Section VI
QCD

QCD

QUANTUM ELECTRODYNAMICS: is the quantum theory of the electromagnetic interaction.

- mediated by massless photons
- photon couples to electric charge
- strength of interaction: $\langle \psi_f | \hat{H} | \psi_i \rangle \propto \sqrt{\alpha}$

$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$$

QUANTUM CHROMODYNAMICS: is the quantum theory of the strong interaction.

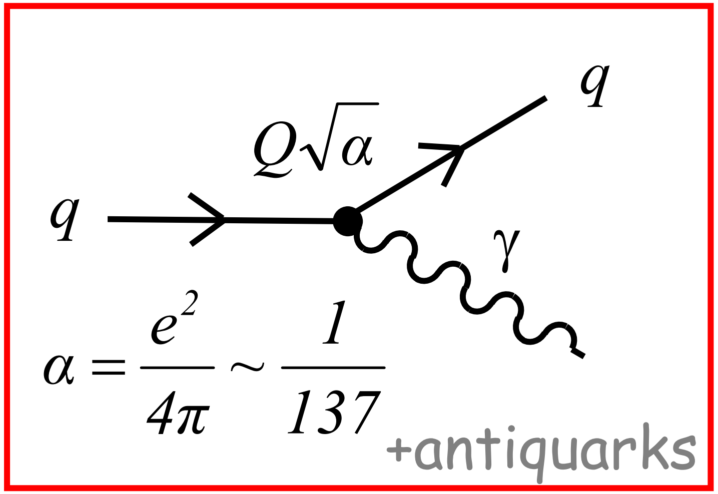
- mediated by massless gluons
- gluon couples to "strong" charge
- only quarks have non-zero "strong" charge, therefore only quarks feel the strong interaction.
- strength of interaction: $\langle \psi_f | \hat{H} | \psi_i \rangle \propto \sqrt{\alpha_s}$

$$\alpha_s = \frac{g_s^{\ 2}}{4\pi} \sim 1$$

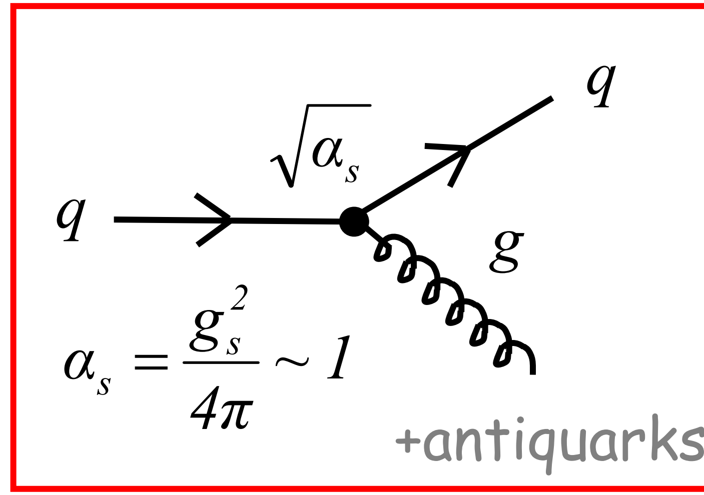
The Strong Vertex

Basic QCD interaction looks like a stronger version of QED:

QED



QCD



- The coupling constant, g_s, couples to the “strong” charge.
- Energy, momentum, angular momentum and charge always conserved.
- QCD vertex NEVER changes quark flavour
- QCD vertex ALWAYS conserves PARITY

Colour

QED:

- Charge of QED is electric charge
- Electric charge - conserved quantum number

QCD:

- Charge of QCD is called "COLOUR"
- COLOUR is a conserved quantum number with 3 VALUES labelled RED, GREEN and BLUE.

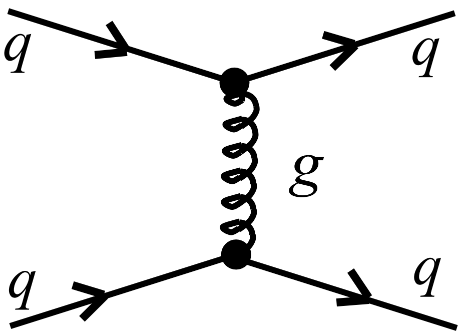
Quarks carry	"COLOUR"	$r \quad g \quad b$
Antiquarks carry	"ANTI-COLOUR"	$\bar{r} \quad \bar{g} \quad \bar{b}$

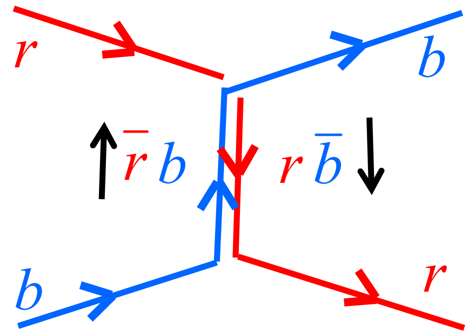
- Leptons, γ, $W^{\pm}$, Z^0 DO NOT carry colour, i.e. "have zero colour charge"
- Leptons DO NOT interact via the STRONG interaction.

Gluons

Gluons are MASSLESS spin 1 bosons, which carry the colour quantum number (unlike γ in QED which is charge neutral).

Consider a red quark scattering off a blue quark. Colour is exchanged, but always conserved.





Expect 9 gluons (3 colours x 3 anticolours): $r\bar{b}$ $r\bar{g}$ $g\bar{r}$ $g\bar{b}$ $b\bar{g}$ $b\bar{r}$
$r\bar{r}$ $g\bar{g}$ $b\bar{b}$

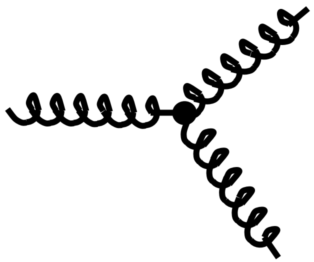
However: Real gluons are orthogonal linear combinations of the above states. The combination $\frac{1}{\sqrt{3}}(r\bar{r}+g\bar{g}+b\bar{b})$ is colourless and does not take part in the strong interaction.

$\Rightarrow$ 8 COLOURED GLUONS

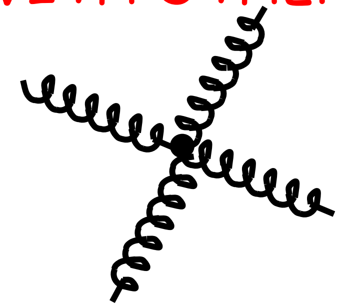
Gluon Self-Interactions

QCD looks like a stronger version of QED. However, there is one BIG difference and that is GLUONS carry colour "charge".

$\Rightarrow$ GLUONS CAN INTERACT WITH OTHER GLUONS

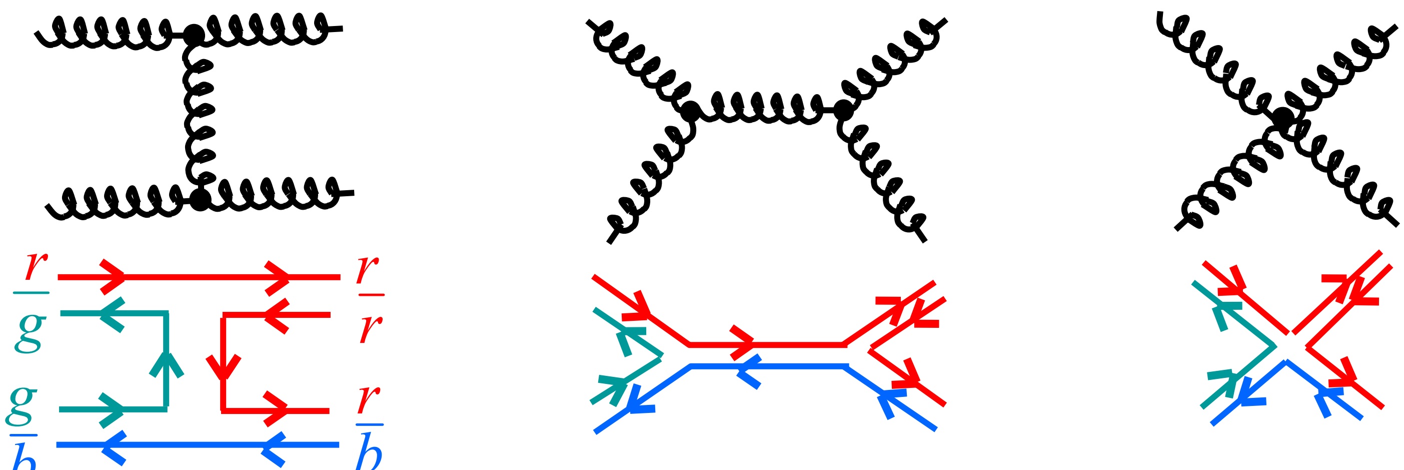


3 GLUON VERTEX



4 GLUON VERTEX

Example: Gluon-gluon scattering $gg \to gg$



e.g. $r\bar{g} + g\bar{b} \to r\bar{r} + r\bar{b}$

QCD Potential

QED Potential:

$$V_{QED} = -\frac{\alpha}{r}$$

QCD Potential:

At short distances QCD potential looks similar

$$V_{QCD} = -\frac{4}{3}\frac{\alpha_s}{r}$$

apart from 4/3 factor.

Note: the colour factor 4/3 arises because more than one gluon can participate in the process $q \rightarrow qg$. Obtain colour factor from averaging over initial colour states and summing over final/intermediate colour states.

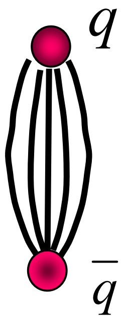
Confinement

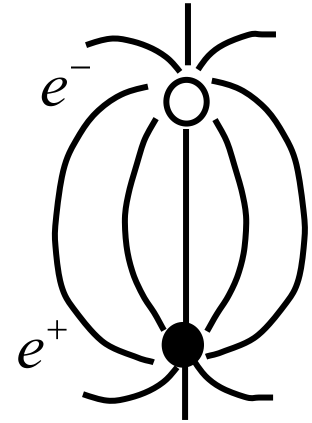
NEVER OBSERVE single FREE quarks or gluons.

- Quarks are always confined within hadrons
- This is a consequence of the strong interaction of gluons.

Qualitatively, compare QCD with QED:

QCD
Colour field





QED
Electric field

Self interactions of the gluons squeezes the lines of force into a narrow tube or STRING. The string has a "tension" and as the quarks separate the string stores potential energy.

Energy stored per unit length in field ~ constant

$$V(r) \propto r$$

Energy required to separate two quarks is infinite. Quarks always come in combinations with zero net colour charge $\Rightarrow$ CONFINEMENT.

How Strong is Strong ?

QCD potential between quarks has two components:

- Short range, Coulomb-like term: $-\frac{4}{3}\frac{\alpha_s}{r}$
- Long range, linear term: $+kr$

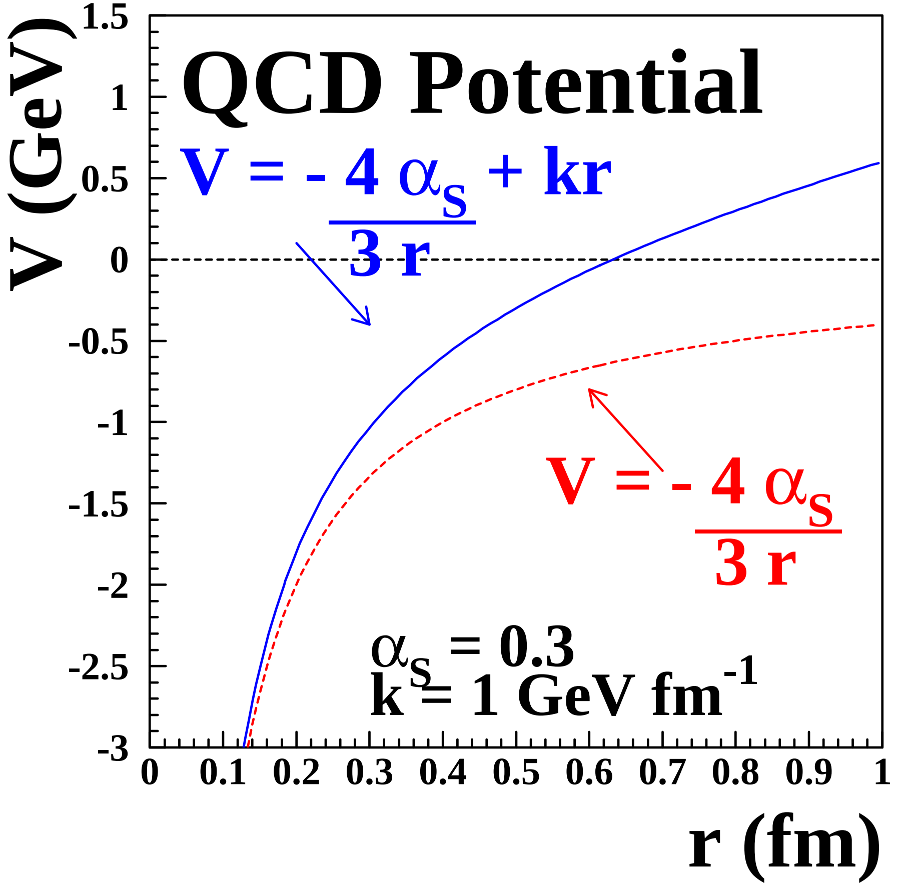
$$V_{QCD} = -\frac{4}{3}\frac{\alpha_s}{r} + kr$$

