Section VIII The Weak Interaction

The Weak Interaction

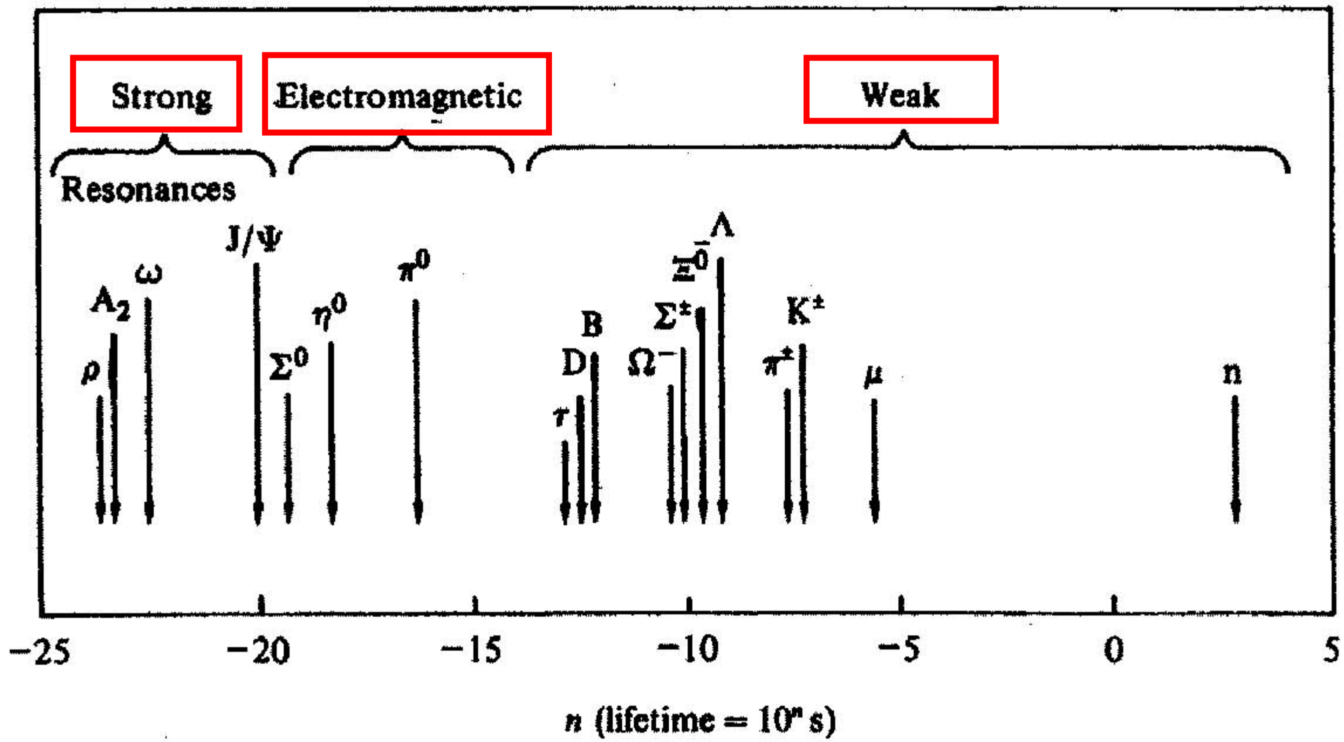
- The WEAK interaction accounts for many decays in particle physics

Examples:

$$\mu^- \to e^- \bar{\nu}_e \nu_\mu \quad ; \quad \tau^- \to e^- \bar{\nu}_e \nu_\tau$$

$$\pi^+ \to \mu^- \bar{\nu}_\mu \quad ; \quad n \to p e^- \bar{\nu}_e$$

- Characterized by long lifetimes and small cross-sections



- Two types of WEAK interaction:

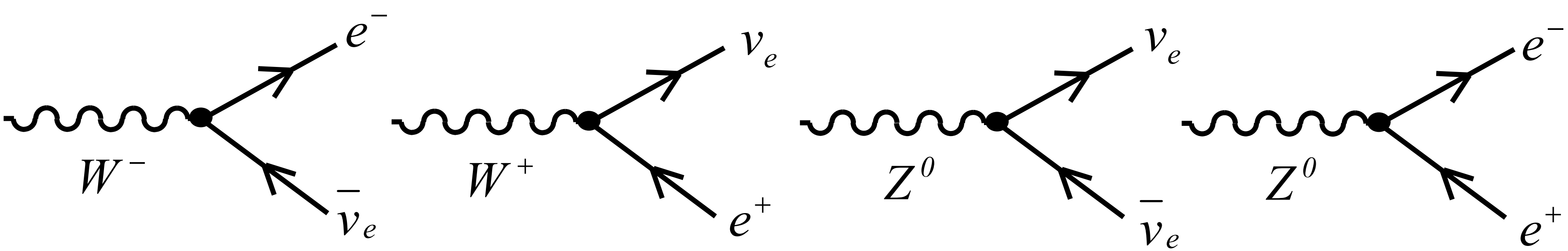
CHARGED CURRENT (CC): $W^{\pm}$ Bosons
NEUTRAL CURRENT (NC): Z^0 Boson

- The WEAK force is mediated by MASSIVE VECTOR BOSONS:

$M_W \sim 80$ GeV
$M_Z \sim 91$ GeV

Examples:

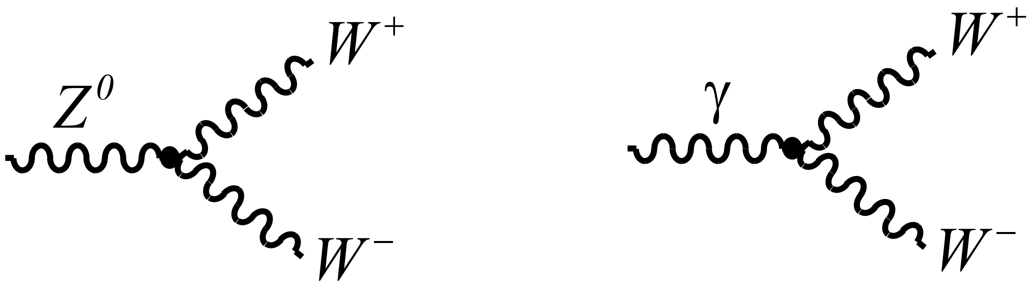
Weak interactions of electrons and neutrinos:

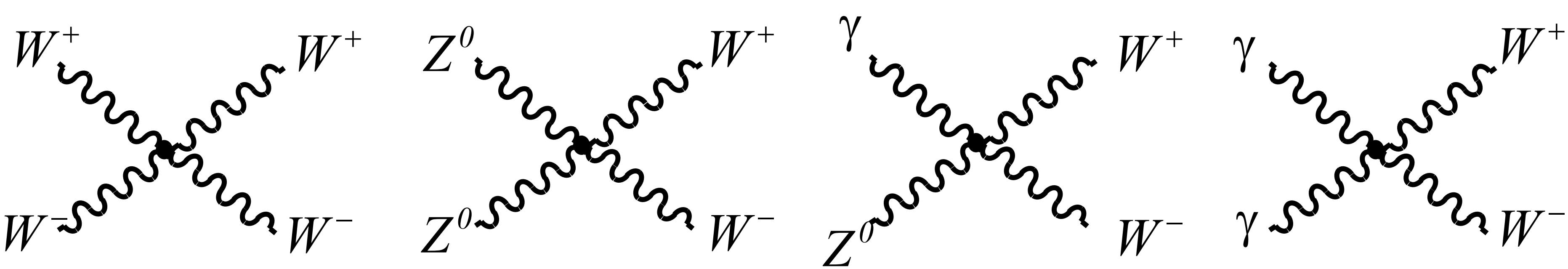


Boson Self-Interactions

- In QCD the gluons carry "COLOUR" charge.
- In the WEAK interaction the $W^{\pm}$ and Z^0 bosons carry the WEAK CHARGE
- $W^{\pm}$ also carry EM charge

$\Rightarrow$ BOSON SELF-INTERACTIONS





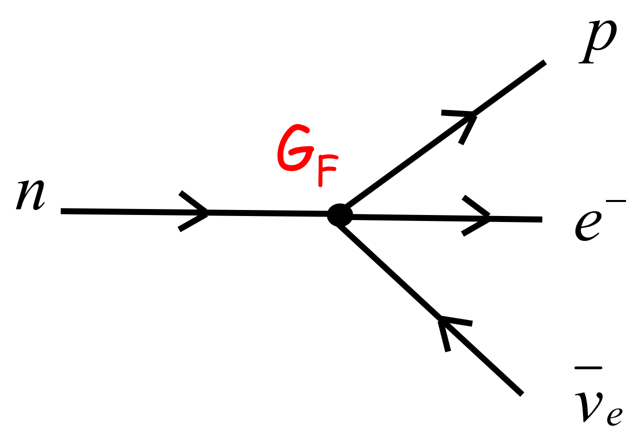
Fermi Theory

Weak interaction taken to be a "4-fermion contact interaction"

- No propagator
- Coupling strength given by the FERMI CONSTANT, G_F
- $G_F = 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$

β Decay in Fermi Theory

$$n \to p e^- \bar{\nu}_e$$



Use Fermi's Golden Rule to get the transition rate

$$\Gamma = 2\pi \left|M_{fi}\right|^2 \rho\left(E_f\right) \qquad \rho\left(E_f\right) = \frac{dN}{dE_f}$$

where M_{if} is the matrix element and $\rho(E_f)$ is the density of final states.

Density of Final States: 2-body vs. 3-body (page 46)

➢ TWO BODY FINAL STATE:

$$dN = \frac{E^2}{(2\pi)^3}\, d\Omega\, dE$$

Relativistic (E ~ p)
i.e. neglect mass of final state particles.

Only consider one of the particles since the other fixed by $(E,\vec{p})$ conservation.

➢ THREE BODY FINAL STATE (e.g. β decay):

$$d^2N = \frac{E_\nu^2}{(2\pi)^3}\, d\Omega_\nu\, dE_\nu\, \frac{E_e^2}{(2\pi)^3}\, d\Omega_e\, dE_e$$

Now necessary to consider two particles – the third is given by $(E,\vec{p})$ conservation.

In nuclear β decay, the energy released in the nuclear transition, E_0, is shared between the electron, neutrino and the recoil kinetic energy of the nucleus:

$$E_0 = E_e + E_\nu + T_{recoil}$$

Since the nucleus is much more massive than the electron /neutrino:

$$E_0 \approx E_e + E_\nu$$

and the nuclear recoil ensures momentum conservation.

