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Part II Particle and Nuclear Physics
Examples Sheet 2

Weak Interactions

19. Show that the electron momentum spectrum in β decay using Fermi theory can be written as

$$\frac{d\Gamma}{dp_e} = \frac{G_F^2}{2\pi^3}\left(E_0 - E_e\right)^2 p_e^2$$

where G_F is the Fermi constant, E_e and p_e are the energy and momentum of the electron and E_0 is the total energy release. You may neglect the mass of the electron and neutrino.

Show that the average kinetic energy carried off by the electron in β decay is $E_0/2$ when the electron is highly relativistic, and $E_0/3$ when the electron is non-relativistic.

20. Show that the τ lepton decay branching fractions should be approximately in the ratios

$$\tau \to e : \tau \to \mu : \tau \to hadrons = 1 : 1 : 3.0$$

Estimate the mean lifetime of the τ lepton, assuming the branching fraction for $\tau^- \to e^-\bar{\nu}_e\nu_\tau$ is 18%.

[$\mathrm{m}_\tau = 1.777$ GeV/c^2, $\mathrm{m}_\mu = 0.106$ GeV/c^2 and the mean μ lifetime is 2.197×10^{-6} s.]

21. In a beam of antineutrinos, it is proposed to search for $\bar{\nu}_\tau$ via their interactions on nucleons in a stationary target to produce τ-leptons.

(a) Draw the Feynman diagram for the simplest such production process.

(b) Calculate the minimum energy of the $\bar{\nu}_\tau$ which would permit τ-lepton production.

(c) What is the energy of the produced τ-lepton when the $\bar{\nu}_\tau$ has this threshold energy ?

(d) How far will the τ-lepton travel on average before decaying, given that its mean lifetime is 290 fs ?

(e) Draw a Feynman diagram for the principal decay modes of the τ^+.

(f) Estimate the branching fraction for the β-decay $\tau^+ \to e^+\bar{\nu}_\tau\nu_e$.

[The masses of τ^+, proton and neutron are 1.777 GeV/c^2, 0.938 GeV/c^2 and 0.940 GeV/c^2 respectively.]

22. The Ω^- baryon (sss), produced in the event shown in example 4, is seen to decay weakly through the decay chain $\Omega^- \to \Xi^0\pi^-$, $\Xi^0 \to \Lambda^0\pi^0$ and $\Lambda^0 \to \mathrm{p}\pi^-$. Draw the Feynman diagrams for the decays of the Ω^-, the Ξ^0 and the Λ^0.

Explain, with the aid of Feynman diagrams, why the Ω^- is not observed to decay via the strong decay $\Omega^- \to \Xi^-\overline{\mathrm{K}^0}$ and that the weak decay $\Omega^- \to \Lambda^0\pi^-$ is strongly suppressed.

[$\Xi^0(uss)$, $\Xi^-(dss)$, $\overline{\mathrm{K}^0}(s\bar{d})$, $\Lambda^0(uds)$]

Electroweak Unification

23. In the OPAL experiment at LEP the cross-section for $e^+e^- \to \tau^+\tau^-$ was measured at various centre-of-mass energies. Some of the results are shown below. Plot these data and make estimates of the Z^0 boson mass, M_{Z^0}, the total width of the Z^0 boson, Γ_Z, and the partial decay width Γ_τ (assuming lepton universality of the Neutral Current).

E_{cm}/GeV	$\sigma(e^+e^- \to \tau^+\tau^-)$/nb
88.481	0.2769 ± 0.0235
89.442	0.4892 ± 0.0091
90.223	0.8331 ± 0.0368
91.283	1.4988 ± 0.0213
91.969	1.1892 ± 0.0235
92.971	0.7089 ± 0.0105
93.717	0.4989 ± 0.0276

Why is the measured resonance curve asymmetric and what else needs to be taken into account when determining accurately M_{Z^0}, Γ_Z and Γ_τ ?

24. Estimate the total decay width, Γ_Z, and the lifetime of the Z^0 boson using the resonant cross-section ratio,

$$\frac{\sigma(e^+e^- \to Z^0 \to \text{hadrons})}{\sigma(e^+e^- \to Z^0 \to \mu^+\mu^-)} = 20.7,$$

and the Z^0 partial decay widths, $\Gamma(Z^0 \to \mu^+\mu^-) = 83.3$ MeV and $\Gamma(Z^0 \to \nu_\mu\bar{\nu_\mu}) = 166.5$ MeV. Make clear any assumptions you make.

