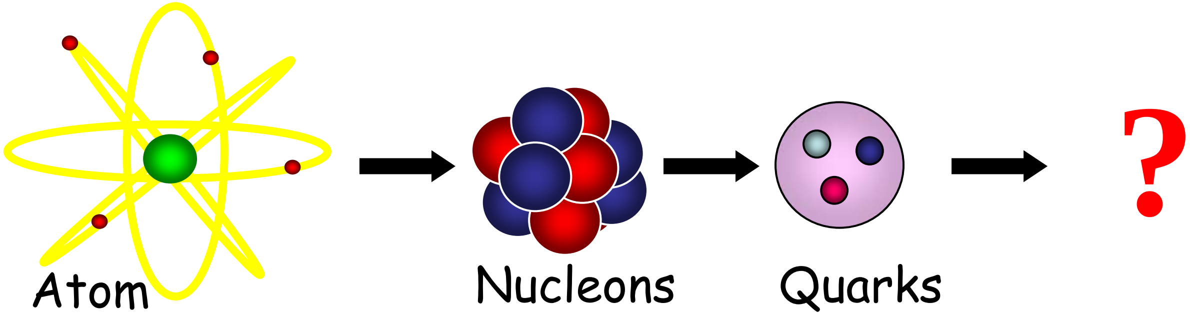
# Section XI
# Nuclear Physics

# Why Study Nuclear Physics ?

Nuclear processes play a fundamental role in the physical world:

- Origin of the Universe
- Creation of chemical elements
- Energy of stars
- Constituents of matter




Nuclear processes also have many practical applications:

- Uses of radioactivity in research, health and industry (e.g. NMR, radioactive dating)
- Nuclear power
- Various tools for the study of materials (e.g. Mössbauer, NMR)

# Nuclear Force

- ALL particle interactions can be explained in terms of 3 basic forces:

  ELECTROMAGNETIC, WEAK, and STRONG.

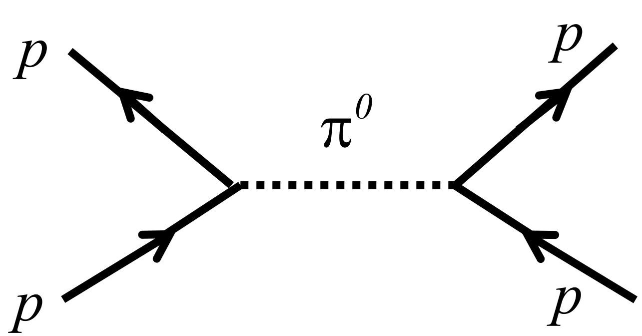
- Nucleons experience the strong interaction at short distances (a few fm).

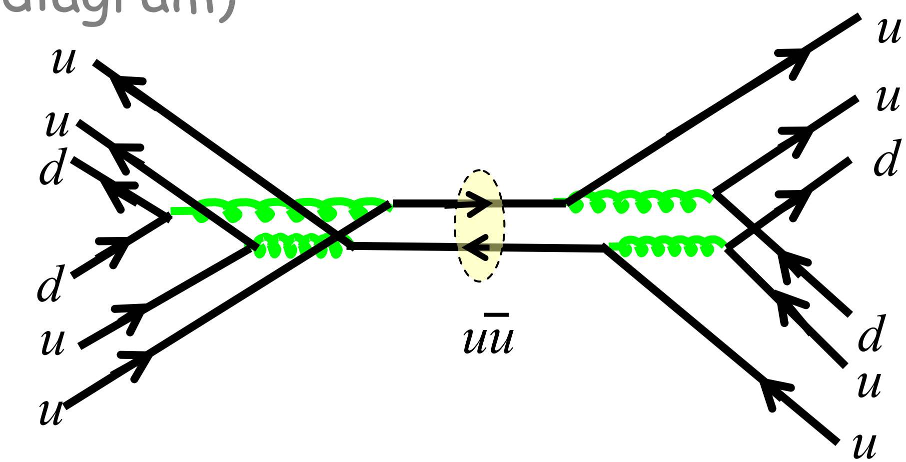
| Force | Boson | | Spin | Strength | Mass (GeV) | Range (m) |
|---|---|---|---|---|---|---|
| Strong | Gluon | $g$ | 1 | 1 | Massless | Massless |
| Strong (Nuclear) | e.g. pion | $\pi^0, \pi^\pm$ | | | 0.14 | $10^{-15}$ |
| Electromagnetic | Photon | $\gamma$ | 1 | $10^{-2}$ | Massless | Massless |
| Weak | W and Z | $W^\pm, Z^0$ | 1 | $10^{-7}$ | 80, 91 | 80, 91 |

The force between nucleons, the STRONG NUCLEAR FORCE, is a many-body problem in which

- quarks do not behave as if they were completely independent inside the nuclear volume
- nor do they behave as if they were completely bound to form protons and neutrons.

e.g. $p$-$p$ interaction (one possible diagram)



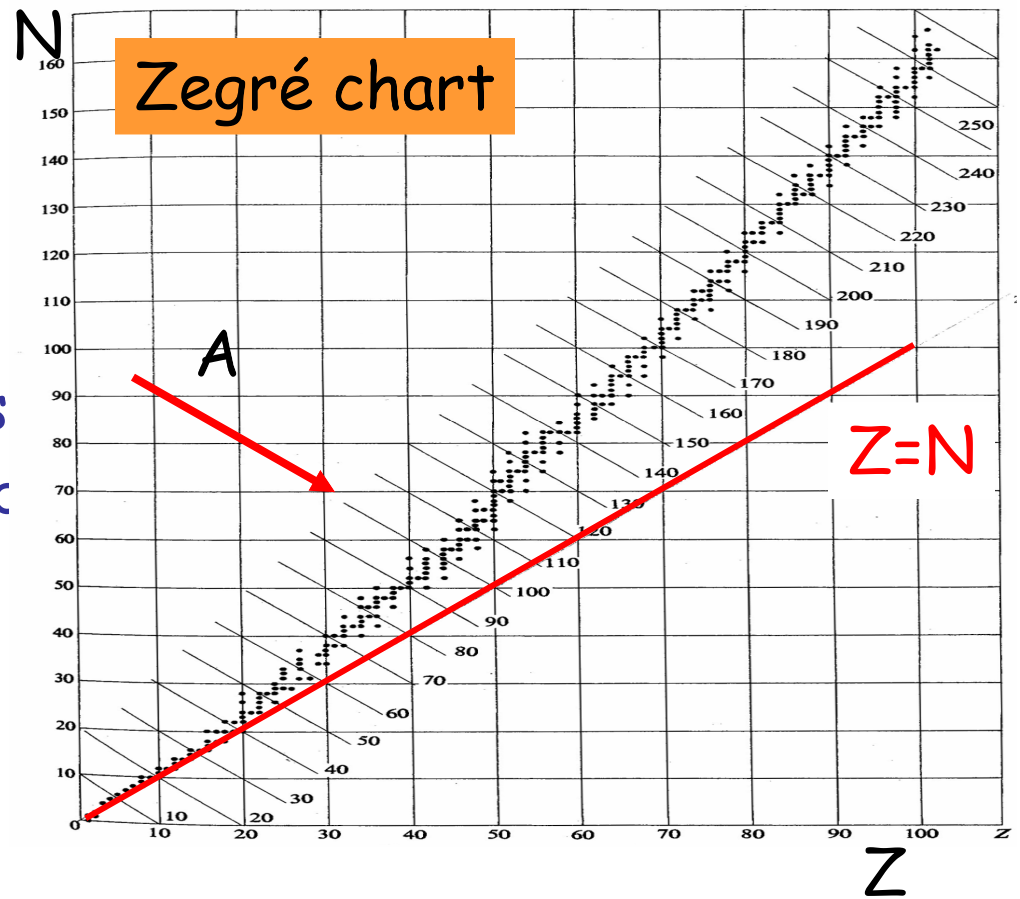

The nuclear force is therefore not calculable in detail at the quark level and can ONLY be deduced empirically from nuclear data.

