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Mass and Spin Measurement Techniques

(for the Large Hadron Collider)

Based on "A review of Mass Measurement Techniques proposed for the Large Hadron Collider", Barr and Lester, arXiv:1004.2732

TASI 2011

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arXiv:1004.2732

A Review of the Mass Measurement Techniques proposed for the Large Hadron Collider

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We review the methods which have been proposed for measuring masses of new particles at the Large Hadron Collider paying particular attention to the kinematical techniques suitable for extracting mass information when invisible particles are expected.

Scope and disclaimers

– am not interested in fully visible final states as standard mass reconstruction techniques apply

– will only consider new particles of unknown mass decaying to invisible particles of unknown mass (and other visible particles)

– selection bias – more emphasis on things I've worked with

  - Transverse masses, MT2, kinks, kinematic methods.
  - (Not Matrix Element / likelihood methods / loops)

– not shameless promotion – focus on faults!

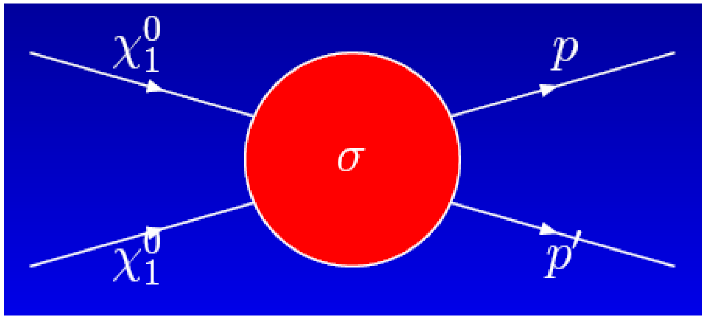
Sneak peek at conclusions

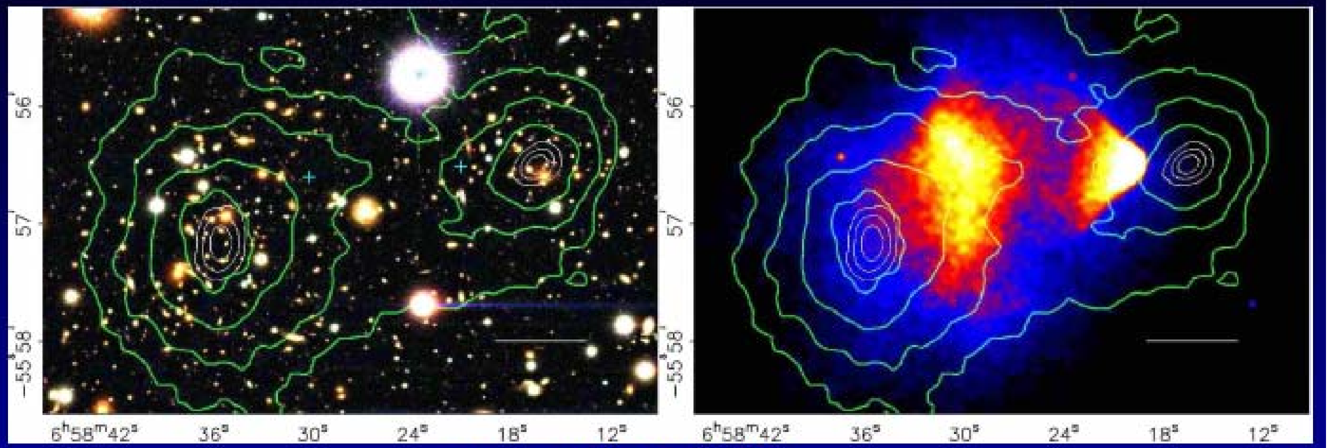
- Don’t trust experimental collaborations. They are probably doing the wrong thing.
- If you can’t understand why the experimental paper says the experiment did, it might be because they don’t know either (sphericity)

Recall there are some problems

Aim was to fix some of these problems with the Standard Model

- Fine-tuning / "hierarchy problem" (technical) – Why are particles light?
- Does not explain Dark Matter
- No gauge coupling unification



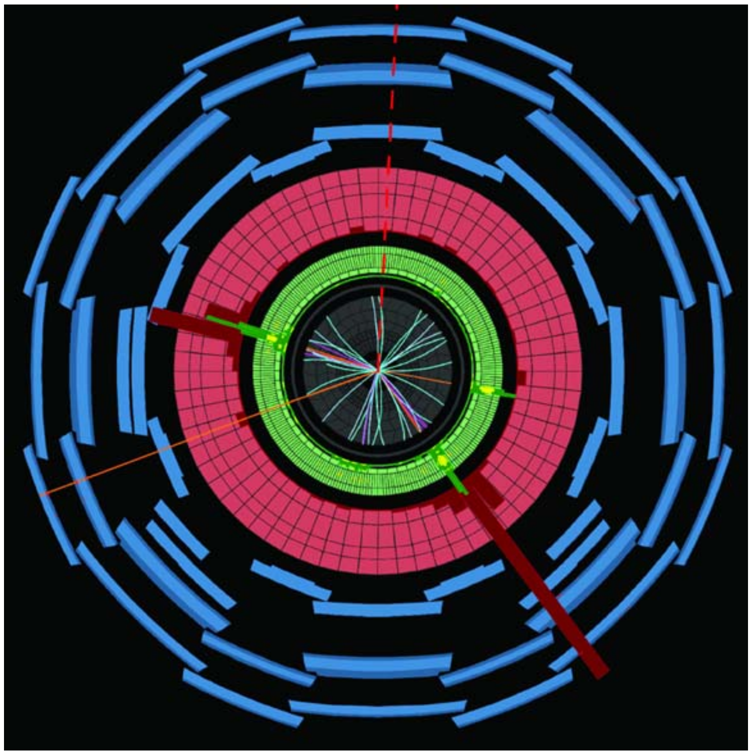


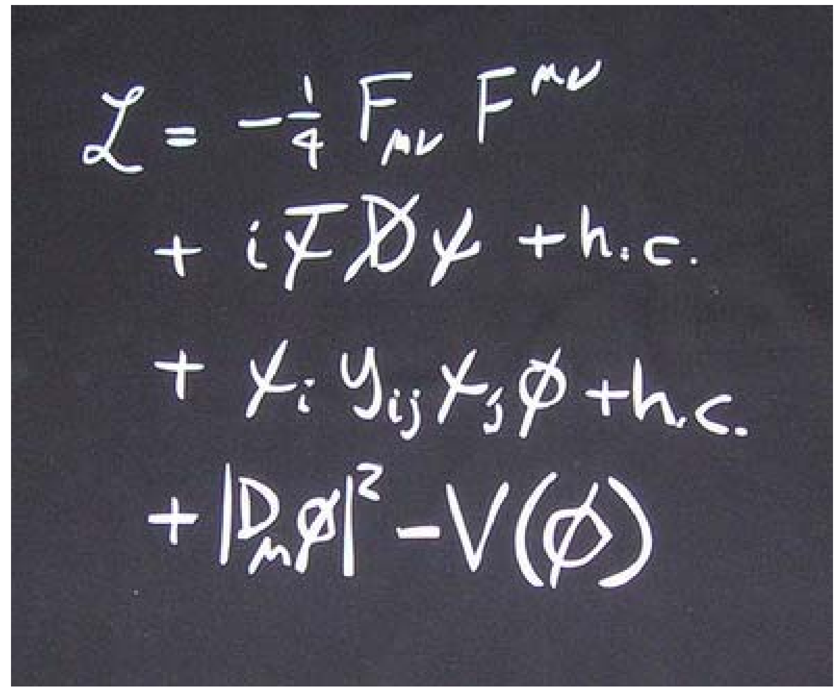
astro-ph/0608407

What are common features of “solutions” to these problems?

- Big increase in particle content
- Longish decay chains
- Missing massive particles
- Large jet/lepton/photon multiplicity

The game…

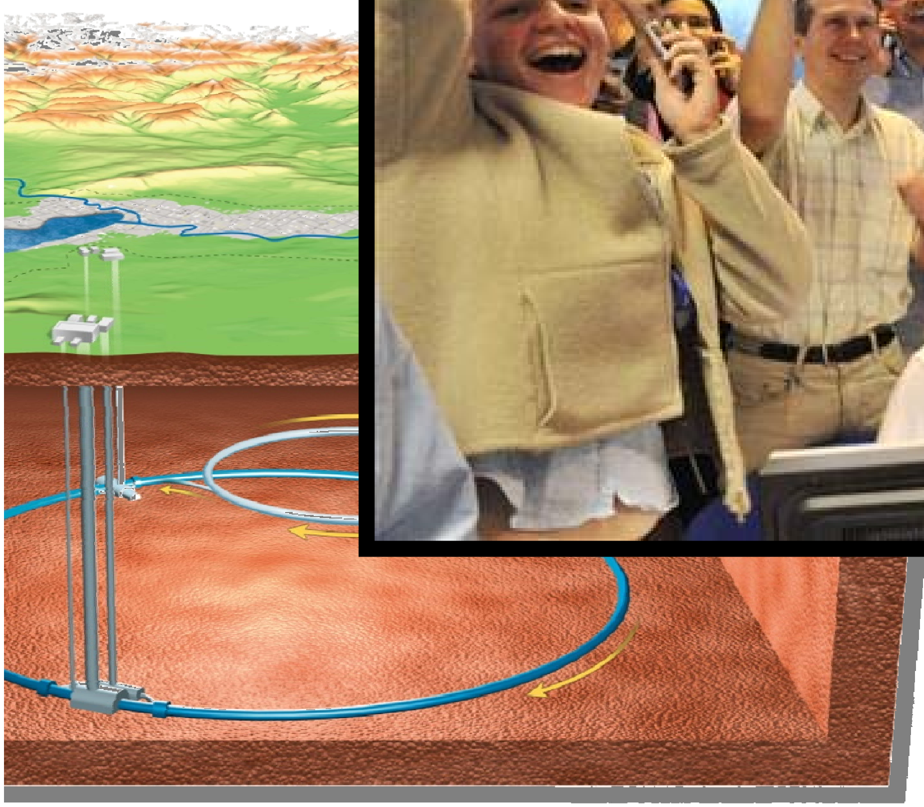




+ more terms…?

40 M / second over 10 years

At some point, 5000 people will shout:



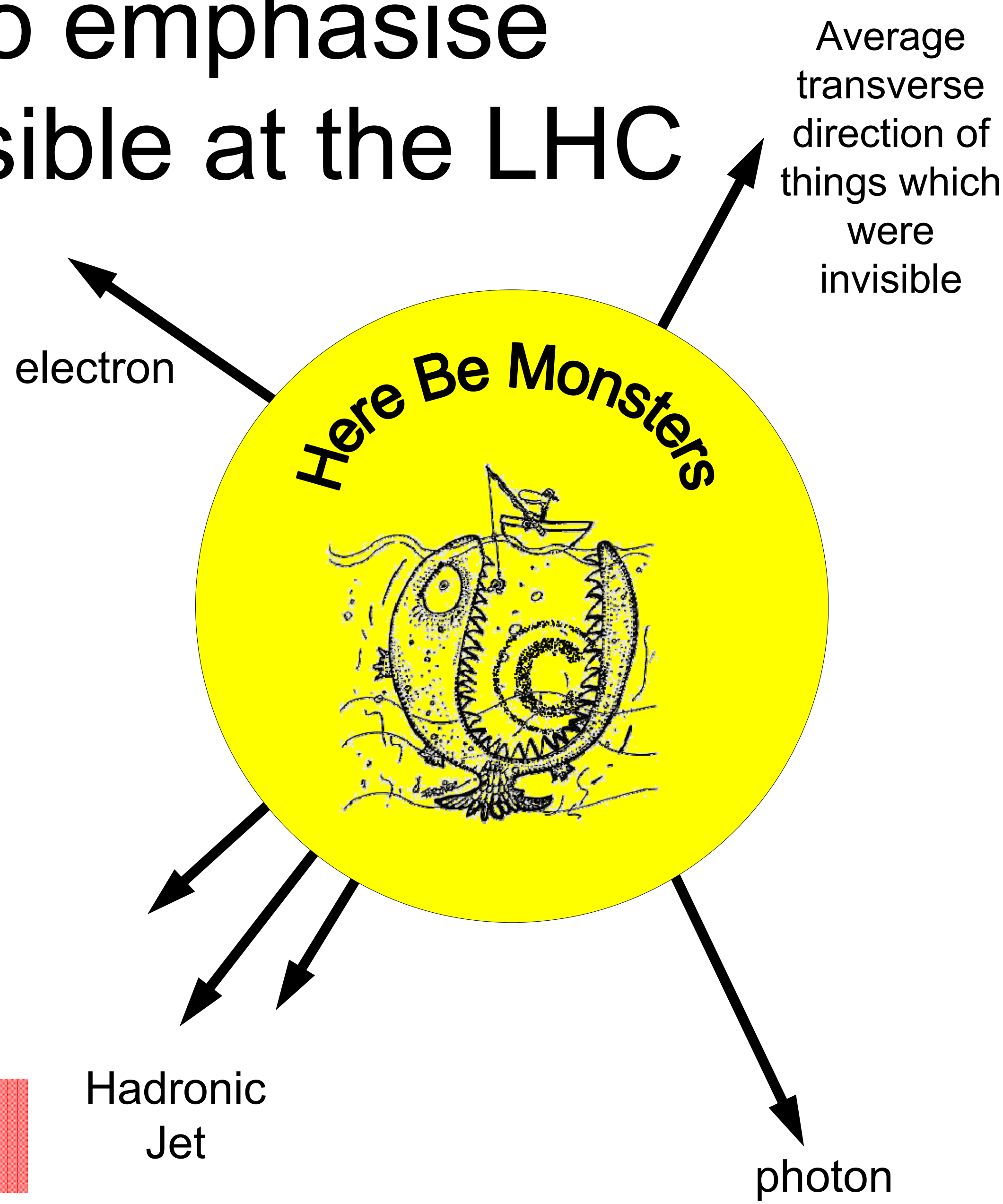
"We've found a …
[long pause]
… SOMETHING!"

A large collider of hadrons …
… not a collider of large hadrons

How hard is it to identify what was found?

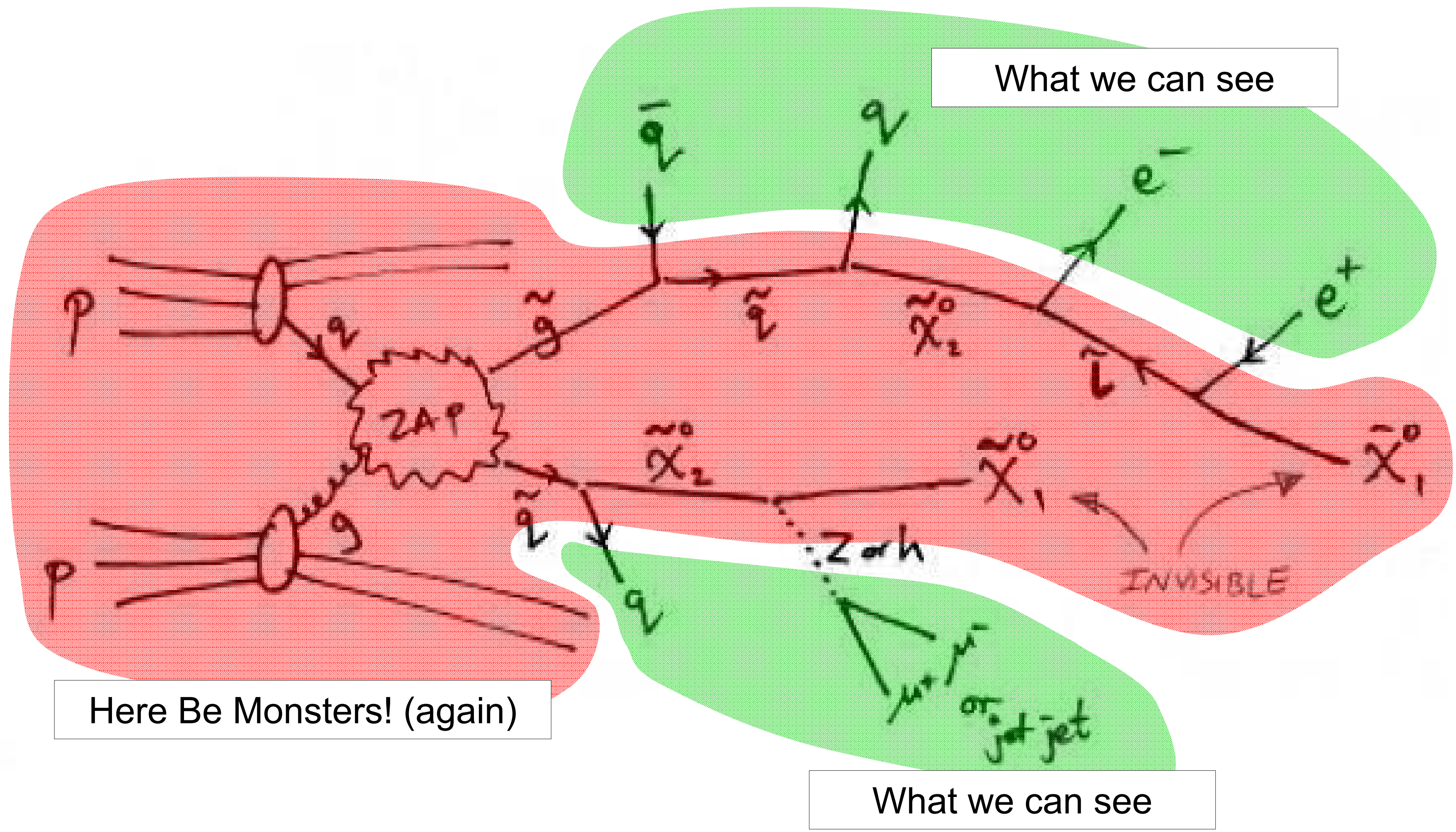
Want to emphasise what is visible at the LHC

- Distinguish the following from each other
  - Hadronic Jets,
    - B-jets (sometimes)
  - Electrons, Positrons, Muons, Anti-Muons
    - Tau leptons (sometimes)
  - Photons
- Measure Directions and Momenta of the above.
- Infer total transverse momentum of invisible particles. (eg neutrinos)



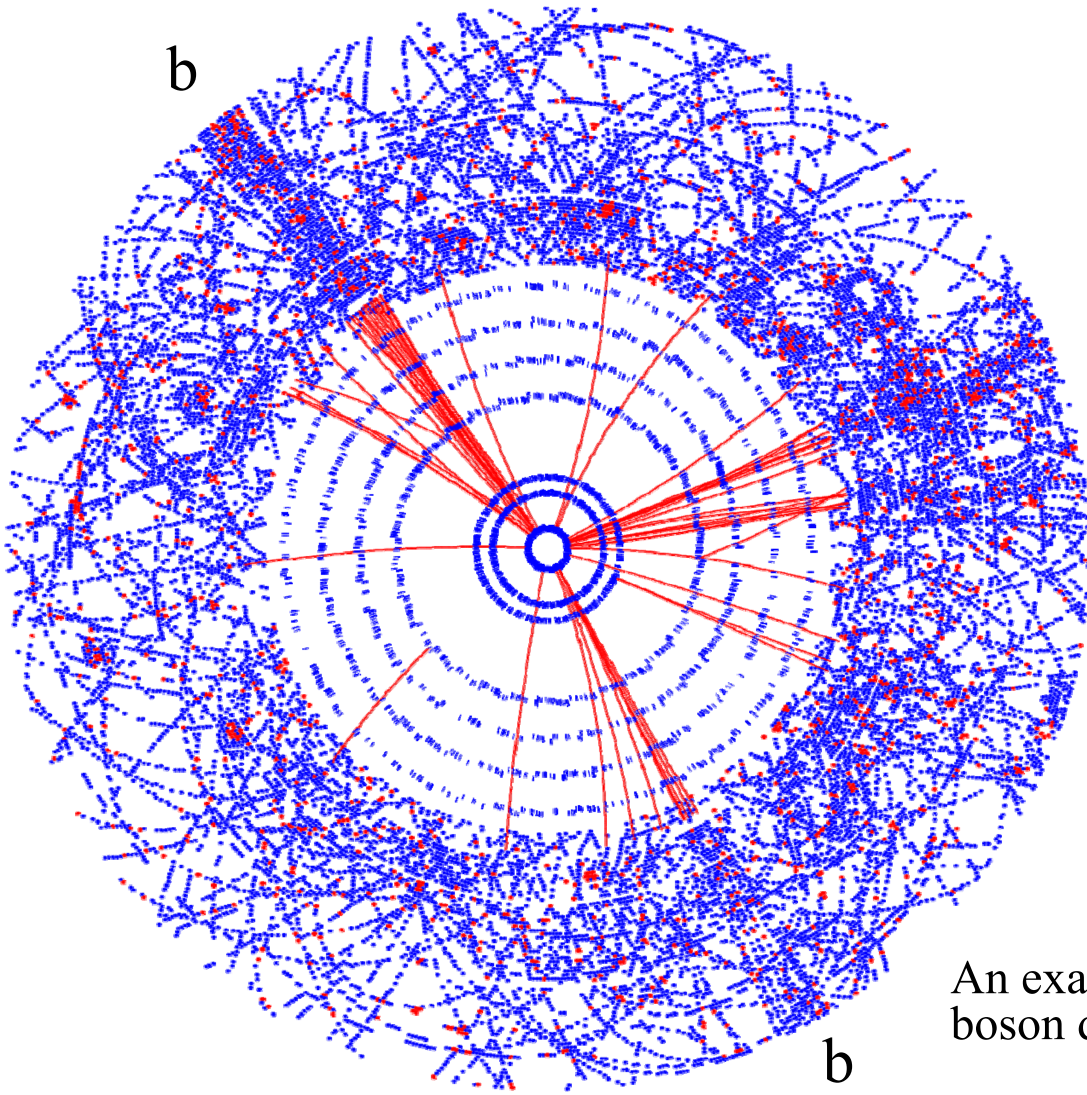
What do we NOT measure?

What might events look like?



This is the high energy physics of the 21st Century!

What events really look like scares me!



An example of an event where a higgs boson decayed to a pair of b-quarks/

Supersymmetry as Lingua Franca

Some possibilities:

- Supersymmetry
  - Minimal
  - Non-minimal
  - R-parity violating or conserving
- Extra Dimensional Models
  - Large (SM trapped on brane)
  - Universal (SM everywhere)
  - With/without small **black holes**
- "Littlest" Higgs ?
- ….

We will look mainly at supersymmetry (SUSY)

Supersymmetry!
CAUTION!

- It may exist
- It may not
- First look for deviations from Standard Model!

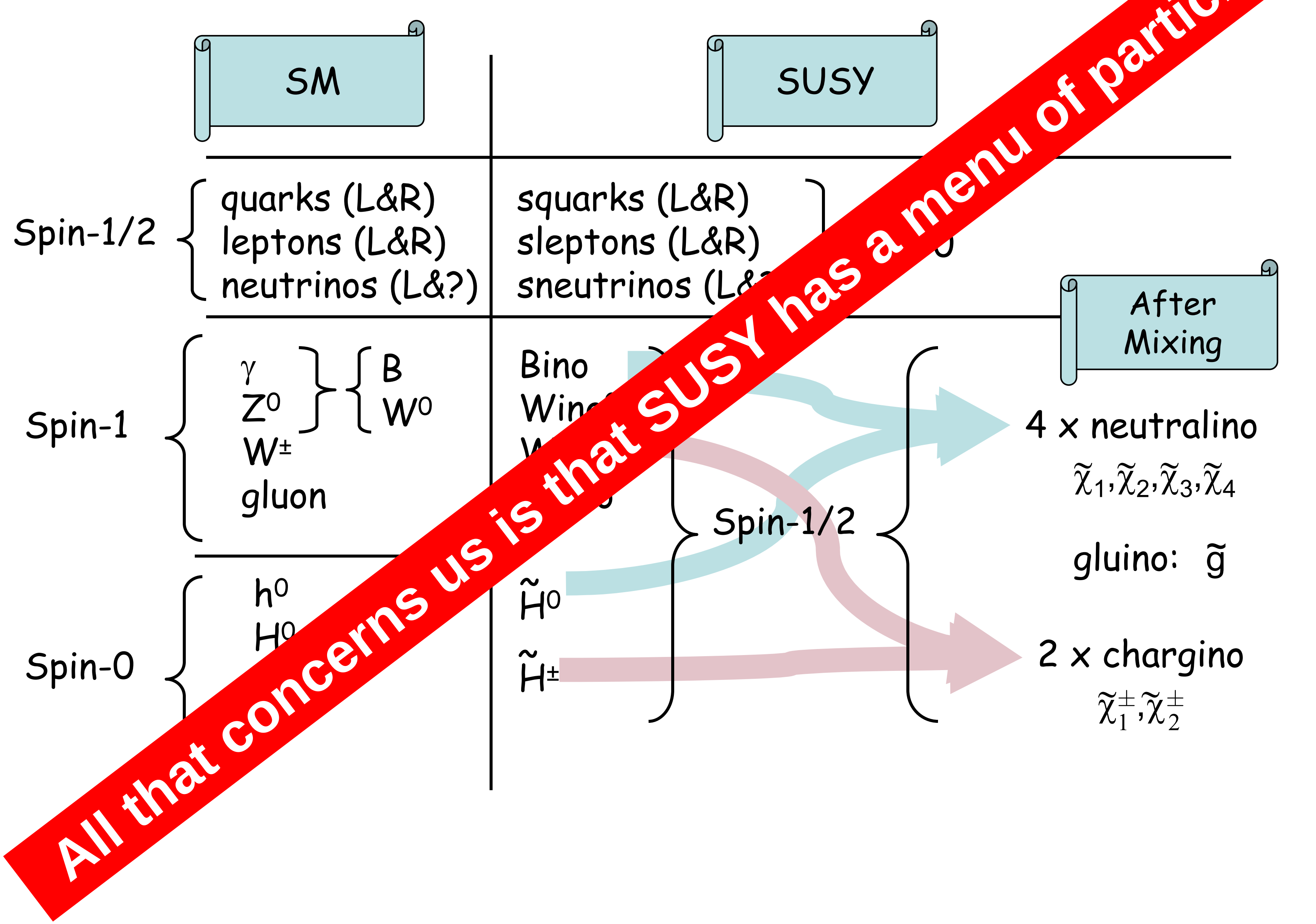
Experiment must lead theory.

Gamble:

IF DEVIATIONS ARE SEEN:

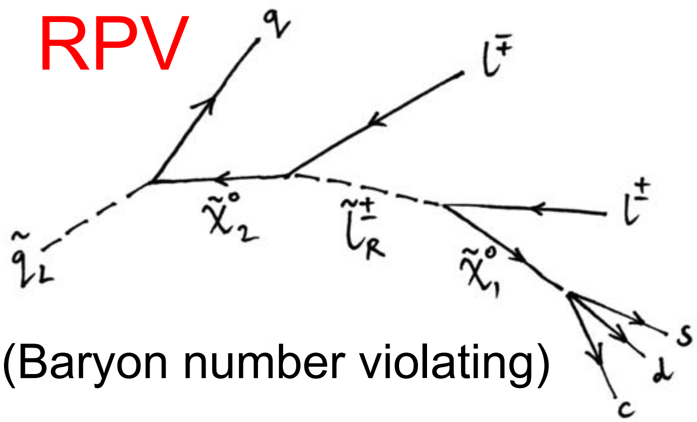
- Old techniques won't work
- New physics not simple
- Can new techniques in SUSY but can apply them elsewhere.

SUSY particle content



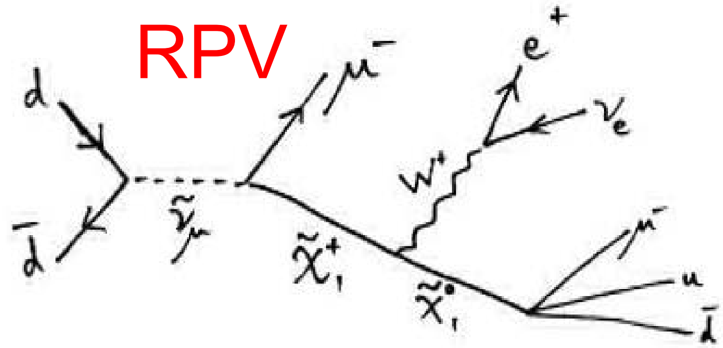
Even in SUSY many possibilities

RPV

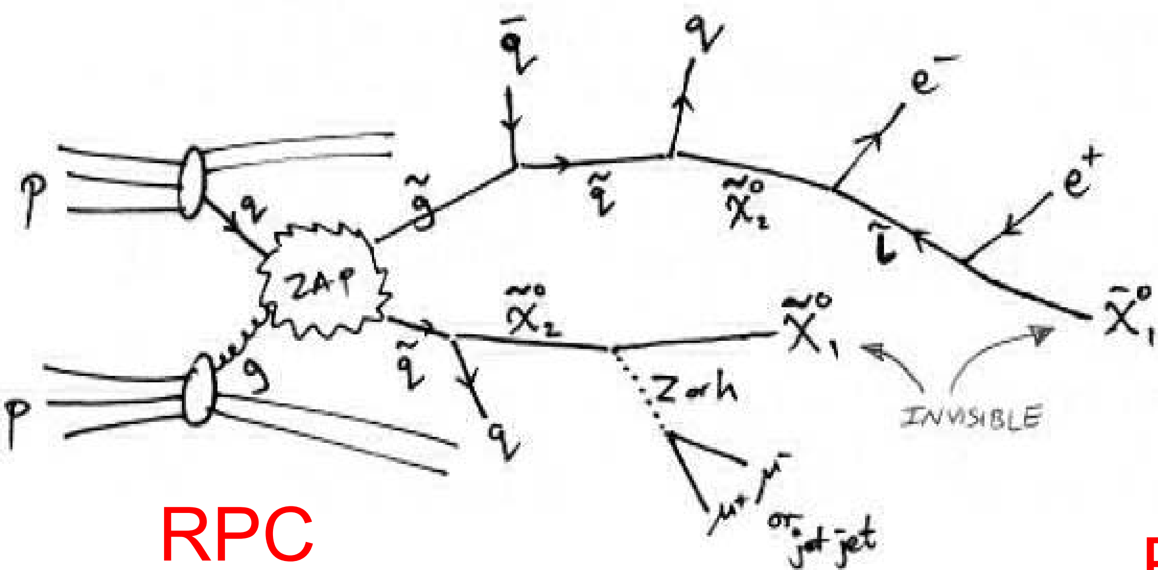


(Baryon number violating)

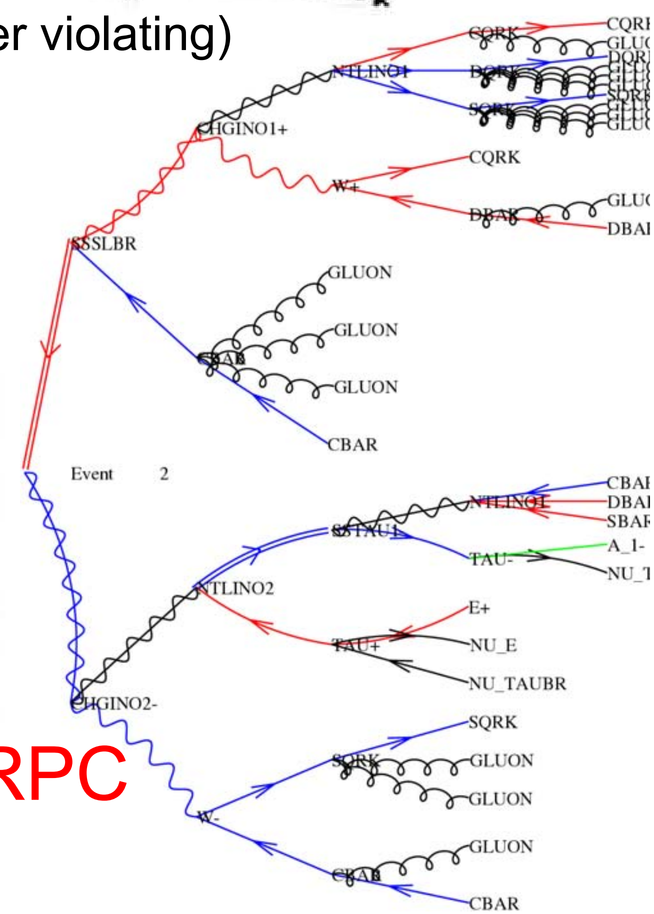
RPV



(Lepton number violating)



RPC



RPC

Do we care about masses?

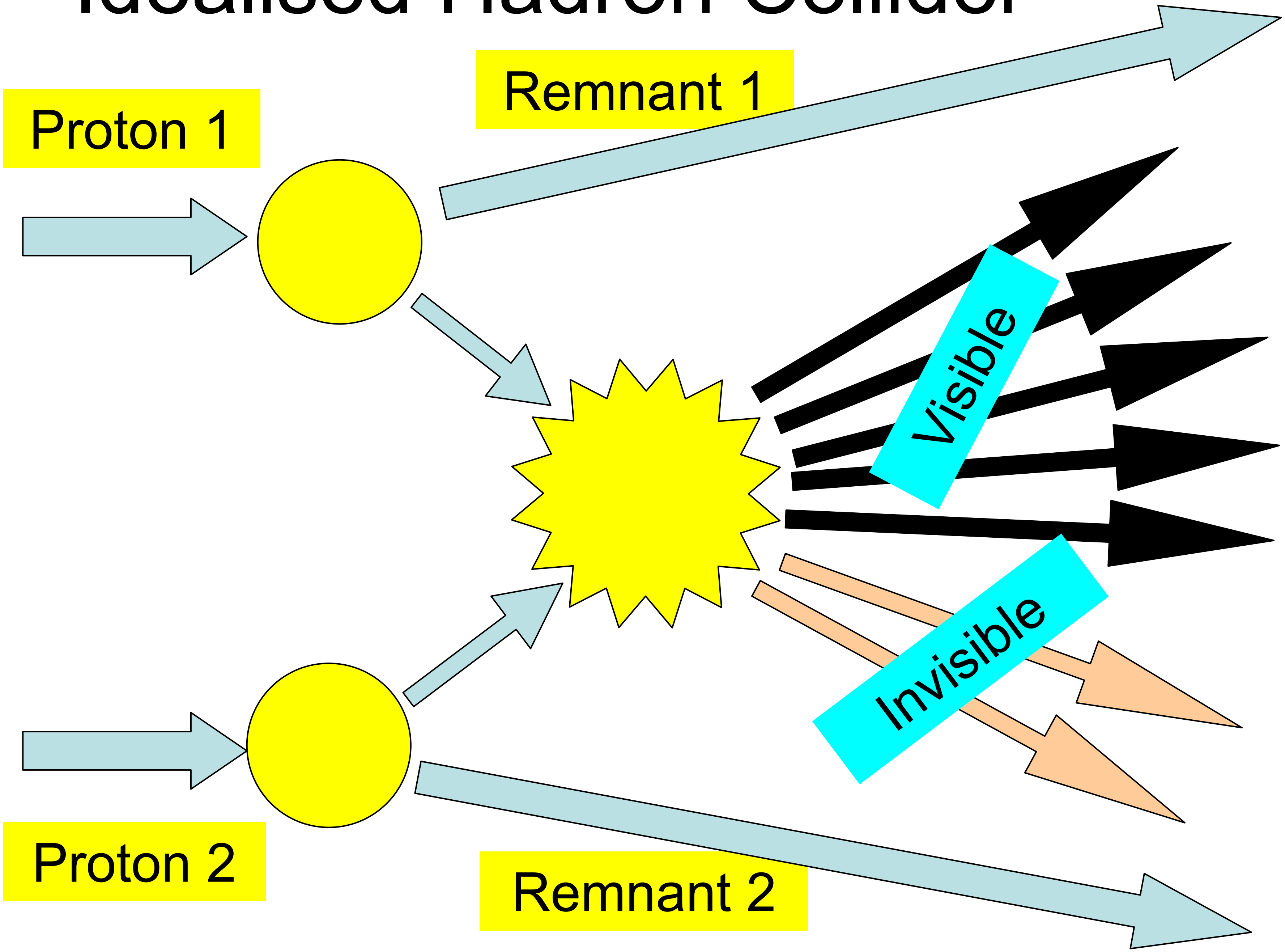
- Common **Parameter** in the Lagrangian
- Expedites **discovery** – optimal **selection**
- **Interpretation**
  (SUSY breaking mechanism,
  Geometry of Extra Dimensions)
- **Prediction** of new things
  Mass of W,Z → indirect top quark mass
  “measurement”

“mass measurement methods”

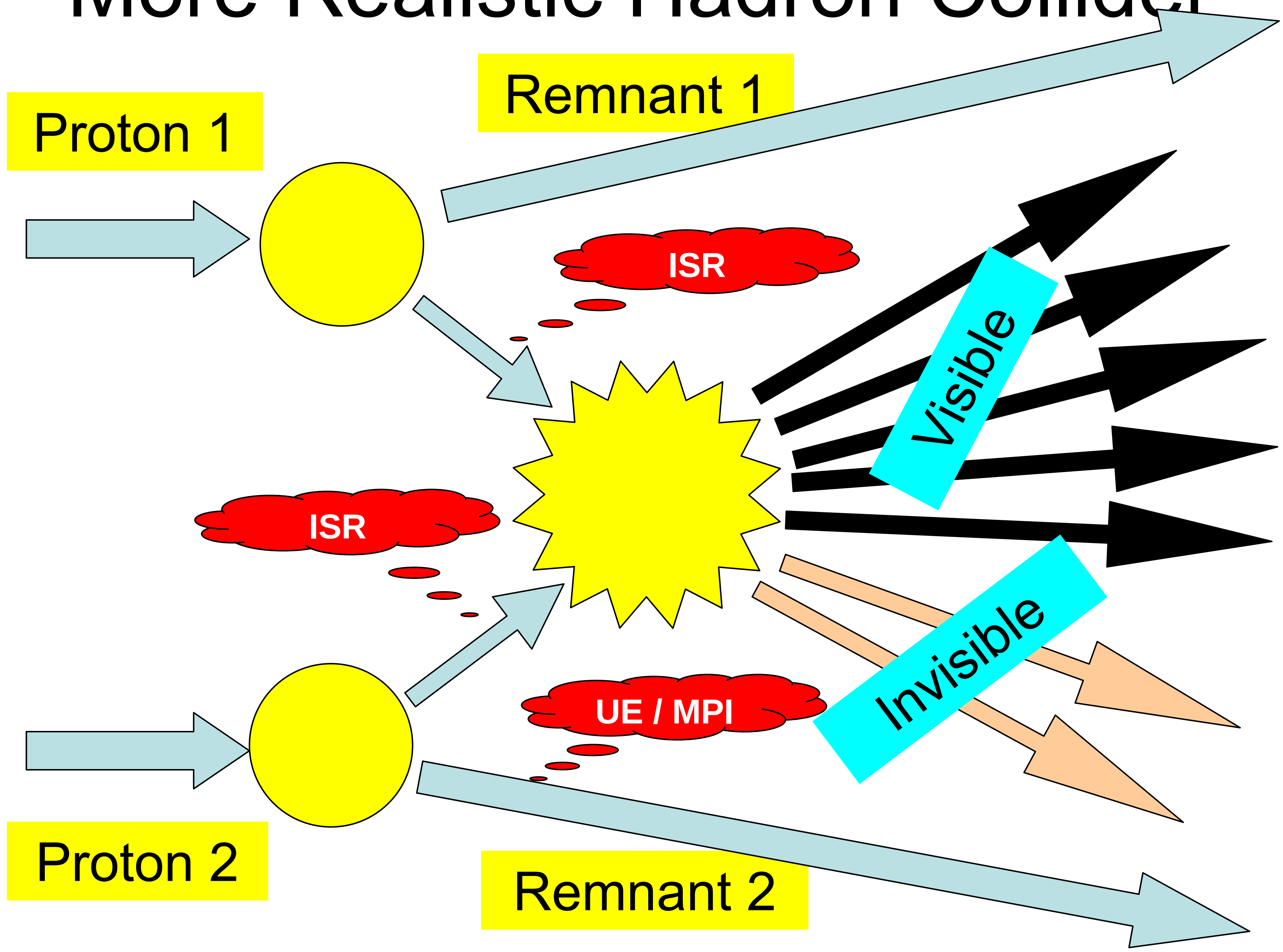
… short for …

“parameter estimation and discovery techniques”

Idealised Hadron Collider



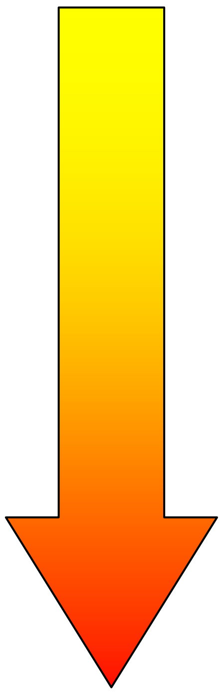
More Realistic Hadron Collider



Types of Technique

Few

assumptions



Many

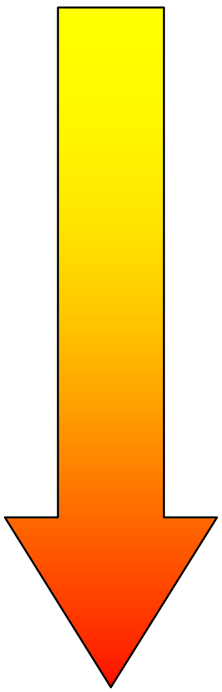
assumptions

- Missing transverse momentum
- M_eff, H_T
- s Hat Min
- M_T
- M_TGEN
- M_T2 / M_CT
- M_T2 (with “kinks”)
- M_T2 / M_CT (parallel / perp)
- M_T2 / M_CT (“sub-system”)
- “Polynomial” constraints
- Multi-event polynomial constraints
- Whole dataset variables
- Cross section
- Max Likelihood / Matrix Element

Types of Technique

Vague

conclusions



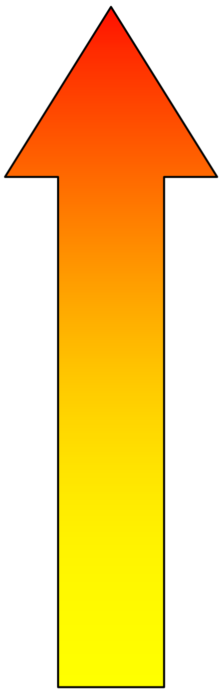
Specific

conclusions

- Missing transverse momentum
- M_eff, H_T
- s Hat Min
- M_T
- M_TGEN
- M_T2 / M_CT
- M_T2 (with “kinks”)
- M_T2 / M_CT (parallel / perp)
- M_T2 / M_CT (“sub-system”)
- “Polynomial” constraints
- Multi-event polynomial constraints
- Whole dataset variables
- Cross section
- Max Likelihood / Matrix Element

Types of Technique

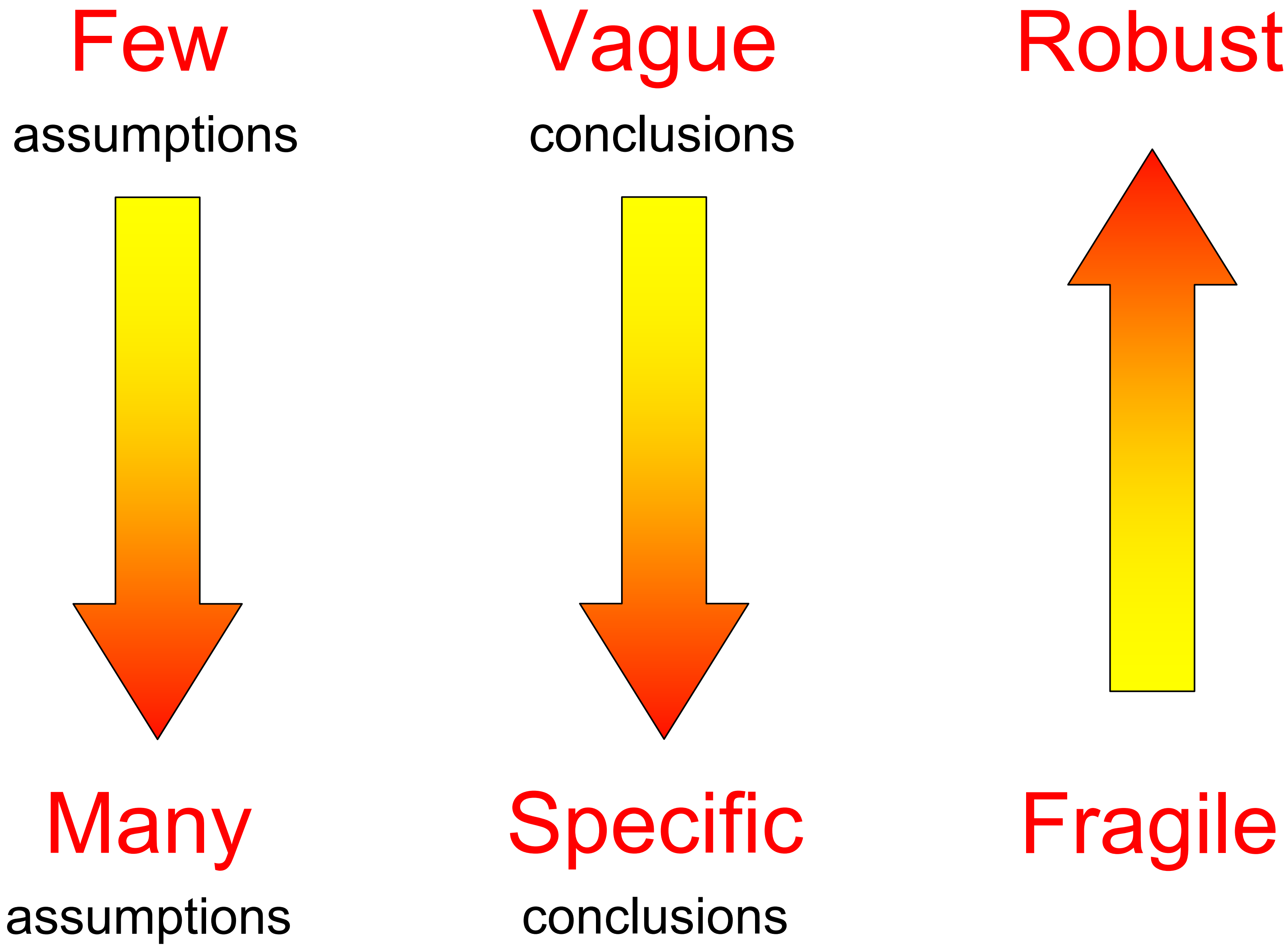
Robust



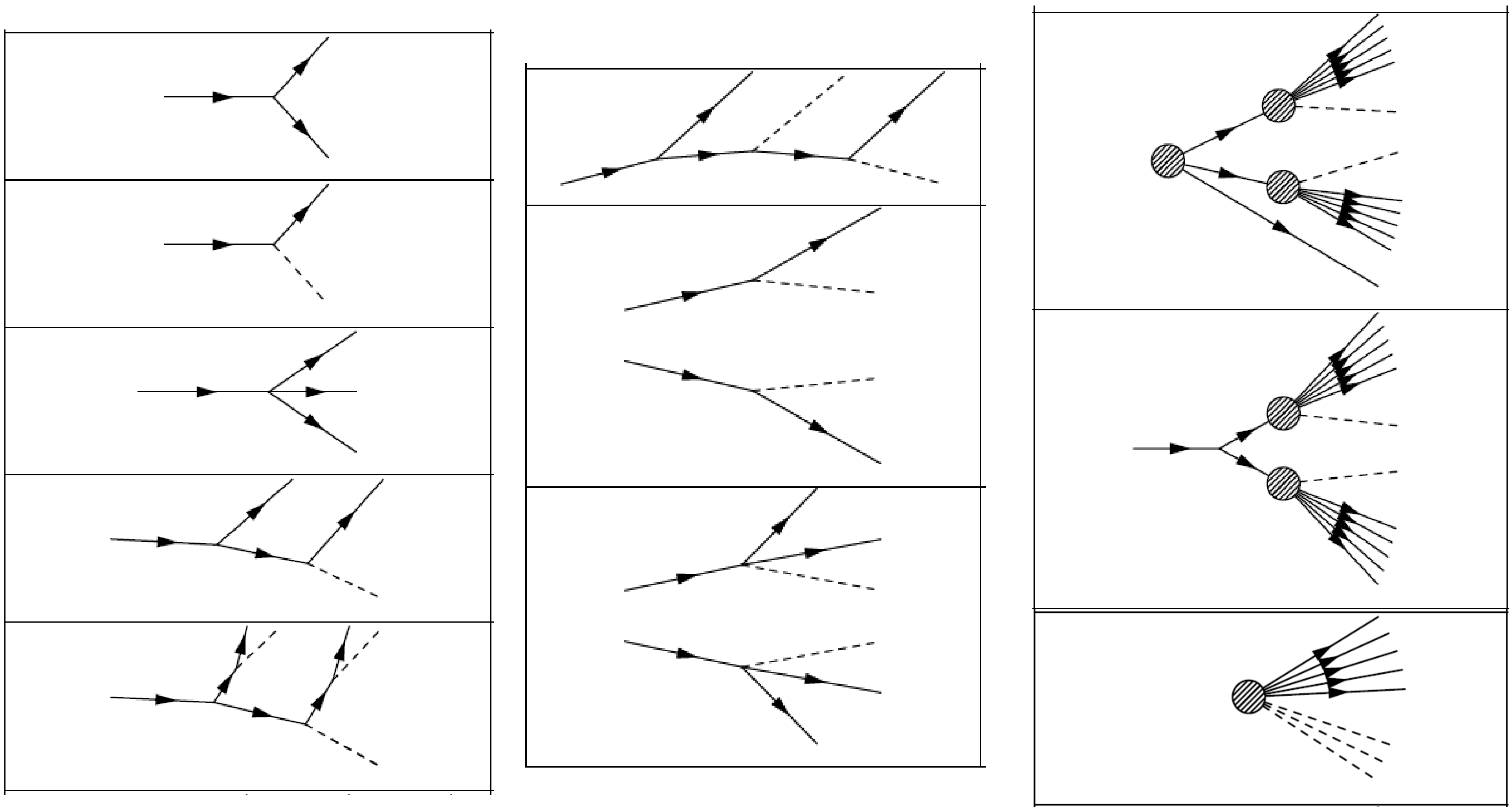
Fragile

- Missing transverse momentum
- M_eff, H_T
- s Hat Min
- M_T
- M_TGEN
- M_T2 / M_CT
- M_T2 (with “kinks”)
- M_T2 / M_CT (parallel / perp)
- M_T2 / M_CT (“sub-system”)
- “Polynomial” constraints
- Multi-event polynomial constraints
- Whole dataset variables
- Cross section
- Max Likelihood / Matrix Element

Interpretation : the balance of benefits

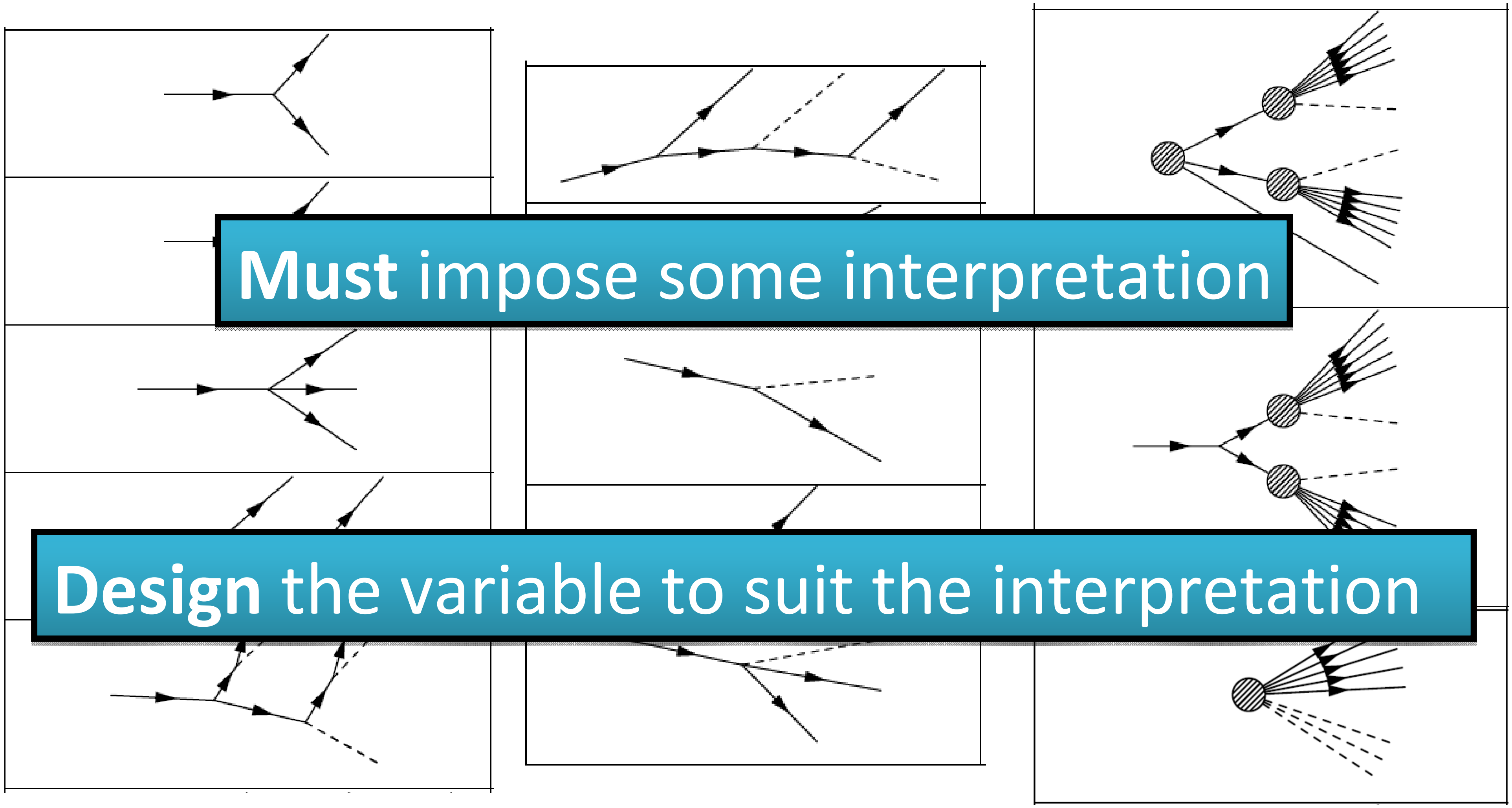


Topology / hypothesis



Full index in [arXiv:1004.2732]

Topology / hypothesis

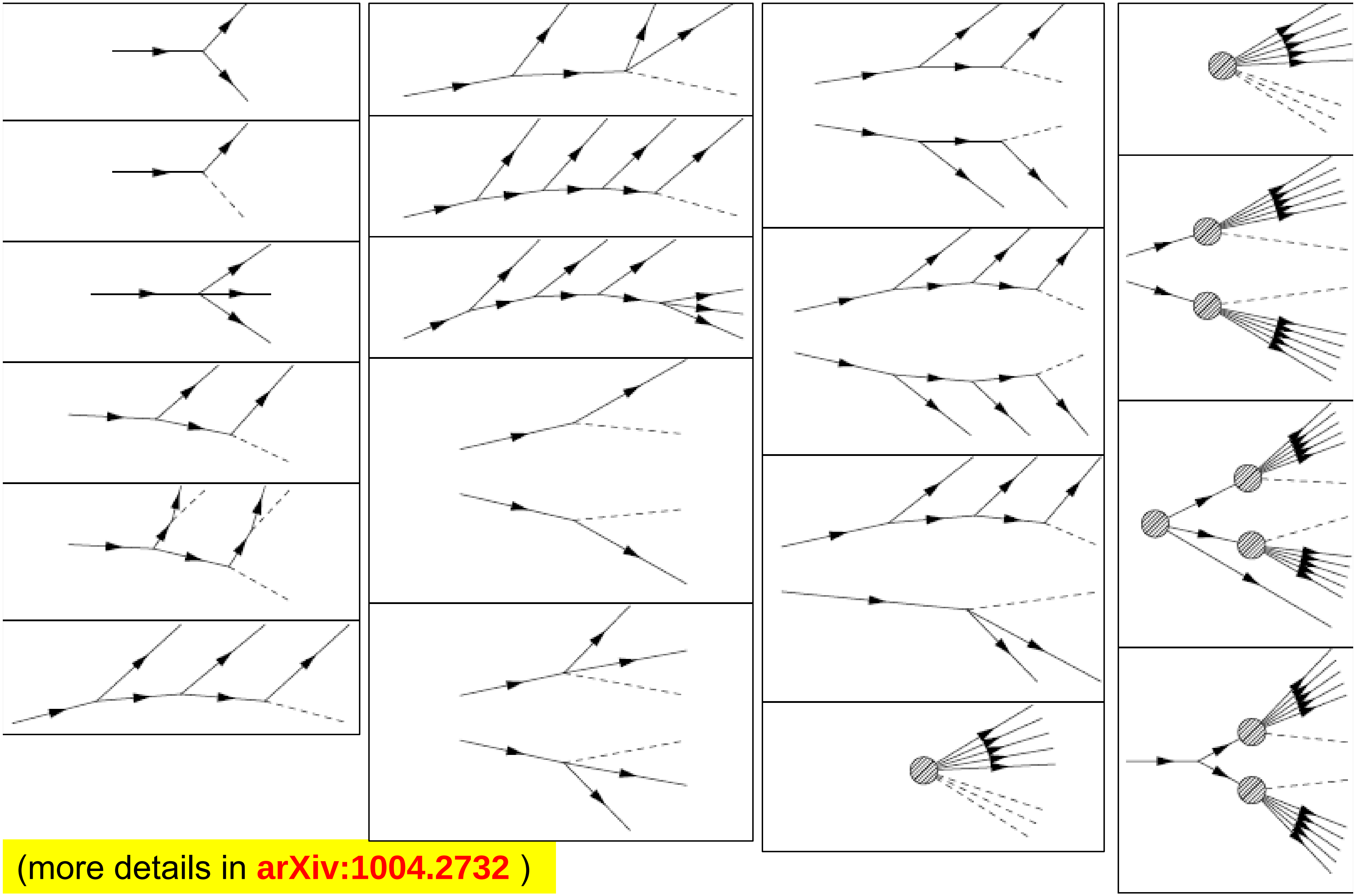


Must impose some interpretation

Design the variable to suit the interpretation

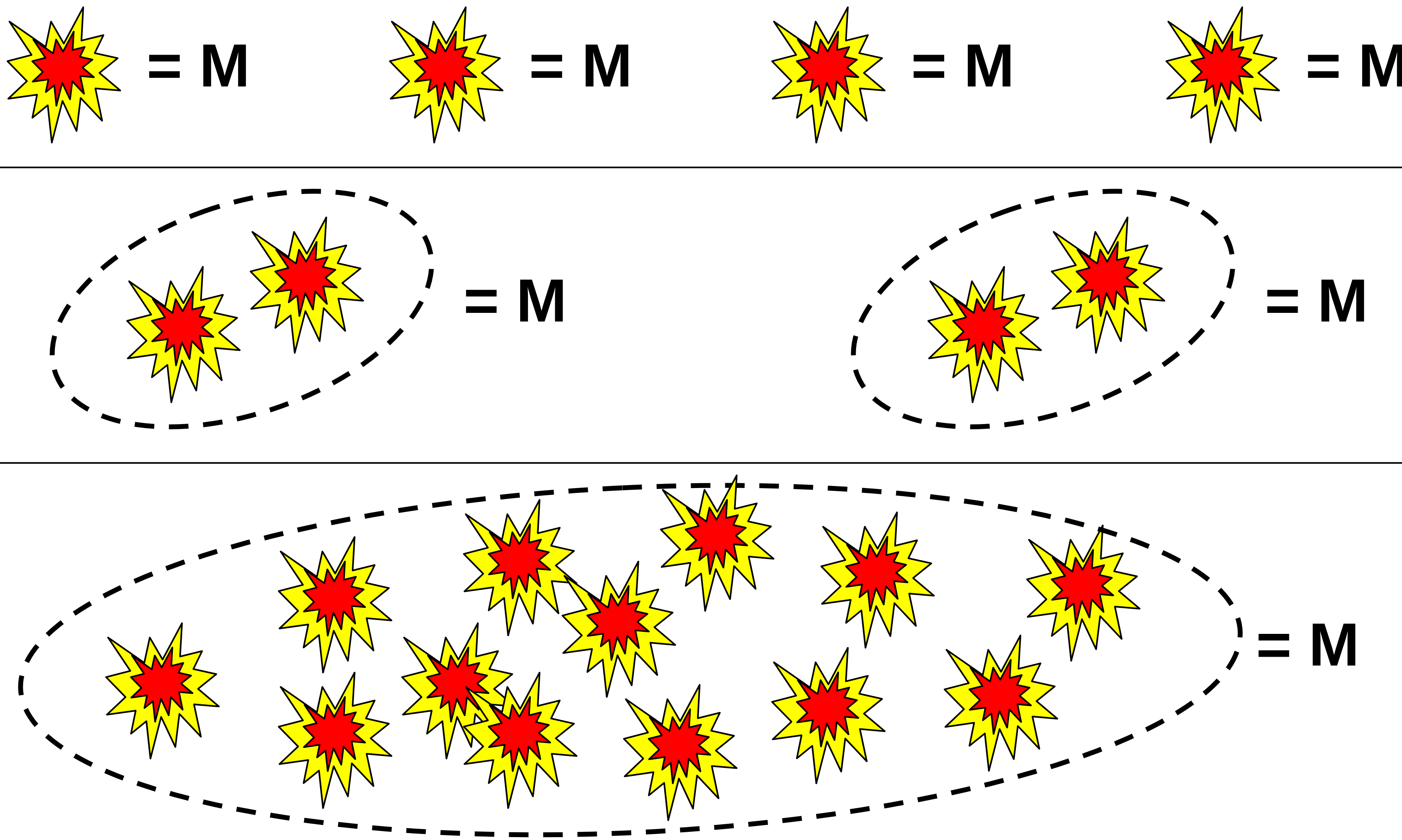
Full index in arXiv:1004.2732

Lectures are roughly ordered from **simple** to **complicated** …

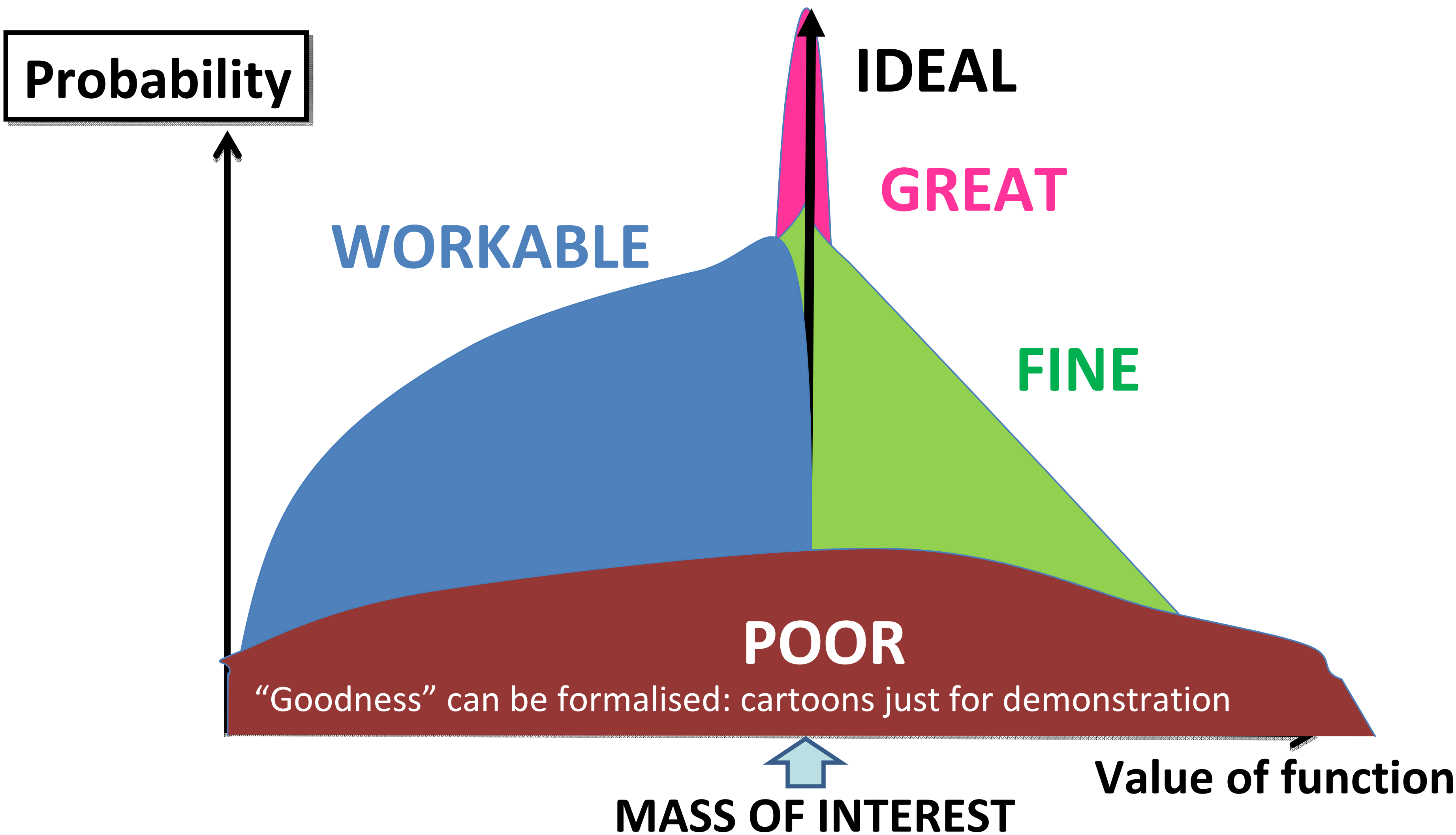


(more details in **arXiv:1004.2732**)

… and from **few** events required, to **many** events required ….



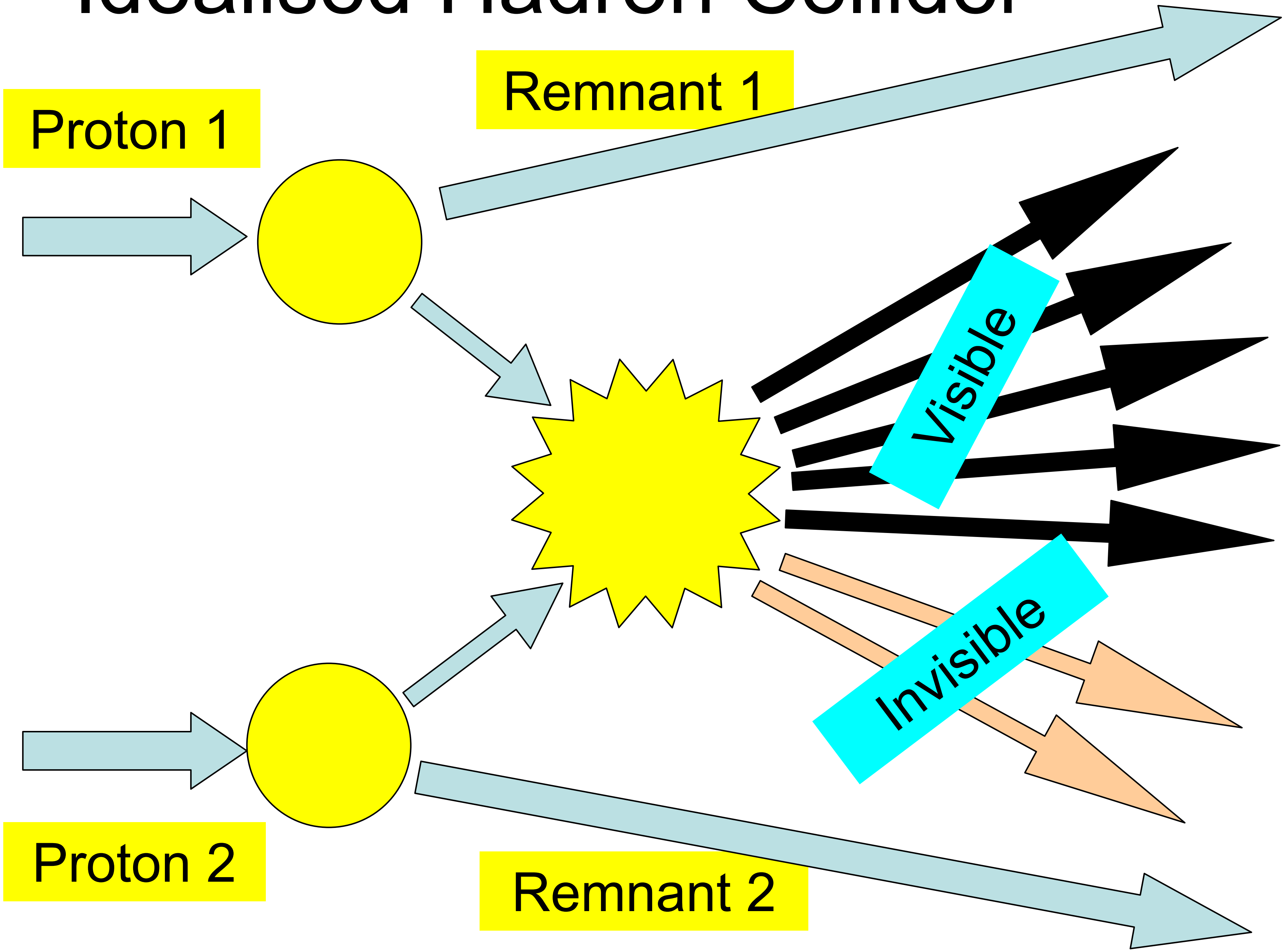
Good vs poor variables



Few assumptions,
Vague Conclusions.

