Section IX Electroweak Unification

Electroweak Unification

Weak Charged Current interactions explained by W[±] exchange.
 W bosons are charged, ∴couple to photon.



The GWS Model

The Glashow, Weinberg and Salam model treats EM and WEAK interactions as different manifestations of a single UNIFIED ELECTROWEAK force (Nobel Prize 1979)



<u>Basic Idea:</u>

Start with 4 massless bosons $\{W^+, W^0, W^-\}$ and B^0 . The neutral bosons MIX to give physical bosons (the particles we see), i.e. the W^{\pm} , Z^0 and γ .

$$\begin{pmatrix} W^{+} \\ W^{0} \\ W^{-} \end{pmatrix}, B^{0} \rightarrow \begin{pmatrix} W^{+} \\ Z^{0} \\ W^{-} \end{pmatrix}, \gamma$$

Physical fields: W^+ , Z^0 , W^- and A (photon)

$$Z^{0} = W^{0} \cos \Theta_{W} - B^{0} \sin \Theta_{W}$$
$$A = W^{0} \sin \Theta_{W} + B^{0} \cos \Theta_{W}$$

 $\vartheta_{\scriptscriptstyle W}$ weak mixing angle

 W^{\pm} , Z^{0} "acquire" mass via the HIGGS MECHANISM.

The beauty of the GWS model is that it makes EXACT predictions of the W^{\pm} and Z⁰ masses and couplings with ONLY 3 free parameters.



If we know { α_{em} , G_F , sin ϑ_W } (from experiment) everything else is FIXED. 211

Evidence for GWS Model

Discovery of Neutral Currents (1973)
 The process $\overline{v}_{\mu}e^{-} \rightarrow \overline{v}_{\mu}e^{-}$ was observed.
 ONLY possible Feynman diagram (no W[±] diagram)
 Indirect evidence for Z⁰



- ➤ Direct Observation of W[±] and Z⁰ (1983)
 First DIRECT observation in pp collisions at √s = 540 GeV via decays into leptons. $pp \to W^{\pm} + X$ $pp \to W^{\pm} + X$ $pp \to Z^{0} + X$ $\downarrow e^{\pm}v_{e}, \mu^{\pm}v_{\mu}$ $\downarrow e^{+}e^{-}, \mu^{+}\mu^{-}$
- Precision Measurements of the Standard Model (1989-2000) LEP e⁺e⁻ collider (see later) provided many precision measurements of the Standard Model.
- Wide variety of different processes consistent with GWS model predictions and measure SAME VALUE of

$$sin^2 \vartheta_W = 0.23113 \pm 0.00015 \qquad \vartheta_W \approx 29^\circ$$

The Weak NC Vertex

All weak neutral current interactions can be described by the Z⁰ boson propagator and the weak vertices:



> Z⁰ couplings are a MIXTURE of EM and WEAK couplings and therefore depend on $sin^2 \vartheta_W$ (exact form treated in Part III Particles course)







Summary of Standard Model Vertices



Drawing Feynman Diagrams

A Feynman diagram is a pictorial representation of the matrix element describing particle decay or interaction

 $a \rightarrow b + c + \dots$ $a + b \rightarrow c + d$

To draw a Feynman diagram and determine whether a process is allowed, follow the FIVE basic steps:



2 Draw the SIMPLEST Feynman diagram using the Standard Model vertices. Bearing in mind:

- Similar diagrams for particles/antiparticles
- NEVER have a vertex connecting a LEPTON to a QUARK
- Only the WEAK CC vertex changes FLAVOUR

within generations for leptons within/between generations for quarks

- Particle scattering
- > If all particles (or all antiparticles), only SCATTERING diagrams involved e.g. $a + b \rightarrow c + d$



> If particles and antiparticles, can have SCATTERING and/or ANNIHILATION diagrams e.g. $a + \overline{b} \rightarrow c + \overline{d}$





- Check that the whole system CONSERVES
- Energy, momentum (trivially satisfied for interactions)
- Charge
- Angular Momentum



Parity

- CONSERVED in EM/STRONG interaction
- CAN be violated in the WEAK interaction



Check SYMMETRY for IDENTICAL particles in the final state

- **BOSONS** $\psi(1,2) = +\psi(2,1)$
- FERMIONS $\psi(1,2) = -\psi(2,1)$

Finally, a process will occur (in order of preference) via the STRONG, EM and WEAK interaction if steps 1 - 5 are satisfied.

