V.Gibson
Easter Term 2006

Part II Particle and Nuclear Physics
Examples Sheet 3

Basic Nuclear Properties

29. The Semi-Empirical mass formula for atomic masses may be written in the form

$$\mathrm{M(A,Z) = Zm_p + (A-Z)m_n + Zm_e - a_V A + a_S A^{\frac{2}{3}} + a_C \frac{Z^2}{A^{\frac{1}{3}}} + a_A \frac{(A-2Z)^2}{A}} + \delta(\mathrm{A,Z})$$

where $\mathrm{m_p, m_n}$ and $\mathrm{m_e}$ are the masses of the proton, neutron and electron respectively.

(a) Explain the physical significance of the various terms.

(b) Treating the nucleus as a sphere of uniform charge density, calculate a numerical value for the constant $\mathrm{a_C}$.

(c) With reference to the Semi-Empirical mass formula explain why nuclear fission and fusion are possible.

(d) Use the Semi-Empirical mass formula to estimate the value of Z of the most stable isobar of a super-heavy nucleus of mass number 300.

(e) Estimate the energy released when a nucleus of $^{235}_{92}\mathrm{U}$ undergoes fission into the fragments $^{87}_{35}\mathrm{Br}$ and $^{145}_{57}\mathrm{La}$ with three prompt neutrons.

[$\mathrm{m_p = 938.3\ MeV, m_n = 939.6\ MeV, m_e = 0.511\ MeV; a_V = 15.8\ MeV, a_S = 18.0\ MeV, a_A =}$ $\mathrm{23.5\ MeV}$. Nuclear radius $\mathrm{R = R_0 A^{1/3}; R_0 = 1.2\ fm}$]

30. A spherically symmetric nucleus has a radial charge density $\rho(r)$ which is normalised such that $\int \rho(r) d\tau = 1$. Show that the form factor can be expanded as:

$$F(q^2) = 1 - \frac{1}{6} q^2 \overline{R^2} + \ldots$$

where $\overline{R^2}$ is the mean square radius of the charge distribution. When elastic scattering of 200 MeV electrons from a gold foil is observed at 11°, it is found that the scattered intensity is 70% of that expected for a point nucleus. Calculate the r.m.s. radius of the gold nucleus.

For larger scattering angles ($> 50°$) it is found that the scattered intensity, instead of falling off monotonically with angle, exhibits definite structure. What does this imply about $\rho(r)$?

31. Use the Semi-Empirical mas formula to show that the difference in Coulomb energies between a pair of mirror nuclei, ΔE_C, can be written as

$$\Delta E_C = \frac{6}{5}\frac{Z\alpha}{R}$$

where $\alpha = e^2/4\pi\epsilon_0$ in natural units and R is the nuclear radius.

The atomic mass difference between two mirror nuclei can be determined from the β^+ spectra of the $(A, Z+1)$ member of the pair,

$$M(A, Z+1) - M(A, Z) = m_e + E_{max}$$

where m_e is the mass of an electron and E_{max} is the maximum kinetic energy of the positron. Calculate the radii of the $(A, Z+1)$ member of the following pairs of mirror nuclei

(a) $({}^{11}_{5}\mathrm{B}, {}^{11}_{6}\mathrm{C})$, $E_{max} = 0.98$ MeV;

(b) $({}^{23}_{11}\mathrm{Na}, {}^{23}_{12}\mathrm{Mg})$, $E_{max} = 2.95$ MeV; and

(c) $({}^{39}_{19}\mathrm{K}, {}^{39}_{20}\mathrm{Ca})$; $E_{max} = 5.49$ MeV;

and comment on the results.

The Nuclear Shell Model

32. What are *magic numbers* ? Outline the basis of the Nuclear Shell Model and show how it accounts for magic numbers. How can the shell model be used to predict the spins, parities and magnetic dipole moments of nuclear ground states ?

