Supersymmetry

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1 Introduction

During the 20th century, a plethora of experiments and mathematics led to the Standard Model of Particle Physics (SM)[1]. This encapsulates our understanding of what indivisible particles exist and how they interact to give us nuclei, atoms and the thus the world we live in. It is one of the most well tested theories of all time — arguably deserving much more public recognition than it receives — and yet some observations mean that it's not quite correct. Physicists seek ways to extend the theory so that such observations can be explained. One of the simplest extensions is called Supersymmetry (SUSY for short). Unlike other options, such as String theory, this has the advantage of being experimentally observable.

2 The Standard Model and its Problems

Mathematically, the SM is a mixture of the ideas of Quantum Field theory, Quantum Mechanics and Special Relativity; alongside an aim to replicate reality. It describes 'particles'. By this, we mean the indivisible objects that make up everything in the universe and the forces that they feel in order to interact with each other. For example, the electron is a particle but the proton is not. It is a Baryon, a composite object made from 3 particles called quarks which are held together by 'gluons' — the particles which manifest the Strong force holding the 3 quarks together.

Particles are categorised by properties such as their mass, electric charge and spin in order to produce a 'periodic table' of particles as in figure 1. Spin is a tricky concept, but essentially it is an intrinsic and unchangeable property of a particle which contributes to its angular momentum. Particles with an integer spin (0,1,...) are called bosons and 1/2-integer spin particles ($\frac{1}{2}$,$\frac{3}{2}$,...) are fermions. Fermions are split into leptons (charge -1, e.g. electrons or charge 0 neutrinos) and quarks (e.g. up and down quarks which make up protons and neutrons). There are 3 'generations' of fermions, which are the identical apart from their masses. In fig1, each column of quarks/leptons is a different generation.

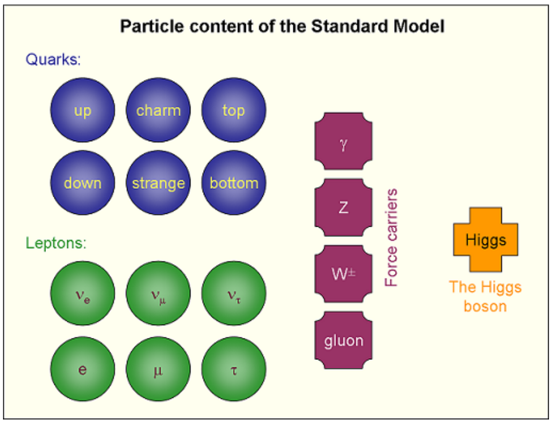

Figure 1: The particle contents of the Standard Model [2].

In the same way that if someone throws a ball at you, they transfer some momentum to you via the ball; spin-1 bosons 'carry' particle forces between particles to represent interactions. Photons carry Electromagnetism, W and Z bosons carry the Weak force which is responsible for radioactive beta decay, and gluons carry the Strong force which holds quarks together in baryons. The only SM spin-0 particle is the Higgs, which interacts with particles to give them mass [3].

The SM has so far seemed correct whenever it has been tested. It correctly describes the particles that we know exist and how they interact - atoms, radioactivity, chemistry are as they seem. However, there are some deep problems. First - it doesn't include gravity! There is no proven theory to describe gravity at the tiny scales of particle physics, leading to a detachment between the two pillars of modern physics: Quantum Mechanics and General Relativity. Some possible further extensions to the SM which describe 'quantum gravity' require SUSY.

One problem that SUSY can solve is the gauge hierarchy problem [4]. Why is gravity so much weaker than other forces? It may not seem like gravity is $10^{23}\times$ weaker than these. But consider that the atoms in your body are held together so strongly that even the gravity of an entire planet isn't enough to flatten you to the ground like pancake. A small magnet can pick up a paperclip which is being pulled down by the same gravity. A way to quantify this problem is in the mass of the Higgs.

The Higgs mass depends on its interactions with particles that appear and disappear in space, in the same way that you would feel heavier if you were trying to walk through a vat of custard instead of air. In the equation for the Higgs mass, there are multiple terms added for each particle in existence. For fermions, the terms are added and so increase it. For bosons, the terms are subtracted and so decrease it. Overall, in the SM you get to an infinite Higgs mass. Given that CERN found a mass of 10^{-25}kg there is clearly something wrong.