with $k \approx 1\ GeV/fm$

$$F = -\frac{dV}{dr} = \frac{4}{3}\frac{\alpha_s}{r^2} + k$$

at large r $F = k \approx \frac{1.6\times10^{-10}}{10^{-15}}\ N$

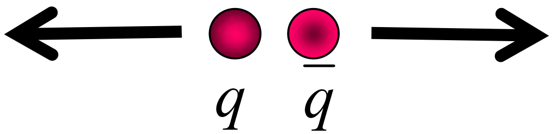
$$= 160000\ \ N$$

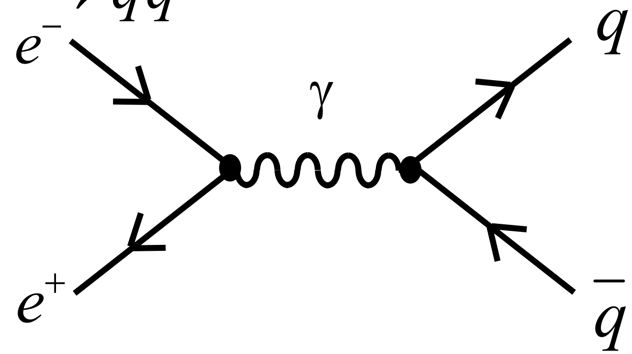


Equivalent to ~150 people

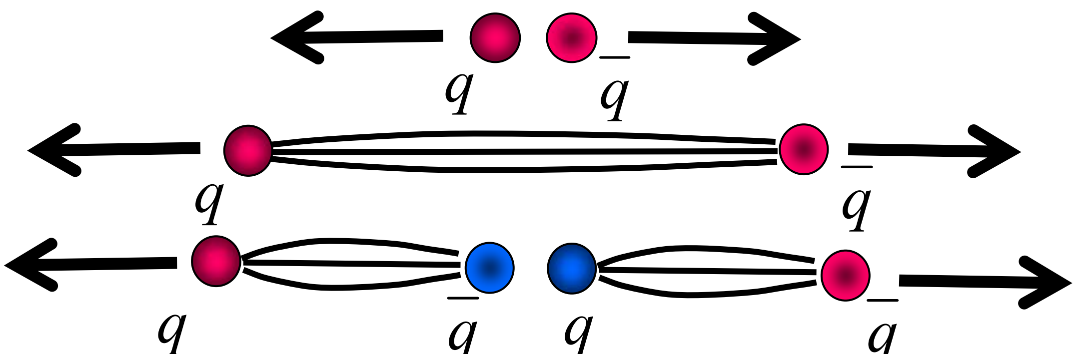
Jets

Consider the $q\bar{q}$ pair produced in $e^+e^- \to q\bar{q}$

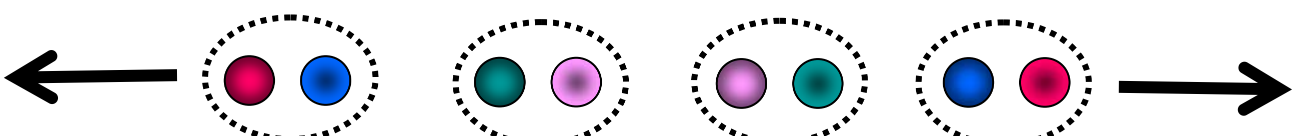




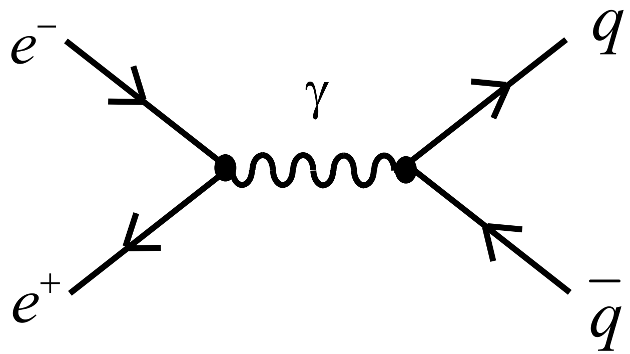
As the quarks separate, the potential energy in the colour field ("string") starts to increase linearly with separation. When the energy stored exceeds $2m_q$, new $q\bar{q}$ pairs can be created.

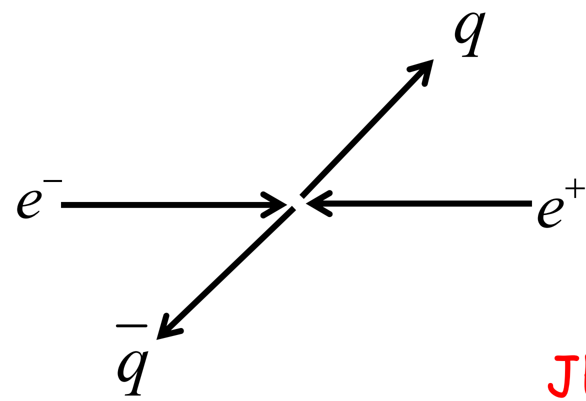


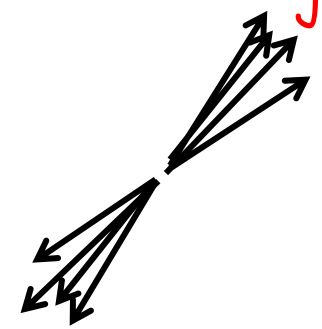
As energy decreases… hadrons (mainly mesons) freeze out

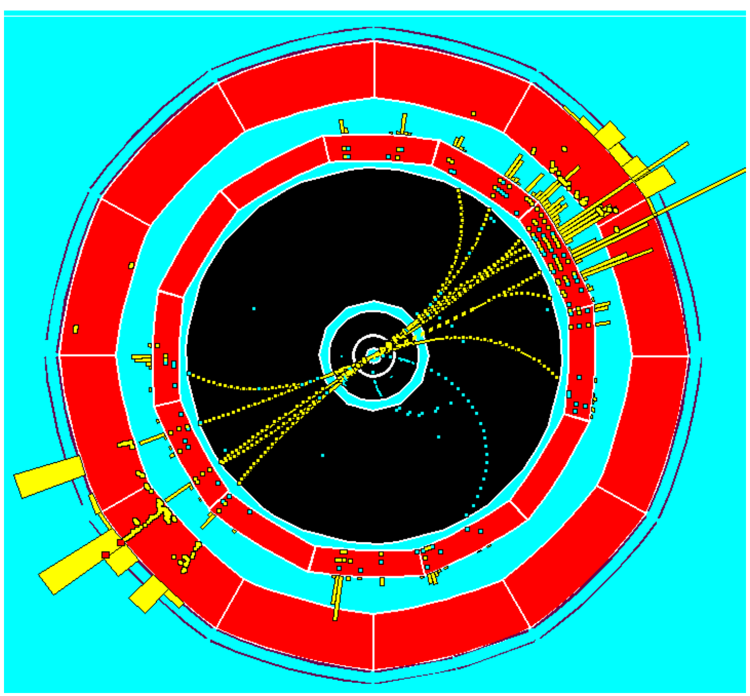


As quarks separate, more $q\bar{q}$ pairs are produced. This process is called HADRONIZATION. Start out with quarks and end up with narrowly collimated JETS of HADRONS.









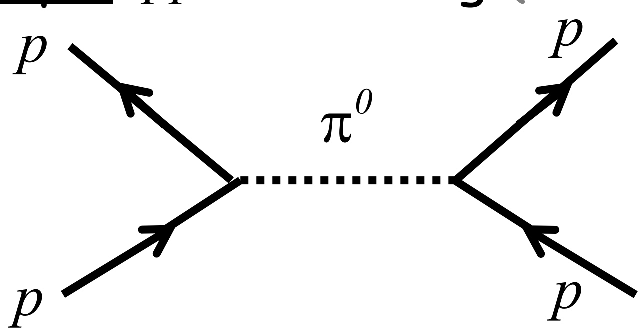
Typical $e^+e^- \rightarrow q\bar{q}$ event

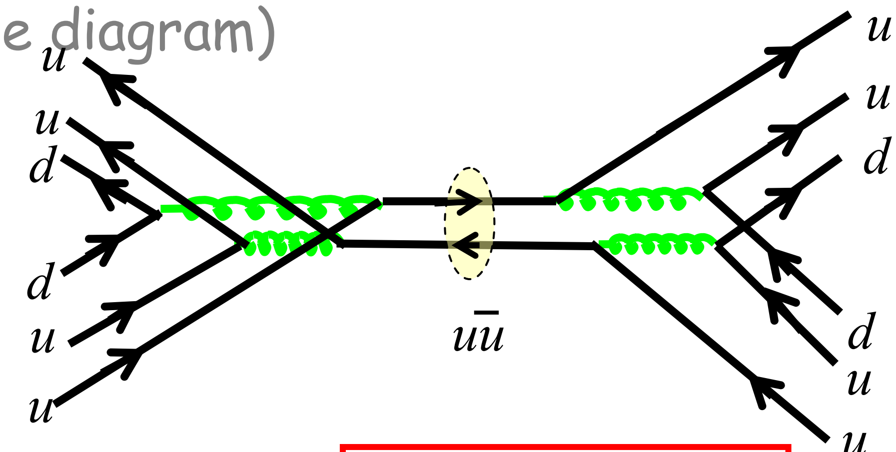
The hadrons in a quark(antiquark) jet follow the direction of the original quark(antiquark). Consequently, $e^+e^- \rightarrow q\bar{q}$ is observed as a pair of back-to-back jets.

Nucleon-Nucleon Interactions

- Bound qqq states (e.g. protons and neutrons) are COLOURLESS (COLOUR SINGLETS)
- They can only emit and absorb another colour singlet state, i.e. not single gluons (conservation of colour charge).
- Interact by exchange of PIONS.

<u>Example</u>: pp scattering (One possible diagram)



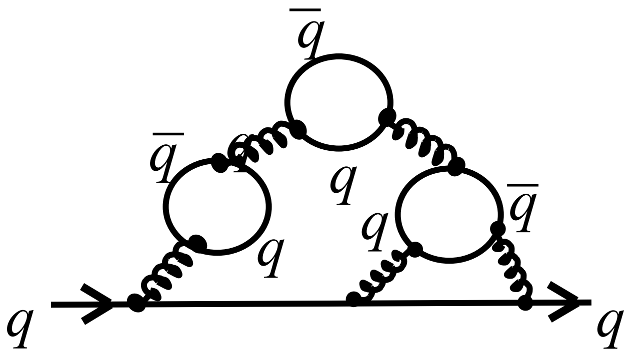


- Nuclear potential is YUKAWA potential with
$$V(r) = -\frac{g^2}{4\pi}\frac{e^{-m_\pi r}}{r}$$

- Short range force: $Range \approx 1/m_\pi = (0.140\ GeV)^{-1} = 7\ GeV^{-1}$
$$= 7 \times (\hbar c)\ fm = \underline{1.4\ \ fm}$$

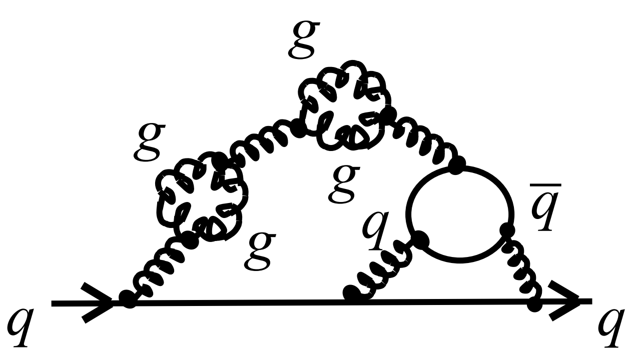
Running of α_s

- α_s specifies the strength of the strong interaction.
- BUT, just as in QED, α_s is not a constant. It "runs" (i.e. depends on energy).
- In QED, the bare electron charge is screened by a cloud of virtual electron-positron pairs.
- In QCD, a similar "colour screening" effect occurs.



In QCD, quantum fluctuations lead to a cloud of virtual $q\bar{q}$ pairs.

One of many (an infinite set) of such diagrams analogous to those for QED.

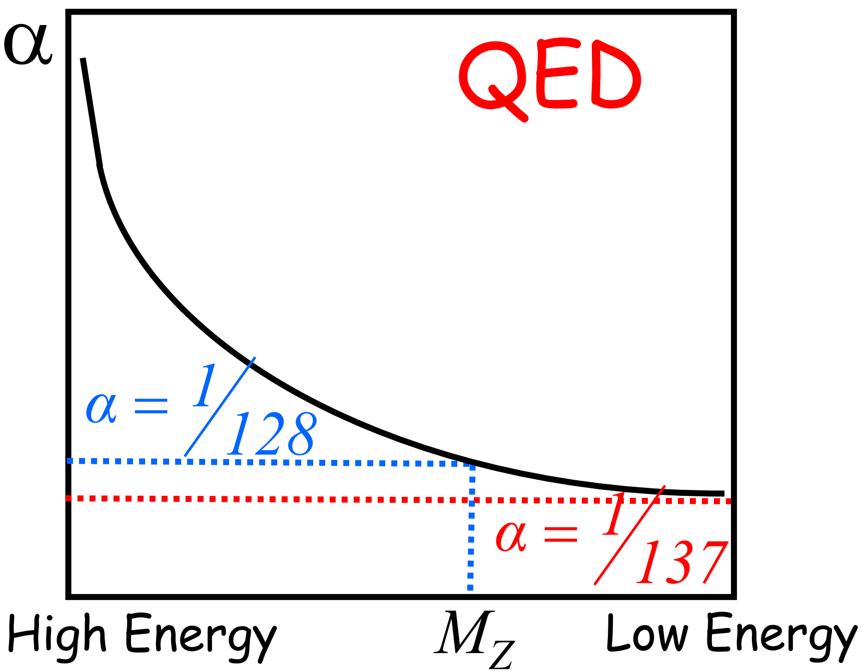


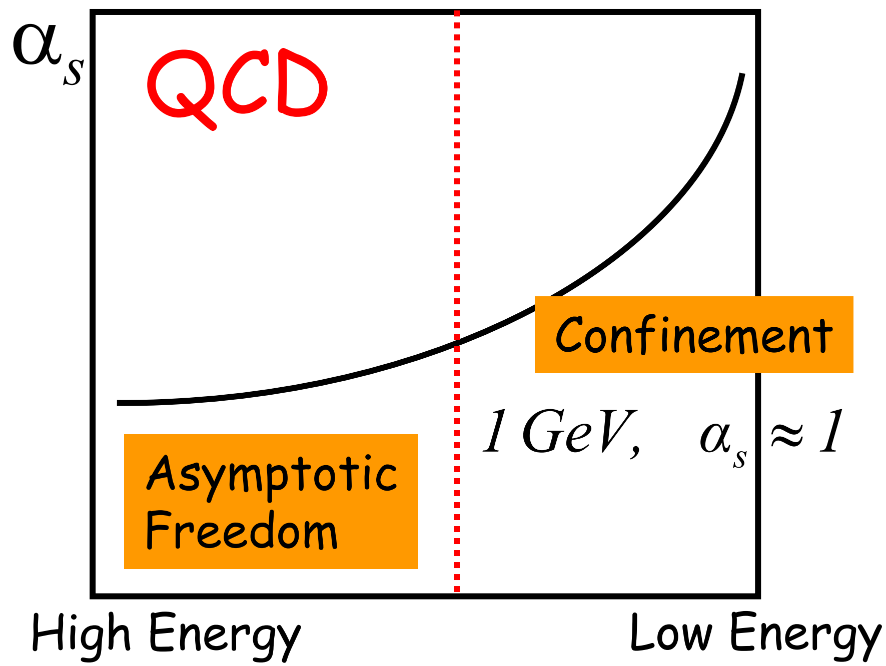
In QCD, the gluon self-interactions ALSO lead to a cloud of virtual gluons.

