Section XIV
Nuclear Decay

α Decay

- α decay is due to the emission of a ${}^{4}_{2}He$ nucleus
- ${}^{4}_{2}He$ is doubly magic and very tightly bound
- α decay is energetically favourable for almost all nuclei having $A \geq 190$ and for many $A \geq 150$.

Why α ?

Consider energy release (E_0) in various possible decays of ${}^{232}U$

	n	p	${}^{2}H$	${}^{3}H$	${}^{3}He$	${}^{4}He$	${}^{5}He$	${}^{6}Li$	${}^{7}Li$
E_0 (MeV)	-7.26	-6.12	-10.70	-10.24	-9.92	+5.41	-2.59	-3.79	-1.94

α easy to form inside nucleus 2p ↑↓+2n ↑↓

(extent to which α exist inside nucleus still unknown)

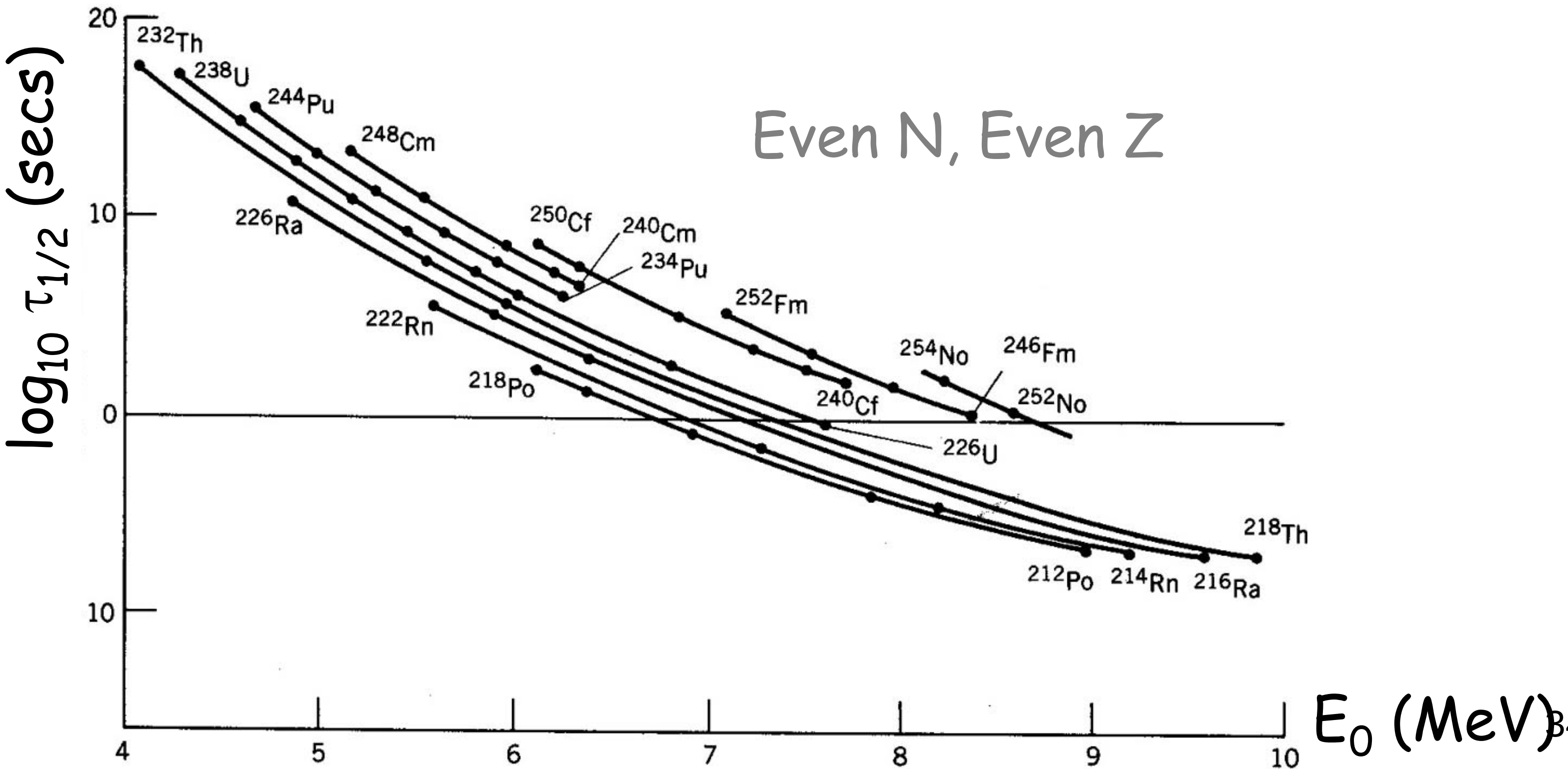
DEPENDENCE of $\tau_{1/2}$ on E_0 (Geiger and Nuttall 1911)

A VERY striking feature of α decay is the strong dependence of lifetime on E_0.

Example:

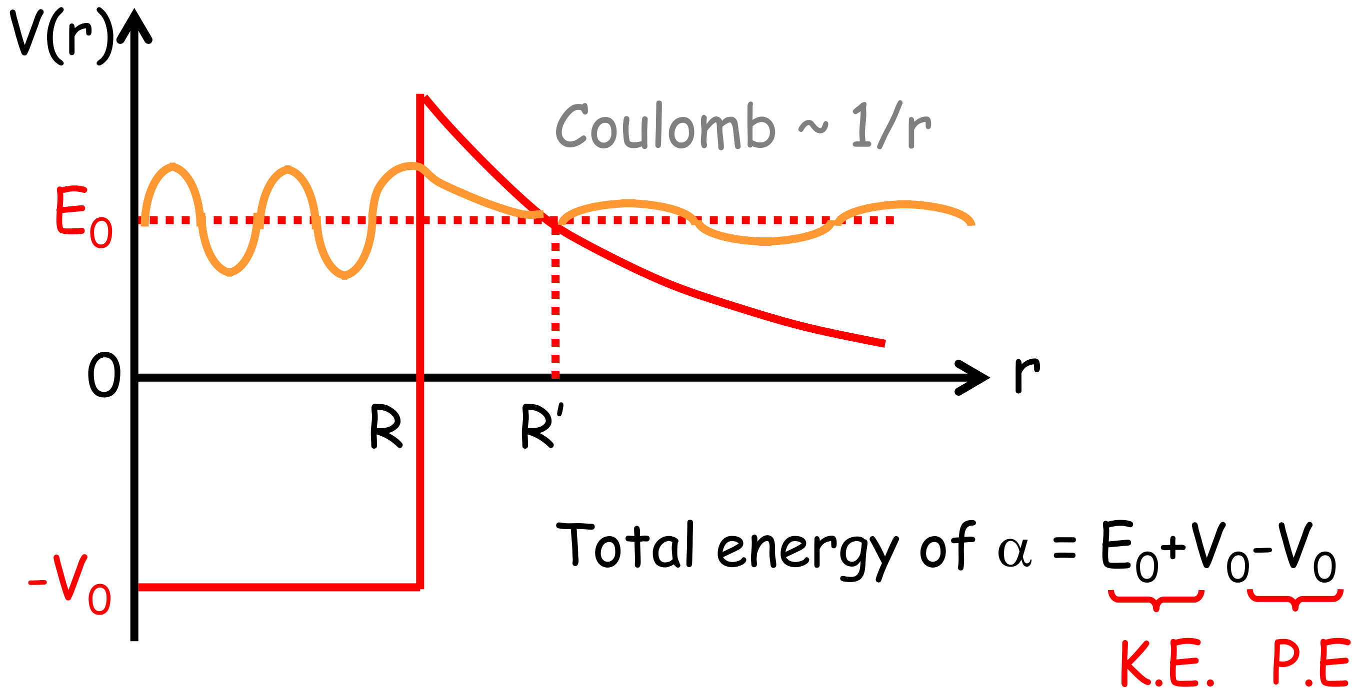
$$^{232}Th \quad E_0 = 4.08\ MeV \quad \tau_{1/2} = 1.4 \times 10^{10}\ yrs$$
$$^{218}Th \quad E_0 = 9.85\ MeV \quad \tau_{1/2} = 1.0 \times 10^{-7}\ secs$$

A factor of ~ 2.5 in E_0 $\Rightarrow$ Factor 10^{24} in $\tau_{1/2}$!