Another cause of dissatisfaction lies in parameters. Numbers such as the electron mass, though predicted to be some finite value, are not explicitly set by the theory and instead are random. It is unpleasant to have a theory which doesn't tell you *why* the charge of an up quark is $-2/3$ the charge of an electron. Theories beyond the SM should attempt to provide mechanisms to explain free parameters.

3 Symmetry and Supersymmetry

The SM is based on a certain set of 'symmetries', such as energy conservation. The equation that describes particle interactions, the Lagrangian, is built up by adding the simplest terms that are allowed under a given symmetry. The entire SM Lagrangian can be seen written out in fig2. In this context, a symmetry is a mathematical 'operation' which when applied to the Lagrangian leaves it unchanged. For example, in $L = x^2$, L remains unchanged if x is acted on by the operator 'times by -1'to give $-x$. If x was a position you would say L has spatial inversion symmetry, or that Parity is conserved.

The three forces are introduced by imposing continuous symmetries onto fermions to reflect reality. A continuous symmetry is something that can be split into a series of infinitesimal steps, like rotation or translation. In order to make the Lagrangian invariant, specific extra terms must be added to cancel out any changes under the symmetry operator. These are interpreted as new boson particles and this idea is called gauge field theory. The SM symmetries are based on electric charge and the analogous concepts of 'colour' for the Strong force and 'weak isospin' and 'hypercharge' for the Weak force.