For a GIVEN electron energy E_e:

$$dE_\nu = dE_0$$

$$\frac{dN}{dE_0} = \frac{dN}{dE_\nu} = \frac{E_\nu^2}{(2\pi)^3}\, d\Omega_\nu \, \frac{E_e^2}{(2\pi)^3}\, d\Omega_e \, dE_e$$

Assuming isotropic decay distributions and integrating over $d\Omega_e d\Omega_\nu$ gives:

$$\frac{dN}{dE_0} = (4\pi)^2 \frac{E_\nu^2}{(2\pi)^3} \frac{E_e^2}{(2\pi)^3} dE_e = \frac{E_\nu^2 E_e^2}{4\pi^4} dE_e = \frac{(E_0 - E_e)^2 E_e^2}{4\pi^4} dE_e$$

$$d\Gamma = 2\pi |M_{fi}|^2 \frac{(E_0 - E_e)^2 E_e^2}{4\pi^4} dE_e$$

$$\frac{d\Gamma}{dE_e} = |M_{fi}|^2 \frac{(E_0 - E_e)^2 E_e^2}{2\pi^3}$$

Matrix Element

In Fermi theory, $M_{fi} = G_F \int \psi_n \; \psi_p^* \; \psi_e^* \; \psi_{\bar{\nu}}^* \; d^3\vec{r}$ 4-point interaction

and treat e, ν as free particles $\psi_e = e^{i\vec{p}_e \cdot \vec{r}}$ $\psi_\nu = e^{i\vec{p}_\nu \cdot \vec{r}}$

$$\therefore \quad M_{fi} = G_F \int \psi_p^* \; e^{-(\vec{p}_e + \vec{p}_\nu)\cdot\vec{r}} \; \psi_n \; d^3\vec{r}$$

- Typically, e and ν have energies ~ MeV, so $\vec{p}\cdot\vec{r} \approx 10^{-2}$ « size of nucleus and

$$\psi_e, \psi_\nu \approx 1$$

Corresponds to zero angular momentum ($l = 0$) states for the e and ν.

$\Rightarrow$ ALLOWED TRANSITIONS

The matrix element is then given by

$$\left|M_{fi}\right|^2 = G_F^2 \left|\int \psi_p^* \, \psi_n \, d^3\vec{r}\right|^2 = G_F^2 \left|M_{nuclear}\right|^2$$

where the nuclear matrix element $\left|M_{nuclear}\right|^2$ accounts for the overlap of the nuclear wave-functions.

- If the n and p wave-functions are very similar, the nuclear matrix element

$$\left|M_{nuclear}\right|^2 = 1$$

$\Rightarrow M_{fi}$ large and β decay is favoured:

$\Rightarrow$ SUPERALLOWED TRANSITIONS

Here, assume $\left|M_{nuclear}\right|^2 = 1$ (superallowed transition):

$$\frac{d\Gamma}{dE_e} = \frac{G_F^2}{2\pi^3}\left(E_0 - E_e\right)^2 E_e^2 \qquad \left|M_{fi}\right|^2 = G_F^2$$

$$\Gamma = \frac{G_F^2}{2\pi^3}\int_0^{E_0}\left(E_0 - E_e\right)^2 E_e^2 \, dE_e = \frac{G_F^2}{2\pi^3}\left[\frac{E_0^5}{3} + \frac{E_0^5}{5} - 2\frac{E_0^5}{4}\right]$$

$$\Gamma = \frac{G_F^2 E_0^5}{60\pi^3}$$

SARGENT RULE: $\tau \propto E_0^{-5}$

e.g. μ^- and τ^- decay (see later)

By studying lifetimes for nuclear beta decay, we can determine the strength of the weak interaction in Fermi theory:

$$G_F^\beta = \left(1.136 \pm 0.003\right)\times 10^{-5}\ \mathrm{GeV}^{-2}$$

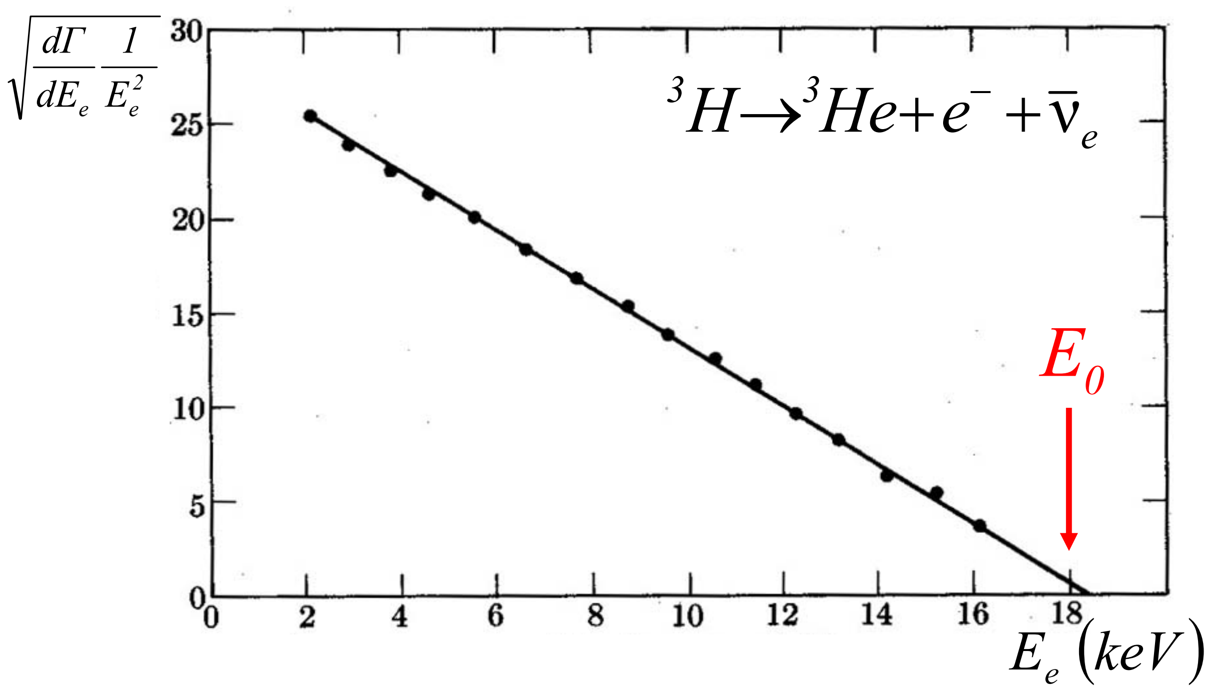
Beta-Decay Spectrum

$$\frac{d\Gamma}{dE_e} = \frac{G_F^2}{2\pi^3}\left(E_0 - E_e\right)^2 E_e^2$$

Plot of $\sqrt{\frac{d\Gamma}{dE_e}\frac{1}{E_e^2}}$ versus $\left(E_0 - E_e\right)$ is linear

$$\sqrt{\frac{d\Gamma}{dE_e}\frac{1}{E_e^2}} \propto \left(E_0 - E_e\right)$$

KURIE PLOT



ν Mass

- $m_\nu = 0$ (neglect mass of final state particles)
  End point of electron spectrum = E_0

- $m_\nu \neq 0$ (allow for mass of final state particles)

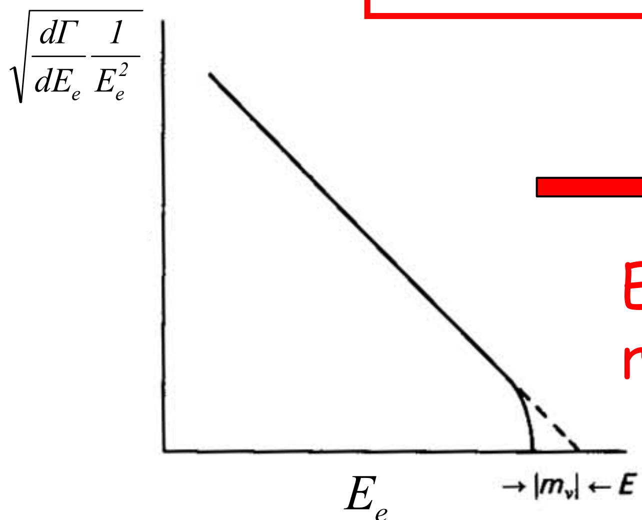
Density of states $dN = \frac{E^2}{(2\pi)^3}\, d\Omega\, dE \Rightarrow dN = \frac{p^2}{(2\pi)^3}\frac{E}{p}\, d\Omega\, dE$ (page 46)

$$\Rightarrow \quad \frac{d\Gamma}{dE_e} = \frac{G_F^2}{2\pi^3}(E_0 - E_e)^2 E_e^2 \sqrt{1 - \frac{m_\nu^2}{(E_0 - E_e)^2}} \sqrt{1 - \frac{m_e^2}{E_e^2}}$$

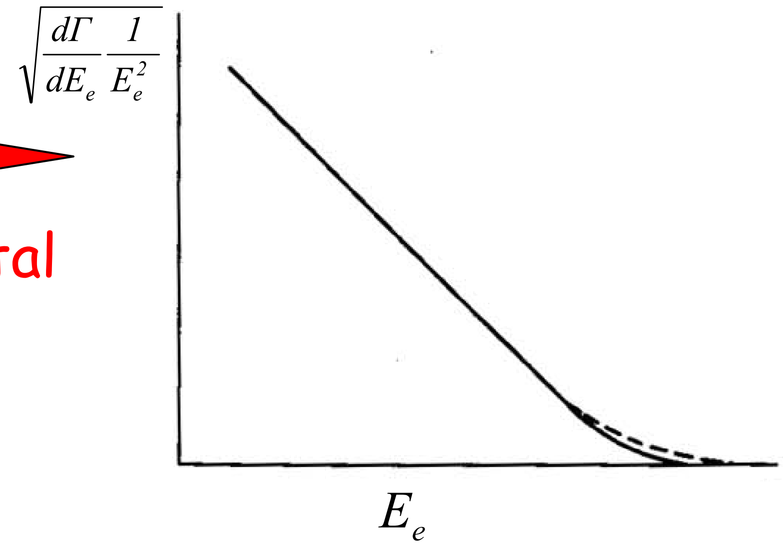
m_e known, m_ν small $\Rightarrow$ only significant effect is where $E_e \approx E_0$

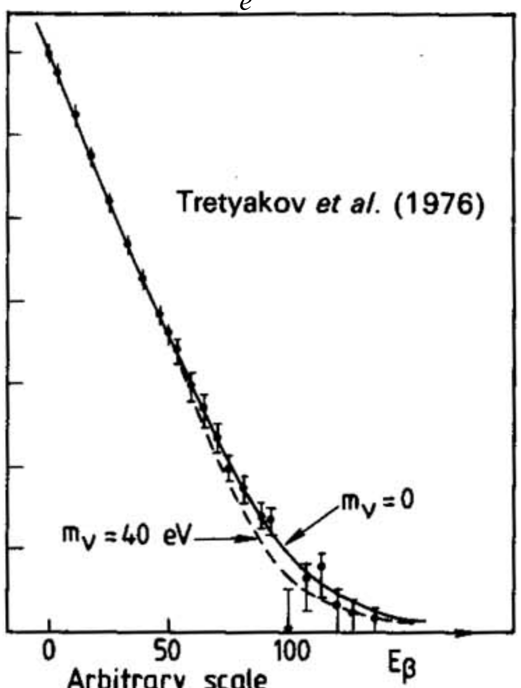
KURIE PLOT

$$\sqrt{\frac{1}{E_e^2}\frac{d\Gamma}{dE_e}} \propto \left(E_0 - E_e\right)\left[1 - {m_\nu^2}/{\left(E_0 - E_e\right)^2}\right]^{\frac{1}{4}}\left[1 - {m_e^2}/{E_e^2}\right]^{\frac{1}{4}}$$



Experimental resolution





Most recent results (1999)
Tritium β decay:

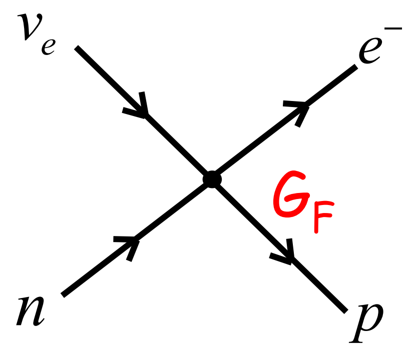
$m_{\nu e} < 3$ eV

If neutrinos have mass, $m_{\nu e} \ll m_e$

Why so small ?