QUANTUM MECHANICAL TUNNELLING

- The nuclear potential for the α particle due to the daughter nucleus includes a Coulomb barrier which inhibits the decay.



- Classically, α particle cannot enter or escape.

- Quantum mechanically, α particle can penetrate the Coulomb barrier

$\Rightarrow$ Quantum Mechanical Tunnelling

Simple Theory (Gamow, Gurney, Condon 1928)

- Assume α exists inside the nucleus and hits the barrier.

α decay probability, $\lambda = f\,P$

f = escape trial frequency, P = probability of tunnelling through barrier

semi-classically, $f \sim v/2R$

v = velocity of a particle inside nucleus , R = radius of nucleus

e.g. $V_0 \sim 35$ MeV, $E_0 = 5$ MeV $\Rightarrow E_\alpha = 40$ MeV

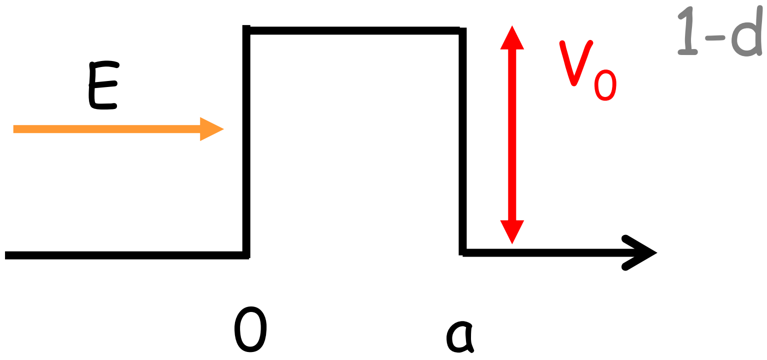
$$f \sim \frac{v}{2R} = \frac{1}{2R}\sqrt{\frac{2E_\alpha}{m_\alpha}} \sim 10^{22}\ s^{-1}$$

$m_\alpha = 3.7$ GeV
$R \sim 2.1$ fm

- Obtain tunnelling probability, P, by solving S.E. in 3 regions and using boundary conditions

- Transmission probability,

$$P = \left[1 + \frac{V_0^2}{4(V_0 - E)E} \sinh^2 ka \right]^{-1}$$

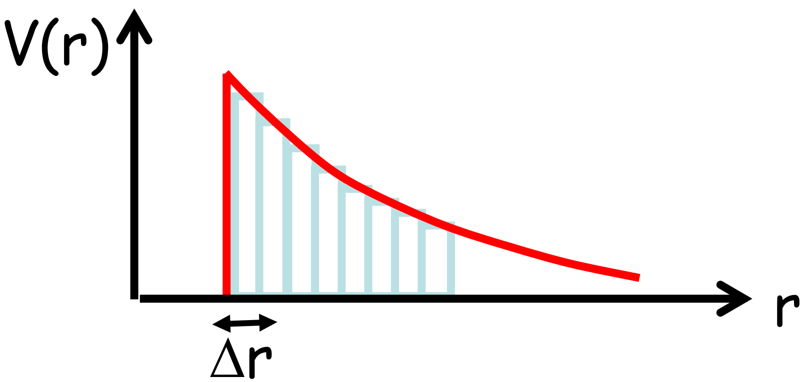


$$\frac{\hbar^2 k^2}{2m} = V_0 - E \quad m = \text{reduced mass}$$

For $ka \gg 1$, P is dominated by exponential decay within barrier

$$\Rightarrow \qquad \boxed{P \sim e^{-2ka}}$$

- Coulomb potential, V, and thus k varies with r. Divide into rectangular pieces and multiply together exponents.



- Probability to tunnel through Coulomb barrier

$$k = \frac{[2m(V(r)-E_0)]^{1/2}}{\hbar}$$

$$P = \prod_i e^{-2k_i \Delta r} = exp\left\{-2\int_R^{R'} \frac{[2m(V(r)-E_0)]^{1/2}}{\hbar} dr\right\} \equiv exp\{-2G\}$$

where $G = \int_R^{R'} \frac{[2m(V(r)-E_0)]^{1/2}}{\hbar} dr$ is the GAMOW FACTOR

- For r > R: $V(r) = \frac{2Z'e^2}{4\pi\varepsilon_0 r} \equiv \frac{B}{r}$ $\quad Z' = Z - 2$

α escapes at r = R', V(R') = E_0 $\Rightarrow$ R' = B/ E_0

$$\therefore G = \int_R^{R'} \left(\frac{2m}{\hbar^2}\right)^{1/2} \left[\frac{B}{r} - E_0\right]^{1/2} dr = \left(\frac{2mB}{\hbar^2}\right)^{1/2} \int_R^{R'} \left[\frac{1}{r} - \frac{1}{R'}\right]^{1/2} dr$$

Appendix H

$$\Rightarrow \quad G = \left(\frac{2m}{E_0}\right)^{1/2} \frac{B}{\hbar}\left[cos^{-1}\left(\frac{R}{R'}\right)^{1/2} - 2\left\{\left(1-\frac{R}{R'}\right)\left(\frac{R}{R'}\right)\right\}^{1/2}\right]$$

- Most practical cases R << R′, so term in [...] ~ π/2

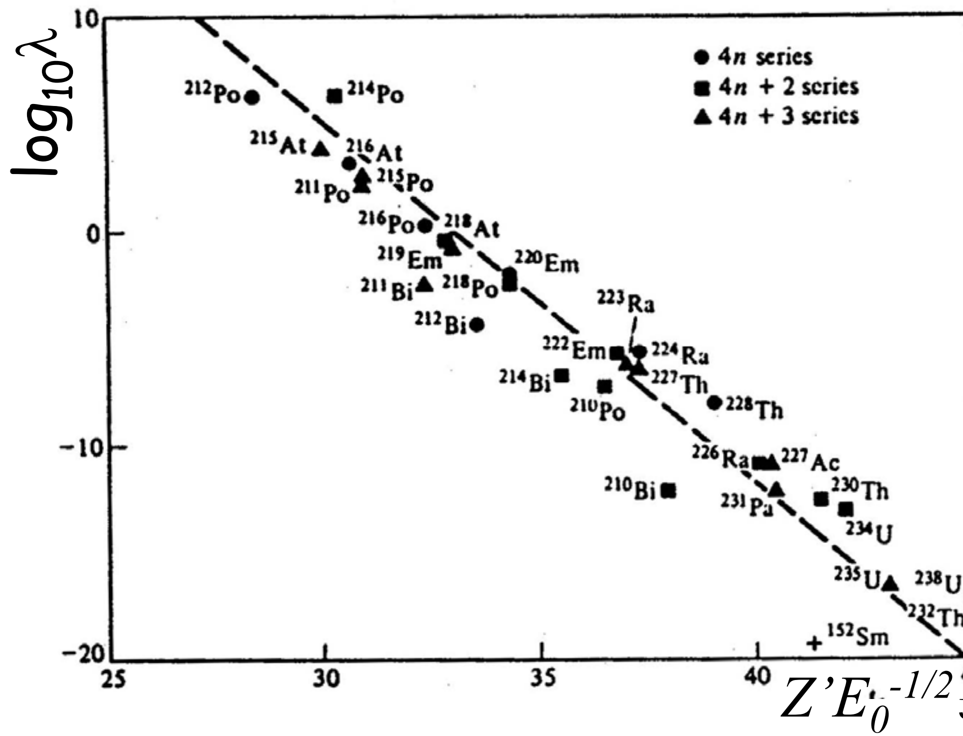
$$G \approx \left(\frac{2m}{E_0}\right)^{1/2} \frac{B}{\hbar}\frac{\pi}{2} \qquad B = \frac{2Z'e^2}{4\pi\varepsilon_0}$$

e.g. Z = 90, E_0 ~ 6 MeV ⇒ R′ ~ 40 fm >> R ⇒ $G \approx Z'\left(\frac{3.9\ MeV}{E_0}\right)^{1/2}$