# Section XII
# Basic Nuclear Properties

# Stable Nuclei

STABLE NUCLEI are those nuclei which do not decay by the strong interaction, although they may transform by $\beta$ and $\alpha$ emission.

- Tend to have N = Z for light nuclei.
- More have even N or Z, $p$'s and $n$'s tend to form pairs (8/284 have both N and Z odd
- Certain values of Z and N large number of isotopes and isotones



# Binding Energy

<u>BINDING ENERGY</u> is the energy required to split a nucleus into its constituents.

$$\boxed{\text{Mass of nucleus} = Z\,m_p + N\,m_n - B}$$

<u>SEPARATION ENERGY</u> of a nucleon is the energy required to remove one nucleon from a nucleus

e.g.

$$n: \quad B({}^{A}_{Z}X) - B({}^{A\text{-}1}_{Z}X) = m({}^{A\text{-}1}_{Z}X) + m_n - m({}^{A}_{Z}X)$$

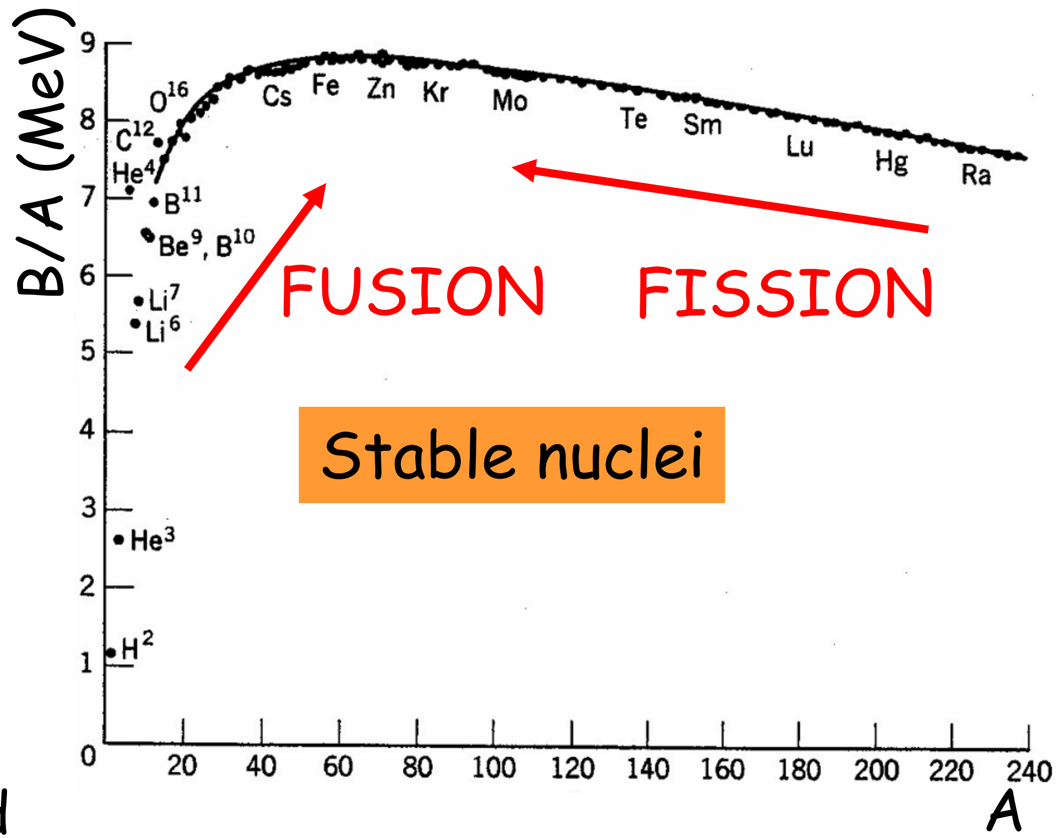
$$p: \quad B({}^{A}_{Z}X) - B({}^{A\text{-}1}_{Z\text{-}1}X') = m({}^{A\text{-}1}_{Z\text{-}1}X') + m({}^{1}H) - m({}^{A}_{Z}X)$$

Binding energy is very important: gives information on

- forces between nucleons
- stability of nucleus
- energy released or required in nuclear decays or reactions.

# Binding Energy /Nucleon

- B/A ≈ constant ≈ 8 MeV per nucleon, A≥20
  (c.f. B of electron per nucleon ≤ 3 keV)




- Broad maxima at A ≈ 60
  (Fe, Co, Ni)

  A ≤ 60 Fusion favoured

  A ≥ 60 Fission favoured

- Light nuclei with $A = 4n$, $n$=integer, show peaks ($\alpha$ stability)

B/A ≈ constant ⇒ in a nucleus, the nucleons are only attracted by nearby nucleons.

⇒ Nuclear force is SHORT RANGE and SATURATED.

# Nuclear Mass

Atomic mass: $M(A,Z) = Z\,(m_p + m_e) + (A - Z)\,m_n - B$

Nuclear mass: $m(A,Z) = Zm_p + (A - Z)m_n - B$

LIQUID DROP MODEL : Approximate the nucleus as a sphere with a uniform interior density, that drops to zero at the surface.

| Liquid drop | Nucleus |
|---|---|
| ♦ Intermolecular forces short range | ♦ Nuclear force short range |
| ♦ Density independent of drop size | ♦ Density independent of nuclear size |
| ♦ Heat required to evaporate fixed mass independent of drop size | ♦ $B/A \approx$ constant |

$$B = a_V A - a_S A^{2/3} - \frac{a_C Z^2}{A^{1/3}}$$

$\boxed{+a_V A}$

VOLUME TERM: Strong force between nucleons increases B and reduces mass by a constant amount per nucleon.

Nuclear volume ~ A

$\boxed{-a_S A^{2/3}}$

SURFACE TERM: Nucleons on surface are not as strongly bound ⇒ decrease B

Surface area ~ $R^2$ ~$A^{2/3}$

$\boxed{-a_C \frac{Z^2}{A^{1/3}}}$

COULOMB TERM: Protons repel each other ⇒ reduce B

Electrostatic P.E. ~ $\frac{Q^2}{R}$ ~ $\frac{Z^2}{A^{1/3}}$

Basic liquid drop model does not account for two other observations:

1. N $\approx$ Z
2. Nucleons tend to pair

Understand these using the FERMI GAS MODEL in which confined nucleons can only have certain discrete energies in accordance with the PAULI EXCLUSION PRINCIPLE.

$$\boxed{-a_A \frac{(N-Z)^2}{A}}$$

ASYMMETRY TERM: Nuclei tend to have N $\approx$ Z.

Kinetic energy of Z protons and N neutrons is minimized if N=Z. The greater the departure from N=Z, the smaller the binding energy. Correction scaled down by 1/A as levels are more closely spaced as A increases.