Neutrino Scattering in Fermi Theory (Inverse β Decay).

$$\nu_e + n \rightarrow p + e^-$$



$$d\sigma = 2\pi \left| M_{fi} \right|^2 \frac{dN}{dE} = 2\pi\, G_F^2 \frac{E_e^2}{(2\pi)^3}\, d\Omega$$

$$\sigma = \frac{G_F^2\, s}{\pi}$$

Appendix F

where E_e is the energy of the e^- in the centre-of-mass system and $\sqrt{s}$ is the energy in the centre-of-mass system.

In the laboratory frame: $s = 2E_\nu m_n$ (see page 26)

$$\Rightarrow \quad \sigma \sim \left(E_\nu \text{ in MeV}\right) \times 10^{-43}\ \text{cm}^2$$

- ν's only interact WEAKLY ∴ have very small interaction cross-sections
- Here WEAK implies that you need approximately 50 light-years of water to stop a 1 MeV neutrino !

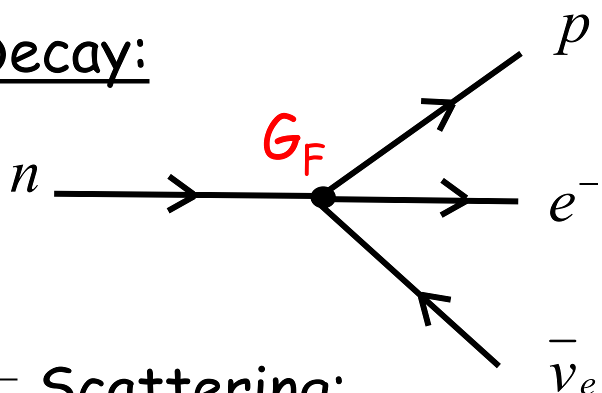
However, as $E_\nu \rightarrow \infty$ the cross-section can become very large. Violates maximum allowed value by conservation of probability at $\sqrt{s} = 740\ \text{GeV}$ (UNITARITY LIMIT).

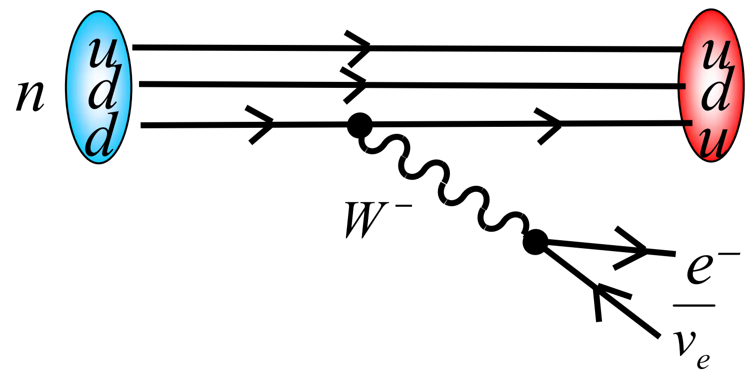
⇒ Fermi theory breaks down at high energies.

Weak Charged Current: $W^{\pm}$ Boson

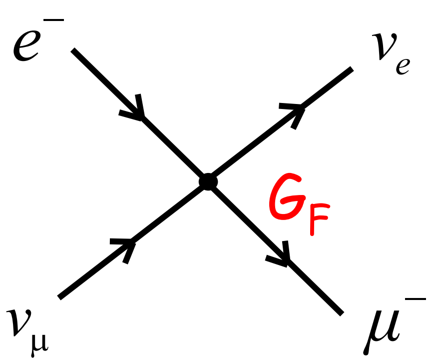
- Fermi theory breaks down at high energy
- True interaction described by exchange of **CHARGED $W^{\pm}$ BOSONS**
- Fermi theory is the low energy $\left(q^2 << m_W^2\right)$ **EFFECTIVE** theory of the **WEAK** interaction.

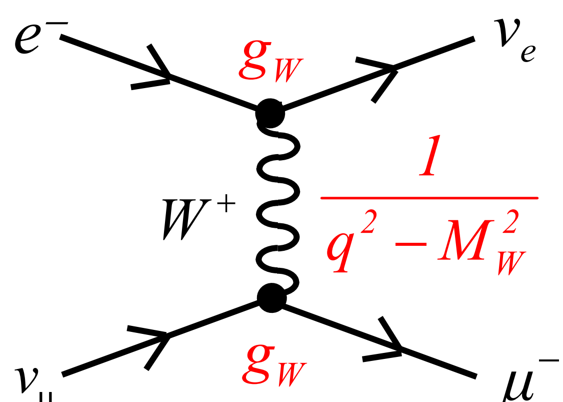
β Decay:





$\nu_\mu e^-$ Scattering:

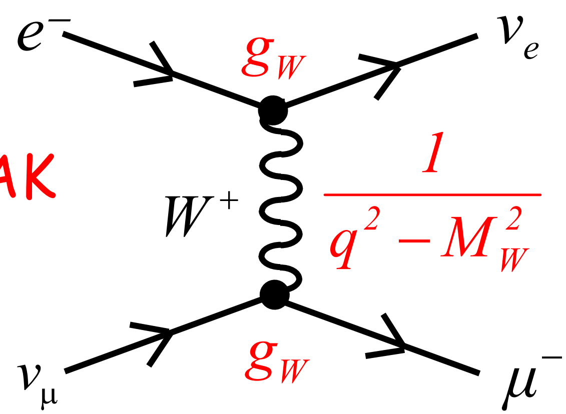




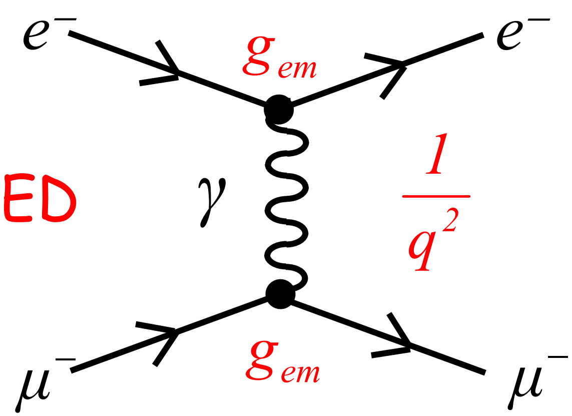
Compare WEAK and QED interactions:

WEAK

$$\alpha_W = \frac{g_W^2}{4\pi}$$



QED



$$g_{em} = e$$

$$\alpha = \frac{e^2}{4\pi}$$

CHARGED CURRENT WEAK INTERACTION

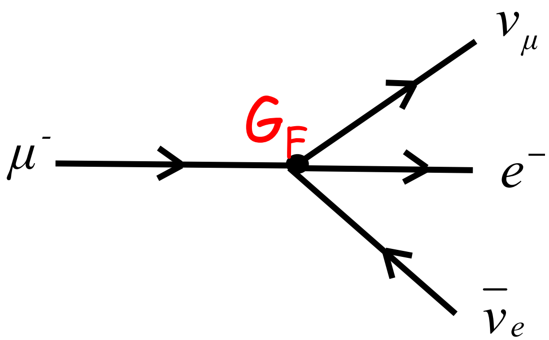
- At low energies, $q^2 << M_W^2$, propagator $\frac{1}{q^2 - M_W^2} \rightarrow \frac{1}{-M_W^2}$
  i.e. appears as POINT-LIKE interaction of Fermi theory.
- Massive propagator $\rightarrow$ short range

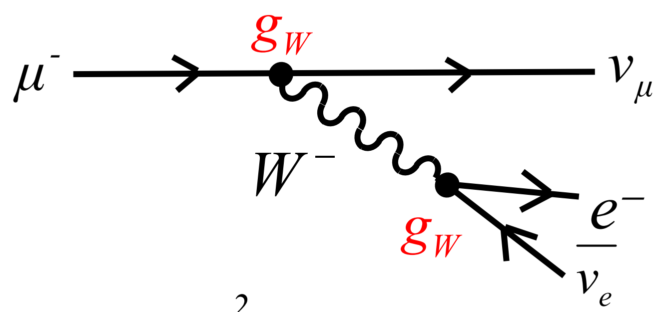
$$M_W = 80.4 \text{ GeV} \quad \Rightarrow \quad \text{Range} \approx \frac{1}{M_W} \sim 0.002 \text{ fm}$$

- Exchanged boson carries electromagnetic charge.

- FLAVOUR CHANGING – ONLY WEAK interaction changes flavour
- PARITY VIOLATING – ONLY WEAK interaction can violate parity conservation.

Compare Fermi theory c.f. massive propagator





For $q^2 << M_W^2$ compare matrix elements:

$$\frac{g_W^2}{M_W^2} \to G_F$$

G_F is small because m_W is large.

The precise relationship is:

$$\boxed{\frac{g_W^2}{8M_W^2} \to \frac{G_F}{\sqrt{2}}}$$

The numerical factors are partly of historical origin (see Perkins 4th ed., page 210).

$$M_W = 80.4\ \text{GeV} \quad \text{and} \quad G_F = 1.166 \times 10^{-5}\ \text{GeV}^{-2}$$

(see later to why different to G_F^β)

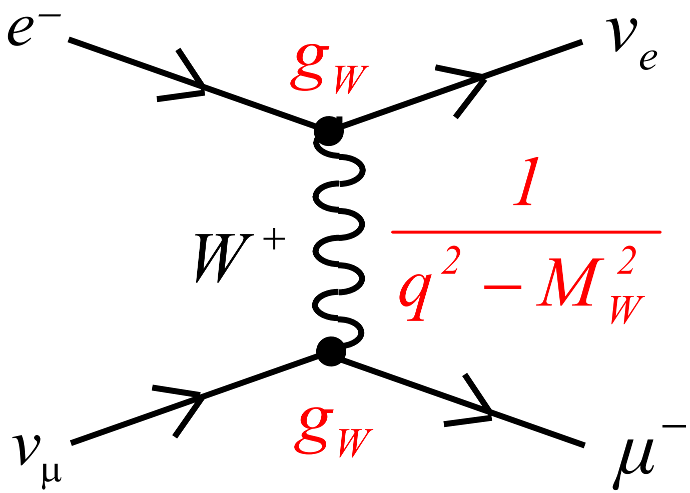
$$\Rightarrow \boxed{g_W = 0.65 \quad \text{and} \quad \alpha_W = \frac{g_W^2}{4\pi} \approx \frac{1}{30}}$$

$$\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}$$

The intrinsic strength of the WEAK interaction is GREATER than that of the electromagnetic interaction. At low energies (low q^2), it appears weak due to the massive propagator.

Neutrino Scattering with a Massive W Boson

Replace contact interaction by massive boson exchange diagram:



$$\frac{d\sigma}{dq^2} = \frac{1}{32\pi} \frac{g_W^4}{\left(q^2 - M_W^2\right)^2}$$

with $|q| = 2E\sin\vartheta/2$ where θ is the scattering angle.

(similar to page 48)

Integrate to give:

$$\sigma = \frac{G_F^2 s}{\pi} \qquad s \ll M_W^2$$

$$\sigma = \frac{G_F^2 M_W^2}{\pi} \qquad s \gg M_W^2$$

Appendix G

Total cross-section now well behaved at high energies.