- Lifetime $\tau_{1/2} = \frac{1}{\lambda} = \frac{1}{f\,P} \sim \frac{2R}{v}e^{2G} \quad \Rightarrow \quad Ln\,\tau_{1/2} \sim 2G + Ln\frac{2R}{v}$

$$Ln\,\lambda \sim -\frac{Z'}{E_0^{\ 1/2}} + constant$$

GEIGER-NUTTALL LAW



- Simple tunnelling model accounts for strong dependence of $\tau_{1/2}$ on E_0.
- Also explains why decay to heavier fragments e.g. ^{12}C disfavoured

$$G \sim m^{1/2} \text{ and } G \sim \text{charge of fragment}$$

<u>Deficiences</u>

- Assumed existence of one a particle in nucleus and have taken no account of probability of formation.

- Assumed "semi-classical" approach to estimate escape trial frequency, $f \sim v/2R$, and make absolute prediction of decay rate.

- If α emitted with some angular momentum, ℓ, radial wave equation must include a centrifugal barrier term

$$V = \frac{\ell(\ell+1)\hbar^2}{2mr^2}$$

ℓ = relative a.m. of α and daughter nucleus
m = reduced mass

which raises the barrier and hence suppresses emission of α in high ℓ states.

SELECTION RULES IN α DECAY

- Nuclear Shell Model: α has $J^P = 0^+$

Angular momentum

$$X \rightarrow Y + \alpha$$

spin J_X J_Y $\underbrace{\qquad}_{\ell}$

$$\boxed{J_X = J_Y \oplus \ell}$$

ℓ ranges from $J_X + J_Y$ to $|J_X - J_Y|$

Parity

- Parity is conserved in a decay.
- Orbital wavefunction has parity $(-1)^{\ell}$

X, Y SAME parity $\Rightarrow \ell$ must be EVEN

X, Y OPPOSITE parity $\Rightarrow \ell$ must be ODD

Example: if X, Y both even-even nuclei in their ground states $\Rightarrow$ shell model $J^P = 0^+ \Rightarrow \ell = 0$

- More generally, if X has $J^P = 0^+$, the states of Y which can be formed in a decay are $J^P = 0^+, 1^-, 2^+, 3^-, 4^+, \ldots$

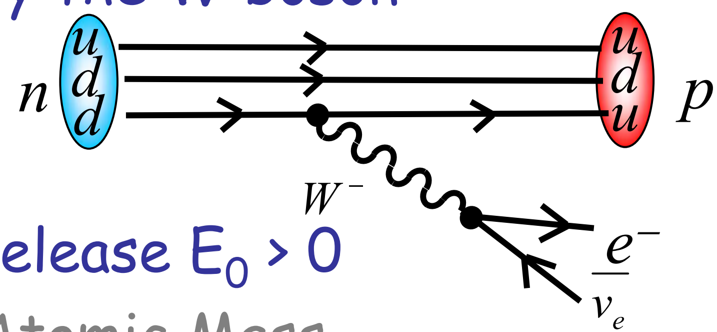
β Decay

β⁻ $n \rightarrow p + e^- + \bar{\nu}_e$ ${}^A_Z X \rightarrow {}^A_{Z+1}Y + e^- + \bar{\nu}_e$

β⁺ $p \rightarrow n + e^+ + \nu_e$ ${}^A_Z X \rightarrow {}^A_{Z-1}Y + e^+ + \nu_e$

electron capture $p + e^- \rightarrow n + \nu_e$ ${}^A_Z X \rightarrow {}^A_{Z-1}Y + \nu_e$

- β decay is a weak interaction mediated by the W boson
- Parity is violated in β decay



- Kinematics: Decay is possible if energy release $E_0 > 0$

	Nuclear Mass	Atomic Mass
β⁻	$E_0 = m_X - m_Y - m_e - m_\nu$	$E_0 = M_X - M_Y - m_\nu$
β⁺	$E_0 = m_X - m_Y - m_e - m_\nu$	$E_0 = M_X - M_Y - 2m_e - m_\nu$
e.c.	$E_0 = m_X - m_Y + m_e - m_\nu$	$E_0 = M_X - M_Y - m_\nu$

$M(A,Z) = m(A,Z) + Zm_e$

- Only $n \rightarrow p e \nu$ can occur outside the nucleus ($m_n > m_p$)
- Electron capture is possible if β⁺ not allowed

Nuclear Stability Against β Decay

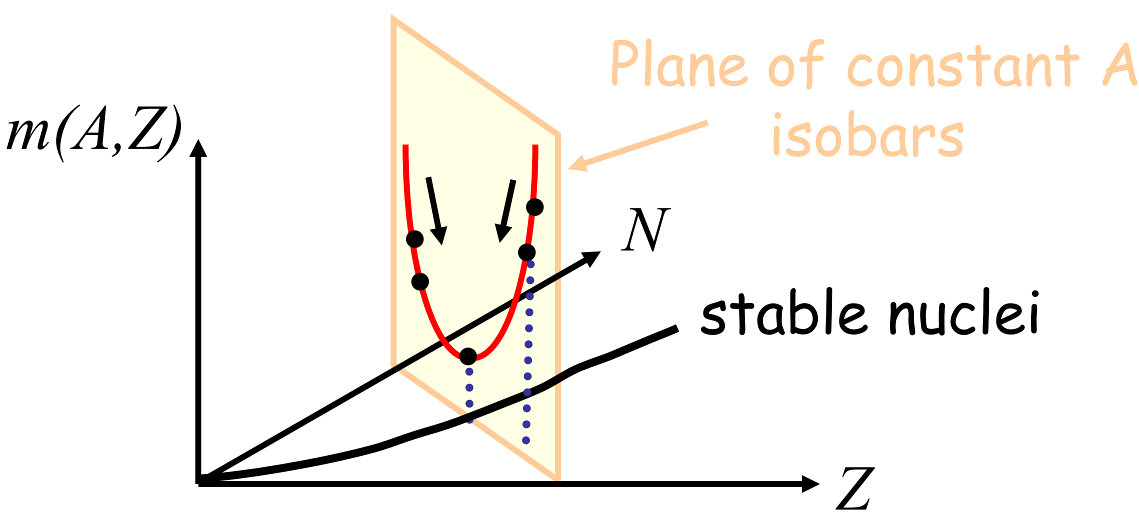
- Plot nuclear mass on an axis ⊥'r to $N - Z$ plane.