One of many (an infinite set) of such diagrams
No analogy in QED, photons do not carry the charge of the interaction.

Colour Anti-Screening

- Due to gluon self-interactions bare colour charge is screened by both virtual quarks and gluons.
- The cloud of virtual gluons carries colour charge and the effective colour charge DECREASES at smaller distances (high energy) !
- At low energies, α_s is large $\rightarrow$ cannot use perturbation theory.
- At high energies, α_s is small. In this regime, can treat quarks as free particles and use perturbation theory $\rightarrow$ ASYMPTOTIC FREEDOM.

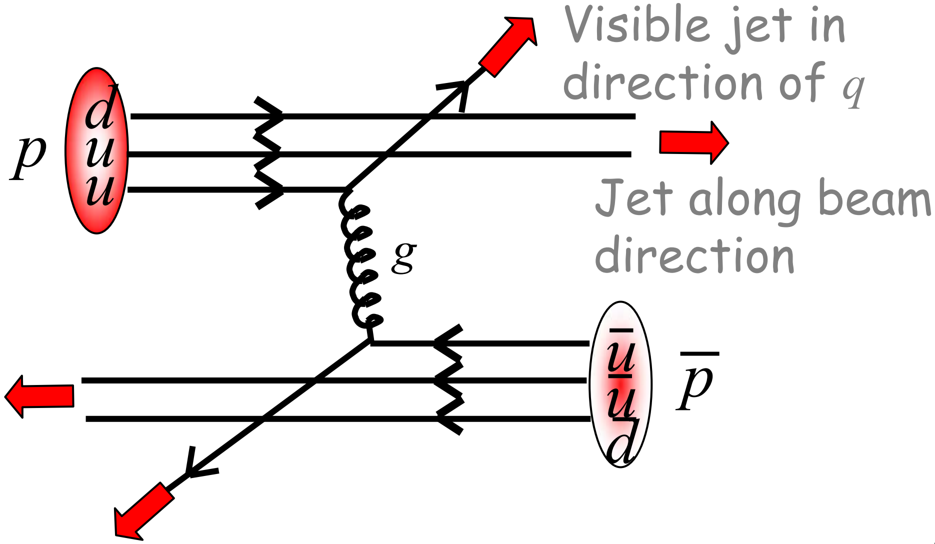


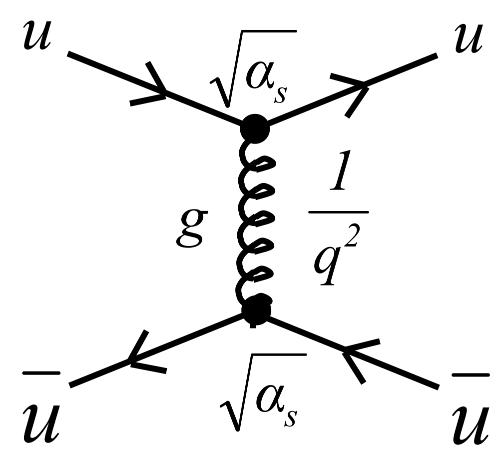


$\sqrt{s} = 100\ GeV, \quad \alpha_s = 0.12$

Scattering in QCD

<u>Example:</u> High energy proton-antiproton scattering.



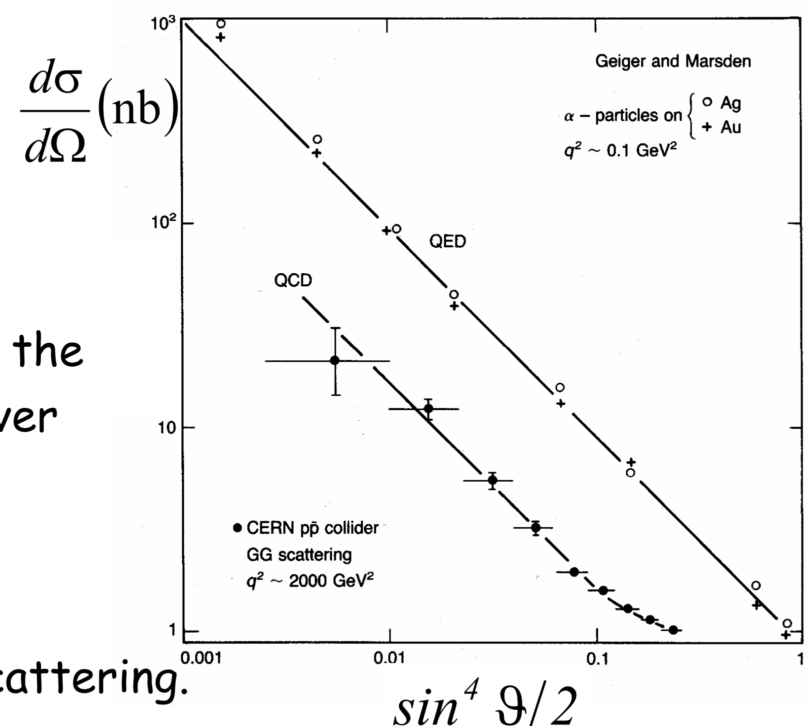


$$M \sim \frac{1}{q^2}\sqrt{\alpha_s}\sqrt{\alpha_s} \Rightarrow \frac{d\sigma}{d\Omega} \sim \frac{(\alpha_s)^2}{\sin^4 \vartheta/2}$$

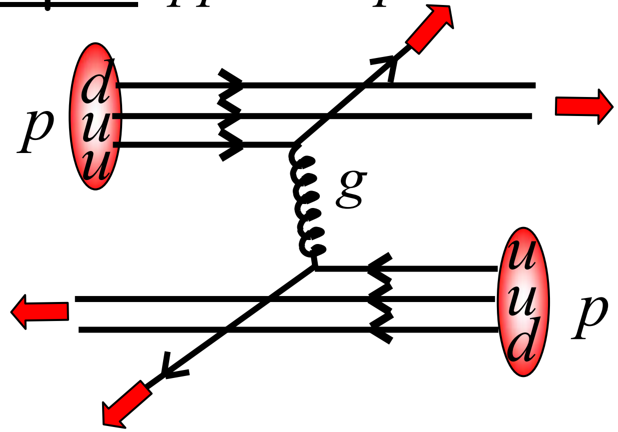
Upper points: Geiger and Marsden data (1911) for the elastic scattering of α particles from gold and silver foils.

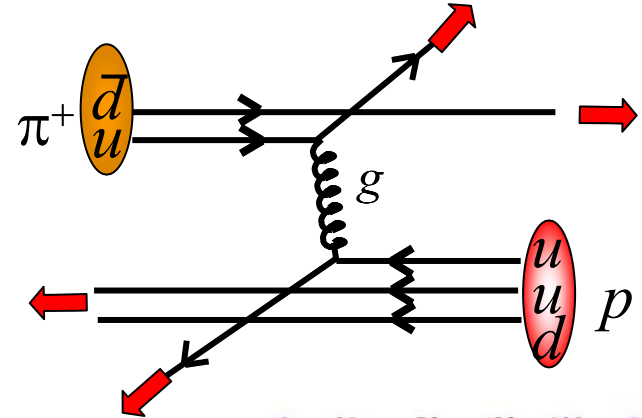
Lower points: angular distribution of quark jets observed in $p\bar{p}$ scattering at $q^2 = 2000\ GeV^2$.

Both follow the Rutherford formula for elastic scattering.



Example: pp vs π^+p scattering

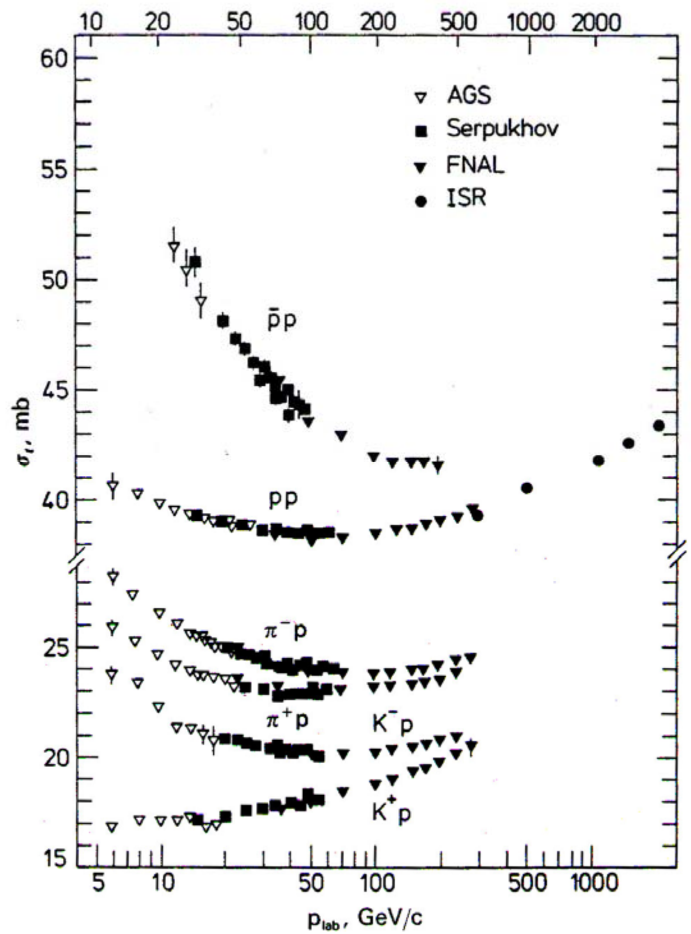




Calculate ratio of $\sigma(pp)_{\text{total}}$ to $\sigma(\pi^+p)_{\text{total}}$

QCD does not distinguish between quark flavours, only COLOUR charge of quarks matters.

At high energy (E>> binding energy of quarks within hadrons), ratio of $\sigma(pp)_{\text{total}}$ and $\sigma(\pi^+p)_{\text{total}}$ depends on number of possible quark-quark combinations.



Predict

$$\frac{\sigma(\pi p)}{\sigma(pp)} = \frac{2\times 3}{3\times 3} = \frac{2}{3}$$

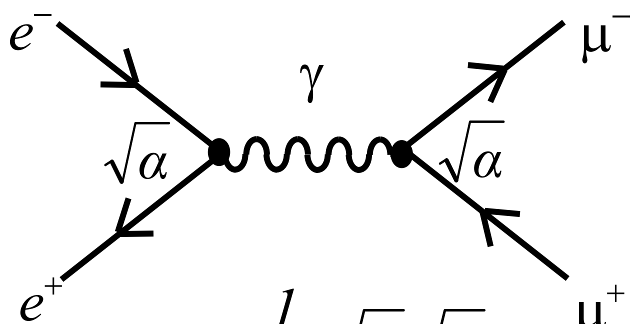
Experiment

$$\frac{\sigma(\pi p)}{\sigma(pp)} \approx \frac{24\ mb}{38\ mb} \approx \frac{2}{3}$$

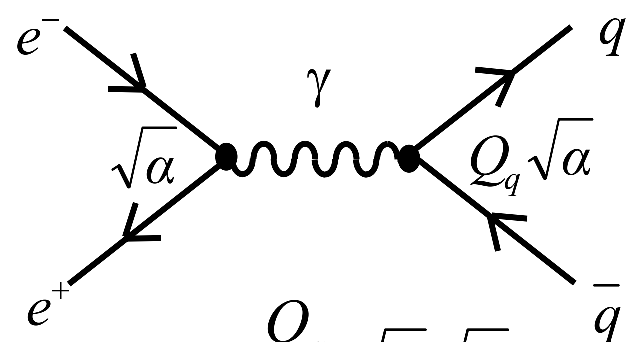
QCD in e^+e^- Annihilation

e^+e^- annihilation at high energies provides direct experimental evidence for COLOUR and for GLUONS.