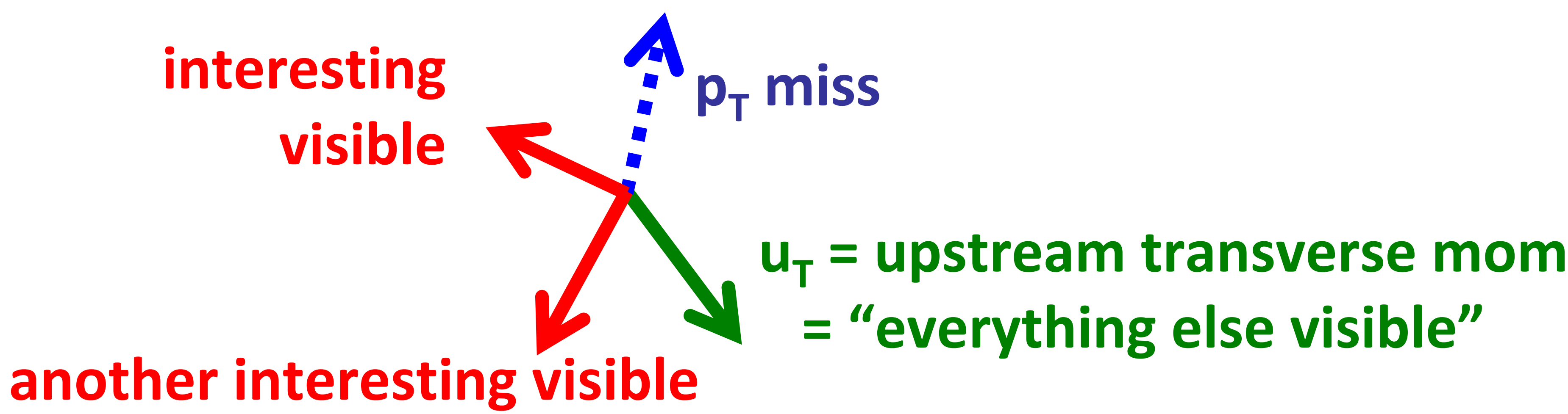
Anything with sensitivity
to mass scales.

Idealised Hadron Collider



Missing transverse momentum

$$\vec{\mathbf{p}}_T^{\,miss} = -\sum_i \vec{\mathbf{p}}_T^{\,i^{th} visible}$$



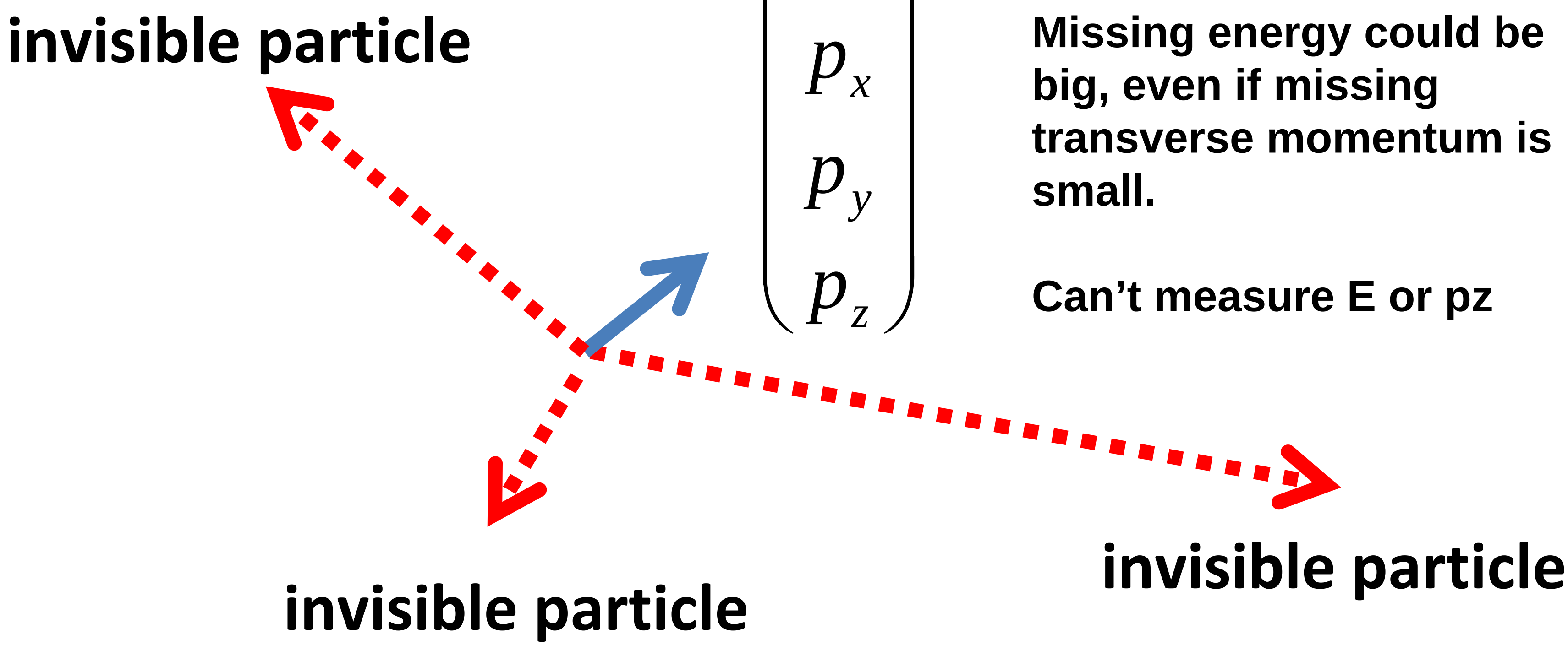
Events have missing energy too, and it’s not missing momentum

$$\begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix}$$

Total 4-momentum of invisbiles.

Missing energy could be big, even if missing transverse momentum is small.

Can’t measure E or pz



Rant about missing transverse momentum

- eTmiss – aaargh
- MET – AAAARGH
- missing energy – AAAAAARRRGH

- Blame LEP?
- Calorimeter apologists?

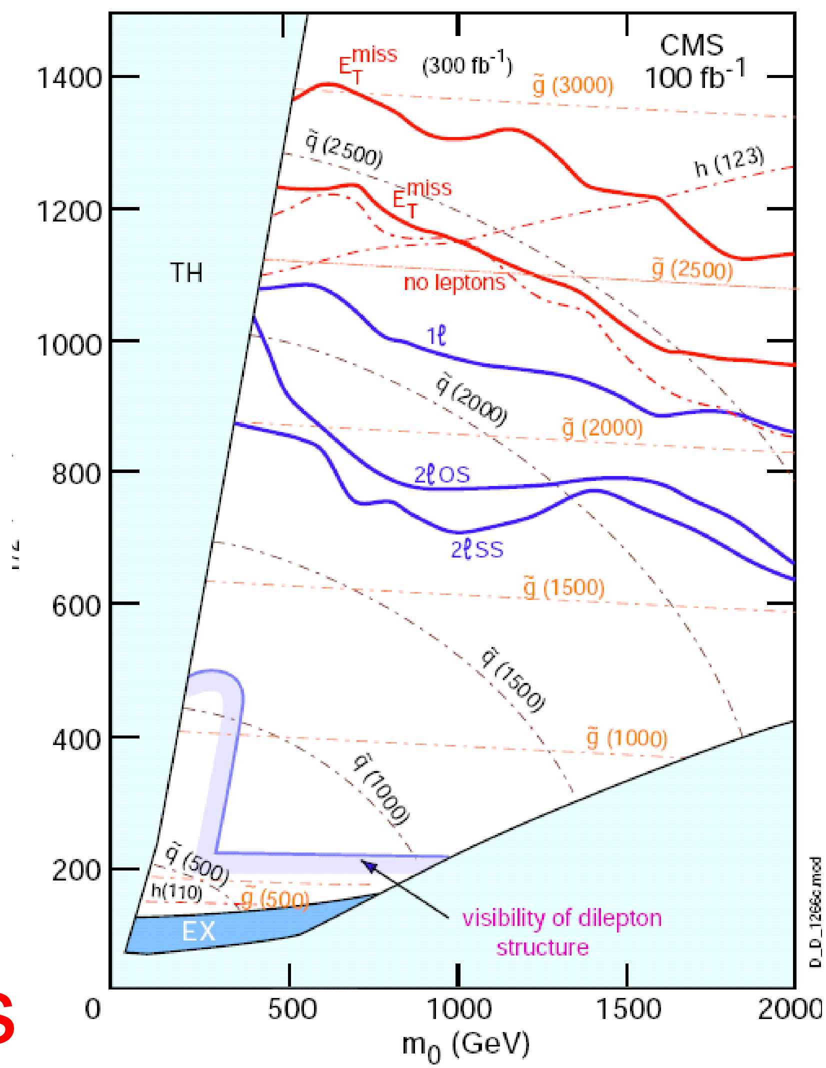
- alphaT

Main EASY signatures are:

- Lots of missing pt
- Lots of leptons
- Lots of jets

Just Count Events!

- $\not\!\!E_T \Leftarrow$ Dominant signature
- $\not\!\!E_T$ with lepton veto
- One lepton
- Two leptons Same Sign (SS)
- Two leptons Opposite Sign (OS)



Simply counting events

Perhaps

simple = best ?

The End

Can attempt to spot susy by counting “strange” events …

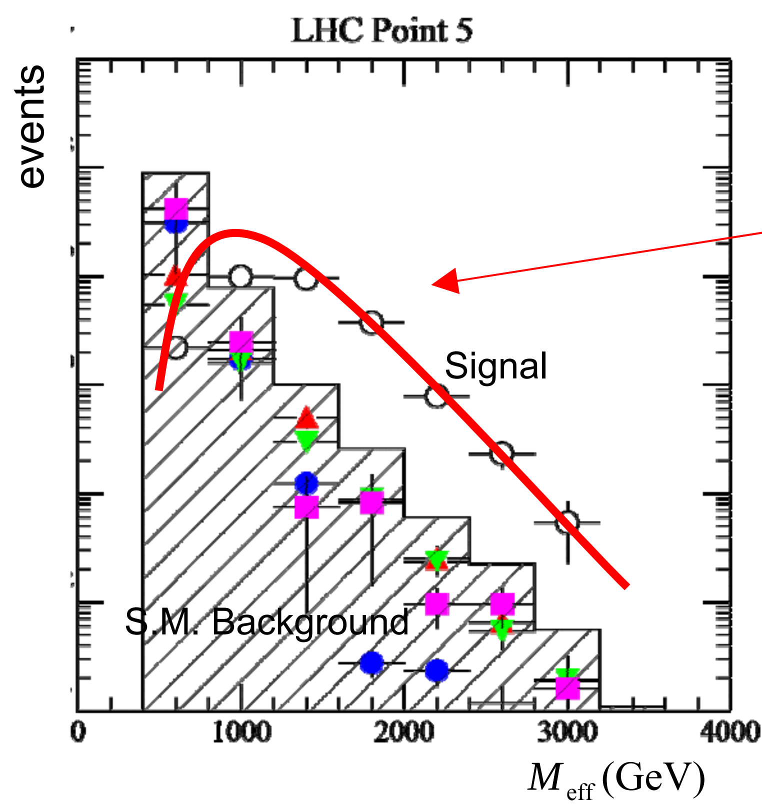
… but can we say anything concrete about a mass scale?

Next example still low-tech ….

Effective mass

What you histogram:

$$M_{\mathrm{eff}} = \mathbf{p}_{\mathrm{T}}^{\mathrm{missing}} + \sum_i \left| \mathbf{p}_{\mathrm{T}}^{\mathrm{jet}_i} \right|$$



You look for position of this peak and call it MeffPeak

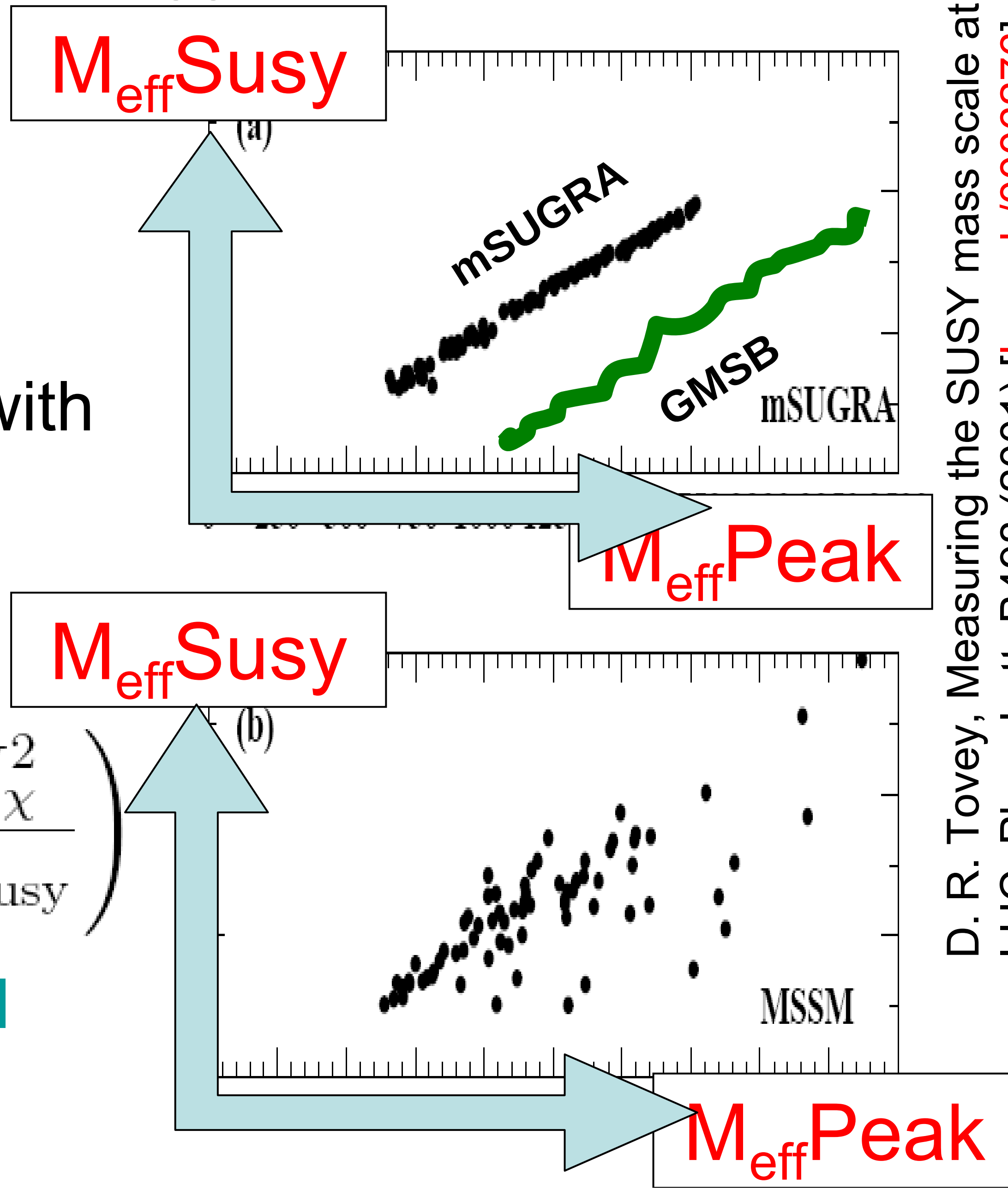
Call it Meff and Mest too (just to confuse people!)

What might Meff peak position correlate with?

Define SUSY scale:

$$M_{\text{susy}}^{\text{eff}} = \left(M_{\text{susy}} - \frac{M_\chi^2}{M_{\text{susy}}} \right), \text{ with } M_{\text{SUSY}} \equiv \frac{\Sigma_i \, M_i \sigma_i}{\Sigma_i \, \sigma_i}$$

$M_{effPeak}$ / M_{est} example



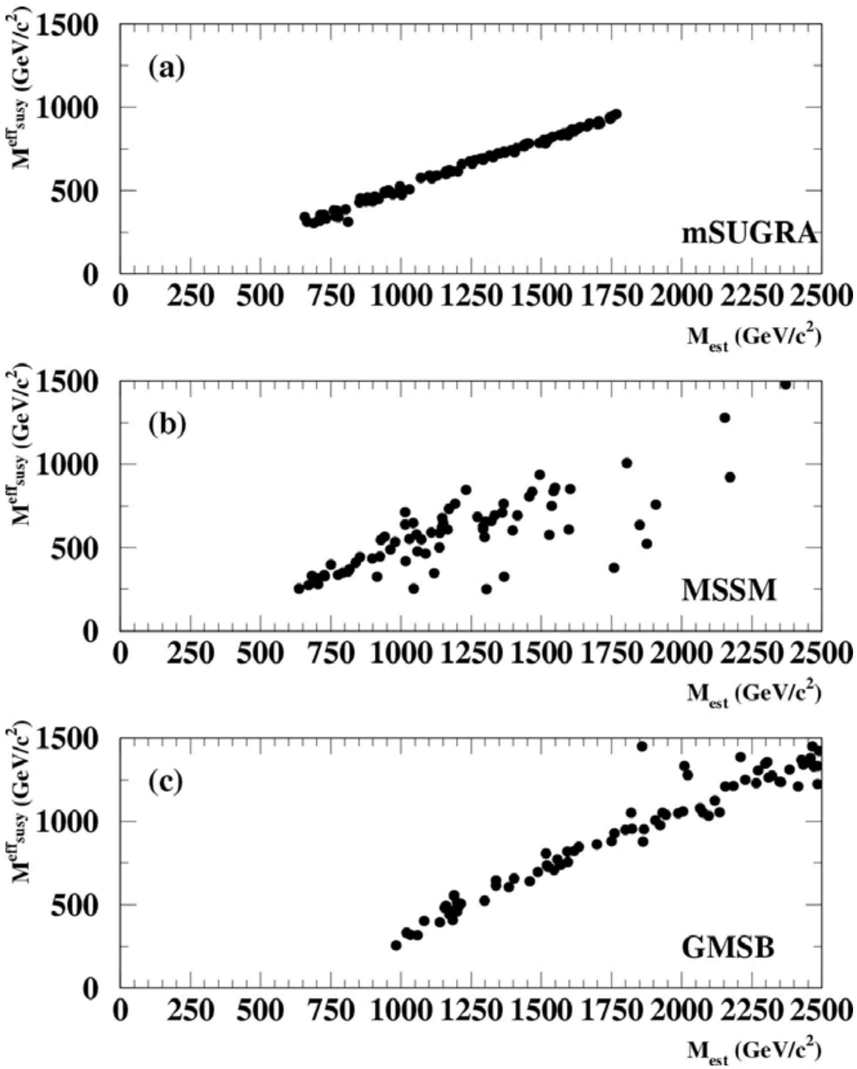
Observable M_{eff}Peak sometimes correlates with property of model M_{eff} defined by

$$M^{\mathrm{eff}}_{\mathrm{susy}} = \left(M_{\mathrm{susy}} - \frac{M_\chi^2}{M_{\mathrm{susy}}} \right)$$

but correlation is model dependent

D. R. Tovey, Measuring the SUSY mass scale at the LHC, Phys. Lett. B498 (2001) [hep-ph/0006276]

Correlations between MeffPeak position and MeffSusy



(Tovey)

M_Hotpants ..

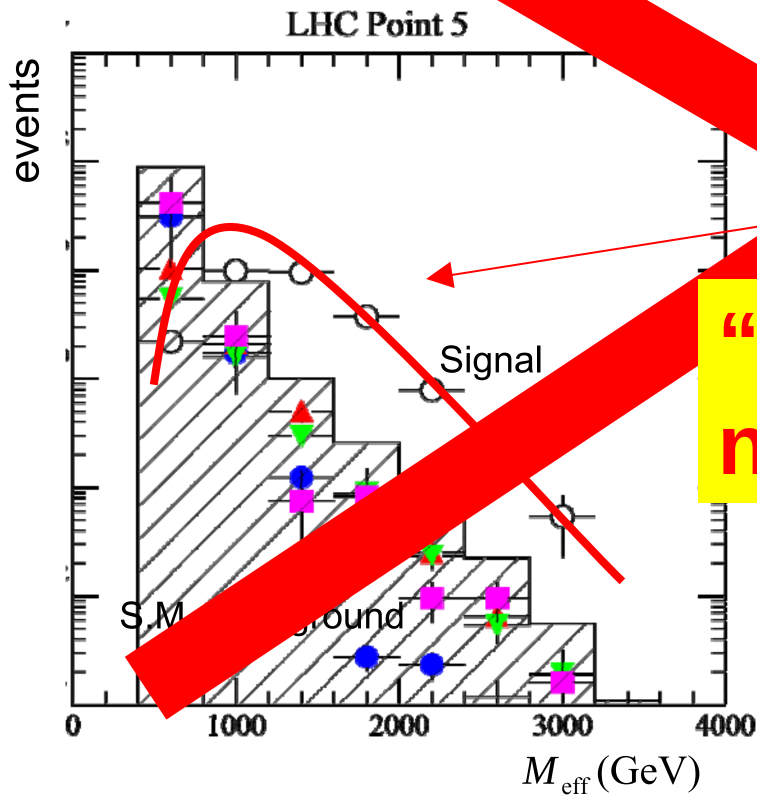
- Can encourage tendency to

- Create your variable, then see what might be able to measure. Oops.

Effective mass

W t you
histog :

$$M_{\mathrm{eff}} = \mathbf{p}_{\mathrm{T}}^{\mathrm{missing}} + \sum_i |\mathbf{p}_{\mathrm{T}}^{i}|$$



You look for position of this peak and call

“It is neither a mass, nor effective” - KM

Call it Meff too (just to confuse people!)

Meff is not alone …

Murky underworld of badly formed twins known variously as HT … the less said the better

$$H_T = E_{T(2)} + E_{T(3)} + E_{T(4)} + |\not{p}_T|$$

$$E_T = E \sin\theta$$

See arXiv:1105.2977 for why sinTheta brings on nightmares.

(There are **no standard definitions** of H_T authors differ in how many jets are used, whether PT miss should be added etc.)

All have *some* sensitivity to the overall mass scales involved, but interpretation requires a model and more assumptions.

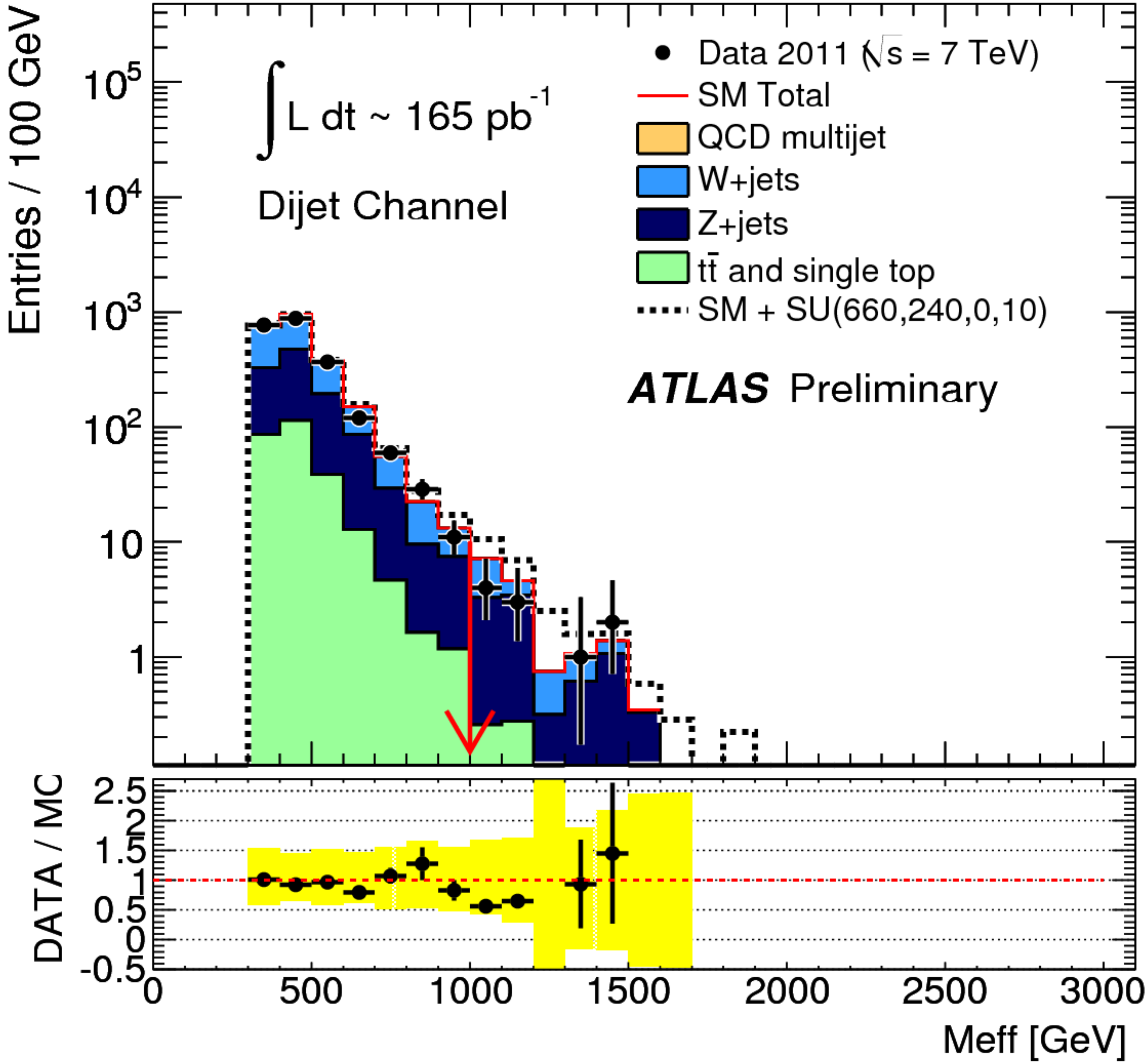
Why are we adding transverse momenta?

- Why not multiply?
  (or add logs)?

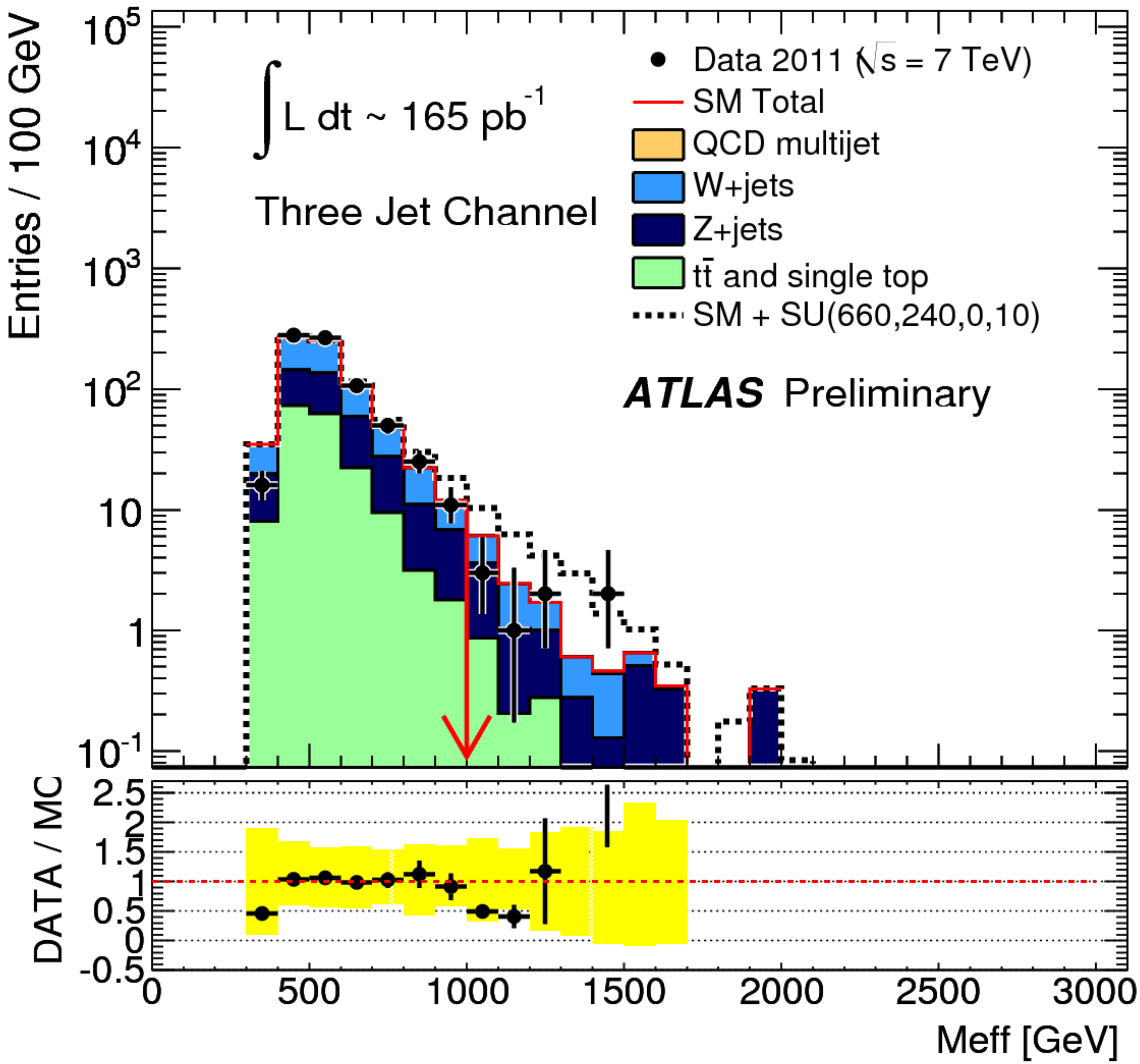
$$M_{happy} = \left(\prod_{i=1}^{n} \mathbf{p}_T^i \right)^{\frac{1}{n}}$$

- Serious proposal to use Meff^2-(u_T)^2 in arXiv:1105.2977
- Why are the signs the same? Why equal weights? Silly?
- How many years would it take ATLAS/CMS to discover the invariant mass for Z -> a b ?

$$\mathrm{M}^2 = \left(\sqrt{m_a^2 + a_x^2 + a_y^2 + a_z^2} + \sqrt{m_b^2 + b_x^2 + b_y^2 + b_z^2} \right)^2$$
$$- \left(a_\mathrm{x} + b_\mathrm{x}\right)^2 - \left(a_\mathrm{y} + b_\mathrm{y}\right)^2 - \left(a_\mathrm{z} + b_\mathrm{z}\right)^2$$



Latest ATLAS 0-lepton, jets, missing transverse momentum data.

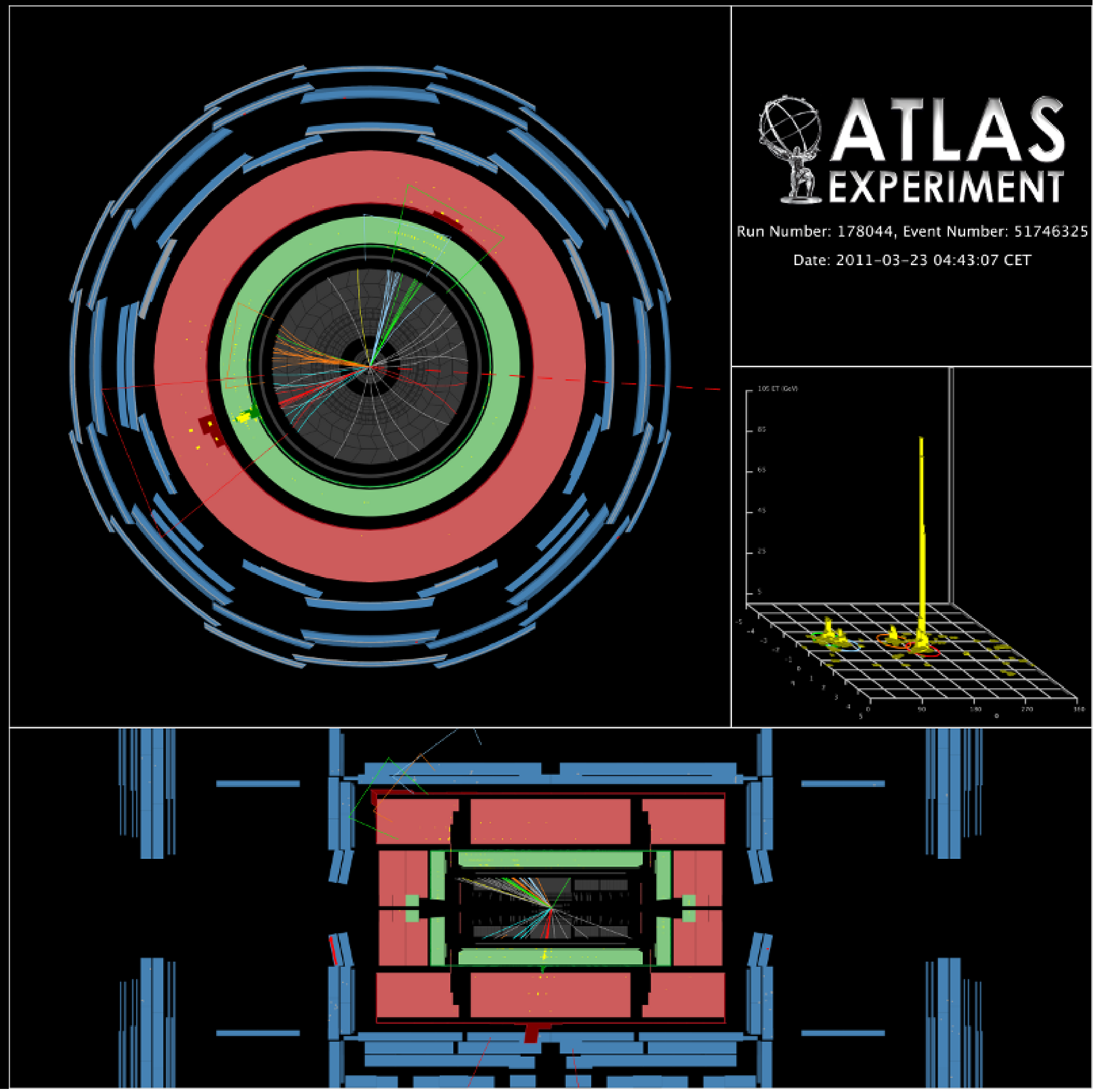


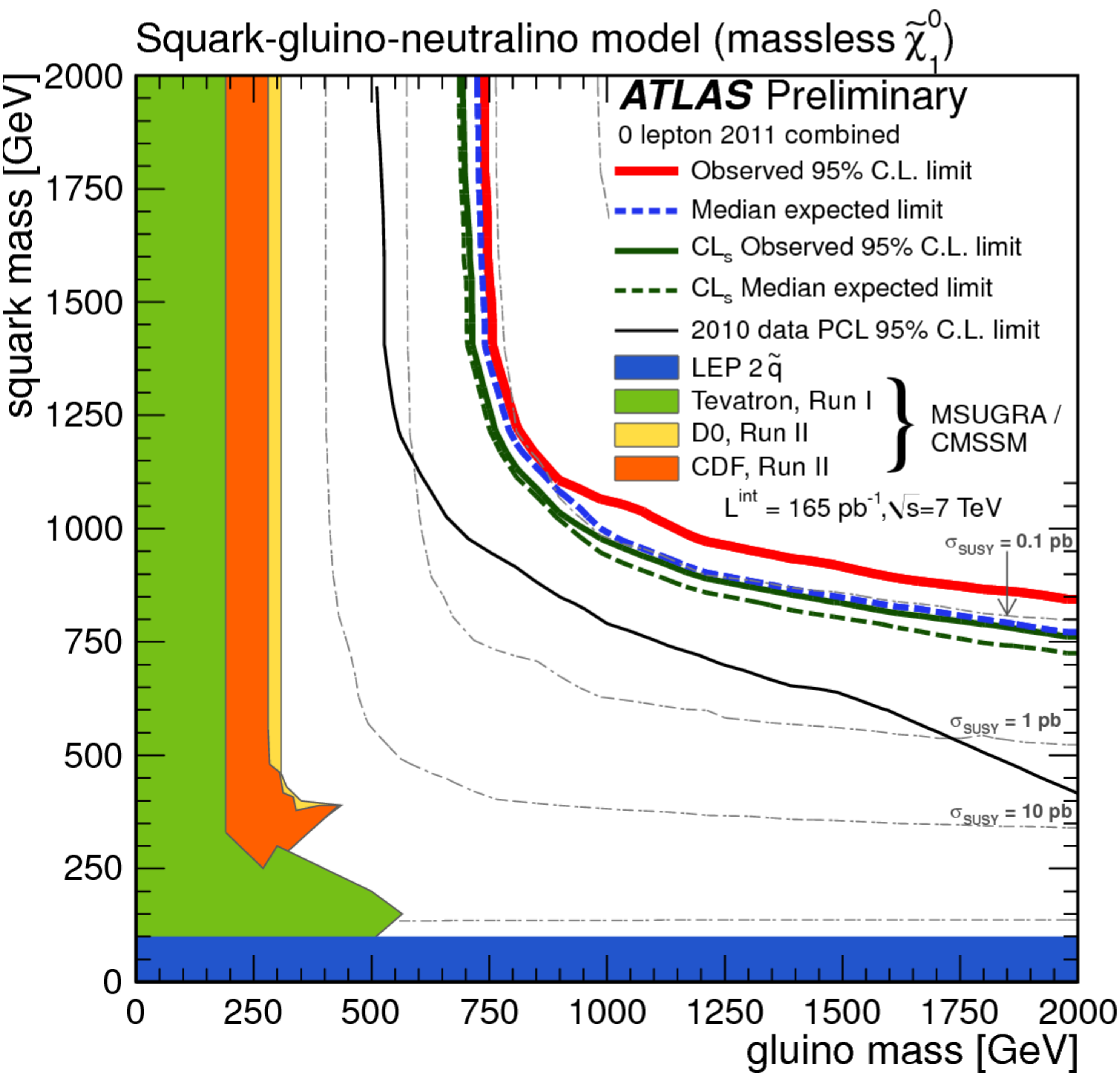
Latest ATLAS 0-lepton, jets, missing transverse momentum data.

Highest Meff event so far ….

The highest Meff in any (supposedly “clean”) ATLAS event is 1548 GeV

- calculated from four jets with pts:
  - 636 GeV
  - 189 GeV
  - 96 GeV
  - 81 GeV
- 547 GeV of missing transverse momentum.





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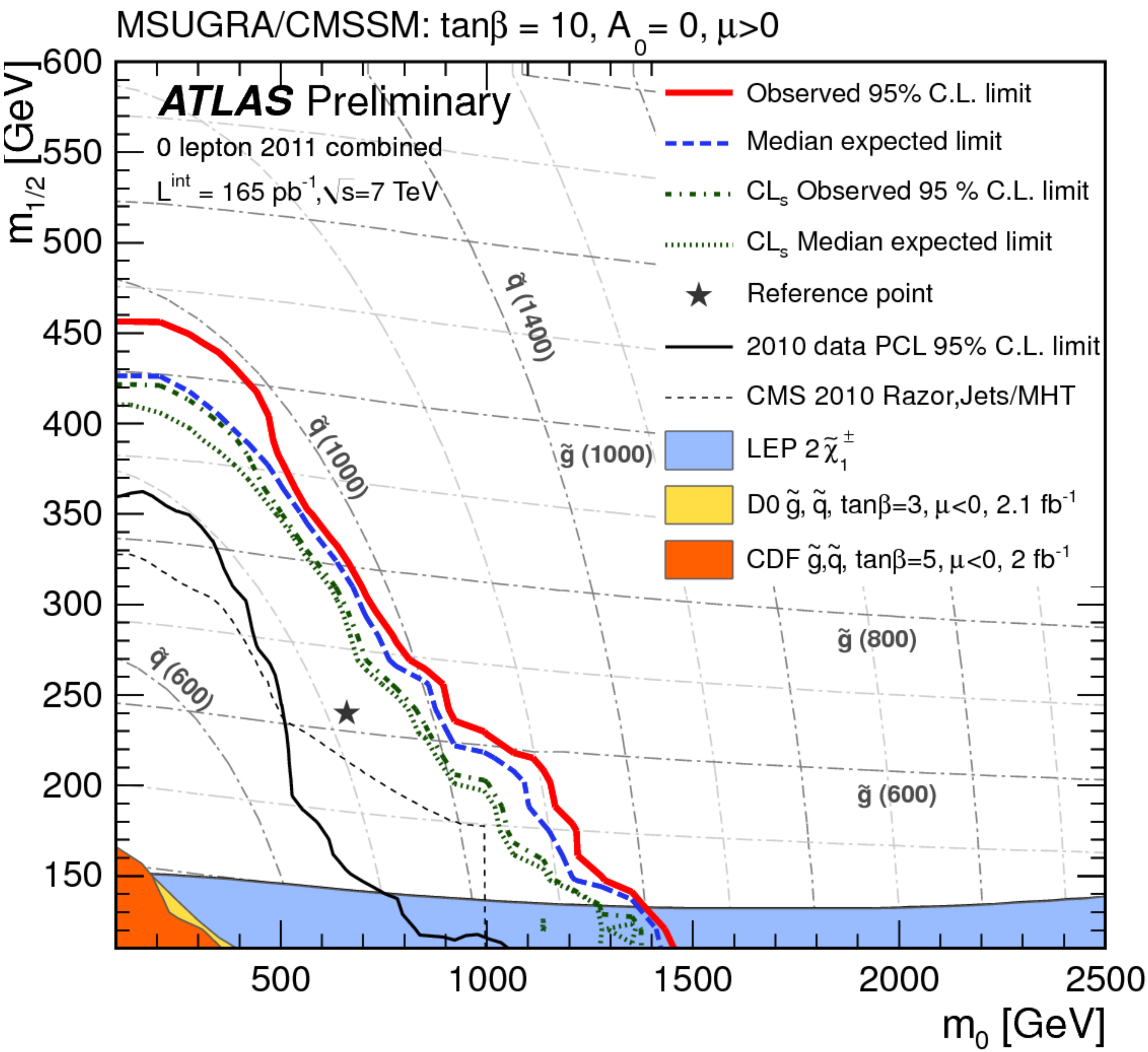


Latest ATLAS 0-lepton, jets, missing transverse momentum data.

Don’t confuse simplicity with complexity … can layer add many layers of interpretation

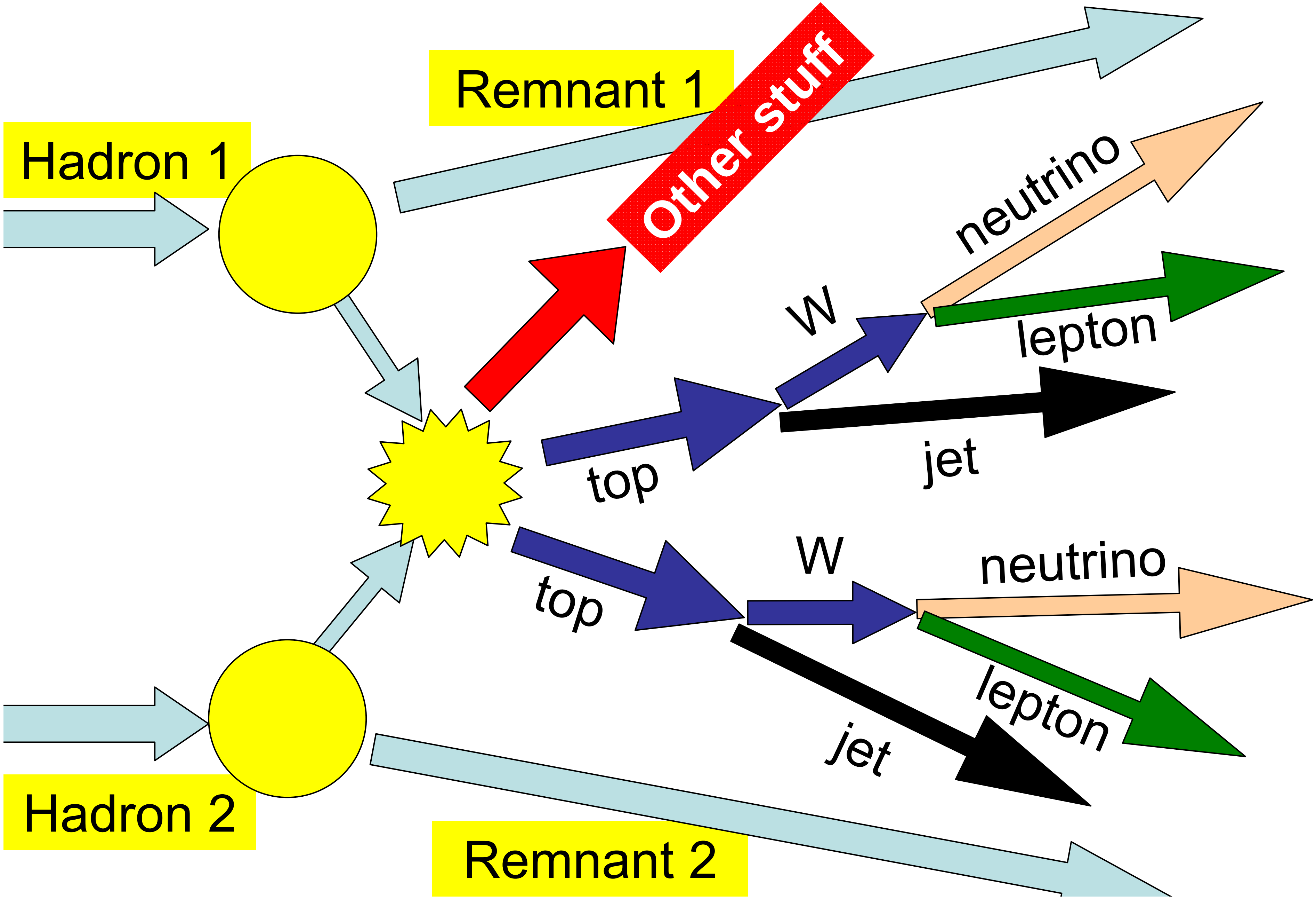
Measure top quark mass from mean lepton PT only!

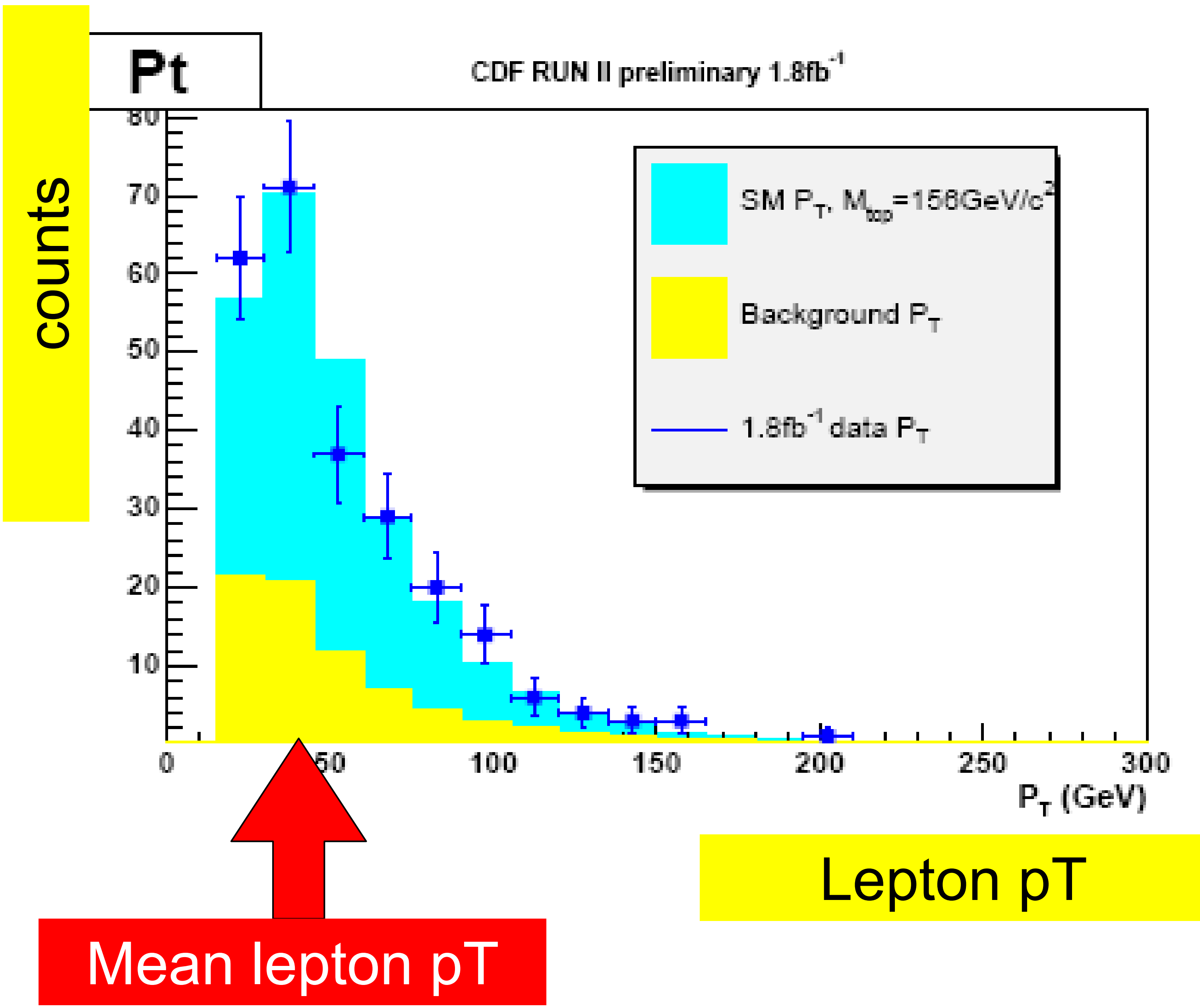
CDF note 8959

Measurement of the top quark mass from the lepton transverse momentum in the $t\bar{t} \rightarrow$ dilepton channel at the Tevatron

A new measurement of the top quark mass at 1.8 fb^{-1} integrated luminosity, using leptons' P_T in the dilepton channel is presented. A top quark mass of $m_{top}=156 \pm 20_{(stat)} \pm 4.6_{(syst)}$ GeV/c^2 is obtained with the Likelihood method and of $149 \pm 21_{(stat)} \pm 5$(syst)GeV/$c^2$ is obtained with the Straight Line method.

Top quark production tevatron - dileptonic

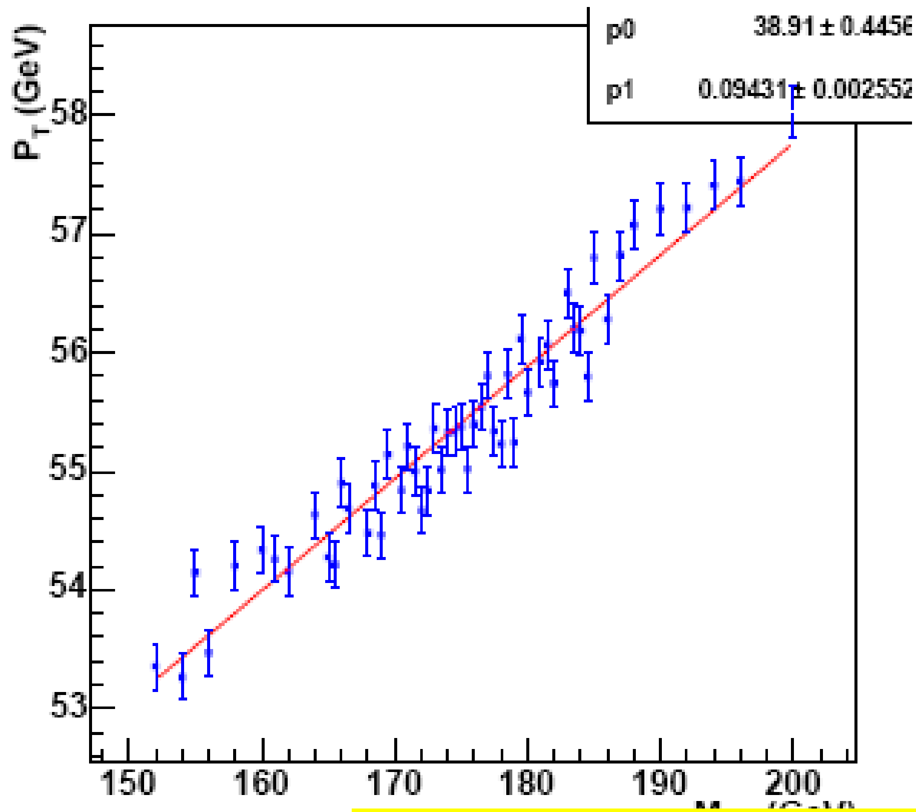




Pt
CDF RUN II preliminary 1.8fb-1
counts
SM P_T, M_top=156GeV/c^2
Background P_T
1.8fb-1 data P_T
P_T (GeV)
0 50 100 150 200 250 300
10 20 30 40 50 60 70 80
Lepton pT
Mean lepton pT

Frightening y-axis!

Mean lepton pT



Simulated top quark mass

Result $m_{top} = 156 \pm 20_{(stat)} \pm 4.6_{(syst)}\,\mathrm{GeV}$

Moral

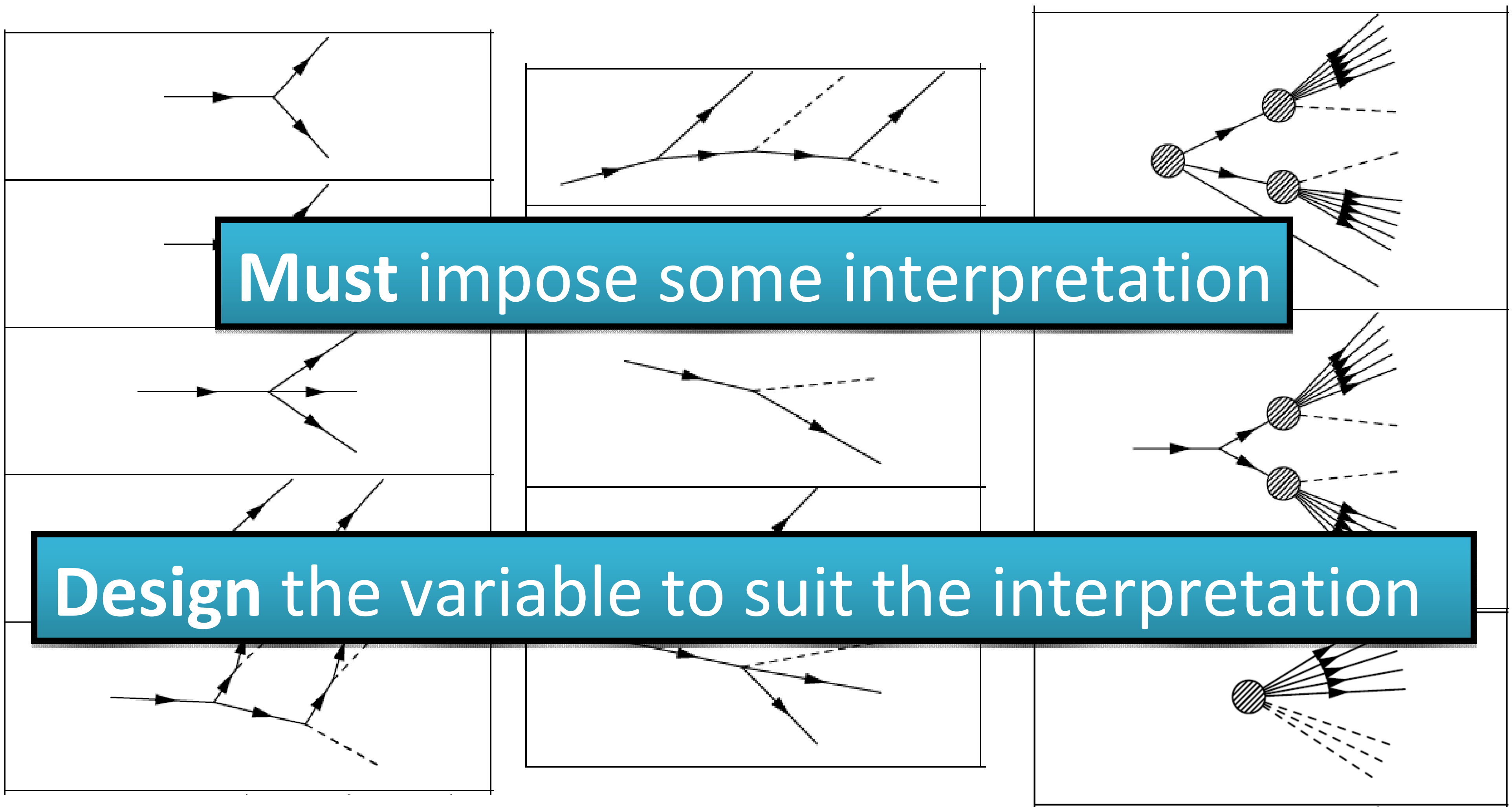
- You can monte-carlo anything.
  - example h->tau tau

- But do you trust it? Is it the best you can do?

More assumptions
Less Vague Conclusions

non-hotpants

Topology / hypothesis



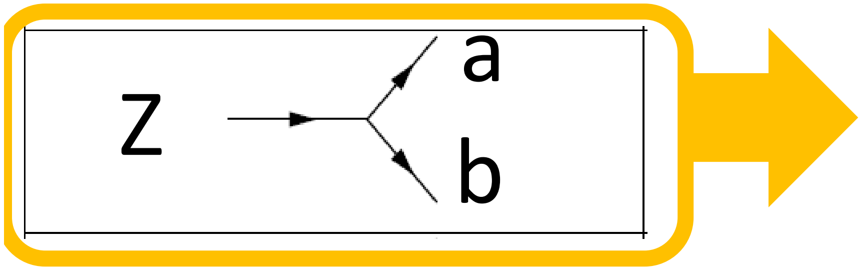
Must impose some interpretation

Design the variable to suit the interpretation

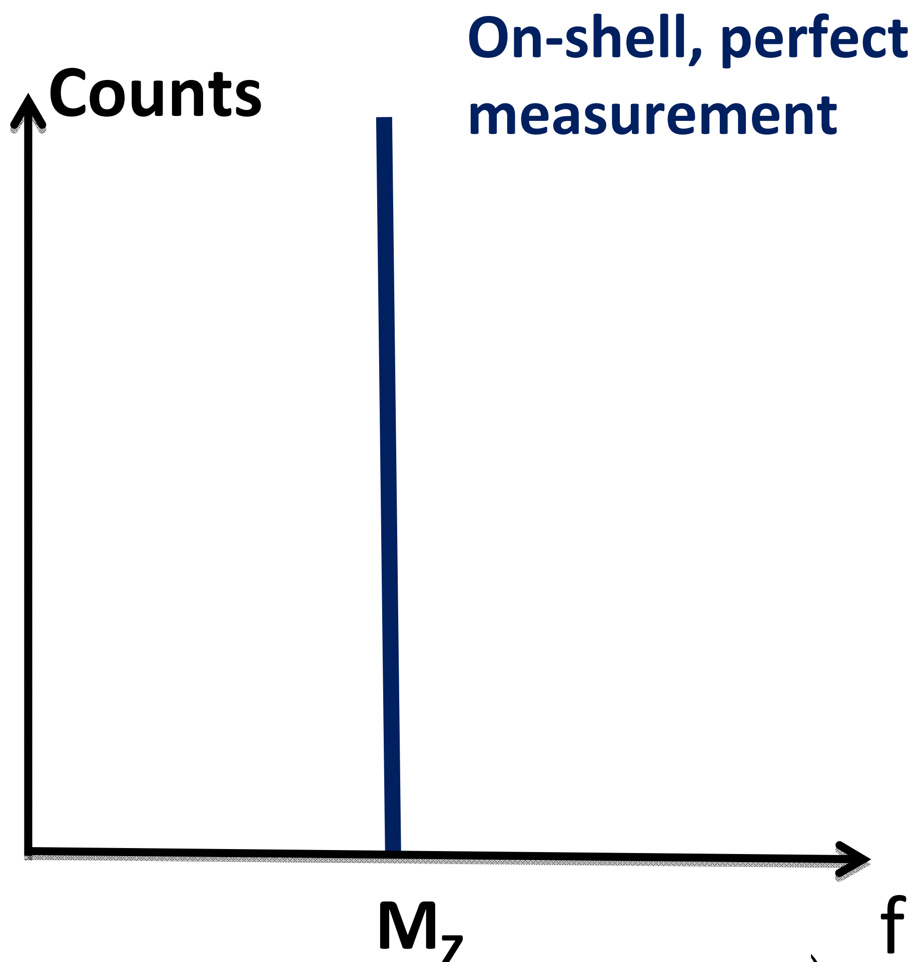
Full index in [arXiv:1004.2732]

All **visible**

$Z^0 \rightarrow e^+ e^-$



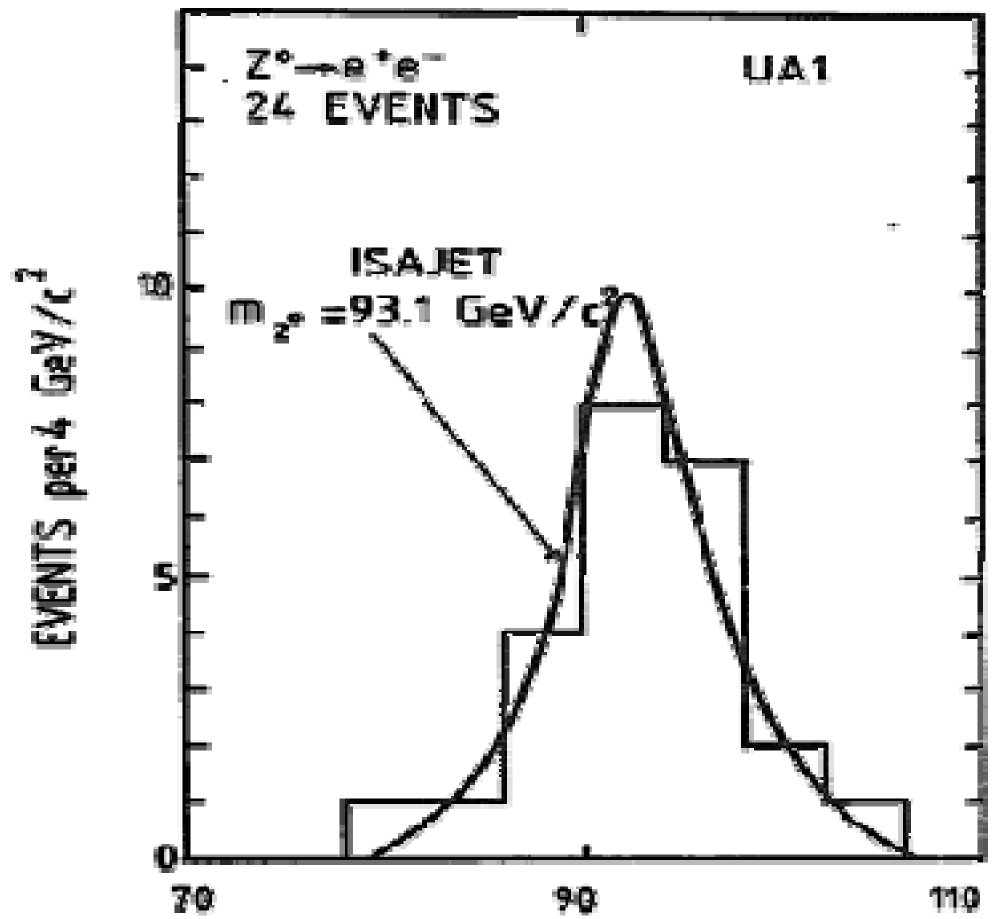
$f^2 = Z^\mu Z_\mu = (a+b)^\mu (a+b)_\mu$



$$M^2 = \left(\sqrt{m_a^2 + a_x^2 + a_y^2 + a_z^2} + \sqrt{m_b^2 + b_x^2 + b_y^2 + b_z^2}\right)^2 - (a_x + b_x)^2 - (a_y + b_y)^2 - (a_z + b_z)^2$$

UA1 CERN 1989

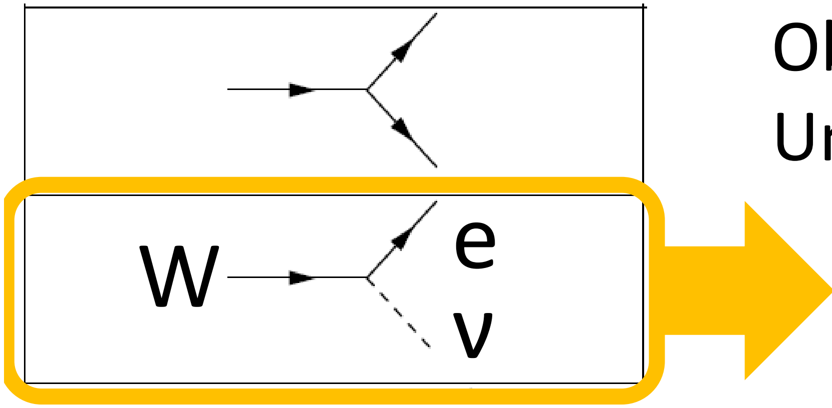
SPS – the Z boson Mass



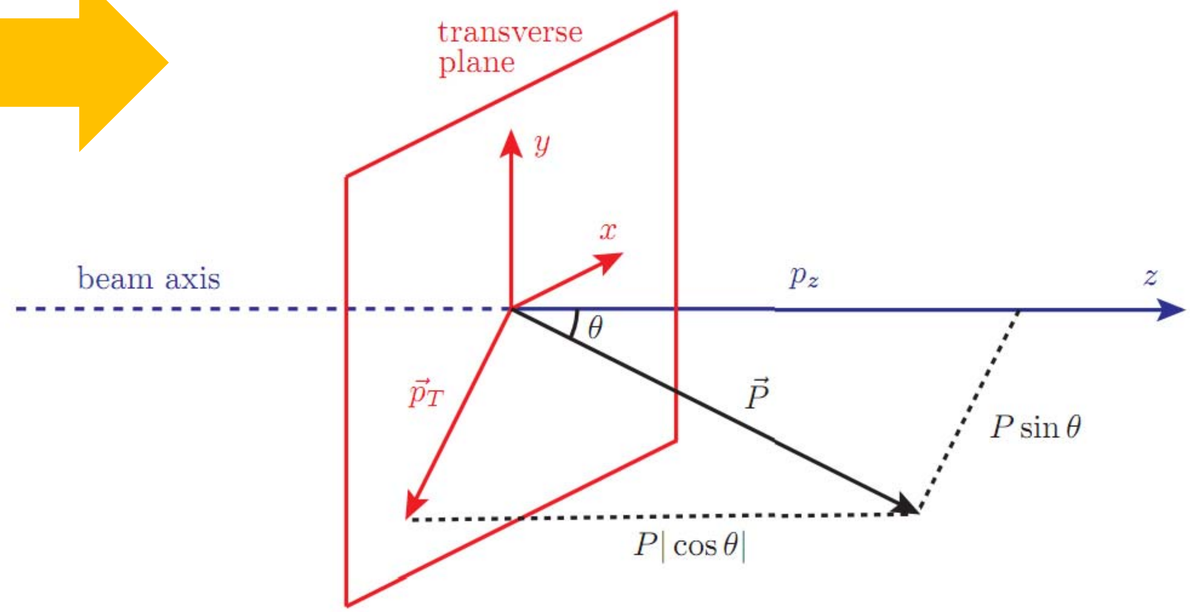
Finite width
Detector resolution

Broaden peak

Dealing with incomplete information

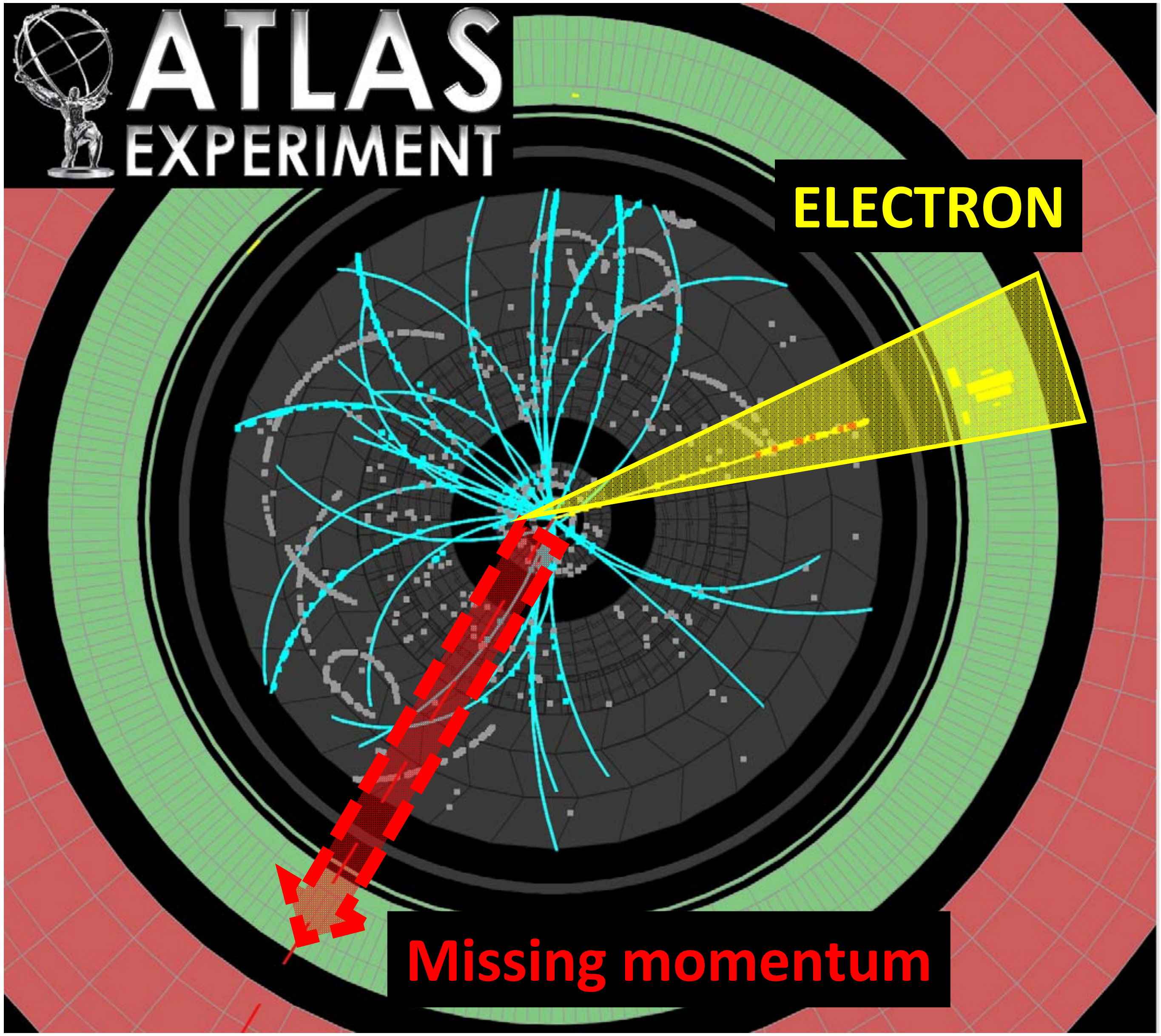


Observe: P_e (four components)
Unobserved: P_ν (does not interact)



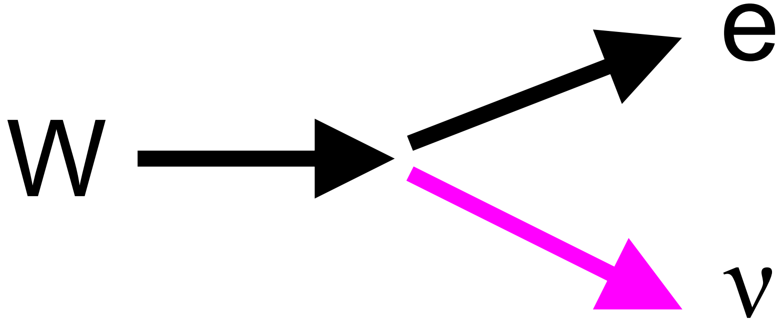
Cannot reconstruct $(P_\nu + P_e)^2$

Unobserved, but not unconstrained…


ATLAS
EXPERIMENT
ELECTRON
Missing momentum

Historical solution:
(full!) W transverse mass

$$m_T^2 = m_e^2 + m_\nu^2 + 2(e_e e_\nu - \mathbf{p}_{\mathbf{T}e} \cdot \mathbf{p}_{\mathbf{T}\nu})$$



$$e_e = \sqrt{m_e^2 + p_{Te}^2}$$

$$e_\nu = \sqrt{m_\nu^2 + p_{T\nu}^2}$$

!! NOT THIS !!

$$m_T = \sqrt{2\left|\vec{P}_{Te}\right|\left|\vec{P}_{T\nu}\right|(1-\cos\vartheta)}$$

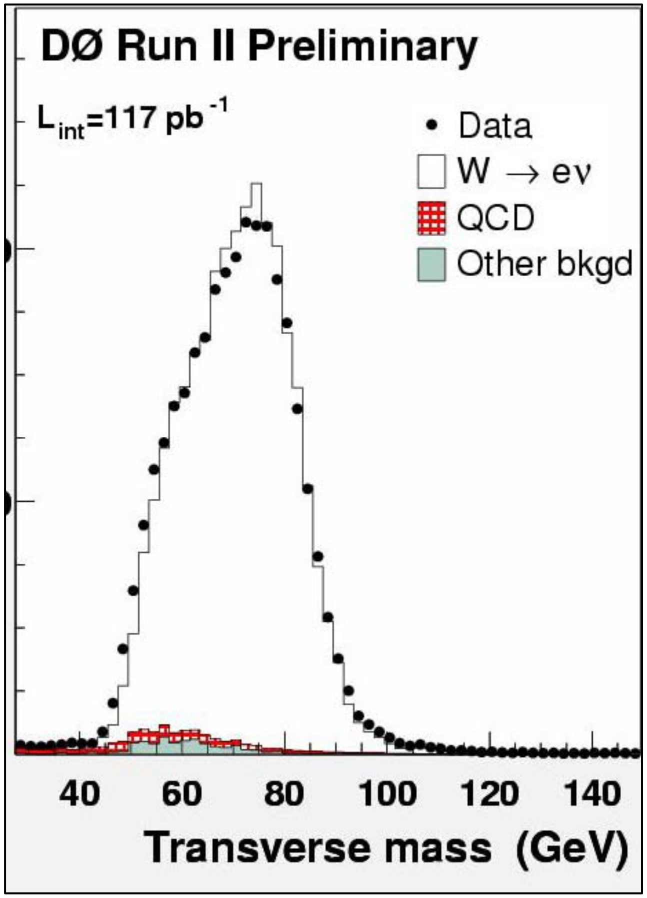
!! This is **NOT** the transverse mass !!