$$
\begin{gathered}
-\tfrac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \tfrac{1}{4}g_s^2 f^{abc}f^{ade}g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \\
\tfrac{1}{2}ig_s^2(\bar{q}^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu + \bar{G}^a\partial^2 G^a + g_s f^{abc}\partial_\mu \bar{G}^a G^b g^c_\mu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - \\
M^2 W^+_\mu W^-_\mu - \tfrac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \tfrac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \tfrac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \tfrac{1}{2}\partial_\mu H \partial_\mu H - \\
\tfrac{1}{2}m_h^2 H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - M^2\phi^+\phi^- - \tfrac{1}{2}\partial_\mu\phi^0\partial_\mu\phi^0 - \tfrac{1}{2c_w^2}M\phi^0\phi^0 - \beta_h[\tfrac{2M^2}{g^2} + \\
\tfrac{2M}{g}H + \tfrac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)] + \tfrac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu\partial_\nu W^-_\mu - \\
W^-_\nu\partial_\nu W^+_\mu)] - igs_w[\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu\partial_\nu W^-_\mu - \\
W^-_\mu\partial_\nu W^+_\mu) + A_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)] - \tfrac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \\
\tfrac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + \\
g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w[A_\mu Z^0_\nu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] - \\
\tfrac{1}{8}g^2\alpha_h[H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2] - \\
gMW^+_\mu W^-_\mu H - \tfrac{1}{2}g\tfrac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \tfrac{1}{2}ig[W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - \\
W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)] + \tfrac{1}{2}g[W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) - W^-_\mu(H\partial_\mu\phi^+ - \\
\phi^+\partial_\mu H)] + \tfrac{1}{2}g\tfrac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) - ig\tfrac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + \\
igs_w MA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) - ig\tfrac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + \\
igs_w A_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \tfrac{1}{4}g^2 W^+_\mu W^-_\mu[H^2 + (\phi^0)^2 + 2\phi^+\phi^-] - \\
\tfrac{1}{4}g^2\tfrac{1}{c_w^2}Z^0_\mu Z^0_\mu[H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2\phi^+\phi^-] - \tfrac{1}{2}g^2\tfrac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + \\
W^-_\mu\phi^+) - \tfrac{1}{2}ig^2\tfrac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \tfrac{1}{2}g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- + \\
W^-_\mu\phi^+) + \tfrac{1}{2}ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\tfrac{s_w}{c_w}(2c_w^2 - 1)Z^0_\mu A_\mu\phi^+\phi^- - \\
g^1 s_w^2 A_\mu A_\mu\phi^+\phi^- - \bar{e}^\lambda(\gamma\partial + m_e^\lambda)e^\lambda - \bar{\nu}^\lambda\gamma\partial\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial + m_u^\lambda)u^\lambda_j - \\
\bar{d}^\lambda_j(\gamma\partial + m_d^\lambda)d^\lambda_j + igs_w A_\mu[-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \tfrac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \tfrac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)] + \\
\tfrac{ig}{4c_w}Z^0_\mu[(\bar{\nu}^\lambda\gamma^\mu(1 + \gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar{u}^\lambda_j\gamma^\mu(\tfrac{4}{3}s_w^2 - \\
1 - \gamma^5)u^\lambda_j) + (\bar{d}^\lambda_j\gamma^\mu(1 - \tfrac{8}{3}s_w^2 - \gamma^5)d^\lambda_j)] + \tfrac{ig}{2\sqrt{2}}W^+_\mu[(\bar{\nu}^\lambda\gamma^\mu(1 + \gamma^5)e^\lambda) + \\
(\bar{u}^\lambda_j\gamma^\mu(1 + \gamma^5)C_{\lambda\kappa}d^\kappa_j)] + \tfrac{ig}{2\sqrt{2}}W^-_\mu[(\bar{e}^\lambda\gamma^\mu(1 + \gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\lambda\kappa}\gamma^\mu(1 + \\
\gamma^5)u^\lambda_j)] + \tfrac{ig}{2\sqrt{2}}\tfrac{m_e^\lambda}{M}[-\phi^+(\bar{\nu}^\lambda(1 - \gamma^5)e^\lambda) + \phi^-(\bar{e}^\lambda(1 + \gamma^5)\nu^\lambda)] - \\
\tfrac{g}{2}\tfrac{m_e^\lambda}{M}[H(\bar{e}^\lambda e^\lambda) + i\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda)] + \tfrac{ig}{2M\sqrt{2}}\phi^+[-m_d^\kappa(\bar{u}^\lambda_j C_{\lambda\kappa}(1 - \gamma^5)d^\kappa_j) + \\
m_u^\lambda(\bar{u}^\lambda_j C_{\lambda\kappa}(1 + \gamma^5)d^\kappa_j] + \tfrac{ig}{2M\sqrt{2}}\phi^-[m_d^\lambda(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1 + \gamma^5)u^\kappa_j) - m_u^\kappa(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1 - \\
\gamma^5)u^\kappa_j] - \tfrac{g}{2}\tfrac{m_u^\lambda}{M}H(\bar{u}^\lambda_j u^\lambda_j) - \tfrac{g}{2}\tfrac{m_d^\lambda}{M}H(\bar{d}^\lambda_j d^\lambda_j) + \tfrac{ig}{2}\tfrac{m_u^\lambda}{M}\phi^0(\bar{u}^\lambda_j\gamma^5 u^\lambda_j) - \\
\tfrac{ig}{2}\tfrac{m_d^\lambda}{M}\phi^0(\bar{d}^\lambda_j\gamma^5 d^\lambda_j) + \bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 - \\
\tfrac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2 Y + igc_w W^+_\mu(\partial_\mu\bar{X}^0 X^- - \partial_\mu\bar{X}^+ X^0) + igs_w W^+_\mu(\partial_\mu\bar{Y}X^- - \\
\partial_\mu\bar{X}^+ Y) + igc_w W^-_\mu(\partial_\mu\bar{X}^- X^0 - \partial_\mu\bar{X}^0 X^+) + igs_w W^-_\mu(\partial_\mu\bar{X}^- Y - \\
\partial_\mu\bar{Y}X^+) + igc_w Z^0_\mu(\partial_\mu\bar{X}^+ X^+ - \partial_\mu\bar{X}^- X^-) + igs_w A_\mu(\partial_\mu\bar{X}^+ X^+ - \\
\partial_\mu\bar{X}^- X^-) - \tfrac{1}{2}gM[\bar{X}^+ X^+ H + \bar{X}^- X^- H + \tfrac{1}{c_w^2}\bar{X}^0 X^0 H] + \\
\tfrac{1-2c_w^2}{2c_w}igM[\bar{X}^+ X^0\phi^+ - \bar{X}^- X^0\phi^-] + \tfrac{1}{2c_w}igM[\bar{X}^0 X^-\phi^+ - \bar{X}^0 X^+\phi^-] + \\
igMs_w[\bar{X}^0 X^-\phi^+ - \bar{X}^0 X^+\phi^-] + \tfrac{1}{2}igM[\bar{X}^+ X^+\phi^0 - \bar{X}^- X^-\phi^0]
\end{gathered}
$$