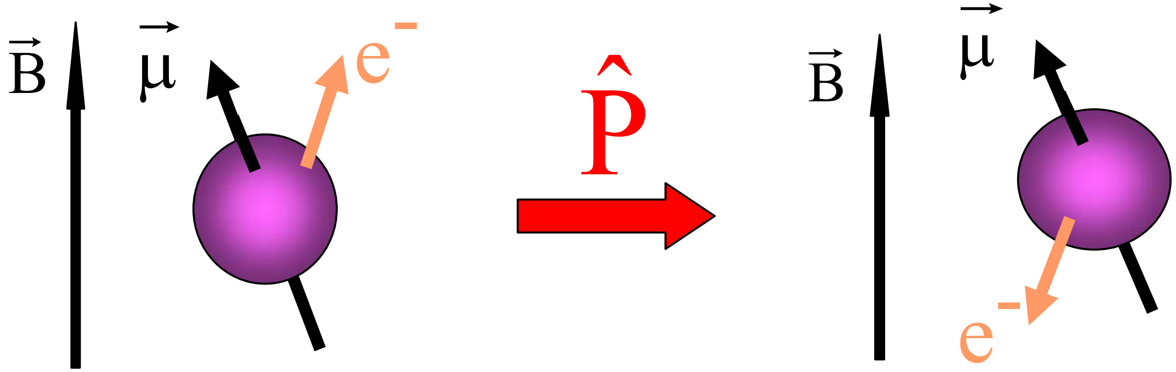
Parity Violation in Beta Decay

Parity violation was first observed in the β decay of ^{60}Co nuclei
(C.S.Wu et. al. Phys. Rev. 105 (1957) 1413)

$$^{60}Co \rightarrow {}^{60}Ni + e^- + \bar{\nu}_e$$

J=5 J=4

Align ^{60}Co nuclei with $\vec{B}$ field and look at direction of emission of electrons



Under parity: $\vec{r} \rightarrow -\vec{r}; \quad \vec{p} \rightarrow -\vec{p}$

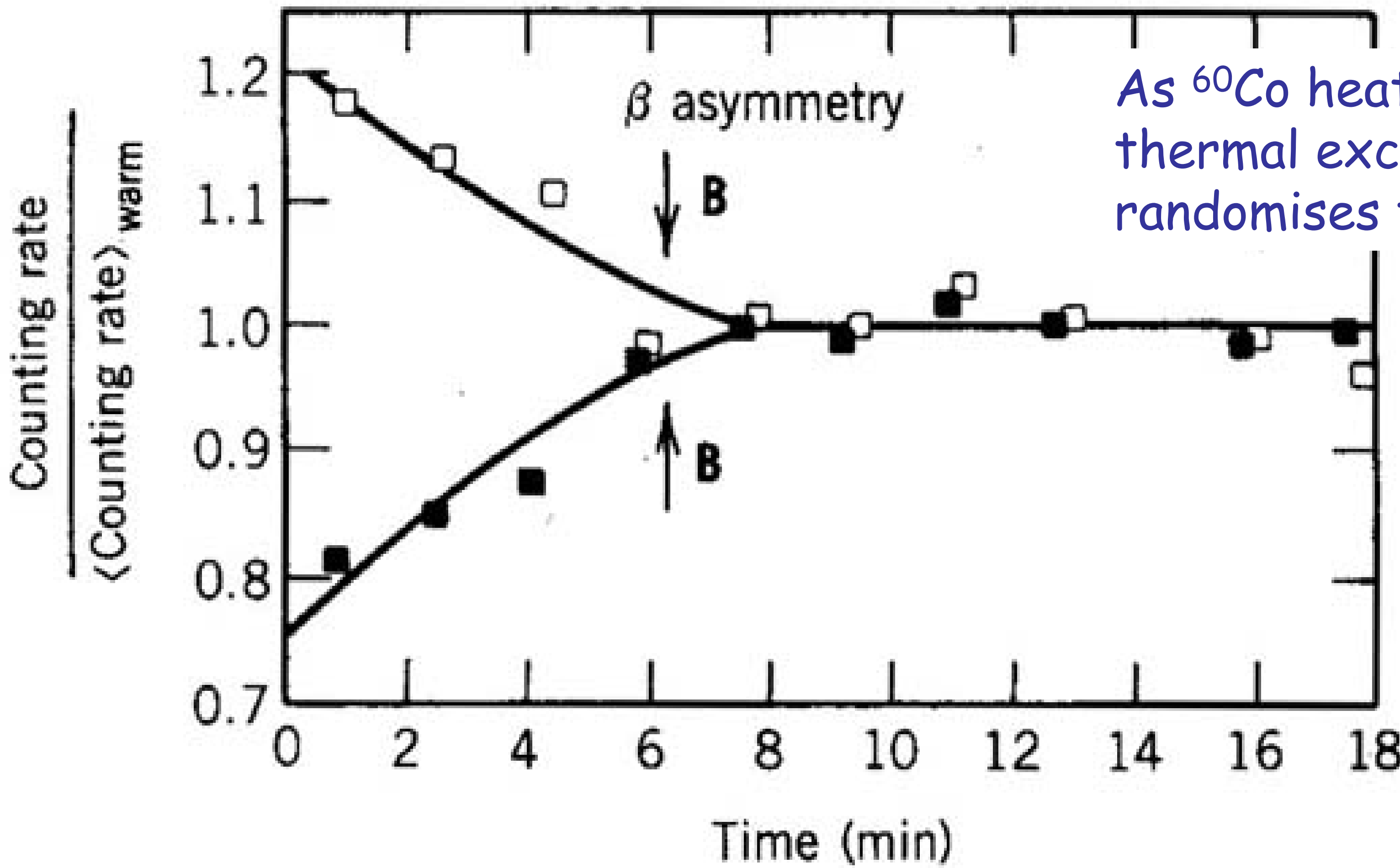
$$\vec{L} = \vec{r} \times \vec{p} \rightarrow \vec{L}; \quad \vec{\mu} \rightarrow \vec{\mu}$$

If PARITY is CONSERVED, expect equal numbers of electrons parallel and antiparallel to $\vec{B}$

Polarized

T = 0.01 K

Unpolarized



As ^{60}Co heats up, thermal excitation randomises the spins

Most electrons emitted opposite to direction of field

⇒ PARITY VIOLATION in β DECAY

Origin of Parity Violation

SPIN and HELICITY

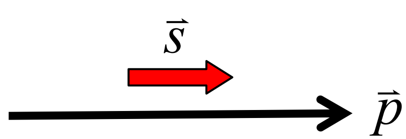
Consider a free particle of constant momentum, $\vec{p}$.

- Total angular momentum, $\vec{J} = \vec{L} + \vec{S}$, is ALWAYS conserved.
- The orbital angular momentum, $\vec{L} = \vec{r} \times \vec{p}$, is perpendicular to $\vec{p}$
- The spin angular momentum, $\vec{S}$, can be in any direction relative to $\vec{p}$
- The value of spin $\vec{S}$ along $\vec{p}$ is always CONSTANT.

Define the sign of the component of spin along the direction of motion as the

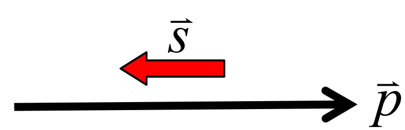
HELICITY

$$h = \frac{\vec{s} \cdot \vec{p}}{|\vec{p}|}$$



$$h = +1$$

"RIGHT-HANDED"



$$h = -1$$

"LEFT-HANDED"

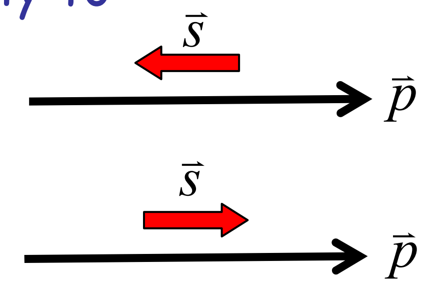
The WEAK interaction distinguishes between LEFT and RIGHT-HANDED states.

The weak interaction couples preferentially to

LEFT-HANDED PARTICLES

and

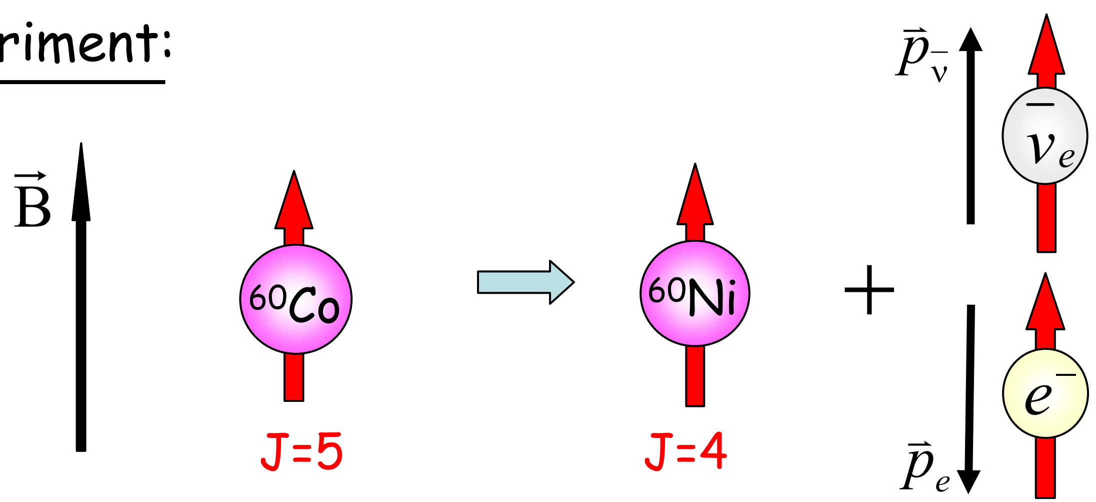
RIGHT-HANDED ANTIPARTICLES



In the ultra-relativistic (massless) limit, the coupling to RIGHT-HANDED particles vanishes.

i.e. even if RIGHT-HANDED ν's exist – they are unobservable !

^{60}Co experiment:

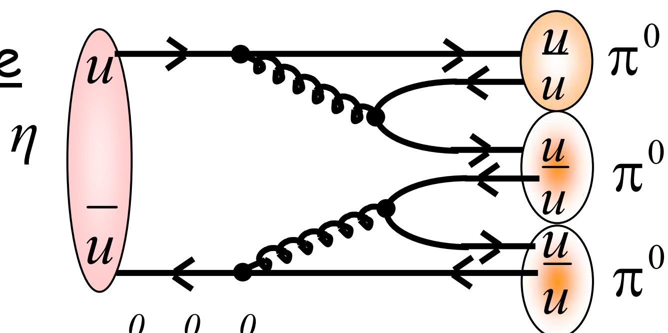


Parity Violation

The WEAK interaction treats LH and RH states differently and therefore can violate PARITY (i.e. the interaction Hamiltonian does not commute with $\hat{P}$).