Start by comparing the cross-sections for $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to q\bar{q}$



$$M \sim \frac{1}{q^2}\sqrt{\alpha}\sqrt{\alpha}$$



$$M \sim \frac{Q_q}{q^2}\sqrt{\alpha}\sqrt{\alpha}$$

$$\Rightarrow \sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}$$

If we neglect the mass of the final state quarks/muons then the ONLY difference is the charge of the final state particles:

$$Q_\mu = -1 \quad \text{and} \quad Q_q = +2/3 \quad or \quad -1/3$$

Evidence for Colour

Consider the ratio

$$R = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

For a single quark of a given colour

$$R = Q_q^2$$

However, we measure $e^+e^- \to jets$ not $e^+e^- \to u\bar{u}$. A jet from a u-quark looks just like a jet from a d-quark etc. Need to sum over all flavours (u, d, c, s, t, b) and colours (r, g, b):

$$R = 3\sum_i Q_i^2$$

(3 colours)

where the sum is over all quark flavours (i) kinematically accessible at centre-of-mass energy, $\sqrt{s}$, of collider.

Expect to see steps in R as energy is increased.

$$R = 3 \sum_i Q_i^2$$

Energy	Ratio R
$\sqrt{s} > 2m_s \sim 1\ GeV$	$3\left(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}\right) = 2$ *uds*
$\sqrt{s} > 2m_c \sim 4\ GeV$	$3\left(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}+\frac{4}{9}\right) = 3\frac{1}{3}$ *udsc*
$\sqrt{s} > 2m_b \sim 10\ GeV$	$3\left(\ldots+\frac{1}{9}\right) = 3\frac{2}{3}$ *udscb*
$\sqrt{s} > 2m_t \sim 350\ GeV$	$3\left(\ldots+\frac{4}{9}\right) = 5$ *udscbt*

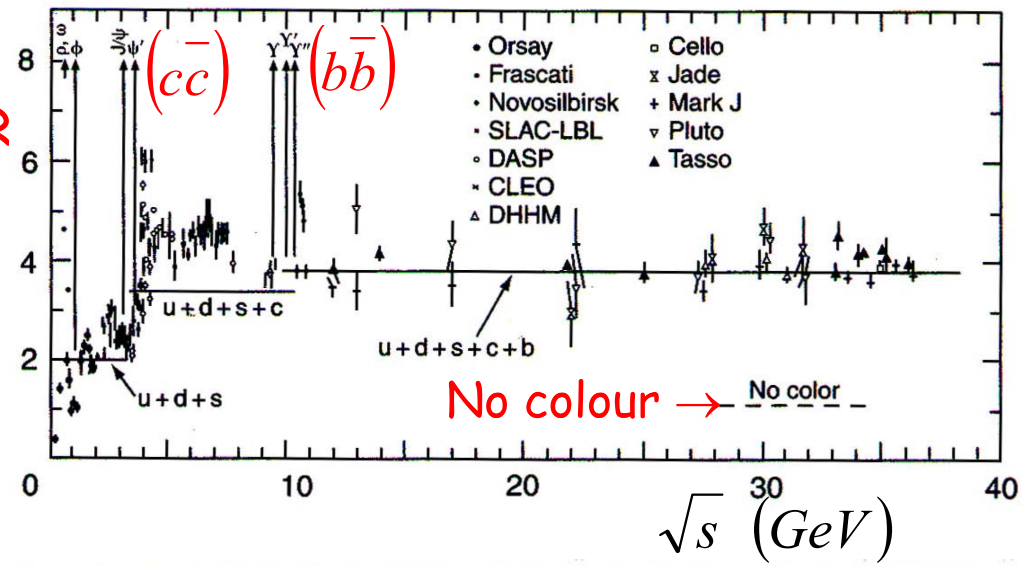
$$R = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

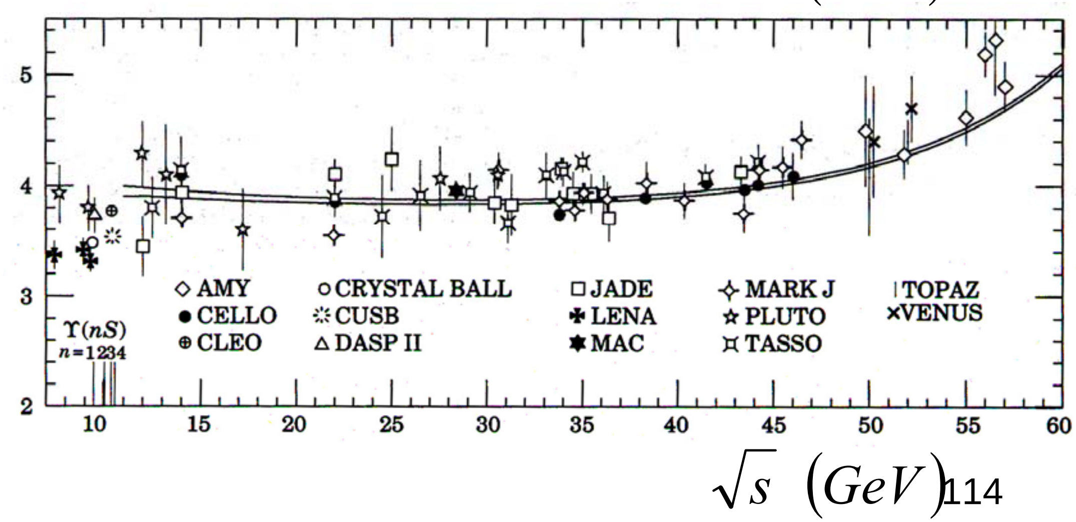
- R increases in steps with $\sqrt{s}$

STRONG EVIDENCE FOR COLOUR

- $\sqrt{s} < 11\ GeV$ region observe bound state resonances: charmomium($c\bar{c}$) and bottomonium ($b\bar{b}$).

- $\sqrt{s} > 50\ GeV$ region observe low edge of Z^0 resonance, $\Gamma \sim 2.5$ GeV.





Experimental Evidence for Colour

- $R = \sigma(e^+e^- \rightarrow hadrons) / \sigma(e^+e^- \rightarrow \mu^+\mu^-)$

- The existence of the $\Omega^-(sss)$

The $\Omega^-(sss)$ is a (L=0) spin 3/2 baryon consisting of 3 s-quarks. The wave-function

$$\psi = s\uparrow s\uparrow s\uparrow$$

is SYMMETRIC under particle interchange. However, quarks are FERMIONS, therefore require an ANTI-SYMMETRIC wave-function, i.e. need another degree of freedom, namely COLOUR.

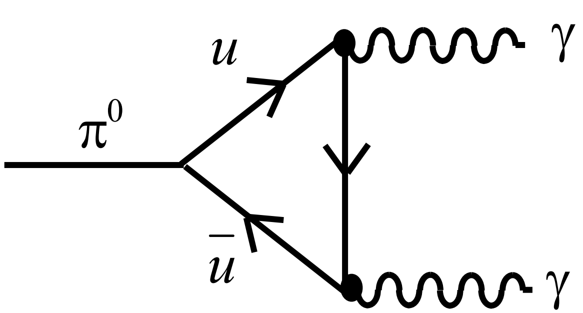
$$\psi = (s\uparrow s\uparrow s\uparrow)\psi_{colour}$$
$$\psi_{colour} = \tfrac{1}{\sqrt{6}}(rgb + gbr + brg - grb - rbg - bgr)$$

- $\pi^0 \rightarrow \gamma\gamma$ decay rate

Need colour to explain $\pi^0 \rightarrow \gamma\gamma$ decay rate.

$$\Gamma(\pi^0 \rightarrow \gamma\gamma) \propto N^2_{colour}$$

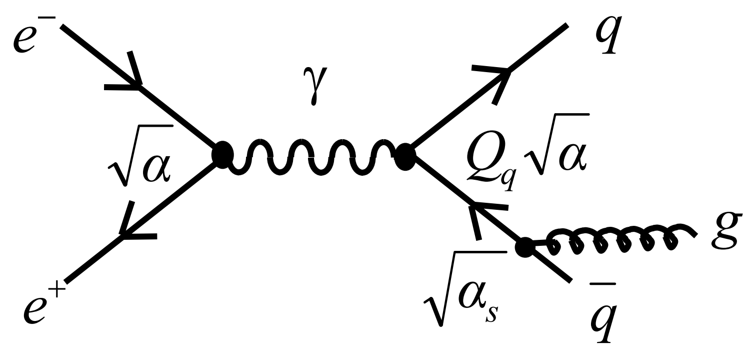
Experiment: $N_{colour} = 2.99 \pm 0.12$



Evidence for Gluons

In QED, electrons can radiate photons. In QCD, quarks can radiate gluons.

<u>Example:</u> $e^+e^- \rightarrow q\bar{q}\,g$



$$M \sim \frac{Q_q}{q^2}\sqrt{\alpha}\sqrt{\alpha}\sqrt{\alpha_s}$$

Giving an extra factor of $\sqrt{\alpha_s}$ in the matrix element, i.e. an extra factor of α_s in the cross-section.

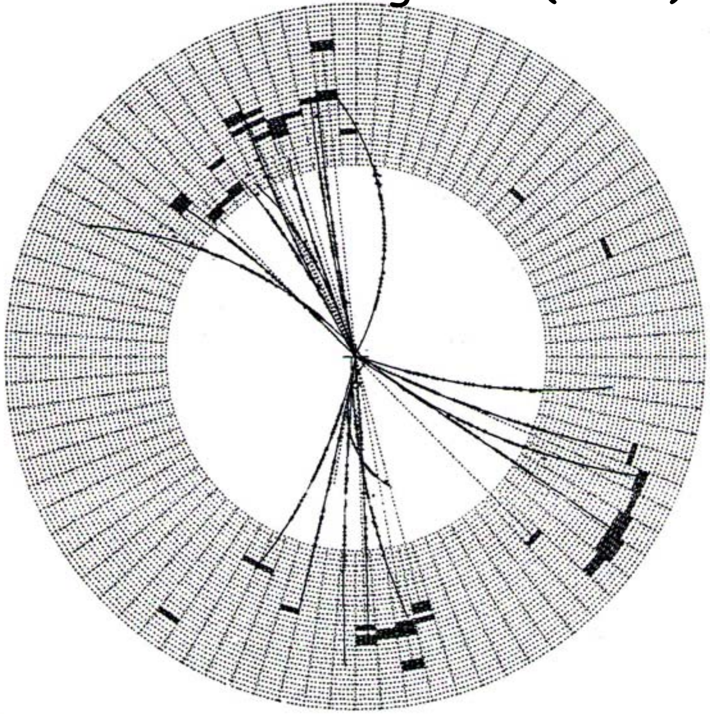
In QED we can detect the photons. In QCD, we never see free gluons due to confinement.

Experimentally, detect gluons as an additional jet: 3-JET events.

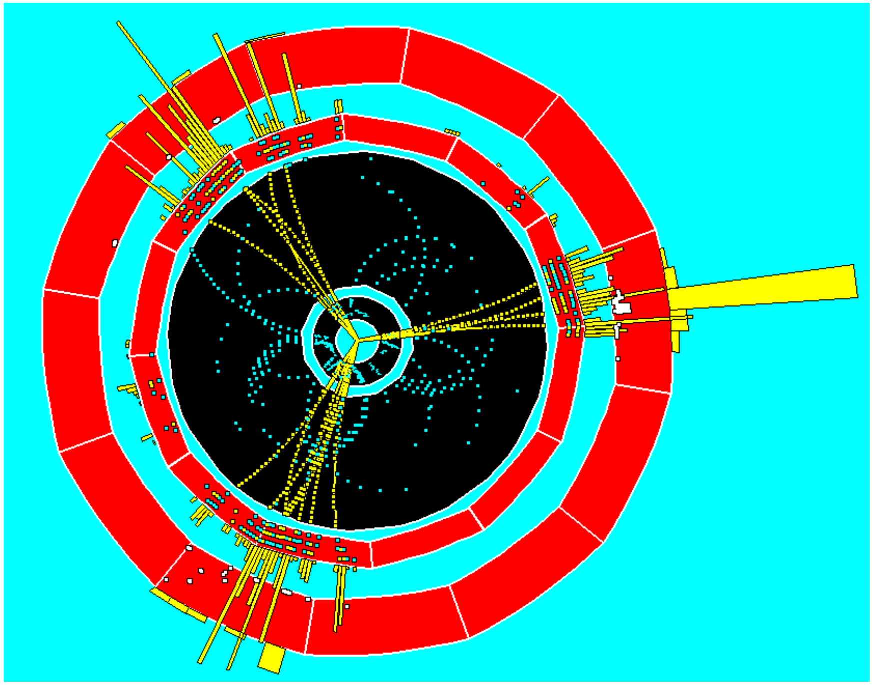
- Angular distribution of gluon jet depends on gluon spin.

JADE Event $\sqrt{s} = 31 GeV$

Direct evidence for gluons (1978)

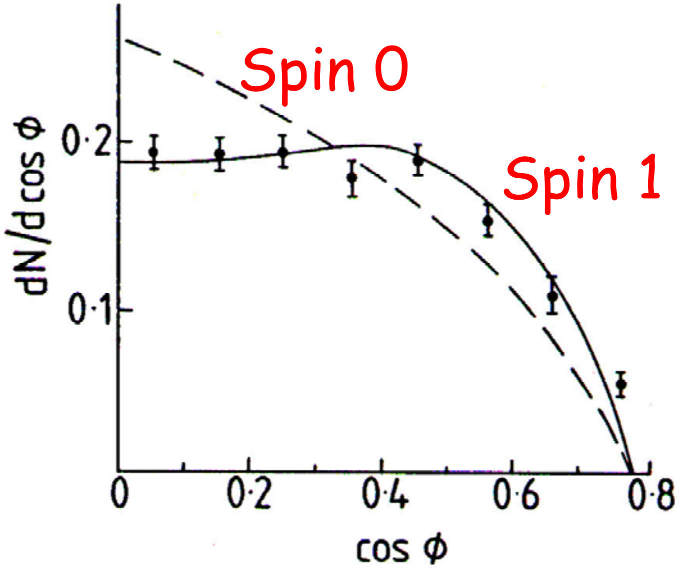


ALEPH Event $\sqrt{s} = 91 GeV$ (1990)



Distribution of the angle, ϕ, between the highest energy jet (assumed to be one of the quarks) relative to the flight direction of the other two (in their cms frame).