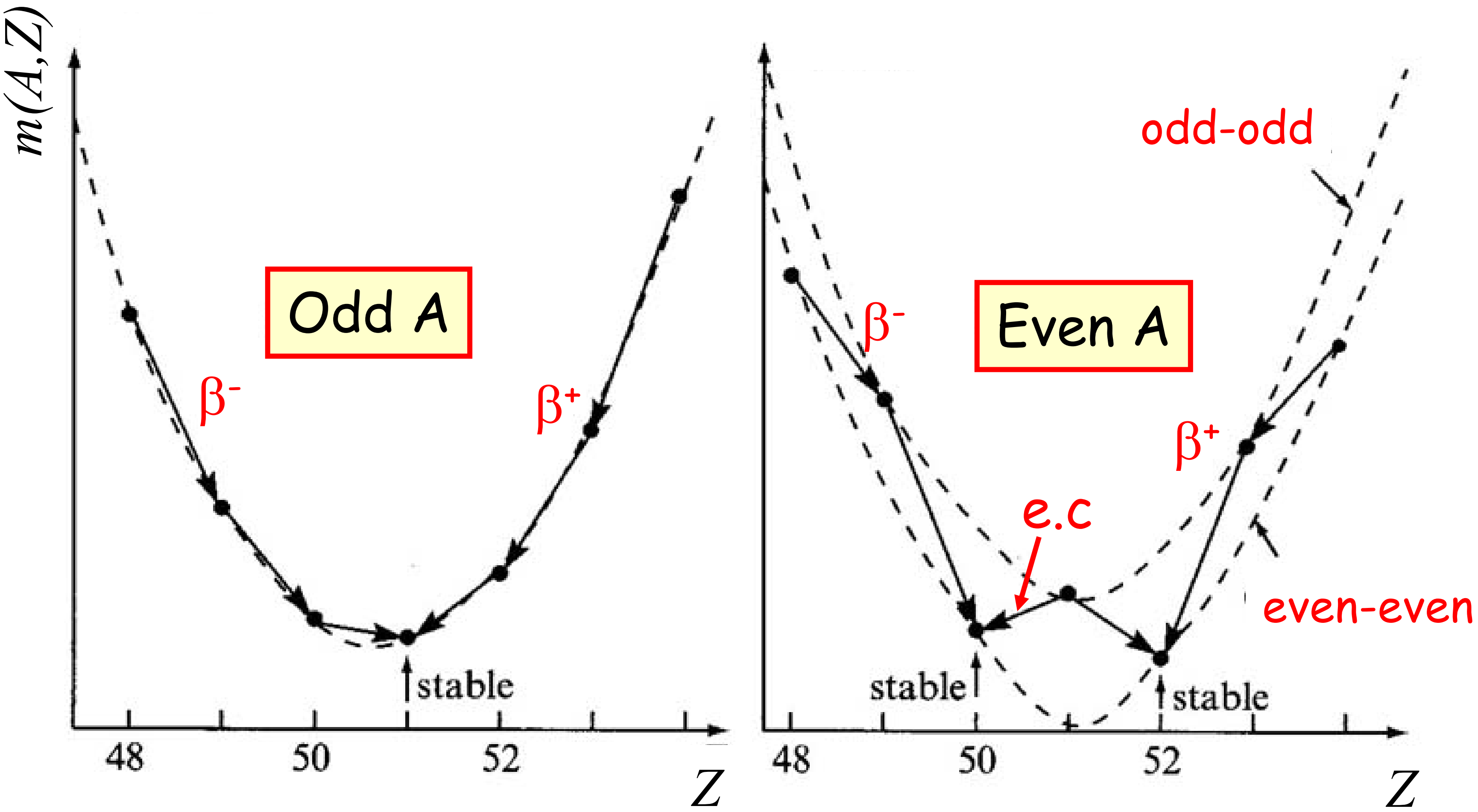
$$m(A,Z) = Z\,m_p + (A-Z)\,m_n - a_V A + a_S A^{2/3} + \frac{a_C Z^2}{A^{1/3}} + a_A \frac{(N-Z)^2}{A} - \delta(A)$$

SEMF (page 288)

- β decay, A is constant, Z changes by ±1 ⇒ m(A,Z) is quadratic in Z.

- Most stable nuclei when $$\frac{\partial m(A,Z)}{\partial Z} = 0$$




m(A,Z)
Odd A
β-
β+
stable
48
50
52
Z
Even A
odd-odd
β-
β+
e.c
even-even
stable
stable
48
50
52
Z

β DECAY RATE (see page 172-178)

- β decay in Fermi Theory

$$\Gamma = 2\pi \left|M_{if}\right|^2 \rho(E_f)$$

where M_{if} is the matrix element

4-point interaction

$$\left|M_{if}\right|^2 = \left|G_F \int \psi_p^* \, e^{-i(\vec{p}_e + \vec{p}_\nu)\cdot\vec{r}} \psi_n d^3\vec{r}\right|^2 = G_F^2 \left|M_{nuclear}\right|^2$$

and $\rho(E_f)$ is the density of final states

$$\rho\left(E_f\right) = \frac{E_\nu^2}{(2\pi)^3} \, d\Omega_\nu \; \frac{E_e^2}{(2\pi)^3} \, d\Omega_e \, dE_e = \frac{\left(E_0 - E_e\right)^2 E_e^2}{4\pi^4} dE_e$$

$$\Rightarrow \quad \Gamma = \frac{G_F^2 \left|M_{nuclear}\right|^2}{2\pi^3} \int_0^Q \left(E_0 - E_e\right)^2 E_e^2 \, dE_e$$

Relativistic
E ~ p

- Total decay rate given by $\Gamma \propto E_0^{\,5}$ SARGENT RULE
- β decay spectrum described by $\sqrt{\frac{d\Gamma}{dE_e} \frac{1}{E_e^2}} \propto \left(E_0 - E_e\right)$ KURIE PLOT

- The momentum of the electron is modified by the Coulomb interaction as it moves away from the nucleus (different for e^- and e^+)

$\Rightarrow$ Multiply spectrum by FERMI FUNCTION, $F(Z,E)$

$$\Rightarrow \quad \Gamma = \frac{G_F^2 \left| M_{nuclear} \right|^2}{2\pi^3} \int_0^{E_0} \left(E_0 - E_e \right)^2 E_e^2 \; F(Z, E_e) \, dE_e$$

- All the information about the nuclear wavefunctions is contained in the matrix element. Values for the complicated FERMI INTEGRAL

$$f(Z, E_e) \equiv \frac{1}{m_e^5} \int_0^{E_0} \left(E_0 - E_e \right)^2 E_e^2 \; F(Z, E_e) \, dE_e$$