Figure 2: The Standard Model Lagrangian [5].

According to Nöether's Theorem, continuous symmetries in a Lagrangian lead to conserved quantities. For the SM this leads to conservation of colour, electric charge and weak isospin as well as energy and angular/linear momentum.

Supersymmetry is just another type of symmetry, which can be manifested in many different ways in many different theories [6]. It considers an operator which acts on a fermion to return a boson; or acts on a boson to return a fermion. It is a symmetry based on spin. This is a simple, natural extension to the SM because spin is just as important and fundamental to quantum physics as electric charge is. This leads to doubling the number of particles; adding a fermion 'superpartner' to each boson and a boson 'superpartner' to each fermion. These are called sparticles, as seen in fig3. They have different interactions with the SM forces and are heavier than SM counterparts.

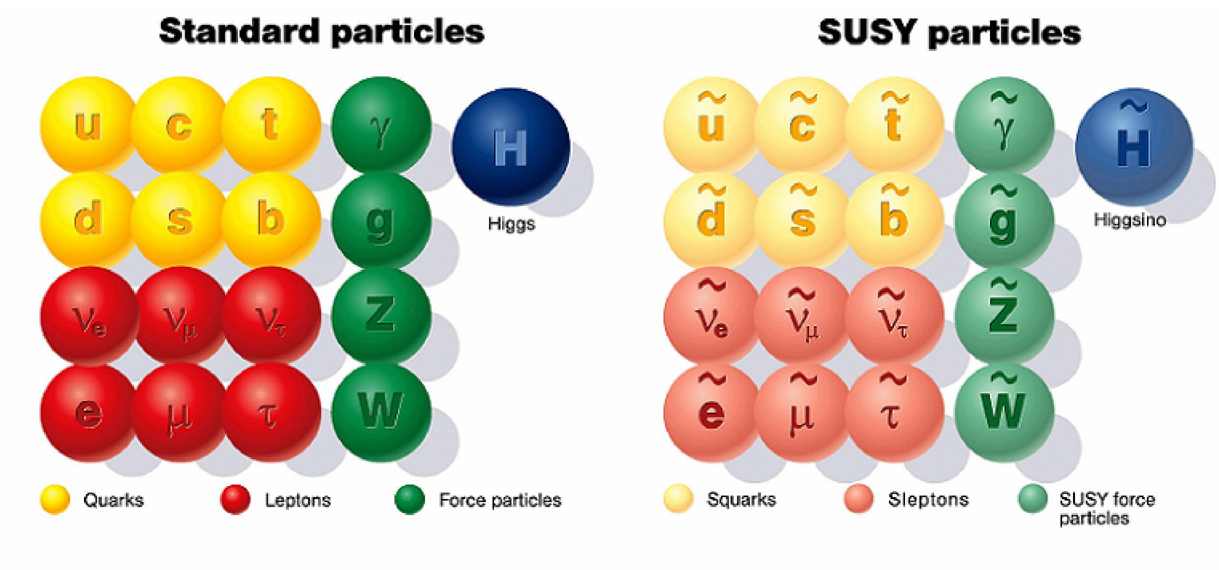

Figure 3: The SUSY particle collection [7] .

Using Nöether's theorem, SUSY can include the conserved quantity R-parity: $R = (-1)^{3B+L+2s}$, where B is the number of baryons (bound groups of quarks), L is number of leptons and s is spin. SM particles have $R = +1$ and sparticles have $R = -1$.