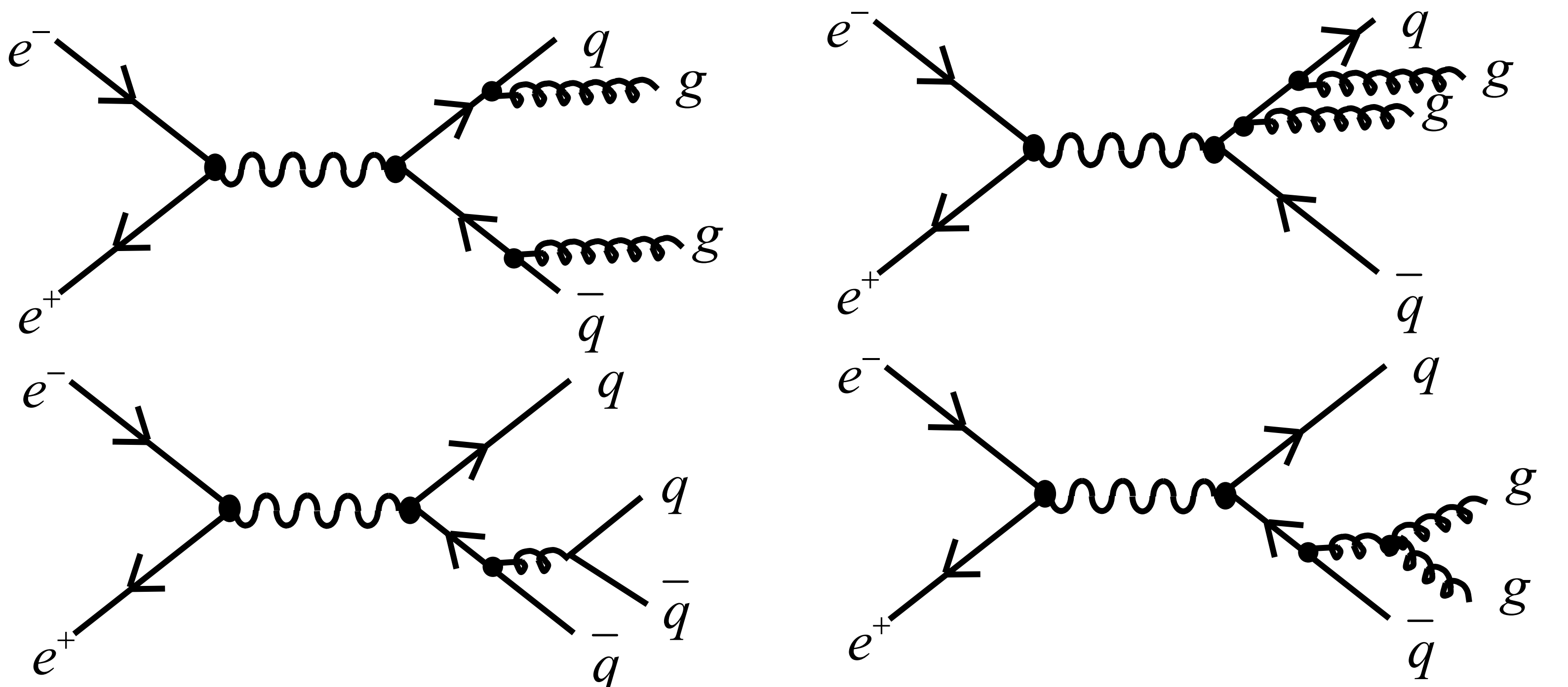
ϕ depends on the spin of the gluon.



$\Rightarrow$ GLUON IS SPIN 1

Evidence for Gluon Self-Interactions

Direct evidence for the existence of the gluon self-interactions from 4-JET events.



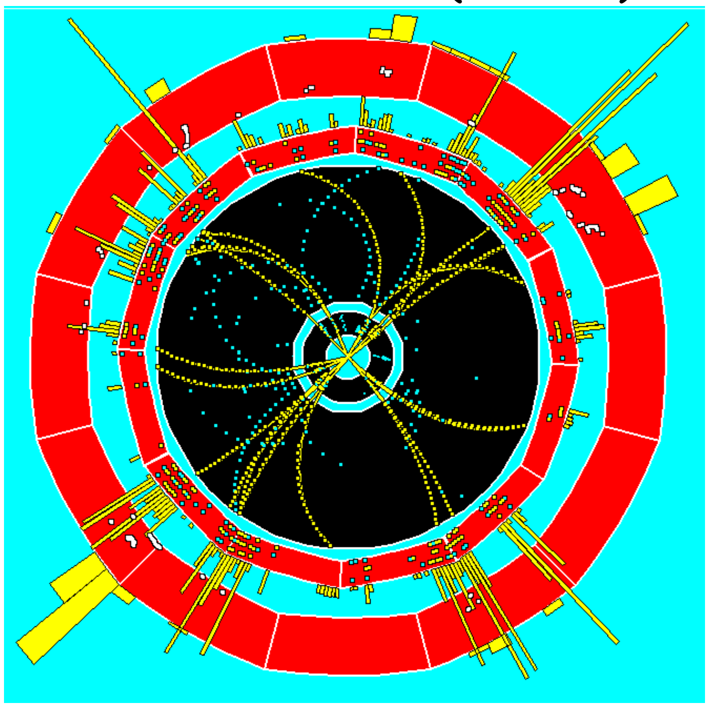
➢ The angular distribution of jets is sensitive to existence of triple gluon vertex

$q\bar{q}g$ vertex consists of 2 spin ½ quarks and 1 spin 1 gluon

ggg vertex consists of 3 spin 1 gluons

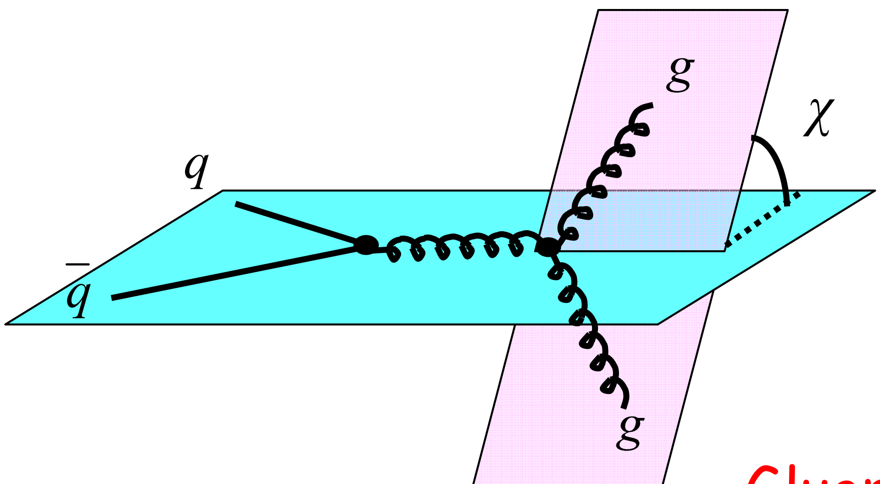
$\Rightarrow$ Different angular distribution.

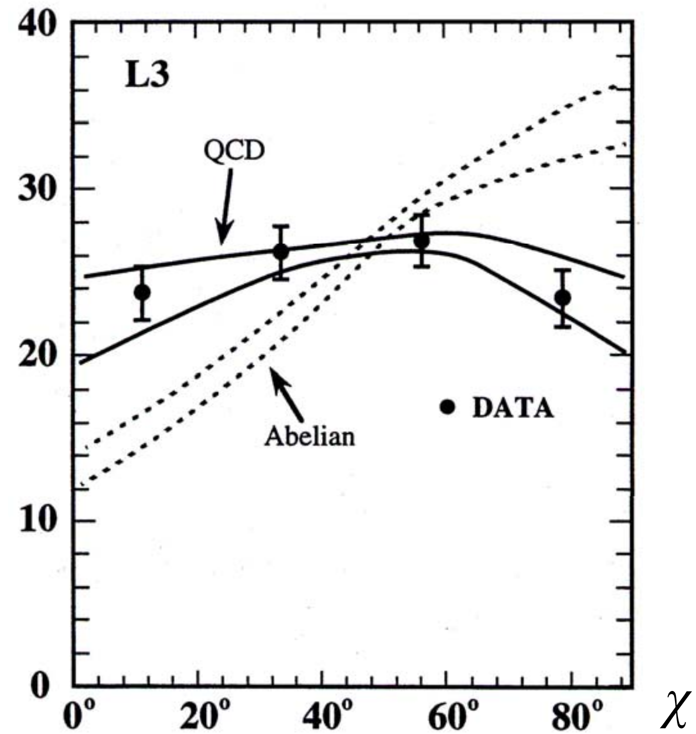
4-JET EVENT (ALEPH)



Experimentally:

- Define the two lowest energy jets as the gluons. (Gluon jets are more likely to be lower energy than quark jets).
- Measure angle between the plane containing the "quark" jets and the plane containing the "gluon" jets, χ.





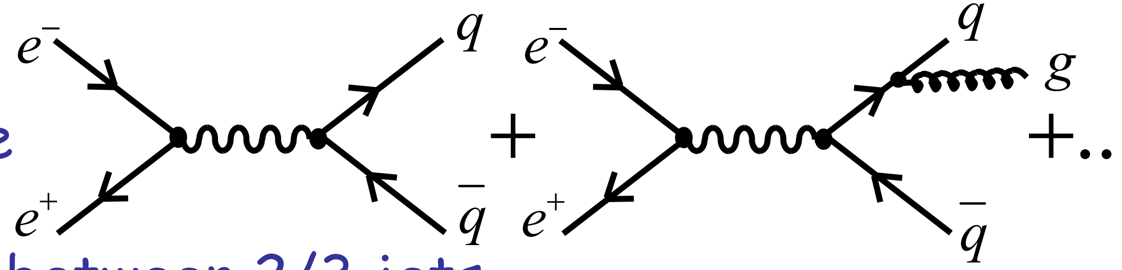
Gluon self-interactions are required to describe the experimental data.

Measurements of α_s

α_s can be measured in many ways. The cleanest is from the ratio

$$R = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}.$$

In practise, measure



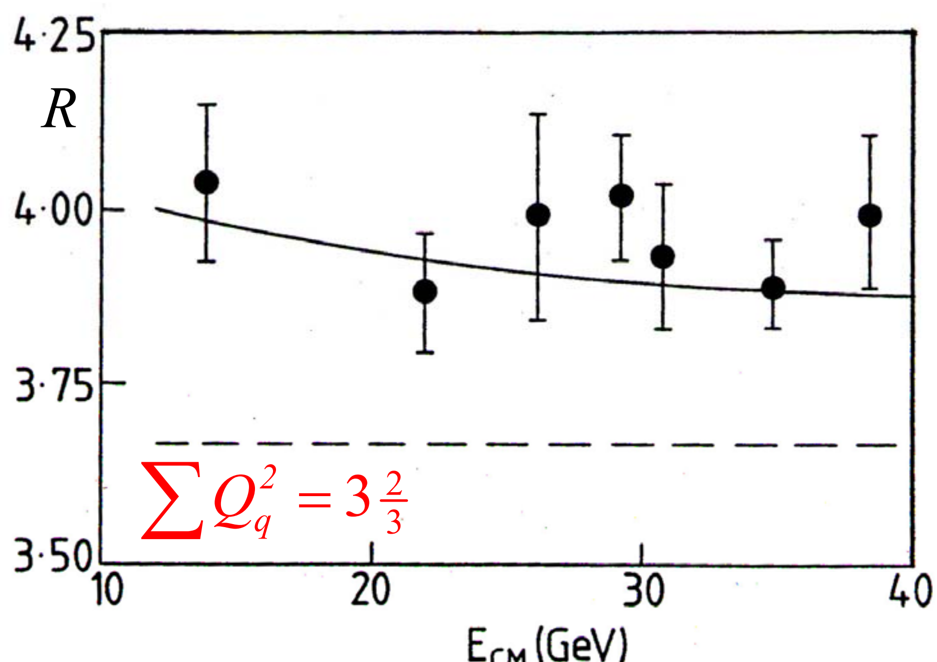
i.e. don't distinguish between 2/3 jets.