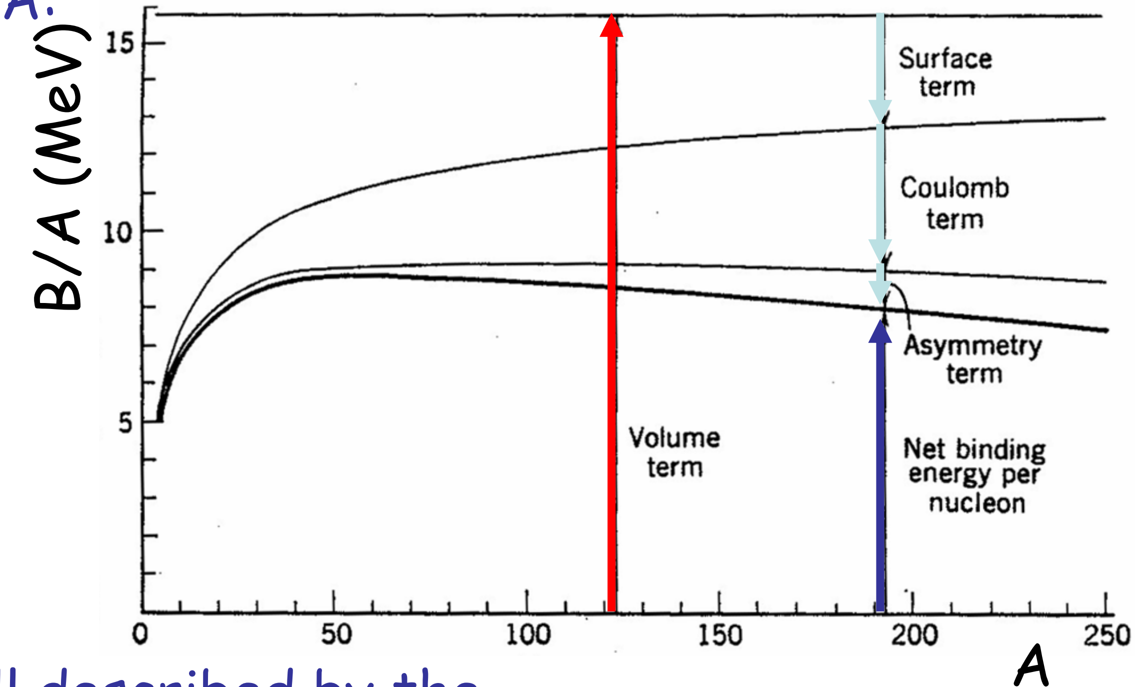
$$\boxed{+\delta(A)}$$

PAIRING TERM: Nuclei tend to have Z even, N even.

Pairing interaction energetically favours the formation of pairs of like nucleons (*pp*, *nn*) with spins ↑↓ and symmetric space wavefunction.

| | N/Z |
|---|---|
| $\delta(A) = +a_P A^{3/4}$ | even-even |
| $= -a_P A^{3/4}$ | odd-odd |
| $= 0$ | even-odd |

Contributions to B/A:




Nuclear mass is well described by the

SEMI-EMPIRICAL MASS FORMULA (Weizsäcker )

$$m(A,Z) = Z\, m_p + (A-Z)\, m_n - B$$

$$B = a_V A - a_S A^{2/3} - \frac{a_C Z^2}{A^{1/3}} - a_A \frac{(N-Z)^2}{A} + \delta(A)$$

with the following coefficients (in MeV)

$a_V = 15.8$, $a_S = 18.0$, $a_C = 0.72$, $a_A = 23.5$, $a_P = 33.5$

# Nuclear Spin and Parity

The nucleus is an isolated system and so has a well defined NUCLEAR SPIN.

Nuclear spin quantum number $= J$

$$|J| = \sqrt{J(J+1)} \qquad \hbar = 1$$

$$m_J = -J, -(J-1), \ldots, J-1, J.$$

Nuclear spin is the sum of the individual nucleons total angular momentum, $j$,

$$\vec{J} = \sum_i \vec{j}_i \qquad \vec{j} = \vec{\ell} + \vec{s}$$

($jj$ coupling)

where the total angular momentum of a nucleon is the sum of its intrinsic spin and orbital angular momentum

- intrinsic spin of $p$ or $n$, $s = 1/2$
- orbital angular momentum of nucleon is integer

A even → J = integer

A odd → J = 1/2 integer

All nuclei with even N and even Z have J=0

# PARITY:

- ALL particles are eigenstates of PARITY $\boxed{\hat{P}|\psi\rangle = P|\psi\rangle \qquad P = \pm 1}$

- The parity of a single PROTON or NEUTRON is

$$\boxed{P = (+1)(-1)^{\ell}}$$

Intrinsic P = +1 (3 quarks)

- The parity of a NUCLEUS is given by the product of the parities of all the neutrons and protons.

- For an ODD A nucleus, the parity is given by UNPAIRED $p$ or $n$.

- Label nuclear states with the NUCLEAR SPIN and PARITY quantum numbers.

Example: $0^+$ (J=0, parity even), $2^-$ (J=2,parity odd)

- Parity is CONSERVED in nuclear processes (strong interaction)

- Parity of nuclear states is extracted from experimental measurements e.g. $\gamma$ transitions

# Nuclear Size

The SIZE of a nucleus is determined using two sorts of interaction:

ELECTROMAGNETIC INTERACTION gives the CHARGE distribution of protons inside the nucleus. e.g

- Electron scattering
- Muonic atoms
- Mirror nuclei

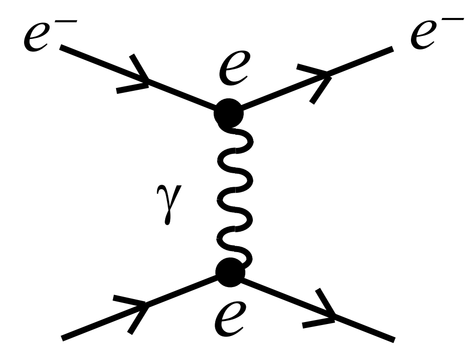
STRONG INTERACTION gives the MATTER distribution of protons and neutrons inside nucleus. N.B. nuclear and charge interactions at the same time $\Rightarrow$ more complex. e.g.

- $\alpha$ particle scattering (Rutherford)
- proton and neutron scattering
- Lifetime of a particle emitters (see later)
- $\pi$- mesic X-rays.

$\Rightarrow$ Find charge and matter radii EQUAL for all nuclei.

# ELECTRON SCATTERING

Use electron as a probe to study deviations from a point-like nucleus.




Nucleus, $Z$ protons

## Electromagnetic Interaction

Coulomb potential $$V(\vec{r}) = -\frac{Z\alpha}{r}$$

Born Approx. $$\frac{d\sigma}{d\Omega} = \frac{E^2}{(2\pi)^2}\left|\int e^{i\vec{q}\cdot\vec{r}}V(\vec{r})d^3\vec{r}\right|^2$$

$\vec{q} = \vec{p}_i - \vec{p}_f$ is the momentum transfer.

Rutherford Scattering $$\boxed{\frac{d\sigma}{d\Omega} = \frac{Z^2\alpha^2}{4E^2\sin^4 \vartheta/2}}$$ (see Scattering in QM page 49)

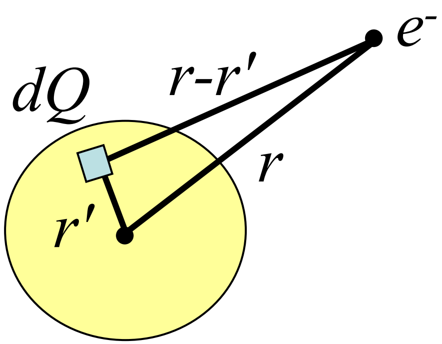
To measure a distance of $\approx$ 1 fm, need energy

$$\underline{E = \frac{1}{\lambda} = 1\,\mathrm{fm}^{-1} \approx 200\,\mathrm{MeV}}$$