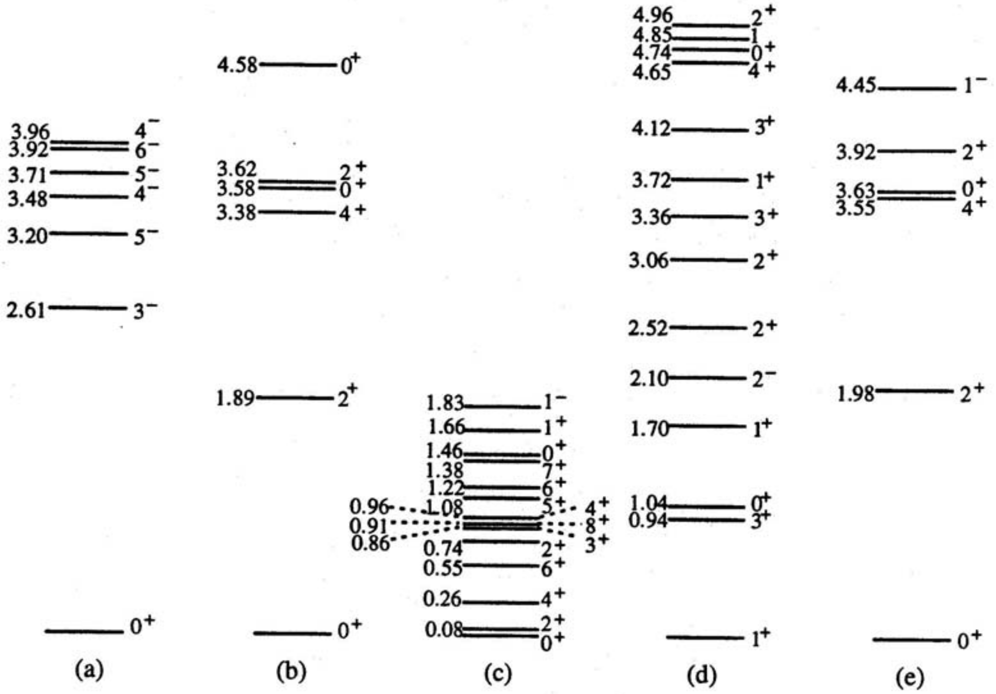
Using the Nuclear Shell Model determine the spins and parities of the ground states of the nuclides listed below and compare them with experimental values. In some cases the magnetic moments in nuclear magnetons are given. In these cases also determine the magnetic moments predicted by the shell model. Comment on any discrepancies you find.

${}^{3}_{2}\mathrm{He}$	${}^{9}_{4}\mathrm{Be}$	${}^{7}_{3}\mathrm{Li}$	${}^{12}_{6}\mathrm{C}$	${}^{13}_{6}\mathrm{C}$	${}^{15}_{7}\mathrm{N}$	${}^{17}_{8}\mathrm{O}$	${}^{23}_{11}\mathrm{Na}$	${}^{131}_{54}\mathrm{Xe}$	${}^{207}_{82}\mathrm{Pb}$
-2.13	-1.17	3.26		0.70	-0.28	-1.89			

33. The diagram overleaf shows the low-lying energy levels for the nuclides:

$${}^{18}_{10}\mathrm{Ne} \quad {}^{166}_{68}\mathrm{Er} \quad {}^{18}_{9}\mathrm{F} \quad {}^{208}_{82}\mathrm{Pb} \quad {}^{18}_{8}\mathrm{O}$$

The schemes are drawn to the same scale, with energies (in MeV) with respect to the ground state and the spin and parity ($\mathrm{J^P}$) values given for each level. Identify which



schemes belong to which nuclei and explain as fully as you can which features of the levels support your choices.

The Decay of Nuclei

34. A nucleus with A=200 can decay by the emission of α particles. The α particles are observed with two different energies, 4.687 MeV and 4.650 MeV. Neither of these decays populates the ground state of the daughter nucleus, but each is followed by a γ ray of energy 266 and 305 keV respectively. No other γ rays are seen.

(a) From this information construct a decay scheme.

(b) The decaying parent state has spin 1 and negative parity and the daughter has ground state $J^P = 0^-$. Explain why there is no direct α decay to the ground state.

35. Outline the Fermi theory of β decay and explain the principal assumptions made.

Explain the difference between Fermi and Gamow-Teller transitions and between super-allowed, allowed and forbidden decays.

Classify each of the following examples of β decay according to whether the decay is super-allowed, allowed, 1st forbidden etc., and whether Fermi or Gamow-Teller matrix elements are involved.

(i) $n \to p$

(ii) ${}^{6}_{2}\mathrm{He}(0^+) \to {}^{6}_{3}\mathrm{Li}(1^+)$ (ft=830s)

(iii) ${}^{14}_{6}\mathrm{O}(0^+) \to {}^{14}_{7}\mathrm{N}^*(0^+)$ (ft=3300s)

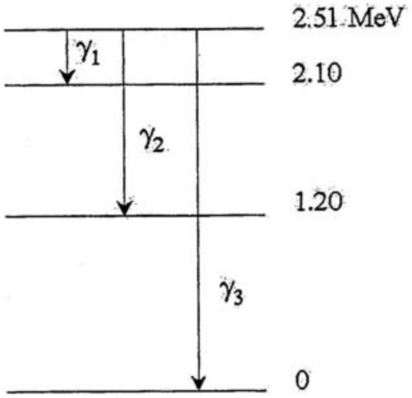
(iv) ${}^{35}_{16}\mathrm{S}(\frac{3}{2}^+) \to {}^{35}_{17}\mathrm{Cl}(\frac{3}{2}^+)$ (ft=1x10^5s)

(v) ${}^{36}_{17}\mathrm{Cl}(2^-) \to {}^{36}_{18}\mathrm{Ar}(0^+)$

(vi) ${}^{76}_{35}\mathrm{Br}(1^-) \to {}^{76}_{34}\mathrm{Se}(0^+)$

(vii) ${}^{137}_{55}\mathrm{Cs}(\frac{7}{2}^+) \to {}^{137}_{56}\mathrm{Ba}(\frac{3}{2}^+)$