When gluon radiation is included:

$$R = 3\sum Q_q^2\left(1+\frac{\alpha_s}{\pi}\right)$$

Therefore, $\left(1+\frac{\alpha_s}{\pi}\right) \approx \frac{3.9}{3.66}$

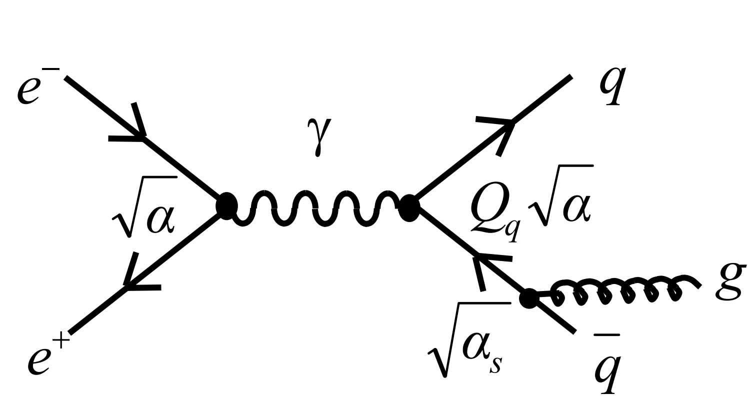
$$\alpha_s\left(q^2 = 25^2\right) \approx 0.2$$

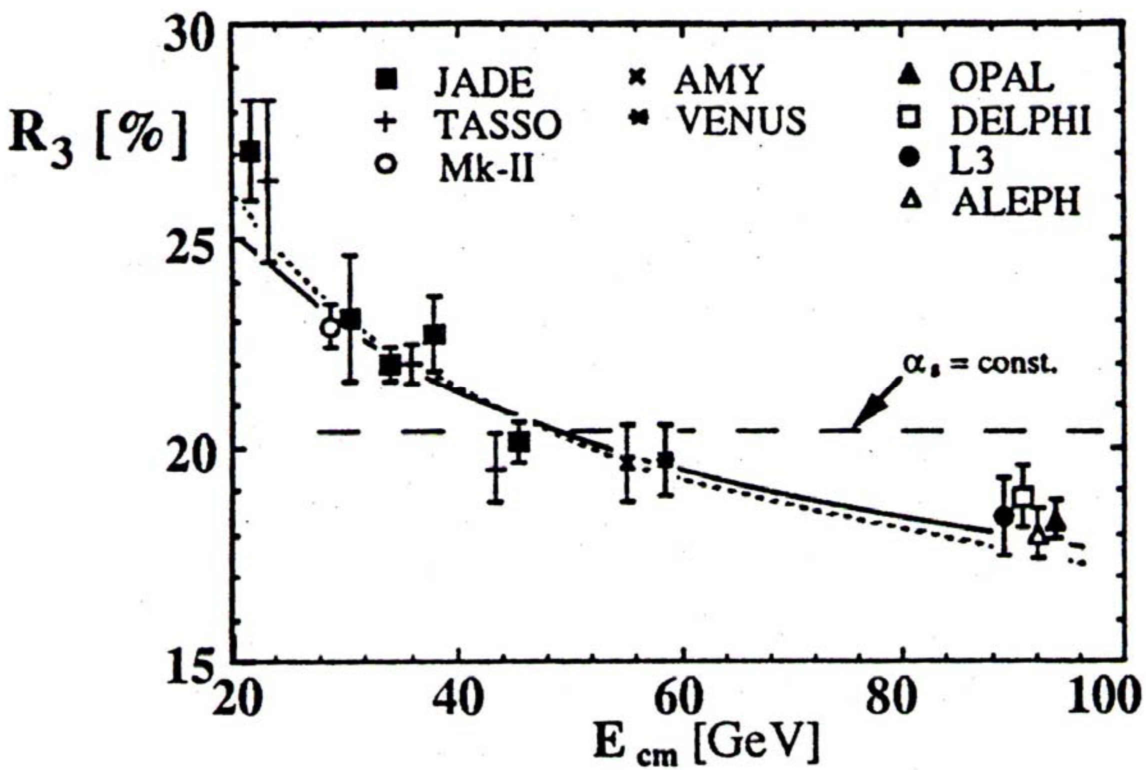


Many other ways to measure α_s…

<u>Example</u>: 3 jet rate $e^+e^- \to q\bar{q}g$

$$R_3 = \frac{\sigma(e^+e^- \to 3\ jets)}{\sigma(e^+e^- \to 2\ jets)} \propto \alpha_s$$

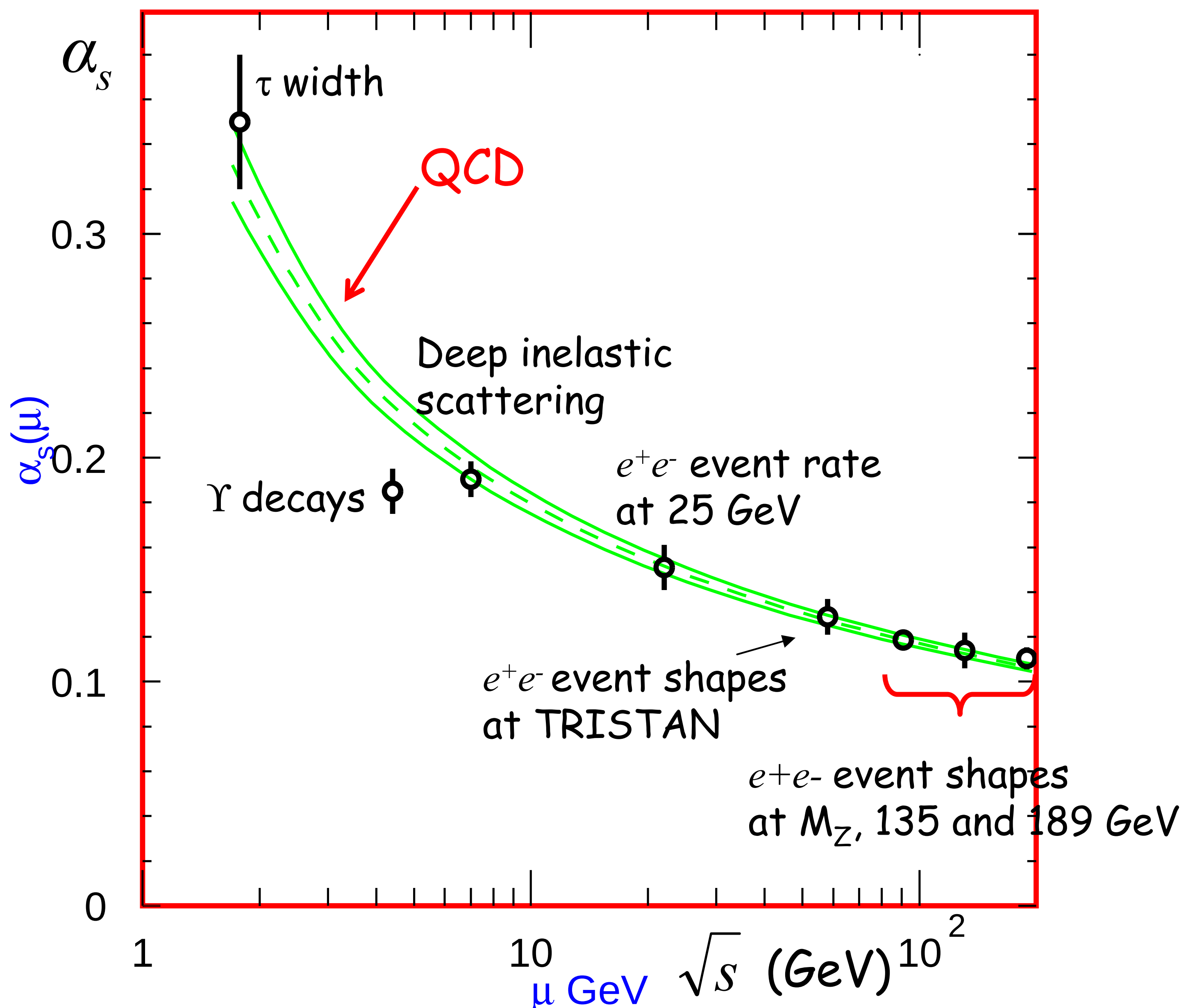




α_s decreases with energy

<u>α_s RUNS !</u>

α_s Summary



Section VII
Quark Model of Hadrons

The Quark Model of Hadrons

EVIDENCE FOR QUARKS

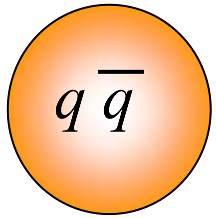
- The magnetic moments of proton and neutron are not $\mu_N = e\hbar/2m_p$ and 0 $\Rightarrow$ not point-like
- Electron-proton scattering at high q^2 deviates from Rutherford scattering $\Rightarrow$ proton has substructure
- Jets are observed in e^+e^- and $p\bar{p}$ collisions
- Symmetries (patterns) in masses and properties of hadron states, "quarky" periodic table $\Rightarrow$ sub-structure
- Steps in $R = \sigma(e^+e^- \to hadrons)/\sigma(e^+e^- \to \mu^+\mu^-)$
- Observation of $c\bar{c}$ and $b\bar{b}$ bound states
- and much, much more….

Here, we will first consider the wave-functions for hadrons formed from light quarks (u, d, s) and deduce their static properties (mass and magnetic moments). Then we will go on to discuss the heavy quarks (c, b). We will cover the t quark later…

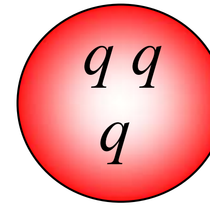
Hadron Wavefunctions

Quarks are always confined in hadrons (i.e. colourless states)

MESONS
Spin 0, 1,...



BARYONS
Spin 1/2, 3/2,...



Treat quarks as IDENTICAL fermions with states labelled with SPATIAL, SPIN, FLAVOUR and COLOUR.

$$\psi = \psi_{space}\,\psi_{flavour}\,\psi_{spin}\,\psi_{colour}$$

All hadrons are COLOUR SINGLETS, i.e. net colour zero

MESONS $\quad \psi^{q\bar{q}}_{colour} = \frac{1}{\sqrt{3}}\left(r\bar{r} + g\bar{g} + b\bar{b}\right)$

BARYONS $\quad \psi^{qqq}_{colour} = \frac{1}{\sqrt{6}}\left(rgb + gbr + brg - grb - rbg - bgr\right)$

Parity

- The PARITY OPERATOR, $\hat{P}$, performs SPATIAL INVERSION.

$$\hat{P}\left|\psi(\vec{r},t)\right\rangle = \left|\psi(-\vec{r},t)\right\rangle$$

- The eigenvalue of $\hat{P}$ is called the PARITY

$$\hat{P}\left|\psi\right\rangle = P\left|\psi\right\rangle, \qquad P = \pm 1$$

- Particles are EIGENSTATES of PARITY and in this case P represents the INTRINSIC PARITY of a particle/antiparticle.

- Parity is a useful concept. If the Hamiltonian for an interaction commutes with $\hat{P}$

$$\left[\hat{P},\hat{H}\right] = 0$$

then PARITY IS CONSERVED in the interaction:

> PARITY CONSERVED in the STRONG and EM interactions but NOT in the WEAK interaction.

- Composite system of 2 particles with orbital angular momentum L:

$$P = P_1 P_2 (-1)^L$$

where $P_{1,2}$ is the intrinsic parity of particle 1,2.

Quantum Field Theory:

Fermions and antifermions : OPPOSITE parity
Bosons and antibosons : SAME parity

Choose:

Quarks and leptons : $P = +1$
Antiquarks and antileptons : $P = -1$

Gauge Bosons (γ, g, W, Z) are vector fields which transform as

$$J^P = 1^-$$

Light Mesons

Mesons are bound $q\bar{q}$ states. Consider ground state mesons consisting of LIGHT quarks (u, d, s).

$$m_u \sim 0.3\ GeV, \quad m_d \sim 0.3\ GeV, \quad m_s \sim 0.5\ GeV$$

- Ground State ($L=0$): Meson "spin" (total angular momentum) is given by the $q\bar{q}$ spin state.

  Two possible $q\bar{q}$ total spin states: $S = 0, 1$

  $S = 0$: pseudo-scalar mesons

  $S = 1$: vector mesons

- Meson Parity: (q and $\bar{q}$ have OPPOSITE parity)

$$P = P_q P_{\bar{q}} (-1)^L = (+1)(-1)(-1)^L = -1 \quad \text{(for } L = 0\text{)}$$

- Flavour States: $u\bar{d}, u\bar{s}, d\bar{u}, d\bar{s}, s\bar{u}, s\bar{d}$, and $u\bar{u}, d\bar{d}, s\bar{s}$ mixtures

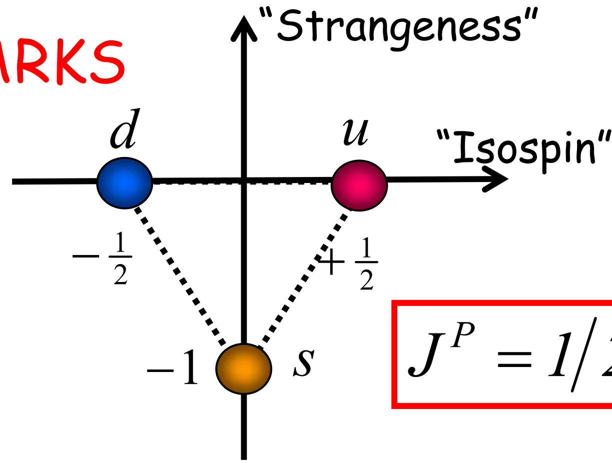
EXPECT: 9 $J^P = 0^-$ mesons: PSEUDO-SCALAR NONET

9 $J^P = 1^-$ mesons: VECTOR NONET

uds Multiplets

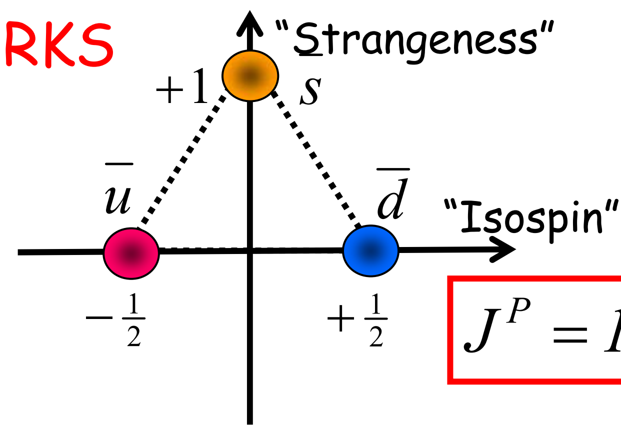
Basic quark multiplet – plot the quantum numbers of (anti)quarks:

QUARKS



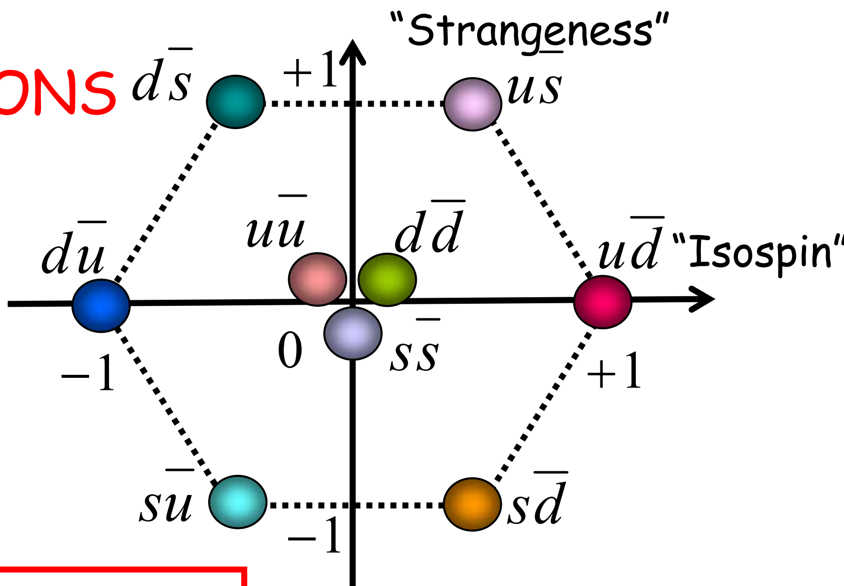
$J^P = 1/2^+$

ANTIQUARKS



$J^P = 1/2^-$

MESONS



SPIN 0 or 1

The ideas of strangeness and isospin are historical quantum numbers assigned to different states. Essentially they count quark flavours (this was all before the formulation of the Quark Model).