$\hbar c = 197\ \mathrm{MeV\,fm}$

# SCATTERING FROM AN EXTENDED NUCLEUS

Let $V(\vec{r})$ depend on the distribution of charge in nucleus




Potential energy of electron due to charge $dQ$

$$dV = -\frac{e\,dQ}{4\pi|\vec{r} - \vec{r}\,'|} \qquad \varepsilon_0=1$$

where

$$dQ = Ze\,\rho(\vec{r}\,')\,d^3\vec{r}\,' \qquad \rho(\vec{r})\text{ charge distribution}$$

$$V(\vec{r}) = \int -\frac{e^2 Z\,\rho(\vec{r}\,')}{4\pi|\vec{r}-\vec{r}\,'|}\,d^3\vec{r}\,' = -Z\alpha\int\frac{\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\,d^3\vec{r}\,' \qquad \alpha = e^2/4\pi$$

$$M_{if} = \int e^{i\vec{q}\cdot\vec{r}}\,V(\vec{r})\,d^3\vec{r} = -Z\alpha\iint e^{i\vec{q}\cdot\vec{r}}\,\frac{\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\,d^3\vec{r}\,'\,d^3\vec{r}$$

$$= -Z\alpha\iint e^{i\vec{q}\cdot\vec{r}\,'}\,\rho(\vec{r}\,')\,\frac{e^{i\vec{q}\cdot(\vec{r}-\vec{r}\,')}}{|\vec{r}-\vec{r}\,'|}\,d^3\vec{r}\,'\,d^3\vec{r}$$

Let $\vec{R} = \vec{r} - \vec{r}\,'$ and set $\vec{r}\,'$ constant i.e. integrate over $\vec{r}$

$$M_{if} = -Z\alpha \underbrace{\int \frac{e^{i\vec{q}\cdot\vec{R}}}{\vec{R}}\, d^3\vec{R}}_{\text{Rutherford scattering}} \underbrace{\int \rho(\vec{r}\,')e^{i\vec{q}\cdot\vec{r}\,'}\, d^3\vec{r}\,'}_{F(q^2)}$$

Hence,

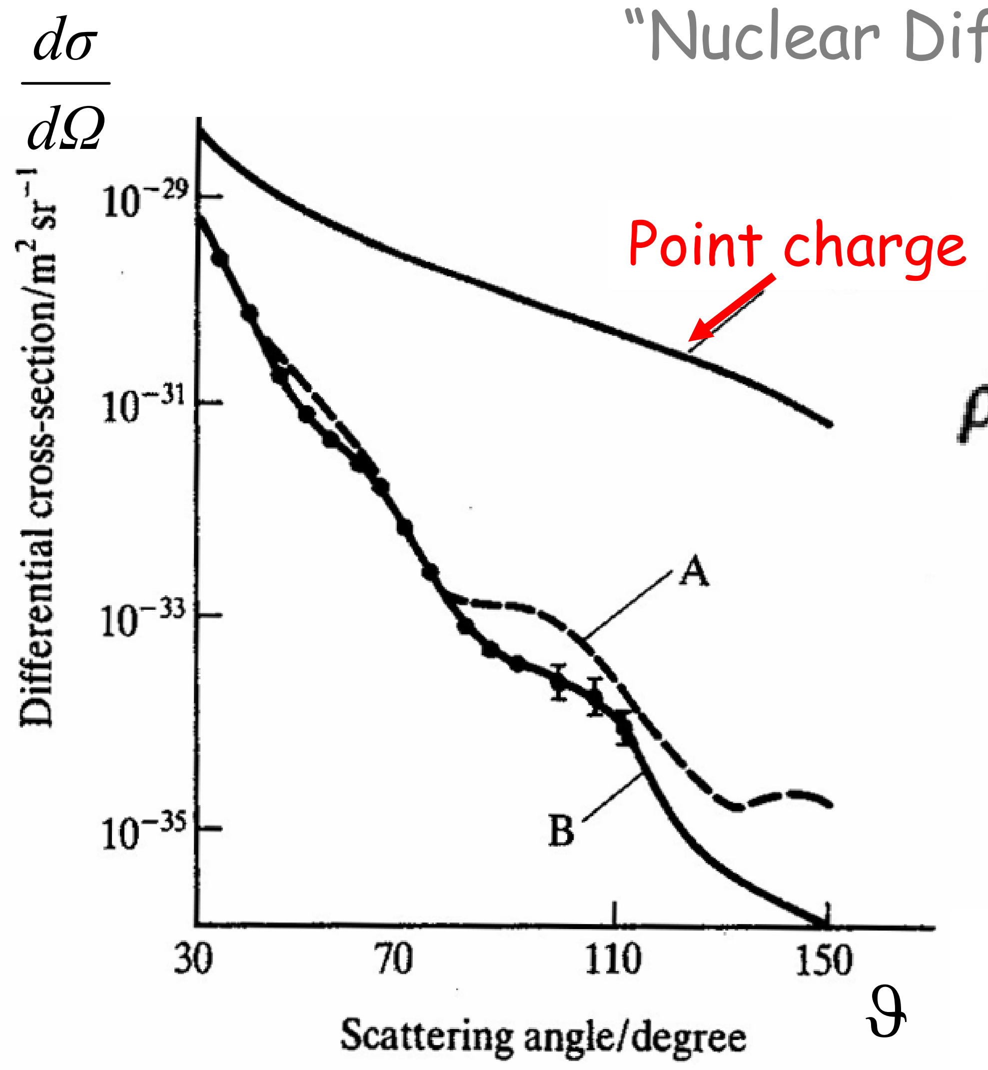
$$\boxed{\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{point} \left|F(q^2)\right|^2}$$

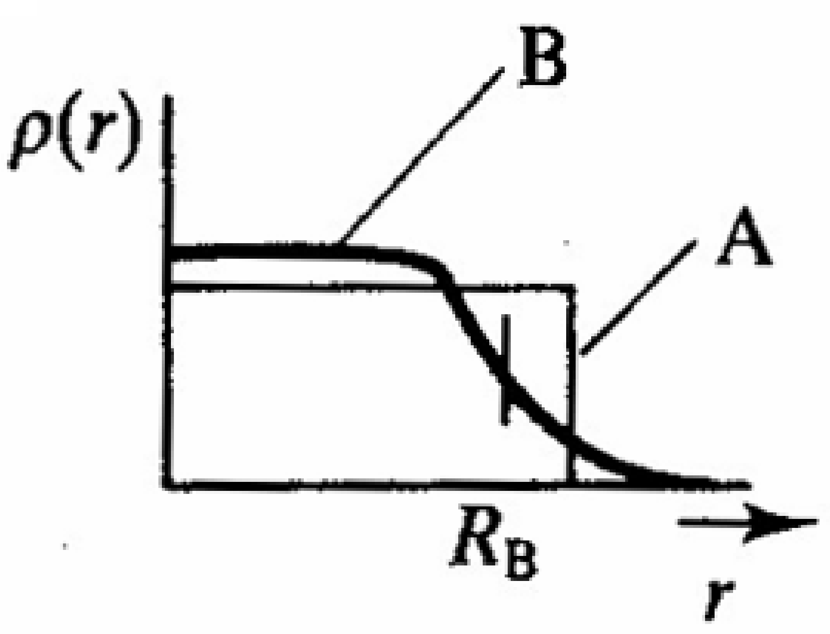
where $F(q^2) = \int \rho(\vec{r}\,')e^{i\vec{q}\cdot\vec{r}\,'} d^3\vec{r}\,'$ is called the FORM FACTOR and is the FOURIER TRANSFORM of the CHARGE DISTRIBUTION.