W transverse mass: nice properties

- In every event $m_T < m_W$ if the W is on shell

- There are events in which m_T can saturate the bound on m_W.

motivate m_T in W discovery and mass measurements.



But where did these properties come from?

Re-examine invariant mass: M→a b

$$\mathrm{M}^2 = \left(\sqrt{m_a^2 + a_x^2 + a_y^2 + a_z^2} + \sqrt{m_b^2 + b_x^2 + b_y^2 + b_z^2}\right)^2$$
$$- \left(a_\mathrm{x} + b_\mathrm{x}\right)^2 - \left(a_\mathrm{y} + b_\mathrm{y}\right)^2 - \left(a_\mathrm{z} + b_\mathrm{z}\right)^2$$

$$= \left(E_a + E_b\right)^2 - \left(a_\mathrm{x} + b_\mathrm{x}\right)^2 - \left(a_\mathrm{y} + b_\mathrm{y}\right)^2 - \left(a_\mathrm{z} + b_\mathrm{z}\right)^2$$

$$= m_a^2 + m_b^2 + 2\left(E_a E_b - a_x b_\mathrm{x} - a_\mathrm{y} b_\mathrm{y} - a_\mathrm{z} b_\mathrm{z}\right)$$

$$\boxed{= m_a^2 + m_b^2 + 2\left(e_a e_b \cosh(\Delta\eta) \; - a_x b_x - a_y b_y\right)}$$

where

$$e_a = \sqrt{m_a^2 + a_x^2 + a_y^2}$$
$$e_b = \sqrt{m_b^2 + a_b^2 + a_b^2}$$

and

$$\eta_a = \tfrac{1}{2}\ln\left(\left(E_a + a_z\right)/\left(E_a - a_z\right)\right)$$
$$\eta_b = \tfrac{1}{2}\ln\left(\left(E_b + b_z\right)/\left(E_b - b_z\right)\right)$$
$$\Delta\eta = \eta_a - \eta_b$$

Comparing invariant and transverse masses:

$$M^2 = m_a^2 + m_b^2 + 2\left(e_a e_b \cosh(\Delta\eta) \; - a_x b_x - a_y b_y\right)$$
$$M_T^2 = m_a^2 + m_b^2 + 2\left(e_a e_b \qquad\qquad\qquad - a_x b_x - a_y b_y\right)$$

Since $\cosh\left(\Delta\eta\right) \geq 1$ have $M_T \leq M$

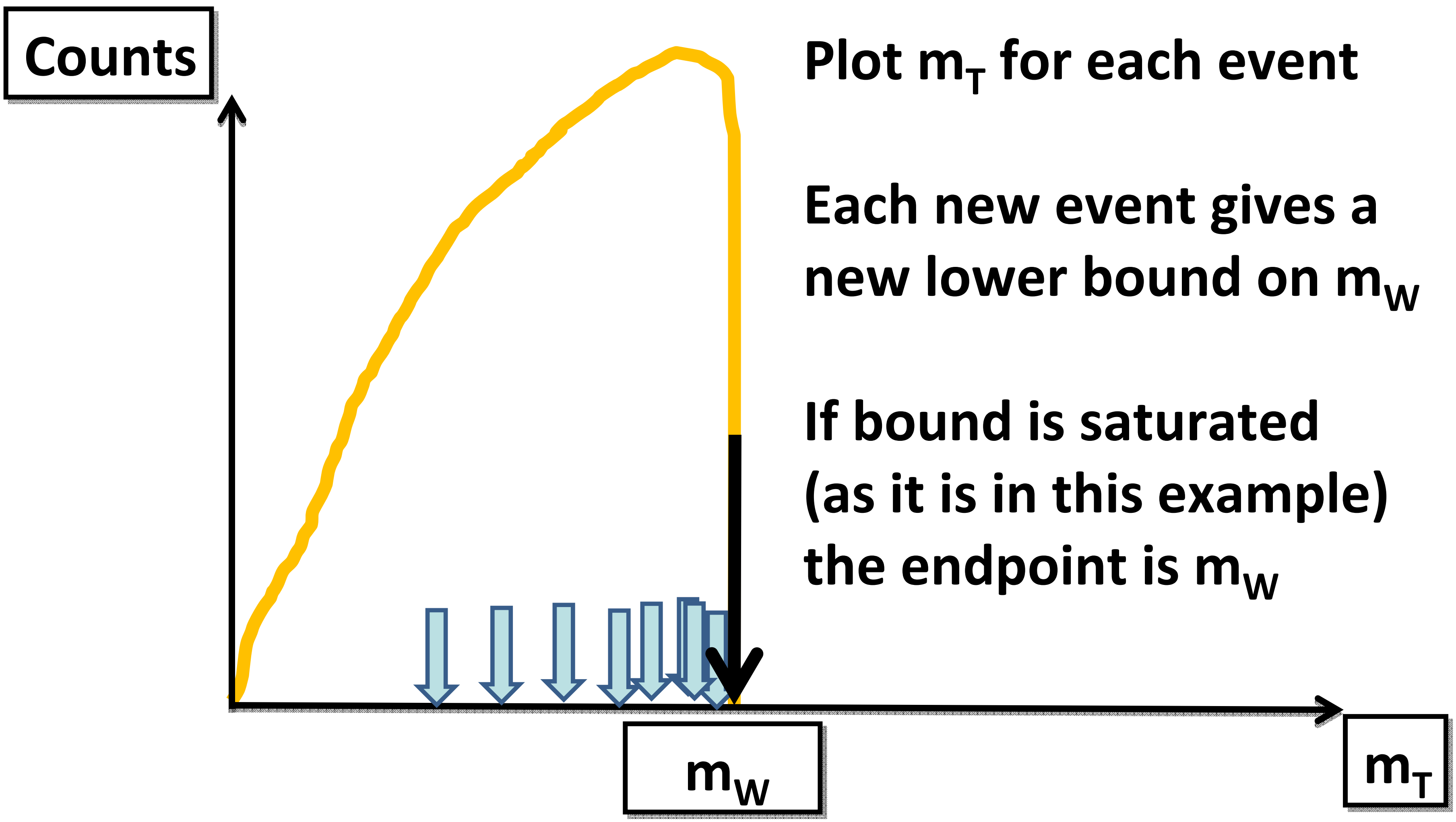
with equality when $\Delta\eta = 0.$

(Not same as throwing away z information!)

But have bound, and bound can be saturated.

Note that at this point we are assuming we know m_b.

W boson mass measurement



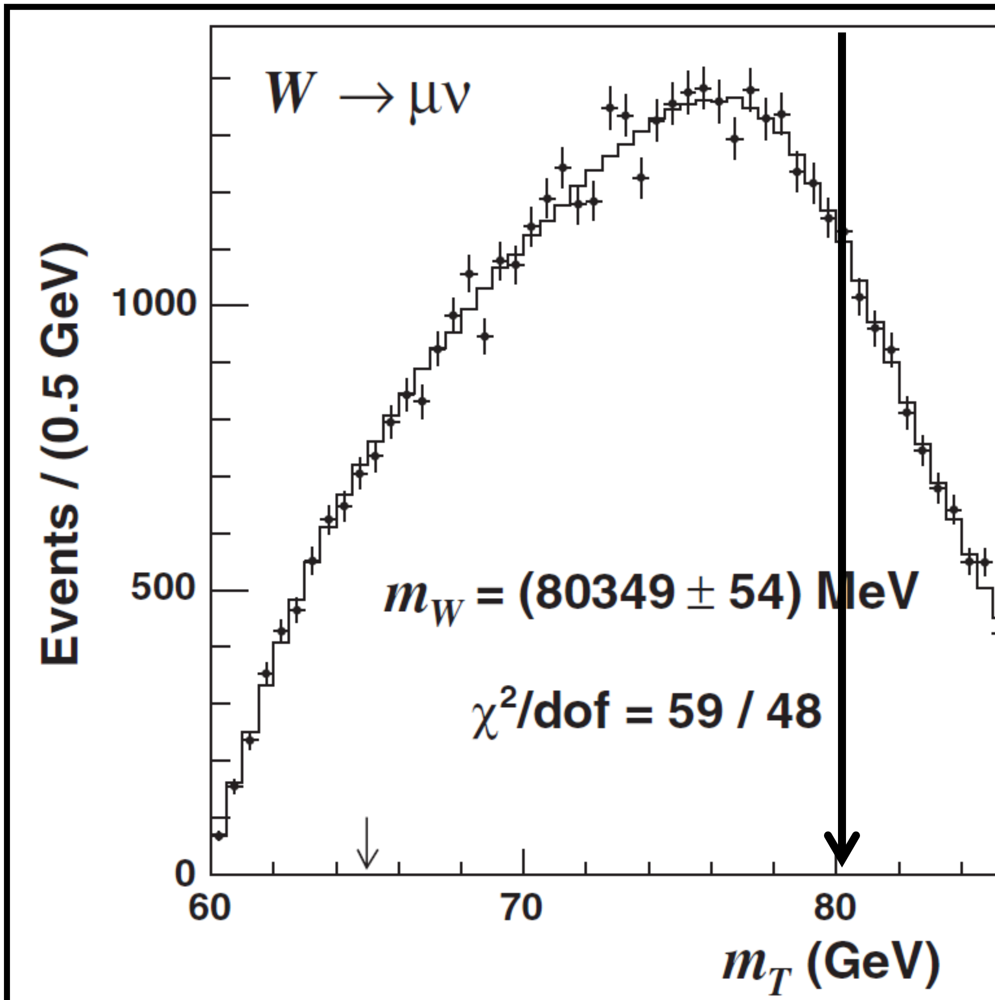
Plot m_T for each event

Each new event gives a new lower bound on m_W

If bound is saturated (as it is in this example) the endpoint is m_W

Phys.Rev.D. **77**, 112001 (2008)

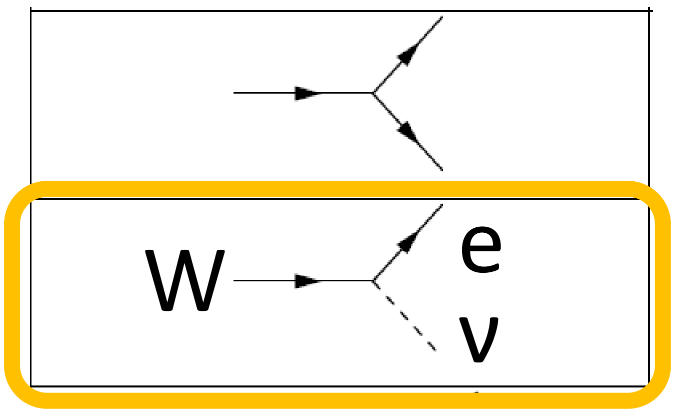
In the data….



Bound at m_W smeared by resolution and finite width effects

➔ Monte Carlo modelling

Alternative way of approaching the problem



Set out **INTENDING** to construct best lower

bound

on $(P_e+P_\nu)^2$

given the constraints

Constraints in this instance:

$0 = (P_\nu)^2$ ***[massless neutrino]***

$0 = \Sigma \mathbf{p}_T = \mathbf{u}_T + \mathbf{p}_T(e) + \mathbf{p}_T(\nu)$

[momentum conservation in transverse plane]

Exercises M→a b

(1) Prove that

$$M^2 = m_a^2 + m_b^2 + 2\left(e_a e_b \cosh(\Delta\eta) \ - a_x b_x - a_y b_y\right)$$

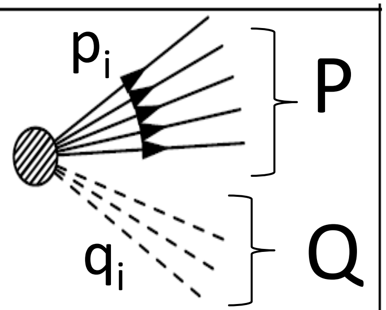
(2) We have shown that M_T (at fixed and correct m_b) is an observable that is bounded above by M for unsmeared signal events M→a b. Go further than this. Prove that it is ***the greatest possible*** lower bound for the mass of the parent.

(3) It is trivial to demonstrate that MT is invariant under longitudinal boosts. Is it invariant under transverse parental boosts? What about the kinematic endpoint of the MT distribution?

Suggests general prescription…

(1) Propose a decay **topology**
(2) Write down your the **Lorentz Invariant** of choice
(3) Write down the **constraints**
(4) **Calculate** the bound (algebraically/numerically/mix)

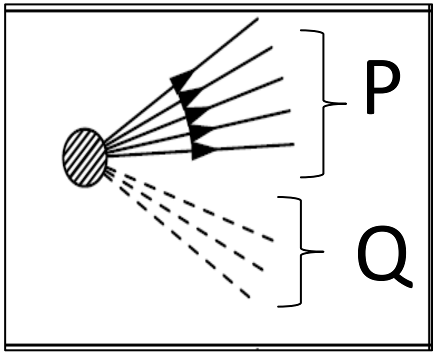
(1)



(2) $$\mathcal{M}_a \equiv \sqrt{g_{\mu\nu}\,(\mathbf{P}_a + \mathbf{Q}_a)^\mu\,(\mathbf{P}_a + \mathbf{Q}_a)^\nu}$$

(3) $$\sum_{i=1}^{N_\mathcal{I}} \vec{q}_{iT} = \vec{\not{p}}_T \equiv -\vec{u}_T - \sum_{i=1}^{N_\mathcal{V}} \vec{p}_{iT}$$

Single parent … multiple daughters



many visibles

many invisibles

$$M_{1\mathrm{T}}^2 = \left(\sqrt{M_P^2 + \vec{\mathbf{p}}_T^2} + \sqrt{M_{\mathrm{slash}}^2 + \vec{\mathbf{q}}_{T\mathrm{miss}}^2}\right)^2 - u_T^2$$

$$M_{\mathrm{slash}} = \sum_i \widetilde{M}_i$$

Bound depends on *GUESS* masses of *all* invisible daughters

Most conservative: **set to zero**

[more later]

Almost exactly same as transverse mass – one small generalization

$$M_{1\mathrm{T}}^2 = \left(\sqrt{M_P^2 + \vec{\mathbf{p}}_T^2} + \sqrt{M_{\mathrm{slash}}^2 + \vec{\mathbf{q}}_{T\mathrm{miss}}^2}\right)^2 - u_T^2$$

$$M_{\mathrm{T}}^2 = \left(\sqrt{M_P^2 + \vec{\mathbf{p}}_T^2} + \sqrt{M_{\mathrm{Q}}^2 + \vec{\mathbf{q}}_{T\mathrm{miss}}^2}\right)^2 - u_T^2$$

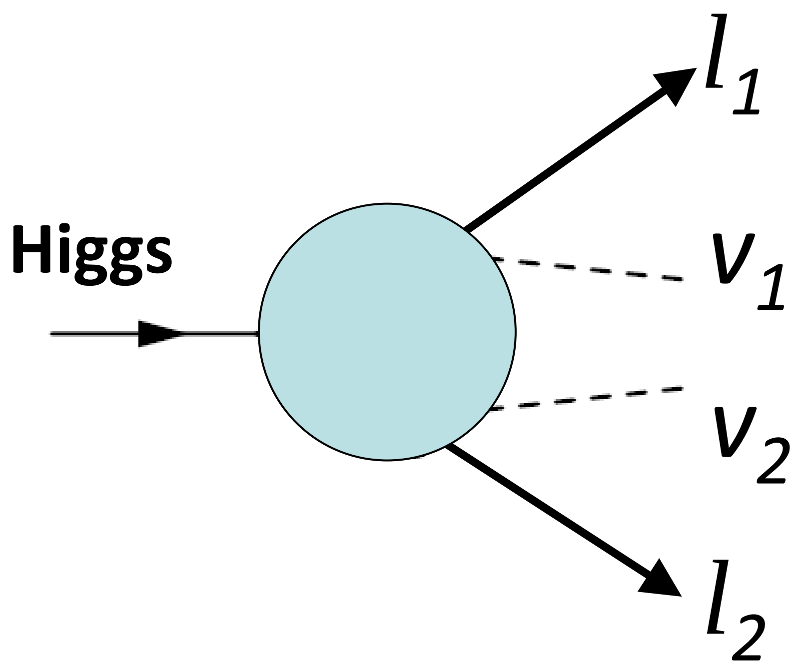
The “invisible mass” has become a parameter …. rather than the actual visible mass.

We will come back to this many times.

Suggests we should think about non-physical parameters a bit more ….

Applications of M_{1T}?

Higgs →WW* → lνlν



$$\begin{aligned} Q_1^\mu Q_{1\mu} &= 0, \\ Q_2^\mu Q_{2\mu} &= 0, \end{aligned}$$

$$\vec{q}_{1\mathrm{T}} + \vec{q}_{2\mathrm{T}} = \vec{\not{p}}_{\mathrm{T}}.$$

Written up in http://arxiv.org/abs/1106.2322

Higgs →WW* → lvlv

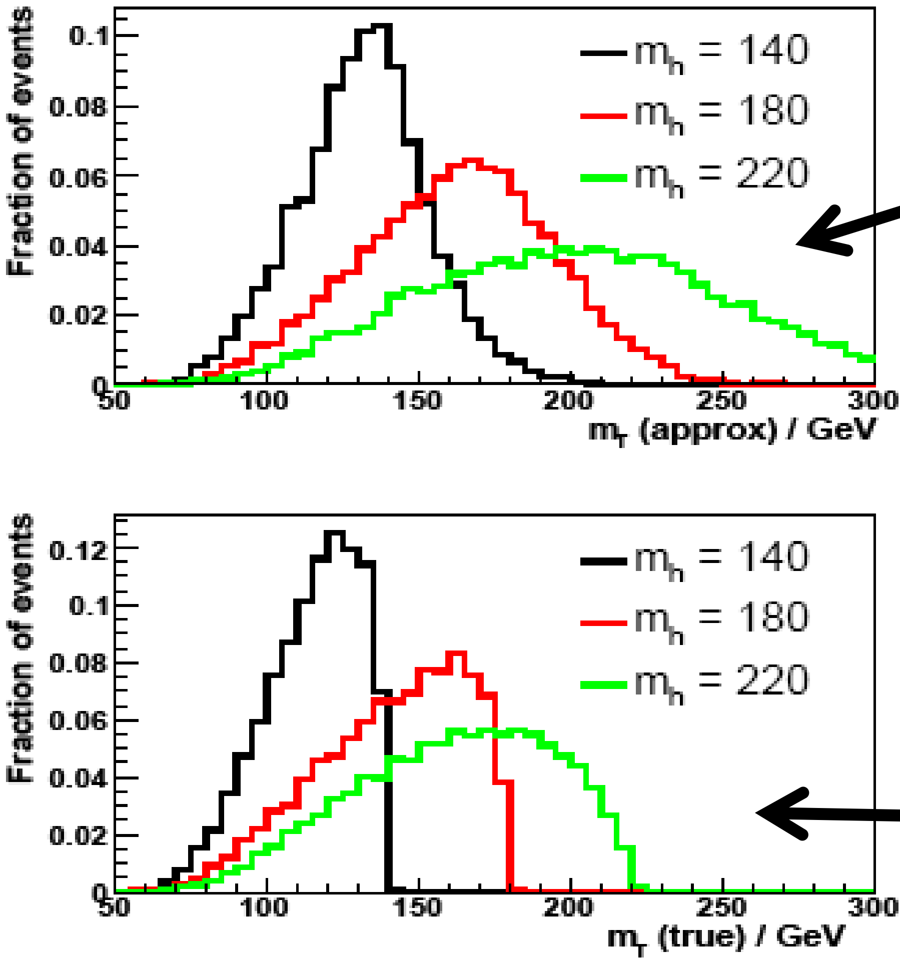


FIG. 1: Signal-only distributions of m_T^{approx} (top) and m_T^{true} (bottom) for various values of m_h (in GeV). No cuts on $\Delta\phi_{\ell\ell}^{\max}$ and $p_{T\,WW}^{\min}$ have been applied.

Previous variable (not a bound)

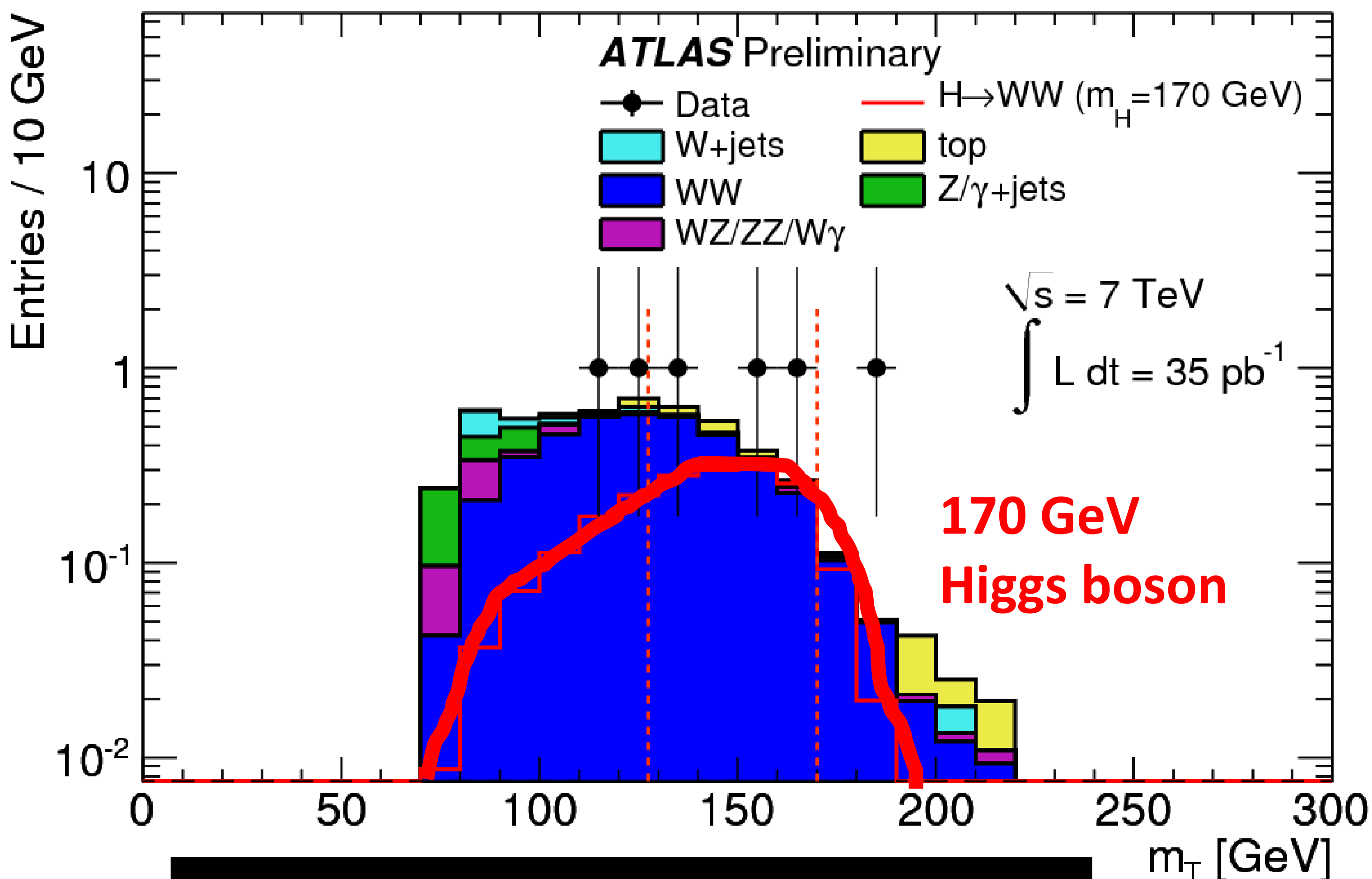
Why are endpoints often more robust than shapes?

Proper bound var MTTrue = M1T

Written up in http://arxiv.org/abs/1106.2322

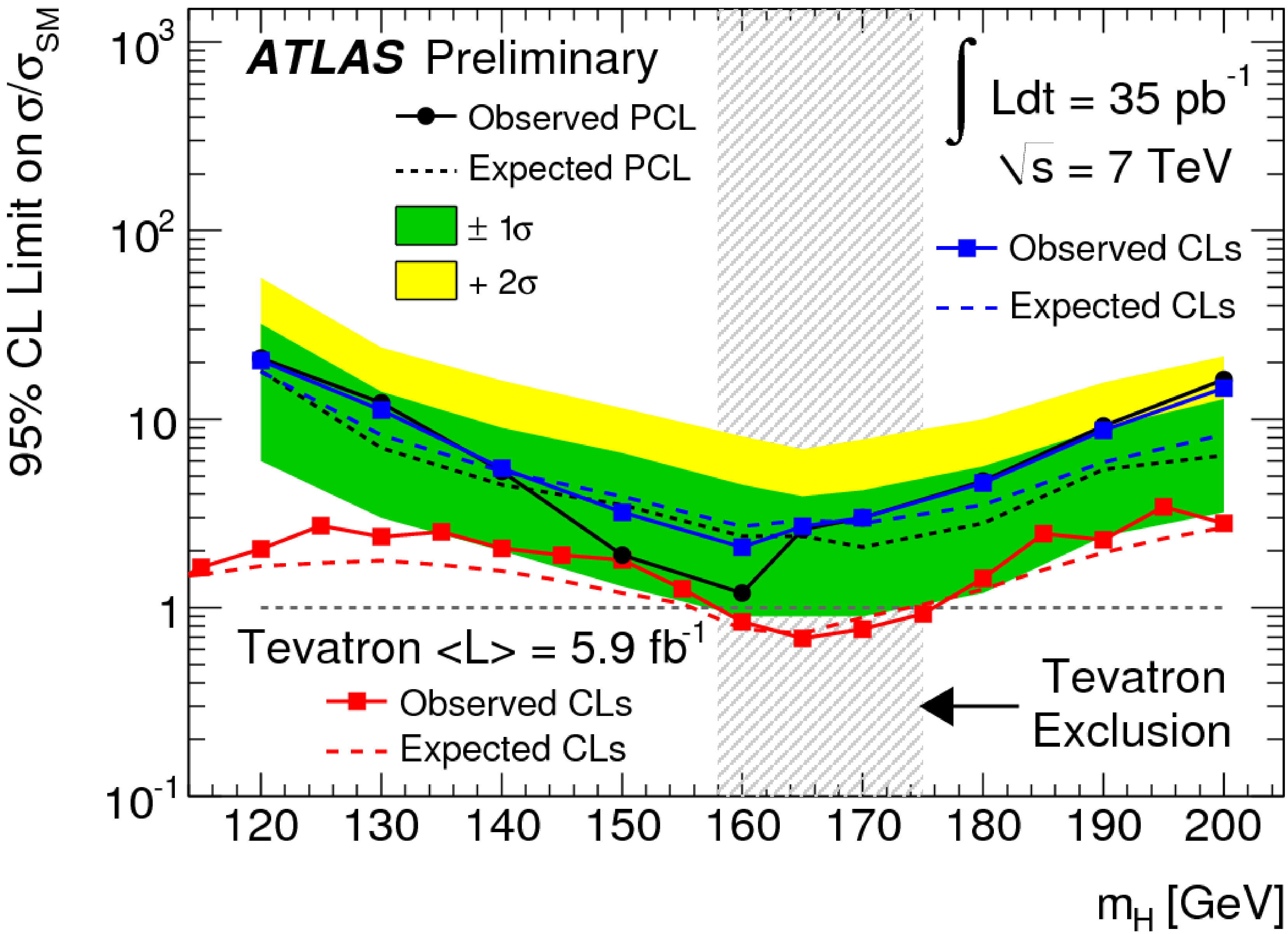
ATLAS-CONF-2011-005

Against the 2010 LHC data…



Big improvement in LHC Higgs Search

ATLAS 35/pb: H → WW → lvlv

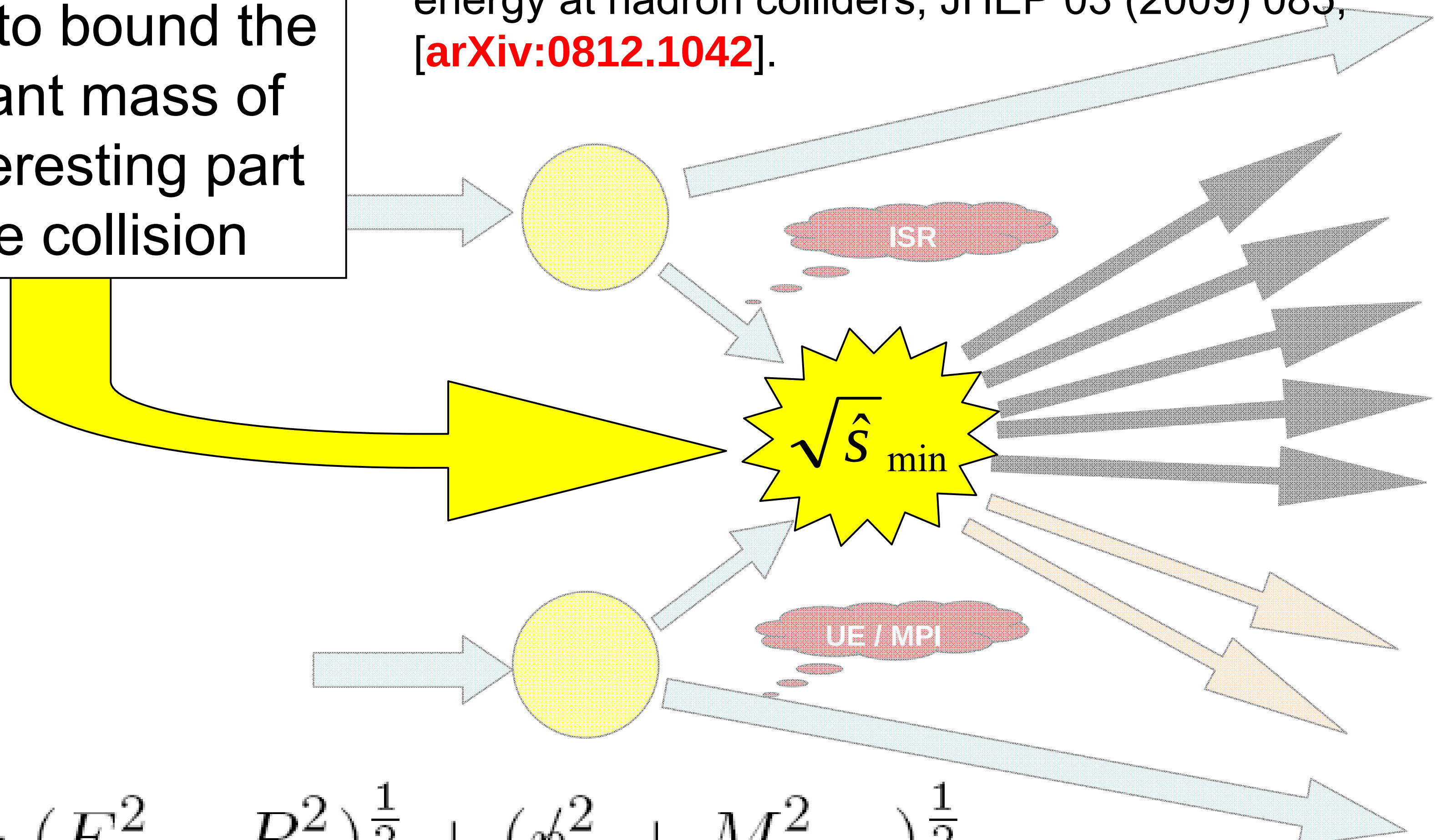


Other applications of M_{1T}?

$\sqrt{\hat{s}}_{\min}$ is fully inclusive M_{1T} (i.e. u_T=0)

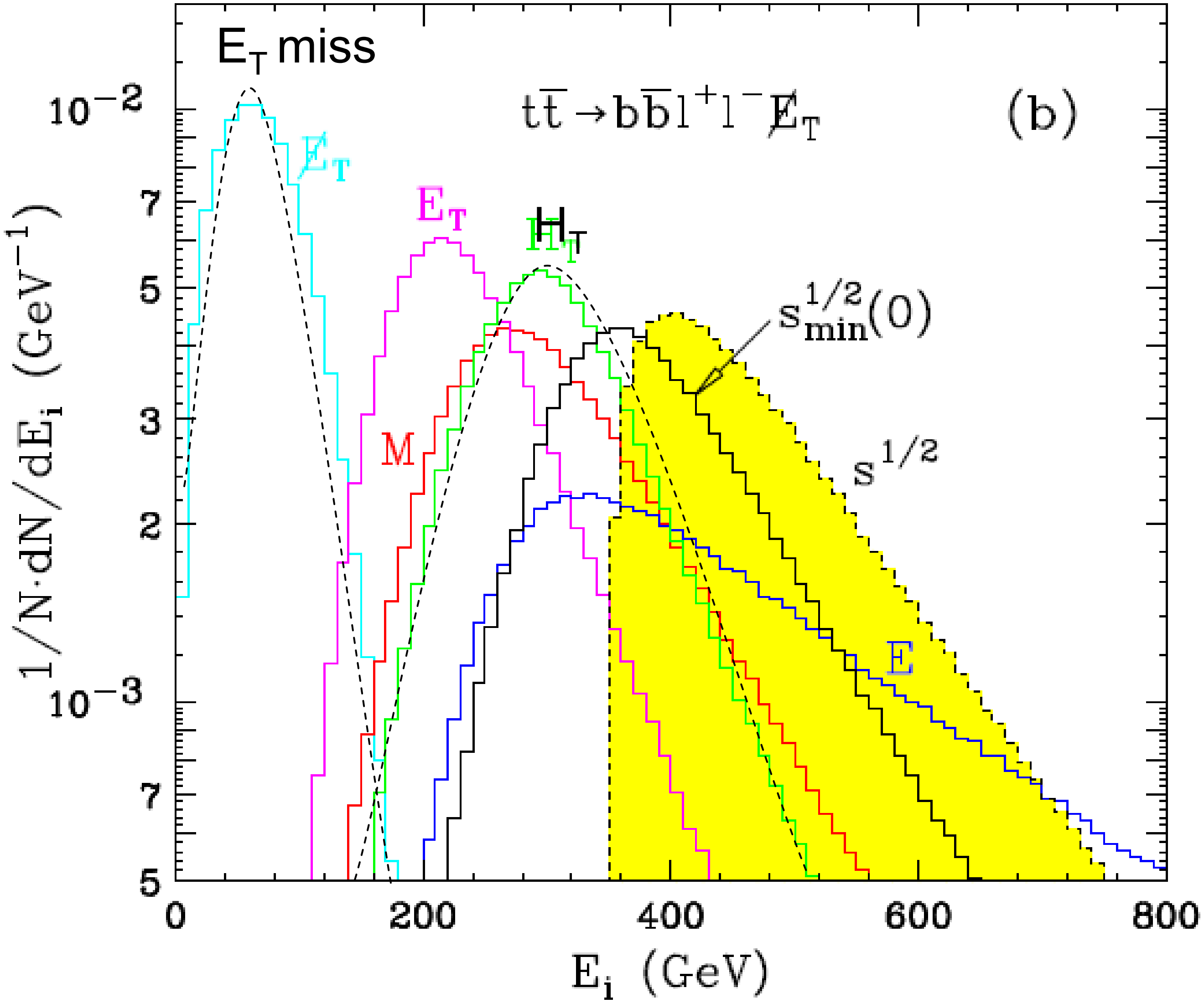
P. Konar, K. Kong, and K. T. Matchev, rootsmin : A global inclusive variable for determining the mass scale of new physics in events with missing energy at hadron colliders, JHEP 03 (2009) 085, [**arXiv:0812.1042**].

$\sqrt{\hat{s}}_{\min}$
seeks to bound the invariant mass of the interesting part of the collision



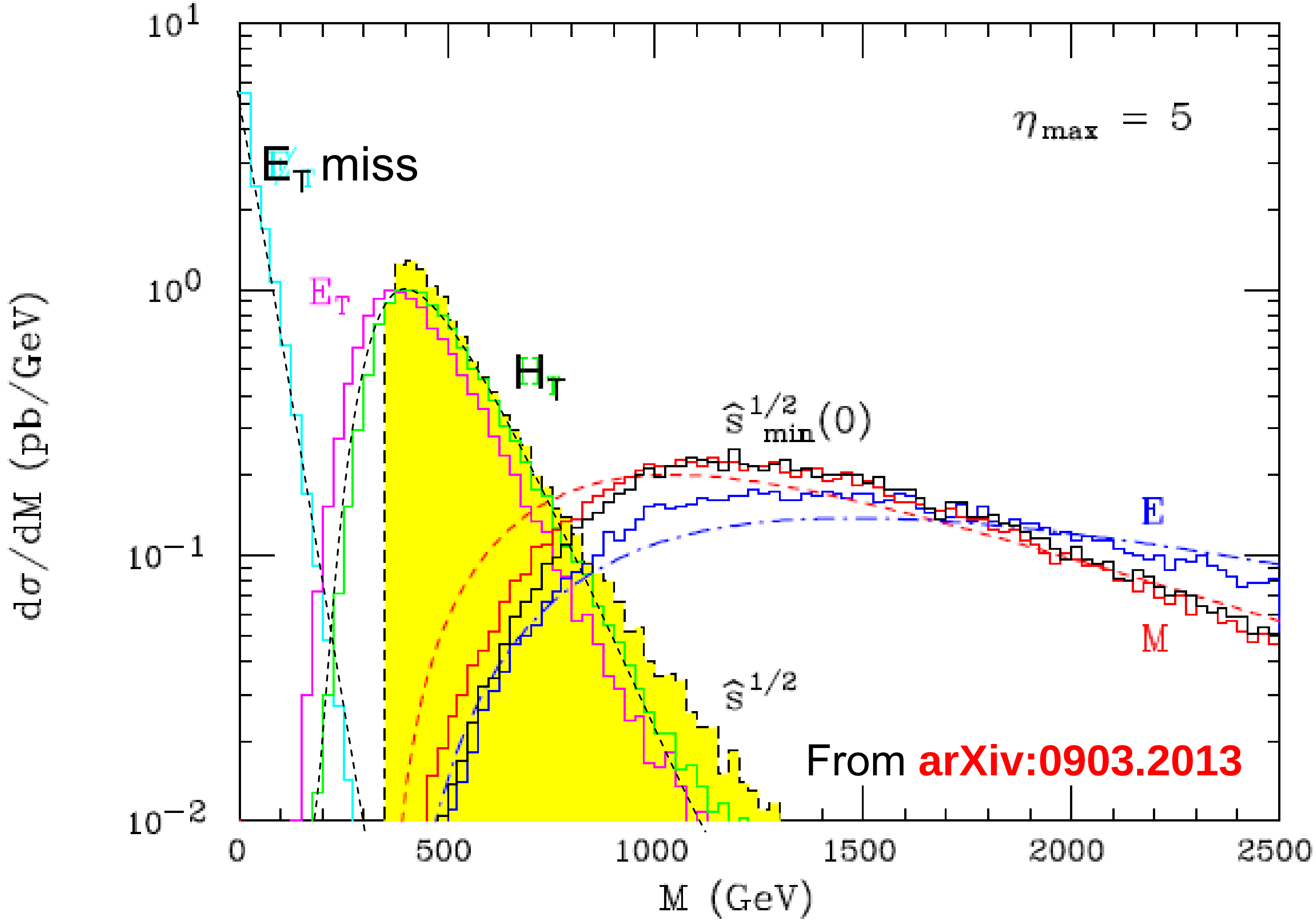
$$\hat{s}_{\min}^{1/2} = (E^2 - P_Z^2)^{\frac{1}{2}} + (\not{p}_T^2 + M_{\text{invis}}^2)^{\frac{1}{2}}$$

Without ISR / MPI



From **arXiv:0812.1042**

Effect of ISR and MPI contamination



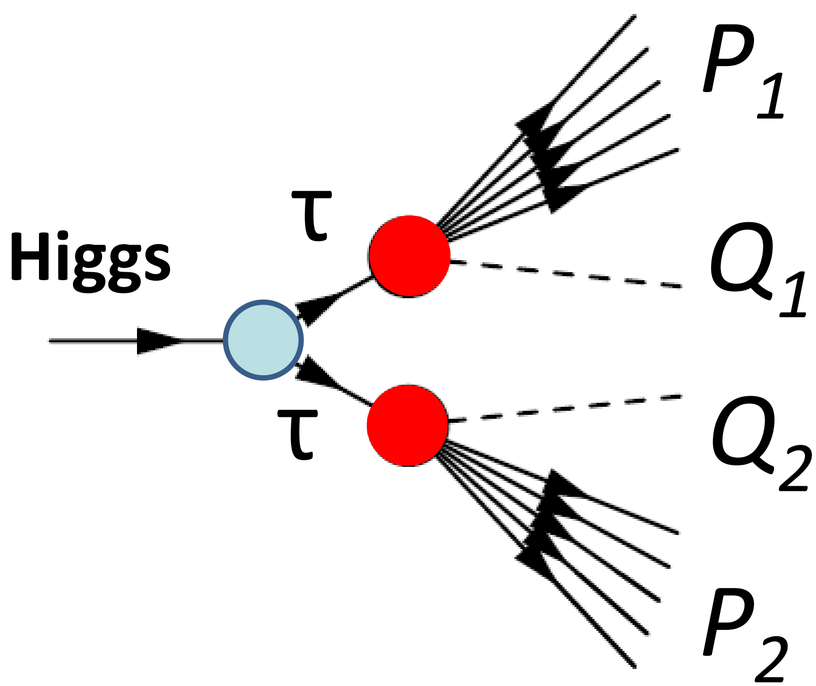
Though dependence on ISR Is large, it is calculable and may offer a good test of our understanding. See arXiv:0903.2013 and 1006.0653

Moral

- Remember our variables are always limited by what we feed them
  - (garbage in garbage out)
- May need alter variable in light of pathologies
  - Try to locate the subsystem that lacks ISR/FSR, e.g. by using reconstructed objects with pt thresholds
  - This takes away $u_T=0$ requirement, and gets us back to M_{1T} (a.k.a. “subsystem root s hat min”)

An example with additional (internal) constrains …

Example with additional internal constraints



$$
\begin{aligned}
Q_1^\mu Q_{1\mu} &= 0, \\
Q_2^\mu Q_{2\mu} &= 0, \\
(Q_1^\mu + P_1^\mu)(Q_{1\mu} + P_{1\mu}) &= m_\tau^2, \\
(Q_2^\mu + P_2^\mu)(Q_{2\mu} + P_{2\mu}) &= m_\tau^2,
\end{aligned}
$$

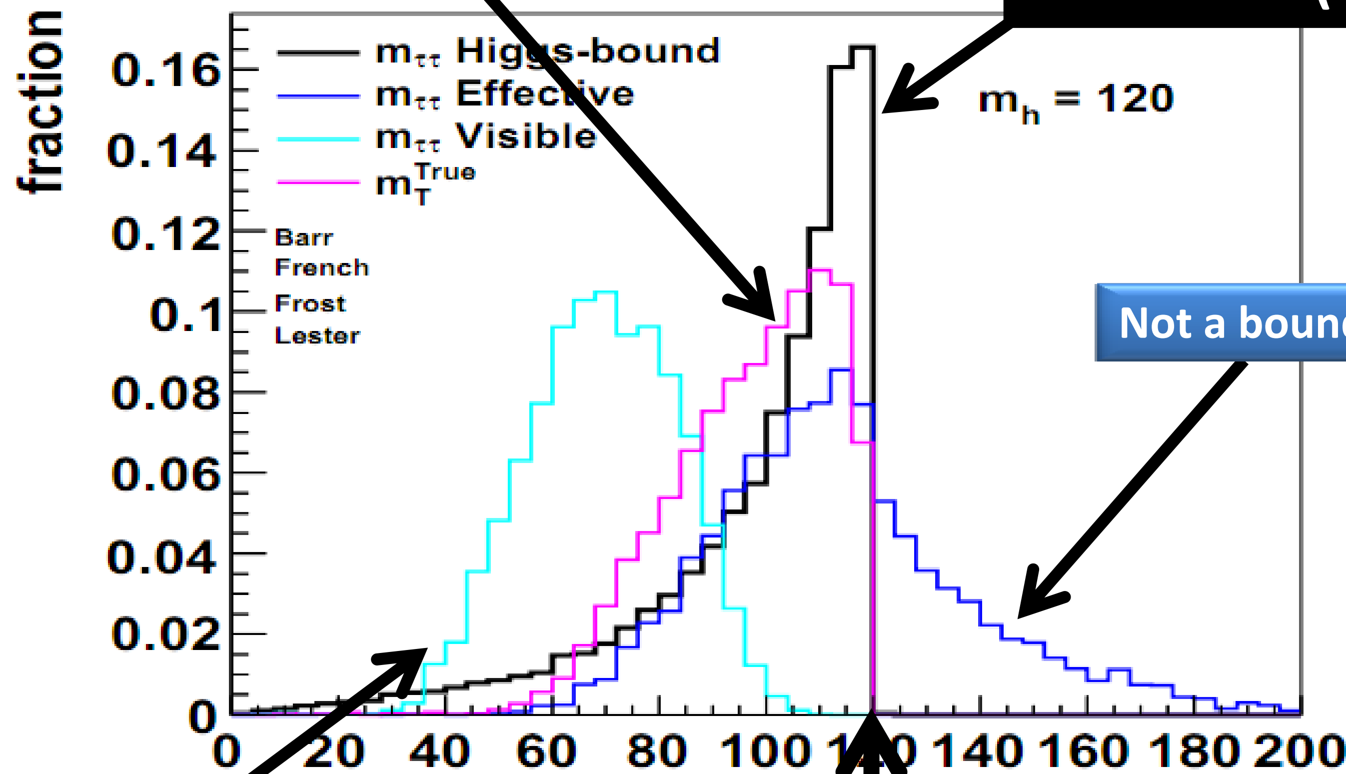
$$\vec{q}_{1\mathrm{T}} + \vec{q}_{2\mathrm{T}} = \vec{\not{p}}_{\mathrm{T}}.$$

Written up in
http://arxiv.org/abs/1106.2322

Result

Parent mas bound (no intermediate constraint) = M1T

Including the intermediate constraint (BEST)



Not a bound (existing var)

Just the visibles (existing var)

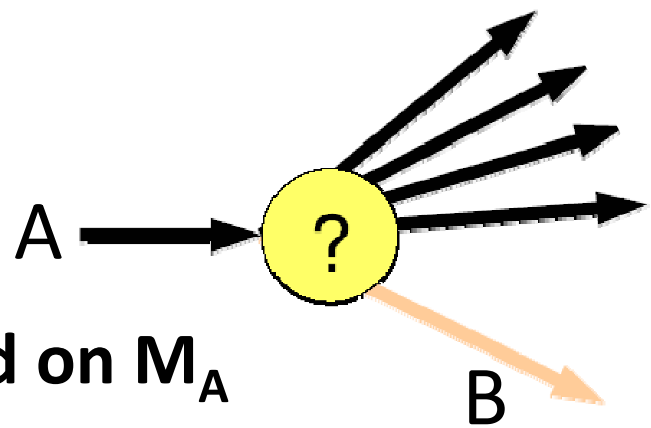
m_H

Dramatic difference to Higgs observability?

http://arxiv.org/abs/1106.2322

change of topic

But what if we don't know the masses of the invisible particle(s)?

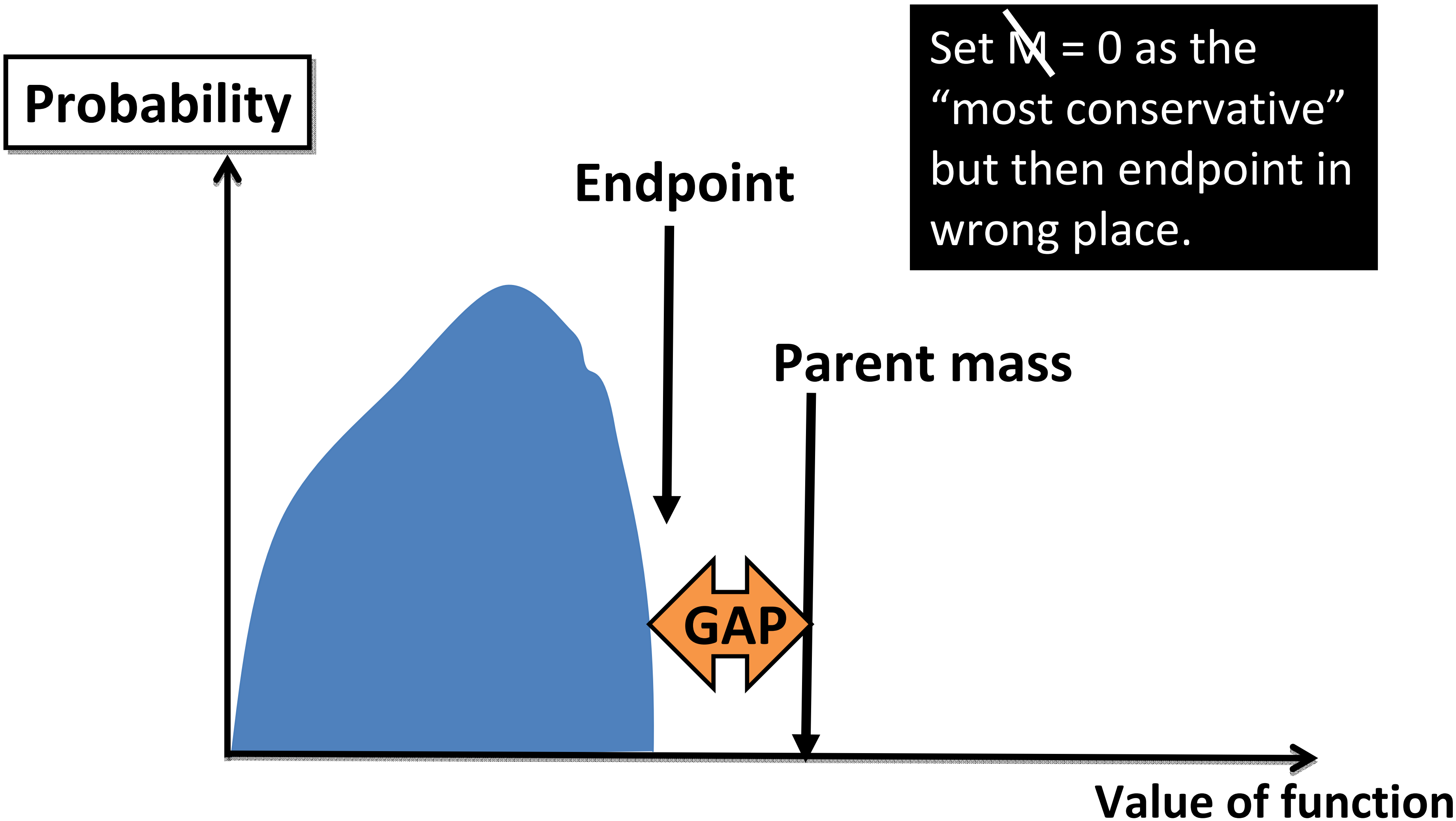


WANT bound on M_A

BUT M_B unknown…

Can we construct a maximal lower bound on M_A that depends on a hypothesis for M_B ?

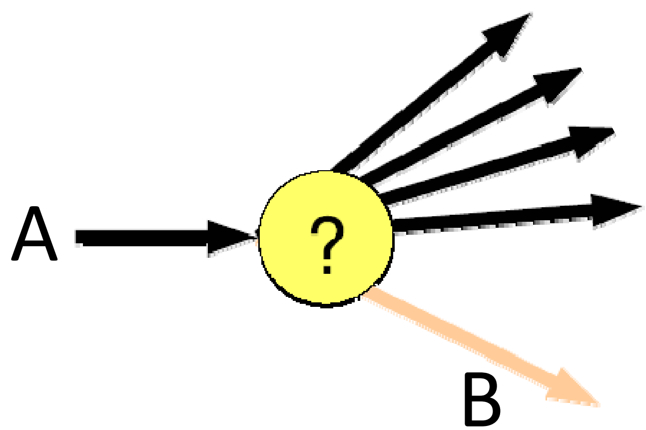
Hmm ….
“wrong M_B” not what M_T was designed for.



Let's go back to the (full) transverse mass again for a closer look!

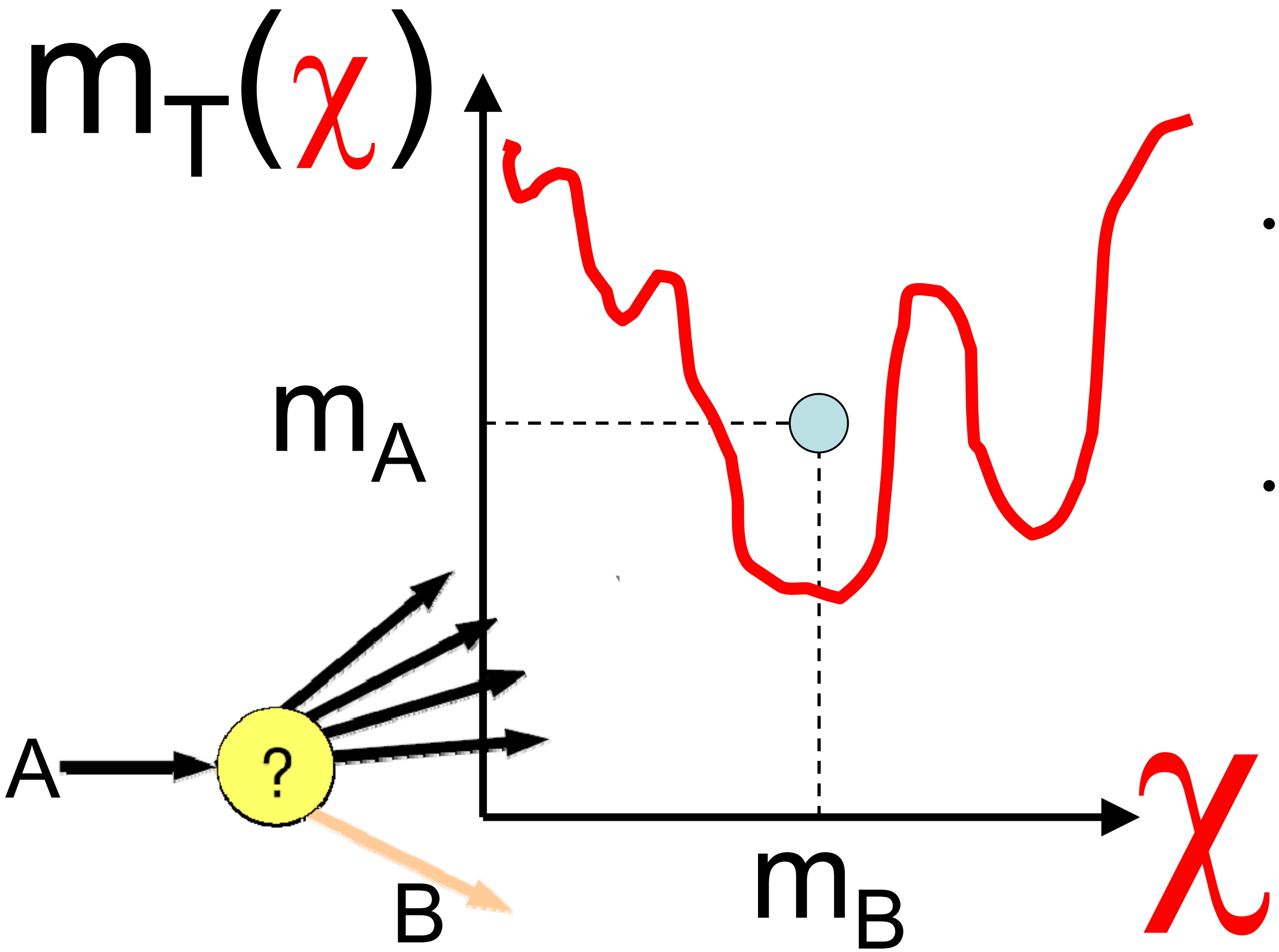
In next few slides:

χ = Guess (i.e. hypothesis) for mass of the invisible daughter



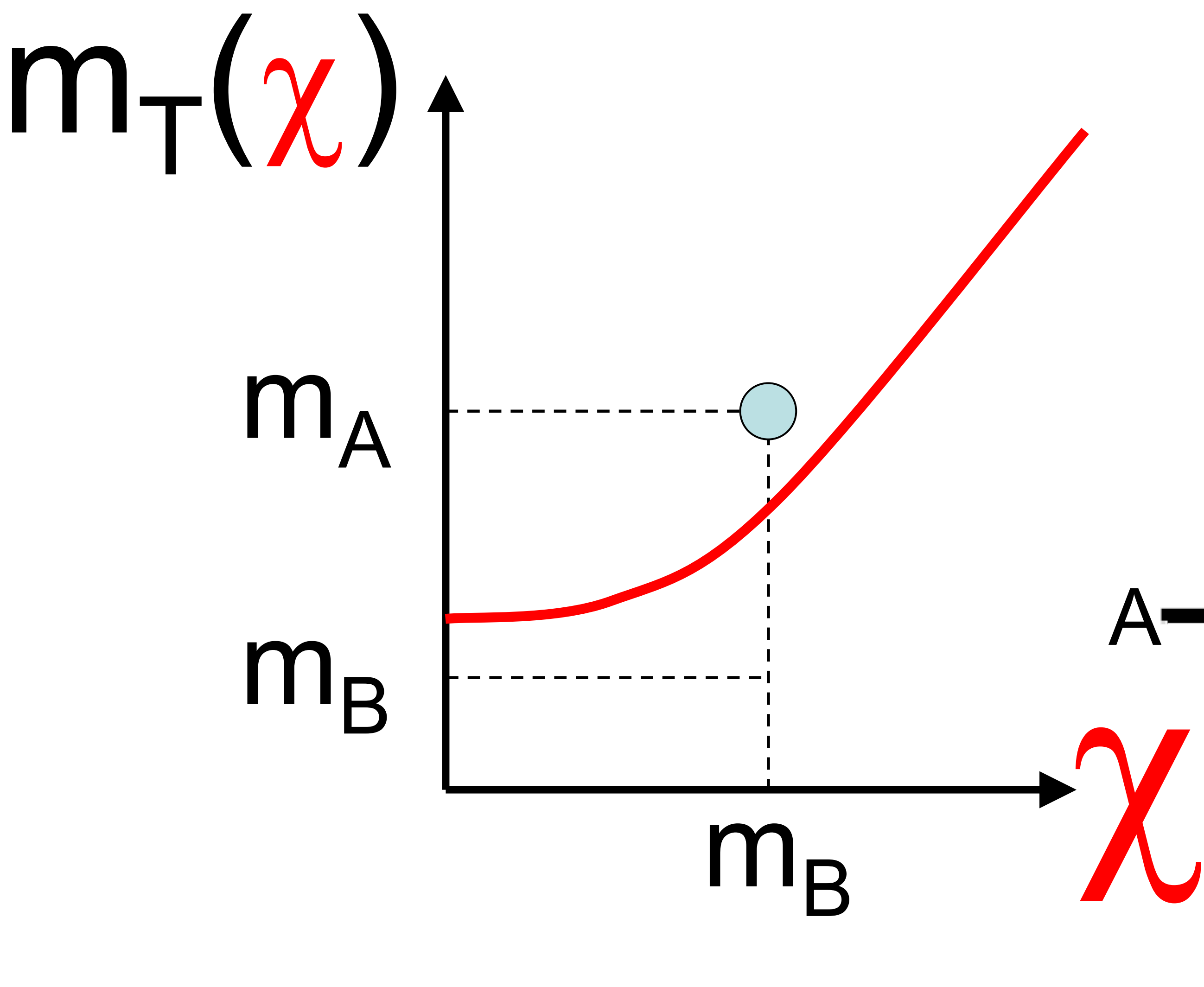
In other words, we will use χ in all the places we previously used M_B.

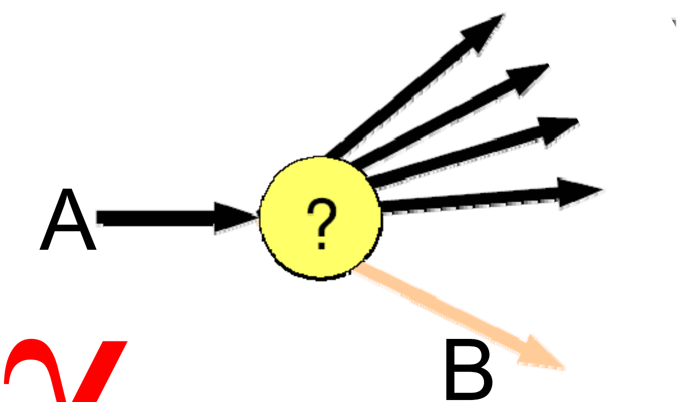
Schematically, all we have guaranteed so far is the picture below:



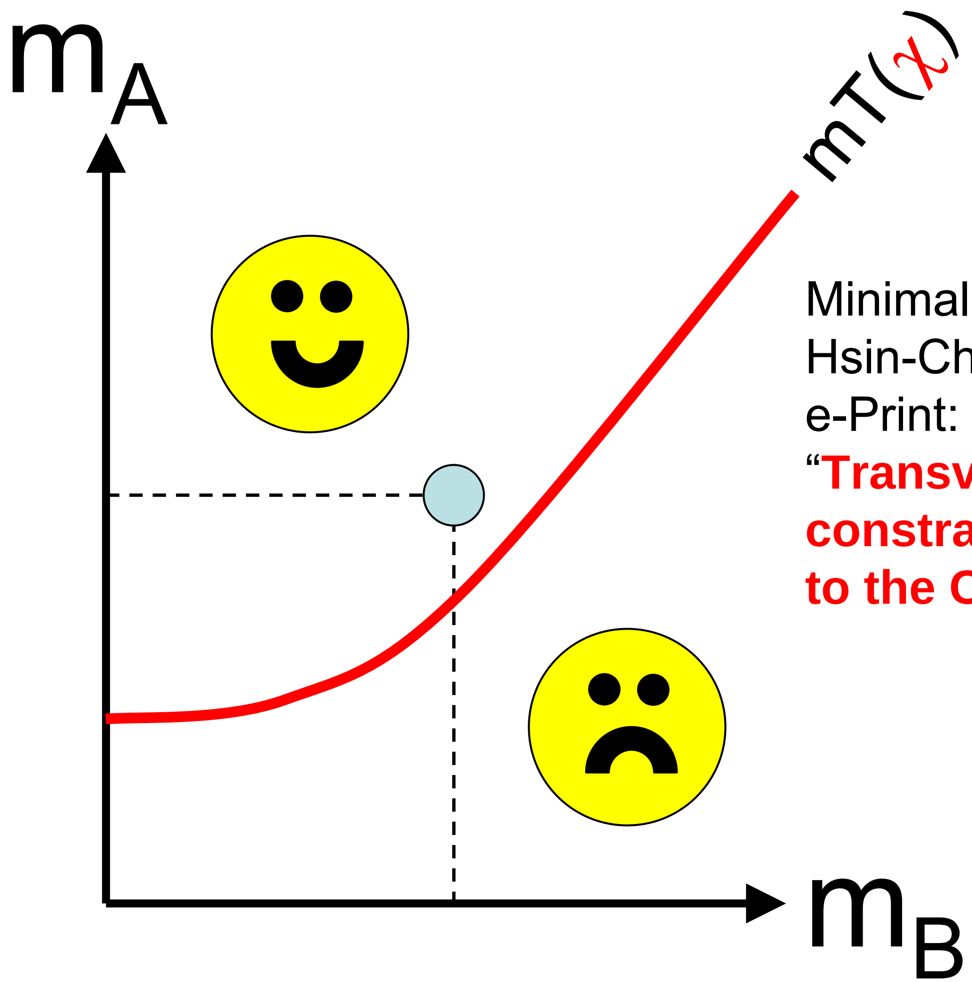
- Since "χ" can now be "wrong", some of the properties of the transverse mass can "break":
- $m_T(\chi)$ max is no longer invariant under transverse boosts! (except when $\chi=m_B$)
- $m_T(\chi)<m_A$ may no longer hold! (however we always retain: $m_T(m_B) < m_A$)

Actual dependence on invisible mass guess χ more like this:



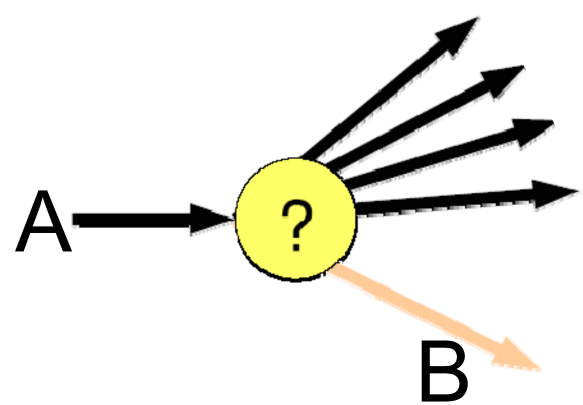


In fact, we get this very nice result:



The "full" transverse mass curve is the boundary of the region of (mother,daughter) masses consistent with the observed event!