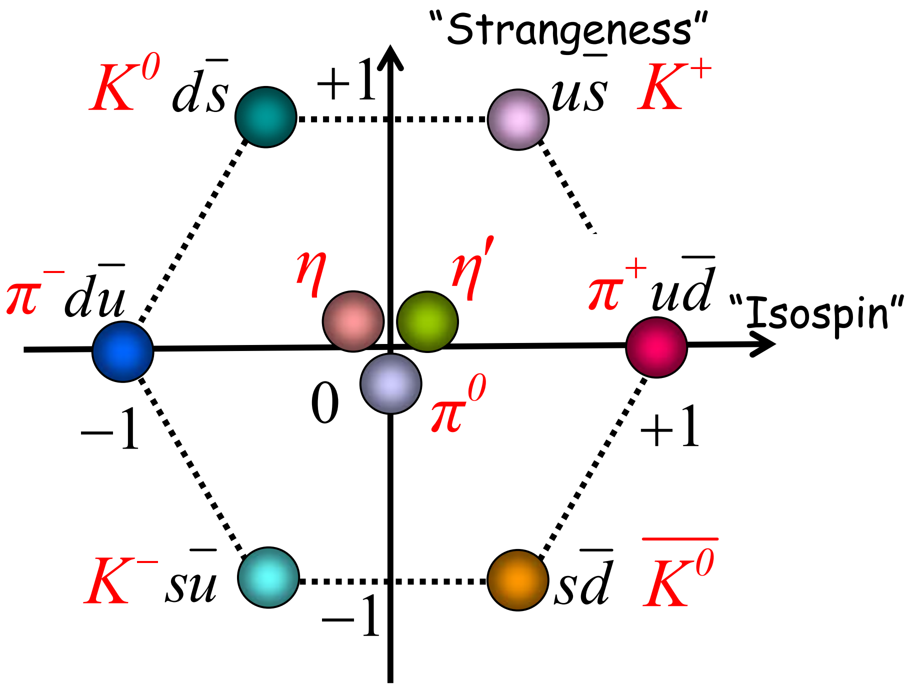
$$Isospin = \tfrac{1}{2}(n_u - n_d - n_{\bar{u}} + n_{\bar{d}})$$

$$Strangeness = n_{\bar{s}} - n_s$$

Light Mesons

PSEUDO-SCALAR NONET

$J^P = 0^-$



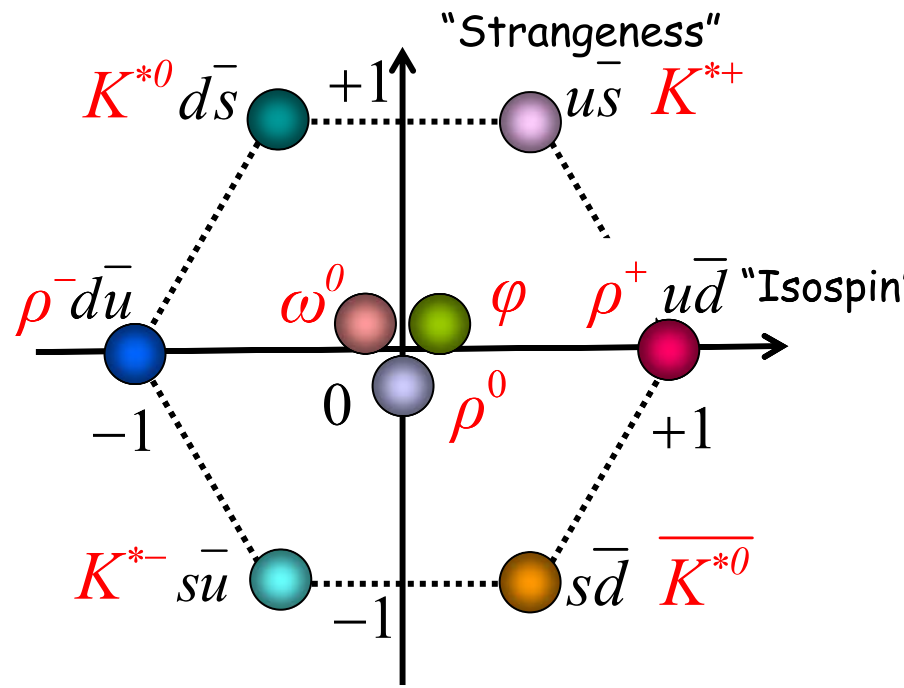
π^0, η, η' are combinations of $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$

Masses/MeV

$\pi(140)$, $K(495)$
$\eta(550)$, $\eta'(960)$

VECTOR NONET

$J^P = 1^-$



ρ^0, ϕ, ω^0 are combinations of $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$

Masses/MeV

$\rho(770)$, $K*(890)$
$\omega(780)$, $\phi(1020)$

$u\bar{u}$, $d\bar{d}$, $s\bar{s}$ States

The states $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ all have zero flavour quantum numbers and can therefore MIX

$$\left.\begin{aligned}\pi^0 &= \tfrac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \\ \eta &= \tfrac{1}{\sqrt{6}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right) \\ \eta' &= \tfrac{1}{\sqrt{3}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right)\end{aligned}\right\} \quad \boxed{J^P = 0^-}$$

$$\left.\begin{aligned}\rho^0 &= \tfrac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \\ \omega^0 &= \tfrac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right) \\ \varphi &= s\bar{s}\end{aligned}\right\} \quad \boxed{J^P = 1^-}$$

Mixing coefficients determined experimentally from meson masses and decays.

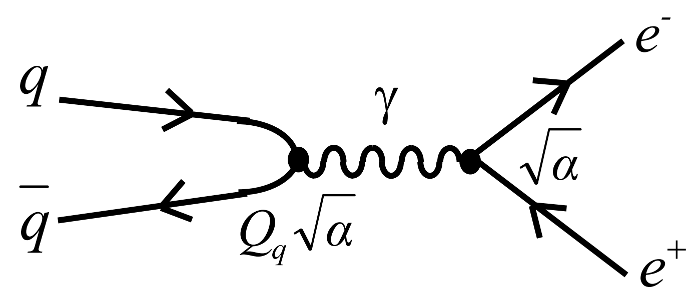
Example: Leptonic decays of vector mesons.

$$M(\rho^0 \to e^+e^-) \sim \frac{e}{q^2}\left[\tfrac{1}{\sqrt{2}}\left(Q_u e - Q_d e\right)\right]$$

$$\Gamma(\rho^0 \to e^+e^-) \propto \left[\tfrac{1}{\sqrt{2}}\left(\tfrac{2}{3} - \left(-\tfrac{1}{3}\right)\right)\right]^2 = 1/2$$

$$\Gamma(\omega^0 \to e^+e^-) \propto \left[\tfrac{1}{\sqrt{2}}\left(\tfrac{2}{3} + \left(-\tfrac{1}{3}\right)\right)\right]^2 = 1/18$$

$$\Gamma(\varphi \to e^+e^-) \propto \left[\tfrac{1}{3}\right]^2 = 1/9$$



$$M \sim Q_q \alpha \qquad \Gamma \sim Q_q^2 \alpha^2$$

Predict: $\Gamma_{\rho^0} : \Gamma_{\omega^0} : \Gamma_{\varphi} = 9:1:2$; Experiment: $\Gamma_{\rho^0} : \Gamma_{\omega^0} : \Gamma_{\varphi} = 8.8 \pm 2.6 : 1 : 1.7 \pm 0.4$

Meson Masses

Meson masses partly from constituent quarks masses:

- $m(K) > m(\pi) \Rightarrow$ suggests $m_s > m_u, m_d$
  495 140 MeV

Not the whole story…

- $m(\rho) > m(\pi) \Rightarrow$ although both are $u\bar{d}$
  770 140 MeV

- Only difference is the orientation of the quark SPINS (↑↑ vs ↓↑)

$\Rightarrow$ SPIN-SPIN INTERACTION

SPIN-SPIN INTERACTION

QED: Hyperfine splitting in H_2 (L=0)

Energy shift due to electron spin in magnetic field of proton

$$\Delta E = \vec{\mu}\cdot\vec{B} = \frac{2}{3}\vec{\mu}_e\cdot\vec{\mu}_p\left|\psi(0)\right|^2 \quad \text{using} \quad \vec{\mu} = \frac{e}{2m}\vec{S}$$

$$\Delta E \propto \alpha\frac{\vec{S}_e\cdot\vec{S}_p}{m_1 m_2}$$

QCD: Colour Magnetic Interaction

Fundamental form of the interaction between a quark and a gluon is identical to that between an electron and a photon.
Consequently, also have a COLOUR MAGNETIC INTERACTION

$$\Delta E \propto \alpha_s\frac{\vec{S}_1\cdot\vec{S}_2}{m_1 m_2}$$

MESON MASS FORMULA (L=0)

$$M_{q\bar{q}} = m_1 + m_2 + A\frac{\vec{S}_1\cdot\vec{S}_2}{m_1 m_2}$$

where A is a constant.

For a state of SPIN $\vec{S} = \vec{S}_1 + \vec{S}_2$ $\quad \vec{S}^2 = \vec{S}_1^2 + \vec{S}_2^2 + 2\vec{S}_1\cdot\vec{S}_2$

$$\vec{S}_1\cdot\vec{S}_2 = \tfrac{1}{2}\left(\vec{S}^2 - \vec{S}_1^2 - \vec{S}_2^2\right)$$

$$\vec{S}_1^2 = \vec{S}_2^2 = S_1(S_1+1) = \tfrac{1}{2}\left(1+\tfrac{1}{2}\right) = \tfrac{3}{4}$$

giving $\quad \vec{S}_1\cdot\vec{S}_2 = \tfrac{1}{2}\vec{S}^2 - \tfrac{3}{4}$

For $J^P = 0^-$ Mesons: $\vec{S}^2 = 0 \quad \Rightarrow \quad \vec{S}_1\cdot\vec{S}_2 = -\tfrac{3}{4}$

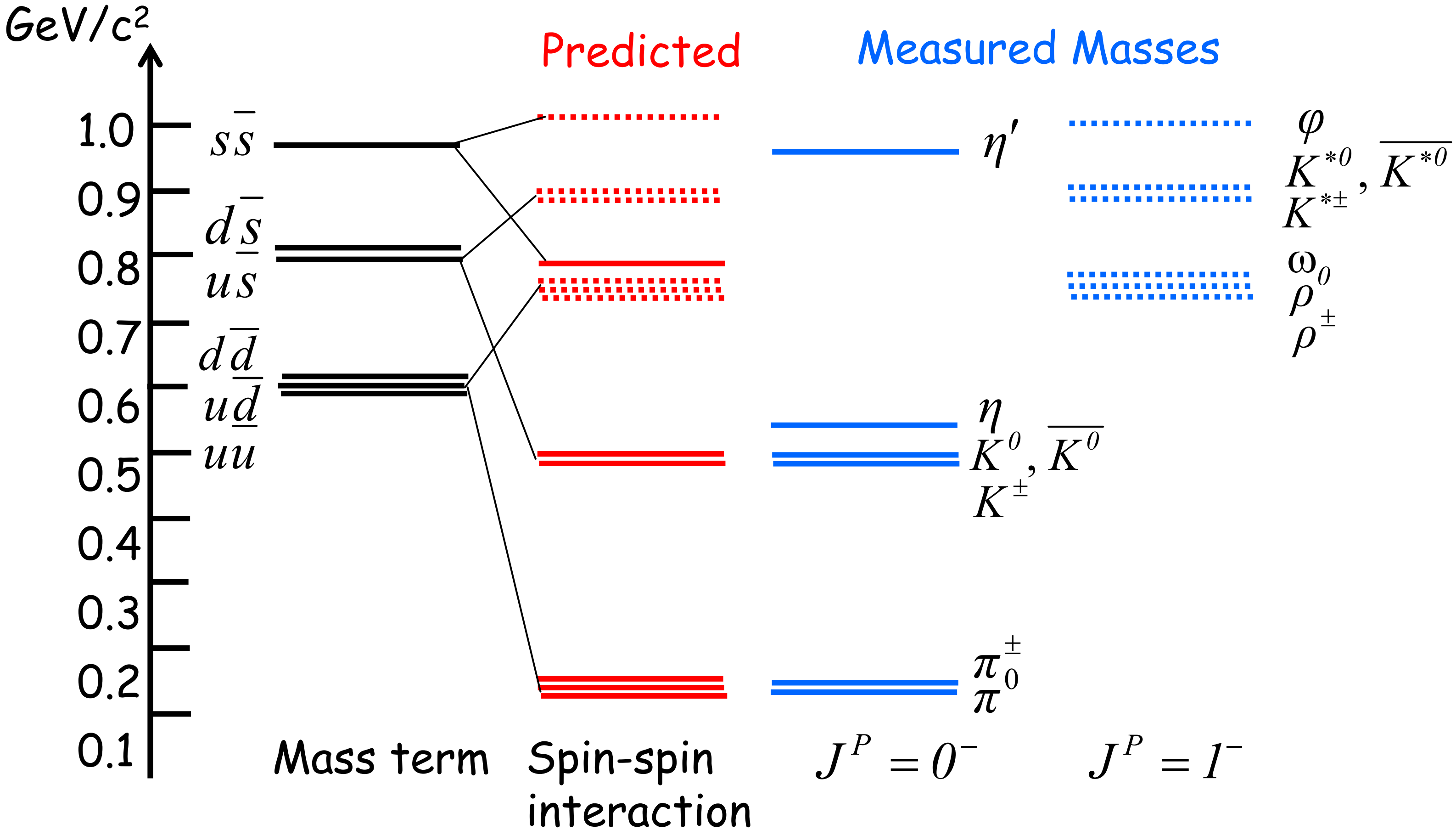
$J^P = 1^-$ Mesons: $\vec{S}^2 = S(S+1) = 2 \quad \Rightarrow \quad \vec{S}_1\cdot\vec{S}_2 = +\tfrac{1}{4}$

Giving the (L=0) Meson Mass formulae

$$M_{q\bar{q}} = m_1 + m_2 - \frac{3A}{4m_1 m_2} \qquad \left(J^P = 0^-\right)$$

$$M_{q\bar{q}} = m_1 + m_2 + \frac{A}{4m_1 m_2} \qquad \left(J^P = 1^-\right)$$

0^- mesons lighter than 1^- mesons.



