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Part II Particle and Nuclear Physics
Examples Sheet 1

Matter and Forces

1. Explain the meaning of the terms *quark, lepton, hadron, nucleus* and *boson* as used in the classification of particles.

Relativistic Kinematics

2. What is meant by *natural* units and the *Heaviside-Lorentz* system ?

(a) The Compton wavelength of a particle can be written in natural units as

$$\lambda = \frac{1}{m}$$

where m is the mass of the particle. Estimate λ for a pion ($m_\pi = 139.6$ MeV/c^2). Quote your answer in natural and SI units.

(b) The total cross-section for e^+e^- annihilation can be written as

$$\sigma = \frac{4}{3}\frac{\pi\alpha^2}{s}$$

where α is the fine structure constant and $\sqrt{s}$ is the centre-of-mass energy. Estimate σ at a centre-of-mass energy equal to the Z^0 mass ($m_{Z^0} = 91.2$ GeV/c^2). Quote your answer in natural units and in barns.

[$\hbar$c = 197 MeVfm, 1 barn = 10^{-28} m^2]

3. Consider the decay of a particle X into two particles a and b.

(a) Show that, in the rest frame of X, the energy of particle a can be written in natural units as

$$E_a = \frac{m_X^2 + m_a^2 - m_b^2}{2m_X}$$

where m_i is the mass of particle i. What is the equivalent expression for the energy of particle b ? What is the energy if the final state particles are the same (or antiparticles of each other)?

(b) Show that the magnitude of the momentum of particle a can be written in natural units as

$$p_a = \frac{\sqrt{m_X^4 + m_a^4 + m_b^4 - 2m_X^2 m_a^2 - 2m_X^2 m_b^2 - 2m_a^2 m_b^2}}{2m_X}.$$

What is the equivalent expression for the momentum of particle b ? What is the value of the momentum if the final state particles are the same (or antiparticles of each other) ?

(c) Show that if one of the final state particles is massless, e.g. $m_b = 0$, then the expression for the momentum simplifies to

$$p_a = \frac{m_X^2 - m_a^2}{2m_X}$$

[Note: replacing m_X by the centre-of-mass energy, $\sqrt{s}$, in the above gives the equivalent expressions for reactions.]

4. The figure shows a photograph and line diagram of the event corresponding to the first observation of the Ω^- baryon ($\Omega^- \to \Xi^0\pi^-$, $\Xi^0 \to \Lambda^0\pi^0$) in a $\mathrm{K^-p}$ interaction in a liquid hydrogen bubble chamber (from Barnes *et al.*, Phys. Rev. Lett. **12** (1964) 204):

(a) The two photons from the $\pi^0 \to \gamma\gamma$ decay are both seen to convert to e^+e^- pairs. Show that the process $\gamma \to e^+e^-$ is kinematically forbidden *in vacuo*.

(b) The π^- and Ξ^0 from the Ω^- decay have momenta of 281 MeV/c and 1906 MeV/c respectively. Their spatial opening angle is 71^0. Estimate the mass of the Ω^- and compute its momentum.

(c) The length of the Ω^- flight path is 2.5 cm. Calculate the proper lifetime of the Ω^-.

[m(π^-)=139.6 MeV/c^2, m(Ξ^0)=1315 MeV/c^2]

Decays and Reactions

5. A sample of gold is exposed to a neutron beam of constant intensity such that 10^{10} neutrons per second are absorbed in the reaction

$$n + {}^{197}\mathrm{Au} \rightarrow {}^{198}Au + \gamma$$

The nuclide ^{198}Au undergoes β decay to ^{198}Hg with a mean lifetime of 4 days.

(a) How many atoms of ^{198}Au will be present after 6 days of irradiation?

(b) How many atoms of ^{198}Hg will be present after 6 days assuming that the neutron beam has no effect on the Hg?

(c) What is the equilibrium number of ^{198}Au nuclei?