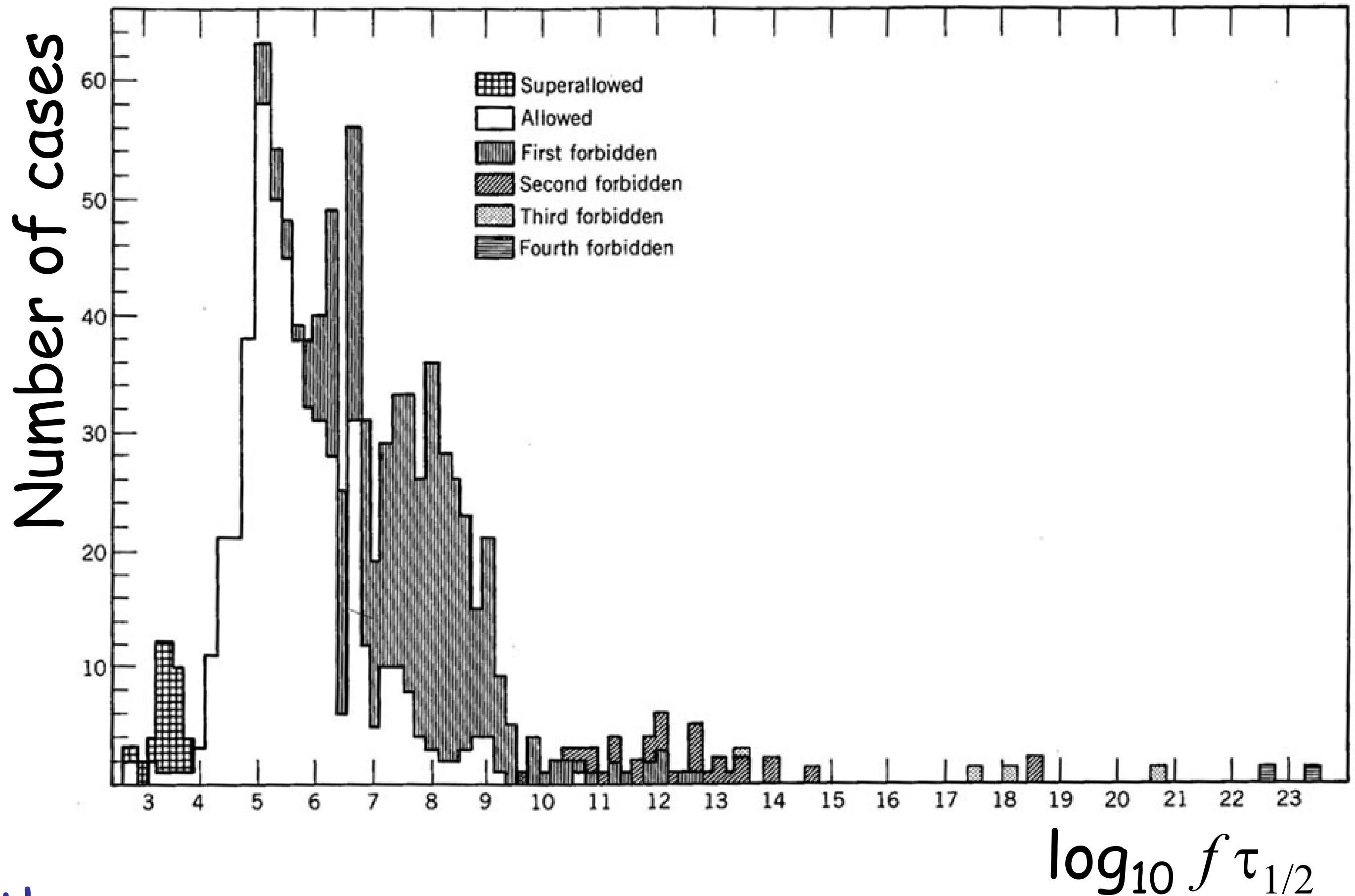
are tabulated.

- Mean lifetime $\tau = \frac{1}{\Gamma}$, half-life $\tau_{1/2} = \frac{Ln2}{\Gamma}$

$$\Rightarrow \quad \boxed{f\tau_{1/2} = Ln2 \, \frac{2\pi^3}{m_e^5 G_F^2 \left| M_{nuclear} \right|^2}}$$

COMPARATIVE HALF-LIFE

Comparative Half-lives



Decays with

$\log_{10} f\tau_{1/2}$ ~3-4 SUPERALLOWED

~4-7 ALLOWED

≥ 6 FORBIDDEN (i.e. suppressed, small M)

SELECTION RULES IN β DECAY

Fermi theory,

$$M_{if} = G_F \int \psi_p^* \underbrace{e^{-i(\vec{p}_e + \vec{p}_\nu)\cdot\vec{r}}}_{e,\ \nu} \psi_n d^3\vec{r}$$

- SUPERALLOWED TRANSITIONS

$$M_{if} \sim \int \psi_p^* \psi_n d^3\vec{r} \approx 1$$ $\log_{10} f\tau_{1/2} \sim 3\text{-}4$

- ALLOWED TRANSITIONS

Angular momentum of $e\nu$ pair relative to nucleus, $\ell = 0$.

$$e^{-i(\vec{p}_e + \vec{p}_\nu)\cdot\vec{r}} \sim 1$$ $\log_{10} f\tau_{1/2} \sim 4\text{-}7$

- There are TWO types of ALLOWED/SUPERALLOWED transitions depending on the relative spin states of the emitted e and ν.

e, ν both have spin ½ ⇒ Total spin of $e\nu$ system, $S_{e\nu}$ = 0 or 1

$$X \rightarrow Y + e + \nu \qquad J_X = J_Y \oplus S_{e\nu}$$

FERMI TRANSITIONS $S_{e\nu} = 0$

$$n\uparrow \rightarrow p\uparrow + \underbrace{e^-\uparrow + \bar{\nu}_e\downarrow}_{S_{e\nu}=0}$$

$J_X = J_Y$ $\quad$ $S_{e\nu} = 0$

$\Delta J = 0$

GAMOW-TELLER TRANSITIONS $S_{e\nu} = 1$

$$n\uparrow \rightarrow p\uparrow + \underbrace{e^-\uparrow + \bar{\nu}_e\uparrow}_{S_{e\nu}=\pm 1}$$

$J_X = J_Y$ $\quad$ $S_{e\nu} = \pm 1$

$\Delta J = \pm 1$

$$n\uparrow \rightarrow p\downarrow + \underbrace{e^-\uparrow + \bar{\nu}_e\uparrow}_{S_{e\nu}=\pm 1}$$

$J_X = J_Y \pm 1$ $\quad$ $S_{e\nu} = \pm 1$

$\Delta J = 0$
$0 \rightarrow 0$ Forbidden

- Total number of spin states of $e\nu$ = 4 (3 G-T, 1 Fermi)
- No change in angular momentum of the $e\nu$ pair relative to the nucleus, $\ell = 0 \Rightarrow$ PARITY UNCHANGED

➢ FORBIDDEN TRANSITIONS $\log_{10} f\tau_{1/2} \geq 6$

Angular momentum of the $e\nu$ pair relative to the nucleus ℓ > 0.

$$e^{-i(\vec{p}_e+\vec{p}_\nu)\cdot\vec{r}} = 1 - i(\vec{p}_e + \vec{p}_\nu)\cdot\vec{r} + [(\vec{p}_e + \vec{p}_\nu)\cdot\vec{r}]^2 - \ldots$$

ℓ	0	1	2	...
P=$(-1)^\ell$	even	odd	even	...
	Allowed	1st forbidden	2nd forbidden	

Transition probabilities for large ℓ are small $\Rightarrow$ forbidden transitions.

Forbidden transitions are only competitive if an allowed transition cannot occur (selection rules). The lowest permitted order of "forbiddeness" will dominate.

In general, n^{th} forbidden $\Rightarrow$ $e\nu$ system carries orbital angular momentum ℓ = n, and $S_{e\nu}$ 0 (Fermi) or 1 (G-T).

Examples:

$$^{34}Cl(0^+) \rightarrow {}^{34}S(0^+)$$

$$^{14}C(0^+) \rightarrow {}^{14}N(1^+)$$

$$n(1/2^+) \rightarrow p(1/2^+)$$

$$^{39}Ar(7/2^-) \rightarrow {}^{39}K(3/2^+)$$

$$^{87}Rb(3/2^-) \rightarrow {}^{87}Sr(9/2^+)$$

γ Decay

Emission of γ-rays (electromagnetic radiation) occur when a nucleus is formed in an excited state (e.g. after α, β decay or collision).

The treatment of radiative transitions in nuclei is generally the same as for atoms, except

Atom $E_\gamma \sim$ eV $\lambda \sim 10^8$ fm $\Gamma \sim 10^9\ s^{-1}$

Only dipole transitions are important.