36. A nucleus in an excited state at 2.51 MeV can decay by emission of three γ rays as shown:

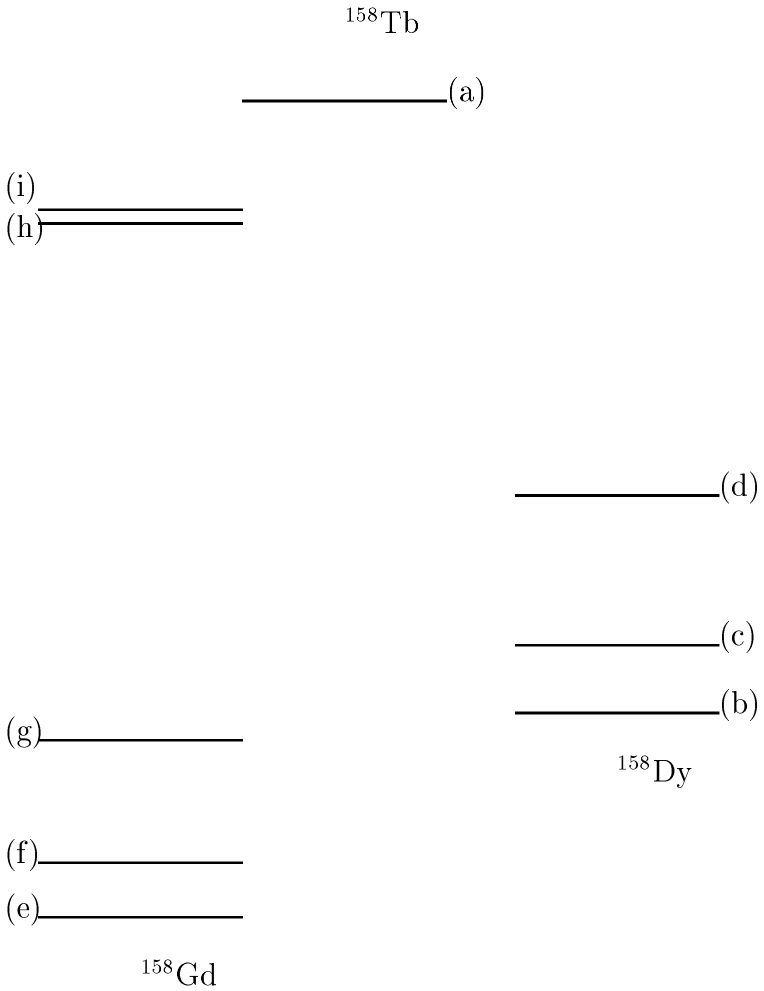


The transitions labelled γ_1, γ_2 and γ_3 are known to proceed via magnetic dipole, electric dipole and electric quadrupole transitions respectively. No other transitions to the ground state, of comparable intensities, are observed. Given that the ground state has a spin of $\frac{3}{2}^+$ what are the most probable spins and parities of these three excited states?

37. The nucleus ${}^{158}_{65}\mathrm{Tb}$ undergoes β^- decay to ${}^{158}_{66}\mathrm{Dy}$. Careful study of the β^- spectrum reveals the presence of two components, with endpoint energies E_0 tabulated overleaf. ${}^{158}_{65}\mathrm{Tb}$ also undergoes electron capture (EC) to four excited states of ${}^{158}_{64}\mathrm{Gd}$. The relative strength f of each component and the energies, E_γ, of the corresponding associated γ-ray(s) are also given.

Decay	f	E_0 /MeV	E_γ / MeV
β^-	1%	0.628	0.218, 0.099
β^-	13%	0.845	0.099
EC	38%		0.963, 0.781, 0.182, 0.079
EC	41%		0.945, 0.079
EC	3%		0.182, 0.079
EC	4%		0.079

The diagram below shows the relevant energy levels for these nuclei. Indicate the observed β and γ transitions on the diagram. What is the likely nature of the lowest-lying excitations of the Dy and Gd nuclei, and hence what are their likely spin-parity values ? Using your knowledge of the relevant selection rules, make spin-parity assignments to all the levels involved. Explain your reasoning, specifying the nature of each of the γ-transitions (i.e. electric dipole, magnetic dipole etc.) and the β-transitions (i.e. allowed, forbidden).



Nuclear Fission and Fusion

38. A mixture of ^{235}U and graphite is to be used for some experiments. The graphite is contaminated with 1 p.p.m. by weight of ^{10}B. What is the maximum fraction by weight of ^{235}U in the mixture if the multiplication factor at infinite volume is not to exceed unity ? The absorption cross-sections for ^{12}C, ^{10}B, and ^{235}U at thermal energies are 0.04 b, 3800 b and 700 b respectively, where fission accounts for 580 b of the cross-section in ^{235}U. Assume that 2.5 neutrons are produced per fission, and that all reactions take place at thermal energies.

Numerical answers

29. (b) 0.7 MeV; (d) 114; (e) 150 MeV.
30. 5 fm.
31. (a) 3.8 fm; (b)4.5 fm; (c)4.8 fm.
38. 1.2×10^{-3}