6. The decay chain ^{139}Cs→^{139}Ba→^{139}La is observed for an initially pure sample of 1mCi of ^{139}Cs. The half life of ^{139}Cs is 9.5 minutes and that of ^{139}Ba is 82.9 minutes; ^{139}La is stable. What is the maximium ^{139}Ba activity, and when does it occur?

[A common unit of activity is the Curie. 1 Curie (Ci)= 3.7x10^{10} disintegrations per second.]

7. Estimate the branching fraction for the decay $K^+ \rightarrow \pi^+\pi^0$, given that the partial width for this decay is 1.2×10^{-8} eV and the mean lifetime of the K^+ meson is 1.2×10^{-8}s.

8. Define the terms *total cross-section* and *differential cross-section* for scattering processes.

A beam of neutrons with an intensity 10^5 particles per second traverses a foil of ^{235}U with a thickness of 10^{-1} kg m^{-2}.‡

A neutron interacting with a ^{235}U nucleus has three possible results:

- Elastic scattering of the neutron. The cross-section for this process is 10^{-2} b.
- The capture of the neutron followed by the emission of a γ-ray. The cross-section for this process is 70 b.

‡To determine the number of particles that interact one must know the density of target particles and the thickness of the target. Instead of giving two numbers, that just have to be multiplied, it is common practise to quote the target thickness multiplied by the target density. This gives a target "thickness" in units of mass/area.

- The capture of the neutron, followed by the resulting nucleus undergoing fission i.e. the splitting of the nucleus into two nuclei of similar masses. The cross-section for this process is 200 b.

Determine:

(a) The attenuation of the neutron beam by the foil.

(b) The number of fission reactions, caused by the incident beam, occurring per second in the foil.

(c) The flux of elastically scattered neutrons at a point 10m from the foil and out of the beam, assuming that the neutrons are scattered isotropically.

9. The Breit-Wigner formula for a reaction cross-section is given by

$$\sigma(E) = \frac{g\lambda^2}{4\pi} \frac{\Gamma_i \Gamma_f}{(E - E_0)^2 + \Gamma^2/4}.$$

Explain the meaning of the symbols in this equation.

The maximum value of the radiative capture cross-section for neutrons in ^{123}Te (J=1/2 in its ground state) is 75 kb and is reached at a neutron energy of 2.2 eV, where the elastic width Γ_n is 0.0104 eV and the radiative width Γ_γ is 0.105 eV. What is the elastic cross-section at resonance and what is the spin of the compound nucleus formed?

10. The Z^0 particle has a width of 2.5 GeV and a mass of 91.2 GeV/c^2. Given that when the Z^0 decays, it decays to $\mu^+\mu^-$ 3.37% of the time, find the reaction cross-section for the process $e^+e^- \to \mu^+\mu^-$ when a beam of 45 GeV electrons collides with a beam of 45 GeV positrons.

[You may assume that the partial widths for $Z^0 \to e^+e^-$ and $Z^0 \to \mu^+\mu^-$ are the same.]

Feynman Diagrams and QED

11. Draw a lowest order Feynman diagram for each of the following processes:

(a) $\gamma \to e^+e^-$

(b) $e^- + e^- \to e^- + e^-$

(c) $e^+ + e^- \to \gamma + \gamma$

12. (a) The π^0 ($J^P = 0^-$) decays predominantly to $\gamma\gamma$ but is also seen to decay to $e^+e^-\gamma$ ("Dalitz decay"), to $e^+e^-e^+e^-$ and to e^+e^- with branching fractions of 1.2%, 3.2×10^{-5} and 2×10^{-7} respectively. Draw the leading order Feynman diagrams for each of these decays. Based on the coupling constants involved (ignore propagator effects etc.), give rough estimates of the branching fractions for each decay.

(b) The ρ^0 ($J^P = 1^-$) decays to e^+e^- with a branching fraction of 4×10^{-5}. Draw the Feynman diagram for this decay and compare the $\pi^0 \to e^+e^-$ and $\rho^0 \to e^+e^-$ partial widths.