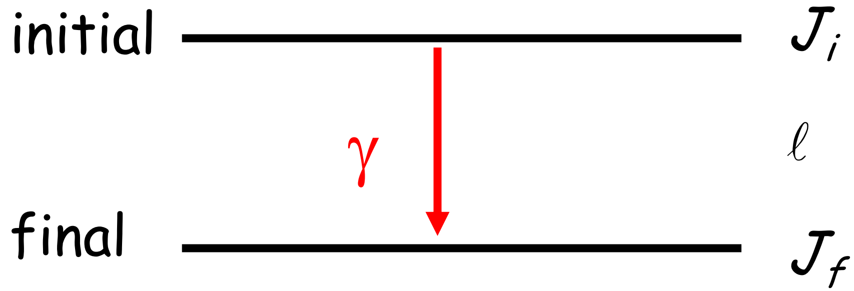
Nuclei $E_\gamma \sim$ MeV $\lambda \sim 10^2$ fm $\Gamma \sim 10^{16}\ s^{-1}$

Higher order transitions also important. Collective motion of many p's lead to higher transition rates.

Two types of transitions:

ELECTRIC (E) TRANSITIONS arise from an oscillating charge which causes an oscillation in the external electric field.

MAGNETIC (M) TRANSITIONS arise from a varying current or magnetic moment which sets up a varying magnetic field.



- In the simplest case, the photon carries away angular momentum ℓ when a proton in the nucleus makes a transition from its initial a.m. state J_i to its final a.m. state J_f.

$$\vec{J}_i = \vec{\ell} \oplus \vec{J}_f \quad \text{and} \quad \left|J_i - J_f\right| \leq \ell \leq \left|J_i + J_f\right|$$

- The photon has intrinsic $J^P = 1^- \Rightarrow \ell \geq 1$

$\Rightarrow$ Single γ emission is FORBIDDEN for a transition between two $J=0$ states. 0→0 transitions can only occur via internal conversion or emission of more than 1 γ.

- The transition probabilities obtained using

$$\Gamma = 2\pi \left|M_{if}\right|^2 \rho(E_f) \qquad \text{FERMI GOLDEN RULE}$$

ELECTRIC DIPOLE TRANSITIONS (E1) ℓ=1

- Insert dipole matrix element into FGR

$$\Gamma_{i\to f} = \frac{\omega^3}{3\pi\varepsilon_0 c^3 \hbar}\left|\langle\psi_f|e\vec{r}|\psi_i\rangle\right|^2$$

see Adv. Quantum
after averaging over initial and summing over final states

- For an order of magnitude estimate of this rate,

$$\left|\langle\psi_f|e\vec{r}|\psi_i\rangle\right|^2 \approx |eR|^2 \quad \Rightarrow \quad \Gamma = \frac{4}{3}\alpha E_\gamma^3 R^2$$

R = radius of nucleus

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} \qquad E_\gamma = \hbar\omega$$

$$\hbar = c = 1$$

$\hbar$c = 197 MeVfm
$\hbar$ = 6.6 x 10^{-22} MeVs

<u>Example</u>: E_γ = 1 MeV, R = 5 fm

$$\Gamma(E1) = 0.24\ MeV^3 fm^2 = \frac{0.24}{(197)^2 6.6\times10^{-22}}\ s^{-1}$$

$$= 10^{16}\ s^{-1}$$

(c.f. atoms $\Gamma \sim 10^9\ s^{-1}$)

- As nuclear wavefunctions have definite parity, the matrix element can only be non-zero if the initial and final states have opposite parity

$$e\vec{r} \xrightarrow{\hat{P}} -e\vec{r}$$

$\Rightarrow$ E1 transition $\Rightarrow$ parity change of nucleus

MAGNETIC DIPOLE TRANSITION (M1) ℓ=1

Matrix element

$$\left|\langle \psi_f | \mu\sigma | \psi_i \rangle\right|^2$$

μ = magnetic moment, σ = Pauli spin matrix

Typically, $\langle \mu\sigma \rangle \sim \dfrac{e\hbar}{2m_p} = \mu_N$ Nuclear Magneton

Proton $\lambda = \dfrac{\hbar}{m_p} \sim 0.2\ fm \quad \sim \dfrac{R}{25} \quad for\ \ R = 5\ fm$

$$\therefore \frac{\Gamma(M1)}{\Gamma(E1)} = \left(\frac{e\hbar}{2m_p}\right)^2 \frac{1}{e^2 R^2} = 10^{-3}$$

Magnetic moment transforms as angular momentum

$$e\vec{r} \times \vec{v} \xrightarrow{\hat{P}} e(-\vec{r}) \times (-\vec{v}) = e\vec{r} \times \vec{v}$$ EVEN

$\Rightarrow$ M1 transition $\Rightarrow$ no parity change of nucleus

HIGHER ORDER TRANSITIONS (Eℓ, Mℓ where ℓ > 1)

- If the initial and final nuclear states differ by more than 1 unit of angular momentum ⇒ HIGHER MULTIPOLE RADIATION

- The perturbing Hamiltonian is a function of electric and magnetic fields and hence of the vector potential

$$\langle \psi_f | H'(\vec{A}) | \psi_i \rangle$$

- $\vec{A}$ for a photon is taken to have the form of a plane wave

$$Ae^{-i\vec{p}\cdot\vec{r}} = 1 \quad - i\vec{p}\cdot\vec{r} \quad + \frac{1}{2}\left(\vec{p}\cdot\vec{r}\right)^2 \quad + \ldots \frac{\left(-i\vec{p}\cdot\vec{r}\right)^n}{n!}$$

	Dipole	Quadrupole	Octopole...
ℓ	1	2	3
	E1, M1	E2, M2	E3, M3

- Each successive term in $\vec{A}$ is reduced from the previous one approx by a factor pR.

Example: p ~ 1 MeV, R~5 fm ⇒ pR~5 MeVfm ~ 0.025 $|pR|^2$ ~ 10^{-3}

$$\therefore \frac{\Gamma(E2)}{\Gamma(E1)} \sim 10^{-3} \sim \frac{\Gamma(M1)}{\Gamma(E1)}$$

➢ The matrix element for E2 transitions ~ r^2 i.e. even under a parity transformation

In general, $E\ell$ transitions Parity = $(-1)^{\ell}$

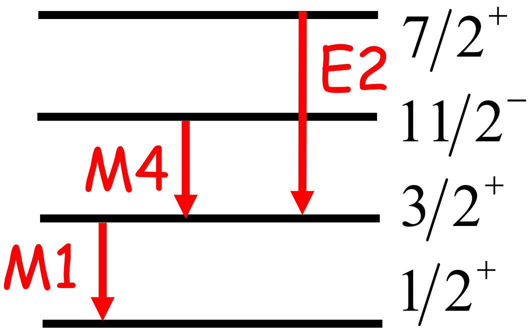
$M\ell$ transitions Parity = $(-1)^{\ell+1}$

Rate	1	10^{-3}	10^{-6}	10^{-9}...
	E1	E2	E3	E4 ...
		M1	M2	M3 ...
Parity change	✓	✗	✓	✗

➢ In general, a decay will proceed dominantly by the highest order process permitted by angular momentum and parity.

<u>Example</u>: if a process has $\Delta J = 2$, no parity change, it will go by E2, even though M3, E4 are also allowed.

Example: $^{117}_{50}Sn$



$3/2^+ \rightarrow 1/2^+$ M1 (E2 also allowed)

$11/2^- \rightarrow 3/2^+$ M4

more likely than $11/2^- \rightarrow 1/2^+$ (E5)

$7/2^+ \rightarrow 3/2^+$ E2

M2, M3 } less likely: $7/2^+ \rightarrow 11/2^-$, $7/2^+ \rightarrow 1/2^+$

- Information about nature of transitions is useful in inferring J^P values of states.

This discussion of rates is very naïve. More complete formulae can be found in books.

