APPENDIX F: NEUTRINO SCATTERING IN FERMI THEORY

Calculation of the cross-section for $\nu_e + n \rightarrow p + e^-$ using Fermi theory. The cross-section is given by Fermi's Golden Rule

$$\Gamma = 2\pi \left|M_{fi}\right|^2 \rho\left(E_f\right)$$

where the matrix element, M_{fi}, is given by the 4-point interaction with a strength equal to the Fermi constant, G_F;

$$\left|M_{fi}\right|^2 \approx G_F^2.$$

There are a total of 4 possible spin states for the spin-$\frac{1}{2}$ e and ν. These correspond to a singlet state $S = 0$ (Fermi transition) and three triplet states $S = 1$ (Gamow-Teller transition). Therefore, the matrix element becomes

$$\left|M_{fi}\right|^2 \approx 4G_F^2.$$

The differential cross-section is then given by,

$$d\sigma = 2\pi 4G_F^2 \frac{E_e^2}{(2\pi)^3} d\Omega$$

where E_e is the energy of the electron. It follows

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2 E_e^2}{\pi^2}.$$

The total energy in the zero-momentum frame, $\sqrt{s} = 2E_e$. Hence, the total cross-section can be written as

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = \frac{4G_F^2 E_e^2}{\pi} = \underline{\frac{G_F^2 s}{\pi}}.$$

APPENDIX G: NEUTRINO SCATTERING WITH A MASSIVE W BOSON

The cross-section for νe scattering with a massive vector boson can be written as

$$\frac{d\sigma}{dq^2} = \frac{g_W^4}{32\pi} \frac{1}{(q^2 - M_W^2)^2}$$

where q^2 is the four-momentum squared carried by the W boson, g_W is the weak coupling constant and M_W is the mass of the W.
At low $q^2 << M_W^2$, the cross-section becomes

$$\frac{d\sigma}{dq^2} = \frac{g_W^4}{32\pi} \frac{1}{M_W^4} = \frac{G_F^2}{\pi}$$

where the weak coupling constant, g_W, and the Fermi constant, G_F, are related by

$$\frac{g_W^4}{M_W^4} = 32 G_F^2.$$

Hence, the total cross-section is

$$\sigma = \frac{G_F^2}{\pi} \int_0^s dq^2 = \frac{G_F^2 s}{\pi}$$

where $\sqrt{s}$ is the centre-of-mass energy. This is consistent with the Fermi theory result given in Appendix F.
At larger values of q^2, the total cross-section becomes

$$\sigma = \frac{g_W^4}{32\pi} \int_s^0 \frac{1}{(q^2 - M_W^2)^2} dq^2.$$

Note: large s corresponds to small q^2. In order to perform the integral, substitute $u = q^2 - M_W^2,\ \ du = dq^2$. Then

$$\begin{aligned}
\sigma &= \frac{g_W^4}{32\pi} \int \frac{1}{u^2} du \\
&= \frac{g_W^4}{32\pi} \left[-\frac{1}{u} \right] \\
&= \frac{g_W^4}{32\pi} \left[-\frac{1}{(q^2 - M_W^2)} \right]_s^0 \\
&= \frac{g_W^4}{32\pi} \left[\frac{1}{M_W^2} + \frac{1}{(s - M_W^2)} \right].
\end{aligned}$$

For $s >> M_W^2$, the cross-section tends towards the constant value

$$\sigma = \frac{g_W^4}{32\pi} \frac{1}{M_W^2} = \frac{G_F^2 M_W^2}{\pi}$$

and is no longer divergent.