[The π^0 and ρ^0 lifetimes are 8.4×10^{-17}s and 4.4×10^{-24}s respectively.]

13. Drell-Yan production of charged lepton pairs in hadron-hadron interactions ($\pi N \to \mu^+\mu^- +$ anything, for example) proceeds via quark-antiquark annihilation into a single virtual photon. Show that the Drell-Yan cross sections in π^+p, π^+n, π^-p and π^-n interactions would be expected to be in the ratio 1 : 2 : 8 : 4.

What would you expect the Drell-Yan cross sections for pp and $\bar{\text{p}}$p collisions to be compared with the Drell-Yan cross section for π^+p interactions ?

QCD and the Quark Model

14. When π^- mesons are stopped in deuterium they form "pionic atoms" (π^-d) which usually undergo transitions to an atomic S-state (L=0), whereupon the capture reaction $\pi^-\text{d}\to\text{nn}$ occurs and destroys them. (The fact that capture normally occurs in an S-state is established from studies of the X rays emitted in the transitions before capture). Given that the deuteron has spin parity 1^+ and the pion has spin 0, show that these observations imply that the pion has negative intrinsic parity.

15. (a) Verify the quark model predictions given in the lectures for the following meson masses:

Meson	Calculated (MeV/c^2)	Observed (MeV/c^2)
π	140	138
K	484	496
η	559	549
ρ	780	776
ω	780	783
K*	896	892
ϕ	1032	1020

[Assume m_u=m_d=310 MeV/c^2, m_s=483 MeV/c^2 and a spin-spin interaction coefficient A= 0.0615 GeV^3.]
What would you predict for the mass of the η' meson (measured mass 958 MeV/c^2)?

(b) What must be the total spin of any pair of quarks in the baryons in the $J^P=\frac{3}{2}^+$ decuplet? Hence predict the masses of the decuplet baryons and compare your predictions with the measured values.

[Assume m_u=m_d=360 MeV/c^2, m_s=540 MeV/c^2 and a spin-spin interaction coefficient A= 0.026 GeV^3.]

16. Derive the magnetic moments of the proton and neutron in the quark model as follows:

(a) Assuming all the quarks are in l=0 states what must be the total spin of the two u quarks in the proton? (Give reasons for your answer).

(b) Hence show that the wave function for a proton in the s_z=$+\frac{1}{2}$ state can be written as

$$\frac{1}{\sqrt{6}}(2u\uparrow u\uparrow d\downarrow - \ u\uparrow u\downarrow d\uparrow - \ u\downarrow u\uparrow d\uparrow)$$

and derive a similar expression for the neutron.

(c) Assuming u and d quarks have equal mass write their magnetic moments in terms of this quark mass.

(d) Hence predict the ratio of the proton and neutron magnetic moments. Compare with the observed values: $\mu_{\rm p} = \ 2.79\mu_N, \ \ \mu_{\rm n} = -1.91\mu_N$. What value for the quark mass is needed to give these values? Is this sensible?

17. Consider the decay of the ρ^0 meson ($J^P = 1^-$) in the following decay modes:

(a) $\rho^0 \to \pi^0\gamma$

(b) $\rho^0 \to \pi^+\pi^-$

(c) $\rho^0 \to \pi^0\pi^0$.

In each case, draw a Feynman diagram and determine whether the process is allowed or forbidden. By considering the strength of the forces involved, list the decay modes in order of expected rate.

18. The figure shown over the page (from Boyarski *et al.*, Phys. Rev. Lett. **34** (1975) 1357) shows the original e^+e^- annihilation cross section measurements from the Mark II Collaboration which contributed to the discovery of the J/ψ meson. The measurements were made during a fine scan of the $e^\pm$ beam energies at the SPEAR storage ring at SLAC which consisted of oppositely circulating e^+ and e^- beams of equal energy.