PARITY is ALWAYS conserved in the STRONG/EM interactions

Example



$$\eta \rightarrow \pi^0\pi^0\pi^0$$

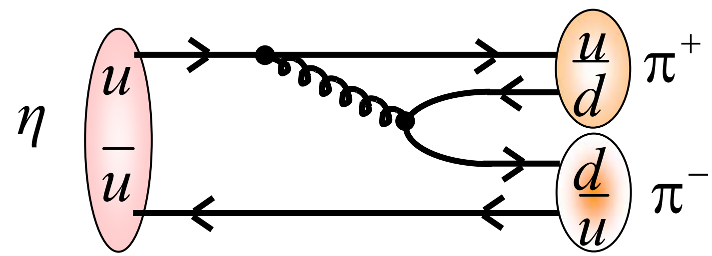
$$J^P \quad 0^- \quad 0^-0^-0^-$$

$$P \quad -1 \quad \underbrace{P(\pi^0)P(\pi^0)P(\pi^0)(-1)^{L_1}(-1)^{L_2}}$$

$$P = -1; \quad L_1 = L_2 = 0$$

PARITY CONSERVED

Branching fraction = 32%



$$\eta \rightarrow \pi^+\pi^-$$

$$J^P \quad 0^- \quad 0^-0^-$$

$$P \quad -1 \quad \underbrace{P(\pi^+)P(\pi^-)(-1)^L}$$

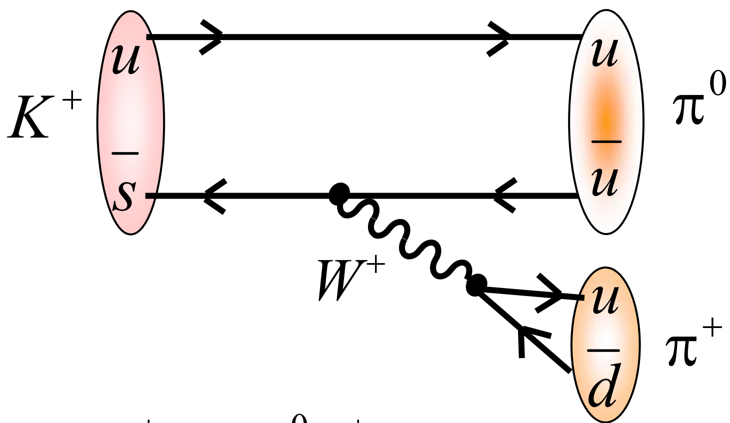
$$P = +1; \quad L = 0$$

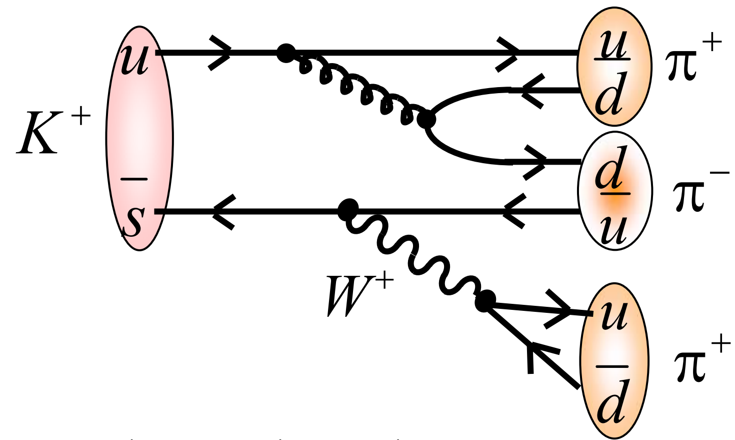
PARITY VIOLATED

Branching fraction < 0.1%

PARITY is USUALLY violated in the WEAK interaction but NOT ALWAYS !

<u>Example</u>





		$K^+ \to \pi^0\pi^+$
J^P	0^-	0^-0^-
P	-1	$P(\pi^0)P(\pi^+)(-1)^L$

$$P = +1; \quad L = 0$$

PARITY VIOLATED

Branching fraction ~ 21%

		$K^+ \to \pi^+\pi^-\pi^+$
J^P	0^-	$0^-0^-0^-$
P	-1	$P(\pi^+)P(\pi^-)P(\pi^+)(-1)^{L_1}(-1)^{L_2}$

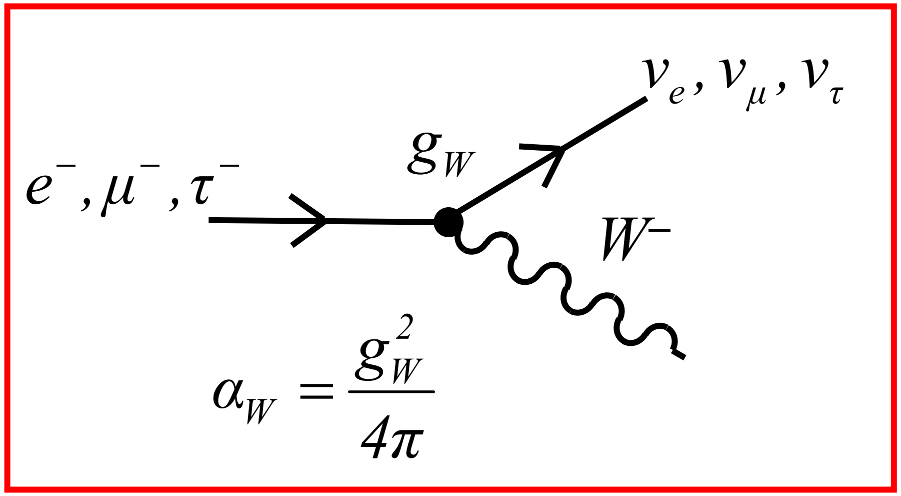
$$P = -1; \quad L_1 = L_2 = 0$$

PARITY CONSERVED

Branching fraction ~ 6%

The Weak CC Lepton Vertex

All weak charged current lepton interactions can be described by the W boson propagator and the weak vertex:



STANDARD MODEL
WEAK CC
LEPTON VERTEX

+antiparticles

- W Bosons only "couple" to the lepton and neutrino within the SAME generation

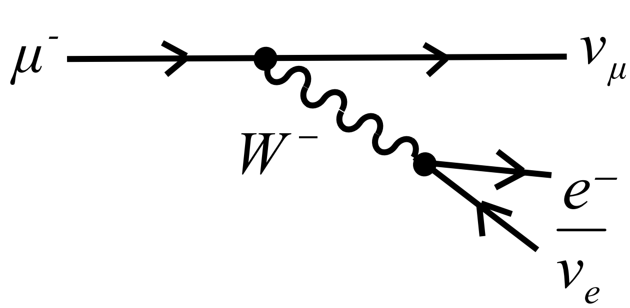
$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \quad \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \quad \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$$

e.g. no $We^-\nu_\mu$ coupling

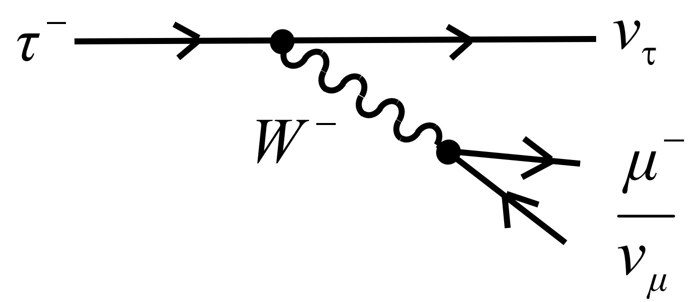
- Universal coupling constant g_W

Examples:

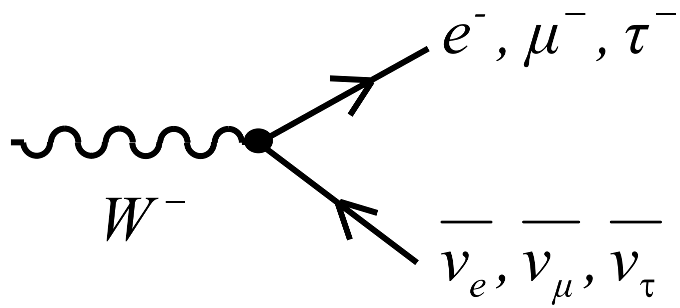
$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$



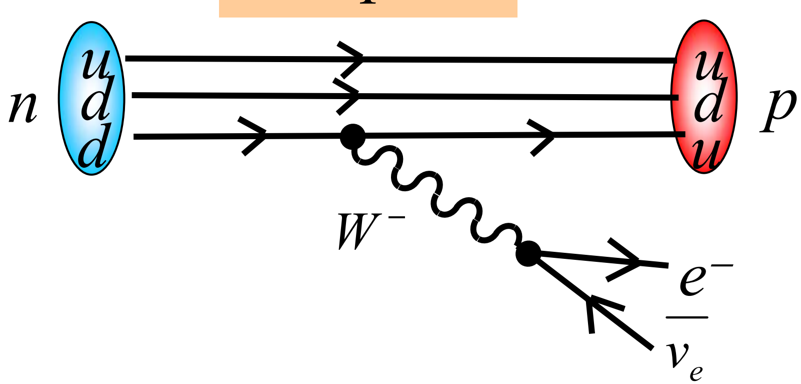
$\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$



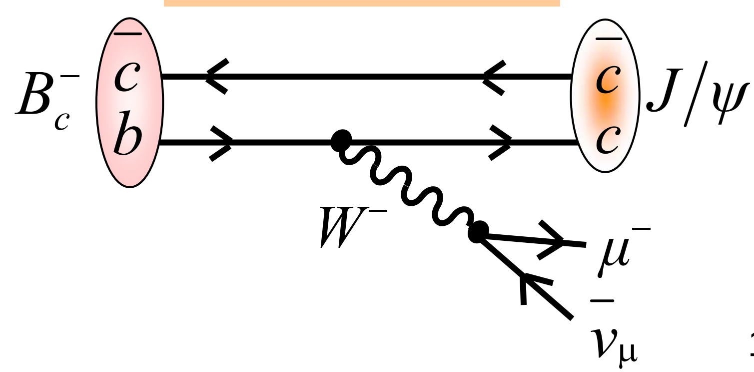
$W^- \rightarrow e^- \bar{\nu}_e, \mu^- \bar{\nu}_\mu, \tau^- \bar{\nu}_\tau$



$n \rightarrow p e^- \bar{\nu}_e$

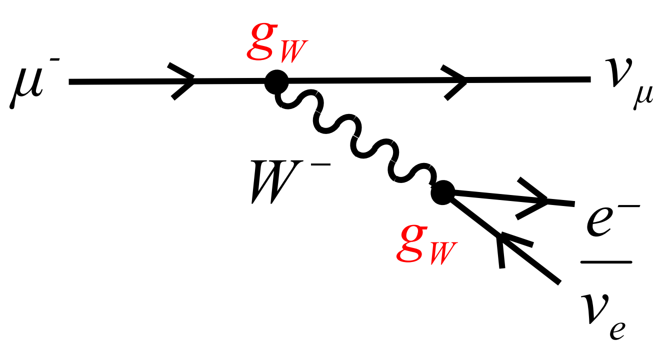


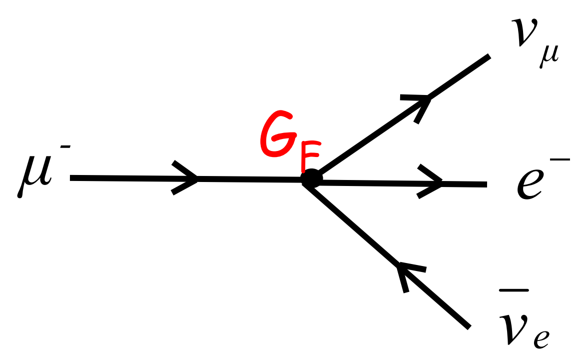
$B_c^- \rightarrow J/\psi \, \mu^- \bar{\nu}_\mu$



μ Decay

- Muons are fundamental leptons ($m_\mu \sim 206\ m_e$).
- Electromagnetic decay $\mu^- \to e^- \gamma$ is NOT observed; the EM interaction does not change flavour.
- Only the WEAK CC interaction changes flavour.
- Muons decay weakly: $\mu^- \to e^- \bar{\nu}_e \nu_\mu$





As $m_\mu^2 << M_W^2 \Rightarrow$ can use FERMI theory to calculate decay width (analogous to β decay).

FERMI theory gives decay width proportional to m_μ^5 (Sargent rule).

