc...xyz}^{\max}$ for the chain $A \to aB, B \to bC, C \to cD$ ... . This would not be essential to the thesis though, XXX