Also collective effects may be important

- many nucleons participate in transitions.
- If nucleus has a large Q → rotational excited states enhance E2 transitions

Mössbauer Effect

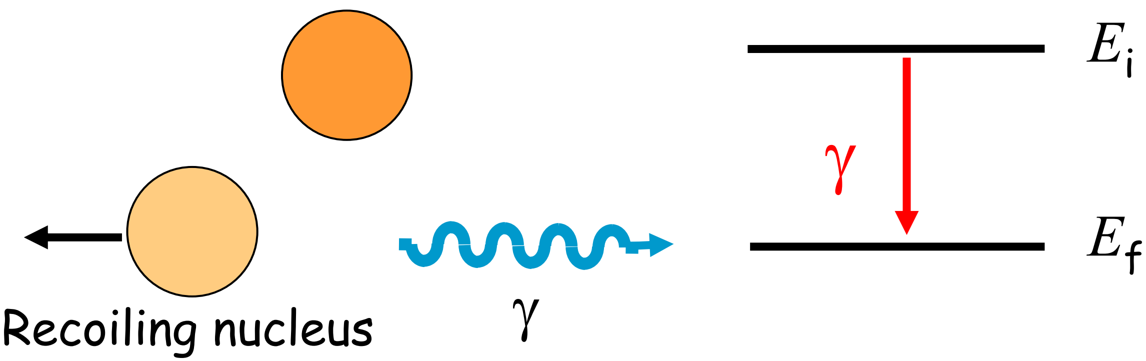
Measurement of small energy differences with extremely high precision using the natural width of nuclear states.

Basic idea:

Nucleus in excited state emits γ to g.s:

initial state

final state



$\Delta E = E_i - E_f$

$$E_i = E_f + E_\gamma + \frac{p^2}{2m}$$

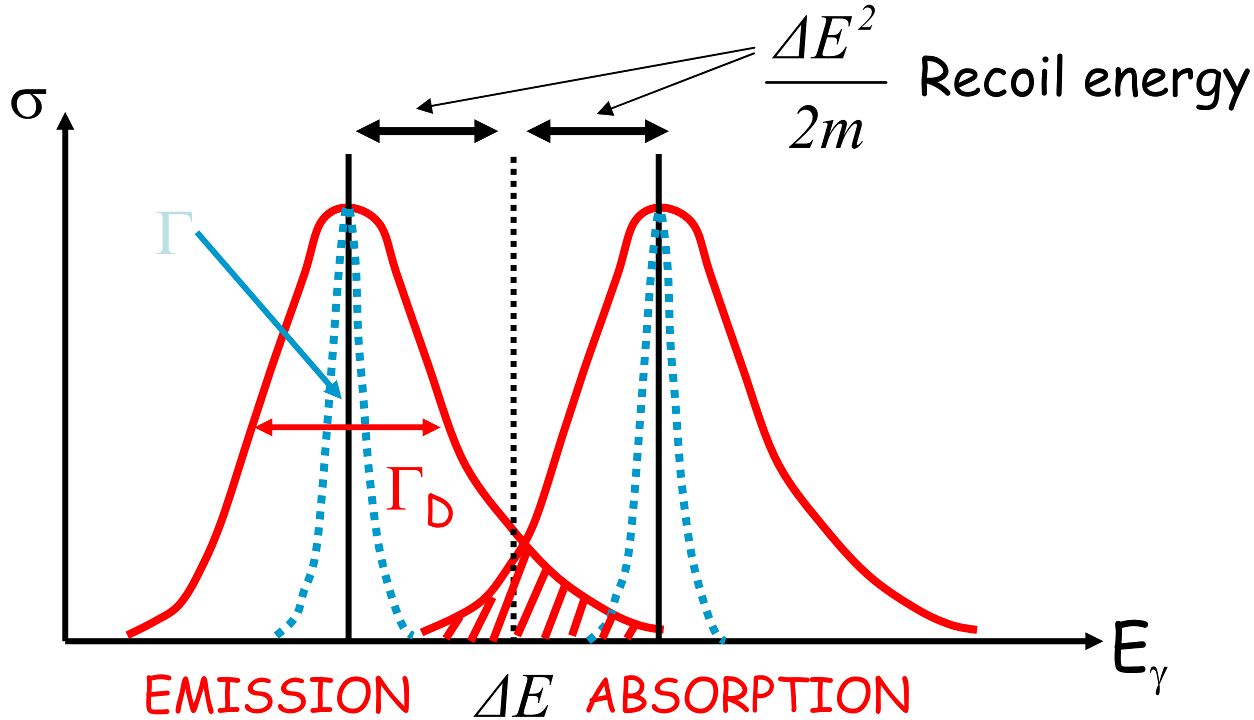
p = nucleus recoil momentum = $p_\gamma = E_\gamma$

c = 1

$$E_\gamma = \Delta E - \frac{E_\gamma^2}{2m} \approx \Delta E - \frac{\Delta E^2}{2m}$$ Emission

$E_\gamma \approx \Delta E$

Similarly, for γ absorption $$E_\gamma \approx \Delta E + \frac{\Delta E^2}{2m}$$



- E_γ varies due to natural width of energy levels

  e.g. ${}^{191}_{77}Ir$ τ ~ 1.4 x 10^{-10} s, E_i = 0.13 MeV

$$\Gamma = \frac{1}{\tau} = 5 \times 10^{-6}\ eV \qquad \frac{\Gamma}{E_i} \sim 10^{-11}$$

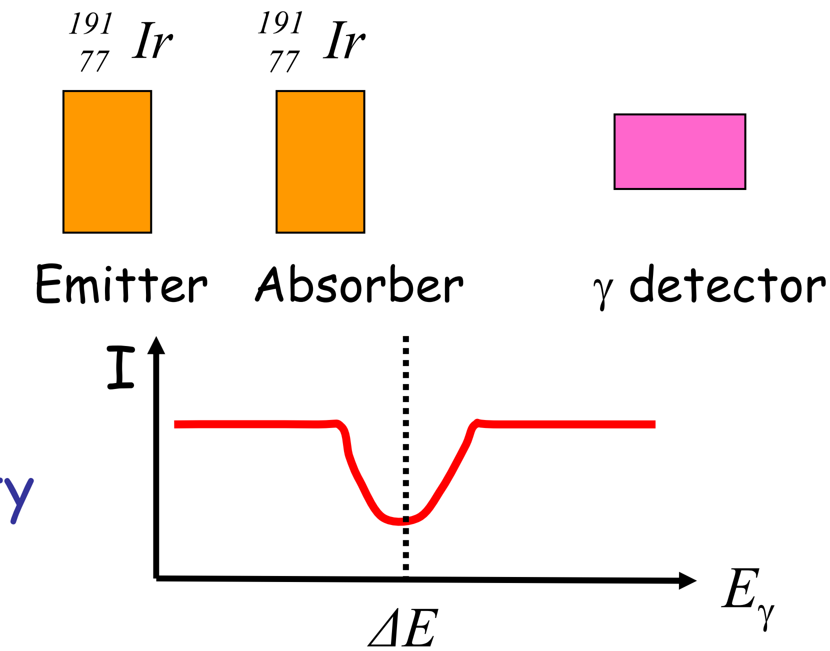
- Initial state is in thermal motion ⇒ Doppler shift

$$\Gamma_D \sim few\ 10^{-1}\ eV >> \Gamma$$

room temperature

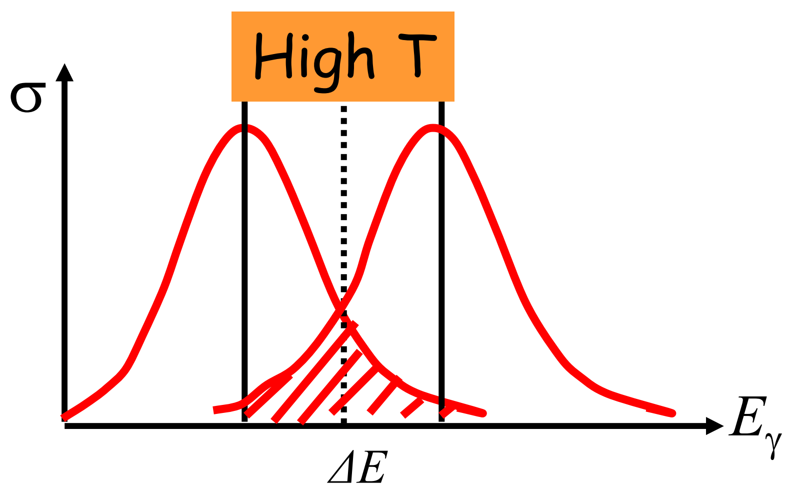
Mössbauer Experiment (1958)

- Absorption of γ's can only occur for energies in the overlap region.