Examples:

1. $\pi^{-} + p \rightarrow K^{0} + \Lambda$

2.
$$v_{\tau} + e^- \rightarrow v_{\tau} + e^ \overline{v}_{\tau} + \tau^- \rightarrow \overline{v}_{\tau} + \tau^-$$

3. $D^+ \rightarrow K^- \pi^+ \pi^+$

4. $\Xi^0 \rightarrow \Sigma^- e^+ v_e$

Experimental Tests of the Standard Model

The Large Electron Positron (LEP) collider at CERN provided precision measurements of the Standard Model (1989-2000).

Designed as a Z^0 and W^{\pm} boson factory



Precise measurements of the properties of Z^0 and W^{\pm} bosons provide the most stringent test of our current understanding of particle physics.



LEP

- > LEP is the highest energy e^+e^- collider ever built $\sqrt{s} = 90 200 \text{ GeV}$
- Large = 27 km circumference
- 4 experiments combined: 16,000,000 Z⁰ and 30,000 W[±] events







A LEP Detector: OPAL



Particle Identification



Typical $e^+e^- \rightarrow Z^0$ Events





In the $e^+e^- \rightarrow Z^0 \rightarrow \tau^+\tau^-$ event, the tau leptons decay within the detector (lifetime ~ 10⁻¹³ s). Here, $\tau^- \rightarrow e^- \bar{\nu}_e v_{\tau}$ and $\tau^+ \rightarrow \mu^+ v_{\mu} \bar{\nu}_{\tau}$ 225

The Z⁰ Resonance

Consider the process of $e^+e^- \rightarrow q\overline{q}$

- > Previously, $\sqrt{s} < 50 \, \text{GeV}$, only considered an intermediate photon
- > At higher energies also have the Z⁰ exchange diagram (plus $Z^{0\gamma}$ interference)



 ➤ The Z⁰ is a decaying intermediate massive states (lifetime ~ 10⁻²⁵ s) ⇒ BREIT-WIGNER RESONANCE

 ➤ At √s ~ M_z the Z⁰ diagram dominates. 226



BREIT-WIGNER formula for $e^+e^- \rightarrow Z^0 \rightarrow f\overline{f}$ (where $f\overline{f}$ is ANY fermion-antifermion pair)

Centre-of-mass energy
$$\sqrt{s} = E_{CM} = E_{e^-} + E_{e^+}$$

$$\sigma(e^+e^- \to Z^0 \to f\bar{f}) = \frac{g\pi}{E_e^2} \frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{(E_{CM} - M_Z)^2 + \frac{\Gamma_Z^2}{4}}$$
with $g = \frac{2J_Z + 1}{(2J_{e^-} + 1)(2J_{e^+} + 1)} = \frac{3}{4}$ $J_Z = 1; J_{e^\pm} = \frac{1}{2}$
giving

$$\sigma\left(e^{+}e^{-} \to Z^{0} \to f\overline{f}\right) = \frac{3\pi}{4E_{e}^{2}} \frac{\Gamma_{ee}\Gamma_{f\overline{f}}}{\left(\sqrt{s} - M_{Z}\right)^{2} + \frac{\Gamma_{Z}^{2}}{4}} = \frac{3\pi}{s} \frac{\Gamma_{ee}\Gamma_{f\overline{f}}}{\left(\sqrt{s} - M_{Z}\right)^{2} + \frac{\Gamma_{Z}^{2}}{4}}$$

 $\succ \Gamma_Z$ is the TOTAL DECAY WIDTH, i.e. the sum over the partial widths for different decay modes.

$$\Gamma_{Z} = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{q\bar{q}} + \Gamma_{v\bar{v}}$$

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At the peak of the resonance $\sqrt{s} = M_Z$:

$$\sigma(e^+e^- \to Z^0 \to f\bar{f}) = \frac{12\pi}{M_Z^2} \frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{\Gamma_Z^2}$$

Hence, for ALL fermion/antifermion pairs in the final state

$$\sigma(e^+e^- \to Z^0 \to anything) = \frac{12\pi}{M_Z^2} \frac{\Gamma_{ee}}{\Gamma_Z} \qquad \Gamma_{f\bar{f}} = \Gamma_Z$$