However, more complicated phase space integration (previously neglected kinetic energy of recoiling nucleus) gives

$$\Gamma_\mu = \frac{1}{\tau_\mu} = \frac{G_F^2}{192\pi^3} m_\mu^5$$

- Muon mass and lifetime known with high precision.

$$\tau_\mu = (2.19703 \pm 0.00004) \times 10^{-6}\ \mathrm{s}$$

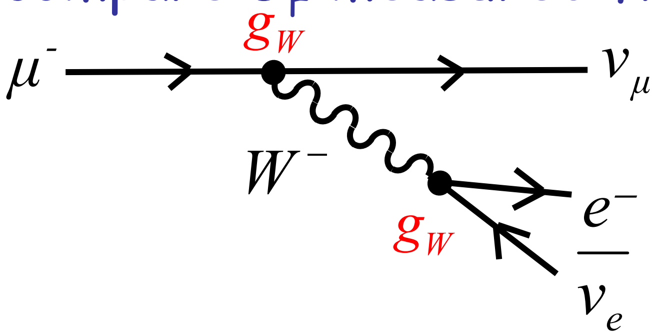
- Use muon decay to fix strength of WEAK interaction G_F

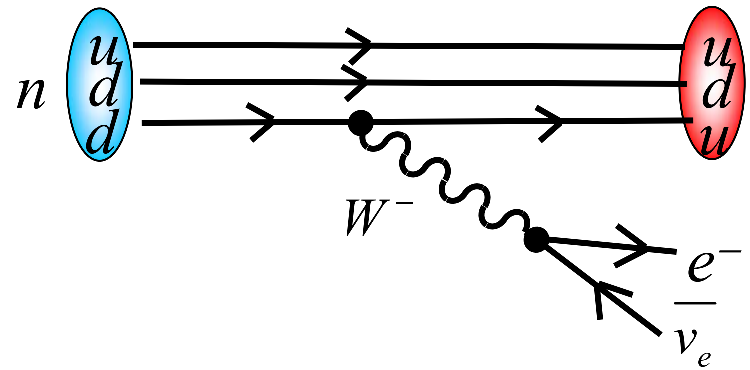
$$G_F = (1.16632 \pm 0.00002) \times 10^{-5}\ \mathrm{GeV}^{-2}$$

- G_F is one of the best determined fundamental quantities in particle physics.

Universality of Weak Coupling

Can compare G_F measured from μ^- decay with that from β decay.





From muon decay measure:

$$G_F^{\mu} = (1.16632 \pm 0.00002) \times 10^{-5}\ \mathrm{GeV}^{-2}$$

From β decay measure:

$$G_F^{\beta} = (1.136 \pm 0.003) \times 10^{-5}\ \mathrm{GeV}^{-2}$$

Ratio

$$\frac{G_F^{\beta}}{G_F^{\mu}} = 0.974 \pm 0.003$$

Conclude that the strength of the weak interaction is **ALMOST** the same for leptons as for quarks. We will come back to the origin of this difference $(\cos\vartheta_C)$

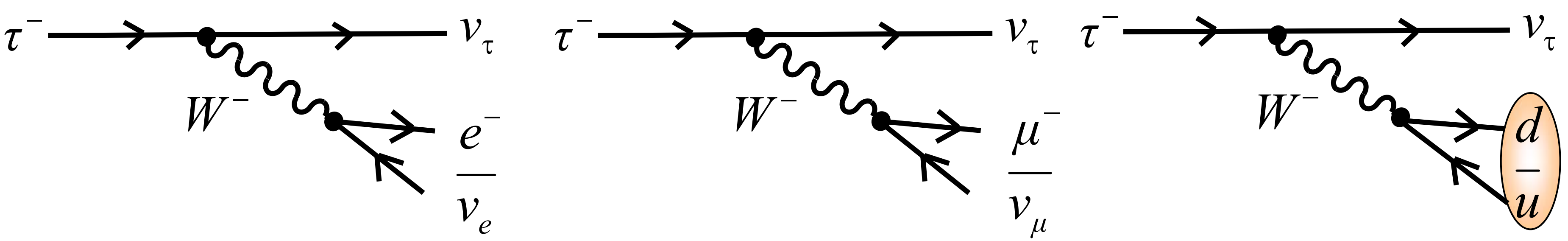
τ Decay

The τ mass is relatively large

$$m_\tau = (1.777 \pm 0.0003)\,\text{GeV}$$

and as
$$m_\tau > \{m_\mu, m_\pi, m_\rho, \ldots\}$$
there are a number of possible decay modes.

Examples



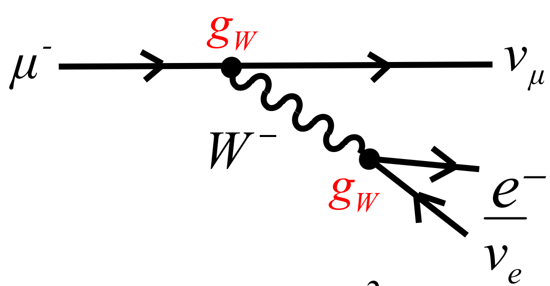
Tau branching fractions:

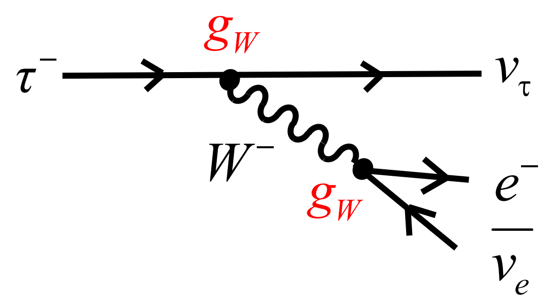
$$\tau^- \to e^- \bar{\nu}_e \nu_\tau \qquad (17.8 \pm 0.1\%)$$
$$\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau \qquad (17.3 \pm 0.1\%)$$
$$\tau^- \to hadrons \qquad (64.7 \pm 0.2\%)$$

Lepton Universality

Test whether all leptons have the same WEAK coupling from measurements of the decay rates and branching fractions.

Compare





$$\frac{1}{\tau_\mu} = \Gamma_{\mu\to e} = \frac{G_F^2}{192\pi^3} m_\mu^5$$

$$\frac{1}{\tau_\tau} = \frac{1}{B(\tau\to e)}\Gamma_{\tau\to e} = \frac{1}{0.178}\frac{G_F^2}{192\pi^3} m_\tau^5$$

$B(\tau\to e) = (17.8 \pm 0.1\%)$

If universal strength of WEAK interaction, expect

$$\frac{\tau_\tau}{\tau_\mu} = 0.178\,\frac{m_\mu^5}{m_\tau^5}$$

m_μ, m_τ, τ_μ are all measured precisely

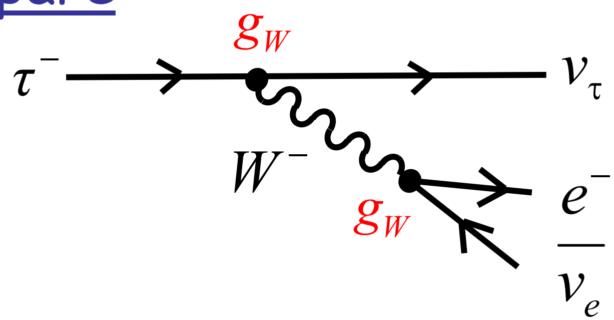
$$\begin{cases} m_\mu = 105.658\ \text{MeV} \\ m_\tau = (1777.0 \pm 0.3)\ \text{MeV} \\ \tau_\mu = (2.19703 \pm 0.00004)\times 10^{-6}\ \text{s} \end{cases}$$

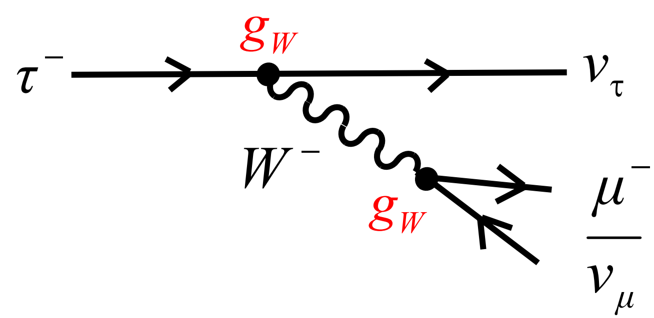
Predict $\tau_\tau = (2.91 \pm 0.01)\times 10^{-13}$ s

Measure $\tau_\tau = (2.91 \pm 0.01)\times 10^{-13}$ s

➡ SAME WEAK CC COUPLING FOR μ AND τ

Also compare





IF same couplings expect:

$$\frac{B(\tau^- \to \mu^- \overline{\nu}_\mu \nu_\tau)}{B(\tau^- \to e^- \overline{\nu}_e \nu_\tau)} = 0.9726$$

(the small difference is due to the slight reduction in phase space due to the non-negligible muon mass).

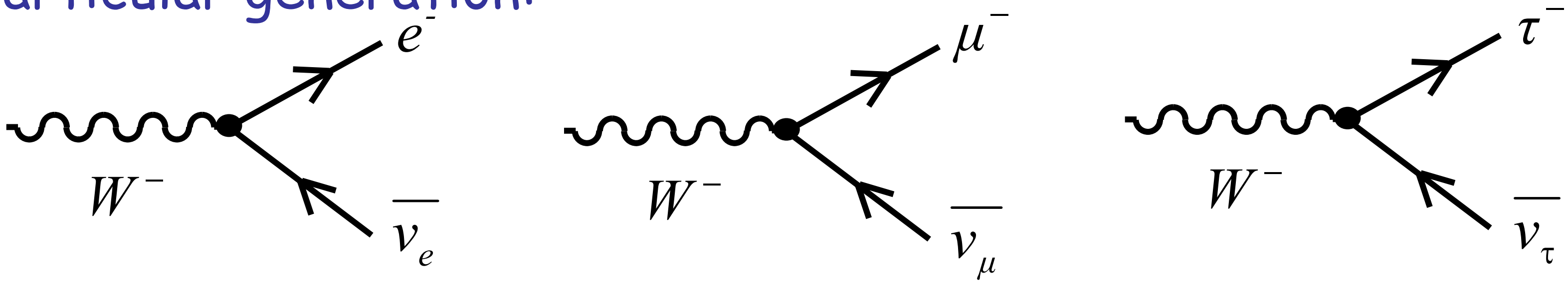
The observed ratio $\frac{B(\tau^- \to \mu^- \overline{\nu}_\mu \nu_\tau)}{B(\tau^- \to e^- \overline{\nu}_e \nu_\tau)} = 0.974 \pm 0.005$ is consistent with the prediction.