The first consequence of SUSY is to increase balance. There are the now the same number of fermions and bosons. Physicists love symmetry and simplicity in their theories because it echoes that which is found in nature. The second consequence returns us to the gauge hierarchy problem. The new particles lead to more terms being added to the Higgs boson mass, and now everything cancels out to give us a mass consistent with the LHC's findings.

4 Dark Matter and Symmetry Breaking

A third consequence of SUSY is to provide a dark matter candidate. Dark matter [8] is an unknown particle that interacts with gravity but not with other forces in the SM. Some extra, new matter is required to account for unexpected observations of galactic dynamics. This dark matter must, in fact, make up 27% of the universe's matter.

If R-parity is conserved, sparticles always decay into other sparticles. This, combined with the impossibility of decay into heavier particles due to energy conservation (and $E = mc^2$), leads to the concept of the LSP (lightest supersymmetric particle). This is stable - there is nothing it can decay into. One possibility for dark matter is a WIMP (weakly interacting massive particle). There are no WIMPs in the SM, but the LSP is a WIMP. In some SUSY theories it is assumed that the SM also contains a particle to carry the gravitational force, the graviton. In these, the LSP is its fermion superpartner, the gravitino.

If R-parity isn't conserved, there is no LSP since it can decay into light SM particles. R-parity violating theories were originally less popular because they don't give an obvious DM candidate and can sometimes allow proton decay - something that has never been observed. But they can be combined with other Dark Matter theories.

5 Evidence and Conclusion

So far, there is no experimental evidence for SUSY. At CERN, SUSY searches are performed by looking for sparticles being created from high energy proton-proton collisions which decay into detectable SM particles. The sparticles produced can decay into a combination of SM particles and lighter sparticles, repeatedly, until the LSP is created. See fig4.

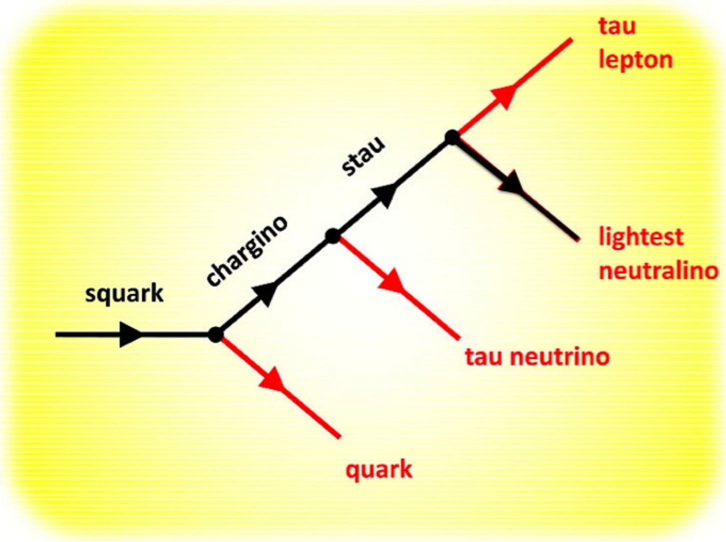

Figure 4: Example SUSY decay chain to LSP [9].

A downside of SUSY is the introduction of additional parameters: the sparticle masses. This makes testing more difficult because if the LSP mass is unknown you can't just eliminate the model if no LSP is found with a certain mass. It just means that the model/parameter combination can be eliminated. Thus, SUSY searches attempt to be model-independent. Rather than one-by-one eliminating thousands of possible model/parameter combinations, CERN looks for more general decays so that the likeliness of many models can be reduced at once, or tighter limits can be set on the free parameters [10]. The lack of positive results so far either means that SUSY is just wrong; that the correct model is more complex than we assume (e.g. not conserving R-parity or having parameters that make it less discernible from SM signals); or that the sparticle masses are too high to be produced at CERN. Currently it seems unlikely that the correct theory is the simplest R-parity conserving SUSY model with fewest parameters introduced, because of limits placed on sparticle masses.

Whatever alternative, it isn't time to give up just yet. Despite the downside of introducing a few more free parameters, SUSY is also a natural extension to the SM and can solve the problems of gauge hierarchy, dark matter and maybe even quantum gravity.

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