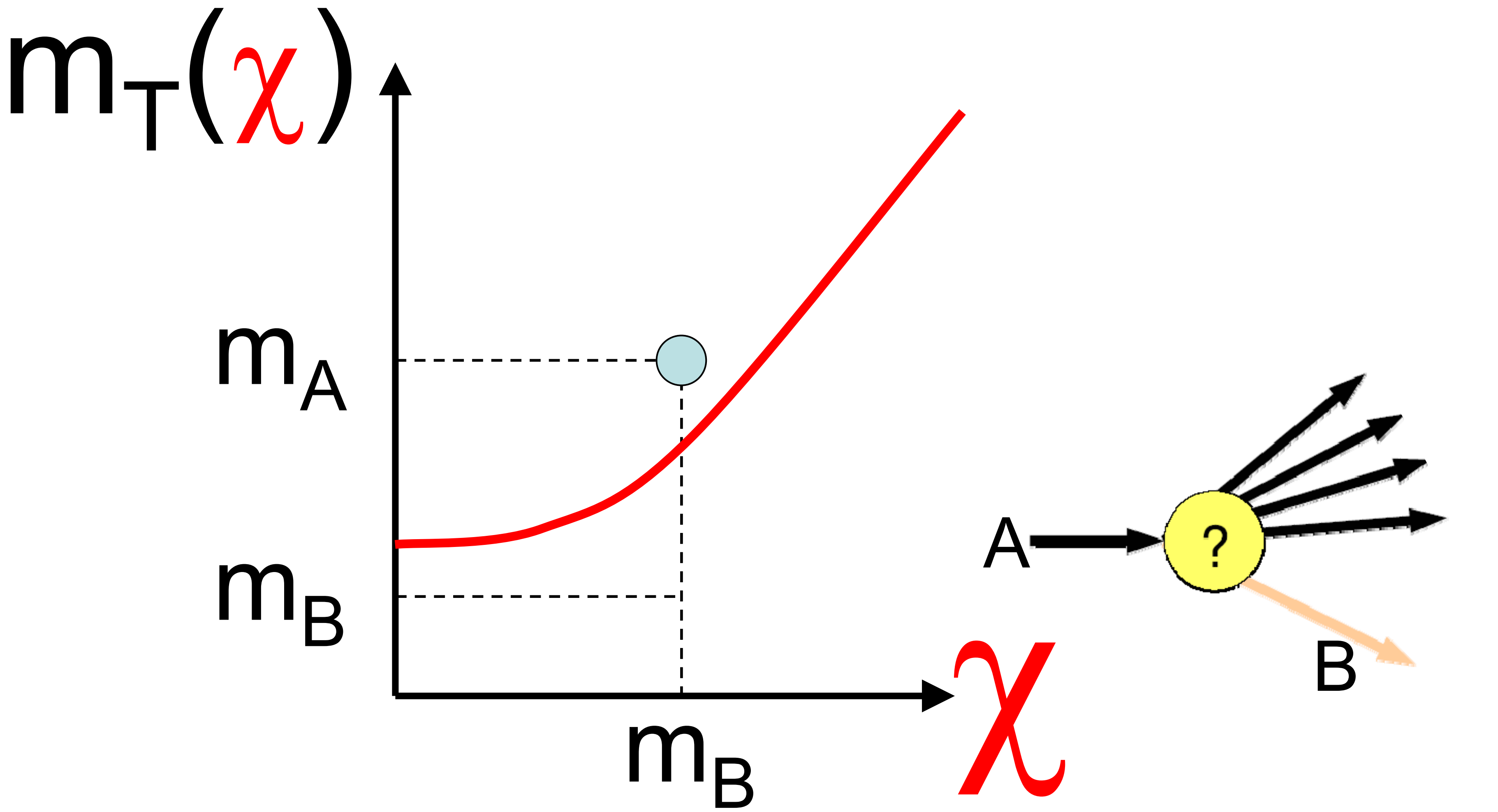
Minimal Kinematic Constraints and m(T2), Hsin-Chia Cheng and Zhenyu Han (UCD) e-Print: arXiv:0810.5178 [hep-ph] and "**Transverse masses and kinematic constraints, from the Boundary to the Crease**" **arXiv:0908.3779**



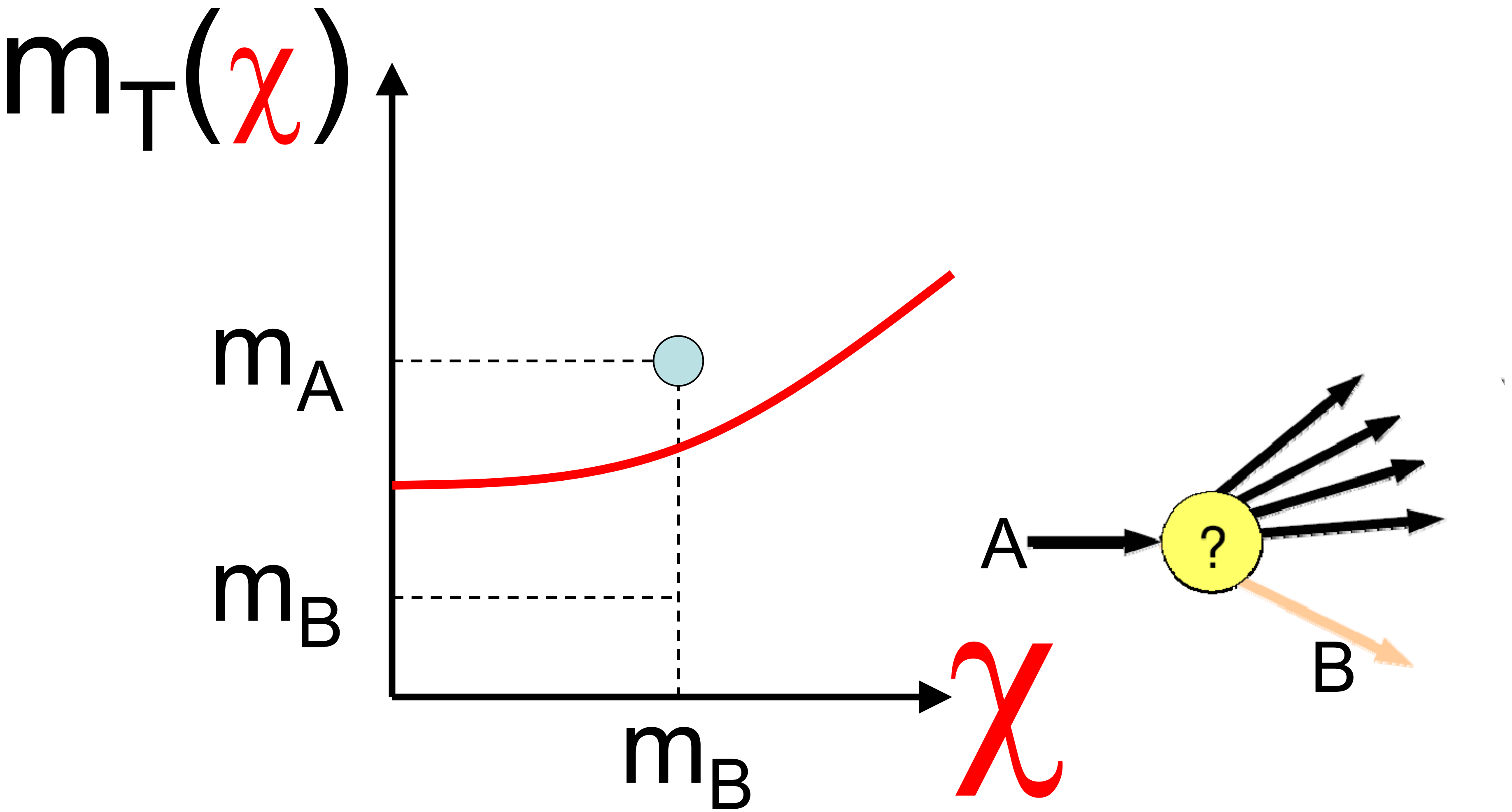
Exercise

- (4) Prove the happy-face/sad-face statement made on the previous slide.
- [Note: not same as exercise (2). There mass of invisible was fixed at true value. Here it is not.]

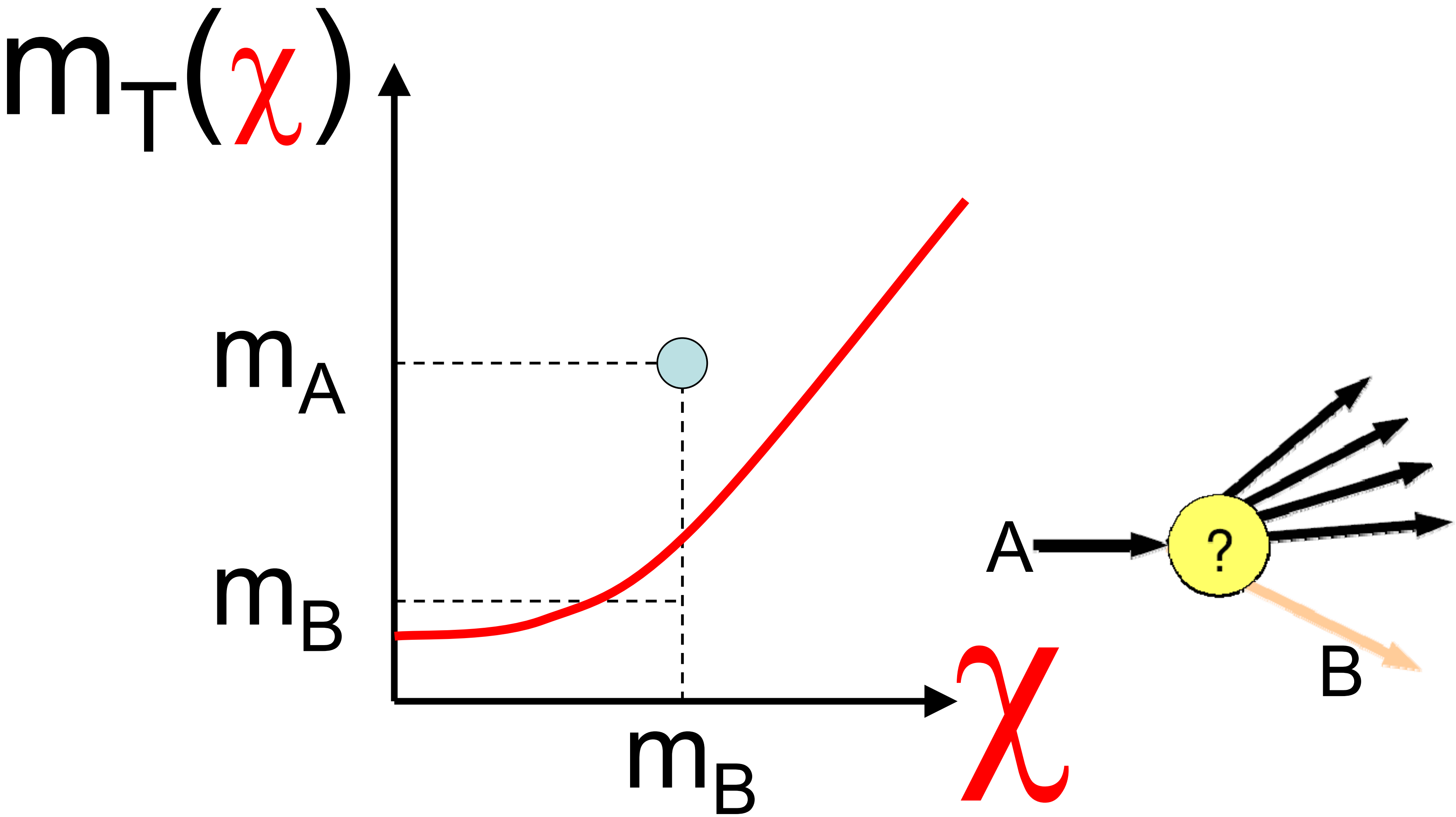
Event 1 of 8



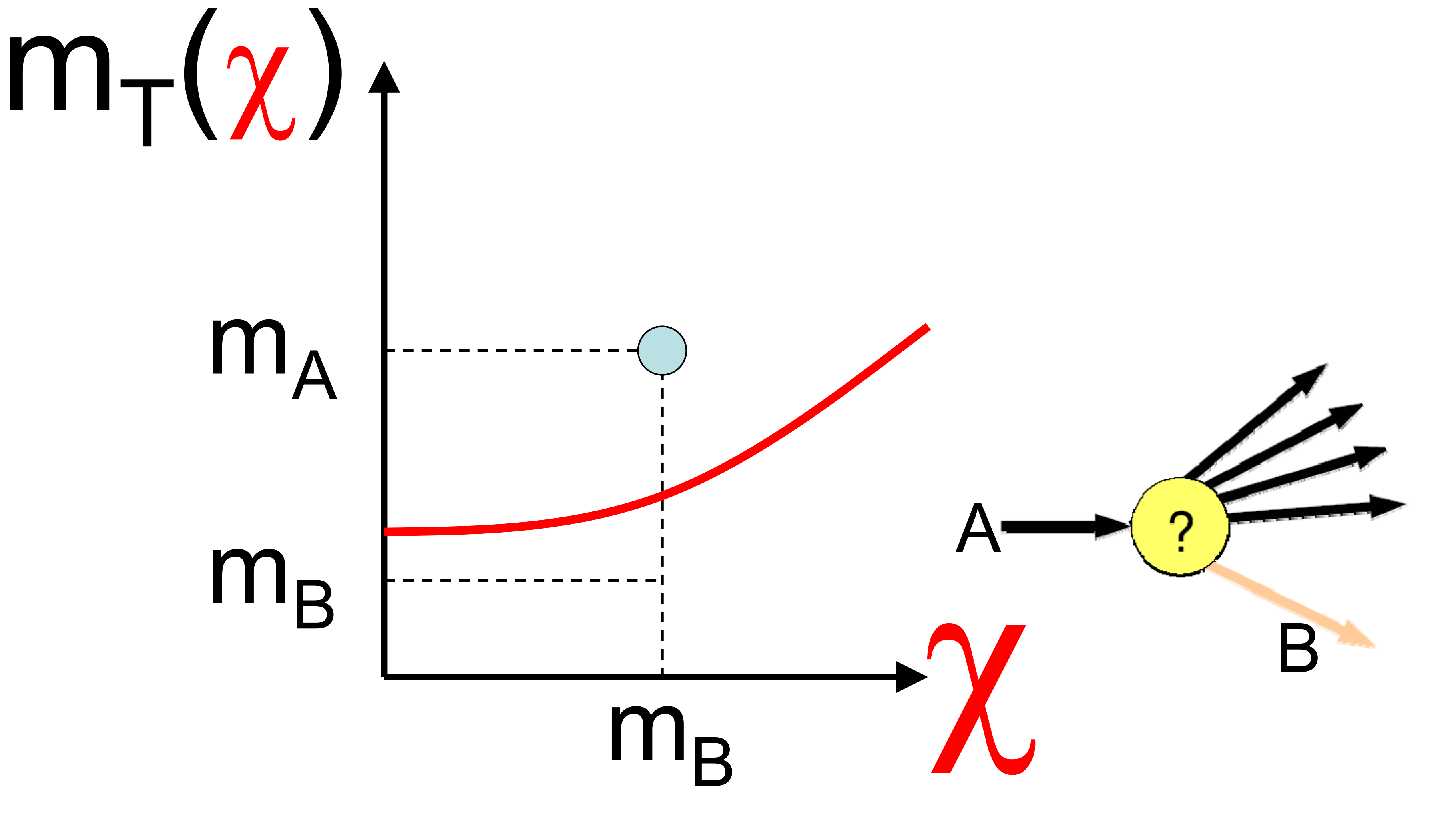
Event 2 of 8



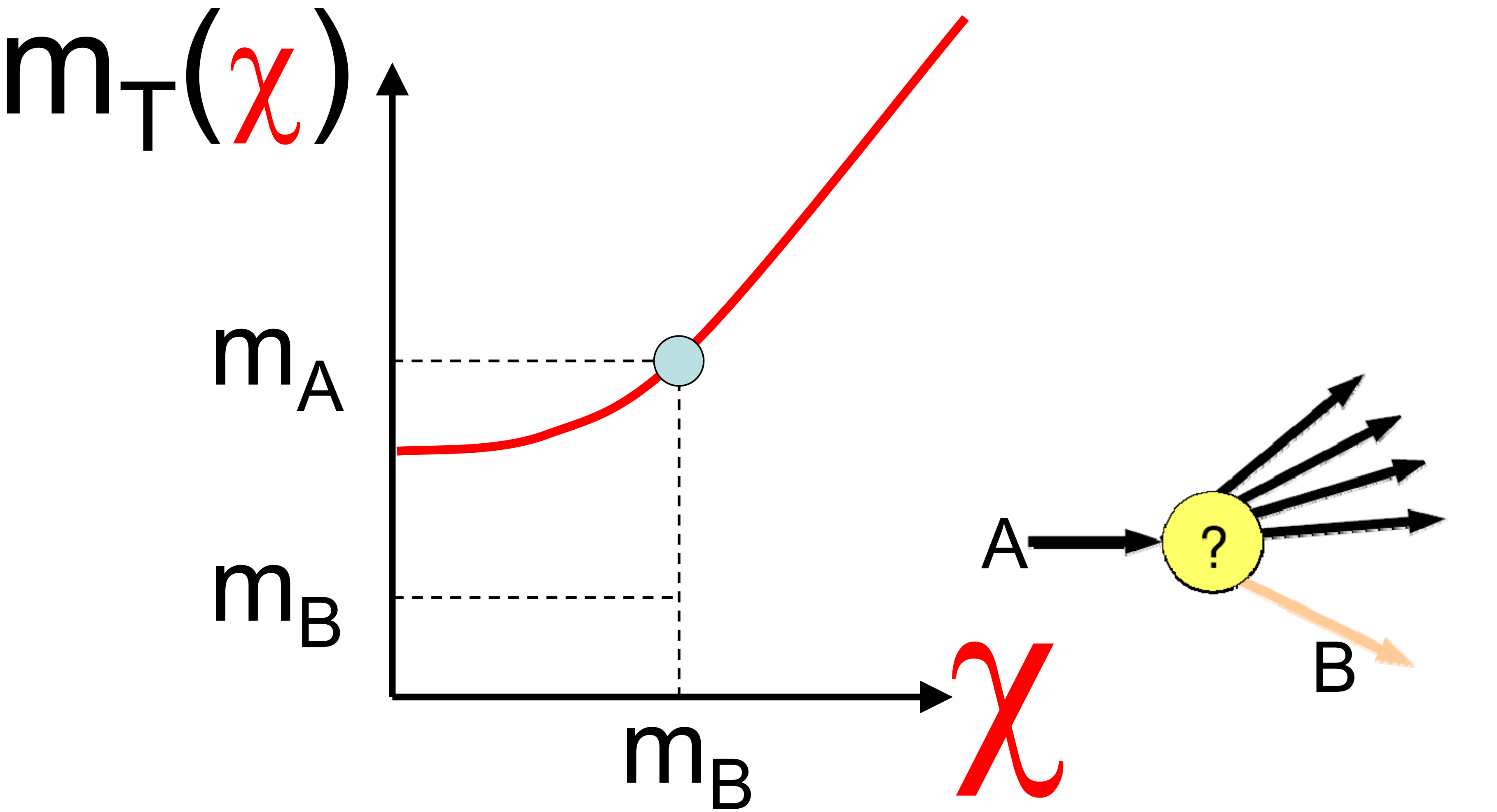
Event 3 of 8



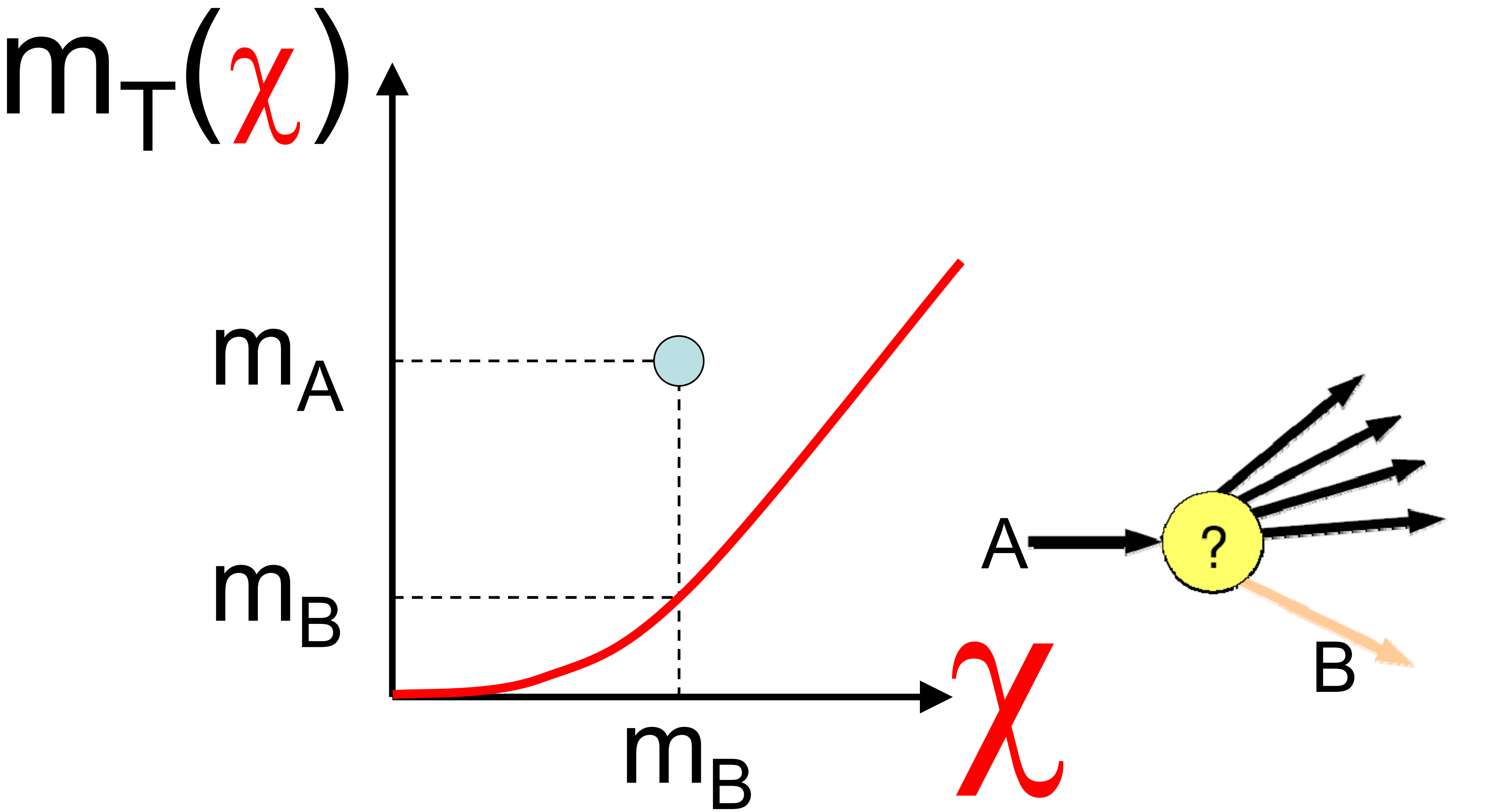
Event 4 of 8



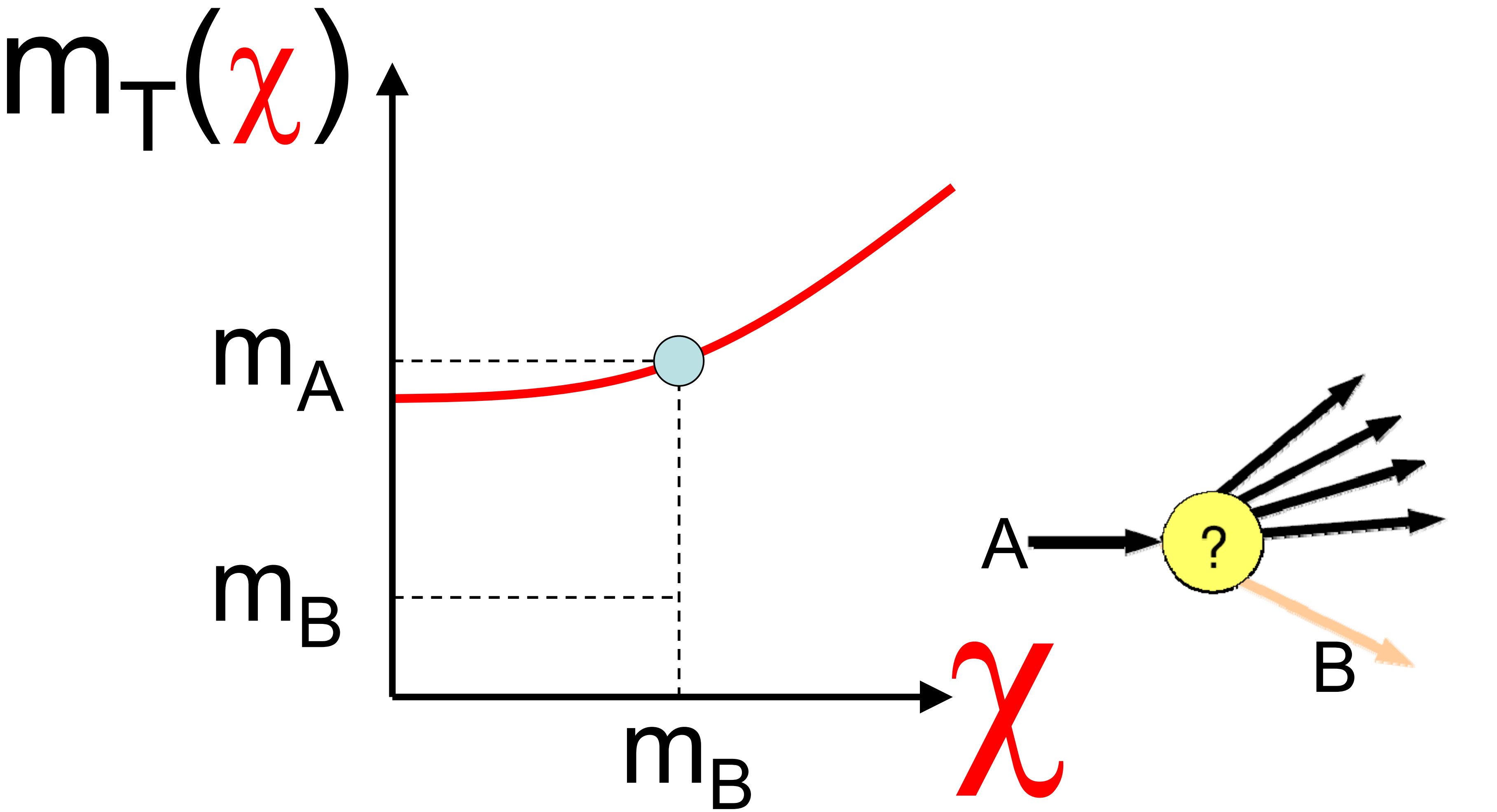
Event 5 of 8



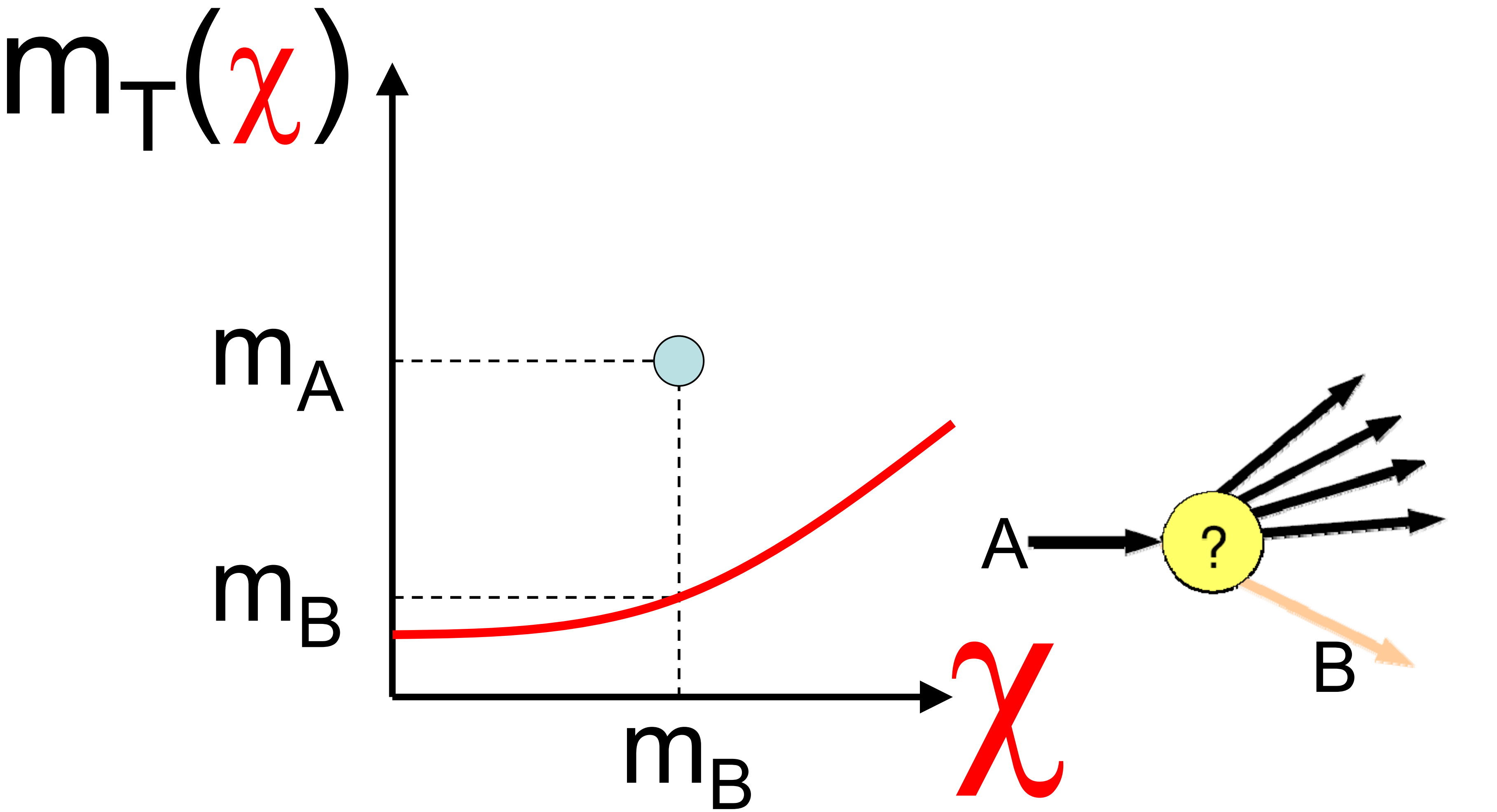
Event 6 of 8



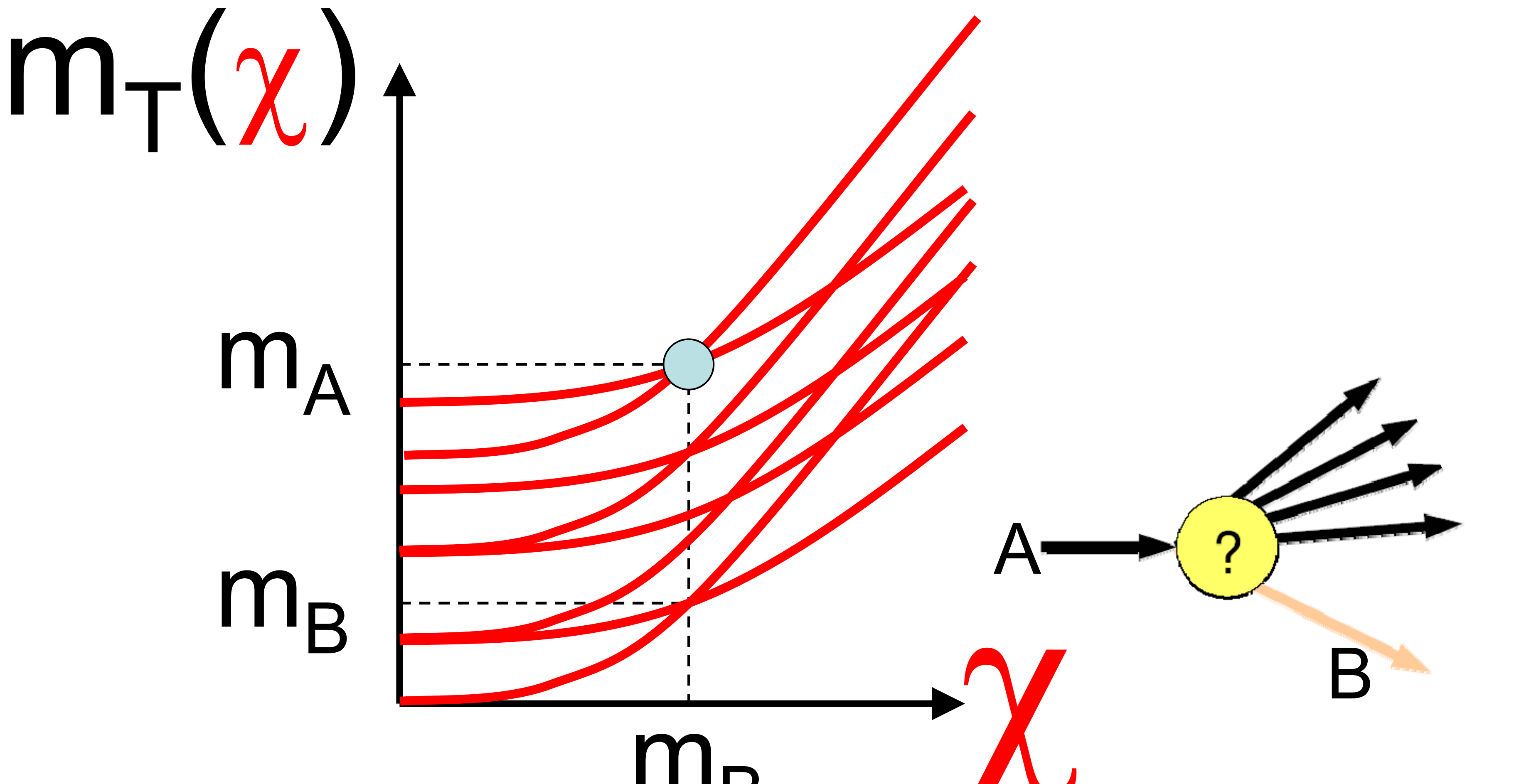
Event 7 of 8



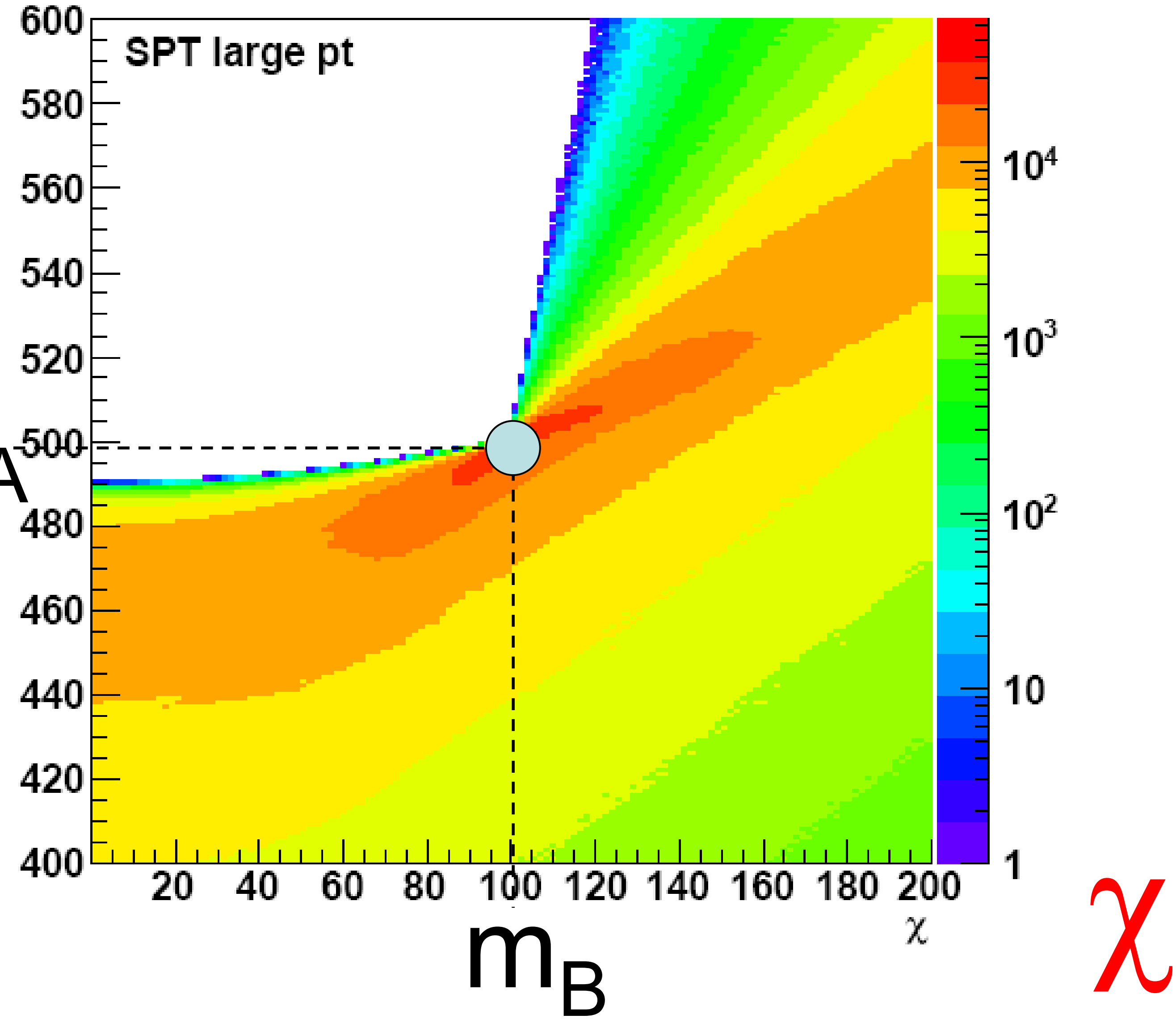
Event 8 of 8



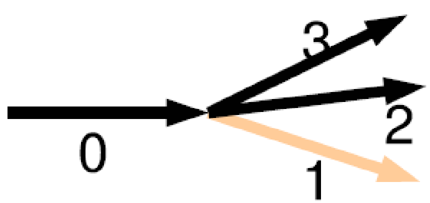
Overlay all 8 events



Overlay many events



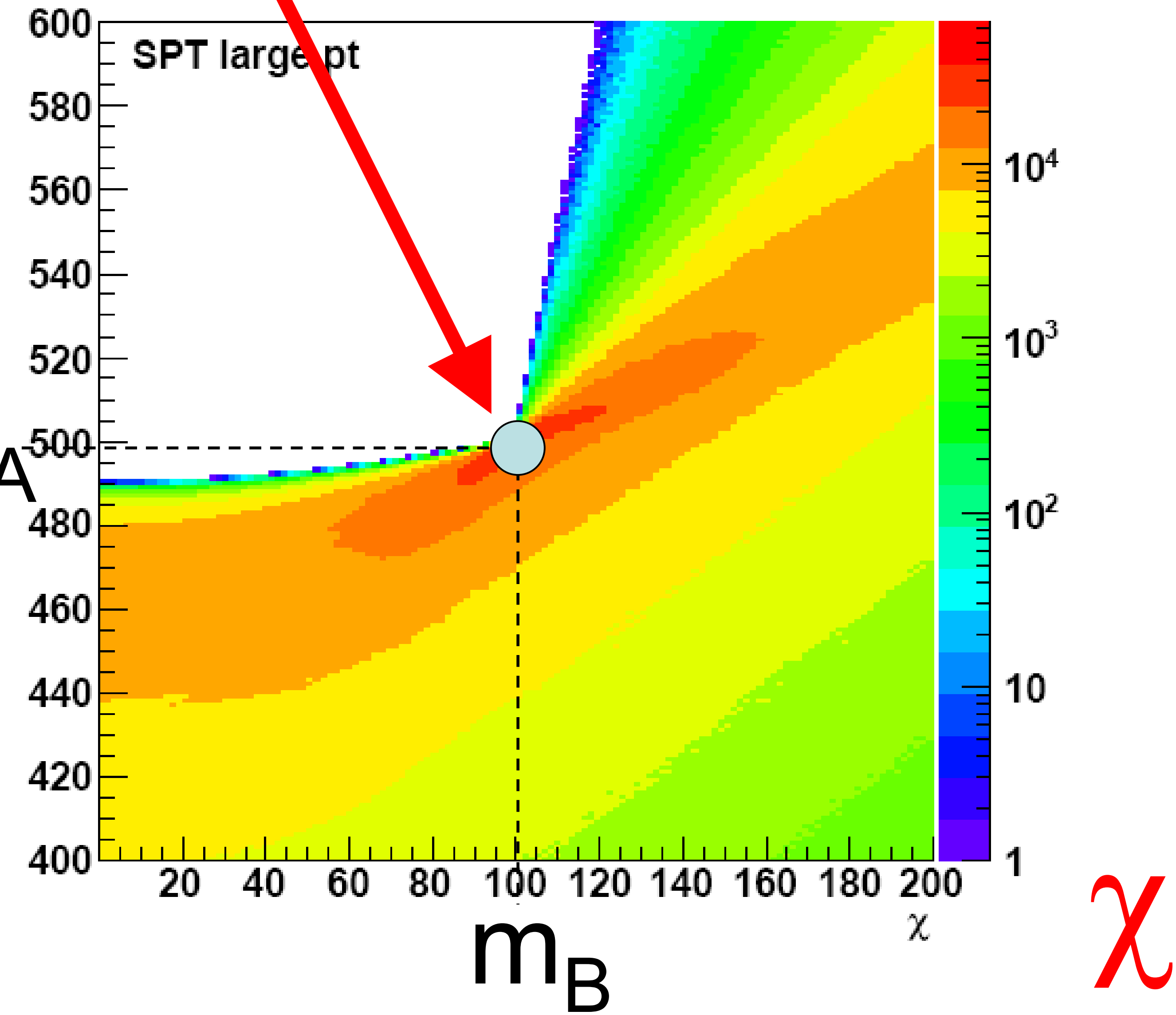
arXiv: 0711.4008



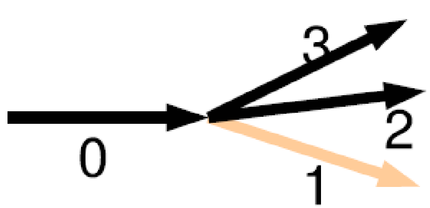
CASE 3v

Weighing Wimps with Kinks at Colliders arXiv: 0711.4008

Here is a transverse mass "KINK"



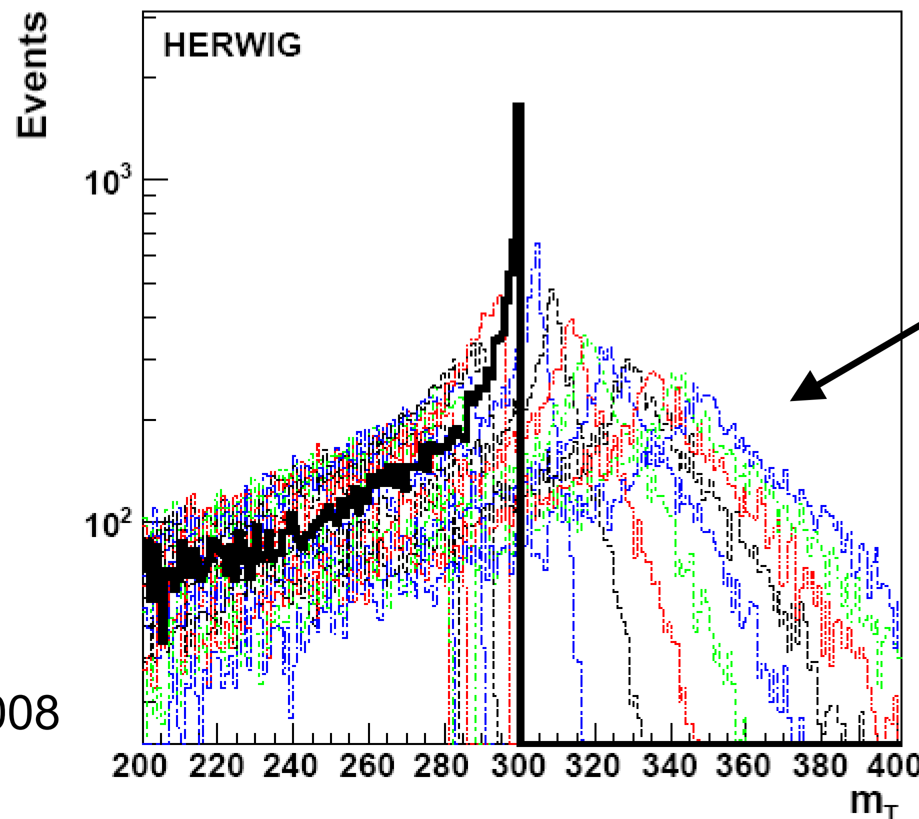
arXiv: 0711.4008



CASE 3v

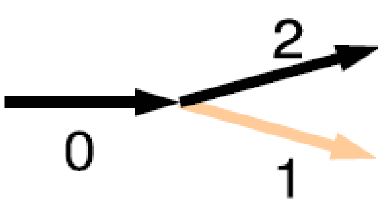
Weighing Wimps with Kinks at Colliders arXiv: 0711.4008

Alternatively, look at M_T distributions for a variety of values of chi.



Each curve has a different value of chi

arXiv: 0711.4008



CASE 2

m_T

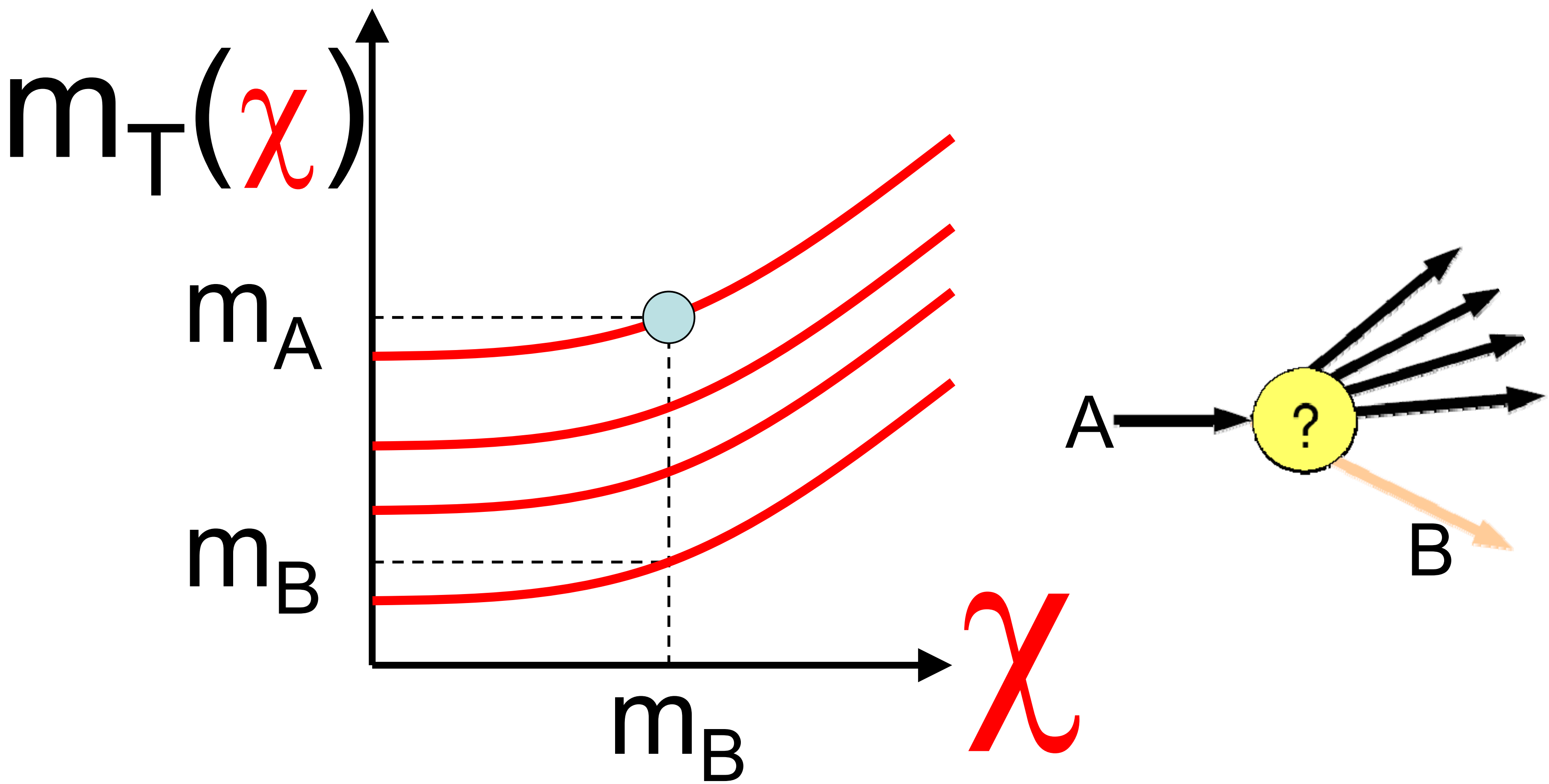
Where is the kink now?

What causes the kink?

- Two entirely independent things can cause the kink:
  - (1) Variability in the "visible mass"
  - (2) Recoil of the "interesting things" against Upstream Transverse Momentum
- Which is the dominant cause depends on the particular situation … let us look at each separately:

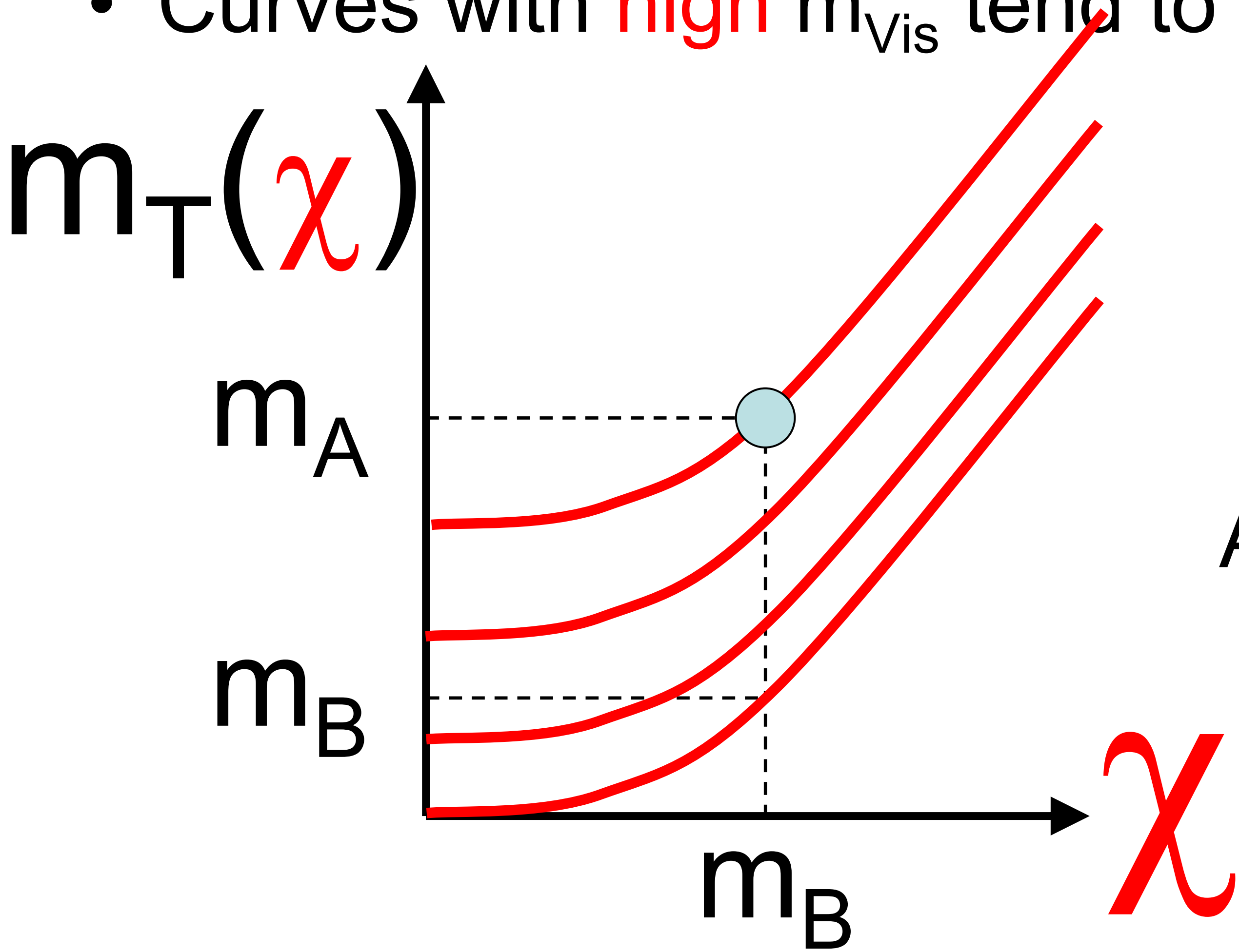
Kink cause 1: Variability in visible mass

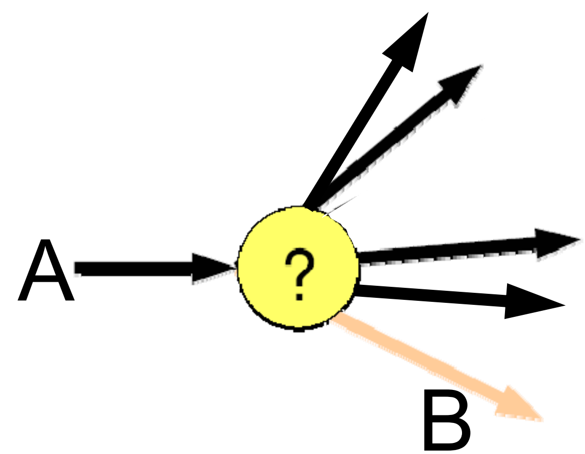
- m_{Vis} can change from event to event
- Gradient of $m_T(\chi)$ curve depends on m_{Vis}
- Curves with low m_{Vis} tend to be "flatter"



Kink cause 1: Variability in visible mass

- m_{Vis} can change from event to event
- Gradient of $m_T(\chi)$ curve depends on m_{Vis}
- Curves with high m_{Vis} tend to be "steeper"

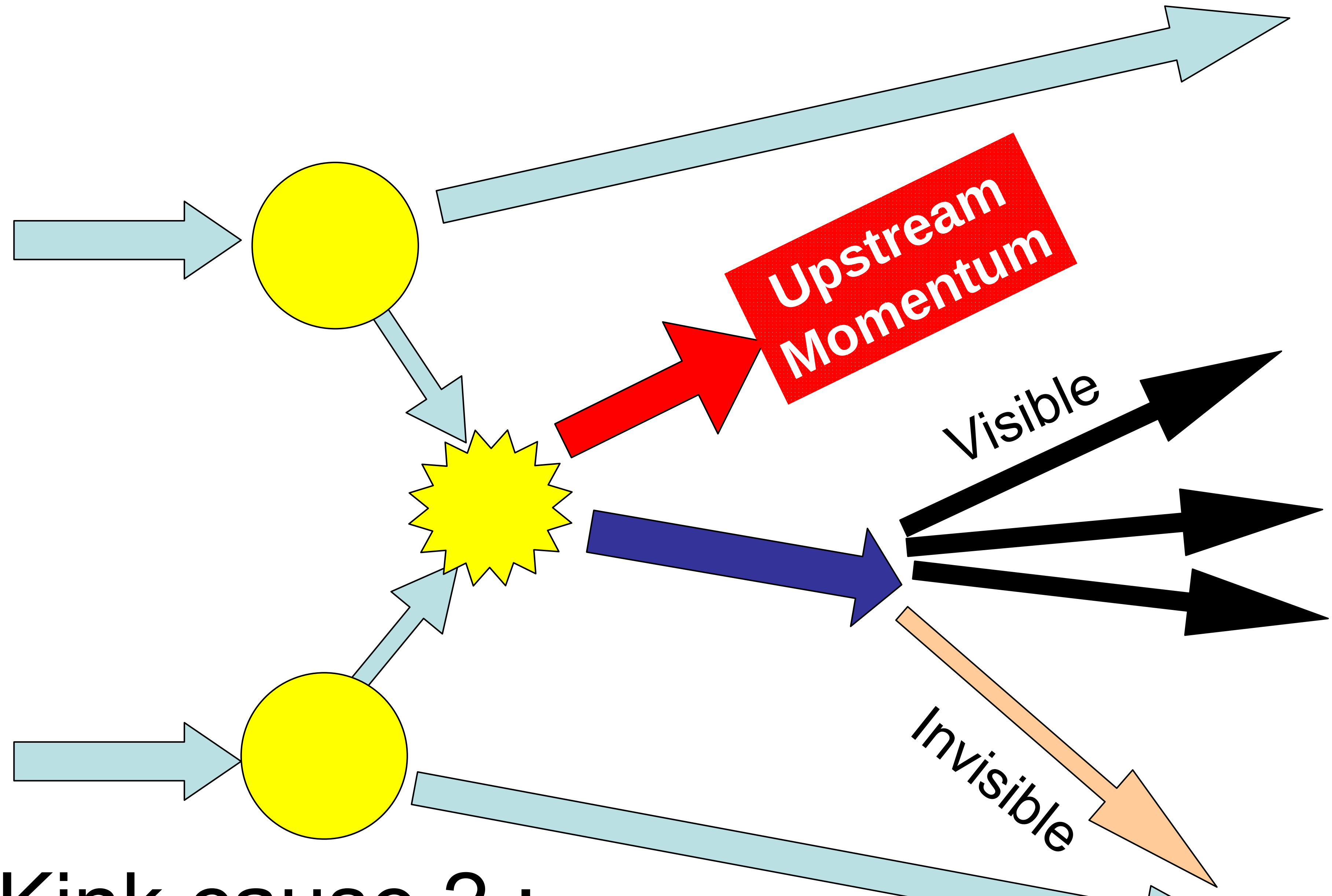




Exercise: $M \to (a_1 a_2) b$

For the three body decay $M \to (a_1 a_2) b$ where a_1 and a_2 are visibles of known masses, while the b is invisible.

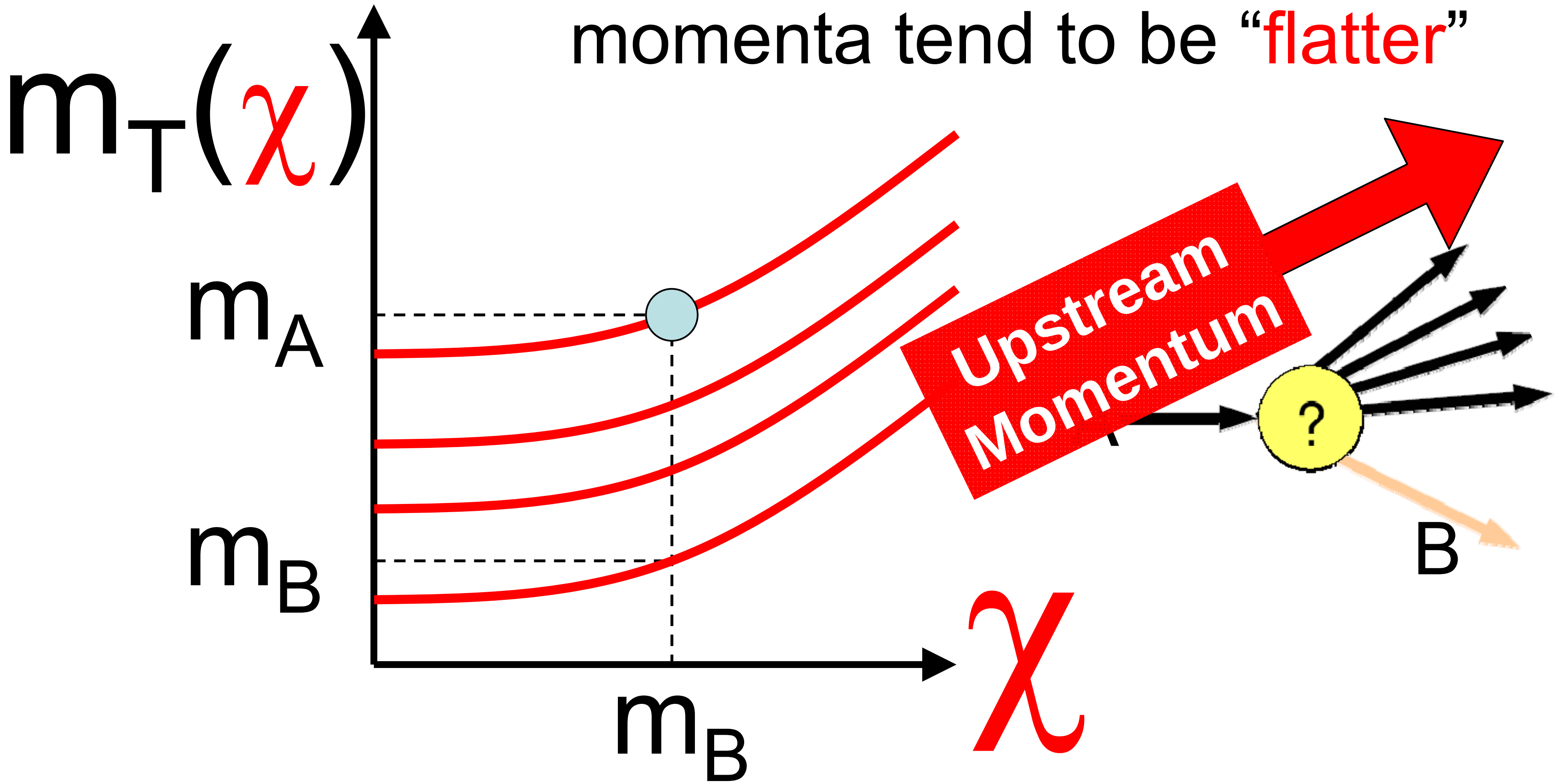
- (5) Satisfy yourself that, at the true value of the invisible mass, events can have M_T values that saturate the bound (i.e. have $M=M_T$) regardless of the invariant mass "m_{vis}" of the $a_1 a_2$ system.
- (6) Sketch a proof of the statements made in the last two slides – in some limit if necessary.



Kink cause 2 :
Recoil against Upstream Momentum

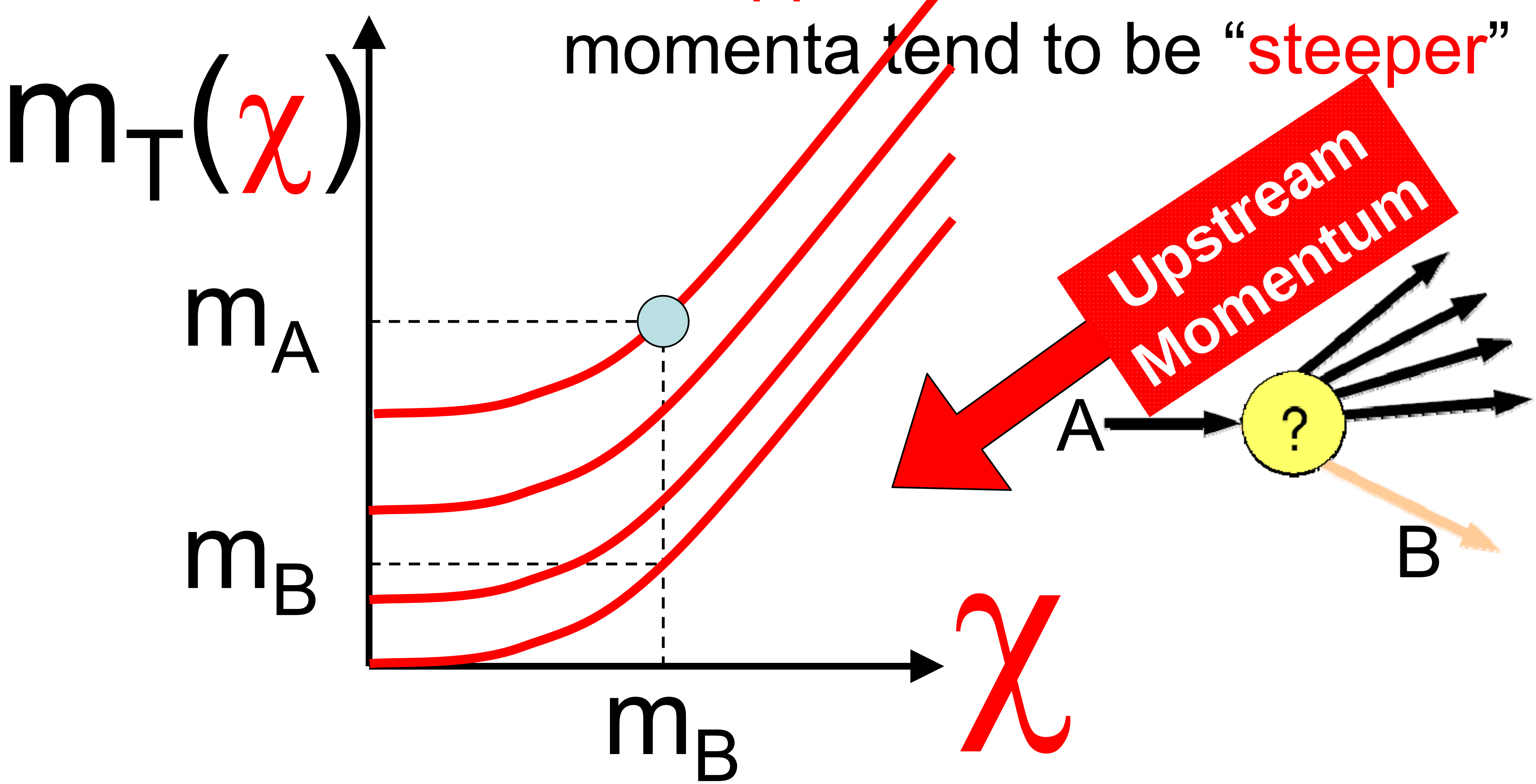
Kink cause 2: Recoil against UTM

- UTM can change from event to event
- Gradient of $m_T(\chi)$ curve depends on UTM
- Curves with UTM parallel to visible momenta tend to be "flatter"



Kink cause 2: Recoil against UTM

- UTM can change from event to event
- Gradient of $m_T(\chi)$ curve depends on UTM
- Curves with UTM opposite to visible momenta tend to be "steeper"



Exercise

- (7) Sketch a proof of the statements of the last two slides (if necessary, only for special cases of your choice)

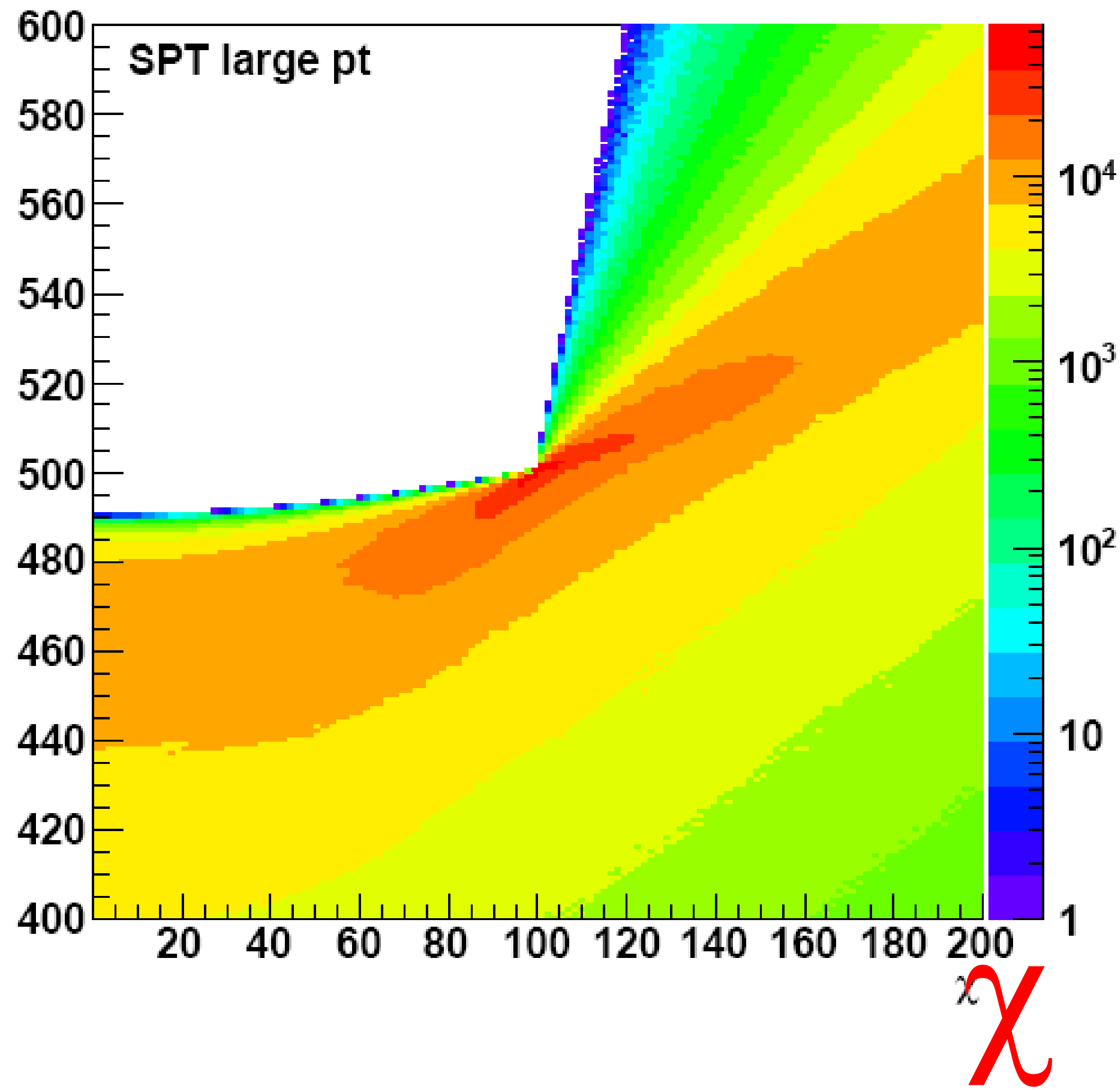
Health warning!

(for those of you interested in LHC dark matter constraints)

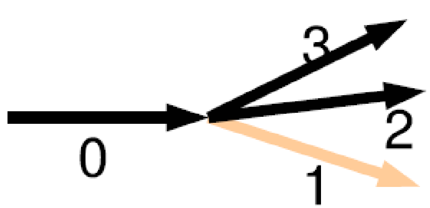
Rather worryingly, M_T kinks are at present the only known kinematic methods which (at least in principle) allow determination of the mass of the invisible particle in short chains at hadron colliders!

[We will see a dynamical method that works for single three+ body decays shortly. Likelihood methods can determine masses in pair decays too, though at cost of model dependence and CPU. See Alwall.]