Spherical symmetry, $\rho = \rho(r)$

$$F(q^2) = \int_0^\infty \rho(r) \frac{\sin qr}{qr} 4\pi r^2\, dr \qquad \rho(r) = \frac{1}{2\pi^2} \int_0^\infty F(q^2) \frac{\sin qr}{qr} q^2\, dq$$

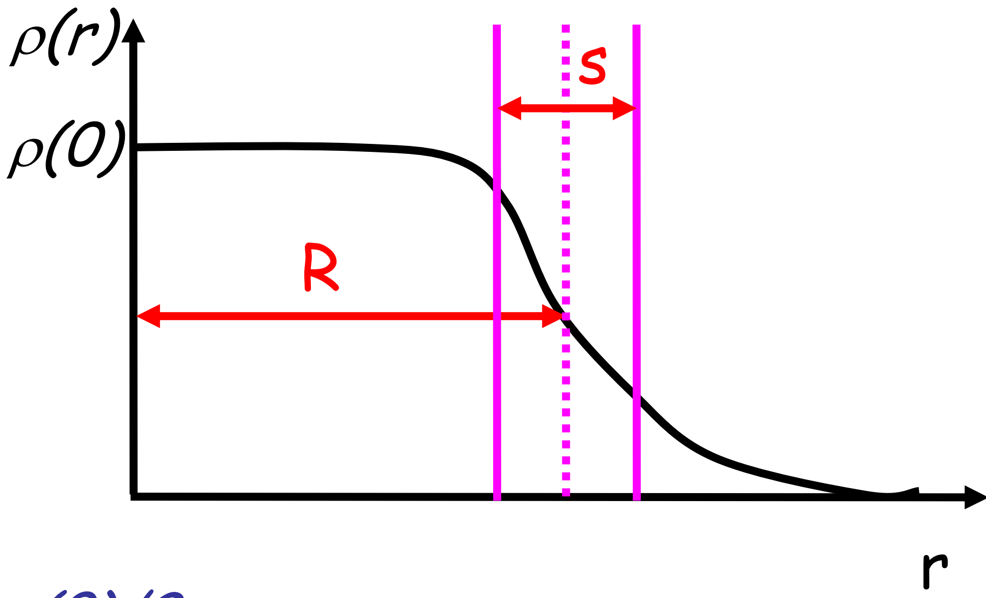
# "Nuclear Diffraction"



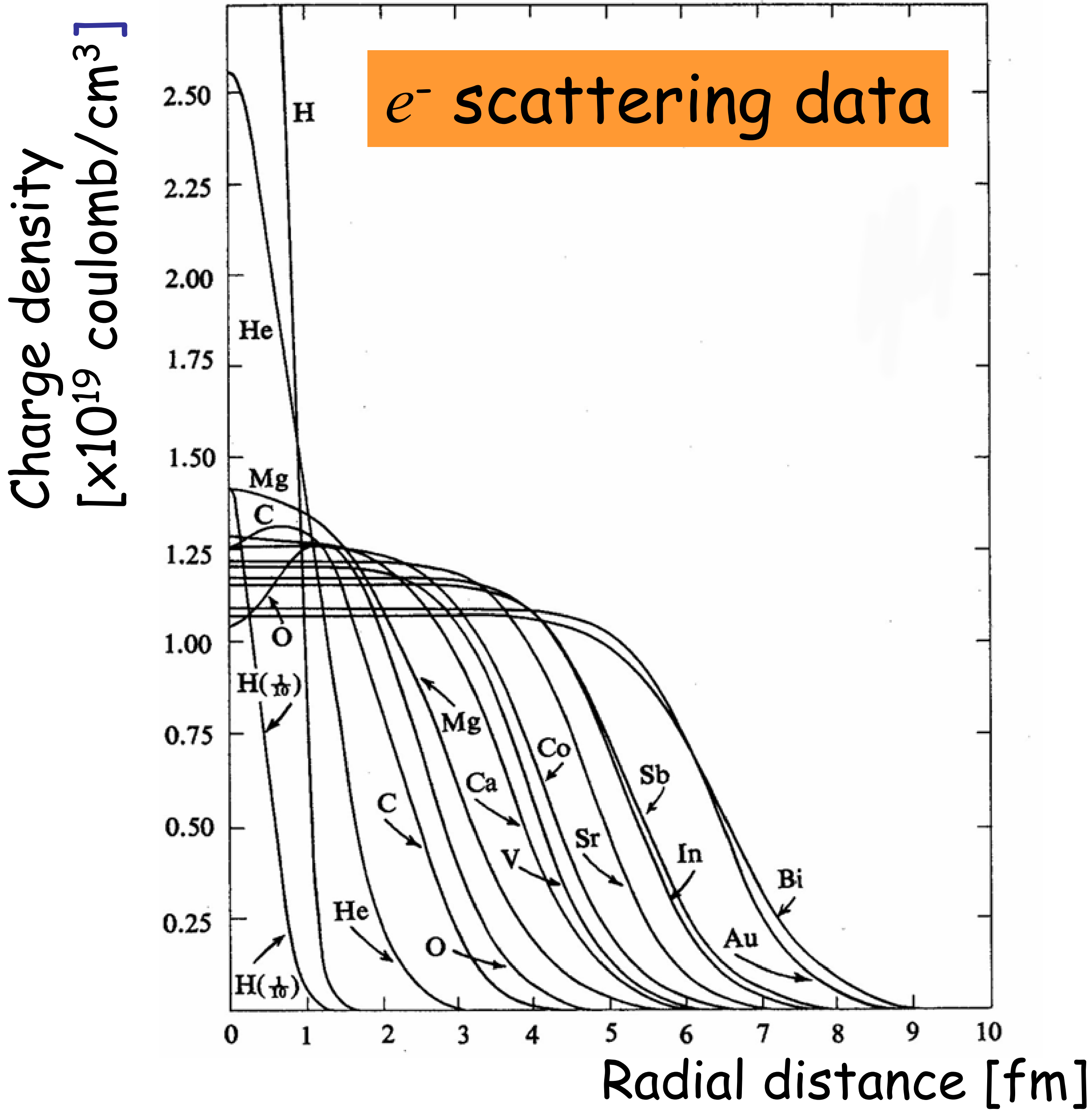



The charge distribution inside a nucleus is well described by the FERMI PARAMETERIZATION

$$\boxed{\rho(r) = \frac{\rho(0)}{\left[1 + e^{(r-R)/s}\right]}}$$




- R is the RADIUS where $\rho(r) = \rho(0)/2$
- R increases with A $\quad \boxed{R = r_0 A^{1/3}} \quad r_0 \approx 1.2$ fm
- s is the SURFACE WIDTH or SKIN THICKNESS where ρ(r) falls from 90%→10%
- s is the same for all nuclei $\quad s \approx 2.5$ fm



$e^-$ scattering data
Charge density [x$10^{19}$ coulomb/cm$^3$]
Radial distance [fm]
2.50
2.25
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0
1
2
3
4
5
6
7
8
9
10
H
He
Mg
C
O
H($\frac{1}{10}$)
Mg
Co
Ca
Sb
C
Sr
In
V
Bi
He
O
Au
H($\frac{1}{10}$)

# MUONIC ATOMS

Muons ($\mu$-) brought to rest in matter, get trapped in atomic orbit and have a higher probability than electrons of spending time inside the nucleus.

Bohr radius ~ $1/Zm$ Energy ~ $Z^2m$

$\mu$ mass ~ 207 $m_e$ $\mu$ lifetime ~ 2$\mu$s

The muons make transitions to low energy levels, emitting X-rays before decaying

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

- For hydrogen and electrons: $r = a_0 = 5\times10^4$ fm (Bohr radius)
- For lead and muons: $r = \dfrac{5\times10^4}{82\times207} = 3$ fm

Transition energy ($2P_{3/2}\rightarrow1S_{1/2}$): 16.41 MeV (Bohr theory),
6.02 MeV (measured)

$\therefore Z_{effective}$ and E are changed relative to electrons

Measure X-ray energies $\rightarrow$ RADIUS

# MIRROR NUCLEI

Mirror nuclei (e.g. ${}^{11}_{6}C,\ {}^{11}_{5}B$ ) have different masses due to the $p$-$n$ difference and the different Coulomb terms in the binding energy:

$$M(A, Z+1) - M(A,Z) = \Delta E_c + m_p + m_e - m_n$$

where $$\Delta E_c = \frac{6}{5}\frac{Z\alpha}{R}$$ (see Example Sheet 3)

The mass difference of 2 mirror nuclei can be determined from the $\beta^+$ decay spectra of the ($A,Z+1$) member of the pair

$${}^{11}_{6}C \rightarrow {}^{11}_{5}B + e^+ + \nu_e$$

$$p \rightarrow n + e^+ + \nu_e$$

$$M(A,Z+1) - M(A,Z) = m_e + E_{max} \qquad m_\nu \sim 0$$

where $E_{max}$ is the maximum kinetic energy of the positron.

$$\boxed{R = \frac{6Z\alpha}{5}\left[\frac{1}{E_{max} - m_p + m_n}\right]}$$

# Nuclear Moments

Static electromagnetic properties of nuclei are specified in terms of ELECTROMAGNETIC MOMENTS which give information about the way magnetism and charge is distributed throughout the nucleus.