$\Rightarrow$ SAME WEAK CC COUPLING FOR e, μ AND τ

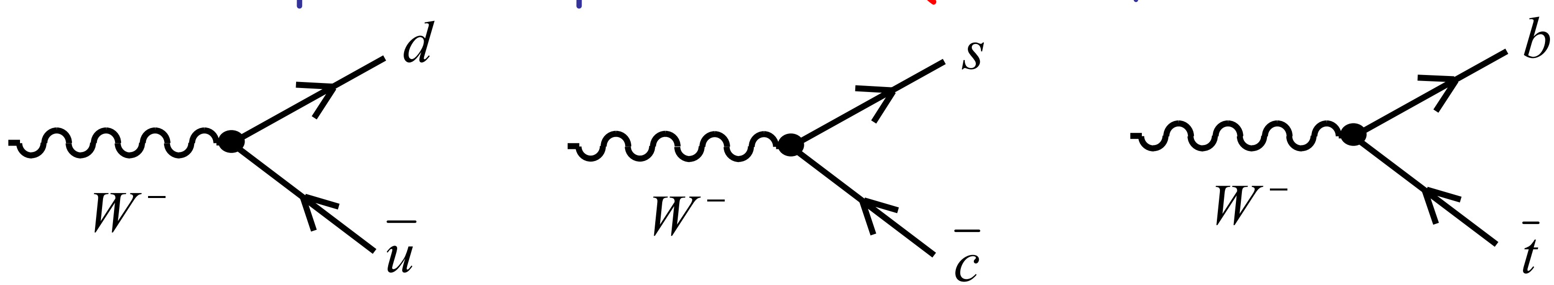
➡ **LEPTON UNIVERSALITY**

Weak Interactions of Quarks

In the Standard Model, the leptonic weak couplings take place within a particular generation:



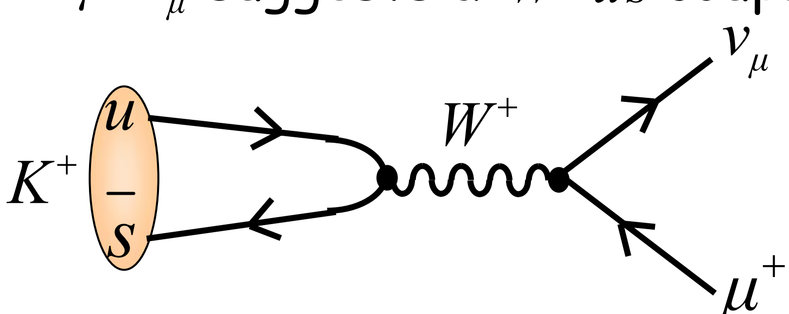
Natural to expect same pattern for QUARKS, i.e.



Unfortunately, not that simple !!

<u>Example:</u>

The decay $K^+ \rightarrow \mu^+ \nu_\mu$ suggests a $W^+ u \overline{s}$ coupling



Cabibbo Mixing Angle

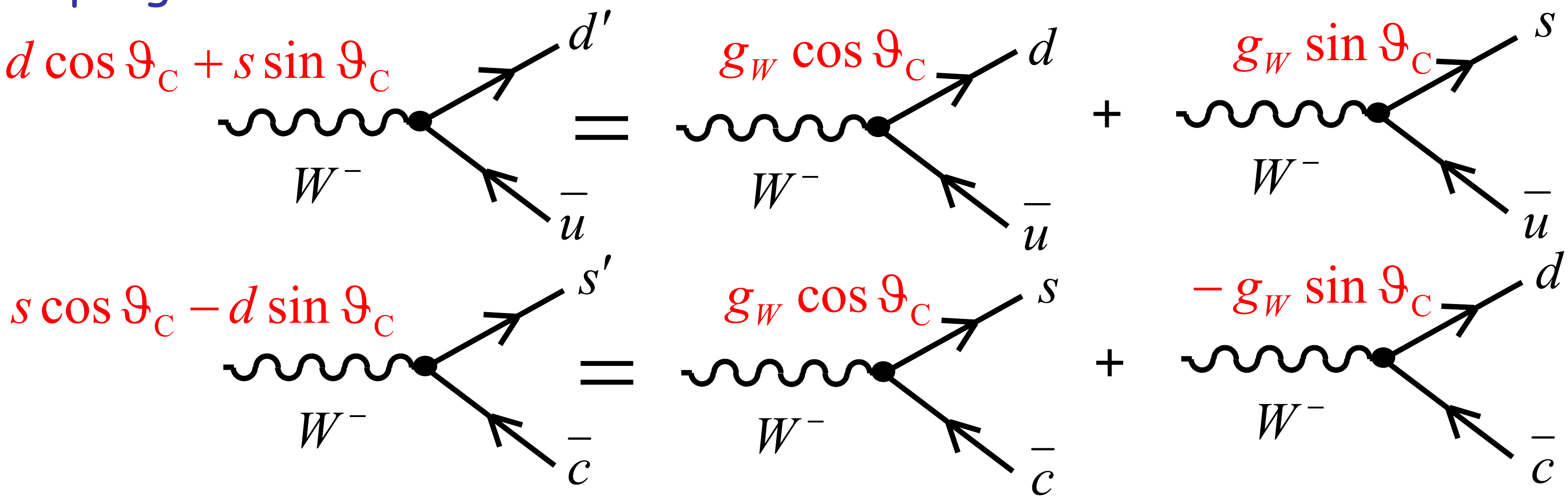
Four-Flavour Quark Mixing

- The states which take part in the **WEAK** interaction are **ORTHOGONAL** combinations of the states of definite flavour (d, s)
- For 4 flavours, {d, u, s and c}, the mixing can be described by a single parameter

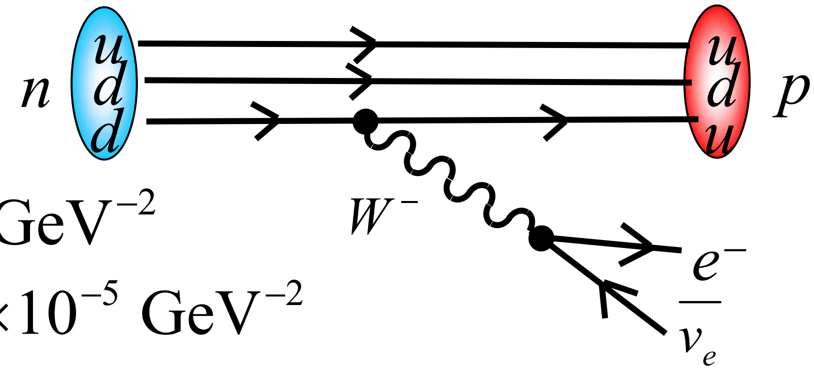
$\Rightarrow$ **CABIBBO ANGLE** $\vartheta_C \approx 13^o$ (from experiment)

Weak Eigenstates $\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\vartheta_C & \sin\vartheta_C \\ -\sin\vartheta_C & \cos\vartheta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$ Flavour Eigenstates

Couplings become:



Example: Nuclear β decay



Recall $G_F^{\beta} = (1.136 \pm 0.003) \times 10^{-5}\ \mathrm{GeV}^{-2}$

$G_F^{\mu} = (1.16632 \pm 0.00002) \times 10^{-5}\ \mathrm{GeV}^{-2}$

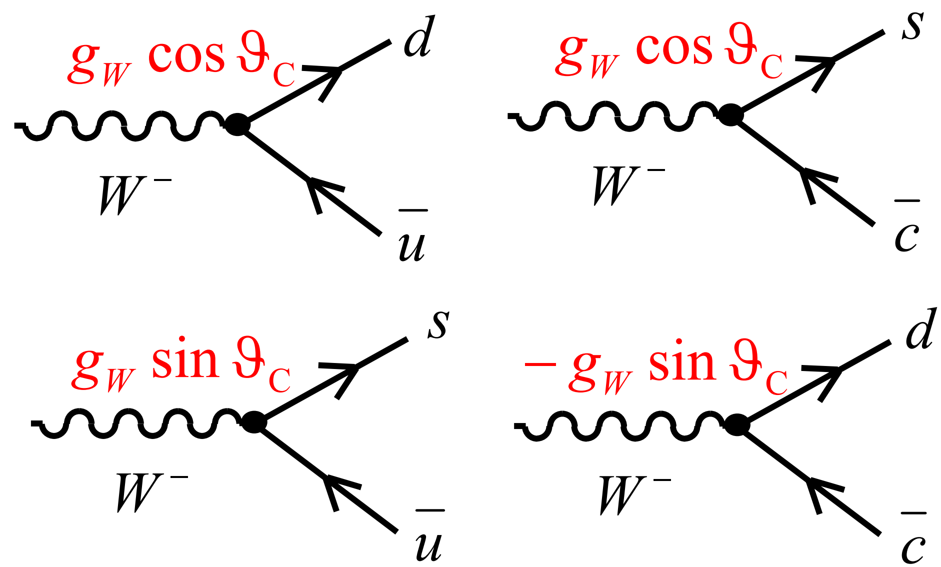
- strength of *ud* coupling $g_W \cos\vartheta_C$
- $\left(G_F^{\beta}\right)^2 \propto |M|^2 \propto \cos^2\vartheta_C$
- Hence, expect $G_F^{\beta} = G_F^{\mu}\cos\vartheta_C$
- It works, $1.136 = 1.16632 \times \cos 13^{\circ}$ $\vartheta_C \approx 13^{\circ}$

Cabibbo Favoured

$|M|^2 \propto \cos^2\vartheta_C$

Cabibbo Suppressed

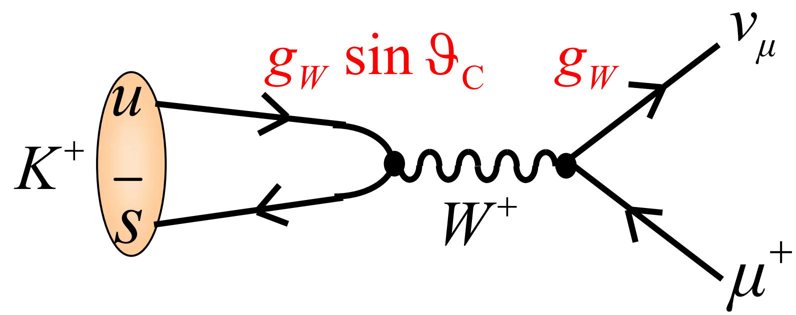
$|M|^2 \propto \sin^2\vartheta_C$



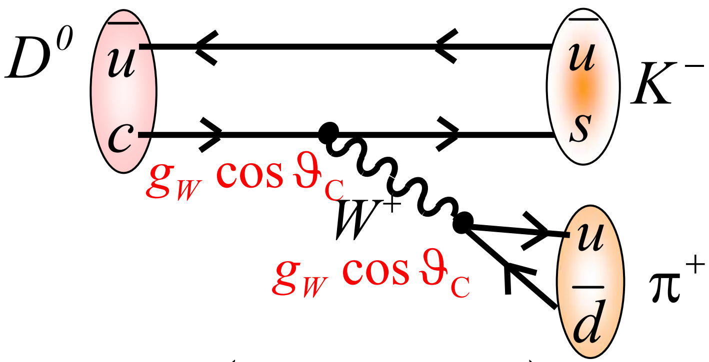
Example: $K^+ \rightarrow \mu^+ \nu_\mu$

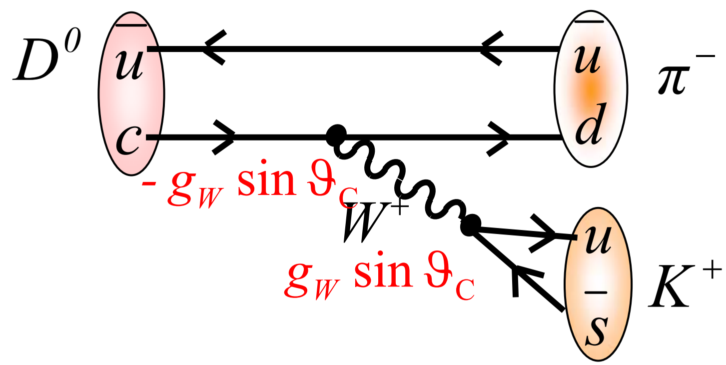
$u\bar{s}$ coupling $\Rightarrow$ Cabibbo suppressed

$$|M|^2 \propto \sin^2 \vartheta_C$$



Example: $D^0 \rightarrow K^-\pi^+$, $D^0 \rightarrow K^+\pi^-$





Expect

$$\frac{\Gamma(D^0 \rightarrow K^+\pi^-)}{\Gamma(D^0 \rightarrow K^-\pi^+)} = \frac{\sin^4\vartheta_C}{\cos^4\vartheta_C} \approx 0.0028$$