25. The number of neutrino species, in principle, can be determined from the total width of the W boson. Using the Standard Model prediction of the partial width for $W \to e\bar{\nu}_e$ decays,

$$\Gamma(W \to e\bar{\nu}_e) = \frac{G_F}{\sqrt{2}}\frac{M_W^3}{6\pi},$$

the mass of the W boson, $M_W = 80$ GeV/c^2 and the total width, $\Gamma_W = 2.1$ GeV, estimate the number of light neutrino species. Make clear any assumptions you make.

[$G_F = 1.2 \times 10^{-5}$ GeV^{-2}.]

26. Draw all possible lowest order Feynman diagrams for the neutrino scattering or annihilation processes:

i) $\nu_e\, e^- \rightarrow \nu_e\, e^-$
ii) $\bar{\nu}_e\, e^- \rightarrow \bar{\nu}_e\, e^-$
iii) $\nu_\mu\, e^- \rightarrow \nu_\mu\, e^-$
iv) $\bar{\nu}_\mu\, e^- \rightarrow \bar{\nu}_\mu\, e^-$
v) $\nu_e\, n \rightarrow e^-\, p$

27. Consider each of the sets of processes given below. In each case, with the aid of Feynman diagrams using the Standard Model vertices, determine which processes are allowed and which are forbidden. By considering the strength of the forces involved, list the processes in each set in order of expected rate.

a) $\pi^0 \rightarrow \gamma\gamma$, $\pi^0 \rightarrow \pi^- e^+ \nu_e$ and $\pi^0 \rightarrow \nu\bar{\nu}$;
b) $e^+e^- \rightarrow \tau^+\tau^-$, $\bar{\nu}_\mu + \tau^- \rightarrow \tau^- + \bar{\nu}_\mu$ and $\nu_\tau + \mathrm{p} \rightarrow \tau^+ + \mathrm{n}$;
c) $\mathrm{B}^0(\overline{\mathrm{b}}\mathrm{d}) \rightarrow \mathrm{D}^-(\overline{\mathrm{c}}\mathrm{d})\pi^+$, $\mathrm{B}^0 \rightarrow \pi^+\pi^-$ and $\mathrm{B}^0 \rightarrow \mathrm{J}/\psi \mathrm{K}^0$.

The Standard Model and Beyond

28. Show that if there are two neutrino mass eigenstates ν_2 and ν_3 with masses m_2 and m_3 and energies E_2 and E_3, mixed so that

$$\begin{aligned} \nu_\mu &= \nu_2 \cos\theta + \nu_3 \sin\theta \\ \nu_\tau &= -\nu_2 \sin\theta + \nu_3 \cos\theta \end{aligned}$$

then the number of muon neutrinos observed at a distance L from the muon source is

$$|\nu_\mu(L)|^2 = |\nu_\mu(L=0)|^2 \times \left[1 - \sin^2(2\theta)\, \sin^2\left\{\left(\frac{E_3 - E_2}{2\hbar}\right)\frac{L}{c}\right\}\right].$$

If m_2 and m_3 are very much less than the neutrino momentum, p, show that

$$|\nu_\mu(L)|^2 \approx |\nu_\mu(L=0)|^2 \times \left[1 - \sin^2(2\theta)\, \sin^2\left\{A\left(\frac{(m_2^2 - m_3^2)L}{p}\right)\right\}\right]$$

where A is a constant.

In 2005 the MINOS experiment will start to study neutrino oscillations by pointing a beam of 1-5 GeV/c muon neutrinos from Fermilab, Illinois, at the 5400 ton MINOS far dectector in the SOUDAN mine in Minnesota, 730 km away. The experiment aims to make a precise measurement of $m_3^2 - m_2^2$.

Sketch the expected energy spectrum of muon neutrinos at the MINOS far detector if $\sin^2(2\theta) = 0.90$ and $m_3^2 - m_2^2 = 2.5 \times 10^{-3}$ $(\mathrm{eV/c^2})^2$. Assume that the energy spectum of neutrinos produced by the beam at Fermilab is of uniform intensity in the range 1-5 GeV and zero elsewhere (i.e. a top-hat function).

If muon neutrinos oscillate into tau neutrinos, will any τ leptons (produced by charged current interactions) be observed in the MINOS far detector ?

[$A = 1.27\ \mathrm{s}^{-1}$ if m_2 and m_3 are measured in eV/c^2, p in GeV/c and L in km. The mass of the τ^- is $1.777\,\mathrm{GeV}/c^2$.]

Numerical answers

20. 0.3 ps
21. (b) 3.47 GeV; (c) 2.88 GeV; (d) 11 μm; (e) $\sim 20\%$
24. 2.47 GeV, 2.66×10^{-25} s
25. 3