That last statement should worry you!



arXiv: 0711.4008

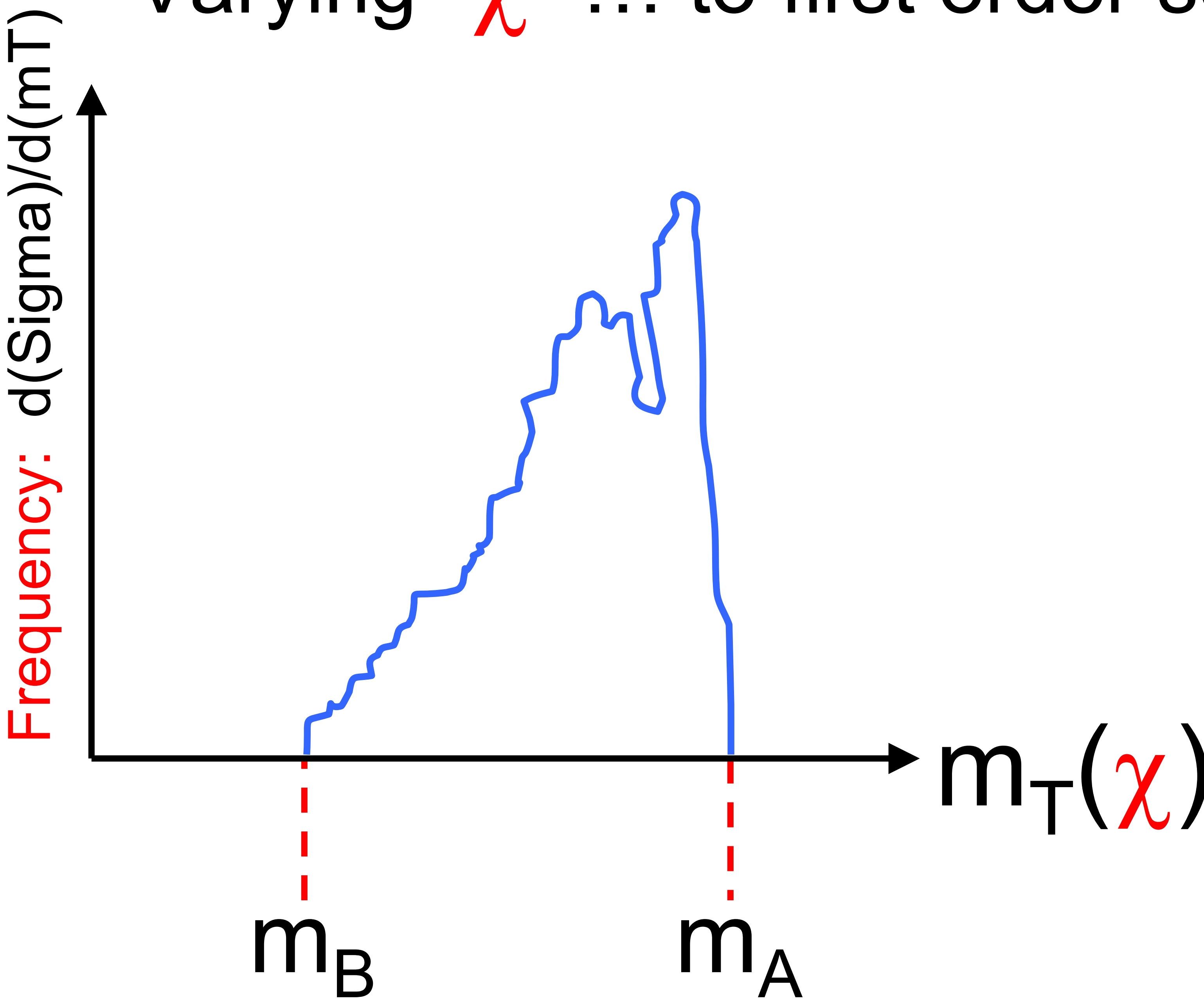


CASE 3v

Weighing Wimps with Kinks at Colliders arXiv: 0711.4008

Spot the kink

Varying “χ” … to first order see:



Take home messages for MT

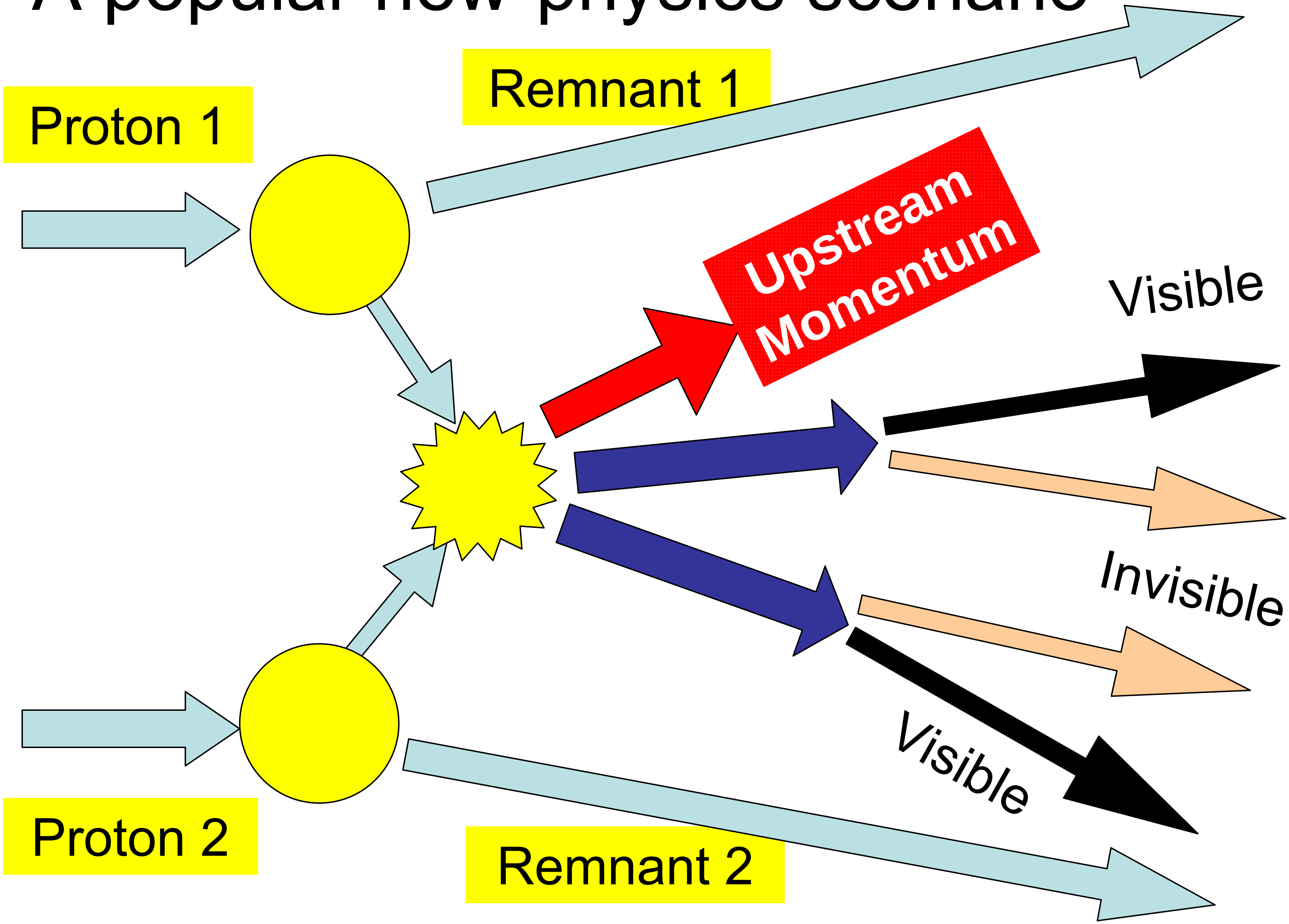
- **EASY to get MASS DIFFERENCE**
- We have two **independent <u>kinematical</u>** opportunities to measure **invisible daughter mass** in single particle decays:
  - “Upstream boost induced” MT kink
    - from ISR alone, useless, from real UTM, possible
  - “Variable visible mass induced” MT kink
    - impossible in 2-body decay, otherwise possible
  - HARD to set absolute mass scale

- We used pT-miss information – so only works with one invisible (so far …)

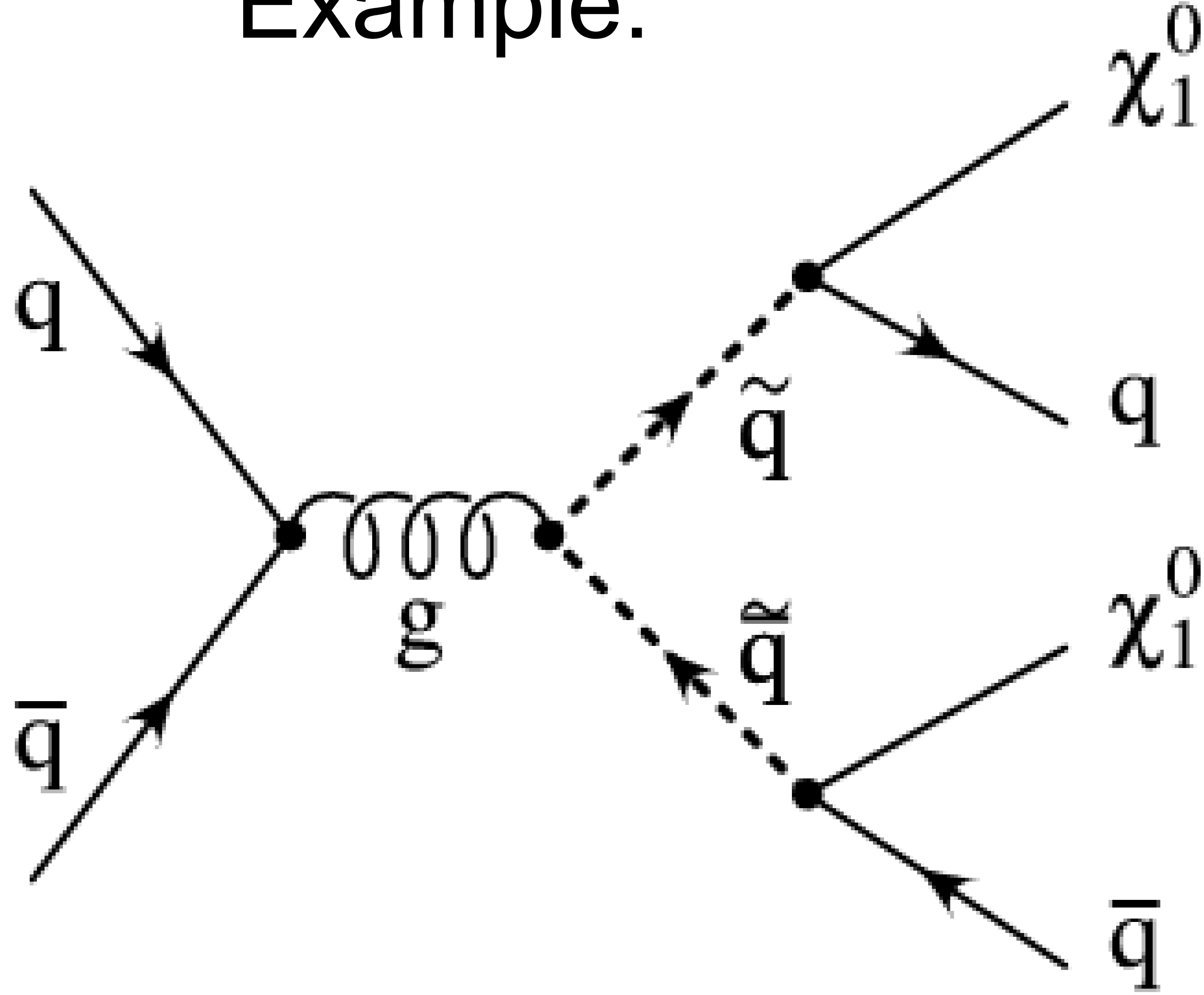
Change of topic:

How do we measure masses when there is Pair Production ?

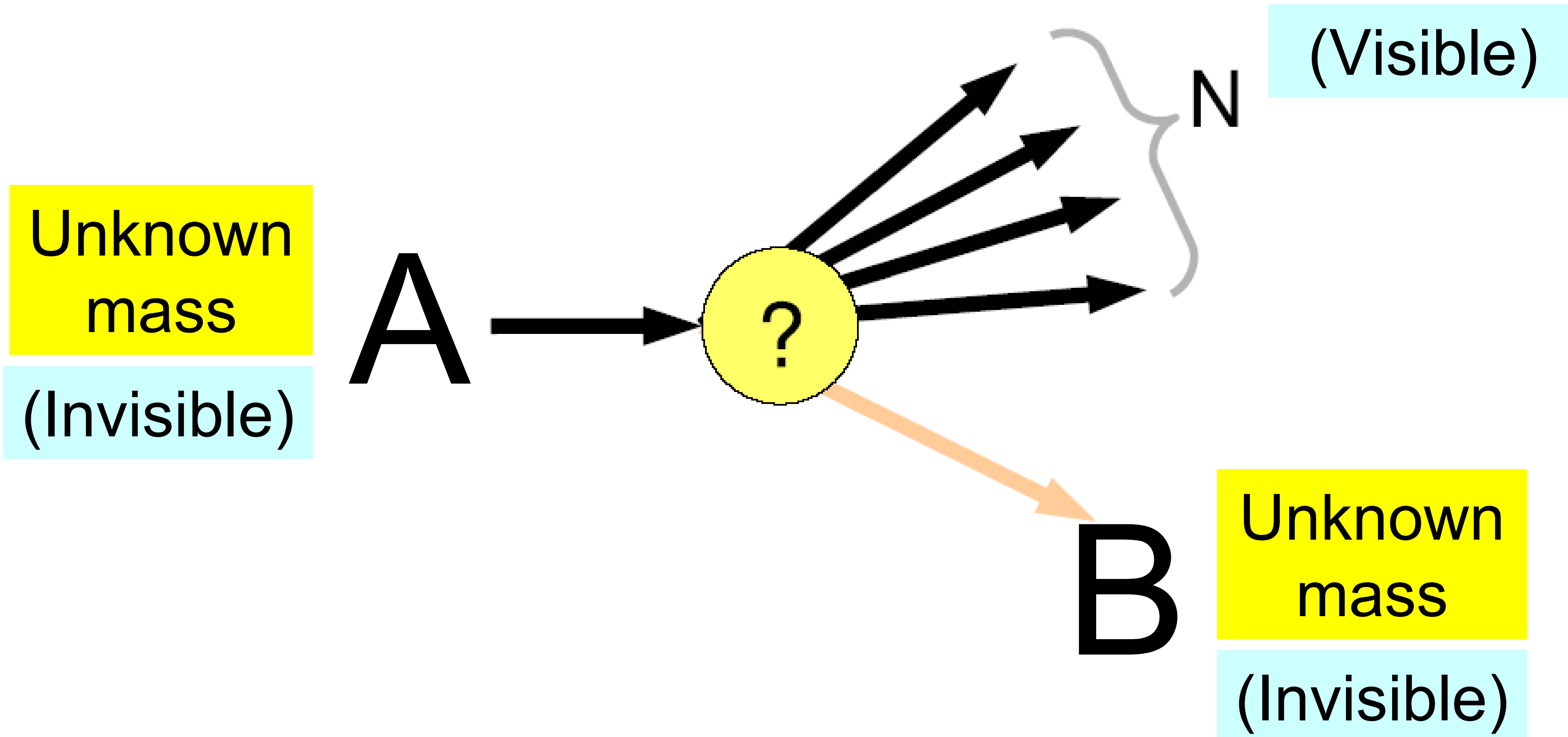
A popular new-physics scenario



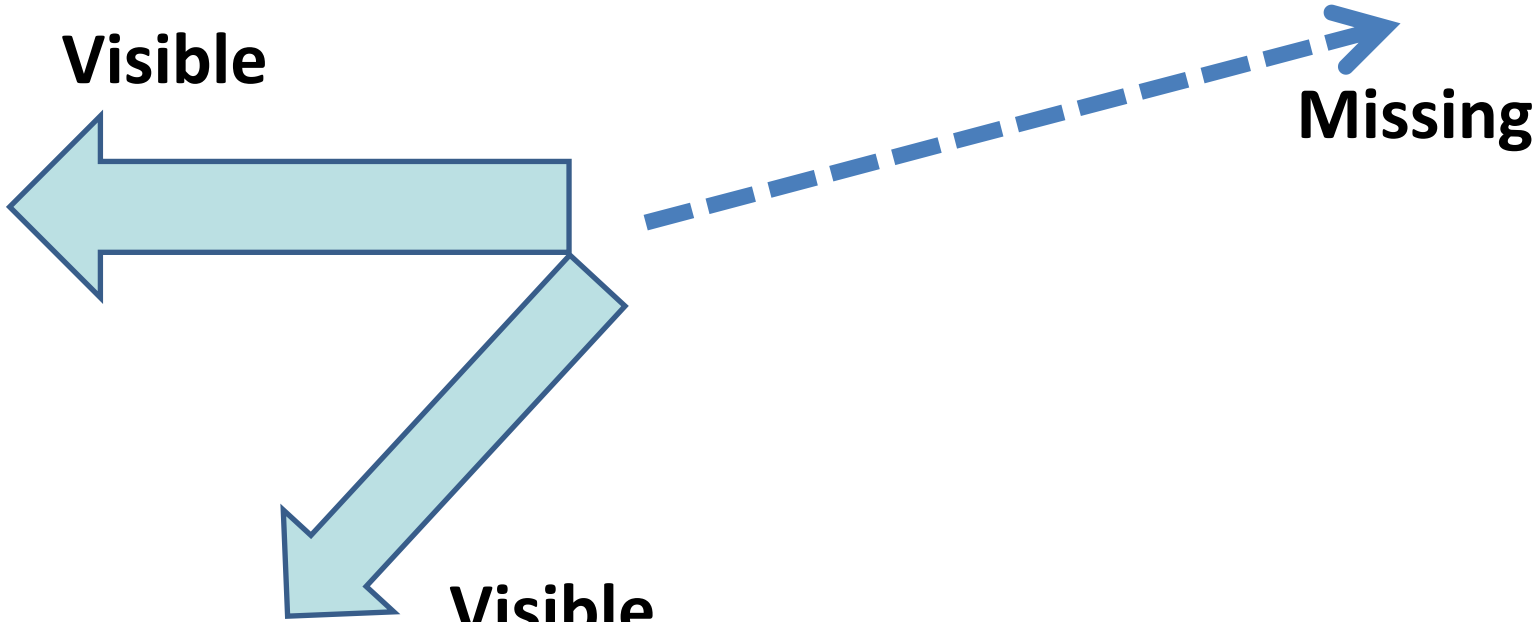
Example:

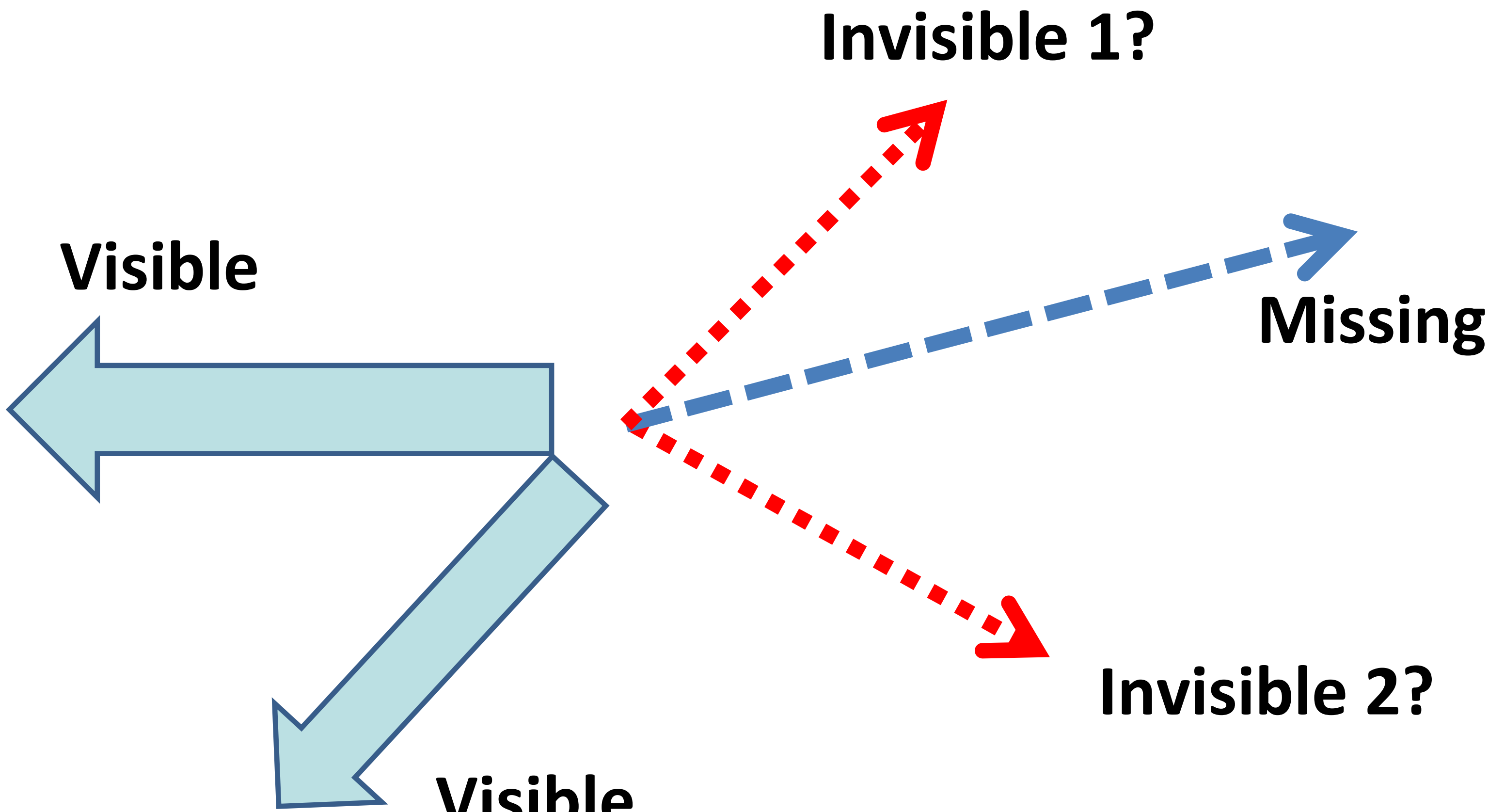


We have two copies of this:

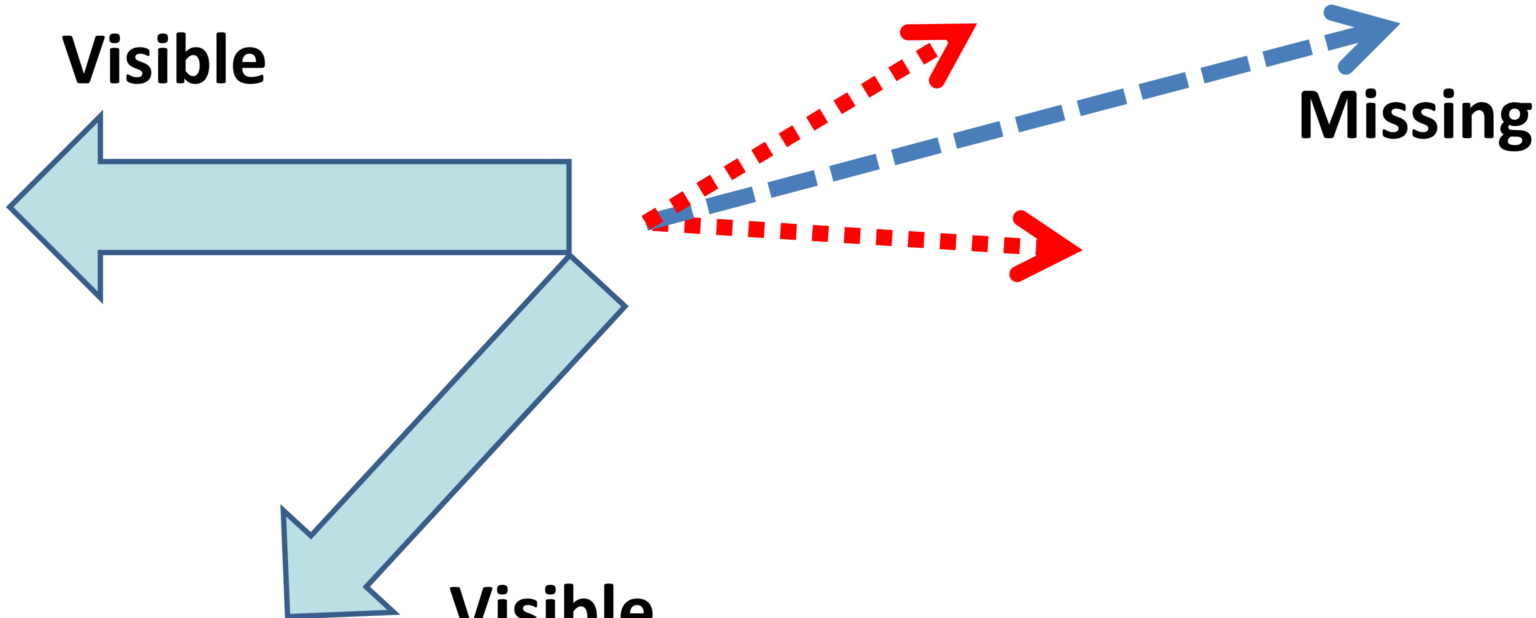


But don't know p_T of B this time! ☹

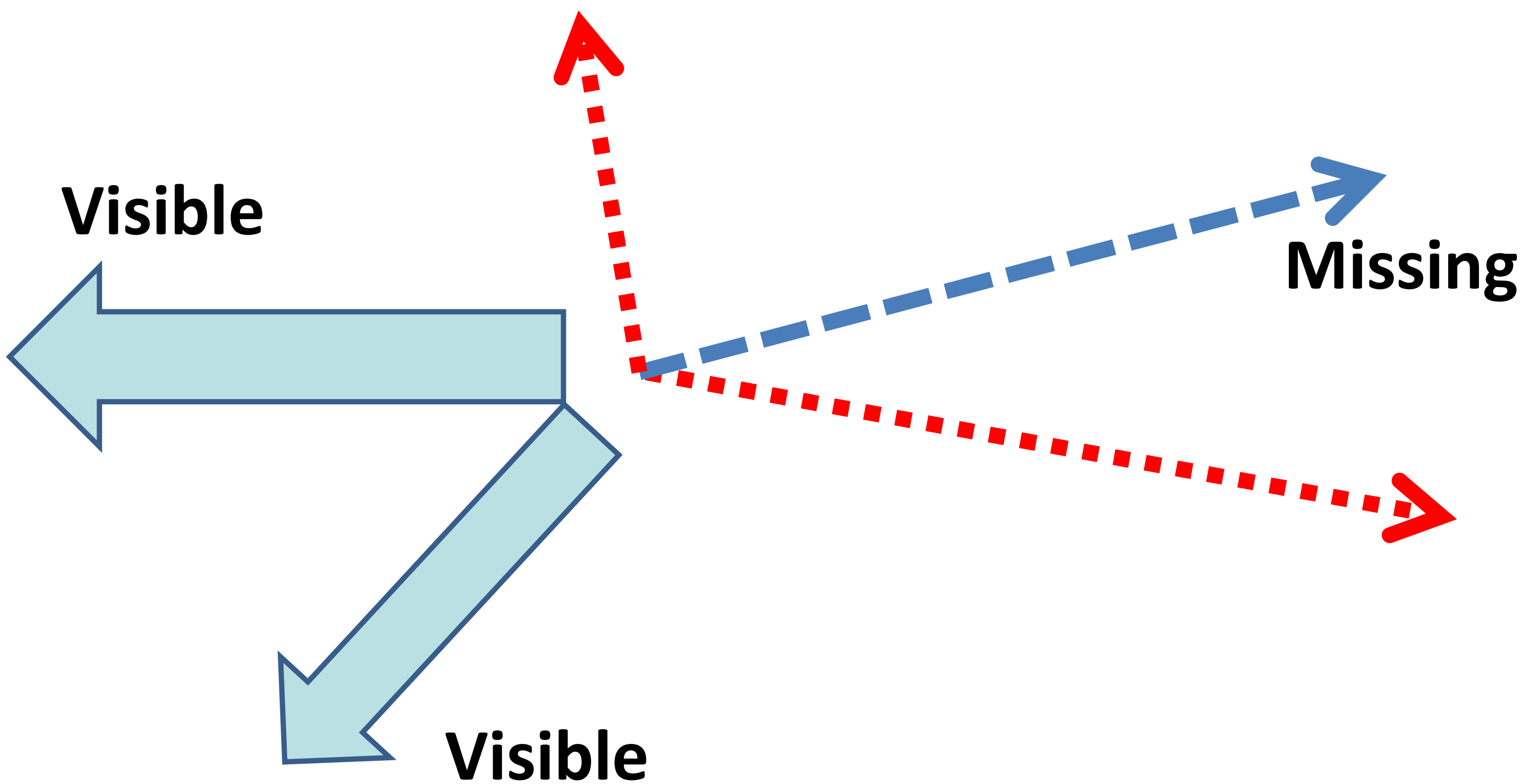

Visible
Missing
Visible



a possible “splitting”



another possible “splitting”



another possible “splitting”

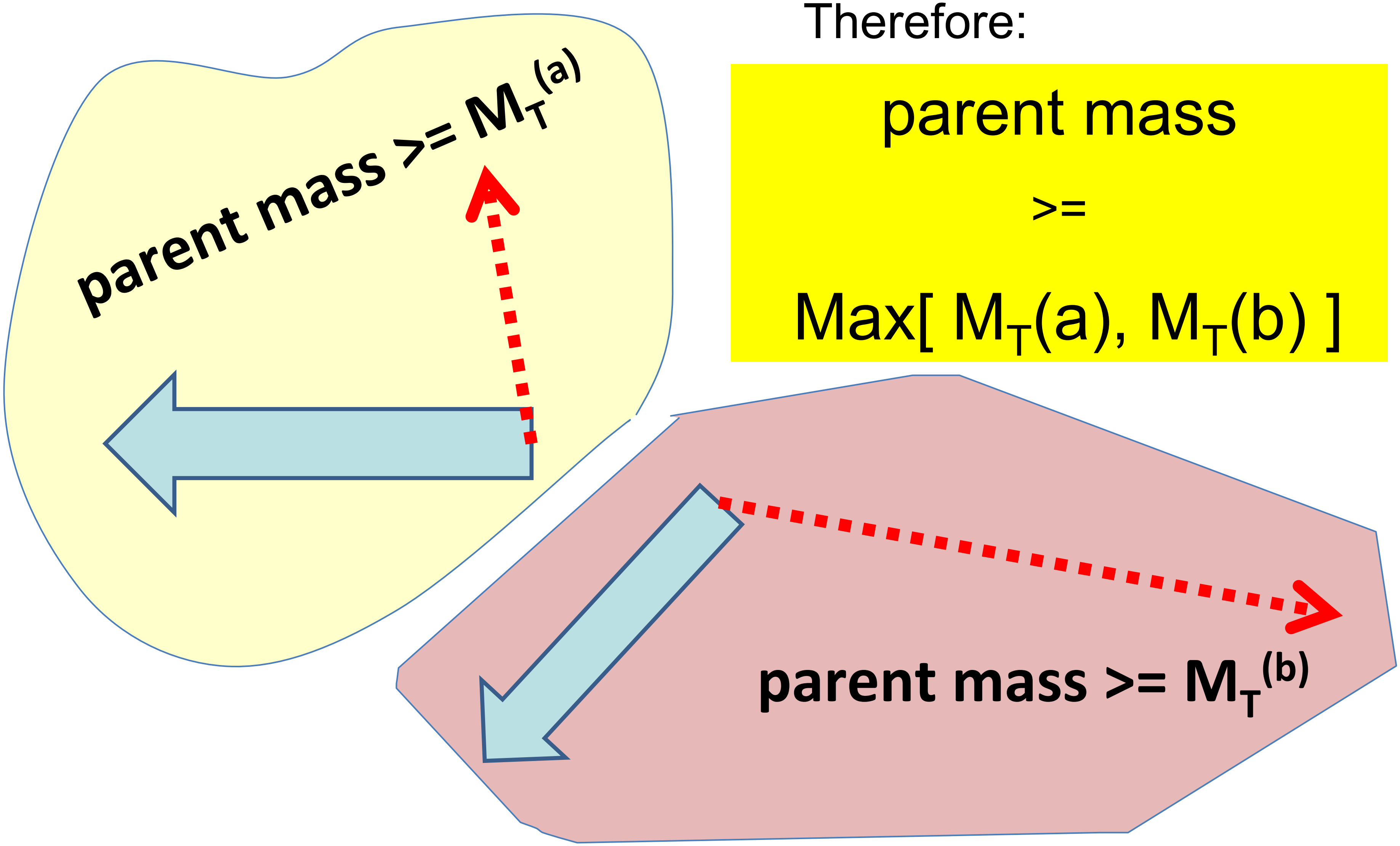
If **<u>this</u>** splitting is "correct":

Therefore:

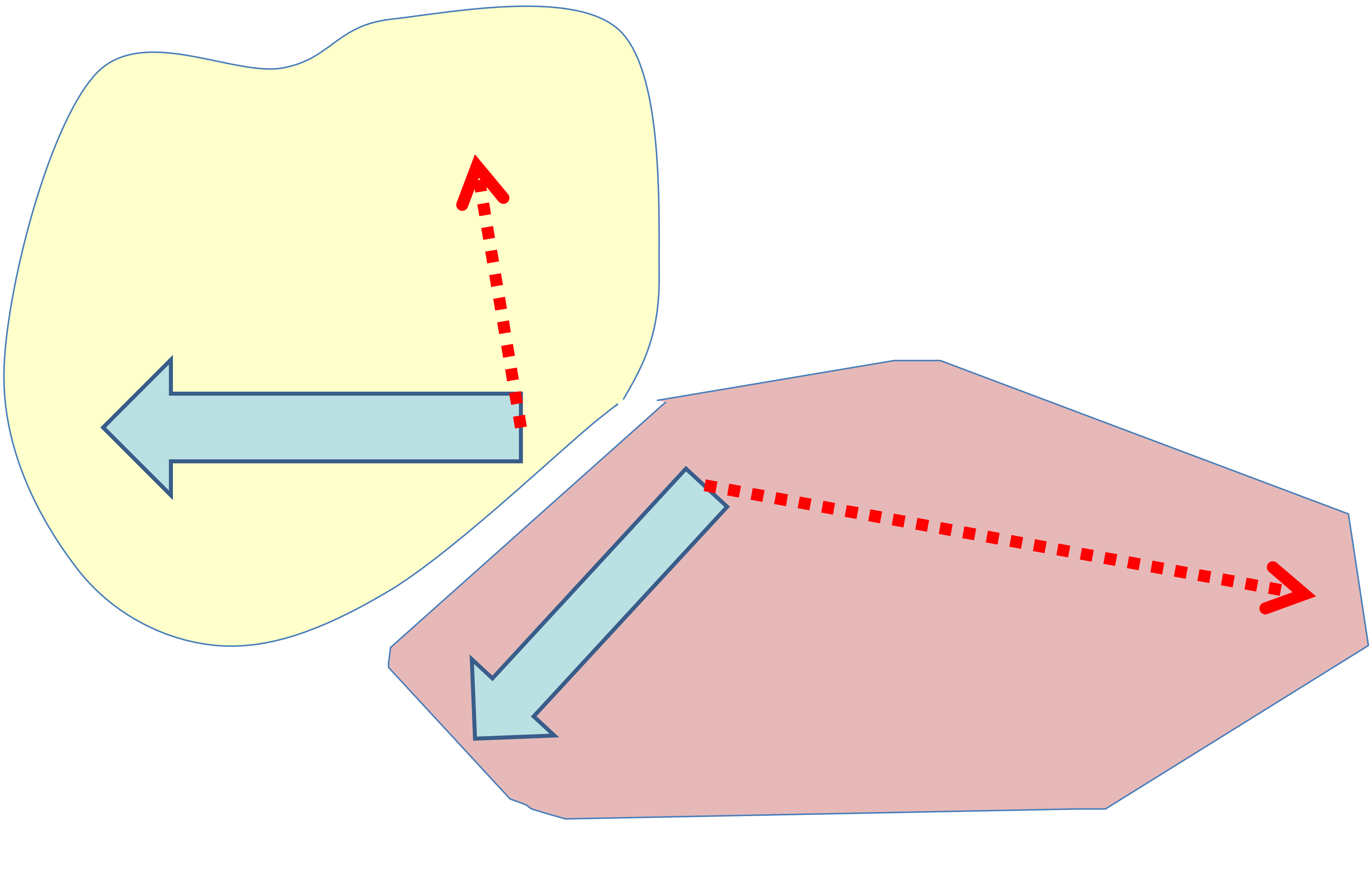
parent mass

$>=$

$\mathrm{Max}[\ M_T(a),\ M_T(b)\ ]$



But this splitting might be wrong!



But can say that:

$$\text{parent mass} \geq \underset{\substack{\text{over all splittings} \\ \text{of ptmiss}}}{\text{Min}}\left\{ \text{Max}[\, M_T(a), M_T(b) \,] \right\}$$

Lester and Summers (hep-ph/9906349)

This is m_{T2} the "Stransverse Mass"

$$\boxed{m_{T2}(v_1, v_2, \not{p}_T, m_i^{(1)}, m_i^{(2)}) \equiv \min_{\sum \mathbf{q}_T = \not{p}_T} \left\{ \max\left(m_T^{(1)}, m_T^{(2)} \right) \right\}}$$

The most conservative partition consistent with the constraint

Take the better of the two lower bounds

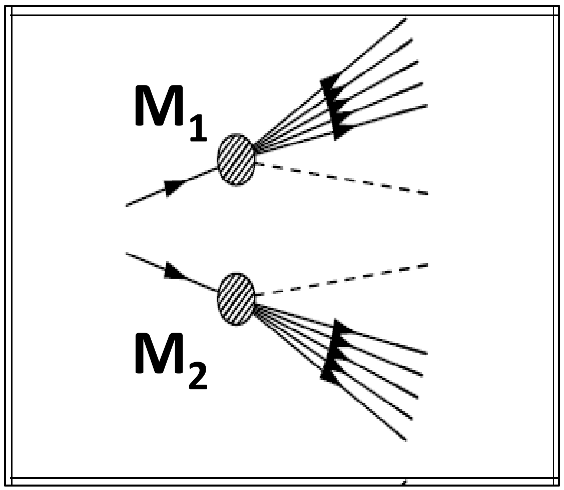
It is the <u>generalisation of transverse mass to pair production</u>.
Clear how to generalise it to any other types of production.

[Received six comments about "mis-spelling" of transverse in ATLAS editorial board!]

Note MT2 def is part of the four-step procedure:

[(1) select topology, (2) parent mass, (3) constraints, (4) find maximal lower bound]

described earlier.



Note, other approaches: MCT, Rogan, etc.

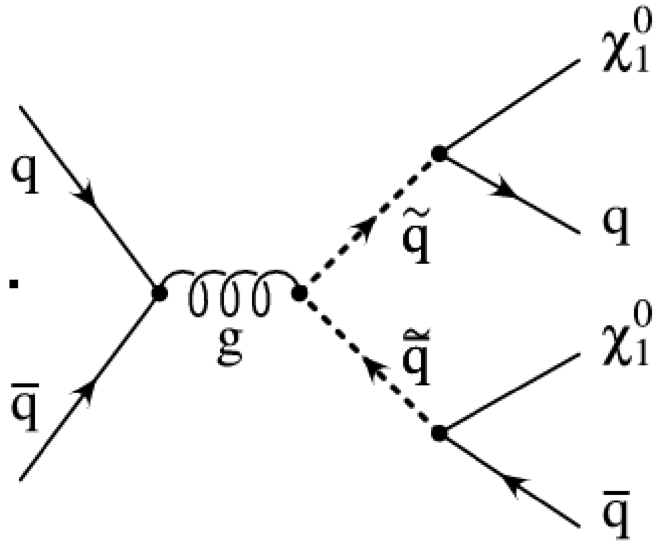
CONSTRAINTS

$$\boxed{M_1 = M_2} \quad + \quad \boxed{\sum_{i=1}^{N_{\mathcal{I}}} \vec{q}_{iT} = \not{\vec{p}}_T \equiv -\vec{u}_T - \sum_{i=1}^{N_{\mathcal{V}}} \vec{p}_{iT}}$$

Momentum conservation in transverse plane

In other words:

- If your event is signal …



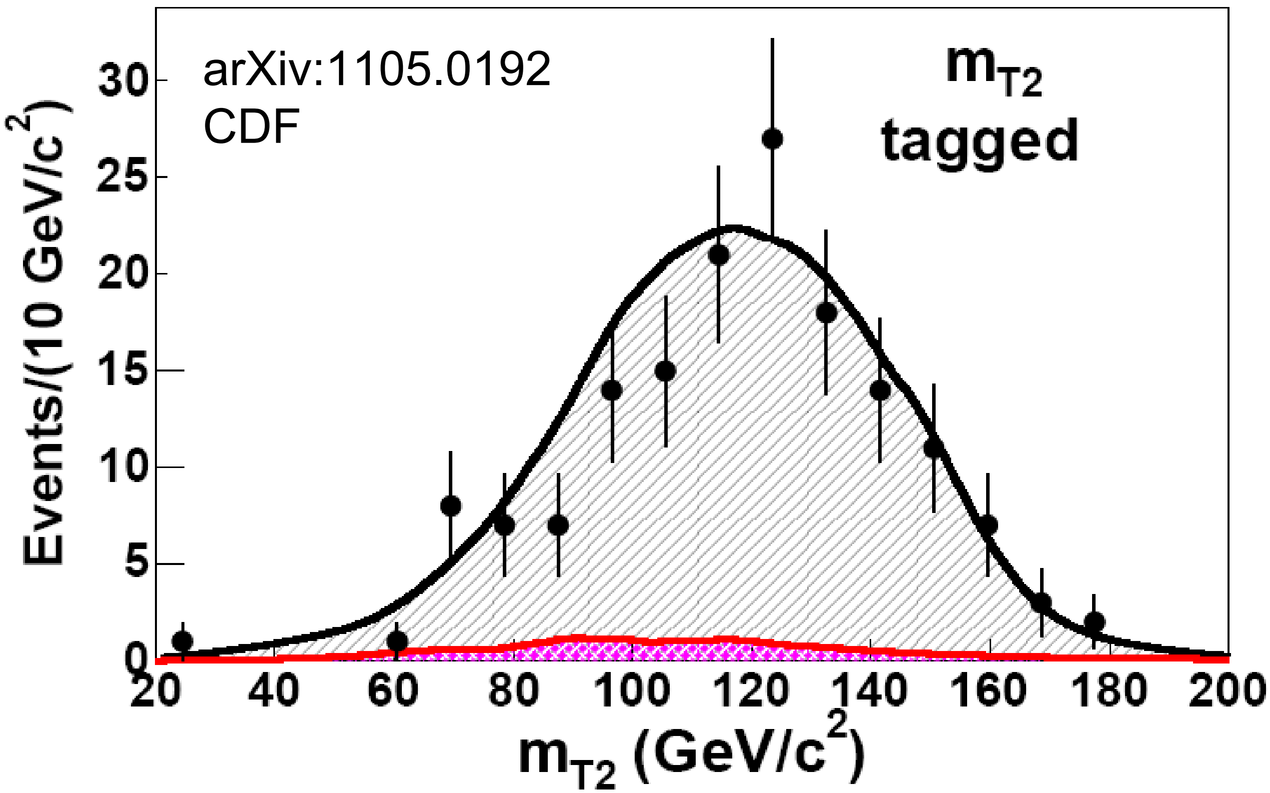
and if MT2 is “350 GeV” …

then the squark mass is >= 350 GeV.

Indeed, can show MT2 is, by construction, the best possible lower bound on the squark mass.

MT2 example in real data .....

- "Top Quark Mass Measurement using mT2 in the Dilepton Channel at CDF" (arXiv:0911.2956 and arXiv:1105.0192) reports that they "achieve **the single most precise measurement of m_{top} in [the dilepton] channel to date**". Also under study by ATLAS.



Top-quark physics is an important testing ground for mT2 methods, both at the LHC and at the Tevatron. If it can't work there, its not going to work elsewhere.

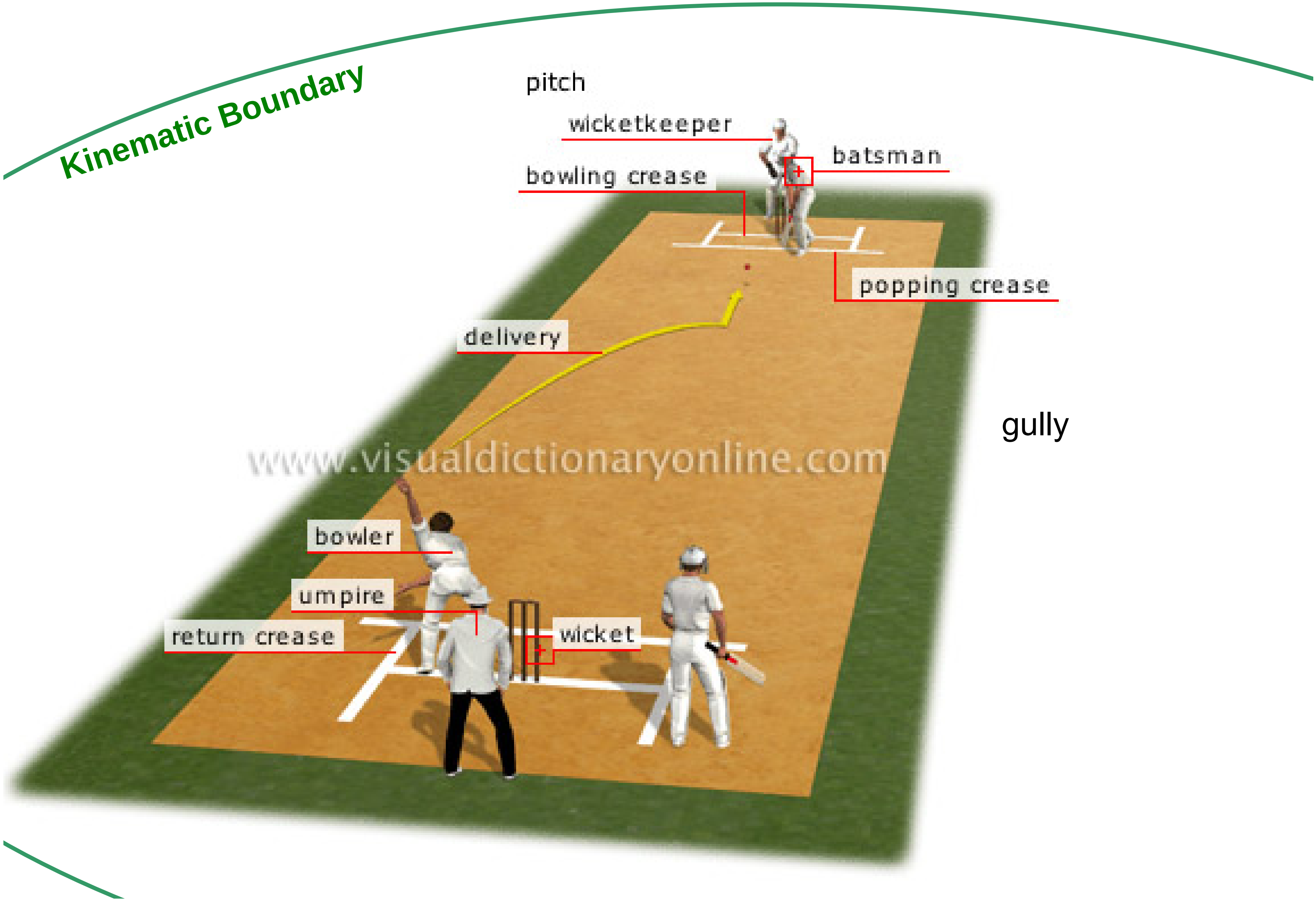
A digression

(Salutary Tale – how not to generalise to dissimilar parent and daughter masses)

Cricket

The Ashes


The Stumps


Kinematic Boundary
pitch
wicketkeeper
batsman
bowling crease
popping crease
delivery
gully
www.visualdictionaryonline.com
bowler
umpire
return crease
wicket

Transverse masses and kinematic constraints: from the boundary to the crease

arXiv:0908.3779v2 [hep-ph]

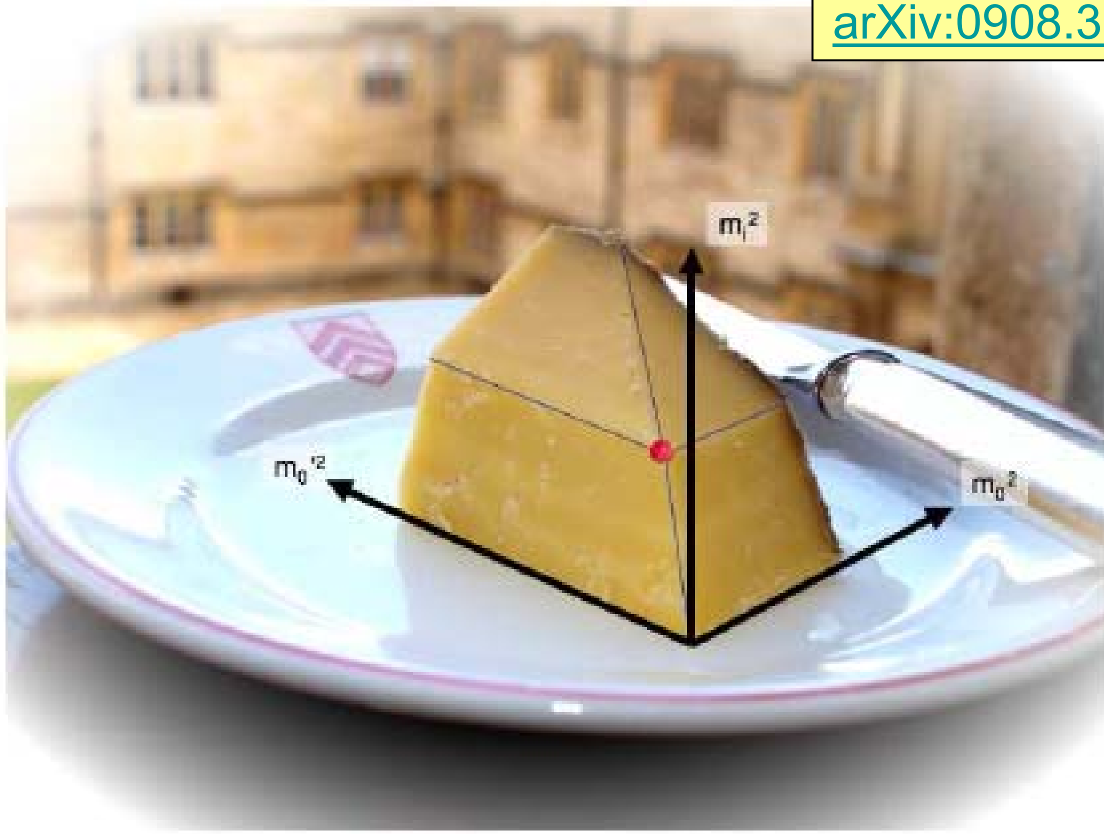


FIG. 1: Representation of the bounding planes (visible faces) and the extremal allowed region (solid) for the case described in the text with $\tilde{m}_i = \tilde{m}_i'$, $m_i = m_i'$, and $m_v = m_v' = 0$. The vertex representing the true values of the masses is indicated with a red ball. The origin of the axes is at the point ($m_0^2 = \tilde{m}_0^2 - \tilde{m}_i^2, m_0'^2 = \tilde{m}_0'^2 - \tilde{m}_i^2, m_i^2 = 0$).

"final test" = "Last cricket match in a series of five or more played over a month when countries' teams compete"

How firm was the wicket?

element – where such calculations are computationally tractable. This final test will show whether it is safe to neglect the effects of spin, determine the character of the creases, and get the desired results by using the boundary.

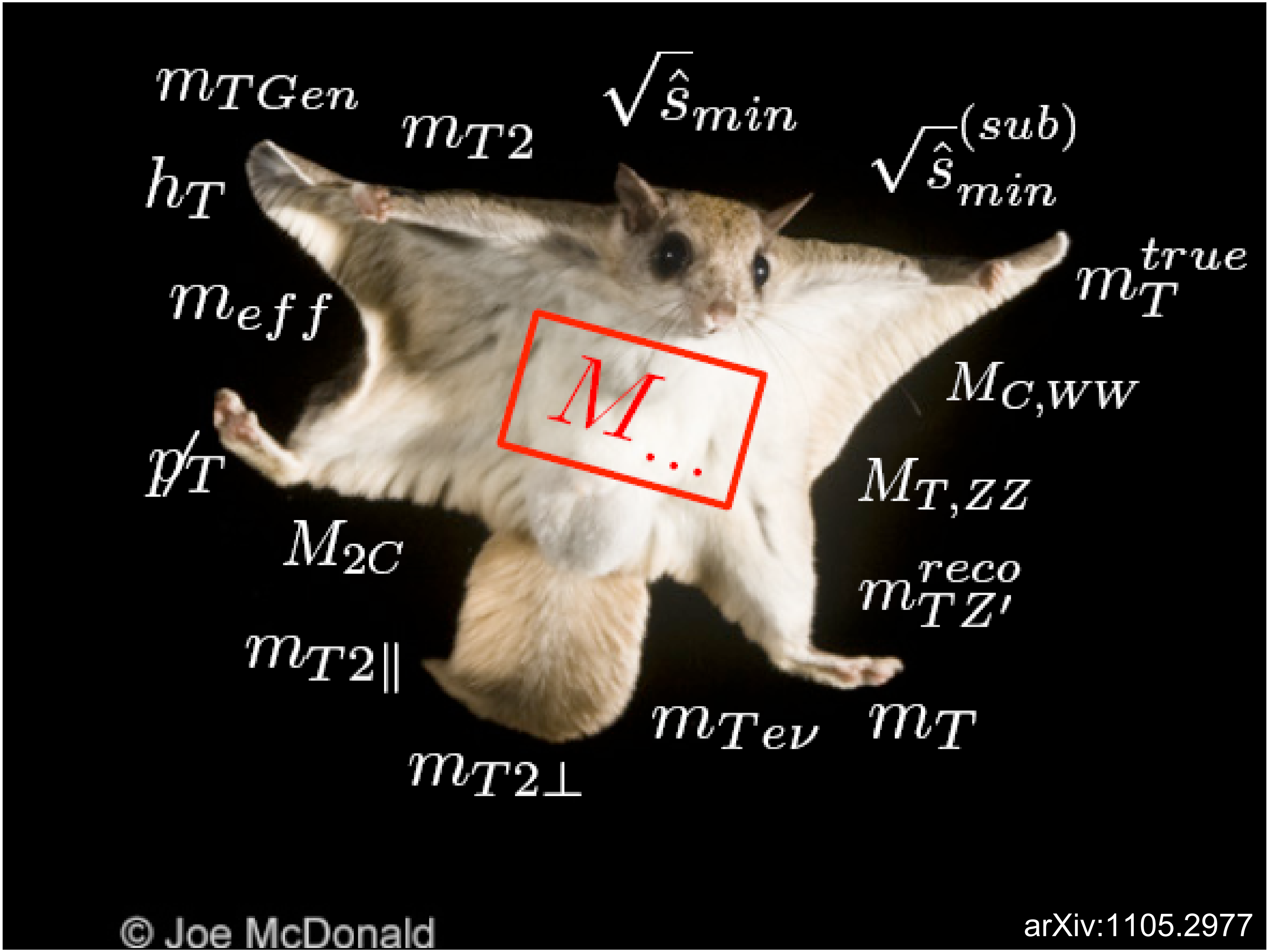
Can England's batsmen defeat the Aussie spin bowlers?

Four runs are scored when the ball reaches the boundary (six if it didn't hit the ground first)

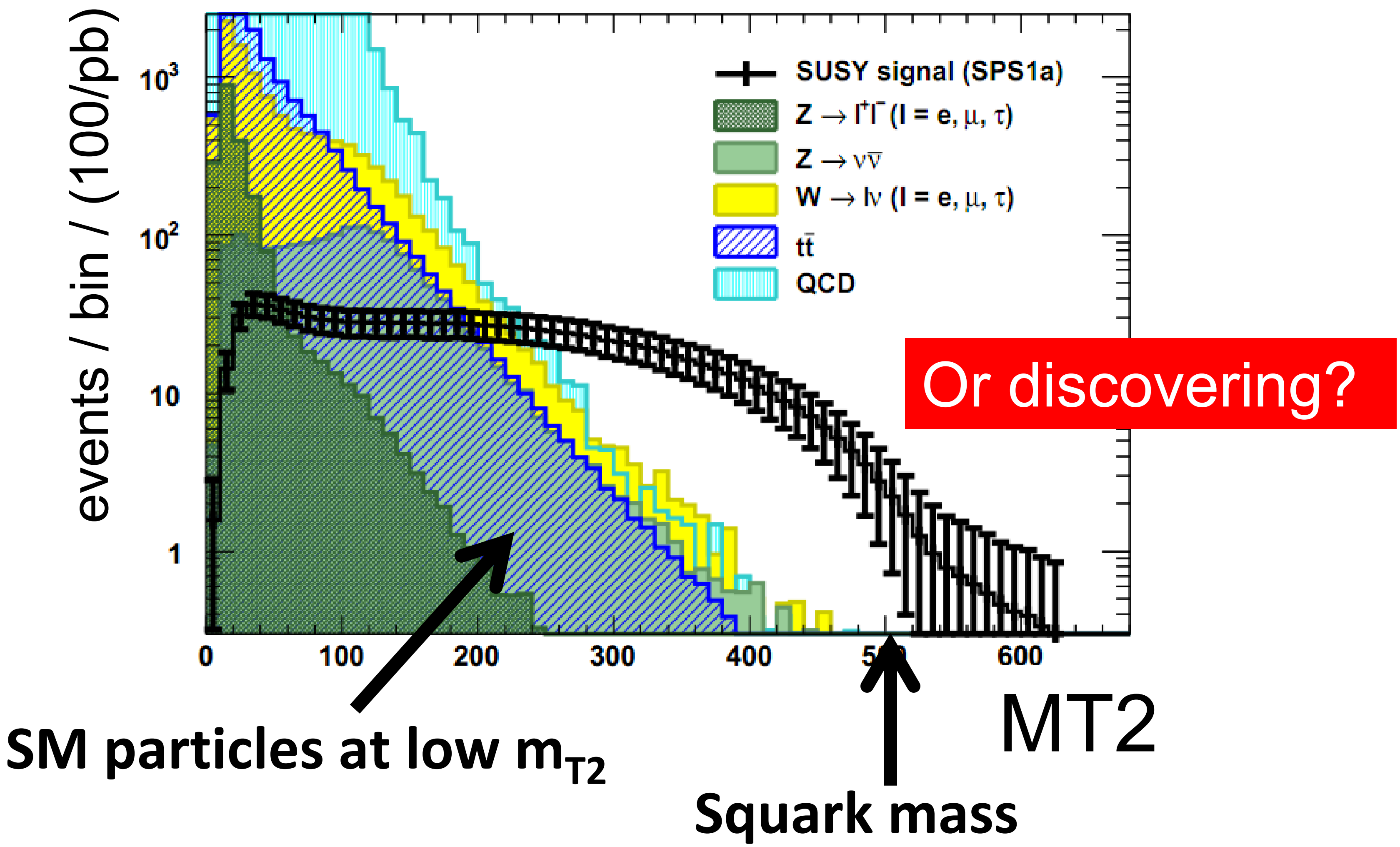
Moral

- Call the paper what it does
- or choose a sport that more people play

- or try for furry animals?


m_{TGen}
m_{T2}
$\sqrt{\hat{s}}_{min}$
$\sqrt{\hat{s}}_{min}^{(sub)}$
h_T
m_T^{true}
m_{eff}
$M_{\ldots}$
$M_{C,WW}$
$\not{p}_T$
$M_{T,ZZ}$
M_{2C}
$m_{TZ'}^{reco}$
$m_{T2\|}$
$m_{Te\nu}$
m_T
$m_{T2\perp}$
© Joe McDonald
arXiv:1105.2977

Example MT2 distribution …
… ?weighing? 500 GeV squarks



arXiv:0907.2713

Properties of the m_{T2} function

1. Identical pair decays

   $m_< < m_{T2} < m_0$

2. Non-identical pair decays

   $m_< < m_{T2} < \max(m_0, m_0')$

3. Small missing momentum

   $m_{T2} \rightarrow m_<$ as $p_T^{miss} \rightarrow 0$

4. Small jet momentum

   $m_{T2} \rightarrow m_<$ as $p_T^{jet} \rightarrow 0$

5. Jet || to missing

   $m_{T2} \rightarrow m_<$ for $p_T^{miss} \parallel p_T^{jet}$

6. $m_{T2} \rightarrow m_<$ for

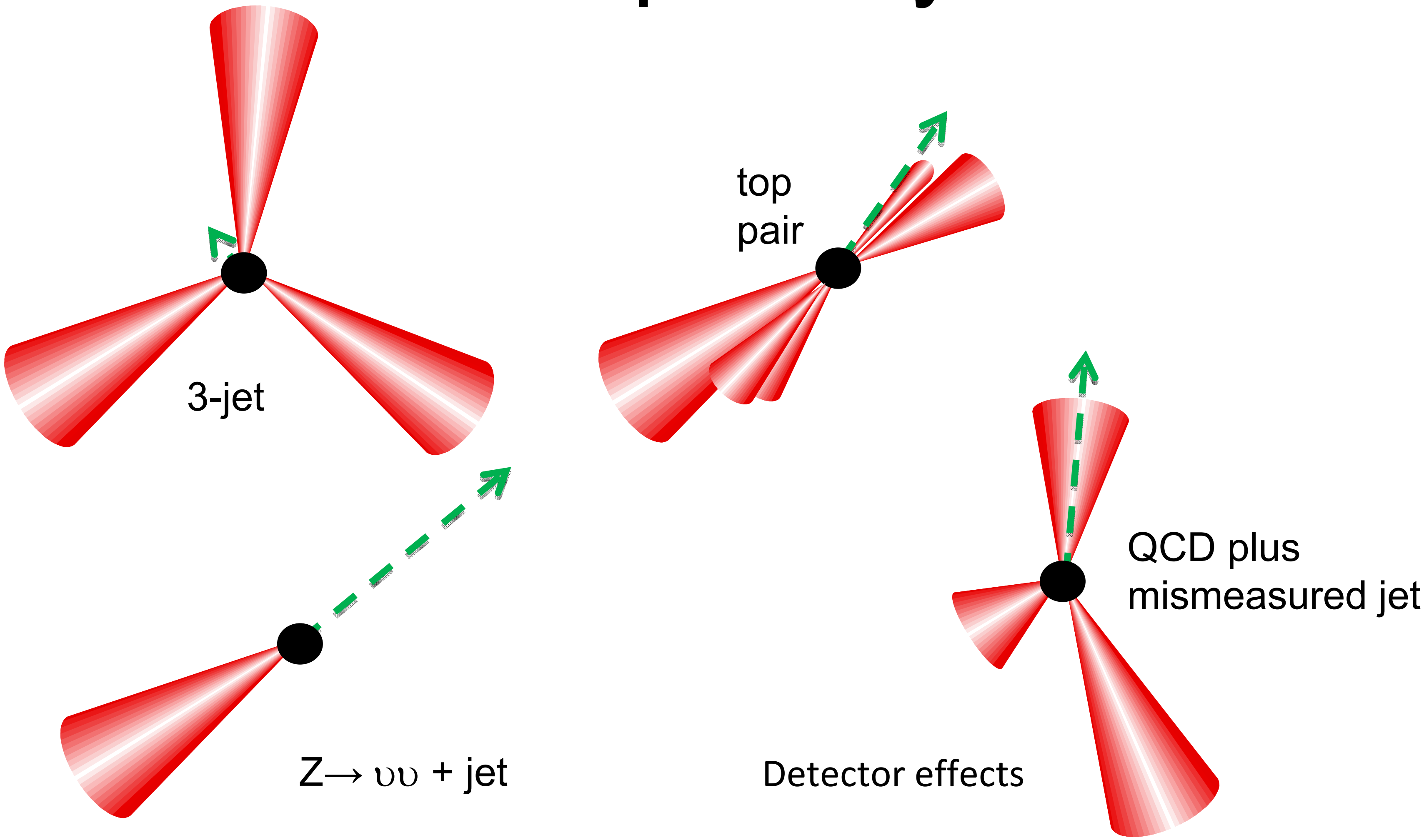
   $p_T^{miss} = \Sigma_i \, \alpha_i \, p_T^{jet(i)}$ for $\alpha_i > 0$

7. 1-6 also true for composite systems

m_{T2} adopts **small** values for a variety of interesting configurations

AJB and Gwenlan
arXiv:0907.2713

Graphically:



All these have m_{T2} either < m_{top} or → $m_{<}$

Example proof

Lemma 4 *When* $\not{\mathbf{p}}_T = \mathbf{0}$ *and* $m_i^{(1,2)} = 0$ *then* $m_{T2} = m_<$.

Proof For $\not{\mathbf{p}}_T = \mathbf{0}$ there exists a trivial partition of the missing momentum with $\mathbf{q}_T^{(1)} = \mathbf{q}_T^{(2)} = \mathbf{0}$. For that partition, $m_T^{(1)} = m_<^{(1)}$ and $m_T^{(2)} = m_<^{(2)}$;

$$m_{T2}(v_1, v_2, \not{\mathbf{p}}_T, m_i^{(1)}, m_i^{(2)}) \equiv \min_{\sum \mathbf{q}_T = \not{\mathbf{p}}_T} \left\{ \max\left(m_T^{(1)}, m_T^{(2)} \right) \right\}$$

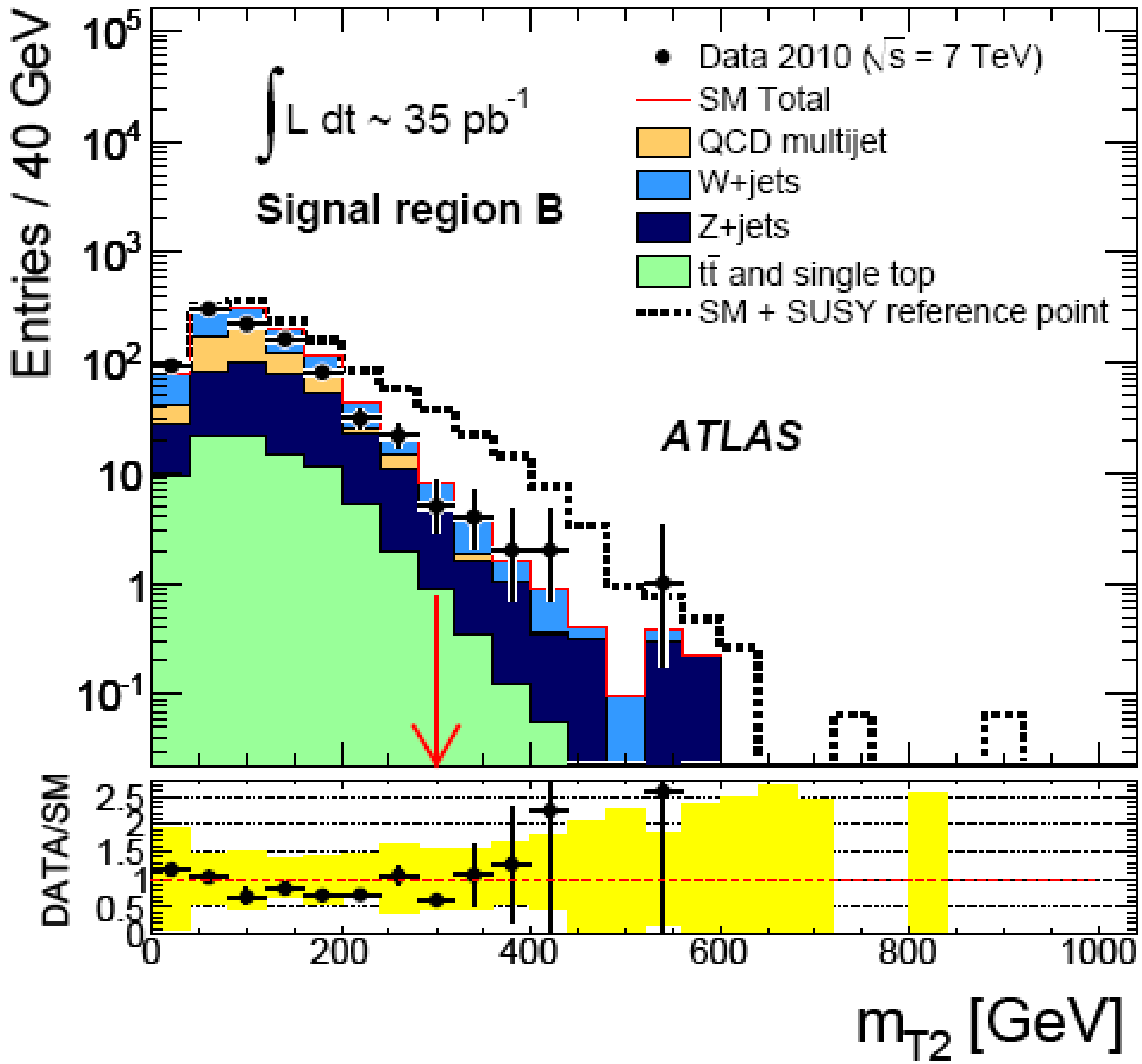
- So small p_T^{miss} ➔ small m_{T2}
- Do we *need* a separate p_T^{miss} cut? (no…)

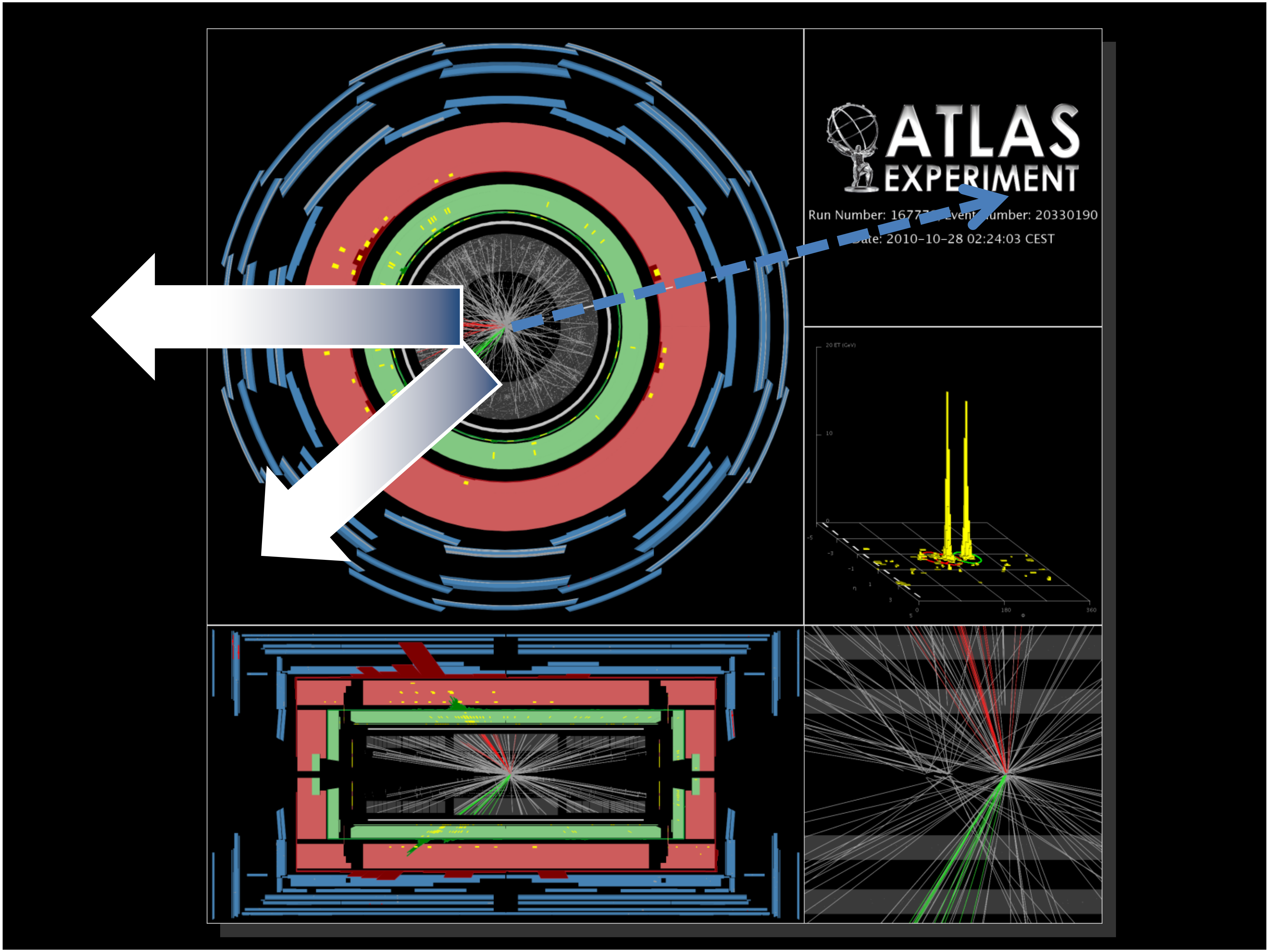
NB the requirement that m_i=0 is on the *input* mass parameter not the *true* LSP mass

Process	$m_{T2}(v_1, v_2, \not{p}_T, 0, 0)$	Comments
QCD di-jet $\rightarrow$ hadrons	$= \max m_j$ by Lemmas 1,4	
QCD multi jets $\rightarrow$ hadrons	$= \max m_j$ by Lemma 4	
$t\bar{t}$ production	$= \max m_j$ by Lemma 4	fully hadronic [illegible]s
	$\leq m_t$ by Lemmas 1,7	any lepto[illegible]cays
Single top / tW	$= \max m_j$ by Lemma 4	fully h[illegible]nic decays
	$\leq m_t$ by Lemmas 2,7	a[illegible]ptonic decays
Multi jets: "fake" $\not{p}_T$	$= \max m_j$ by Lemma 5	[illegible]gle mismeasured jet[a]
	$= \max m_j$ by Lem[illegible]	two mismeasured jets[a]
Multi jets: "real" $\not{p}_T$	$= \max m_j$ by L[illegible] 5	single jet with leptonic b decay[a]
	$= \max m_j$ [illegible]mma 6	two jets with leptonic b decays[a]
$Z \rightarrow \nu\bar{\nu}$	$= 0$ b[illegible]ma 3	
$Z\,j \rightarrow \nu\bar{\nu}\,j$	$=$ [illegible] Lemma 3	one ISR jet[a]
$W \rightarrow \ell\nu$ [b]	[illegible]$_\ell$ by Lemma 3	
$W\,j \rightarrow \ell\nu\,j$ [b]	$\leq m_W$ by Lemma 2	one ISR jet[a]
$WW \rightarrow \ell\nu\ell\nu$ [b]	$\leq m_W$ by Lemma 1	
$ZZ \rightarrow \nu\bar{\nu}\nu\bar{\nu}$	$= 0$ by Lemma 3	also $= m_j$ for one ISR jet[a]
$LQ\,\overline{LQ} \rightarrow q\nu\bar{q}\bar{\nu}$	$\leq m_{LQ}$	} i.e. can take large values
$\tilde{q}\,\bar{\tilde{q}} \rightarrow q\tilde{\chi}_1^0\,\bar{q}\tilde{\chi}_1^0$	$\leq m_{\tilde{q}}$	
$q_1, \bar{q}_1 \rightarrow q\gamma_1, \bar{q}\gamma_1$	$\leq m_{q1}$	

So good for low multiplicity pair production signal discovery – disleptons?