Measure

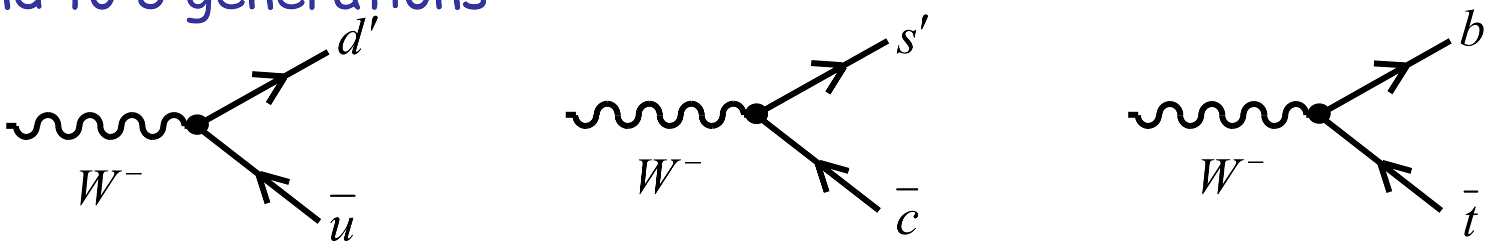
$$0.0038 \pm 0.0008$$

$D^0 \rightarrow K^+\pi^-$ is DOUBLY Cabibbo suppressed

CKM Matrix

Cabibbo-Kobayashi-Maskawa Matrix

Extend to 3 generations

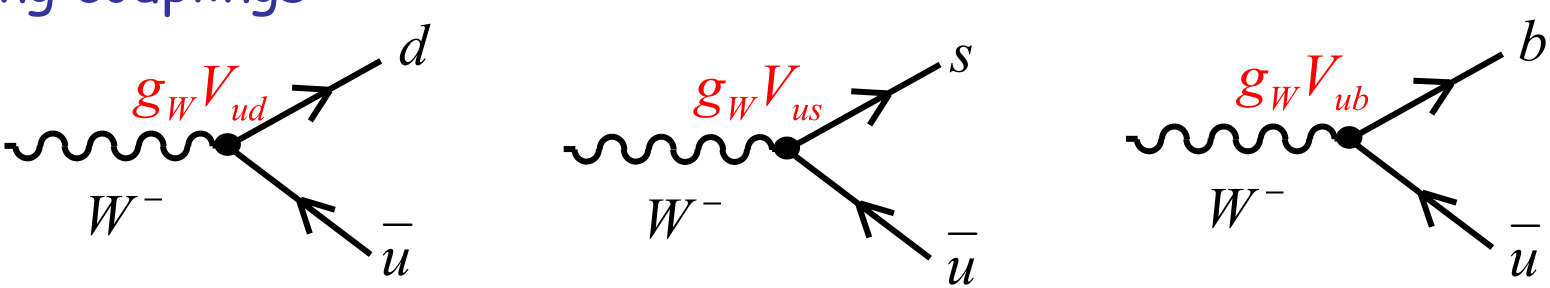


Weak Eigenstates $\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$ Flavour Eigenstates

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} \cos\vartheta_C & \sin\vartheta_C & 0.01 \\ -\sin\vartheta_C & \cos\vartheta_C & 0.05 \\ 0.01 & -0.05 & 1 \end{pmatrix} \approx \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & \lambda^3 \\ -\lambda & 1-\frac{\lambda^2}{2} & \lambda^2 \\ \lambda^3 & -\lambda^2 & 1 \end{pmatrix}$$

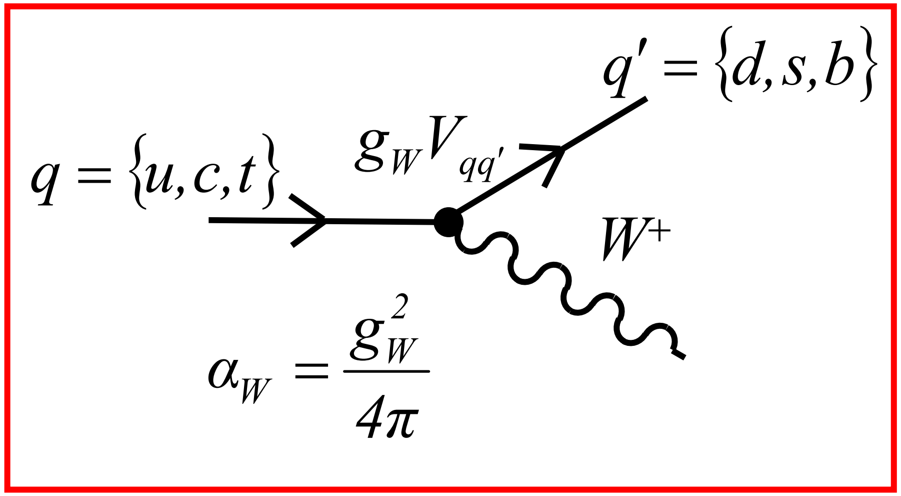
$$\lambda = \sin\vartheta_C$$

Giving couplings



The Weak CC Quark Vertex

All weak charged current quark interactions can be described by the W boson propagator and the weak vertex:



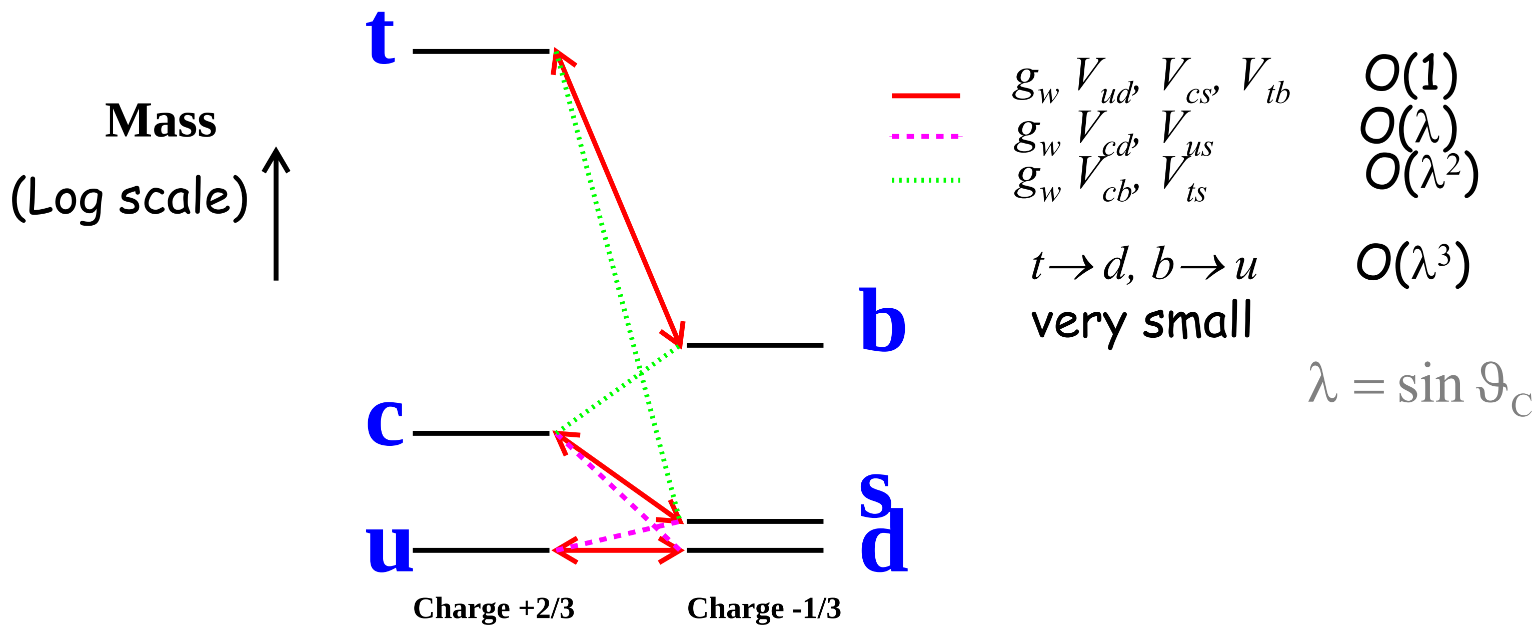
STANDARD MODEL WEAK CC QUARK VERTEX

+antiparticles

- W bosons CHANGE quark flavour
- W likes to couple to quarks in the SAME generation, but quark state mixing means that CROSS-GENERATION coupling can occur.

W-Lepton coupling constant → g_W

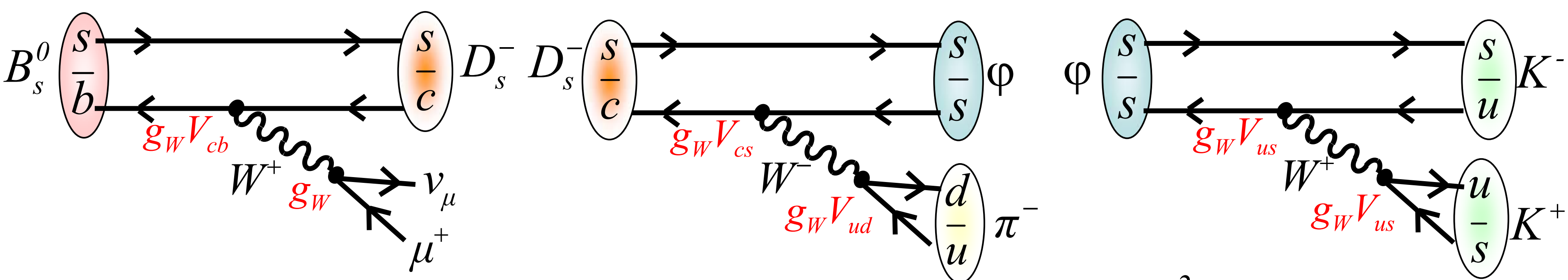
W-Quark coupling constant → $g_W V_{CKM}$



Example: $B_s^0 \to D_s^- \mu^+ \nu_\mu$

$\quad\quad \hookrightarrow \varphi\, \pi^-$

$\quad\quad\quad \hookrightarrow K^+ K^-$



$|M|^2 \propto g_W^4 \sin^4 \vartheta_C$ $\quad |M|^2 \propto g_W^4 \cos^4 \vartheta_C$ $\quad |M|^2 \propto g_W^4 \sin^4 \vartheta_C$

Summary

WEAK INTERACTION (CHARGED CURRENT)

- Weak force mediated by massive W bosons

$$M_W = (80.423 \pm 0.038)\,\mathrm{GeV}$$

- Weak force intrinsically stronger than EM interaction

$$\alpha_W \approx \frac{1}{30} \quad \mathrm{cf} \quad \alpha_{em} \approx \frac{1}{137}$$

- Universal coupling to quarks and leptons
- Quarks take part in the interaction as mixtures of the flavour eigenstates
- Parity can be VIOLATED due to the HELICITY structure of the interaction
- Strength of the weak interaction given by

$$G_F^{\mu} = (1.16632 \pm 0.00002) \times 10^{-5}\ \mathrm{GeV}^{-2}$$

from muon decay.