The observed width (a few MeV) of the J/ψ resonance is due dominantly to the energy spread inherent in the e^+, e^- beams at each scan point. The *relative* centre of mass energy between scan points is known very precisely however (to about 1 part in 10^4). The actual J/ψ width is much smaller than the observed width, but can be extracted from the data as follows:

(a) The Breit-Wigner formula for the scattering of two particles of spin s_1 and s_2 in the region of a resonance of spin J is:

$$\sigma(E) = \frac{\lambda^2}{4\pi}\frac{(2J+1)}{(2s_1+1)(2s_2+1)}\frac{\Gamma_i\Gamma_f}{[(E-E_0)^2+\Gamma^2/4]}$$

where λ is the de Broglie wavelength of the incoming particles in the centre of mass, E is the centre of mass energy, E_0 is the resonance energy, Γ is the total width of the resonance and Γ_i (Γ_f) is the partial width for decay into the initial (final) state. Show that, for the production of the J/ψ resonance in e^+e^- collisions, the integrated elastic cross section under the resonance peak is given by

$$\sigma' \equiv \int \sigma_{el}(E)dE = \frac{3}{8}\lambda^2 B^2 \Gamma$$

where B is the branching fraction for the decay J/$\psi \to e^+e^-$.

(b) Assume that, at each scan point, the beam energy spread produces a spread of centre of mass energies E' distributed about the average centre of mass energy E according to a probability distribution $f(E' - E)$. Show that the measured area under the resonance peak, $\int \sigma_{meas}(E)dE$, is the same as the true area under the peak, $\int \sigma(E)dE$.

(c) Given that the differential cross section dσ/dΩ for the process $e^+e^- \to J/\psi \to e^+e^-$ is proportional to $1+\cos^2\theta$ where θ is the angle between the final e^- and the beam direction, calculate the fraction of J/ψ decays contained within the acceptance region $|\cos\theta| < 0.6$ imposed for the e^+e^- and $\mu^+\mu^-$ channels.

(d) Use the data in the figure to estimate the quantities σ' and B defined above [§]. Hence estimate Γ and Γ_{ee} for the J/ψ.

(e) The corresponding widths for the ϕ meson are Γ=4.4 MeV and Γ_{ee}=1.37 keV. Explain why the J/ψ and ϕ mesons have similar leptonic widths Γ_{ee} but very different total widths Γ.

[§]Note that the measured cross sections contain a significant non-resonant contribution which must be subtracted.

Numerical answers

2. (a) 7.16×10^{-3} MeV^{-1}, 1.41 fm

   (b) 2.68×10^{-8} GeV^{-2}, 0.0104 nb

4. (b) 1689 MeV/c^2, 2015 MeV/c

   (c) 70 ps

5. (a) 2.7x10^{15}; (b) 2.5x10^{15}; (c) 3.46x10^{15}

6. 0.087 mCi, 33.5 minutes

7. 21.8%

8. (a) 99,311 particles/s; (b) 510 /s; (c) 2.03x10^{-5} particles/m^2/s

9. 7.4 kb; J=1

10. 1.07 nb

12. (a) 1, 0.028, 5×10^{-5}, 5×10^{-5}

    (b) 1.56×10^{-6} eV, 6 keV

13. $\sigma(\mathrm{p\bar{p}}) = 17\sigma(\pi^+\mathrm{p})$

15. (a) $m(\eta') = 349$ MeV/c^2

    (b) S=1, $m(\Delta) = 1.230$ GeV/c^2, $m(\Sigma^*) = 1.383$ GeV/c^2, $m(\Xi) = 1.535$ GeV/c^2, $m(\Omega^-) = 1.687$ GeV/c^2

16. (a) S=1

    (d) $\mu_p/\mu_n = -1.5$, Mass of quark = 330 MeV/c^2

18. (c) 50.4%

    (d) $\sigma' \approx 800$ nb MeV, $B \approx 0.055$, $\Gamma \approx 75$ keV, $\Gamma_{ee} \approx 5.3$ keV