Putting it to work for discovery

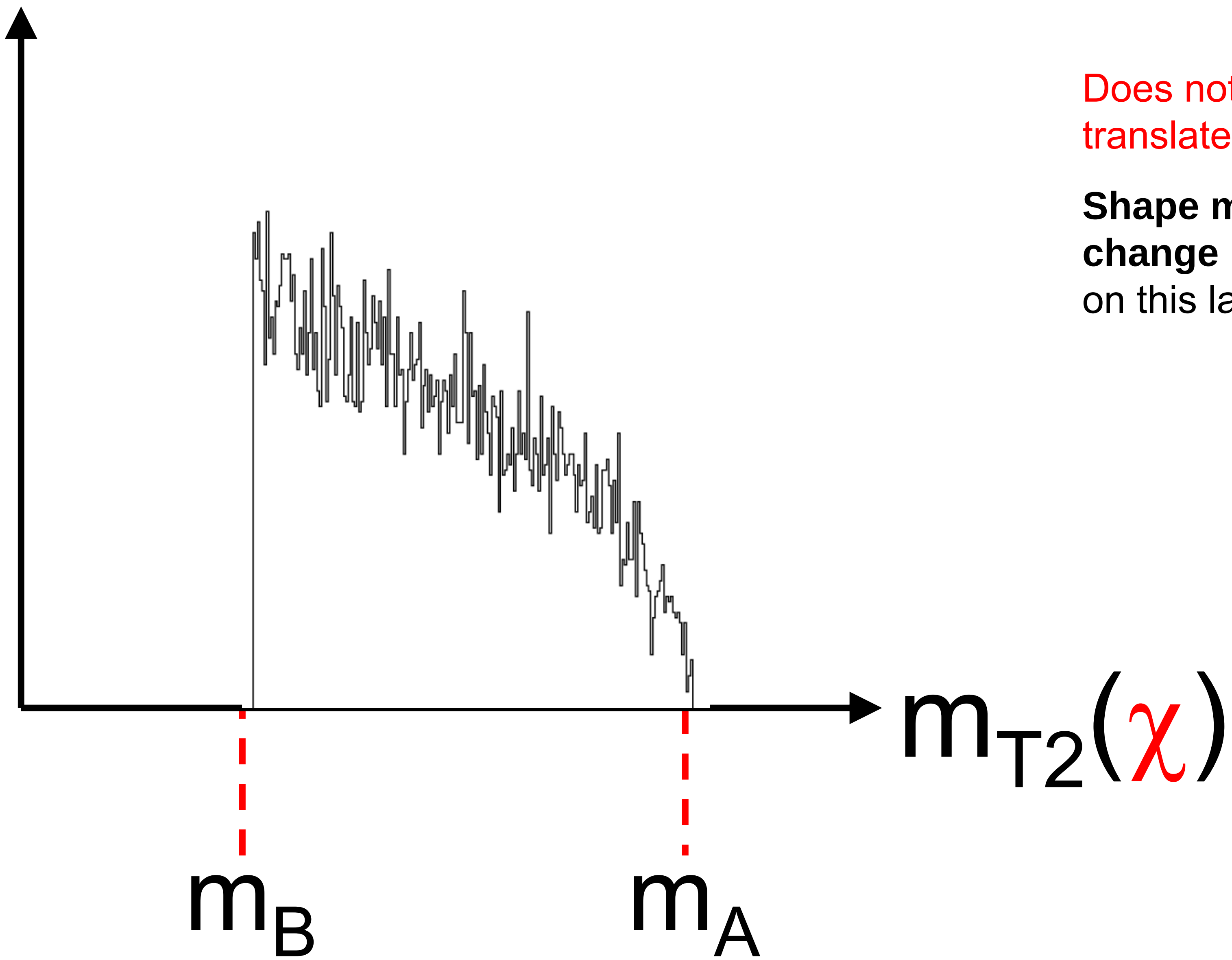



ATLAS
EXPERIMENT
Run Number: 1677[illegible] Event Number: 20330190
Date: 2010-10-28 02:24:03 CEST

Have dodged question of mass of invisible daughters.

What if we don’t know their masses?

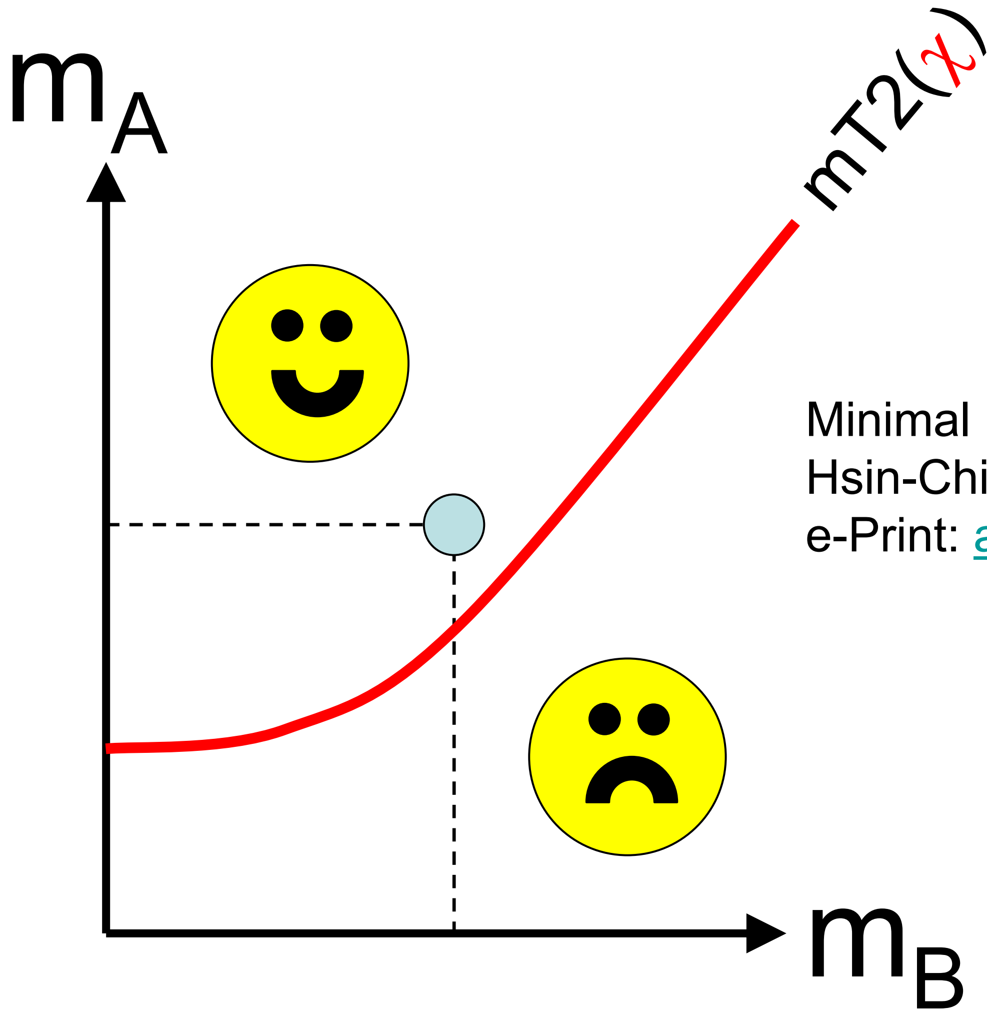
Varying "χ" … to first order



Does not just translate …

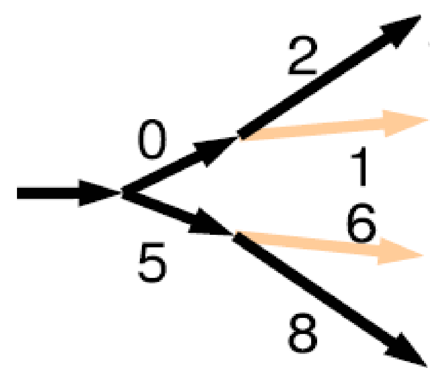
Shape may also change … more on this later.

MT2 inherits mass-space boundary from MT



The MT2(chi) curve is the **boundary** of the region of (mother, daughter) mass-space consistent with the observed event!

Minimal Kinematic Constraints and m(T2),
Hsin-Chia Cheng and Zhenyu Han (UCD)
e-Print: arXiv:0810.5178 [hep-ph]



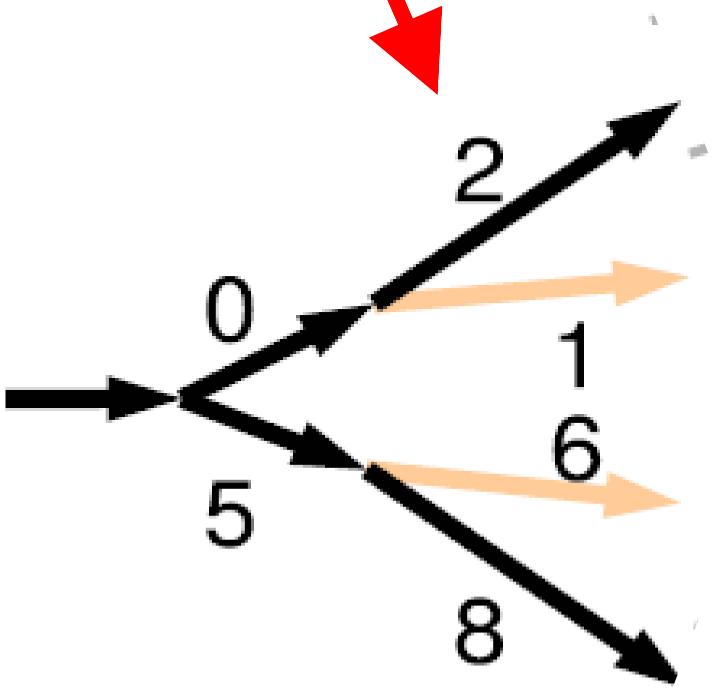
MT2 is defined in terms of MT

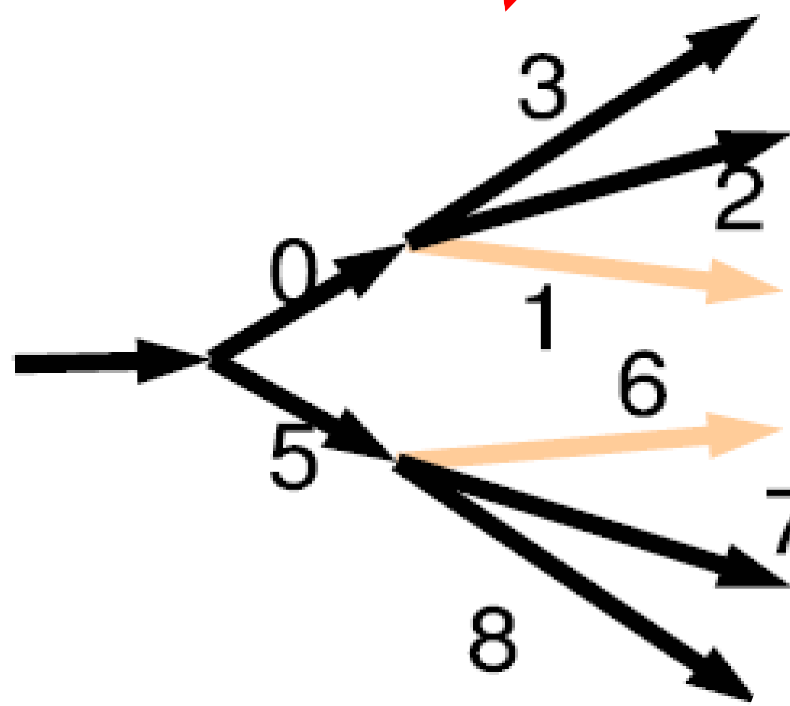
- Consequently, MT2 inherits the "kink structure" of MT and can (in principle) be used to:
  - EASILY measure the parent-daughter mass difference,
  - might PERHAPS measure the absolute mass scale using utm boosts kinks or variable visible mass kinks (HARD)

Are MT2 kinks observable ?

Expect KINK only from UTM Recoil (perhaps only from ISR!)

Expect stronger KINK due to both UTM recoil, AND variability in the visible masses.

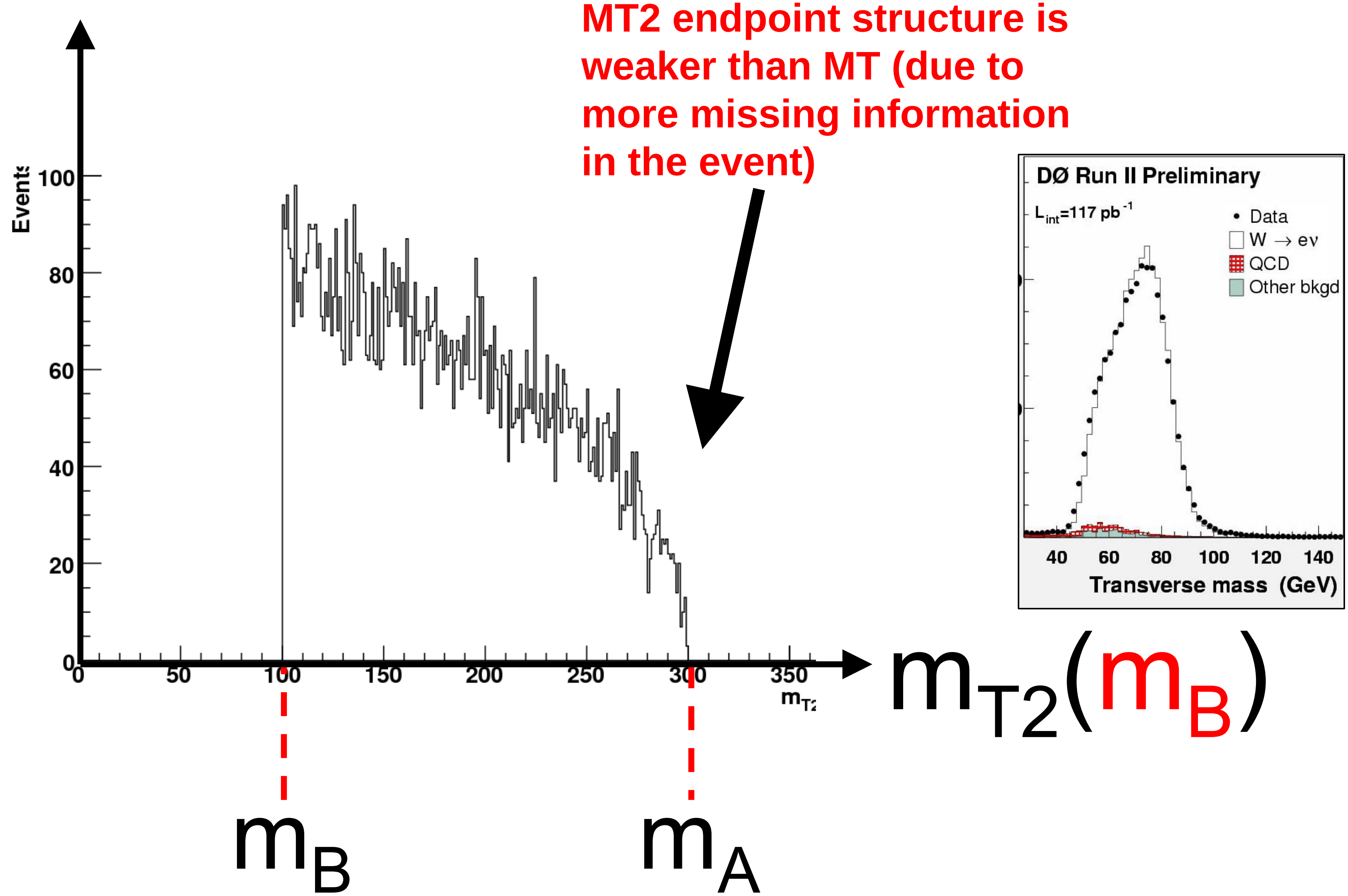




arXiv: 0711.4008

Perhaps: MT2’s endpoint structure is weaker than MT’s.

MT2 endpoint structure is weaker than MT (due to more missing information in the event)



Caveat Mensor!

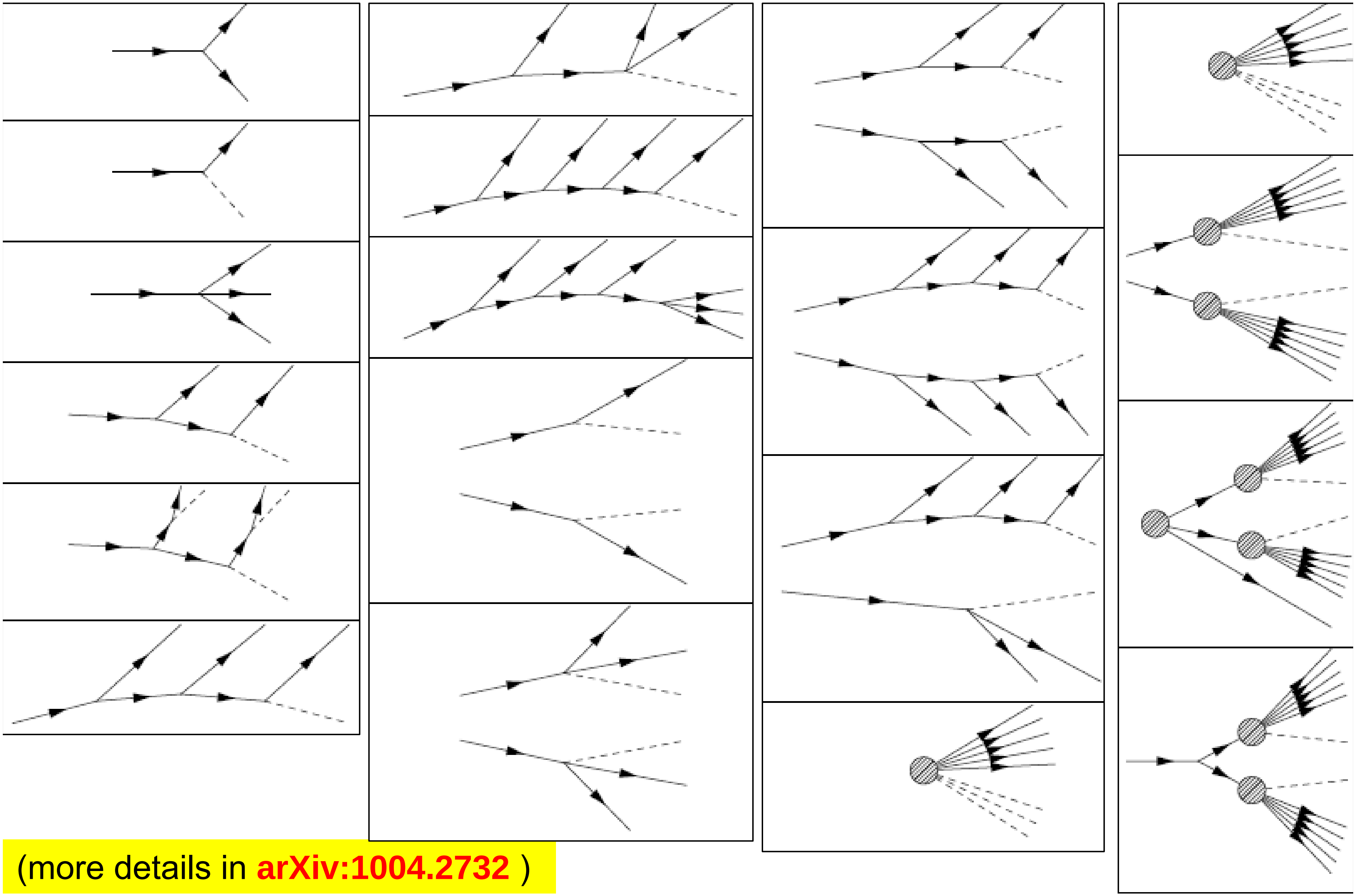
(for those of you interested in LHC dark matter constraints)

Disappointingly, M_{T2} kinks, are the only known kinematic methods which (at least in principle) allow determination of the mass of the invisible daughters of pair produced particles in short chains.

[We will see a dynamical method that works for three+ body decays shortly. Likelihood methods can determine masses in pair decays too, though at cost of model dependence and CPU. See Alwall.]

change of topic!

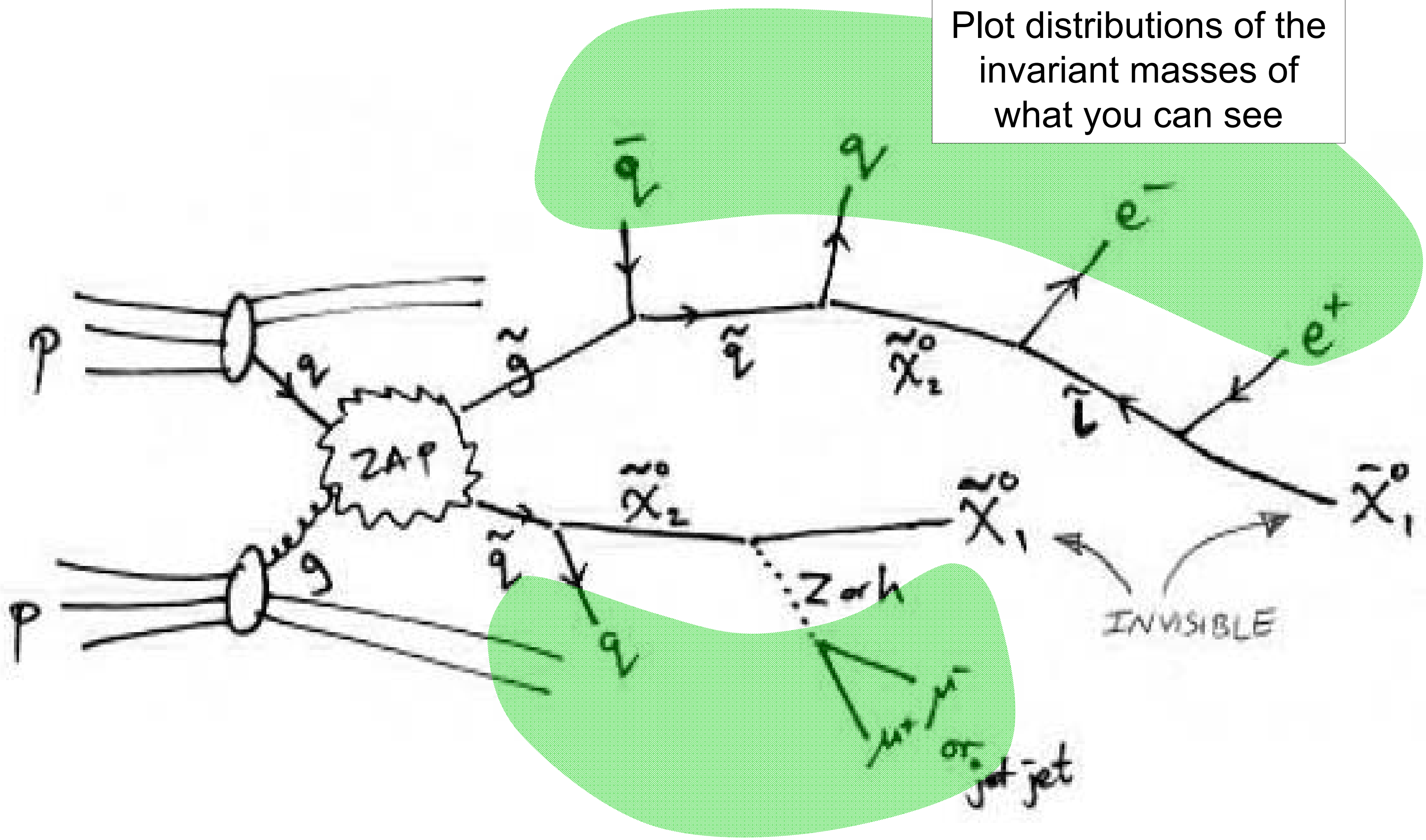
Not all proposed new-physics chains are short!



(more details in **arXiv:1004.2732**)

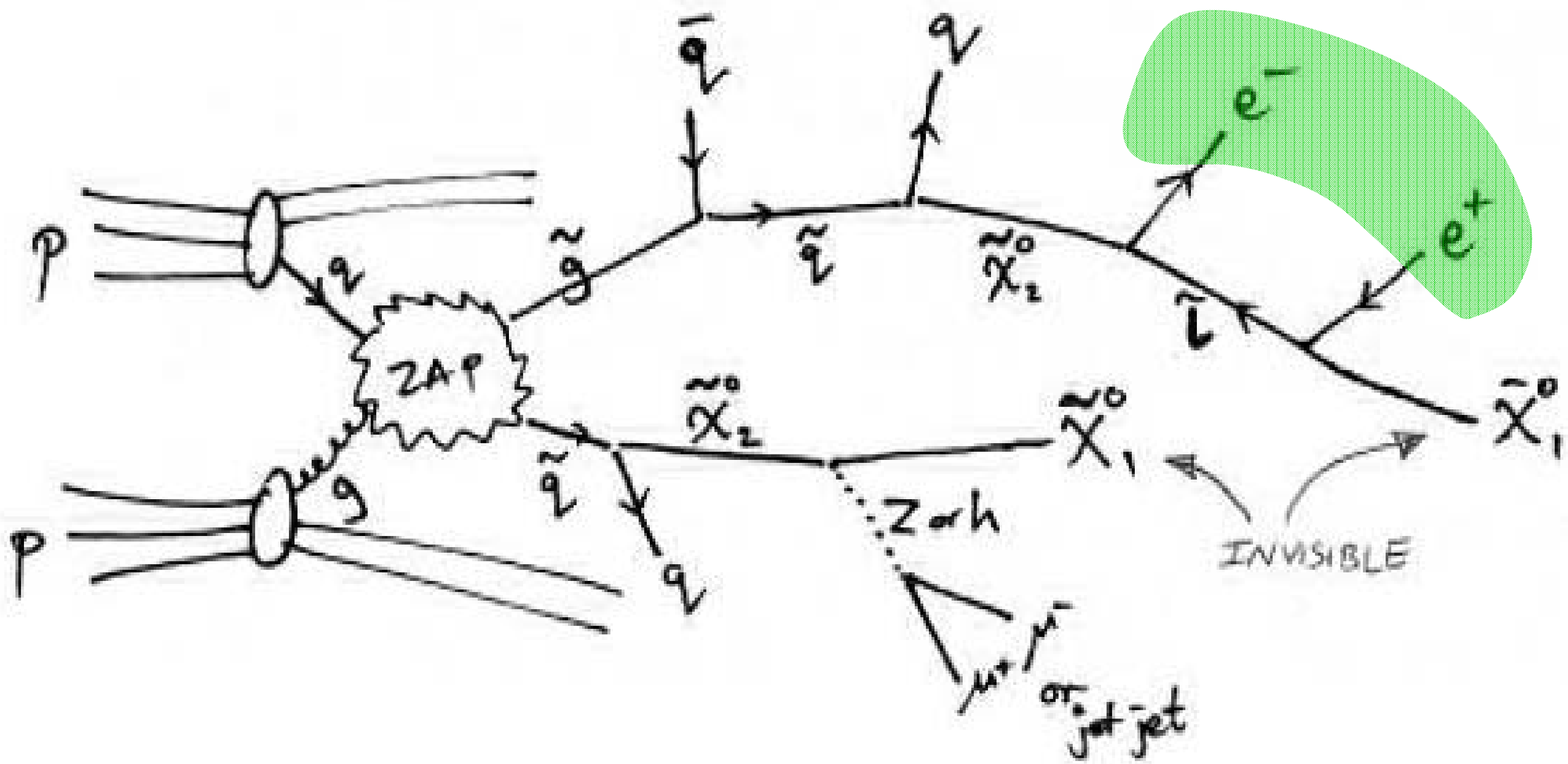
If chains a longer use "edges" or "Kinematic endpoints"

Plot distributions of the invariant masses of what you can see



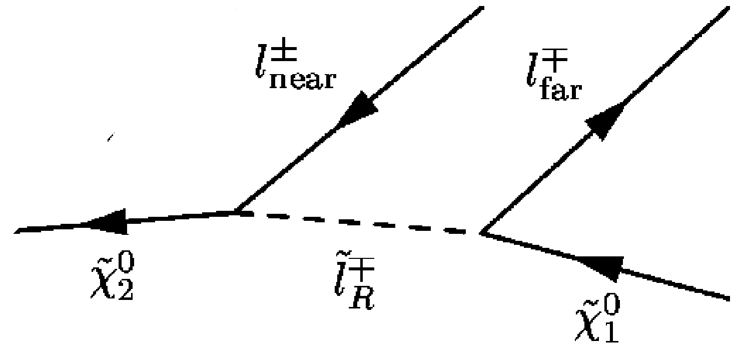
What is a kinematic endpoint?

- Consider M_{LL}

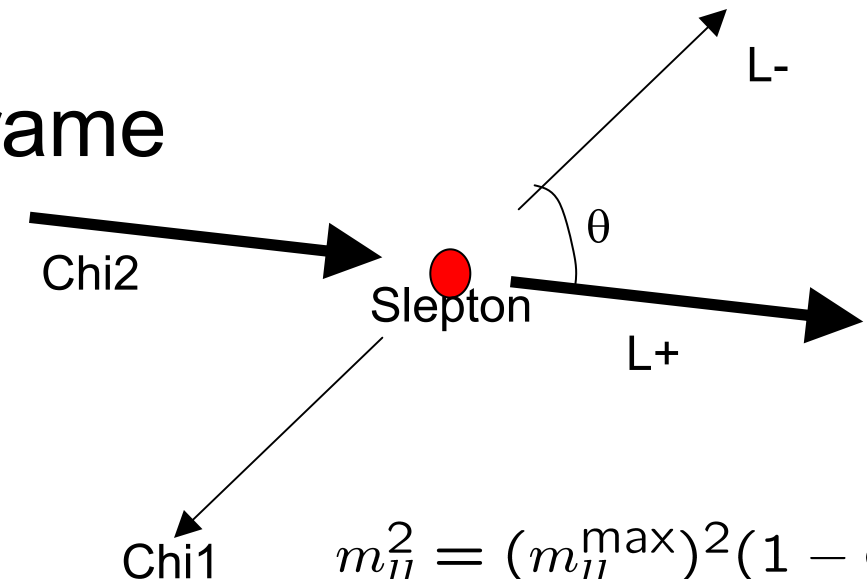


What is a kinematic endpoint?

- Zoom in on di-leptons to calculate m_{LL}

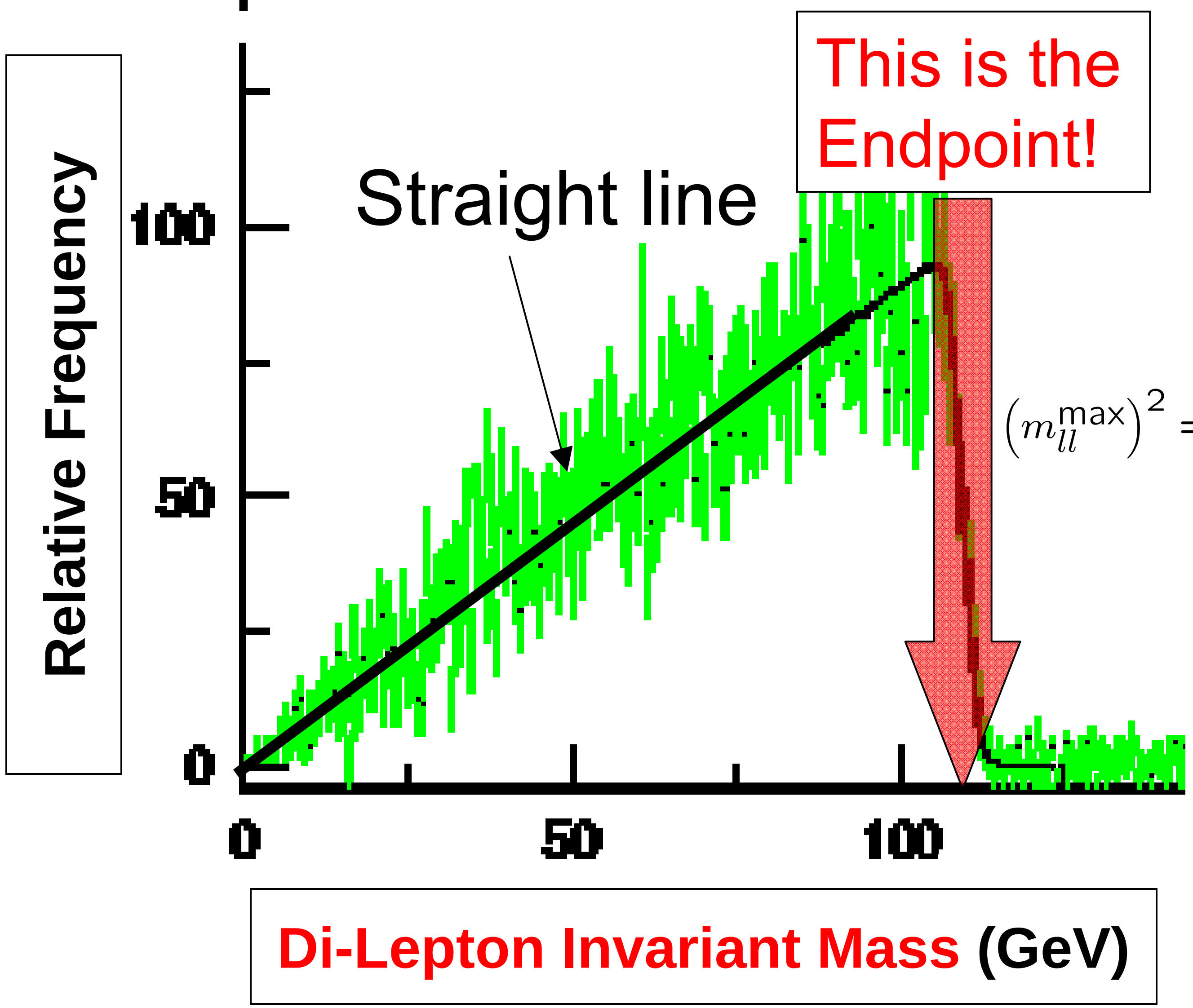


- In slepton rest-frame



$$m_{ll}^2 = (m_{ll}^{\max})^2 (1 - \cos\theta)/2$$

Dilepton invariant mass distribution



$$\left(m_{ll}^{\max}\right)^2 = \frac{\left(m^2_{\tilde{\chi}^0_2} - m^2_{\tilde{l}_R}\right)\left(m^2_{\tilde{l}_R} - m^2_{\tilde{\chi}^0_1}\right)}{m^2_{\tilde{l}_R}}$$

Exercises

- (8) Prove that the phase space distribution for the M_{LL} invariant mass is has the triangular shape shown on the previous slide, and

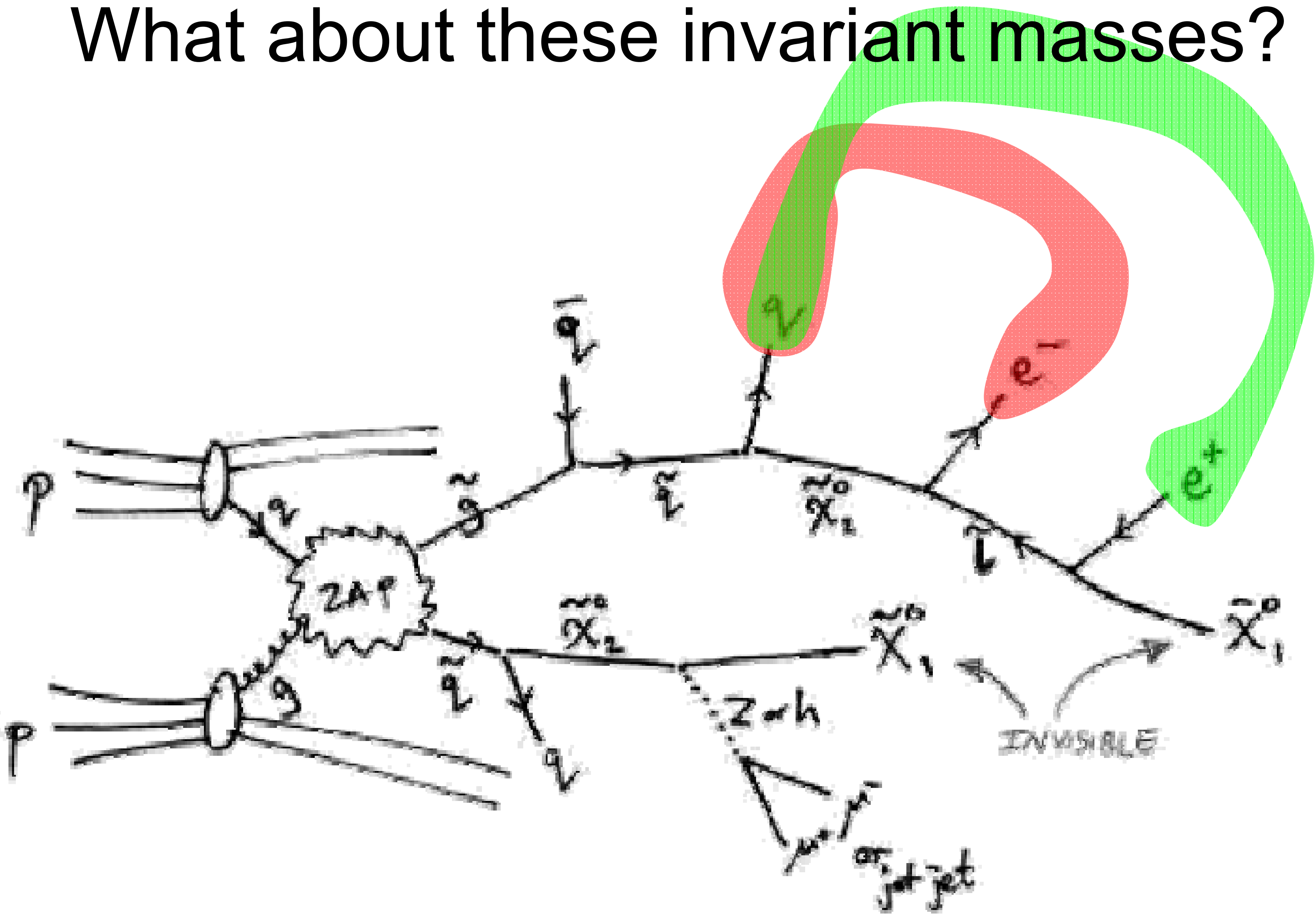
- (9) Show that the endpoint is located at

$$\left(m_{ll}^{\mathrm{max}}\right)^2 = \frac{\left(m^2_{\tilde{\chi}^0_2} - m^2_{\tilde{l}_R}\right)\left(m^2_{\tilde{l}_R} - m^2_{\tilde{\chi}^0_1}\right)}{m^2_{\tilde{l}_R}}$$

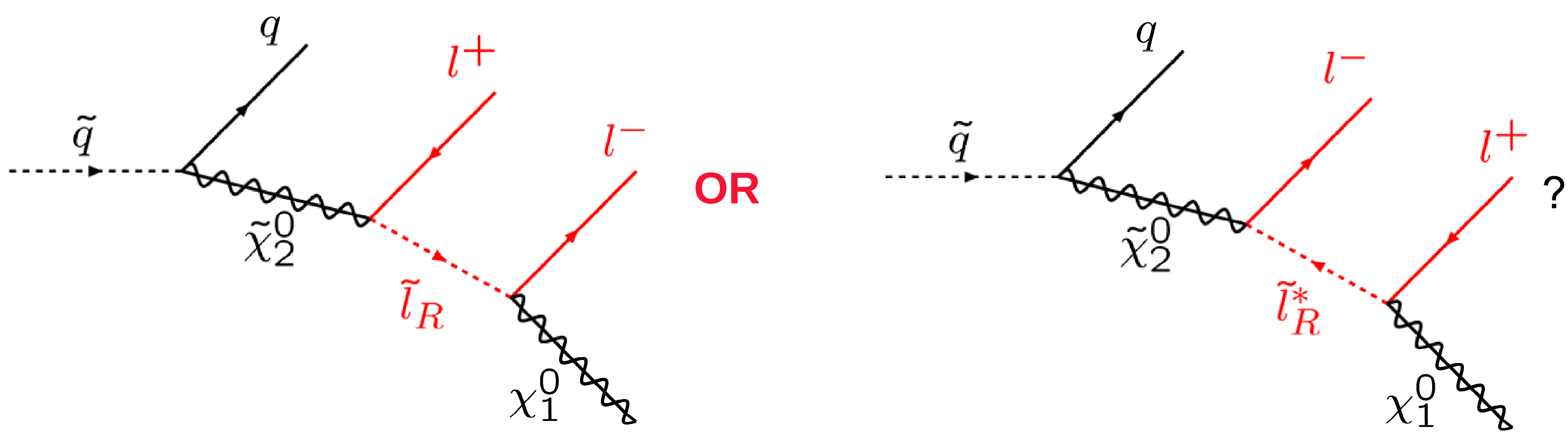
Note key difference to bounding vars

- With the bounding vars you place a bound on a property/parameter/invariant of the hypothesis or model by construction.
- With the kinematic edges and enpoints, you look for a kinematic strucure in a distribution, and use it to constrain one or more parameters of the hypothesis or model.

What about these invariant masses?



Some extra difficulties – may not know order particles were emitted



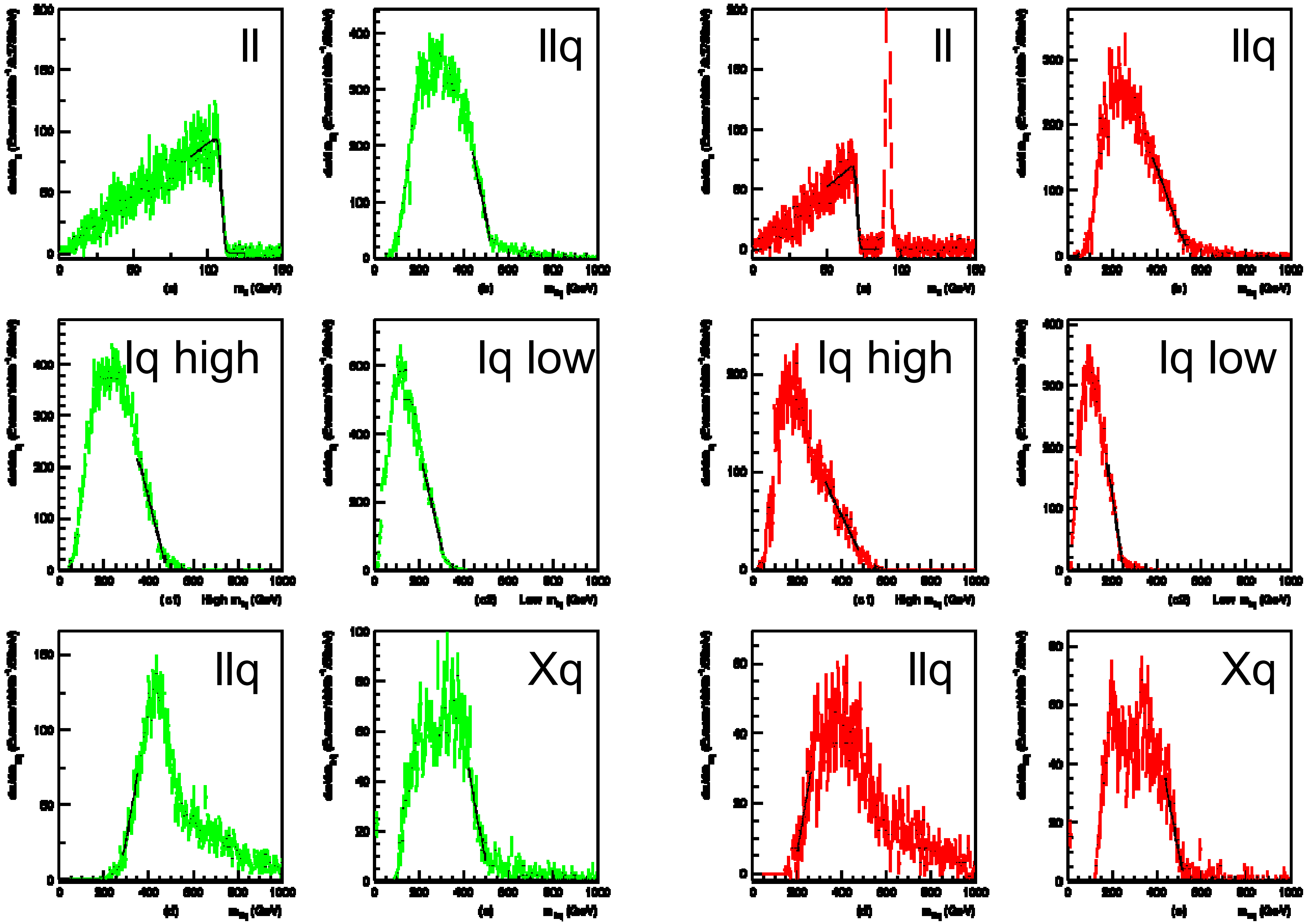
Therefore need to define order-blind variables such as

$$m_{ql}^{high} = \max[m_{ql+}, m_{ql-}]$$
$$m_{ql}^{low} = \min[m_{ql+}, m_{ql-}]$$

$$m^2_{j\ell(s)}(\alpha) \equiv \left(m^{2\alpha}_{j\ell_n} + m^{2\alpha}_{j\ell_f}\right)^{\frac{1}{\alpha}} \qquad m^2_{j\ell(d)}(\alpha) \equiv \left|m^{2\alpha}_{j\ell_n} - m^{2\alpha}_{j\ell_f}\right|^{\frac{1}{\alpha}}$$

There are many other possibilities for resolving problems due to position ambiguity. Compare hep-ph/0007009 and **hep-ph/0510356** with arXiv:0906.2417

Measure Kinematic Edge Positions



Determine how edge positions depend on sparticle masses

Related edge	Kinematic endpoint
l^+l^- edge	$(m_{ll}^{\max})^2 = (\tilde\xi - \tilde l)(\tilde l - \tilde\chi)/\tilde l$
l^+l^-q edge	$(m_{llq}^{\max})^2 = \begin{cases} \max\left[\frac{(\tilde q-\tilde\xi)(\tilde\xi-\tilde\chi)}{\tilde\xi}, \frac{(\tilde q-\tilde l)(\tilde l-\tilde\chi)}{\tilde l}, \frac{(\tilde q\tilde l-\tilde\xi\tilde\chi)(\tilde\xi-\tilde l)}{\tilde\xi\tilde l}\right] \\ \text{except for the special case in which } \tilde l^2 < \tilde q\tilde\chi < \tilde\xi^2 \text{ and} \\ \tilde\xi^2\tilde\chi < \tilde q\tilde l^2 \text{ where one must use } (m_{\tilde q} - m_{\tilde\chi^0_1})^2. \end{cases}$
Xq edge	$(m_{Xq}^{\max})^2 = X + (\tilde q - \tilde\xi)\left[\tilde\xi + X - \tilde\chi + \sqrt{(\tilde\xi - X - \tilde\chi)^2 - 4X\tilde\chi}\right]/(2\tilde\xi)$
l^+l^-q threshold	$(m_{llq}^{\min})^2 = \begin{cases} [\ 2\tilde l(\tilde q - \tilde\xi)(\tilde\xi - \tilde\chi) + (\tilde q + \tilde\xi)(\tilde\xi - \tilde l)(\tilde l - \tilde\chi) \\ -(\tilde q - \tilde\xi)\sqrt{(\tilde\xi + \tilde l)^2(\tilde l + \tilde\chi)^2 - 16\tilde\xi\tilde l^2\tilde\chi}\ ]/(4\tilde l\tilde\xi) \end{cases}$
$l^\pm_{\text{near}}q$ edge	$(m_{l_{\text{near}}q}^{\max})^2 = (\tilde q - \tilde\xi)(\tilde\xi - \tilde l)/\tilde\xi$
$l^\pm_{\text{far}}q$ edge	$(m_{l_{\text{far}}q}^{\max})^2 = (\tilde q - \tilde\xi)(\tilde l - \tilde\chi)/\tilde l$
$l^\pm q$ high-edge	$(m_{lq(\text{high})}^{\max})^2 = \max\left[(m_{l_{\text{near}}q}^{\max})^2, (m_{l_{\text{far}}q}^{\max})^2\right]$
$l^\pm q$ low-edge	$(m_{lq(\text{low})}^{\max})^2 = \min\left[(m_{l_{\text{near}}q}^{\max})^2, (\tilde q - \tilde\xi)(\tilde l - \tilde\chi)/(2\tilde l - \tilde\chi)\right]$
M_{T2} edge	$\Delta M = m_{\tilde l} - m_{\tilde\chi^0_1}$

Table 4: The absolute kinematic endpoints of invariant mass quantities formed from decay chains of the types mentioned in the text for known particle masses. The following shorthand notation has been used: $\tilde\chi = m^2_{\tilde\chi^0_1}, \tilde l = m^2_{\tilde l_R}, \tilde\xi = m^2_{\tilde\chi^0_2}, \tilde q = m^2_{\tilde q}$ and X is m^2_h or m^2_Z depending on which particle participates in the "branched" decay.

hep-ph/0007009

updated version at arXiv:1004.2732

So now we have:

Large set of measurements

Endpoint	S5 Fit	S5 Fit error
l^+l^- edge	109.10	0.13
l^+l^-q edge	532.1	3.2
$l^\pm q$ high-edge	483.5	1.8
$l^\pm q$ low-edge	321.5	2.3
l^+l^-q threshold	266.0	6.4
Xq edge	514.1	6.6
ΔM (M_{T2} edge)	——	——

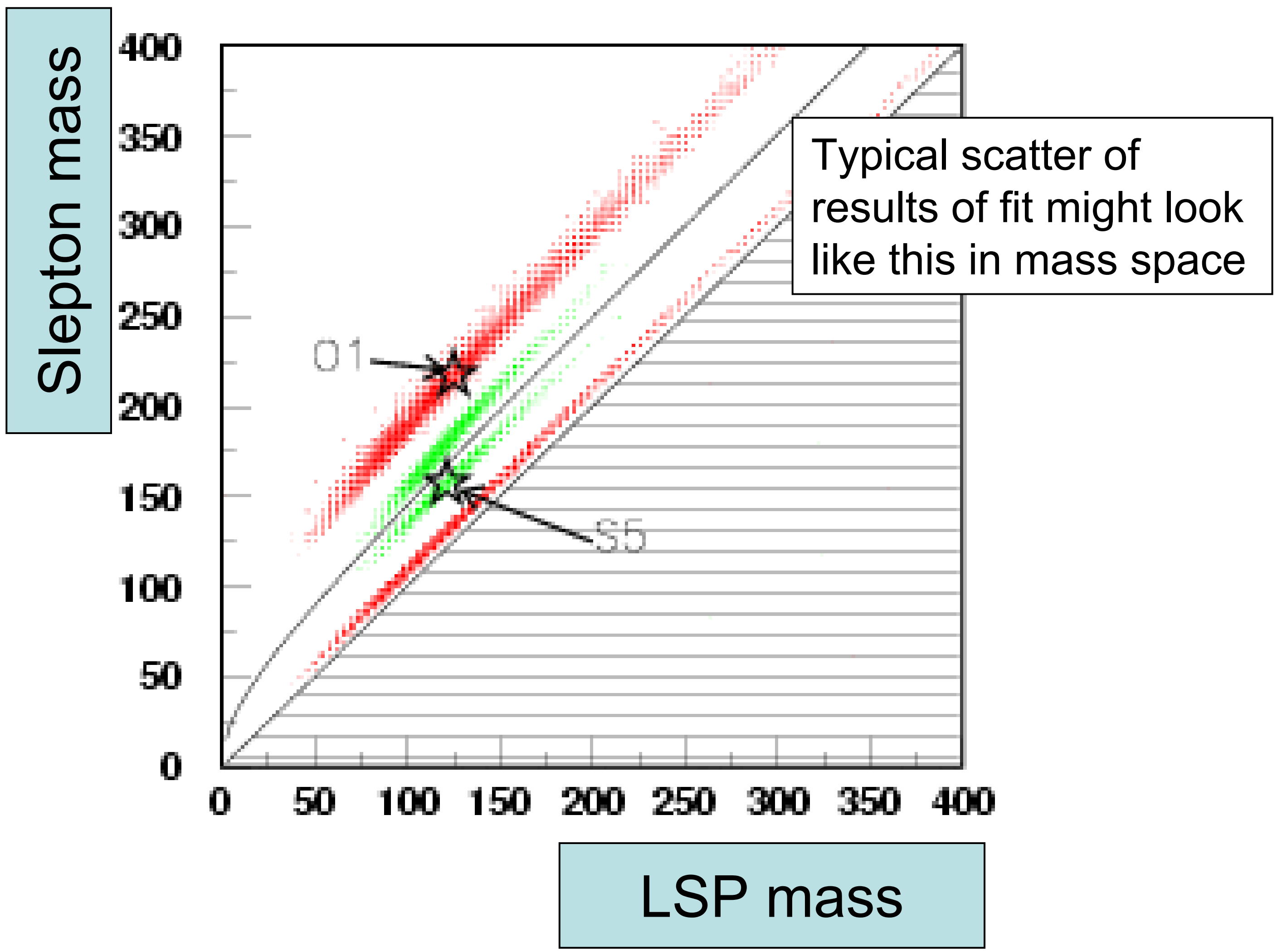
+

Theoretical expressions for edge positions in terms of masses

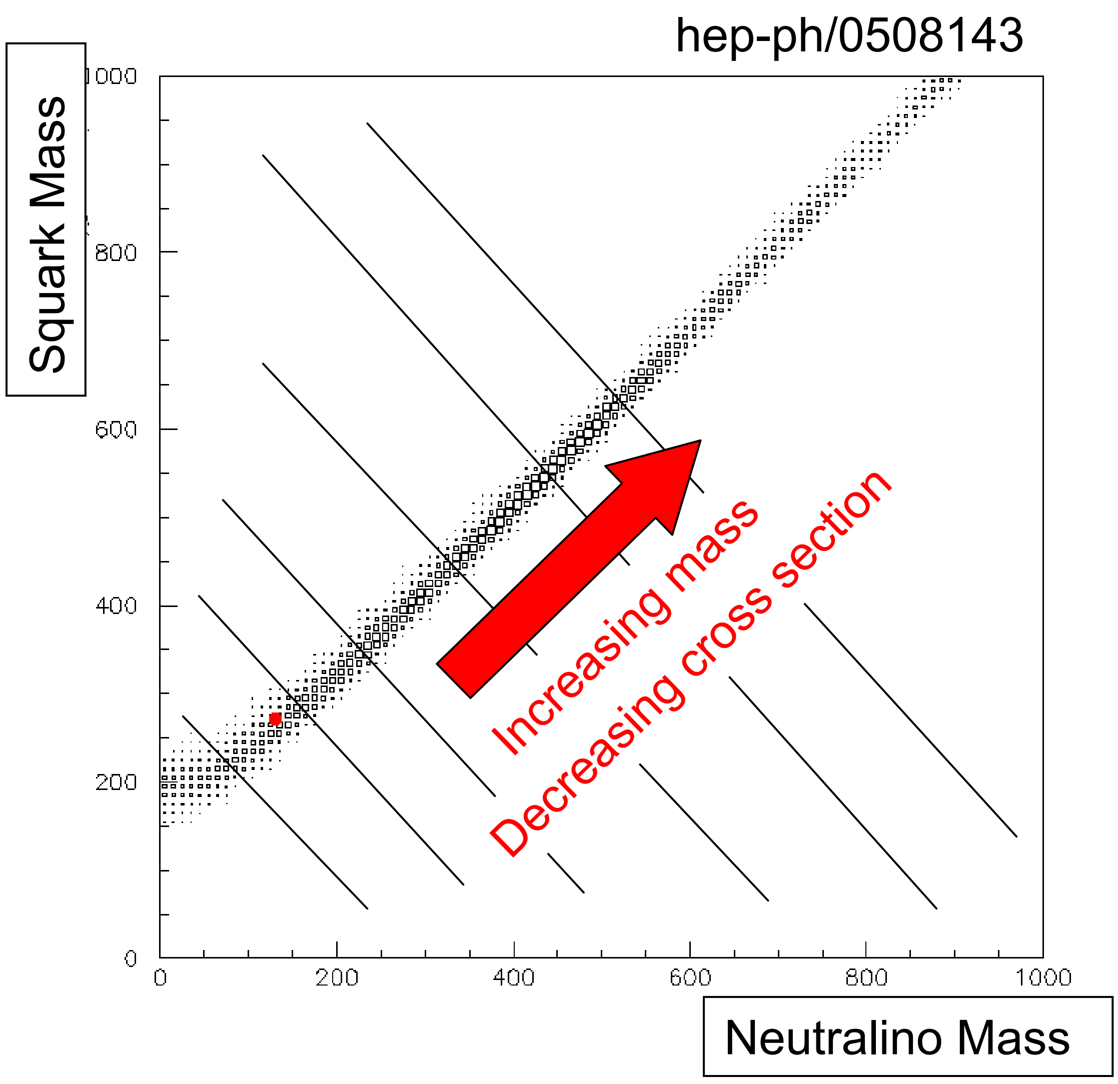
Related edge	Kinematic endpoint
l^+l^- edge	$(m_{ll}^{\max})^2 = (\tilde{\xi} - \tilde{l})(\tilde{l} - \tilde{\chi})/\tilde{l}$
l^+l^-q edge	$(m_{llq}^{\max})^2 = \begin{cases} \max\left[\frac{(\tilde{q}-\tilde{\xi})(\tilde{\xi}-\tilde{\chi})}{\tilde{\xi}}, \frac{(\tilde{q}-\tilde{l})(\tilde{l}-\tilde{\chi})}{\tilde{l}}, \frac{(\tilde{q}\tilde{l}-\tilde{\xi}\tilde{\chi})(\tilde{\xi}-\tilde{l})}{\tilde{\xi}\tilde{l}}\right] \\ \text{except for the special case in which } \tilde{l}^2 < \tilde{q}\tilde{\chi} < \tilde{\xi}^2 \text{ and} \\ \tilde{\xi}^2\tilde{\chi} < \tilde{q}\tilde{l}^2 \text{ where one must use } (m_{\tilde{q}} - m_{\tilde{\chi}_1^0})^2. \end{cases}$
Xq edge	$(m_{Xq}^{\max})^2 = X + (\tilde{q} - \tilde{\xi})\left[\tilde{\xi} + X - \tilde{\chi} + \sqrt{(\tilde{\xi} - X - \tilde{\chi})^2 - 4X\tilde{\chi}}\right]/(2\tilde{\xi})$
l^+l^-q threshold	$(m_{llq}^{\min})^2 = \begin{cases} [\ 2\tilde{l}(\tilde{q}-\tilde{\xi})(\tilde{\xi}-\tilde{\chi}) + (\tilde{q}+\tilde{\xi})(\tilde{\xi}-\tilde{l})(\tilde{l}-\tilde{\chi}) \\ -(\tilde{q}-\tilde{\xi})\sqrt{(\tilde{\xi}+\tilde{l})^2(\tilde{l}+\tilde{\chi})^2 - 16\tilde{\xi}\tilde{l}^2\tilde{\chi}}\ ]/(4\tilde{l}\tilde{\xi}) \end{cases}$
$l^\pm_{\text{near}}q$ edge	$(m_{l_{\text{near}}q}^{\max})^2 = (\tilde{q} - \tilde{\xi})(\tilde{\xi} - \tilde{l})/\tilde{\xi}$
$l^\pm_{\text{far}}q$ edge	$(m_{l_{\text{far}}q}^{\max})^2 = (\tilde{q} - \tilde{\xi})(\tilde{l} - \tilde{\chi})/\tilde{l}$
$l^\pm q$ high-edge	$(m_{lq(\text{high})}^{\max})^2 = \max\left[(m_{l_{\text{near}}q}^{\max})^2, (m_{l_{\text{far}}q}^{\max})^2\right]$
$l^\pm q$ low-edge	$(m_{lq(\text{low})}^{\max})^2 = \min\left[(m_{l_{\text{near}}q}^{\max})^2, (\tilde{q} - \tilde{\xi})(\tilde{l} - \tilde{\chi})/(2\tilde{l} - \tilde{\chi})\right]$
M_{T2} edge	$\Delta M = m_{\tilde{l}} - m_{\tilde{\chi}_1^0}$

Fit all edge position for masses!
...mainly constrain mass differences

Recommend
“fit” not “solve”



Cross section information is orthogonal to mass differences



How applicable are these long chain techniques ?

For the chain $\tilde{q} \to q\tilde{\chi}_2^0 \to ql\tilde{l}_R \to qll\tilde{\chi}_1^0$
we need:

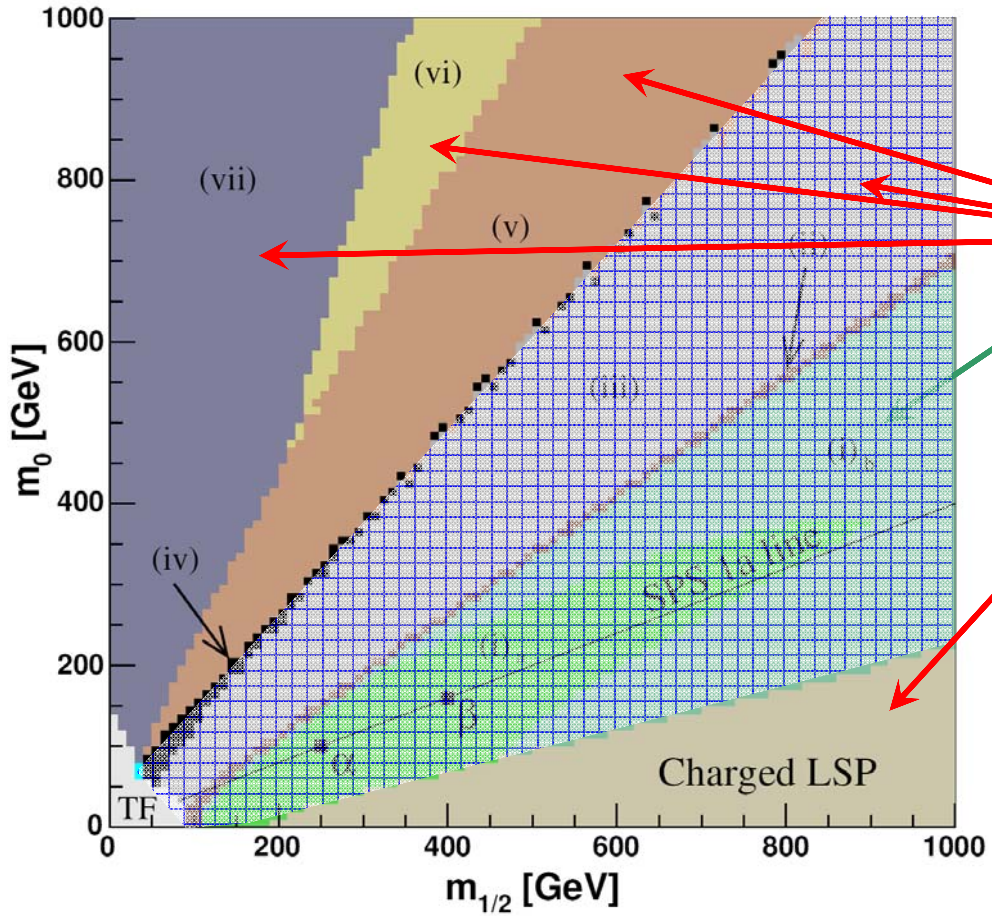
- $m_{\tilde{\chi}_2^0} > m_{\tilde{l}_R} > m_{\tilde{\chi}_1^0}$
- $m_{\tilde{g}} > m_{\tilde{q}}$

This is possible over a wide range of parameter space.

If this chain is not open, the method is still valid, but we need to look at other decay chains.

Example mSUGRA inspired scenario: $-A_0 = m_0,\ \tan\beta = 10,\ \mu > 0$

[See Allanach et al, Eur.Phys.J.**C**25 (2002) 113, hep-ph/0202233]



$\tilde{q} \to q\tilde{\chi}_2^0 \to q l \tilde{l}_R \to q l l \tilde{\chi}_1^0$

Decay chain doesn't work, but others are possible.

The hatched area is amenable to this method in some form.

$$\begin{cases} m_{\tilde{g}} > m_{\tilde{q}} \\ m_{\tilde{l}_R} > m_{\tilde{\chi}_2^0} > m_{\tilde{\chi}_1^0} \end{cases}$$

Its pretty hard to do $\frac{1}{10}\sigma_\alpha$ for other mSUGRA inspired scenarios.

Lighter green is where

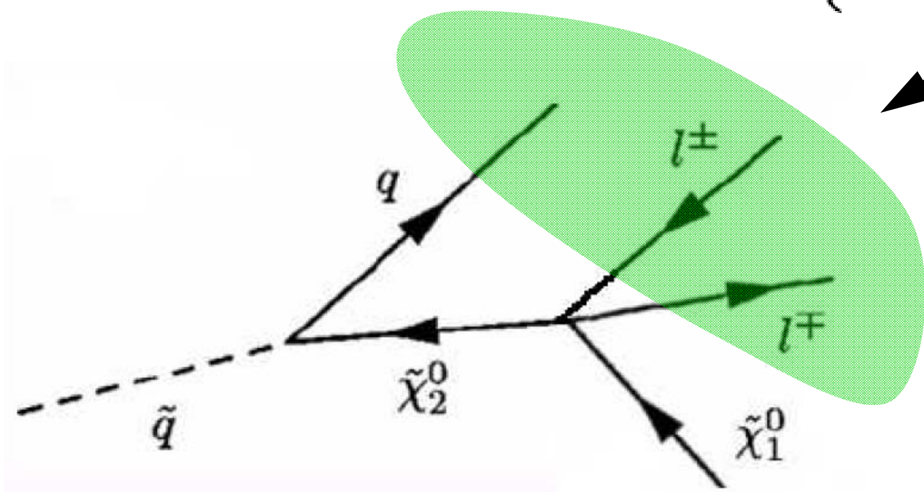
This area doesn't change much / anything with this!