The two most important moments are:

Electric Quadrupole Moment $Q$
Magnetic Dipole Moment $\mu$

## ELECTRIC MOMENTS

Depend on the charge distribution inside the nucleus and are a measure of NUCLEAR SHAPE (contours of constant charge density).

| | |
|---|---|
| $Q = 0$ | Spherical nucleus |
| Large $Q$ | Highly deformed nucleus |

NUCLEAR SHAPE is parameterized by a multipole expansion of the external electric field

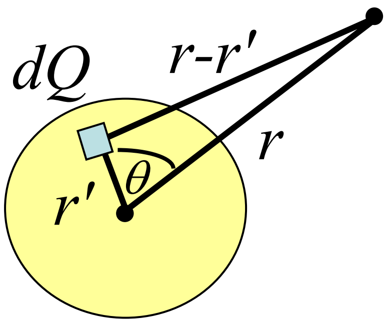
$$V(r) = \frac{1}{4\pi}\int \frac{\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\, d^3\vec{r}\,' \qquad \int \rho(\vec{r}\,') d^3\vec{r}\,' = Ze \qquad \varepsilon_0 = 1$$

$r$ ($r'$)= distance to observer (charge element) from origin.

$$|\vec{r}-\vec{r}\,'| = \left[r^2 + r'^2 - 2rr'\cos\vartheta\right]^{1/2}, \qquad |\vec{r}-\vec{r}\,'|^{-1} = r^{-1}\left[1+\frac{r'^2}{r^2} - 2\frac{r'}{r}\cos\vartheta\right]^{-1/2}$$

$$|\vec{r}-\vec{r}\,'|^{-1} = r^{-1}\left[1-\frac{1}{2}\left(\frac{r'^2}{r^2} - 2\frac{r'}{r}\cos\vartheta\right) + \frac{3}{8}\left(\frac{r'^2}{r^2} - 2\frac{r'}{r}\cos\vartheta\right)^2 + \ldots\right]$$

$$\approx r^{-1}\left[1+\frac{r'}{r}\cos\vartheta + \frac{1}{2}\frac{r'^2}{r^2}\left(3\cos^2\vartheta - 1\right) + \ldots\right] \qquad r' \ll r$$




$$V(r) = \frac{1}{4\pi r}\left[Ze + \frac{1}{r}\int r'\cos\vartheta\rho(r')\, d^3\vec{r}\,' + \frac{1}{2r^2}\int r'^2(3\cos^2\vartheta - 1)\rho(r')\, d^3\vec{r}\,' + \ldots\right]$$

Let $r$ define z-axis: $z = r'\cos\vartheta$

$$V(r) = \frac{1}{4\pi r}\left[ Ze + \frac{1}{r}\int z\,\rho(r')\,d^3\vec{r}\,' + \frac{1}{2r^2}\int (3z^2 - r'^2)\rho(r')\,d^3\vec{r}\,' + \ldots \right]$$

Quantum limit: $\rho(\vec{r}\,') = |\psi(\vec{r}\,')|^2$

E0 MOMENT $\int \psi^*\psi\, d^3\vec{r}\,' = Ze$ charge

E1 MOMENT $\int \psi^* z\,\psi\, d^3\vec{r}\,'$ electric dipole

E2 MOMENT $\frac{1}{e}\int \psi^*(3z^2 - r'^2)\,\psi\;\; d^3\vec{r}\,'$ electric quadrupole

Nuclear wavefunctions have definite parity $|\psi(r)|^2 = |\psi(-r)|^2$

<u>ELECTRIC DIPOLE MOMENT IS ZERO</u>

# Electric Quadrupole Moment

$$\boxed{Q = \frac{1}{e}\int \psi^* \,(3z^2 - r^2)\,\psi \; d^3\vec{r}}$$

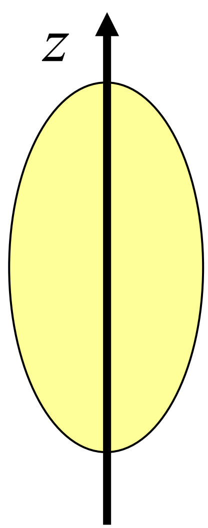
Units: m$^2$ or barns    Area

Spherical symmetry, $z^2 = \frac{1}{3}r^2 \quad \Rightarrow$ Q = 0

ALL J=0 NUCLEI HAVE Q = 0

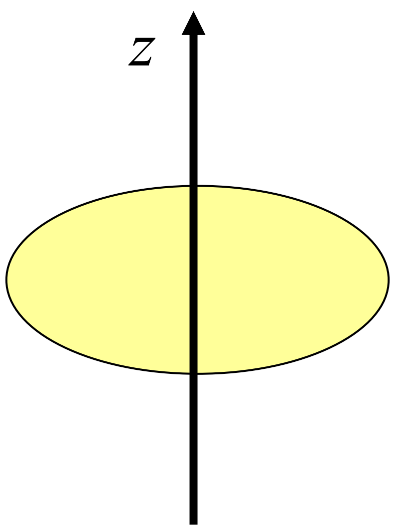
Prolate spheroid

Q > 0




Oblate spheroid

Q < 0



# MAGNETIC MOMENTS

Nuclear magnetic dipole moments arise from the intrinsic spin magnetic dipole moments of the protons and neutrons in the nucleus and from currents circulating in the nucleus due to the motion of the protons:

$$\vec{\mu} = \frac{\mu_N}{\hbar} \sum_i \left[ g_\ell \vec{\ell}_z + g_s \vec{s}_z \right] \qquad \text{sum over all } p,n$$

where $\mu_N = e\hbar / 2m_p$ is the Nuclear Magneton

The NUCLEAR MAGNETIC DIPOLE MOMENT can be written as

$$\boxed{\mu = g_J \, \mu_N \, J}$$

where $J$ total nuclear spin

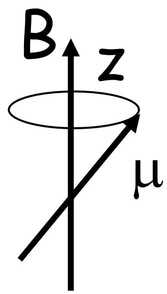
$g_J$ nuclear g-factor

$g_J$ will be determined using the Nuclear Shell Model (see later)

ALL EVEN-EVEN NUCLEI: $\mu$=0 as J=0

# Nuclear Magnetic Resonance

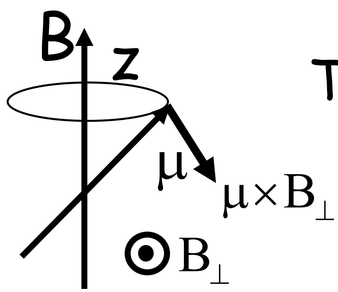
Nuclei with magnetic moment (not J=0 nuclei) in a steady uniform magnetic field, B, exhibit classical Larmor precession



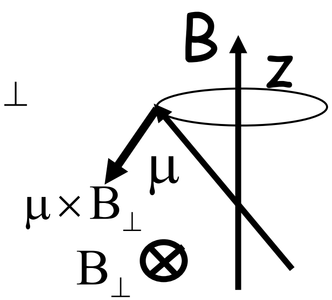

Precession frequency $\omega_L = \dfrac{\mu B}{\hbar J}$

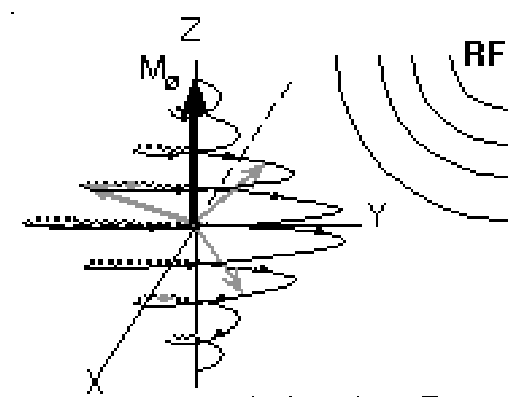
Interaction energy $E = -\vec{\mu} \cdot \vec{B}$ $\quad \Delta E = 2\mu B$

Apply an oscillatory field (frequency ω) ⊥'r to B




Torque = $\mu \times B_\perp$




Laboratory Frame

Maximum energy absorbed at resonance: $\boxed{\omega = \omega_L}$

<u>Example</u>: proton in 1T field, $\Delta E \sim 1.8\times10^{-7}$ eV,

$\nu$ = 43 MHz (radio frequency)

Observe resonance in r.f. power absorbed

- Measure $\mu$
- If $\mu$ known, probe lattice/molecular binding.