Compare to the QED cross-section at $\sqrt{s} = M_Z$ $\sigma_{QED} = \frac{4\pi\alpha^2}{3s}$

$$\frac{\sigma(e^+e^- \to Z^0 \to anything)}{\sigma_{QED}} = \frac{9}{\alpha^2} \frac{\Gamma_{ee}}{\Gamma_Z} \approx 5700$$

$$\Gamma_{ee} = 85 \text{ MeV}$$

 $\Gamma_Z = 2.5 \text{ GeV}$
 $\alpha = \frac{1}{137}$

Measurement of M_Z and Γ_Z

- > Run LEP at various centre-of-mass energies (\sqrt{s}) close to the peak of the Z⁰ resonance and measure $\sigma(e^+e^- \rightarrow q\overline{q})$
- > Determine the parameters of the resonance:



M_Z measured with precision 2 parts in 10⁵

- To achieve this required a detailed understanding of the accelerator and astrophysics! Tidal distortions of the Earth by the Moon cause the rock surrounding LEP to be distorted. The nominal radius of LEP changes by 0.15 mm compared to radius of 4.3 km. This is enough to change the centre-of-mass energy !
- Also need a train timetable ! Leakage currents from the TGV rail via lake Geneva follow the path of least resistance...using LEP as a conductor.

Accounting for these effects (and many others):

$$M_{Z} = (91.1875 \pm 0.0021) \text{ GeV}$$

$$\Gamma_{Z} = (2.4952 \pm 0.0023) \text{ GeV}$$

$$\sigma_{q\bar{q}}^{0} = (41.540 \pm 0.037) \text{ nb}$$

Number of Generations

 $\begin{cases} u & e^{-} \\ d & v_{e} \end{pmatrix} \begin{pmatrix} c & \mu^{-} \\ s & v_{\mu} \end{pmatrix} \begin{pmatrix} t & \tau^{-} \\ b & v_{\tau} \end{pmatrix}$

 Currently know of THREE generations of fermions.
 Masses of quarks and leptons INCREASE

with generation. Neutrinos are ~ massless (?)

- > Could there be more generations ? e.g. $\begin{pmatrix} t' & L \\ b' & v_L \end{pmatrix}$
- $\begin{array}{ll} \blacktriangleright & \mbox{The }Z^0 \mbox{ boson couples to } \mbox{ALL fermions, including neutrinos. Therefore,} \\ & \mbox{the total decay width, } \Gamma_Z \mbox{, has contributions from all fermions with} \\ & \mbox{$m_f < M_Z/2$} \\ & \mbox{$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{q\bar{q}} + \Gamma_{\nu\bar{\nu}}$ \end{array}$
 - with $\Gamma_{v\bar{v}} = \Gamma_{v_e \bar{v}_e} + \Gamma_{v_\mu \bar{v}_\mu} + \Gamma_{v_\tau \bar{v}_\tau}$
- If there were a FOURTH generation, it seems likely that the neutrino would be light, and, if so would be produced at LEP

$$e^+e^- \rightarrow Z^0 \rightarrow v_L \bar{v}_L$$

The neutrinos would not be observed directly, but could infer their presence from the effect on the Z⁰ resonance curve.
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At the peak of the Z⁰ resonance $\sqrt{s} = M_Z$

$$\sigma_{f\overline{f}}^{0} = \frac{12\pi}{M_{Z}^{2}} \frac{\Gamma_{ee}\Gamma_{f\overline{f}}}{\Gamma_{Z}^{2}}$$

A FOURTH generation neutrino would INCREASE the Z⁰ decay rate and thus INCREASE Γ_Z . As a result a DECREASE in the measured peak cross-sections for the VISIBLE final states would be observed.

> Measure the $e^+e^- \rightarrow Z^0 \rightarrow f \overline{f}$ cross-sections for all visible decay modes (i.e. all fermions apart from vv)



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> Have already measured M_Z and Γ_Z from the shape of the Breit-Wigner resonance. Therefore, obtain $\Gamma_{f\overline{f}}$ from the peak crosssections in each decay mode using

$$\sigma_{f\overline{f}}^{0} = \frac{12\pi}{M_{Z}^{2}} \frac{\Gamma_{ee}\Gamma_{f\overline{f}}}{\Gamma_{Z}^{2}}$$

Note, obtain Γ_{ee} from

$$\sigma_{ee}^{0} = \frac{12\pi}{M_Z^2} \frac{\Gamma_{ee}^2}{\Gamma_Z^2}$$

 \succ Can relate the partial widths to the measured TOTAL width (from the resonance curve)

$$\Gamma_{Z} = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{q\bar{q}} + N_{v}\Gamma_{v\bar{v}}$$

where N_{ν} is the NUMBER OF NEUTRINO SPECIES and $\Gamma_{\nu\nu}$ is the partial width for a single neutrino species.