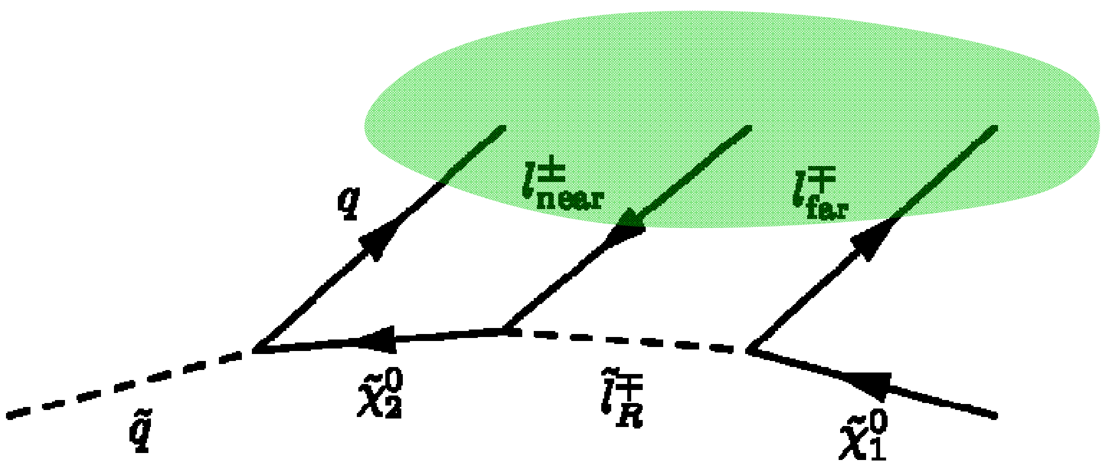
Figure from hep-ph/0410303

Other ambiguities

$$(m^2_{llq})^{\max} = \begin{cases} (m_{\tilde{q}} - m_{\tilde{\chi}^0_1})^2 \text{ if } m^2_{\tilde{\chi}^0_2} > m_{\tilde{q}} m_{\tilde{\chi}^0_1} \\ (m^2_{\tilde{q}} - m^2_{\tilde{\chi}^0_2})(m^2_{\tilde{\chi}^0_2} - m^2_{\tilde{\chi}^0_1})/m^2_{\tilde{\chi}^0_2} \text{ otherwise.} \end{cases}$$

hep-ph/0609298





Both look the same to the detector

(Though shape differs – see later)

Endpoints are not always linearly independent

e.g. if $m_{\tilde{q}_L} > m^2_{\tilde{\chi}^0_2}/m_{\tilde{\chi}^0_1}$ and $m^2_{\tilde{\chi}^0_1} + m^2_{\tilde{\chi}^0_2} > 2m_{\tilde{\chi}^0_1}m_{\tilde{\chi}^0_2} > 2m^2_{\tilde{q}_L}$

then the endpoints are

$$\begin{aligned}
\left(m_{ll}^{\max}\right)^2 &= (m^2_{\tilde{\chi}^0_2} - m^2_{\tilde{l}_R})(m^2_{\tilde{l}_R} - m^2_{\tilde{\chi}^0_1})/m^2_{\tilde{l}_R} \\
\left(m_{qll}^{\max}\right)^2 &= (m^2_{\tilde{q}_L} - m^2_{\tilde{l}_R})(m^2_{\tilde{l}_R} - m^2_{\tilde{\chi}^0_1})/m^2_{\tilde{l}_R} \\
\left(m_{ql_n}^{\max}\right)^2 &= (m^2_{\tilde{q}_L} - m^2_{\tilde{\chi}^0_2})(m^2_{\tilde{\chi}^0_2} - m^2_{\tilde{l}_R})/m^2_{\tilde{\chi}^0_2} \\
\left(m_{ql_f}^{\max}\right)^2 &= (m^2_{\tilde{q}_L} - m^2_{\tilde{\chi}^0_2})(m^2_{\tilde{l}_R} - m^2_{\tilde{\chi}^0_1})/m^2_{\tilde{l}_R}
\end{aligned}$$

$$\Rightarrow \quad \left(m_{qll}^{\max}\right)^2 = \left(m_{ll}^{\max}\right)^2 + \left(m_{ql_f}^{\max}\right)^2$$

Four endpoints not always sufficient to find the masses

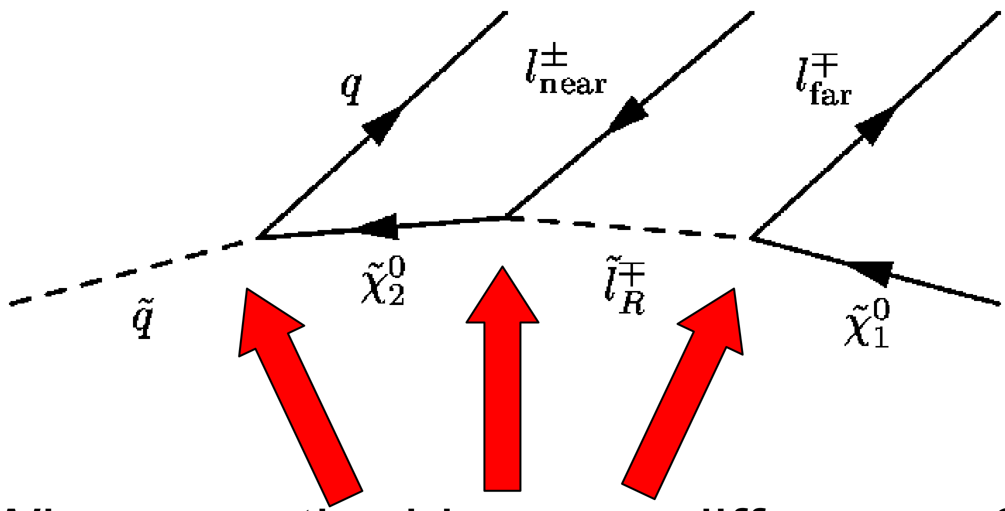
angle between leptons in slepton rest frame

- Introduce new distribution $m_{qll\ \theta>\pi/2}$ identical to m_{qll} except require $\theta>\pi/2$

It is the minimum of this distribution which is interesting

Slide from David Miller

Different parts of model space behave differently: m_{QLL}^{max}



Where are the big mass differences?

hep-ph/0007009

$$(m_{llq}^{\max})^2 = \begin{cases} \max\left[\frac{(\tilde{q}-\tilde{\xi})(\tilde{\xi}-\tilde{\chi})}{\tilde{\xi}}, \frac{(\tilde{q}-\tilde{l})(\tilde{l}-\tilde{\chi})}{\tilde{l}}, \frac{(\tilde{q}\tilde{l}-\tilde{\xi}\tilde{\chi})(\tilde{\xi}-\tilde{l})}{\tilde{\xi}\tilde{l}} \right] \\ \text{except for the special case in which } \tilde{l}^2 < \tilde{q}\tilde{\chi} < \tilde{\xi}^2 \text{ and} \\ \tilde{\xi}^2\tilde{\chi} < \tilde{q}\tilde{l}^2 \text{ where one must use } (m_{\tilde{q}} - m_{\tilde{\chi}^0_1})^2. \end{cases}$$

Exercise

- (10) Prove either

$$
(m_{llq}^{\max})^2 = \begin{cases}
(m_{\tilde{q}}^2 - m_{\tilde{\chi}_2^0}^2)(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{\chi}_1^0}^2)/m_{\tilde{\chi}_2^0}^2 & \text{iff} \quad {m_{\tilde{\chi}_2^0}}^2 < m_{\tilde{\chi}_1^0} m_{\tilde{q}}, \\
(m_{\tilde{q}}^2 - m_{\tilde{l}}^2)(m_{\tilde{l}}^2 - m_{\tilde{\chi}_1^0}^2)/m_{\tilde{l}}^2 & \text{iff} \quad m_{\tilde{\chi}_1^0} m_{\tilde{q}} < {m_{\tilde{l}}}^2, \\
(m_{\tilde{q}}^2 m_{\tilde{l}}^2 - m_{\tilde{\chi}_2^0}^2 m_{\tilde{\chi}_1^0}^2)(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{l}}^2)/(m_{\tilde{\chi}_2^0}^2 m_{\tilde{l}}^2) & \text{iff} \quad {m_{\tilde{l}}}^2 m_{\tilde{q}} < m_{\tilde{\chi}_1^0} {m_{\tilde{\chi}_2^0}}^2, \\
(m_{\tilde{q}} - m_{\tilde{\chi}_1^0})^2 & \text{otherwise.}
\end{cases}
$$

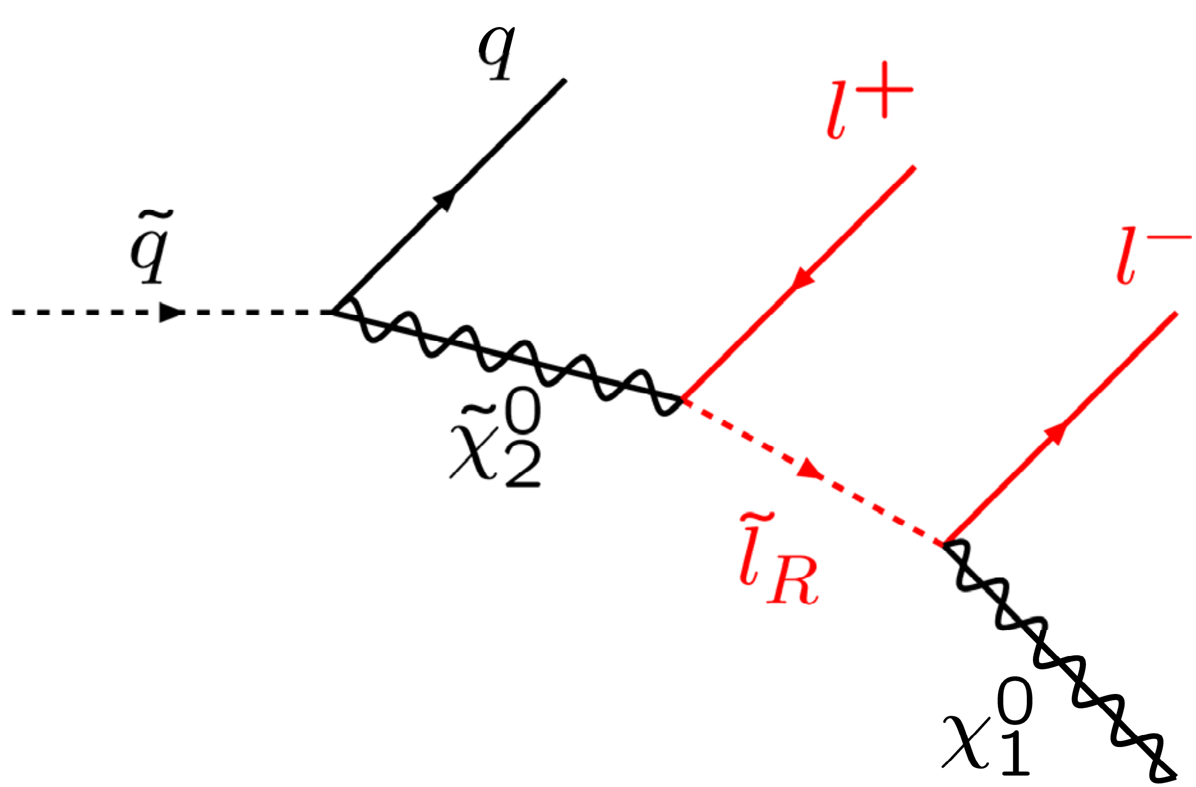
or

$$
(m_{ll_q}^{\max})^2 = \begin{cases}
\max\left[\frac{(\tilde{q}-\tilde{\xi})(\tilde{\xi}-\tilde{\chi})}{\tilde{\xi}}, \frac{(\tilde{q}-\tilde{l})(\tilde{l}-\tilde{\chi})}{\tilde{l}}, \frac{(\tilde{q}\tilde{l}-\tilde{\xi}\tilde{\chi})(\tilde{\xi}-\tilde{l})}{\tilde{\xi}\tilde{l}}\right] \\
\text{except for the special case in which } \tilde{l}^2 < \tilde{q}\tilde{\chi} < \tilde{\xi}^2 \text{ and} \\
\tilde{\xi}^2\tilde{\chi} < \tilde{q}\tilde{l}^2 \text{ where one must use } (m_{\tilde{q}} - m_{\tilde{\chi}_1^0})^2.
\end{cases}
$$

and show that they are equivalent.

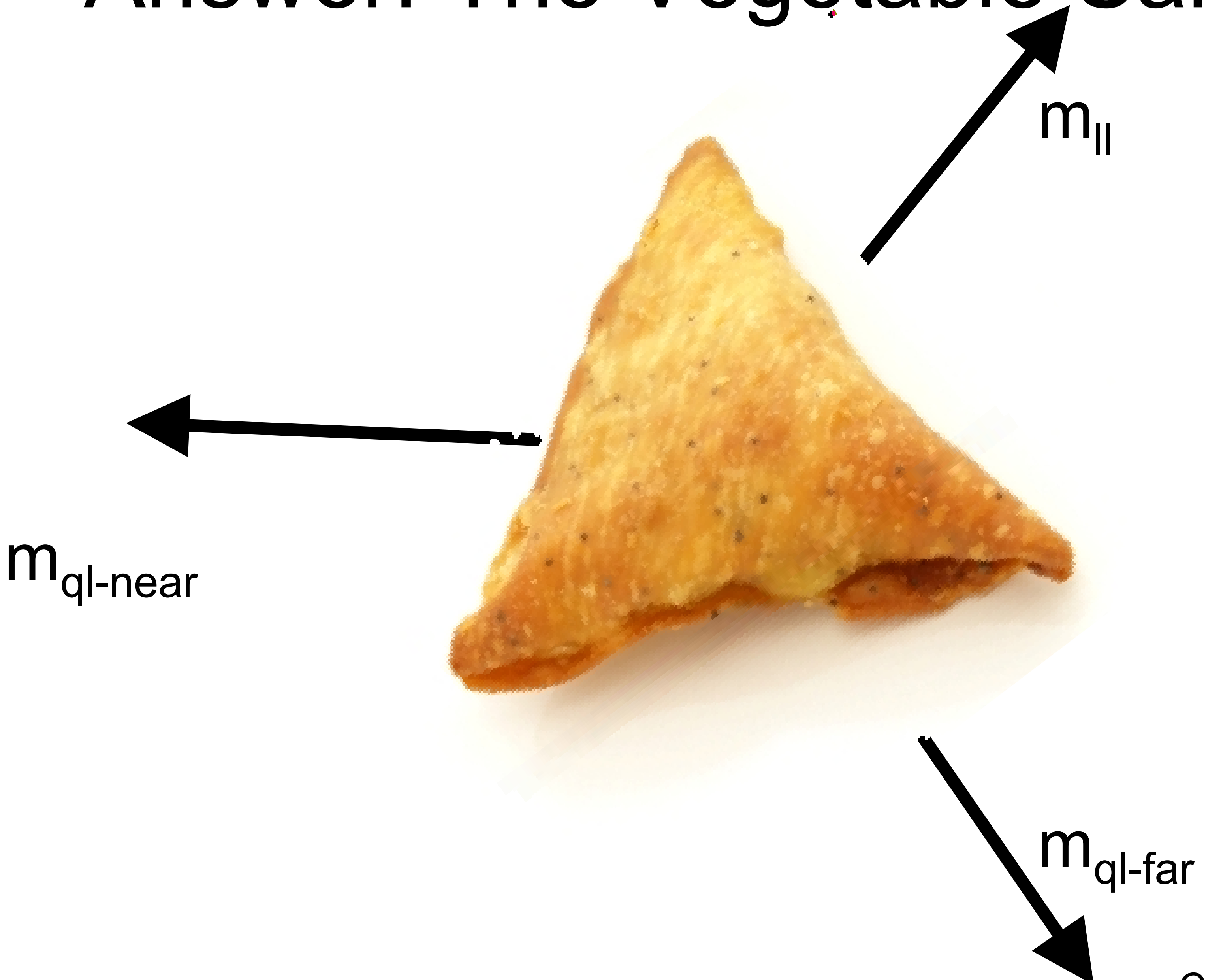
(See definitions of symbols approx three slides back).

Which parts of
$(m^2_{qlnear}, m^2_{qlfar}, m^2_{ll})$-space
are populated by these events:



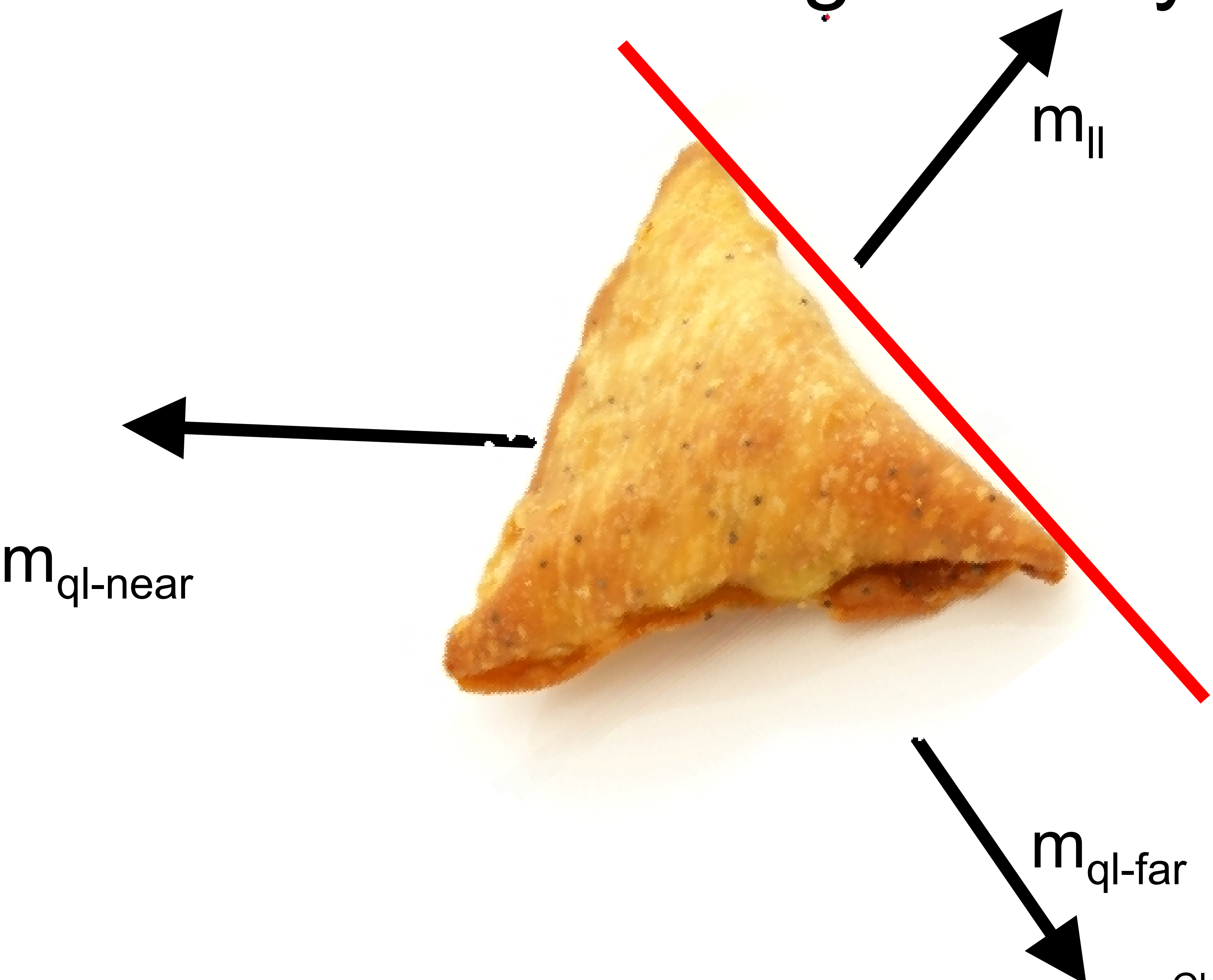
arXiv:0902.2331

Answer: The Vegetable Samosa



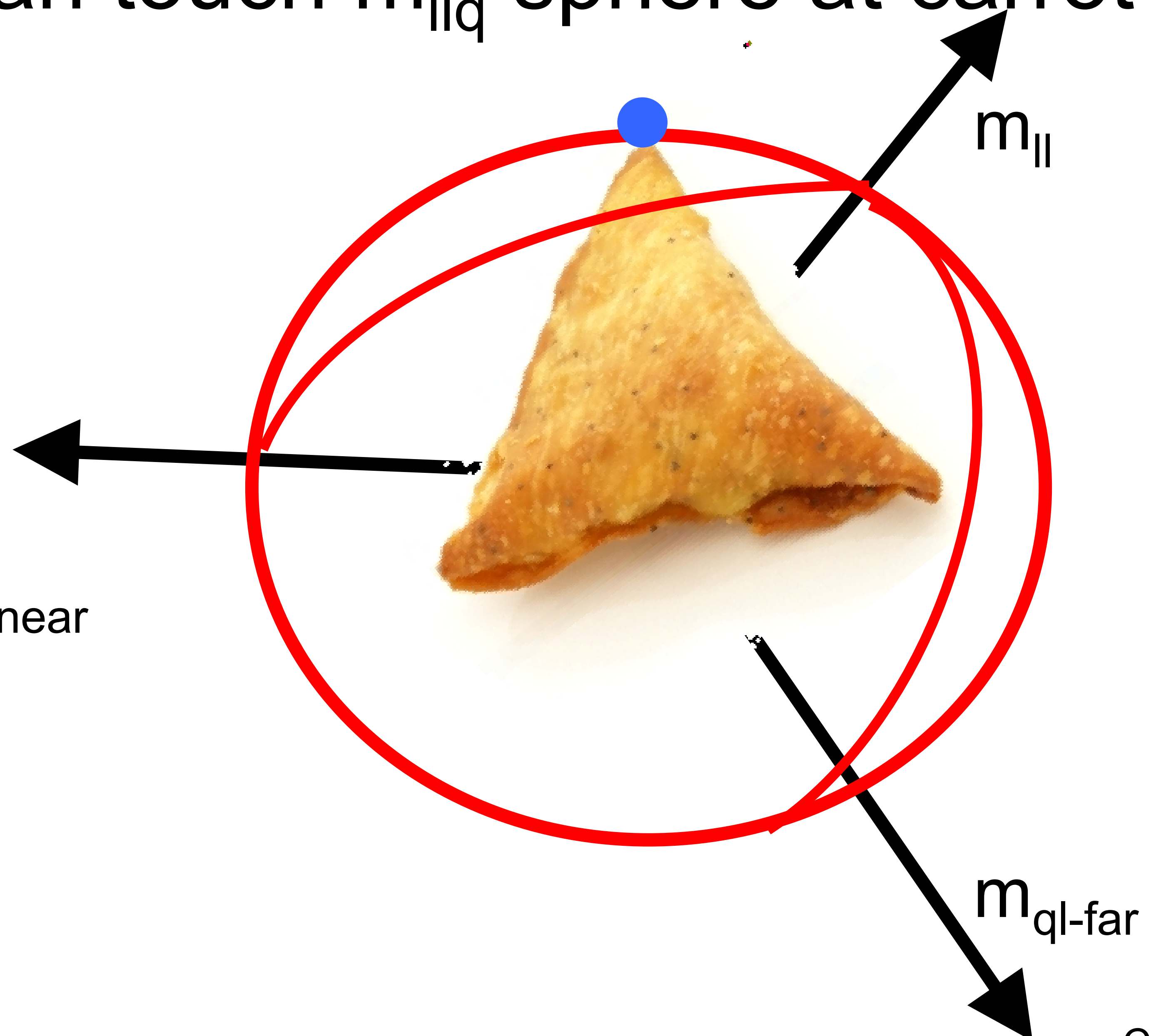
Christopher Lester

Can see ll edge clearly.



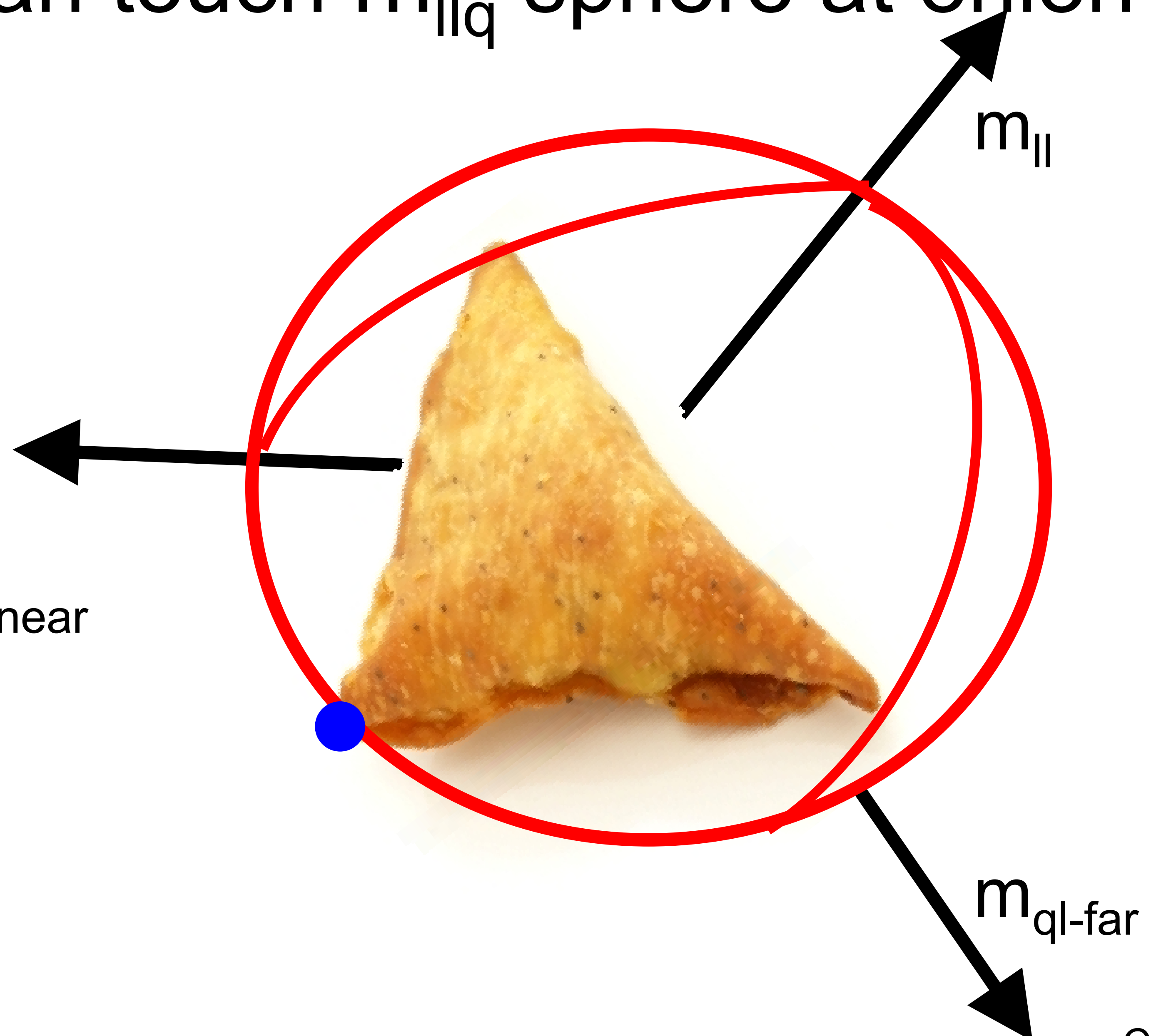
Christopher Lester

Can touch m_{llq} sphere at carrot corner



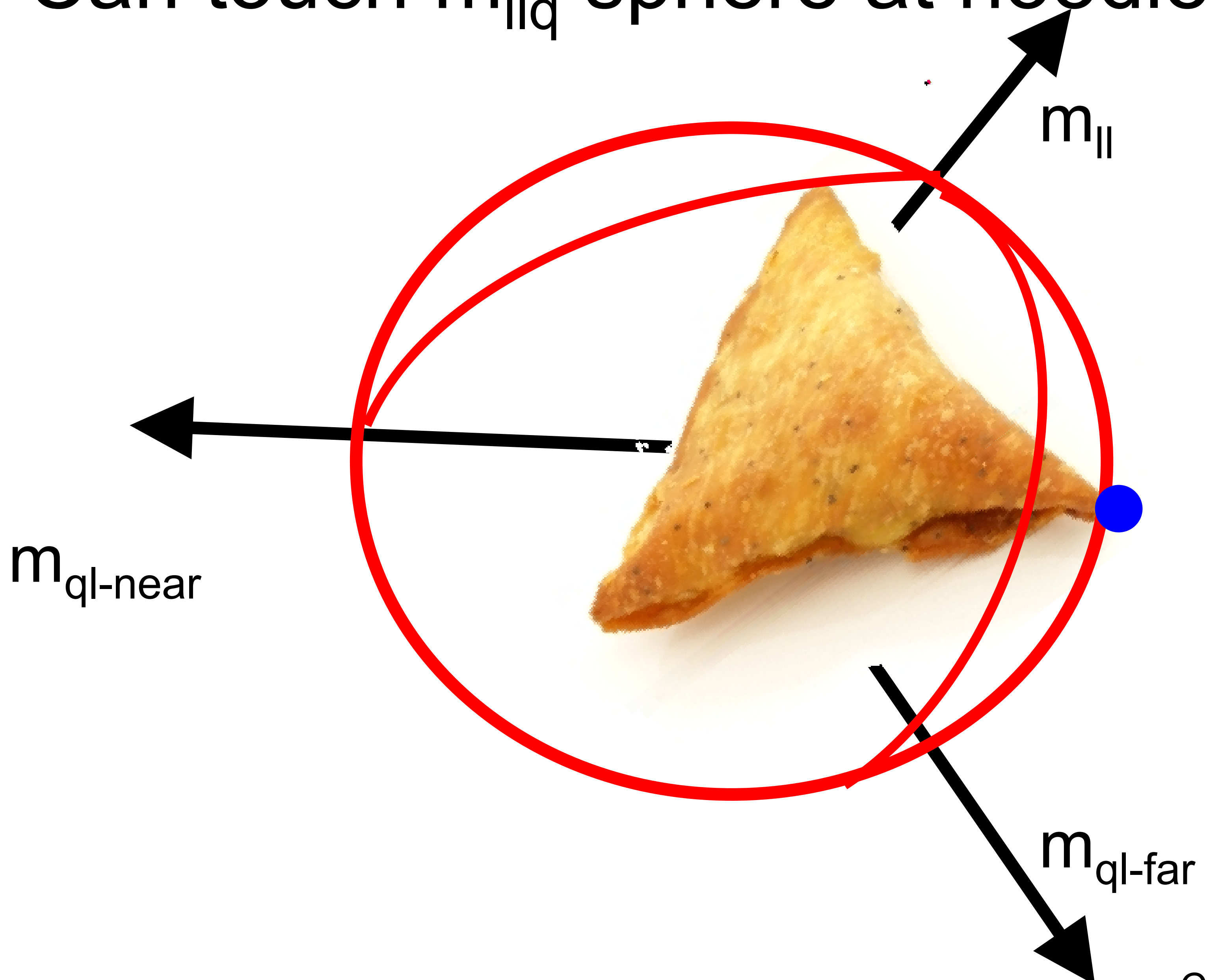
Christopher Lester

Can touch m_{llq} sphere at onion corner



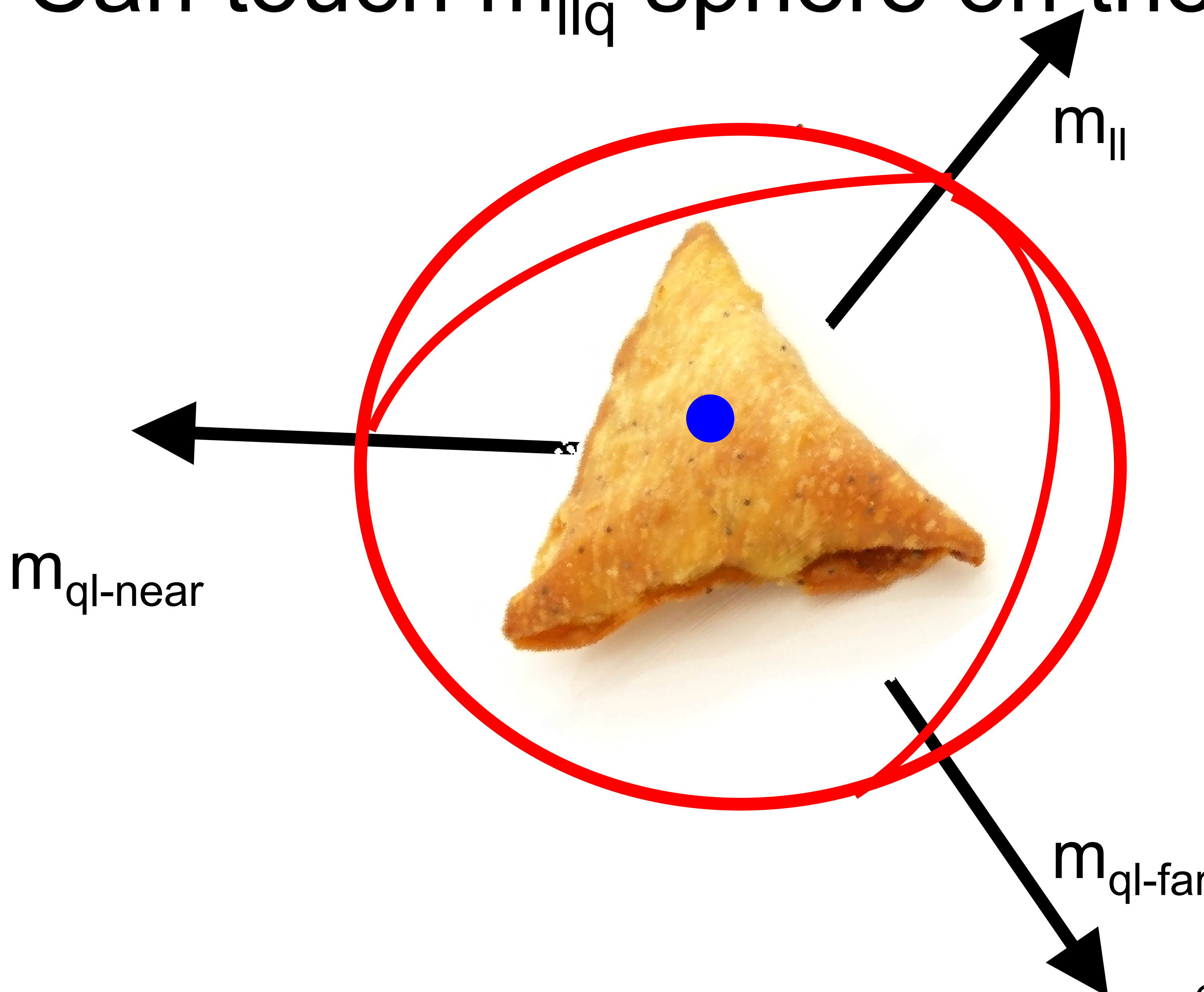
Christopher Lester

Can touch m_{llq} sphere at noodle corner



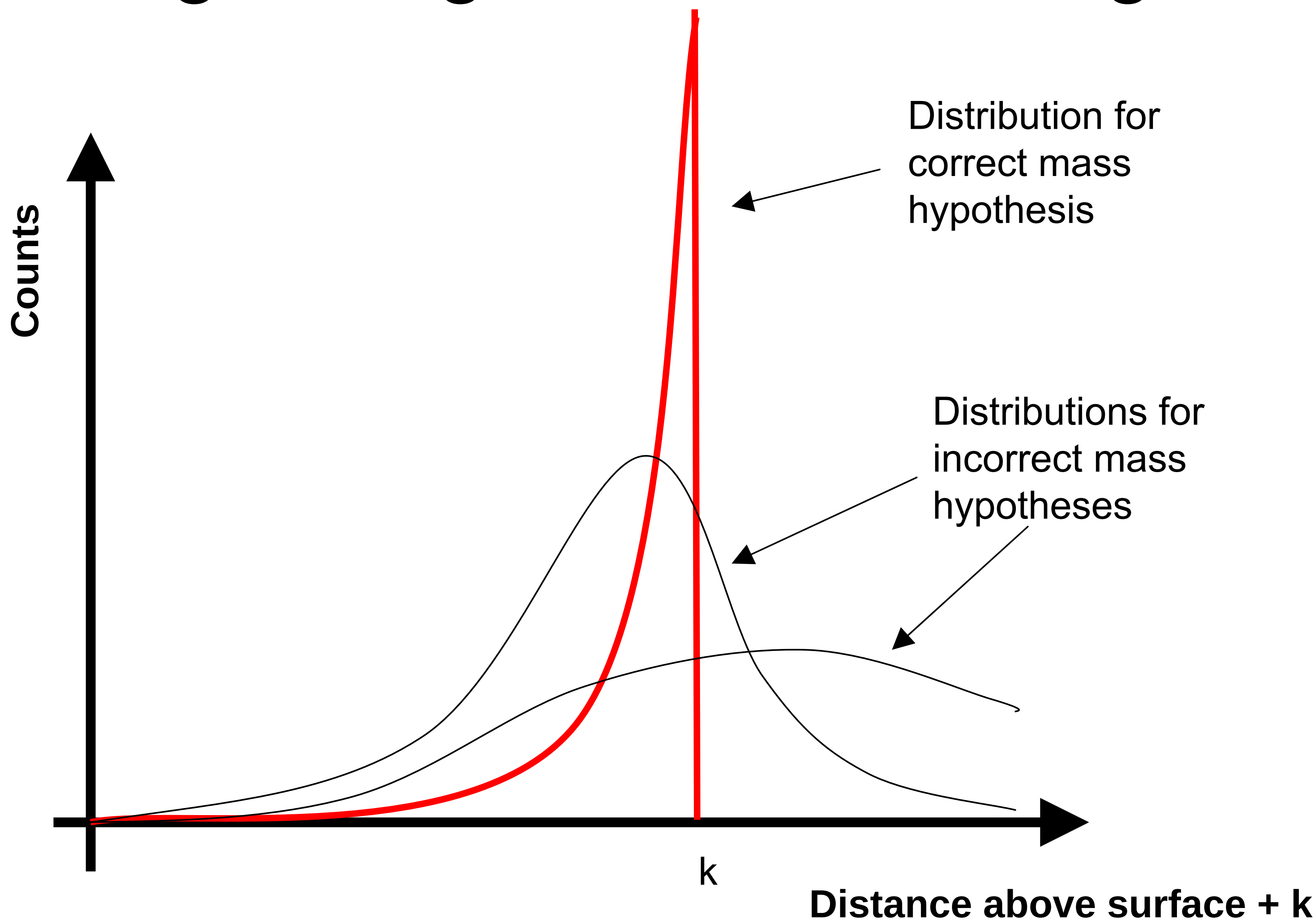
Christopher Lester

Can touch m_{llq} sphere on the “front”



Christopher Lester

So, in principle, find masses by looking for highest contrast edge.



Exercise

(11) For fixed masses of the four particles on the SUSY backbone, find a function $f(q^\mu, l_{near}^\mu, l_{far}^\mu)$ that is zero on the surface of the samosa, and is non-zero elsewhere.

[Hint: I suggest you try to solve for the invisible LSP momentum as a linear combination of the three visible four-momenta q^μ, l_{near}^μ, l_{far}^μ and a fourth four-vector that is a totally antisymmetric combination of them $\Omega_\mu = \varepsilon_{\mu\nu\sigma\rho}\, q^\nu\, l_{near}^\sigma\, l_{far}^\rho$. Then see under what conditions this solution is meaningful.]

arXiv:0902.2331

The “shadow” (projection) of the samosa is useful for origami too

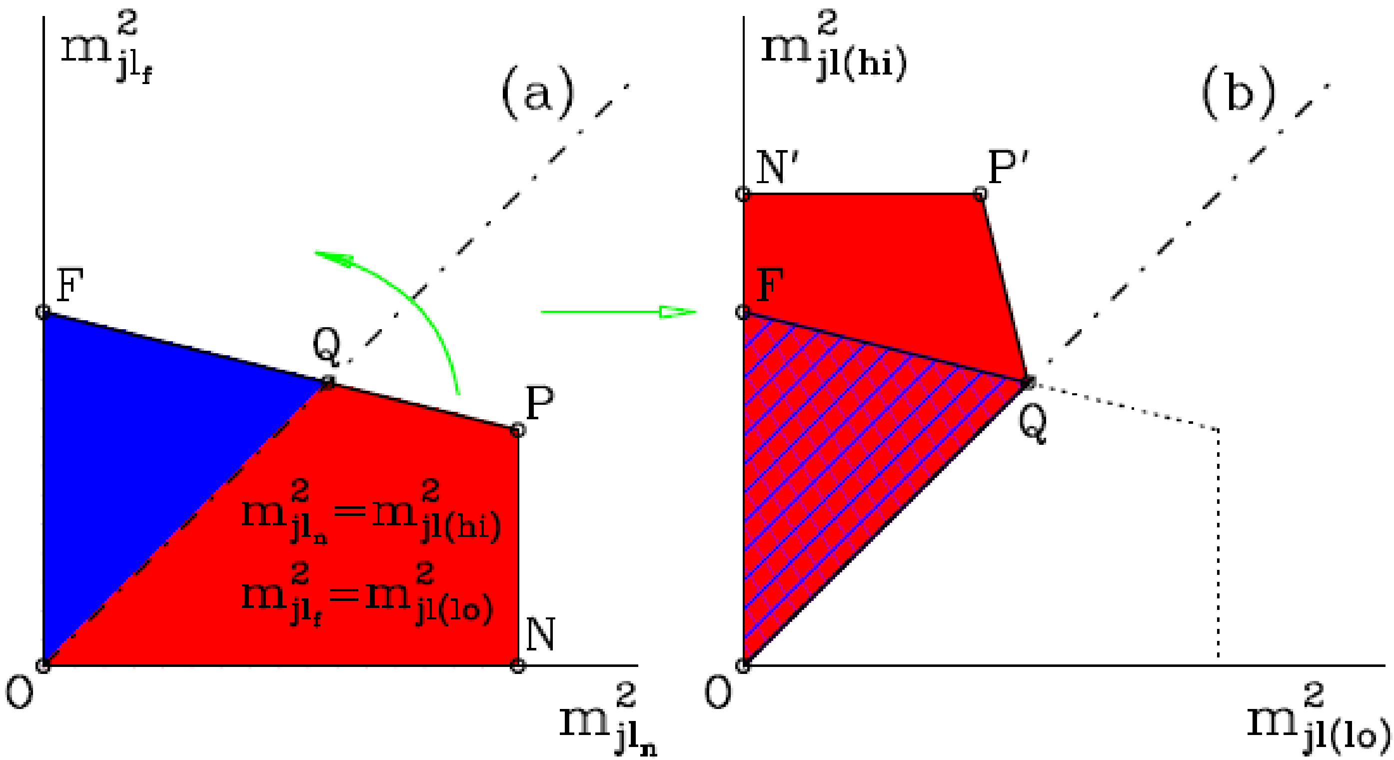
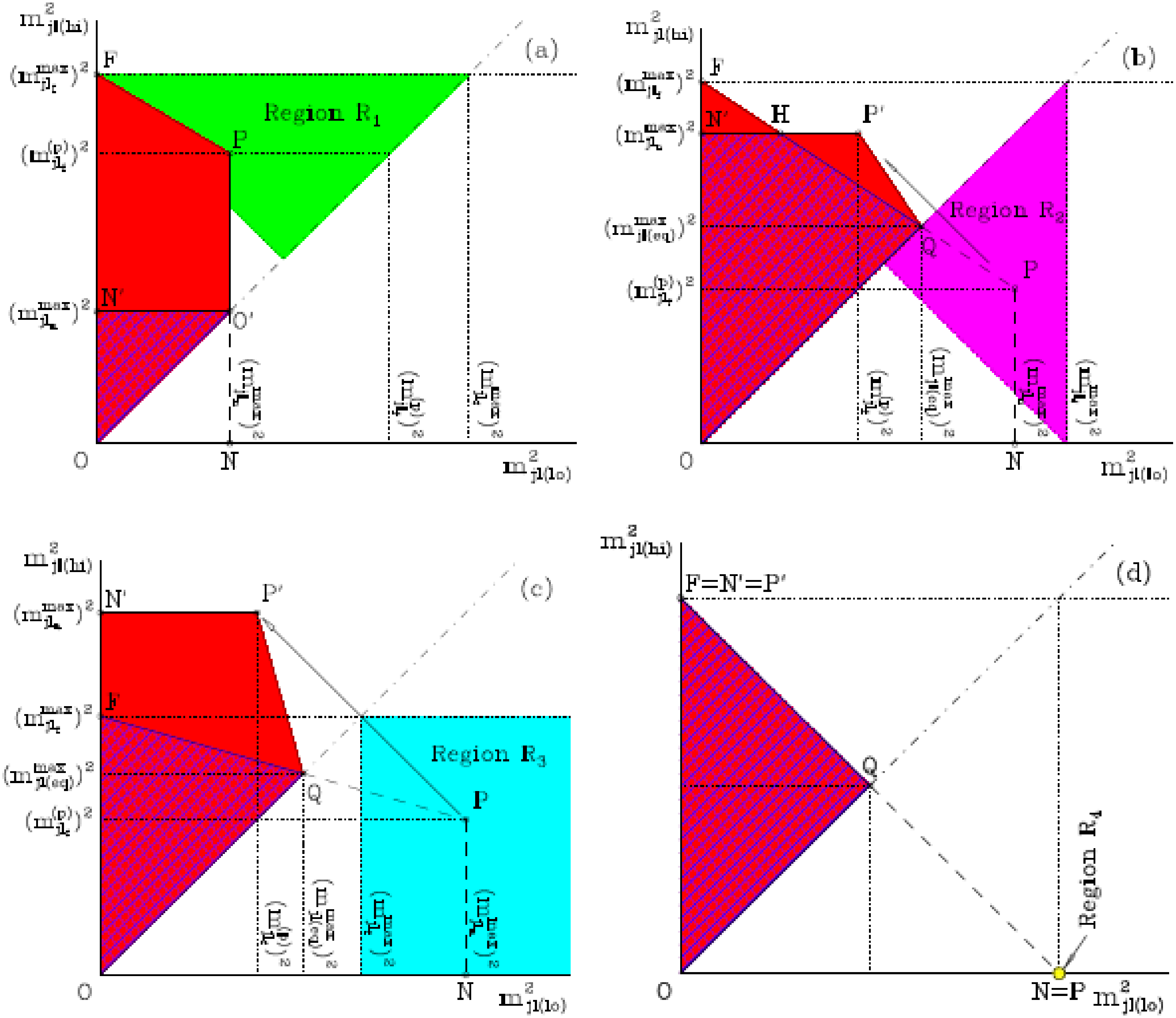


Figure 7: Obtaining the shape of the $m^2_{jl(lo)}$ versus $m^2_{jl(hi)}$ bivariate distribution by folding the $m^2_{jl_n}$ versus $m^2_{jl_f}$ distribution across the line $m^2_{jl_n} = m^2_{jl_f}$. This particular example applies to region $\mathcal{R}_3$. For the other three regions, refer to Figs. 8(a), 8(b) and 8(d).

arXiv:0903.4371


(a)
Region R_1
F
P
N'
O'
O
N
(b)
Region R_2
F
N'
H
P'
Q
P
O
N
(c)
Region R_3
N'
P'
F
Q
P
O
N
(d)
F=N'=P'
Q
Region R_4
O
N=P

Formalising an old idea … kinematic boundaries, creases, edges, cusps etc

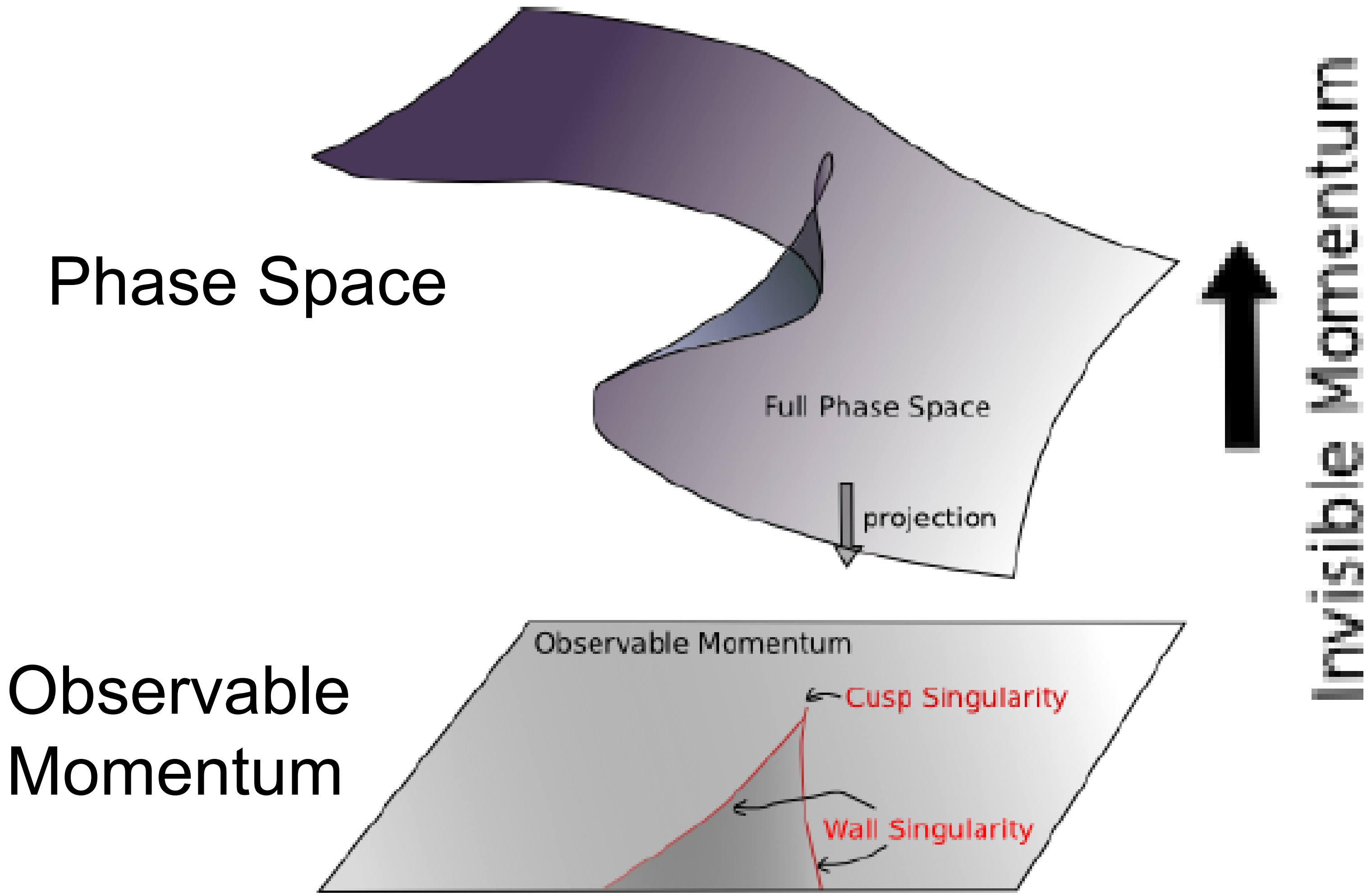
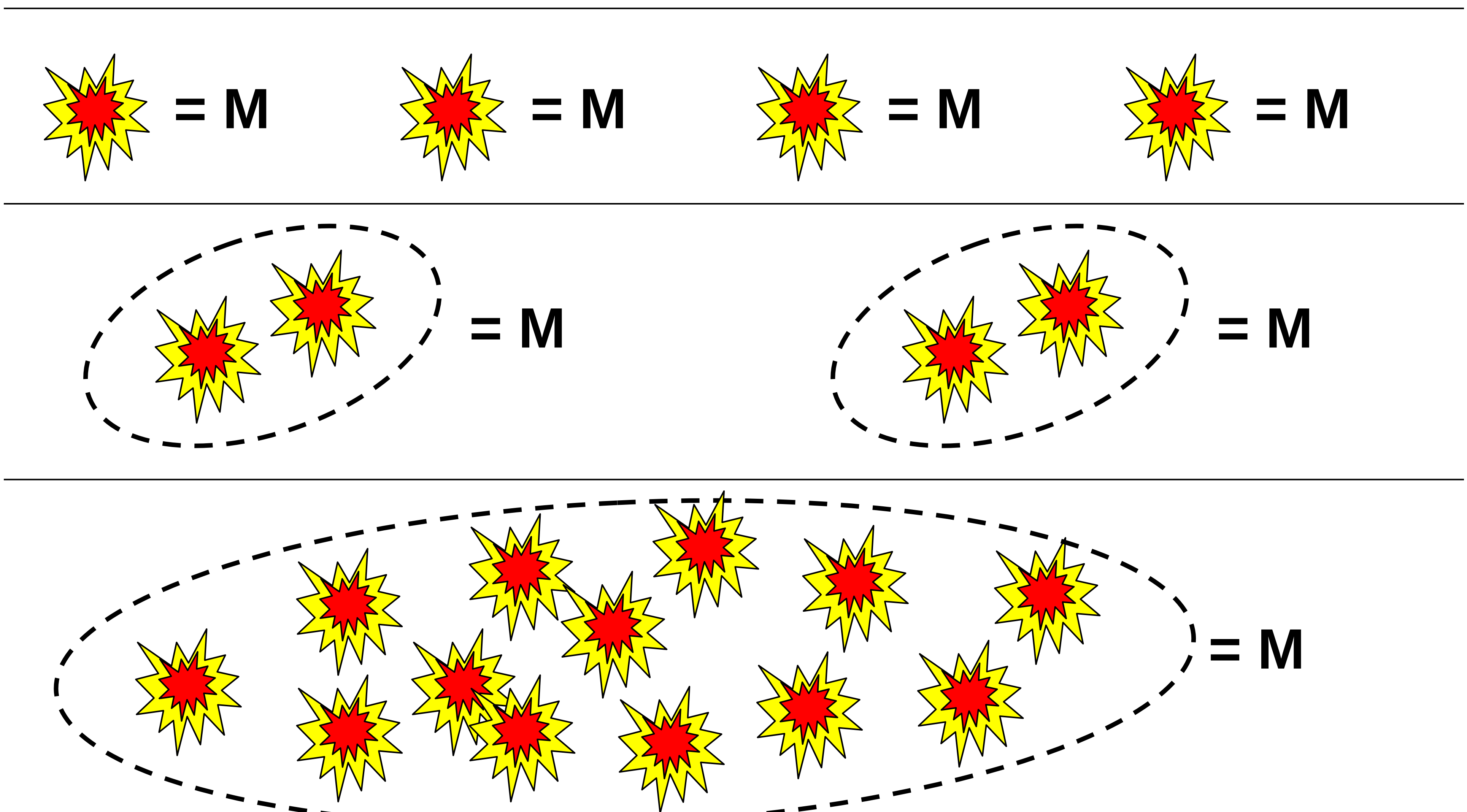


FIG. 1: A schematic diagram describing the relation between the full phase space and the projected observable phase space.

I.W.Kim: "Algebraic singularity method of mass measurement with missing energy" arXiv:0910.1149v2 [hep-ph]

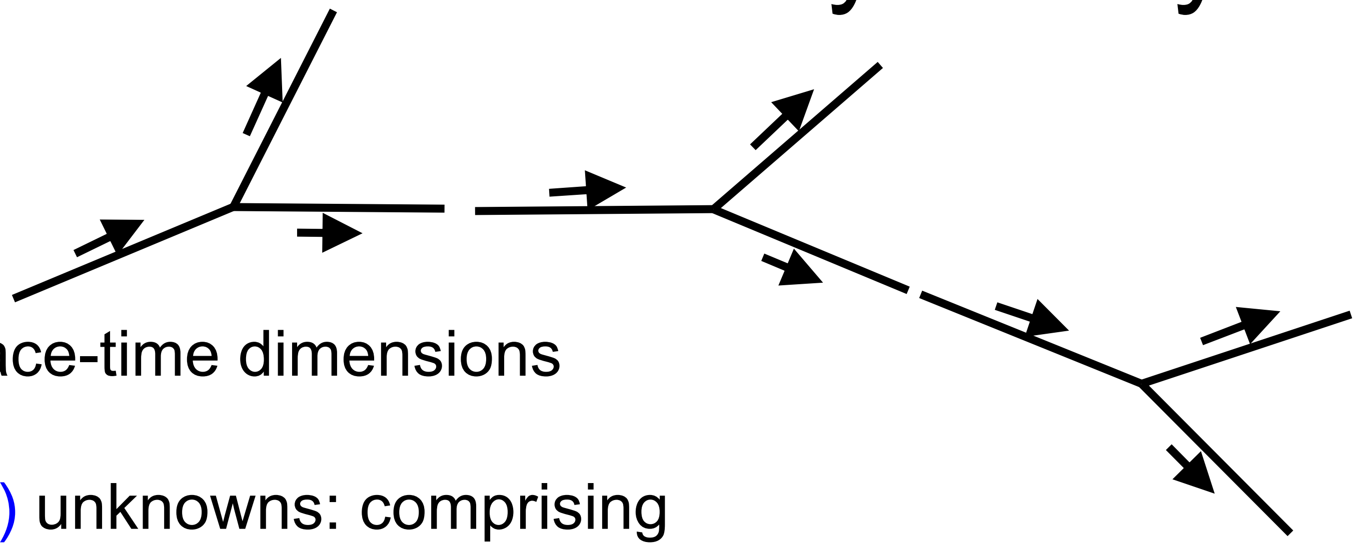
Adding even more assumptions …

Let’s consider what happens when we allow ourselves to look at more than one event ….



See sections X and IX of hep-ph/0402295

N successive 2-body decays



- In D space-time dimensions

- D+(N+1) unknowns: comprising
  - D unknown momentum-components for final "missing particle"
  - (N+1) unknown backbone-particle masses
- N+1 constraints:
  - Invariant masses of the backbone-*momenta* must match the "unknown" masses

- UNKNOWNS - CONSTRAINTS = D > 0
  - Cannot solve for unknowns! ☹

Why not look at K events?

- K events, each (N successive 2-body decays)

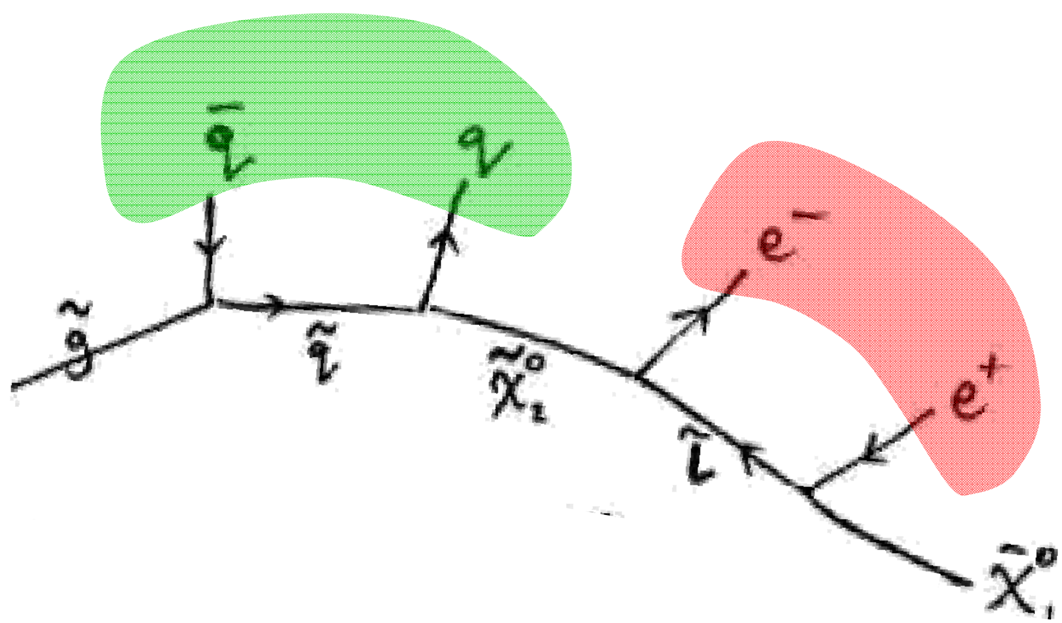
- KD+(N+1) **unknowns**: comprising
  - KD unknown momentum-components for final "missing particle"
  - (N+1) unknown backbone-particle masses
- K(N+1) **constraints**:
  - Invariant masses of the backbone-*momenta* must match the "unknown" massses

- UNKNOWNS - CONSTRAINTS = $K(D-(N+1))+(N+1)$
- System solvable for $K \geq \dfrac{N+1}{N+1-D}$ provided

$$N+1>D \text{ i.e. } N \geq 4.$$

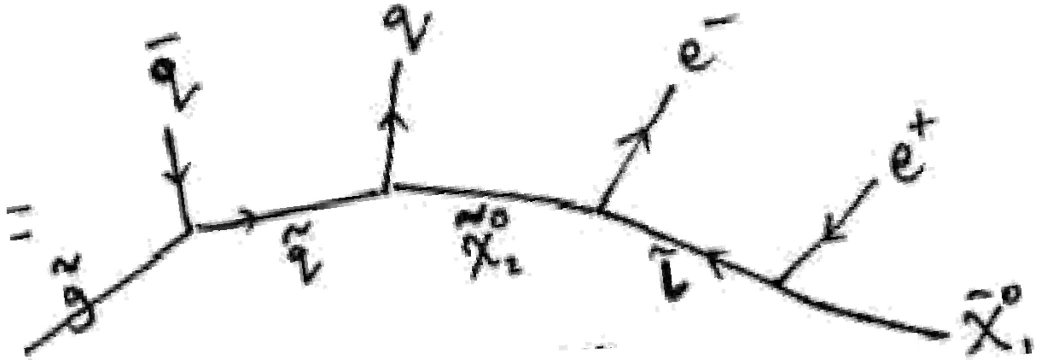
Ambiguities

- Which jet is which?
- Which lepton is which?



- So will need more events than the last calculation suggests ~ x4 ?

“Mass relation” method: summary



- Can:
  - reconstruct complete decay kinematics
  - Measure all sparticle masses
- provided that:
  - Chain has N≥4 successive two-body decays
  - One simultaneously examines at least

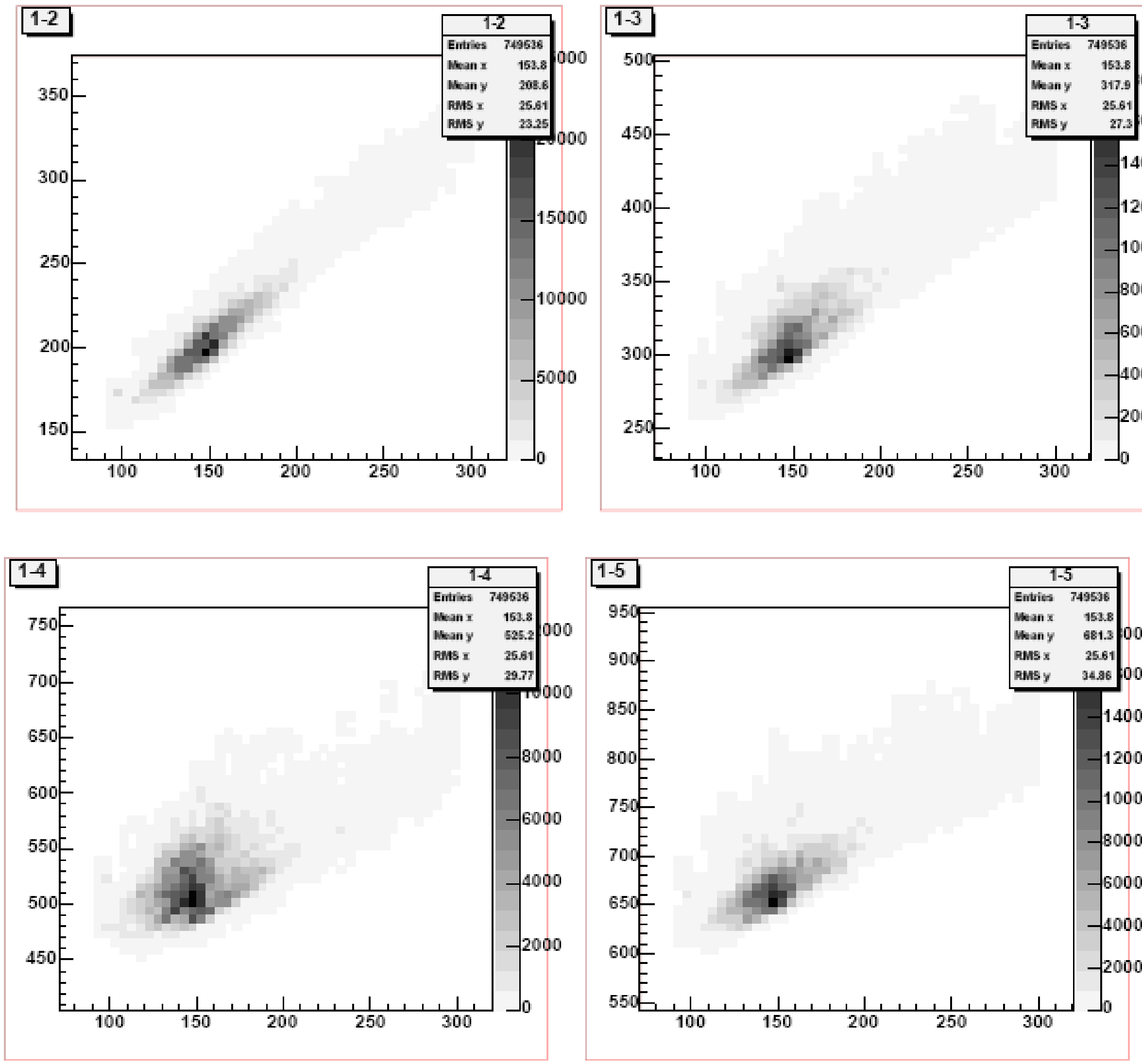
$$\frac{N+1}{N+1-D} = \frac{N+1}{N-3}$$

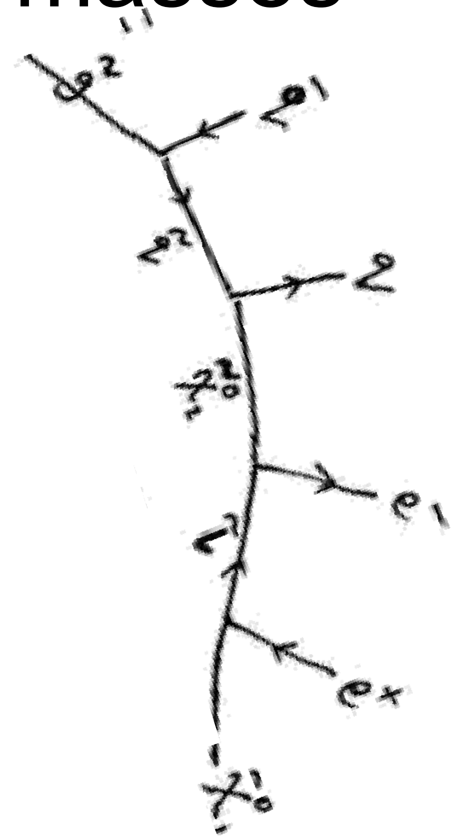
events sharing the same sparticles.

See sections X and IX of hep-ph/0402295

Some example reconstructed masses

(100 events, toy MC)





Caveats:

Though see Miller hep-ph/0501033

Nobody has shown that this will work for real data.
Sample purity. Bias.
Heavily model dependent?

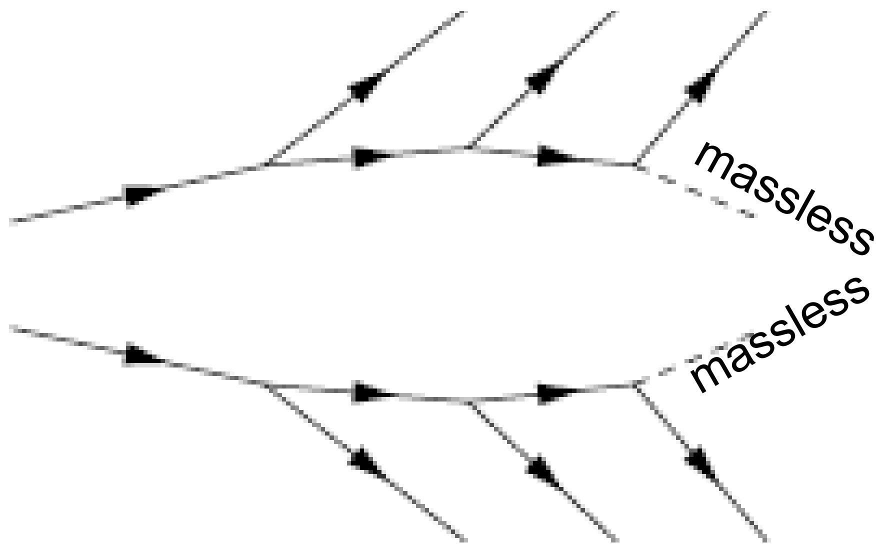
Dependence on reconstruction resolution.

N=4 two-body decays

- Fewer than 5 events
  - Under constrained, cannot solve
- 5 events
  - Can solve in principle (ignoring ambiguities)
  - Can treat events as “ideal”
- More than 5 events
  - Over constrained. Potential for inconsistency.
  - Reconstructed events will not “make sense” until resolutions are taken into account.