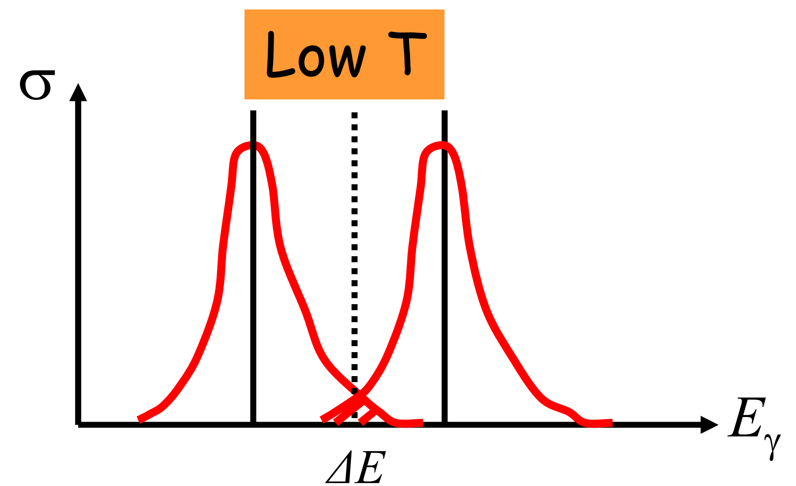


- If absorption occurs, re-emission is isotropic ∴ expect reduced intensity

- Expect



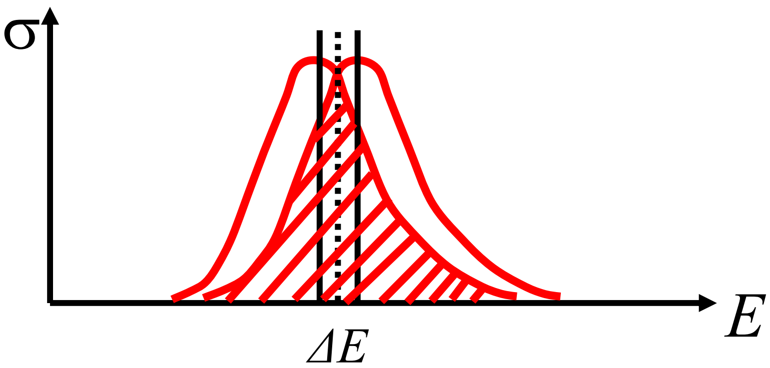
High T → Γ_D increases
Larger overlap
More absorption



Low T → Γ_D decreases
Smaller overlap
Less absorption

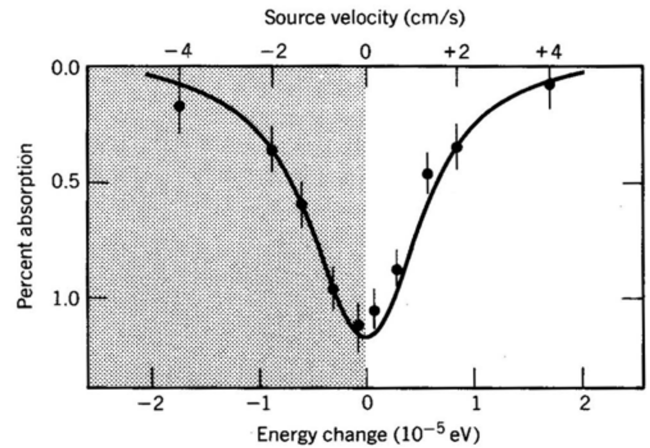
- Mössbauer found that at LOW TEMP the absorption INCREASED.

- Γ_D reduced BUT nucleus bound in crystal lattice



$\therefore M_{nucleus} \rightarrow M_{crystal}$

negligible recoil



γ

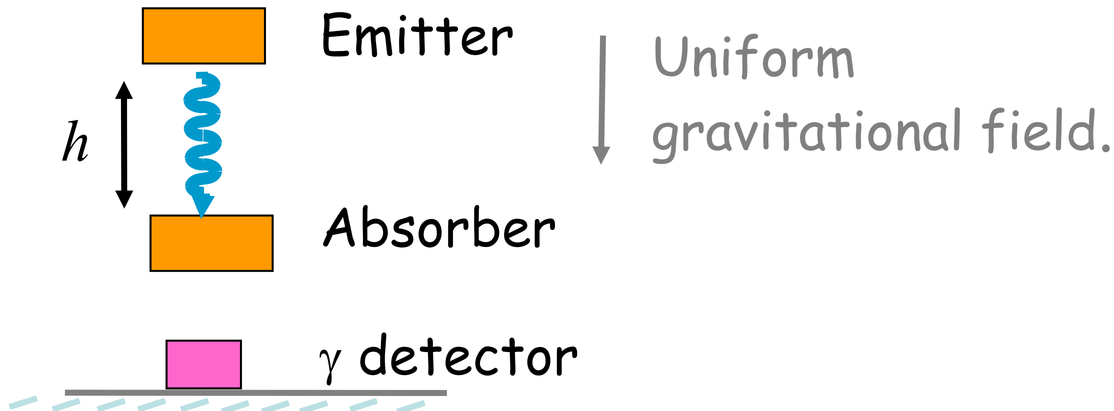
MÖSSBAUER EFFECT

- The Mössbauer effect can be used to measure energy differences ~ same order as width of the resonance (e.g. 10^{-5} eV Ir, 10^{-8} eV Fe).

- Most applications determine the properties of the physical or chemical environment of a nucleus. When the emitter and absorber are in different environments, the emission and absorption peaks do not occur at precisely the same energy. The relative velocity required to obtain maximum absorption is measured.

GRAVITATIONAL RED SHIFT

Principle of Equivalence: Effects of a local uniform gravitational field cannot be distinguished from those of a uniformly accelerated reference frame.



➤ Nucleus in emitter has additional $P.E. = mgh = Egh$ c = 1

$$E_i(1+gh) - E_f(1+gh) = \Delta E(1+gh)$$

➤ Radiated photons are Doppler red-shifted

$$E'_\gamma = E_\gamma(1+gh) \quad \Rightarrow \quad \Delta E_\gamma / E_\gamma = gh \sim 10^{-16} \ m^{-1} \qquad \Delta E_\gamma = E'_\gamma - E_\gamma$$

Example: Harvard-Tower: Pound and Rebka: ^{57}Fe sensitivity Γ/E_γ~3x10^{-13}

$$\Delta E_\gamma / E_\gamma = (4.902 \pm 0.041) \times 10^{-15} \qquad (4.905 \times 10^{-15} \text{ expected})$$

$\Rightarrow$ One of the most precise tests of GR