The difference between the measured value of $\Gamma_{\rm Z}$ and the sum of the partial widths for visible final states gives the INVISIBLE WIDTH $N_{\rm v}\Gamma_{\rm v\bar{v}}$

Γ_{Z}	$2494 \pm 4.1 \text{ MeV}$
Γ_{ee}	83.7 ± 0.2 MeV
${\Gamma}_{_{\mu\mu}}$	84.0 ± 0.3 MeV
$\Gamma_{ au au}$	83.9 ± 0.4 MeV
$\Gamma_{q \overline{q}}$	1745.3 ± 3.5 MeV
$N_{ m v} \Gamma_{ m v ar v}$	$497.3\pm3.5~\text{MeV}$

In the Standard Model, calculate $\Gamma_{v\bar{v}} = 167 \text{ MeV}$

Therefore

$$N_{\nu} = \frac{497.3 \pm 3.5}{167} = \underline{2.98 \pm 0.02}$$

 \Rightarrow THREE generations of light neutrinos

$$m_v < \frac{M_Z}{2}$$

Most likely that ONLY 3 GENERATIONS OF QUARKS AND LEPTONS EXIST



In addition:

> $\Gamma_{ee}, \Gamma_{\mu\mu}, \Gamma_{\pi}$ are consistent \Rightarrow universality of the lepton couplings to the Z⁰

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> $\Gamma_{q\bar{q}}$ is consistent with the expected value which assumes 3 COLOURS - yet more evidence for colour

W⁺W⁻ at LEP

- \triangleright e^+e^- collisions W bosons are produced in pairs.
- Standard Model 3 possible diagrams:



- > LEP operated above the threshold for W⁺W⁻ production (1996-2000) $\sqrt{s} > 2M_W$
- $\succ Cross-section sensitive to the presence of the Triple Gauge Boson vertex <math>Z^0W^+W^-$



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W⁺W⁻ Decay at LEP

In the Standard Model $W^{\pm}\ell v$ and $W^{\pm}q\overline{q}$ couplings are \approx equal.





Measurement of M_W and Γ_W

- > Unlike $e^+e^- \rightarrow Z^0$, W boson production at LEP is NOT a resonant process
- M_W measured by reconstructing the invariantismassion and eventby-event basis.



W Boson Decay Width

In the Standard Model, the W boson decay width is given by

$$\Gamma_W = \frac{g_W^2 M_W}{48\pi} = \frac{G_F M_W^3}{6\sqrt{2}\pi}$$

 $\mu\text{-decay: } G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2} \qquad \text{LEP: } M_W = (80.423 \pm 0.038) \text{ GeV.}$ Therefore, predict partial width: $\Rightarrow \quad \Gamma(W^- \to e^- \bar{v}_e) = 227 \text{ MeV}$

Total width is the sum over all partial widths: $W^- \rightarrow e^- \overline{v}_e, \mu^- \overline{v}_\mu, \tau^- \overline{v}_\tau$ $W^- \rightarrow d\overline{u}, s\overline{c}$ ×3 FOR COLOUR

<u>IF</u> the W coupling to leptons and quarks is EQUAL, and there are 3 colours: $\Gamma = \sum_{i} \Gamma_{i} = (3 + 2 \times 3) \Gamma (W^{-} \rightarrow e^{-} \overline{v_{e}}) \approx 2.1 \text{ GeV}$ Compare with measured value (LEP): $\Gamma_{W} = (2.12 \pm 0.11) \text{ GeV}$

- Universal coupling constant
- Yet more evidence for colour !

Summary

Now have 5 precise measurements of fundamental parameters of the Standard Model

$$\alpha_{em} = 1/(137.03599976 \pm 0.00000050)$$

$$G_F = (1.16632 \pm 0.00002) \times 10^{-5} \text{ GeV}^{-2}$$

$$M_W = (80.423 \pm 0.038) \text{ GeV}$$

$$M_Z = (91.1875 \pm 0.0021) \text{ GeV}$$

$$\sin^2 \vartheta_W = 0.23143 \pm 0.00015$$

In the Standard Model, ONLY 3 are independent.

Their consistency is an incredibly powerful test of the Standard Model of Electroweak Interactions !

 $(at q^2=0)$