Another sort of “just”-constrained event

– get constraint from other “side”

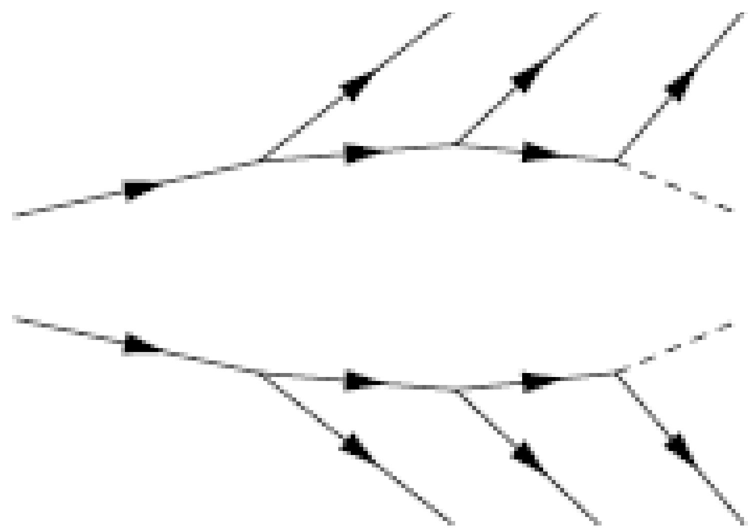


Left: case considered in hep-ph/9812233

- Even if there are invisible decay products, events can often be fully reconstructed if decay chains are long enough.
- (mass-shell constraints must be >= unknown momenta)
- Since we can use ptmiss constraint, chains can be shorter than N=4 now.

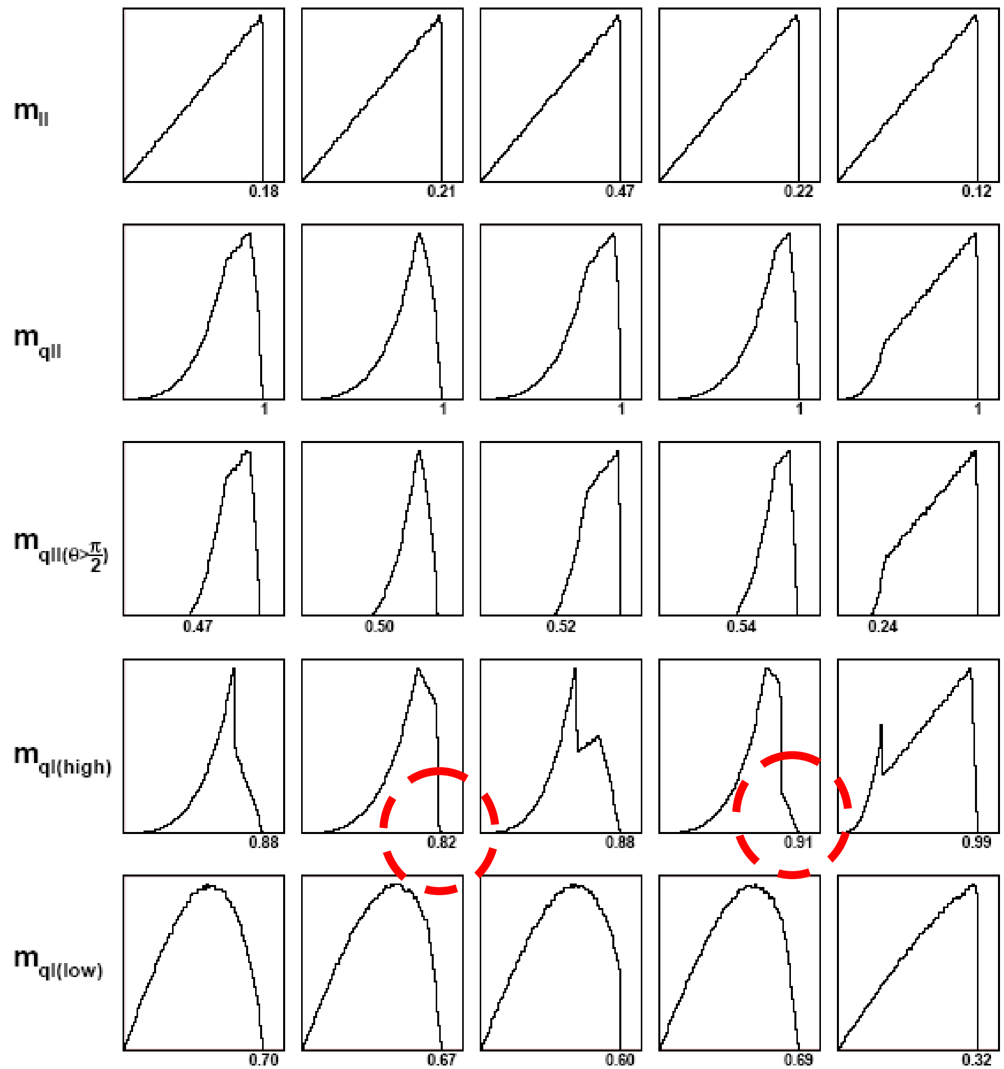
Or do both at once – pairs of double events!

- Pairs of events of the form:



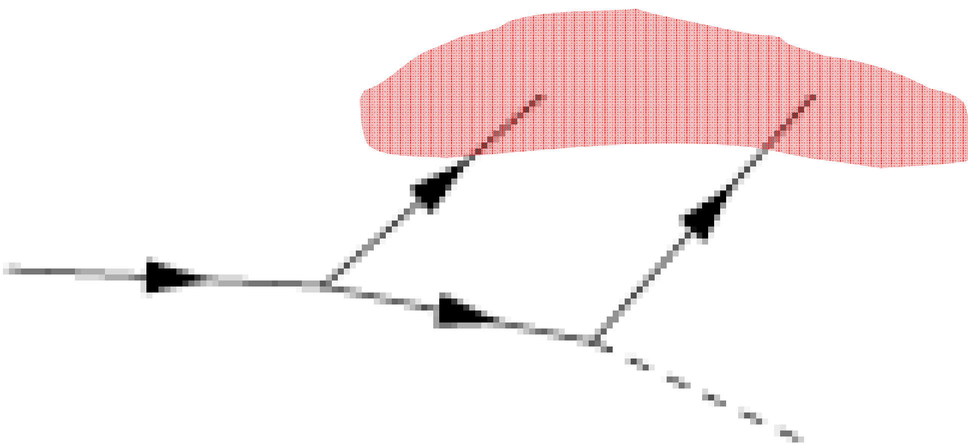
are exactly constrained. (arXiv:0905.1344)

What about shapes of distributions?

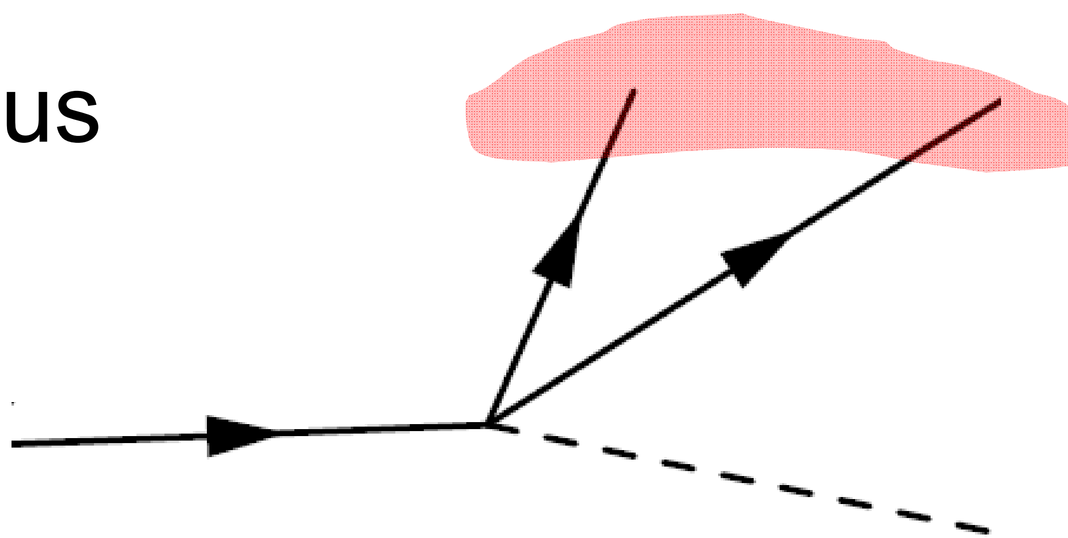


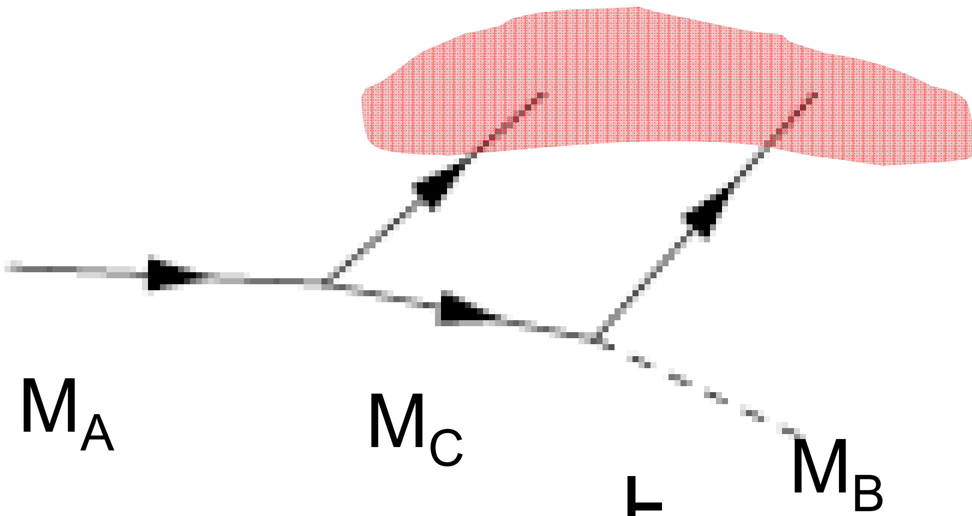
Gjelsten, Miller, Osland: hep-ph/0410303

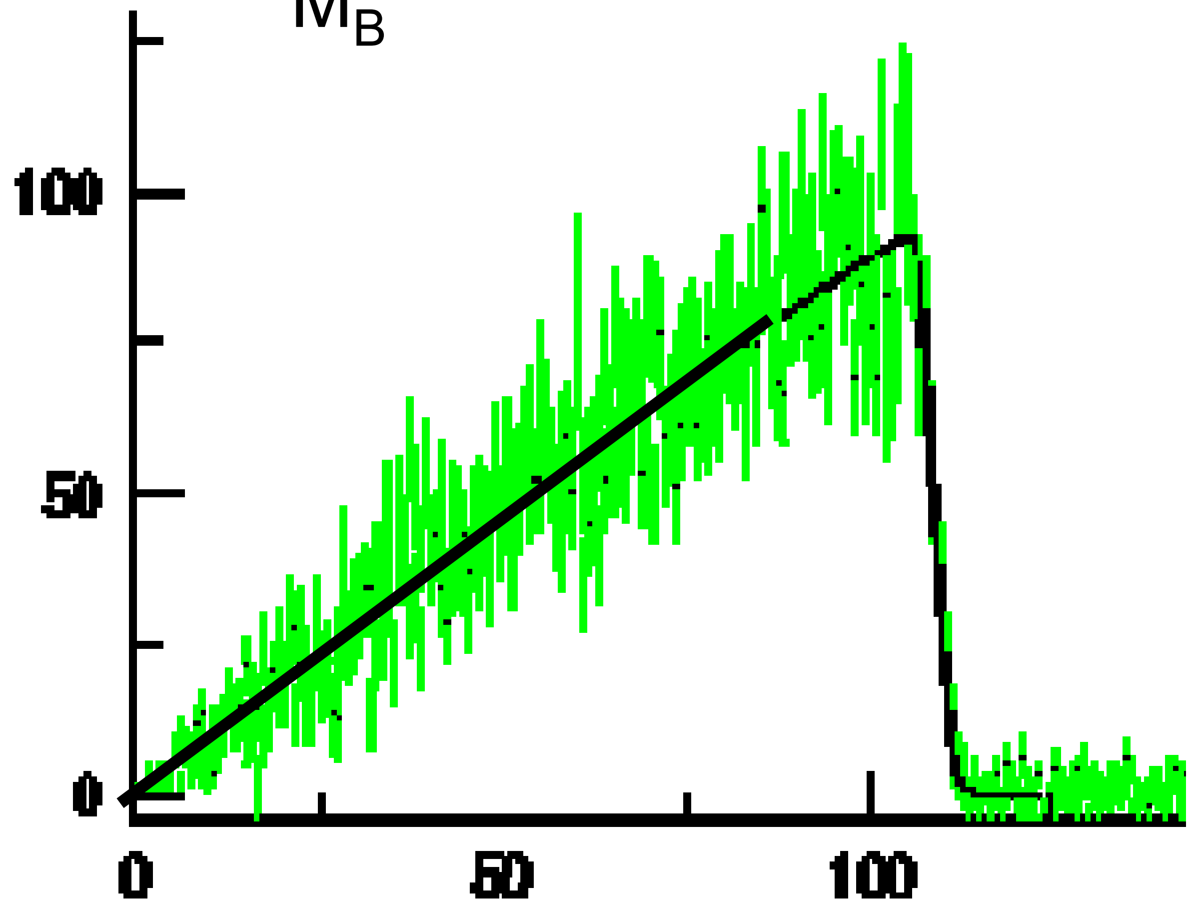
Compare shapes of invariant mass distributions for the highlighted pairs of visible massless momenta:



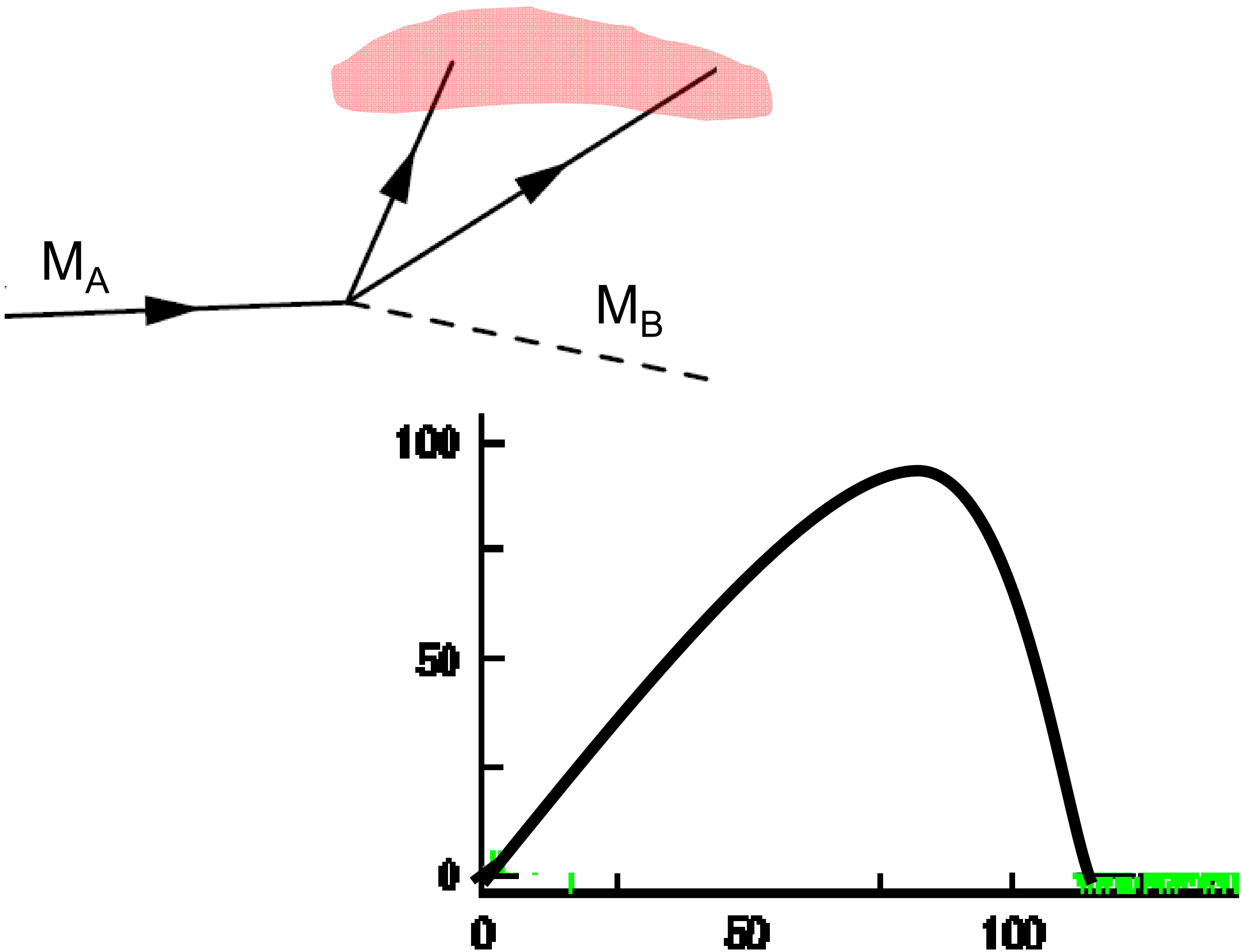
versus



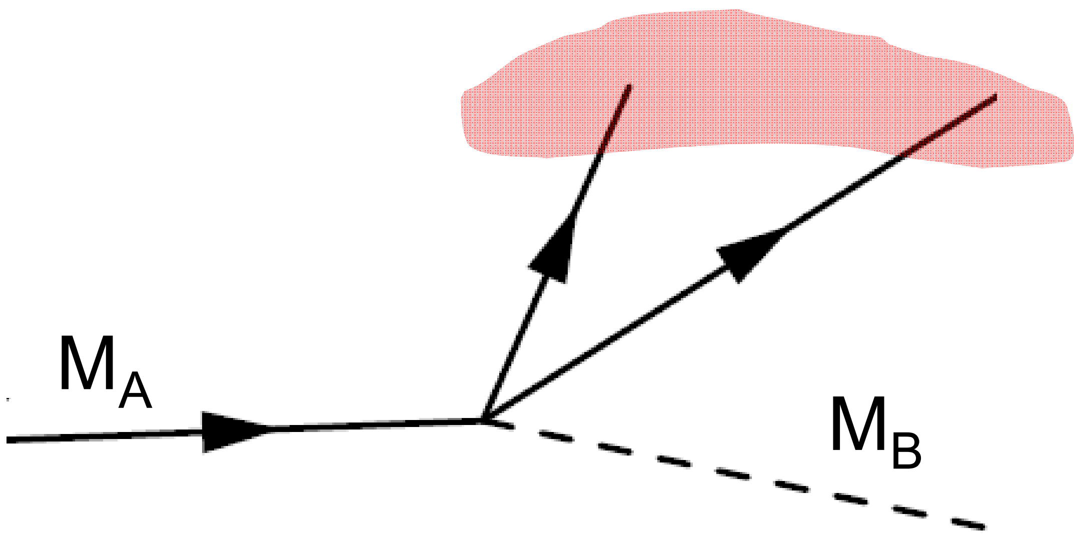




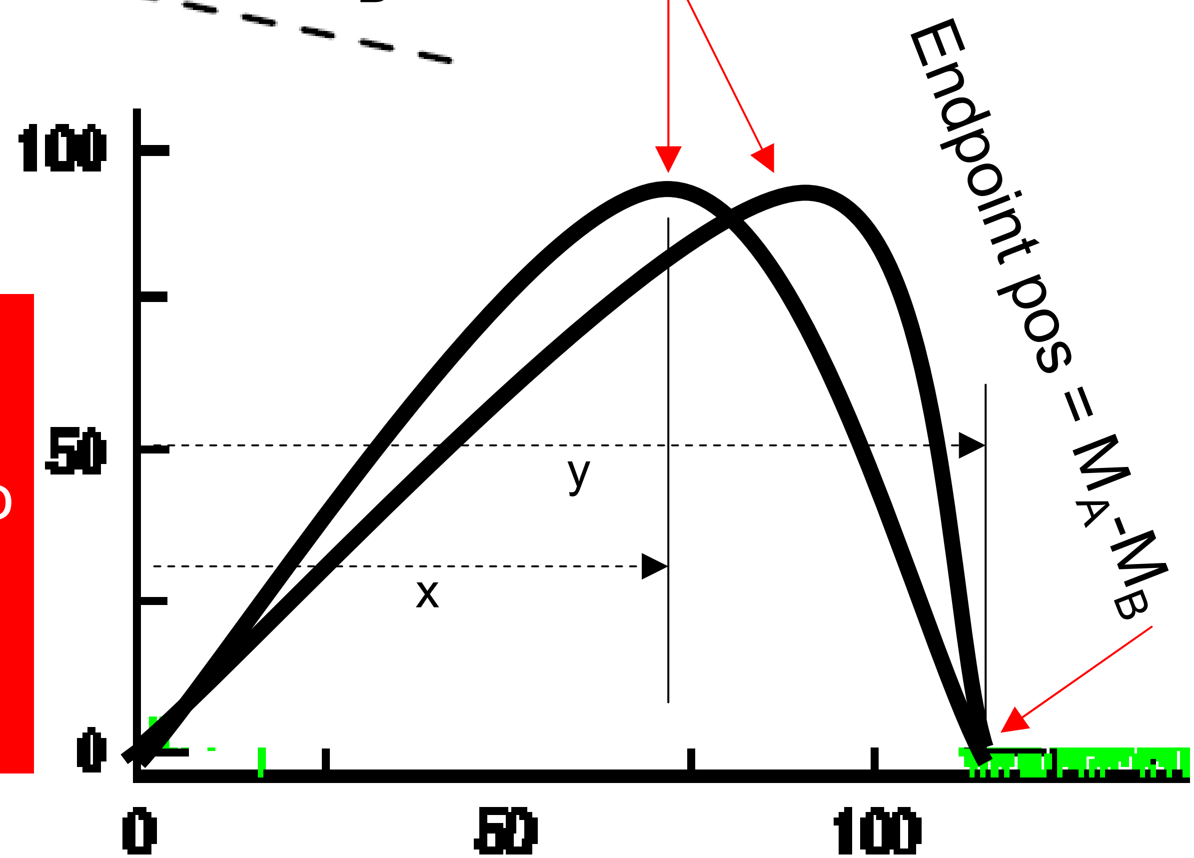
One piece of information (the endpoint position) is not sufficient to determine M_A, M_B and M_C .



M_A
M_B
100
50
0
0
50
100



Shape has dependence on M_A and M_B.



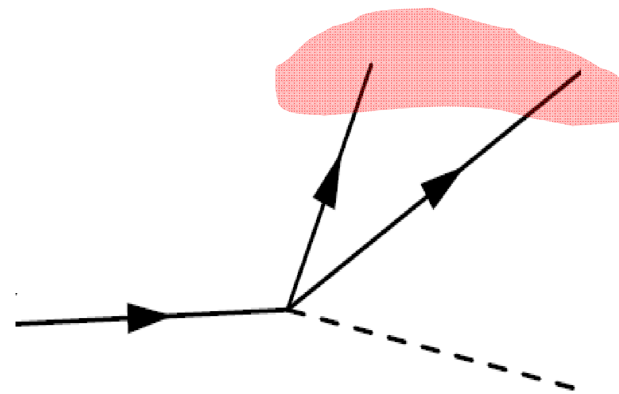
Do we have enough information from shape alone to find M_A and M_B in this three body decay, then?

Yes and no ..

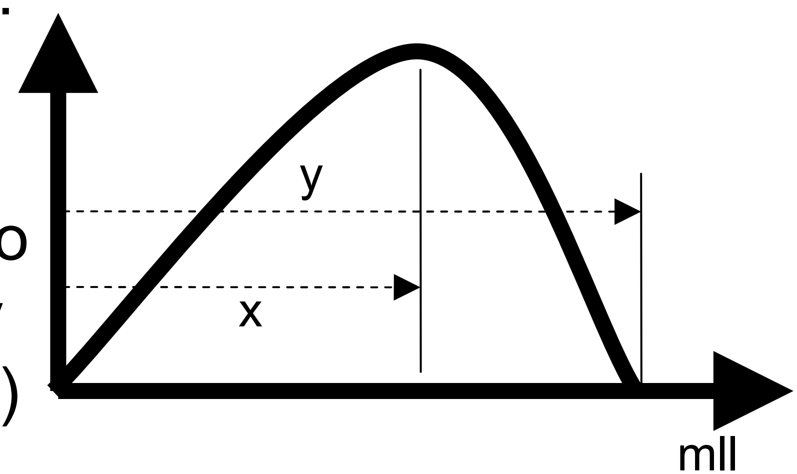
- Putting aside experimental fears concerning efficiency and acceptance corrections …
- … huge errors in the fit, and very poor sensitivity to absolute mass scale. See next exercises.
- This is why endpoints, edges and resonances are good, but shapes less so

Exercises

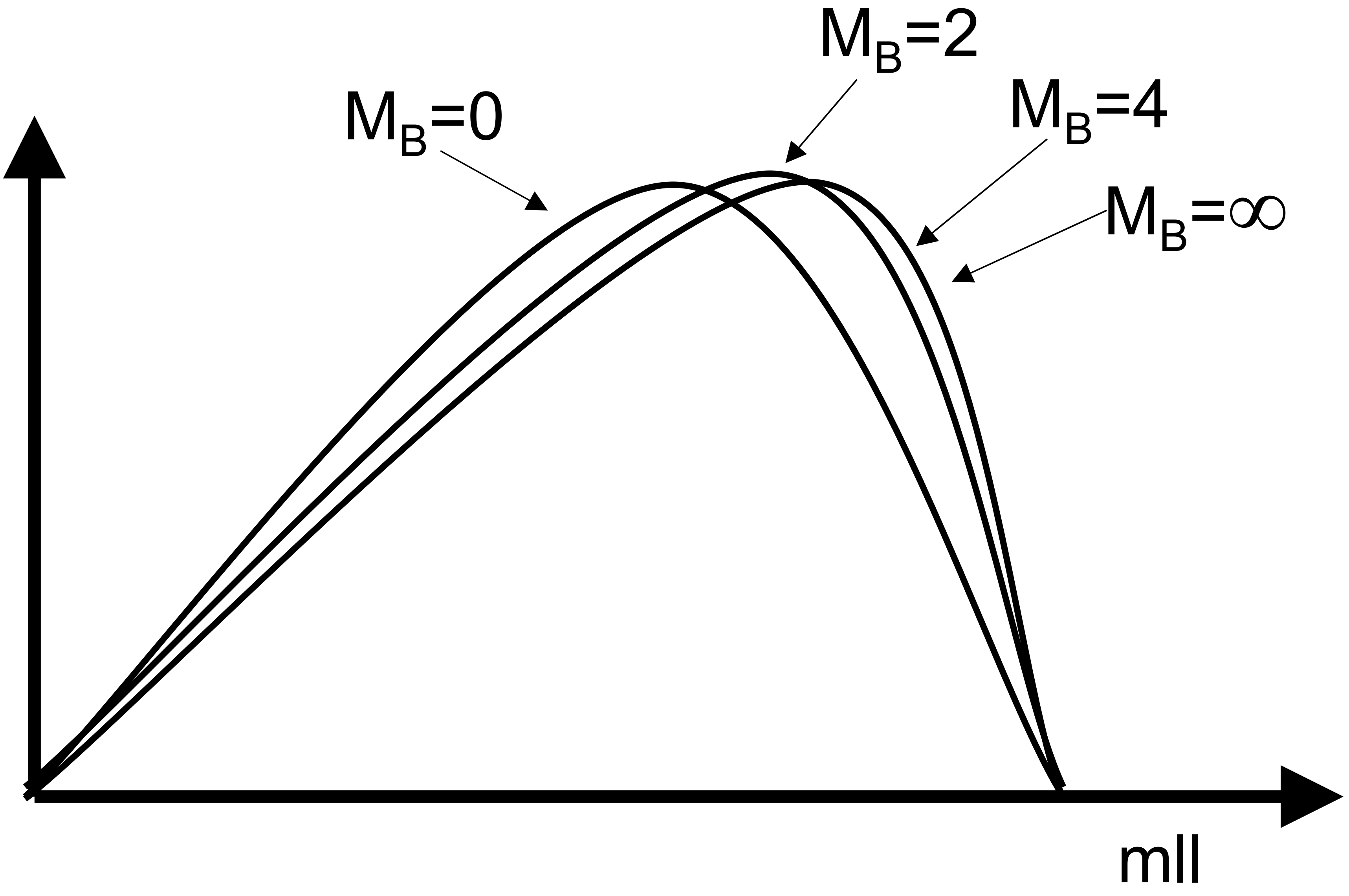
- (12) Determine the shape of the phase space distribution dσ/d(mll) (up to an arbitrary normalizing constant) for the three-body decay shown below. Assume massless visibles, and arbitrary masses for the parent and invisible.



- (13) Prove that r=x/y must lie in the range $1/\sqrt{3} \le r \le 1/\sqrt{2}$. (Note this means r can only move by ±0.06 … not far!)
- (14) Estimate how many events (approximately) would be needed to distinguish two r values differing by 0.012 (i.e. ~1/10th of allowed range)

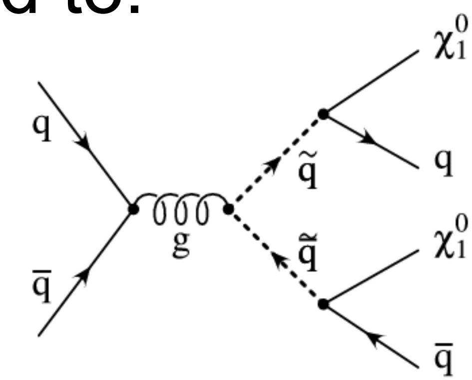


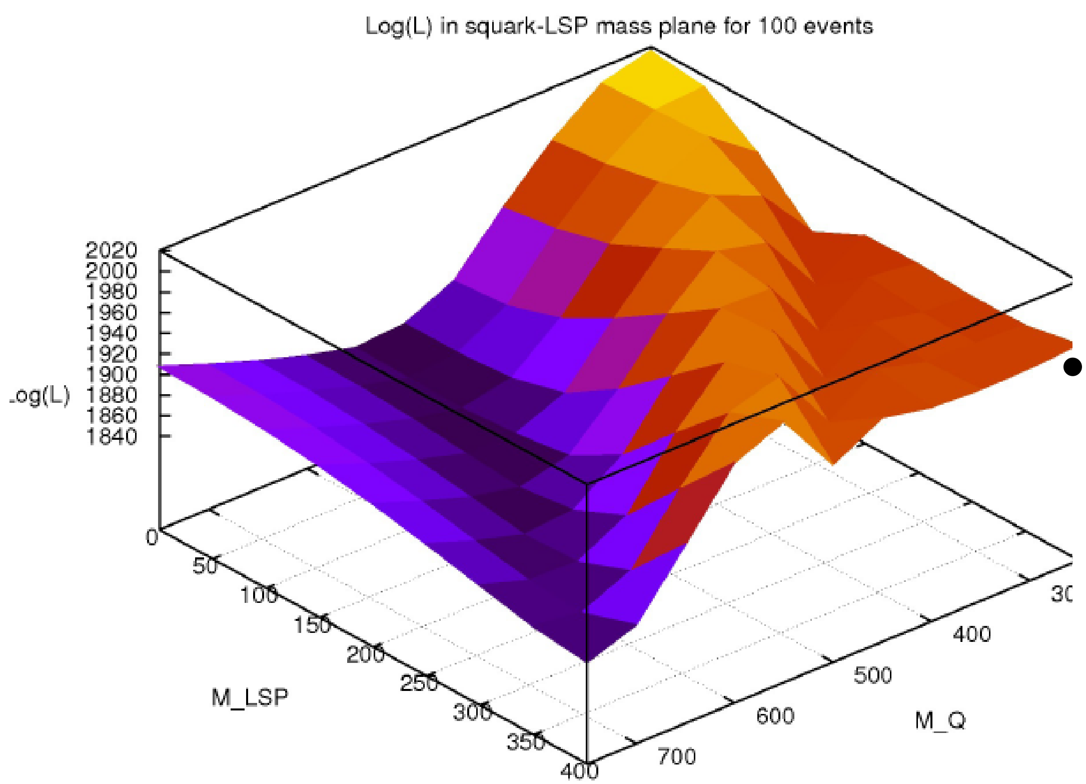
At fixed M_A-M_B you should find



The most detailed "shape" of all is the complete likelihood of the data

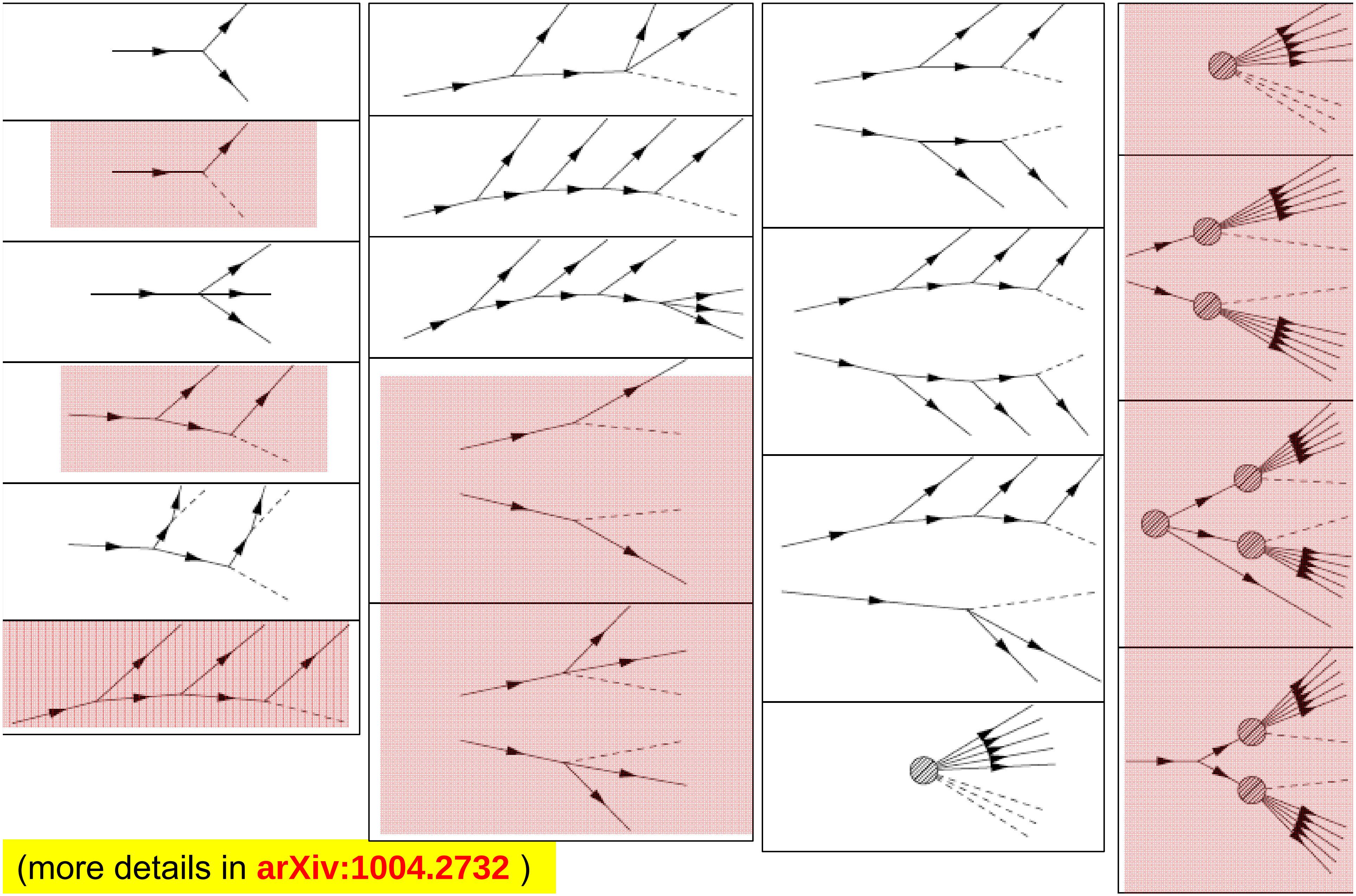
- Alwall et.al. (arXiv:0910.2522, arXiv:1010.2263) applied matrix element method to:





- For ~ 100 events get valley in likelihood surface with same shape as boundary of MT2 distribution

Have only begun to scrape the surface. Need an index.



(more details in **arXiv:1004.2732**)

Not time to talk about many things

- Parallel and perpendicular MT2 and MCT
- Subsystem MT2 and MCT methods
- Solution counting methods (eg arXiv:0707.0030)
- Hybrid Variables
- Phase space boundaries (arXiv:0903.4371)
- Cusps and Singularity Variables (Ian-Woo Kim)
- Why wrong solutions are often near right ones (arXiv:1103.3438)
- Razors
- and many more!

I have only scratched the surface of the variables that have been discussed. Even the recent review of mass measurement methods arXiv:1004.2732 makes only a small dent in 70+ pages. However it provides at least an index …

Let’s stop here!

Take home messages

- Lots of approaches to kinematic mass measurement
  - some very general, some very specific.
  - very little of the “detailed stuff” is tested in anger. Experimentalists not universally convinced of utility!
  - very often BGs present serious impediment.
  - theorists and experimenters should pay close attention to zone of applicability
- BUT
  - Finding sensible variables buys more than just mass measurements - e.g. signal sensitivity

Extras if time …

Other MT2 related variables (1/3)

- MCT ("Contralinear-Transverse Mass")
  (arXiv:0802.2879)
  - Is equivalent to MT2 in the special case that there is no missing momentum (and that the visible particles are massless).
  - Proposes an interesting multi-stage method for measuring additional masses
  - Can be calculated fast enough to use in ATLAS trigger.

Other MT2 related variables (2/3)

- MTGEN ("MT for GENeral number of final state particles") (arXiv:0708.1028)
  - Used when
    - each "side" of the event decays to MANY visible particles (and one invisible particle) and
    - it is not possible to determine which decay product is from which side … all possibilities are tried
- Inclusive or Hemispheric MT2 (Nojirir + Shimizu) (arXiv:0802.2412)
  - Similar to MTGEN but based on an assignment of decay product to sides via hemisphere algorithm.
  - Guaranteed to be >= MTGEN

Other MT2 related variables (3/3)

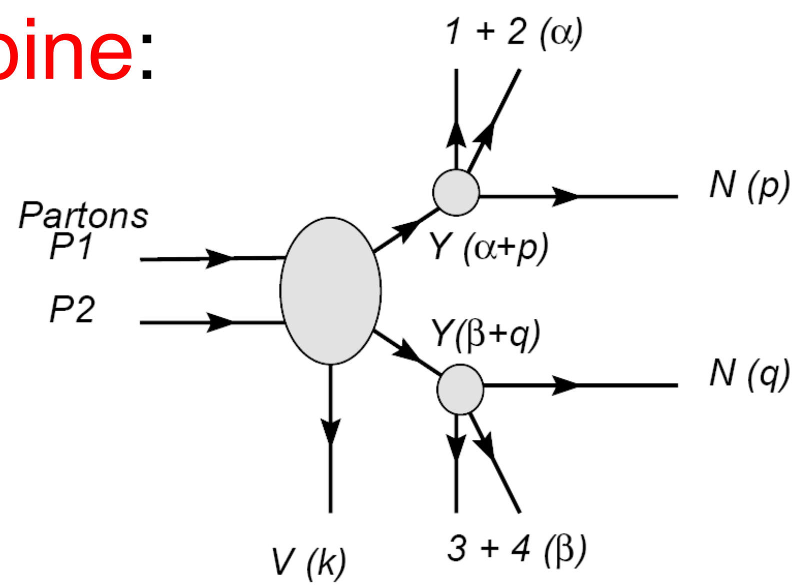
- M2C ("MT2 Constrained") arXiv:0712.0943 (wait for v3 ... there are some problems with the v1 and v2 drafts)
- M2CUB ("MT2 Constrained Upper Bound") arXiv:0806.3224

- There is a sense in which these two variables are really two sides of the same coin.
  – if we could re-write history we might name them more symmetrically
  – I will call them m_{Small} and m_{Big} in this talk.

m_{Small} and m_{Big}

- Basic idea is to combine:

  – MT2

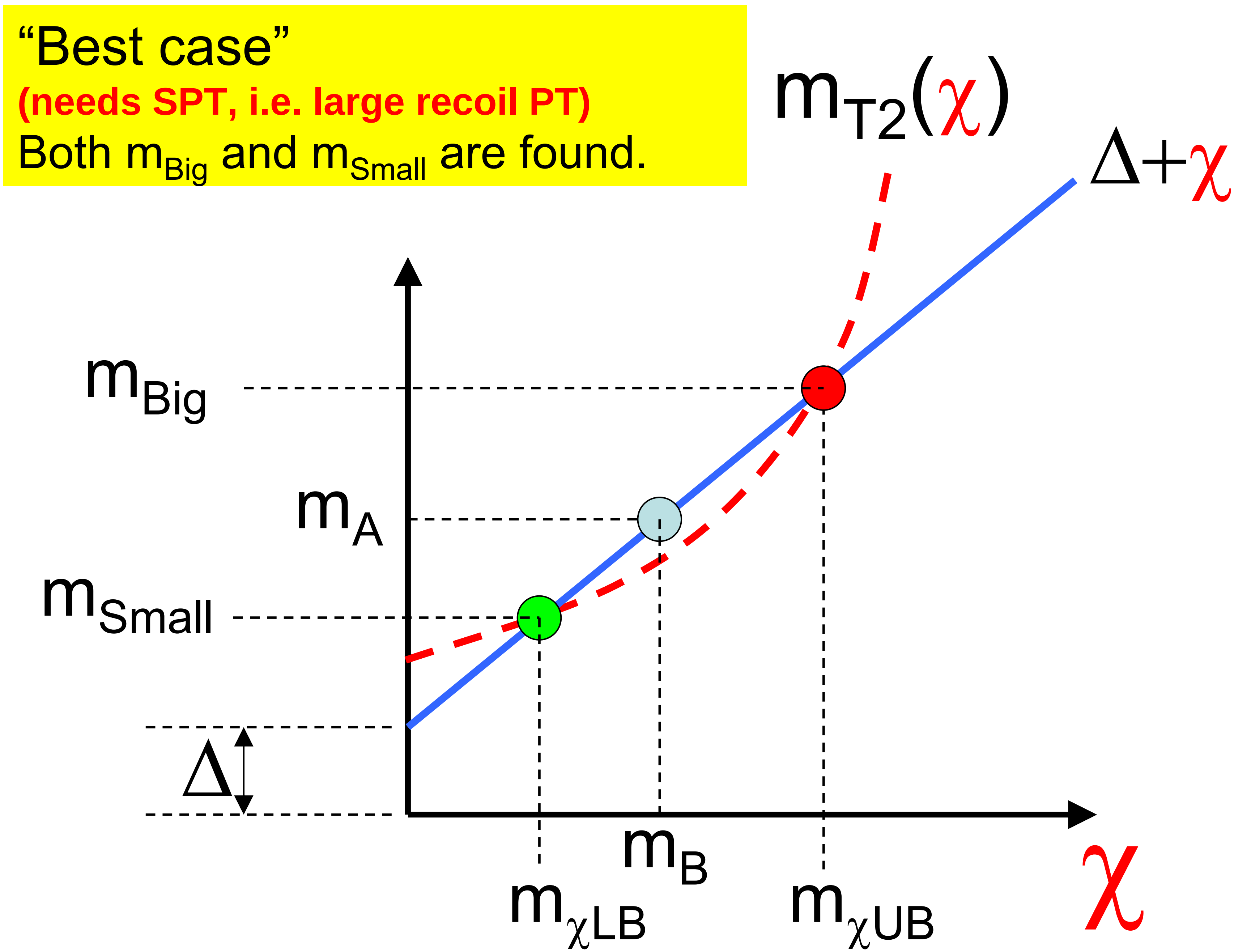


- with

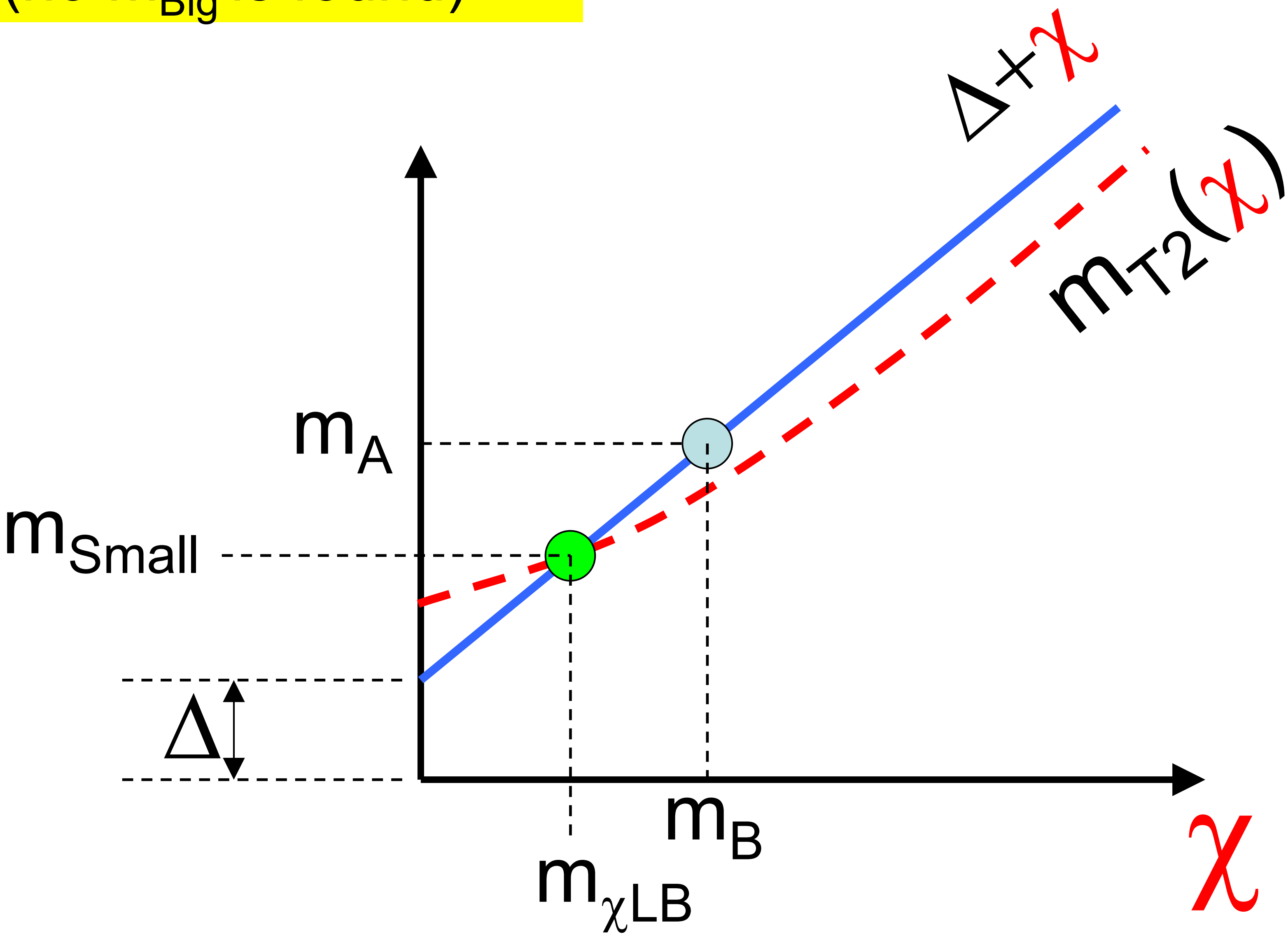
  – a di-lepton invariant mass endpoint measurement (or similar) providing:

$$\Delta = M_A - M_B$$

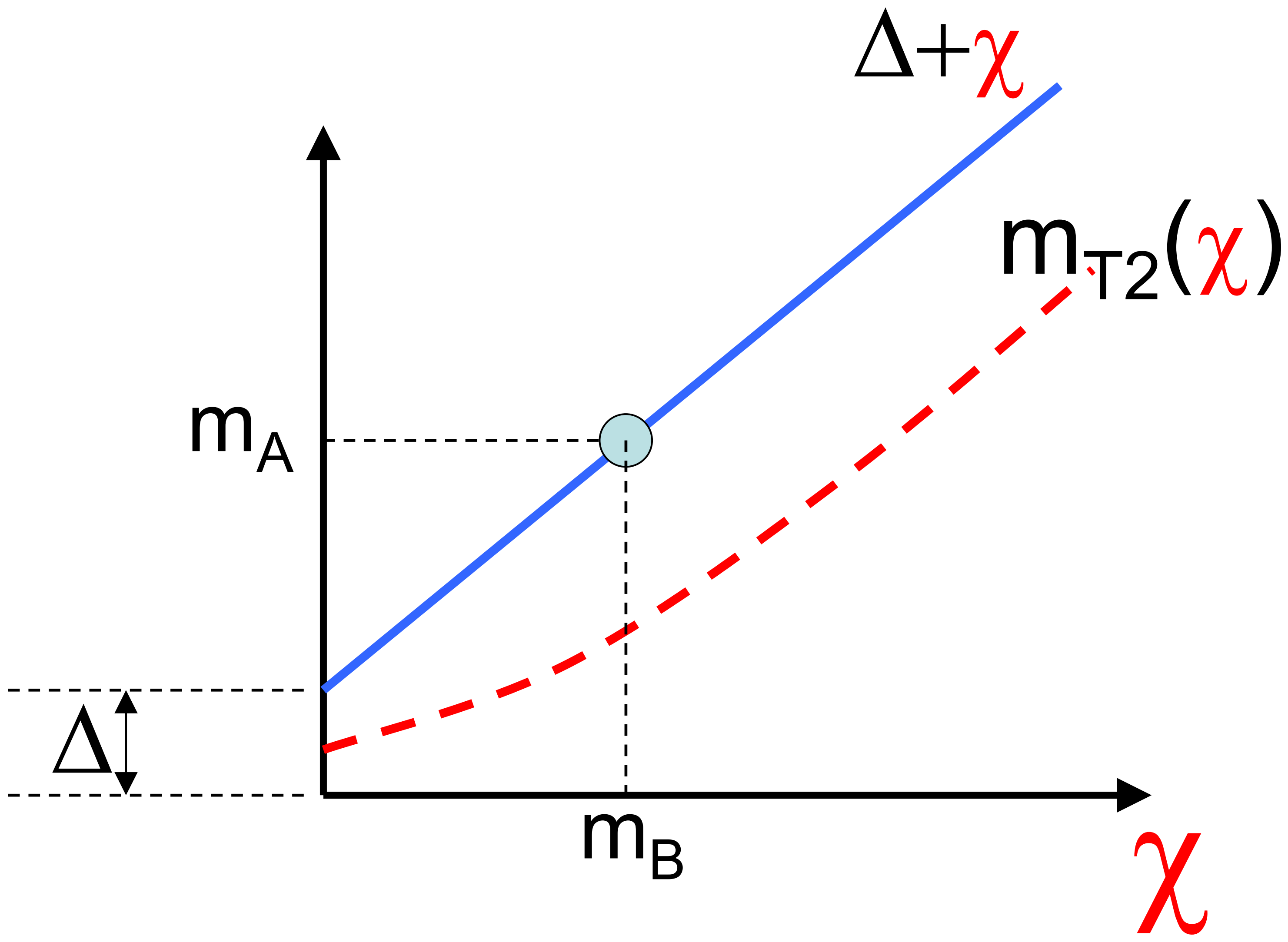
(or M_Y-M_N in the notation of their figure above)


"Best case"
(needs SPT, i.e. large recoil PT)
Both m_{Big} and m_{Small} are found.
$m_{T2}(\chi)$
$\Delta+\chi$
m_{Big}
m_A
m_{Small}
Δ
m_B
$m_{\chi LB}$
$m_{\chi UB}$
χ

"Typical ZPT case"
(no m_{Big} is found)



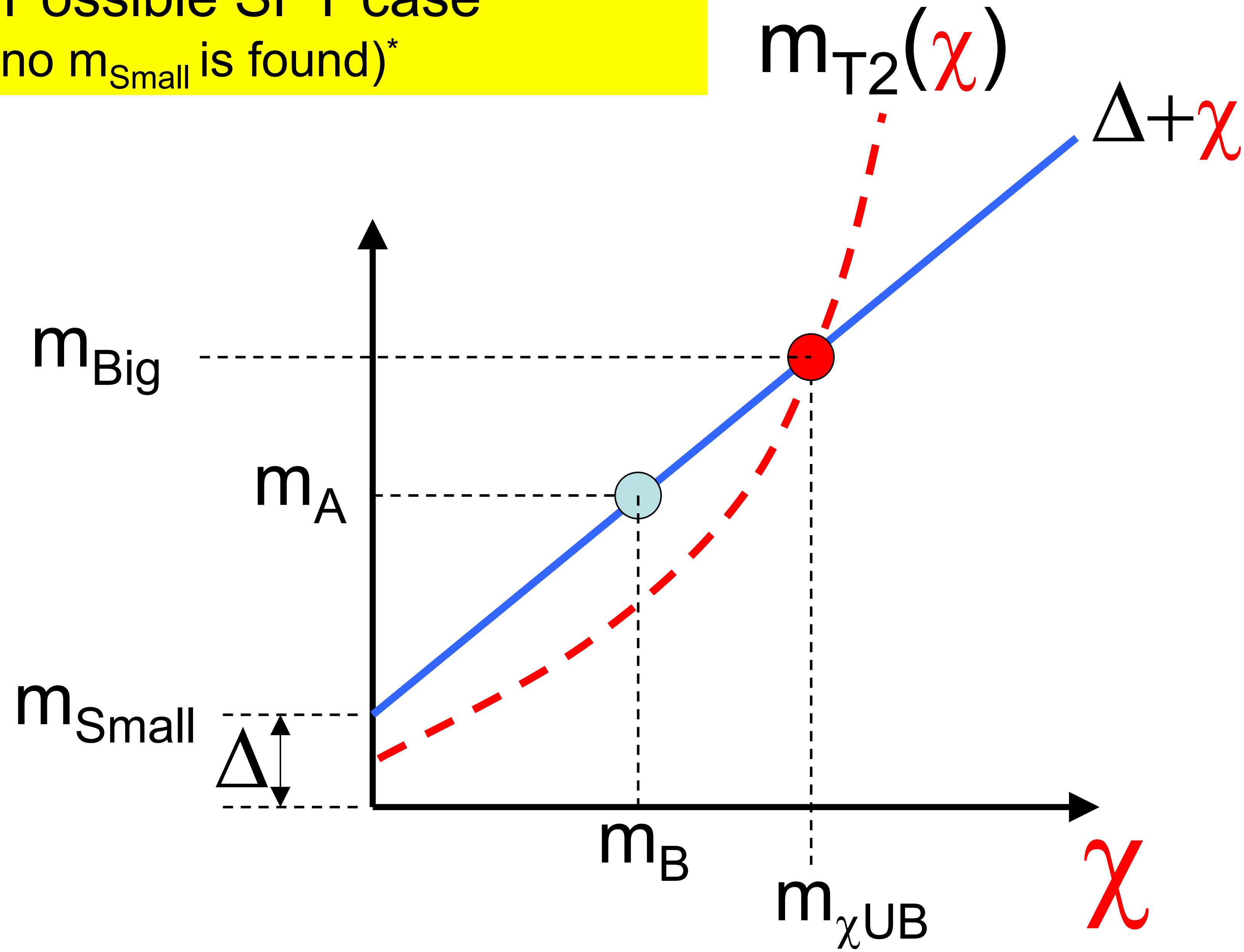
"Possible ZPT case"
(neither m_{Big} nor m_{Small} is found)*



* Except for conventional definition of m_{Small} to be Δ in this case.

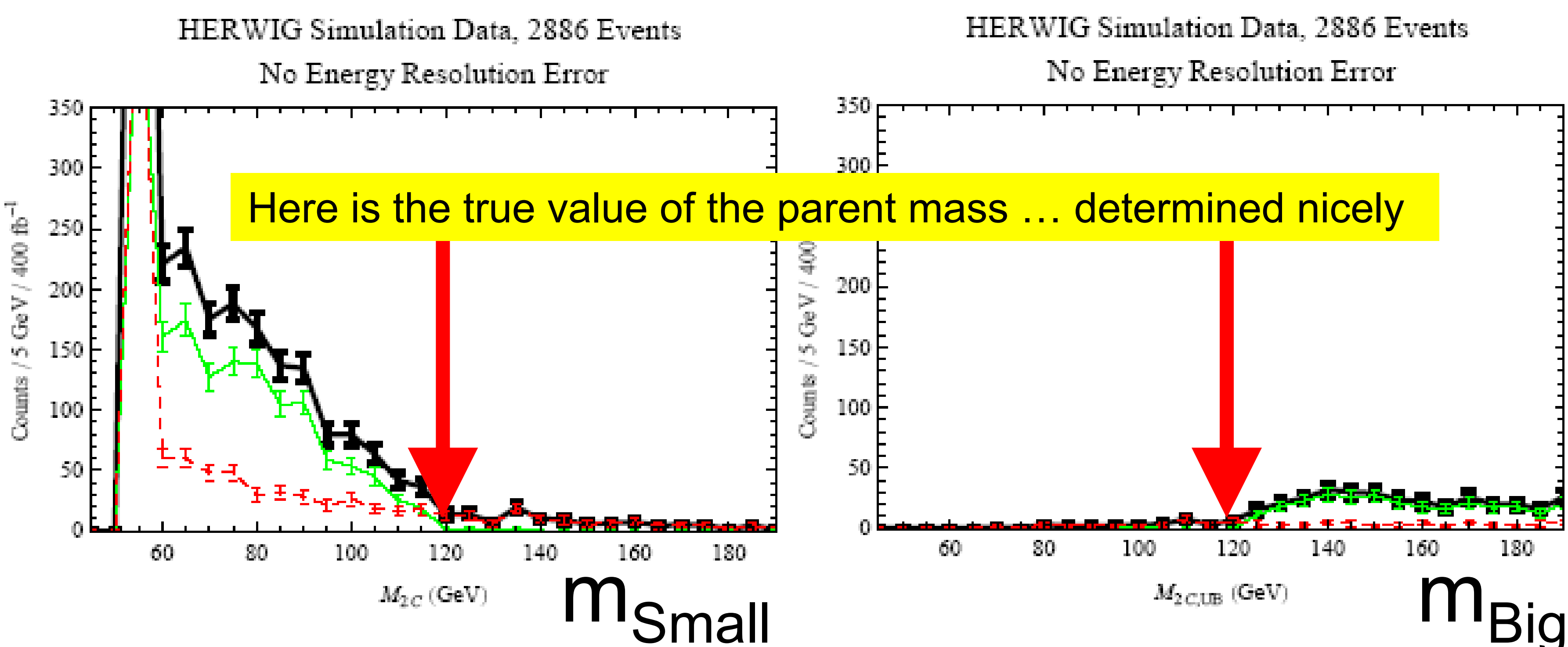
"Possible SPT case"

(no m_{Small} is found)*



* Except for conventional definition of m_{Small} to be Δ in this case.

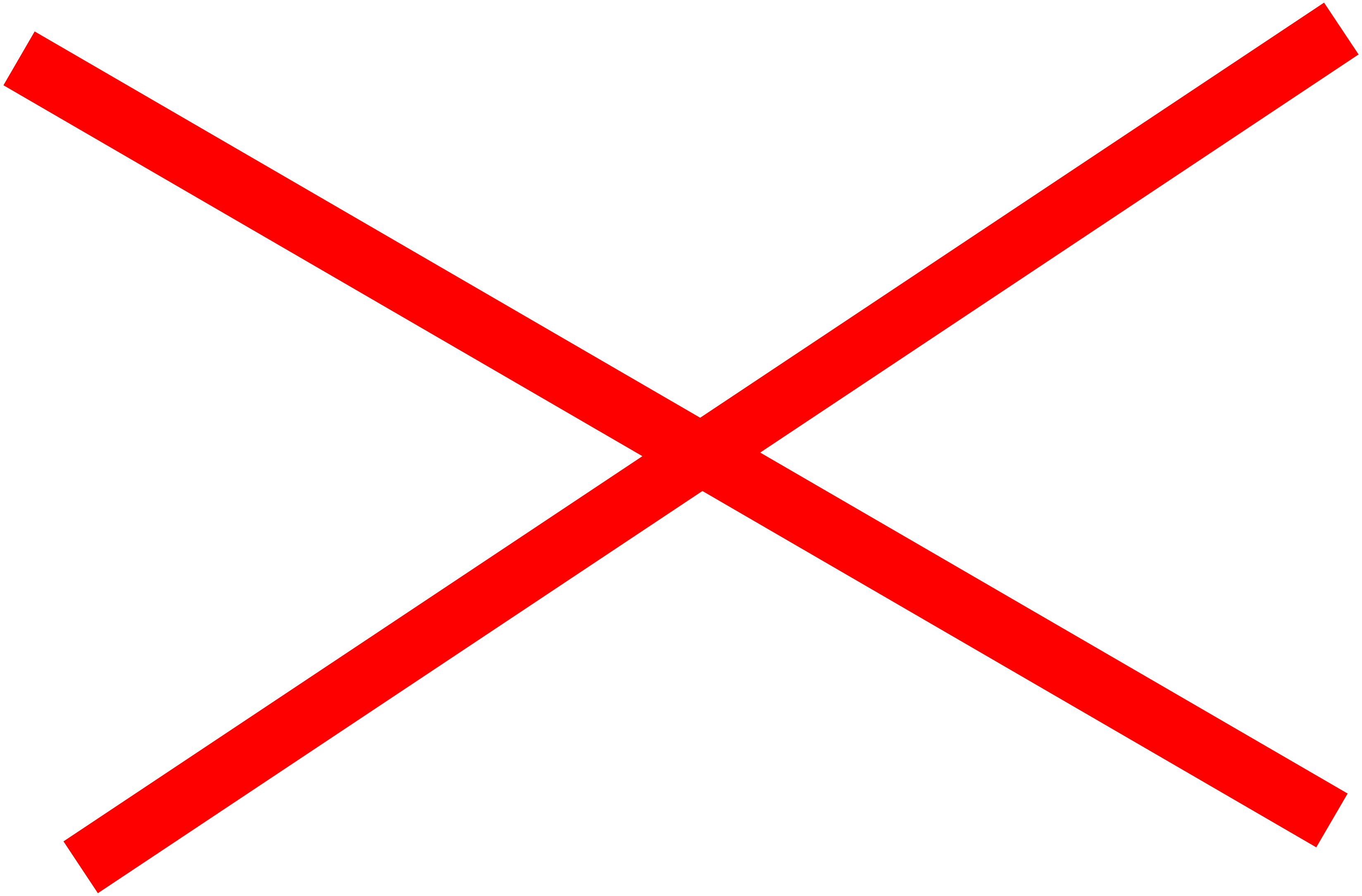
What m_{Small} and m_{Big} look like,
and how they determine the parent mass



arXiv:0806.3224

Outcome:

- m_{Big} provides the first potentially-useful event-by-event upper bound for m_A
  - (and a corresponding event-by-event upper bound for m_B called $m_{\chi UB}$)
- m_{Small} provides a new kind of event-by-event lower bound for m_A which incorporates consistency information with the dilepton edge
- m_{Big} is always reliant on SPT (large recoil of interesting system against "up-stream momentum") – cannot ignore recoil here!



LHC Specific problems

- Hadron Collider – z-boost of COM unknown
- Pile up, multiple interactions
- Production of many new particles at once?

- Multiple massive stable invisible particles?

What sort of parameter spaces?

- High dimensional
- At the very least, 8 dims
- More like ~100 dims

- No really compelling reasons to believe in any particular simple model

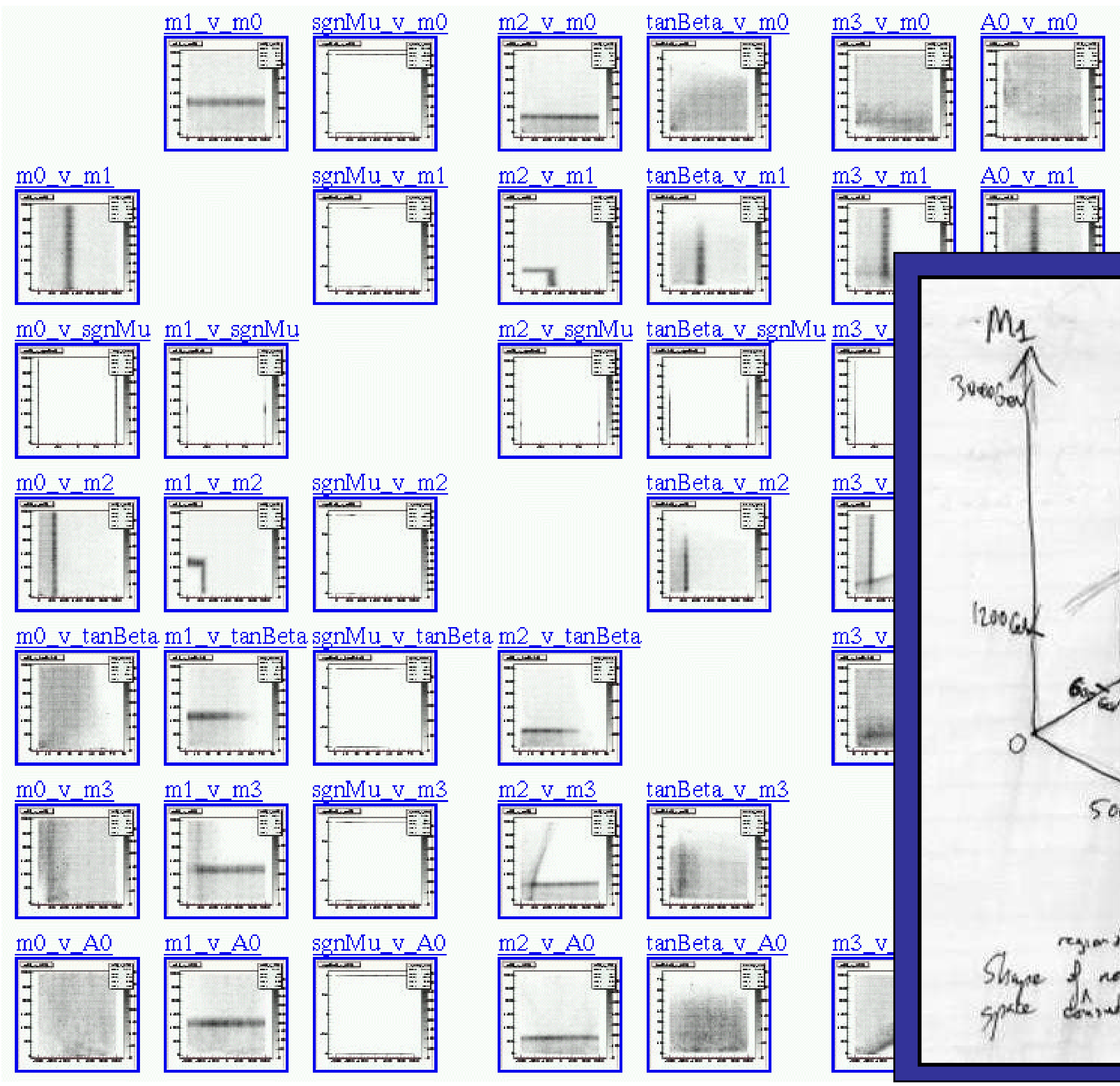
SUSY params

- m_0
- $M_{1/2}$
- A_0
- Tan beta
- Sgn μ

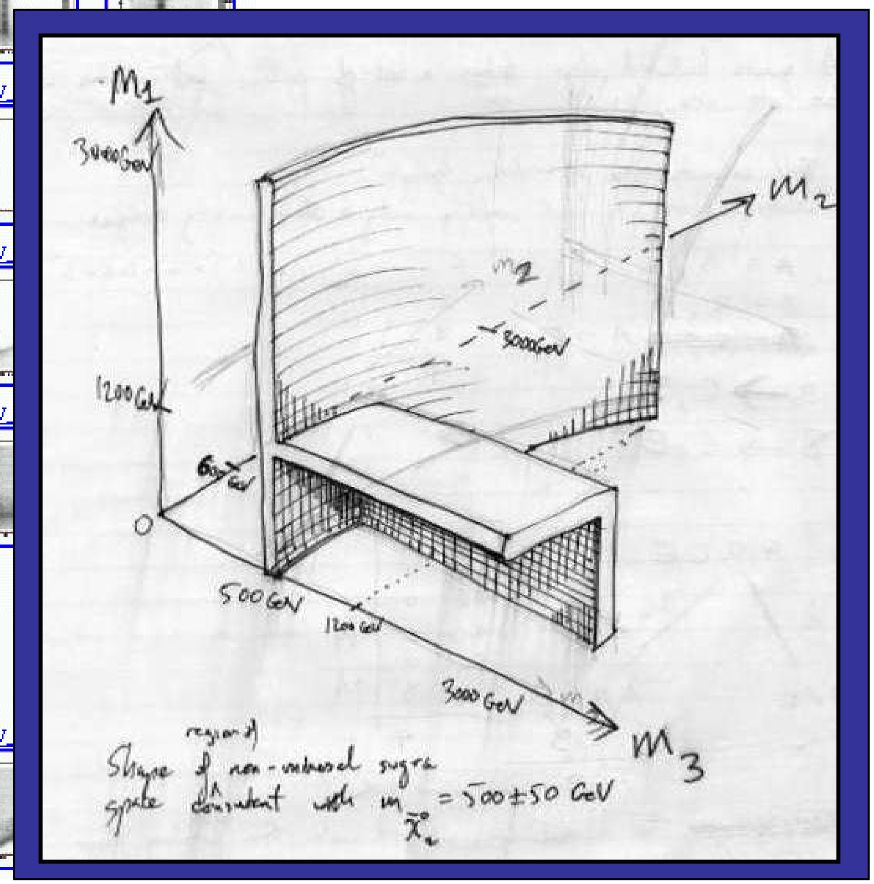
SM params

- m_b
- m_t
- $\alpha_s(M_z)$

Unusual parameter spaces!



Shape of typical set is often something quite horrible.



Contrast with UA1/UA2

- Glashow Wienberg Salam: Phys Rev Lett 19, 1264 (1967)
  - Predictions in terms of (then) unknown θ_W:
  - M_Z > 75 GeV/c^2, M_W > 35 GeV/c^2
- By 1982 θ_W much constrained, giving:
  - $M_Z \approx 92 \pm 2$ GeV/c^2, $M_W \approx 82 \pm 2$ GeV/c^2
- CERN able to build UA1+UA2 (~1980) knowing the above.
- In 1983 UA1+UA2 observe W and Z at expected masses:
  - $M_Z \approx 95 \pm 3$ GeV/c^2, $M_W \approx 81 \pm 5$ GeV/c^2