In nuclear medicine, Magnetic Resonance Imaging (MRI) used to measure distribution of proton-rich tissue.

0.5 - 2 T Magnetic field

# Radioactivity

Natural radioactivity α, β, γ decay

**α DECAY**: ${}^{4}_{2}He$ nucleus

A ≥ 210 $\quad {}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$

For decay to occur, energy must be released (Q > 0)

$$Q = m_X - m_Y - m_{He} = B_Y + B_{He} - B_X$$

**β DECAY**: $e^-$ electron, $e^+$ positron

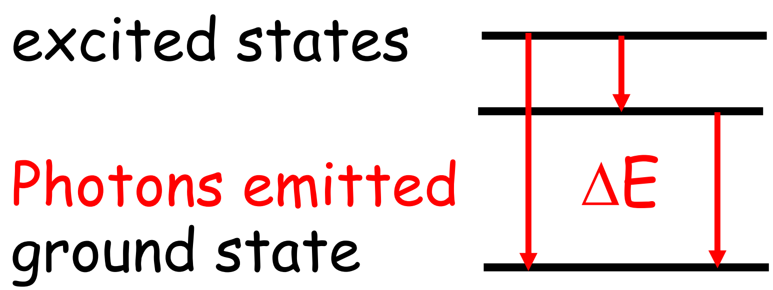
β⁻ $\quad n \rightarrow p + e^- + \bar{\nu}_e \qquad {}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^- + \bar{\nu}_e$

β⁺ $\quad p \rightarrow n + e^+ + \nu_e \qquad {}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + e^+ + \nu_e$

electron capture $\quad p + e^- \rightarrow n + \nu_e \qquad {}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + \nu_e$

Only $n \rightarrow pe\nu$ can occur outside nucleus

$\gamma$ DECAY: Nuclei with excited states can decay by emission of a $\gamma$.

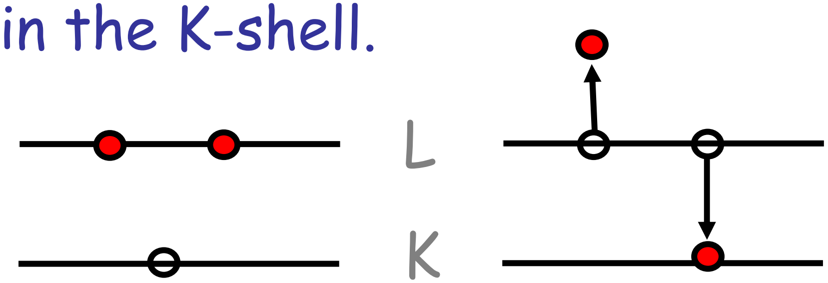



| | ΔE | λ | |
|---|---|---|---|
| Atom | ~10 eV | ~$10^{-7}$ m | optical |
| | ~10 keV | ~$10^{-10}$ m | X-ray |
| Nucleus | ~MeV | ~$10^{-12}$ m | $\gamma$-ray |

INTERNAL CONVERSION: occurs when nuclear excitation energy is lost by the ejection of an atomic $e$- (usually from the K-shell).

The vacancy left by the emission of an $e$- leads to X-ray or Auger $e$- emission as the atom returns to its neutral state.

An Auger $e$- is an atomic $e$- receiving enough KE to be ejected, usually from the L-shell, when another $e$- falls from the same shell to fill a vacancy in the K-shell.



# NATURAL RADIOACTIVITY

The HALF-LIFE, $\tau_{1/2}$, is the time over which 50% of the nuclei decay.

$$\tau_{1/2} = \frac{Ln\ 2}{\lambda} = 0.693\ \tau$$

$\lambda$ Transition rate

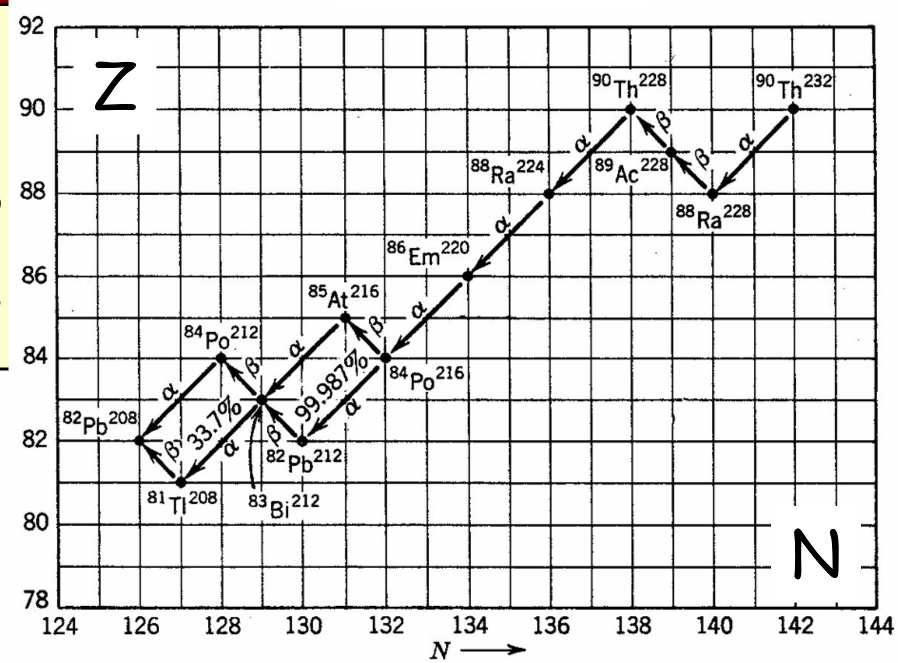
$\tau$ Average lifetime

Some $\tau_{1/2}$ long compared to age of Earth.

| Series Name | Type | Final Nucleus (stable) | Longest-Lived Nucleus | $\tau_{1/2}$ (years) |
|---|---|---|---|---|
| Thorium | 4n | $^{208}Pb$ | $^{232}Th$ | $1.41 \times 10^{10}$ |
| Neptunium | 4n+1 | $^{209}Bi$ | $^{237}Np$ | $2.14 \times 10^{6}$ |
| Uranium | 4n+2 | $^{206}Pb$ | $^{238}U$ | $4.47 \times 10^{9}$ |
| Actinium | 4n+3 | $^{207}Pb$ | $^{235}U$ | $7.04 \times 10^{8}$ |

n is an integer

4n series



# Radioactive Dating

Consider a sample of radioactive PARENT nuclei ($P$) which decay to DAUGHTER nuclei ($D$).

<u>Assumptions</u>:

- know $\tau_P$ from previous studies
- P trapped when sample came into existence
- no P or D entered or left by other means
- At $t=0$, $N_D=0$

$$N_P(t) + N_D(t) = N_P(0)$$

$$N_P(t) = N_P(0)e^{-\lambda \Delta t}$$ Exponential decay law

$$N_P(t) + N_D(t) = N_P(t)e^{\lambda \Delta t}$$

$$e^{\lambda \Delta t} = 1 + \frac{N_D(t)}{N_P(t)}$$

Age $$\Delta t = \tau_P \, Ln\left[1 + \frac{N_D(t)}{N_P(t)}\right]$$

Count $N_P(t)$ and $N_D(t)$ e.g. chemically

Complication when $N_D(0) \neq 0$:

$$N_P(t) + N_D(t) = N_P(0) + N_D(0)$$

2 equations, 3 unknowns

If there is another isotope of $D$, say $D'$, for which

$$N_{D'}(t) = N_{D'}(0)$$

(i.e. D' stable)

Then,

$$\frac{N_P(t) + N_D(t)}{N_{D'}(t)} = \frac{N_P(0) + N_D(0)}{N_{D'}(0)}$$

$$\frac{N_D(t)}{N_{D'}(t)} = \frac{N_P(0) + N_D(0)}{N_{D'}(0)} - \frac{N_P(t)}{N_{D'}(t)}$$

$$\frac{N_D(t)}{N_{D'}(t)} = \frac{N_P(t)}{N_{D'}(t)}\left(e^{\lambda \Delta t} - 1\right) + \frac{N_D(0)}{N_{D'}(0)}$$

With several mineral sources from the same source expect

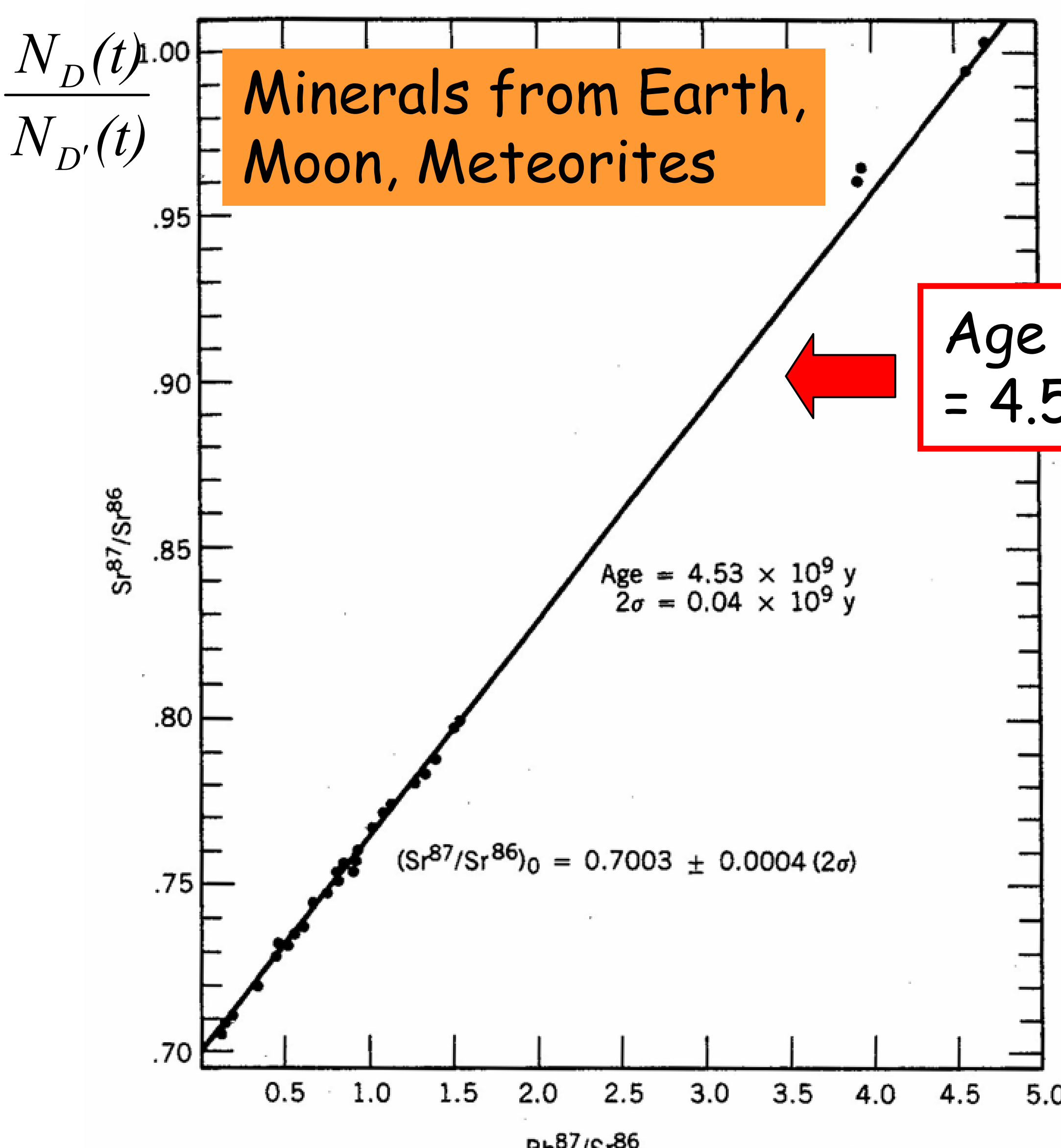
- same age $\Delta t$
- same $N_D(0)/N_{D'}(0)$
- different $N_P(0)$

Plot $N_D(t) / N_{D'}(t)$ versus $N_P(t) / N_{D'}(t)$

slope $e^{\lambda \Delta t} - 1$ intercept $N_D(0) / N_{D'}(0)$

# Example: Use $\beta^-$ decay

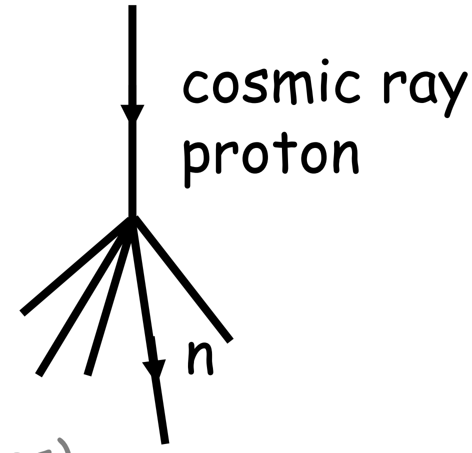
$^{87}Rb \rightarrow {}^{87}Sr \quad (\tau_{1/2} = 4.8 \times 10^{10}$ years) $\qquad D' = {}^{86}Sr$ (stable)




Age of Earth from slope = $4.5 \times 10^9$ years

# RADIO-CARBON DATING

Most recent organic matter: use $^{14}C$ DATING




- $^{14}C$ is continuously formed in Earths atmosphere

$$^{14}N + n \rightarrow {}^{14}C + p$$

Production rate of $^{14}C$ is approx constant

(e.g. checked by comparison with dating from tree rings)

- The carbon in living organisms is continuously exchanged with atmospheric carbon.

Equilibrium: ~1 atom of $^{14}C$ to every $10^{12}$ atoms of other carbon isotopes (98.9% $^{12}C$, 1.1% $^{13}C$).

- $^{14}C$ decays in dead organisms. No more $^{14}C$ from atmosphere.

$$^{14}_{6}C \rightarrow {}^{14}_{7}N + e^{-} + \bar{\nu}_e$$

$\beta^-$ decay, $\tau_{1/2}$=5730 yrs

- Measure the specific activity to obtain age. (Number decays per second per unit mass)

- Complications from burning of fossil fuels, nuclear bomb tests etc.

<u>Example:</u> Turin Shroud [ref: Nature 337 (1989) 611.]

Latest results see: http://www.shroud.com/