Excellent fit obtained to masses of the different flavour pairs ($u\bar{d}$, $u\bar{s}$, $d\bar{u}$, $d\bar{s}$, $s\bar{u}$, $s\bar{d}$) with

$$m_u = 0.305\ GeV, \quad m_d \sim 0.308\ GeV, \quad m_s \sim 0.487\ GeV, \quad A = 0.06\ GeV^3$$

η and η' are mixtures of states, e.g.

$$\eta = \tfrac{1}{\sqrt{6}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right) \qquad M_\eta = \frac{1}{6}\left(2m_u - \frac{3A}{4m_u^2}\right) + \frac{1}{6}\left(2m_d - \frac{3A}{4m_d^2}\right) + \frac{4}{6}\left(2m_s - \frac{3A}{4m_s^2}\right)$$

Baryons

Baryons made from 3 indistinguishable quarks (flavour treated as another quantum number in the wave-function)

$$\psi_{baryon} = \psi_{space}\,\psi_{flavour}\,\psi_{spin}\,\psi_{colour}$$

ψ_{baryon} must be ANTI-SYMMETRIC under interchange of any 2 quarks.

Example: $\Omega^-(sss)$ wavefunction $L=0$

$\psi_{spin}\,\psi_{flavour} = s\uparrow s\uparrow s\uparrow$ is symmetric $\Rightarrow$ REQUIRE antisymmetric ψ_{colour}

Ground State (L=0)

We will only consider the baryon ground states, which have zero orbital angular momentum

ψ_{space} symmetric

- All hadrons are COLOUR SINGLETS

$$\psi_{colour} = \frac{1}{\sqrt{6}}\left(rgb + gbr + brg - grb - rbg - bgr\right)$$ antisymmetric

Therefore, $\psi_{spin}\,\psi_{flavour}$ must be SYMMETRIC

BARYON SPIN WAVE-FUNCTIONS (ψ_{spin})

- **Combine 3 spin ½ quarks:** Total Spin $J = \frac{1}{2} \oplus \frac{1}{2} \oplus \frac{1}{2} = \frac{1}{2} \;\; or \;\; \frac{3}{2}$

- **Consider $J = 3/2$**

Trivial to write down the spin wave-function for the $\left|\frac{3}{2},\frac{3}{2}\right\rangle$ state:

$$\left|\frac{3}{2},\frac{3}{2}\right\rangle = \uparrow\uparrow\uparrow$$

Generate other states using the ladder operator $\hat{J}_-$

$$\hat{J}_-\left|\frac{3}{2},\frac{3}{2}\right\rangle = \left(\hat{J}_-\uparrow\right)\uparrow\uparrow + \uparrow\left(\hat{J}_-\uparrow\right)\uparrow + \uparrow\uparrow\left(\hat{J}_-\uparrow\right)$$

$$\sqrt{\frac{3}{2}\frac{5}{2}-\frac{3}{2}\frac{1}{2}}\left|\frac{3}{2},\frac{1}{2}\right\rangle = \downarrow\uparrow\uparrow + \uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow$$

$$\hat{J}_-\left|j,m\right\rangle = \sqrt{j(j+1)-m(m-1)}\left|j,m-1\right\rangle$$

$$\left|\frac{3}{2},\frac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}\left(\downarrow\uparrow\uparrow + \uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow\right)$$

Giving the $J = 3/2$ states:

$$\left|\frac{3}{2},\frac{3}{2}\right\rangle = \uparrow\uparrow\uparrow$$

$$\left|\frac{3}{2},\frac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}\left(\downarrow\uparrow\uparrow + \uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow\right)$$

$$\left|\frac{3}{2},-\frac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}\left(\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow + \downarrow\downarrow\uparrow\right)$$

$$\left|\frac{3}{2},-\frac{3}{2}\right\rangle = \downarrow\downarrow\downarrow$$

All SYMMETRIC under interchange of any two spins.

- Consider $J = 1/2$

First consider case where first 2 quarks are in a $|0,0\rangle$ state

$$|0,0\rangle_{(12)} = \tfrac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$$

$$|\tfrac{1}{2},\tfrac{1}{2}\rangle_{(123)} = |0,0\rangle_{(12)}|\tfrac{1}{2},\tfrac{1}{2}\rangle = \tfrac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)$$

$$|\tfrac{1}{2},-\tfrac{1}{2}\rangle_{(123)} = |0,0\rangle_{(12)}|\tfrac{1}{2},-\tfrac{1}{2}\rangle = \tfrac{1}{\sqrt{2}}(\uparrow\downarrow\downarrow - \downarrow\uparrow\downarrow)$$

ANTI-SYMMETRIC under interchange $1 \leftrightarrow 2$.

3-quark $J = 1/2$ states can ALSO be formed from the state with the first two quarks in a SYMMETRIC spin wave-function.

Can construct a 3-particle $|\tfrac{1}{2},\tfrac{1}{2}\rangle_{(123)}$ state from

$$|1,0\rangle_{(12)}|\tfrac{1}{2},\tfrac{1}{2}\rangle_{(3)}$$ and

$$|1,1\rangle_{(12)}|\tfrac{1}{2},-\tfrac{1}{2}\rangle_{(3)}$$

Taking linear combination:

$$\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle = a\,|1,1\rangle\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle + b\,|1,0\rangle\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle$$

with $a^2+b^2=1$. Act upon both sides with $\hat{J}_+$

$$\hat{J}_+\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle = a\left[\left(\hat{J}_+|1,1\rangle\right)\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle + |1,1\rangle\left(\hat{J}_+\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle\right)\right]$$
$$+\, b\left[\left(\hat{J}_+|1,0\rangle\right)\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle + |1,0\rangle\left(\hat{J}_+\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle\right)\right]$$

$$0 = a|1,1\rangle\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle + \sqrt{2}\,b\,|1,1\rangle\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle$$

$$a = -\sqrt{2}\,b$$

$$\hat{J}_+|j,m\rangle = \sqrt{j(j+1)-m(m+1)}\,|j,m+1\rangle$$

which with $a^2+b^2=1$ implies: $a=\sqrt{\tfrac{2}{3}},\quad b=-\sqrt{\tfrac{1}{3}}$

Giving

$$\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle = \sqrt{\tfrac{2}{3}}\,|1,1\rangle\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle - \sqrt{\tfrac{1}{3}}\,|1,0\rangle\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle$$

$$\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle = \tfrac{1}{\sqrt{6}}\left(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow\right)$$

Similarly,

$$\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle = \tfrac{1}{\sqrt{6}}\left(2\downarrow\downarrow\uparrow - \downarrow\uparrow\downarrow - \uparrow\downarrow\downarrow\right)$$

$$|1,1\rangle = \uparrow\uparrow$$

$$|1,0\rangle = \tfrac{1}{\sqrt{2}}\left(\uparrow\downarrow + \downarrow\uparrow\right)$$

SYMMETRIC under interchange $1 \leftrightarrow 2$.

3 QUARK SPIN WAVE-FUNCTIONS

$$J = \frac{3}{2}$$

$$\left|\tfrac{3}{2},\tfrac{3}{2}\right\rangle = \uparrow\uparrow\uparrow$$
$$\left|\tfrac{3}{2},\tfrac{1}{2}\right\rangle = \tfrac{1}{\sqrt{3}}\left(\downarrow\uparrow\uparrow + \uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow\right)$$
$$\left|\tfrac{3}{2},-\tfrac{1}{2}\right\rangle = \tfrac{1}{\sqrt{3}}\left(\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow + \downarrow\downarrow\uparrow\right)$$
$$\left|\tfrac{3}{2},-\tfrac{3}{2}\right\rangle = \downarrow\downarrow\downarrow$$

SYMMETRIC under interchange of any 2 quarks

$$J = \frac{1}{2}$$

$$\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle = \tfrac{1}{\sqrt{2}}\left(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow\right)$$
$$\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle = \tfrac{1}{\sqrt{2}}\left(\uparrow\downarrow\downarrow - \downarrow\uparrow\downarrow\right)$$

ANTI-SYMMETRIC under interchange of $1 \leftrightarrow 2$

$$J = \frac{1}{2}$$

$$\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle = \tfrac{1}{\sqrt{6}}\left(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow\right)$$
$$\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle = \tfrac{1}{\sqrt{6}}\left(2\downarrow\downarrow\uparrow - \downarrow\uparrow\downarrow - \uparrow\downarrow\downarrow\right)$$

SYMMETRIC under interchange of $1 \leftrightarrow 2$

$\psi_{spin}\,\psi_{flavour}$ must be symmetric under interchange of any 2 quarks

Consider 3 cases:

1 Quarks all SAME flavour: *uuu, ddd, sss*

- $\psi_{flavour}$ is SYMMETRIC under interchange of any two quarks.
- REQUIRE ψ_{spin} to be SYMMETRIC under interchange of any two quarks.
- ONLY satisfied by $J = 3/2$ states.
- no *uuu, ddd, sss* $J = 1/2$ baryons with $L=0$.

> THREE $J = 3/2$ states: *uuu, ddd, sss*

2 Two quarks have same flavour: *uud, uus, ddu, dds, ssu, ssd*

- For the like quarks, $\psi_{flavour}$ is SYMMETRIC.
- REQUIRE ψ_{spin} to be SYMMETRIC under interchange of LIKE quarks $1 \leftrightarrow 2$.
- Satisfied by $J = 3/2$ and $J = 1/2$

> SIX $J = 3/2$ states and SIX $J = 1/2$ states: *uud, uus, ddu, dds, ssu, ssd*

3 All quarks have DIFFERENT flavours: uds

Two possibilities for the (ud) part:

i) FLAVOUR SYMMETRIC $\frac{1}{\sqrt{2}}(ud + du)$

- require spin wave-function to be SYMMETRIC under interchange of ud
- satisfied by $J = 3/2$ and $J = 1/2$ states

ONE $J = 3/2$ and ONE $J = 1/2$ state: uds

ii) FLAVOUR ANTI-SYMMETRIC $\frac{1}{\sqrt{2}}(ud - du)$

- require spin wave-function to be ANTI-SYMMETRIC under interchange of ud
- only satisfied by $J = 1/2$ state

ONE $J = 1/2$ uds state.

Quark Model predicts that Baryons appear in

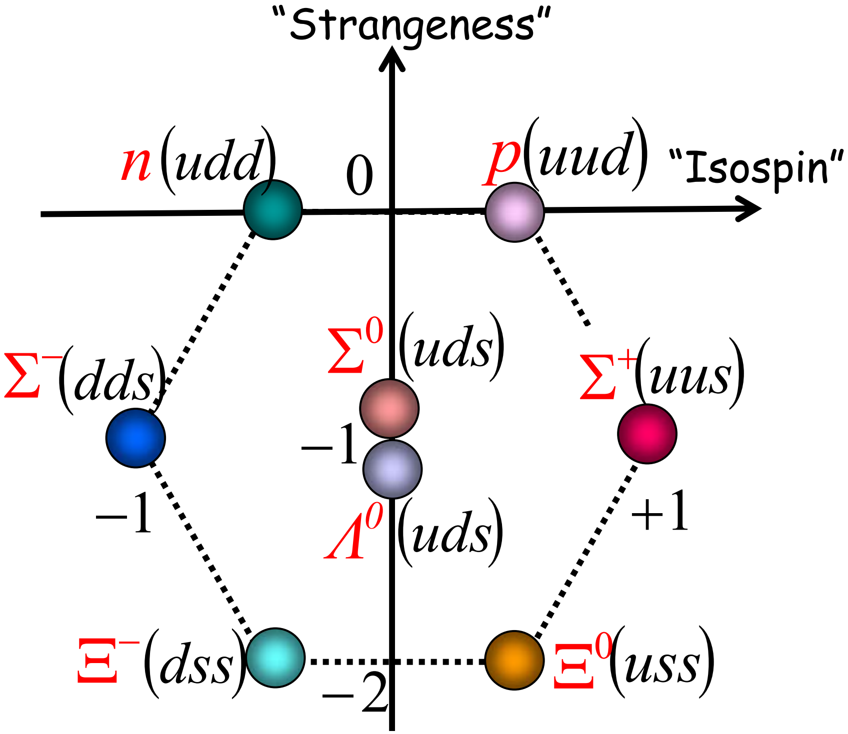
DECUPLETS (10) of SPIN 3/2 states

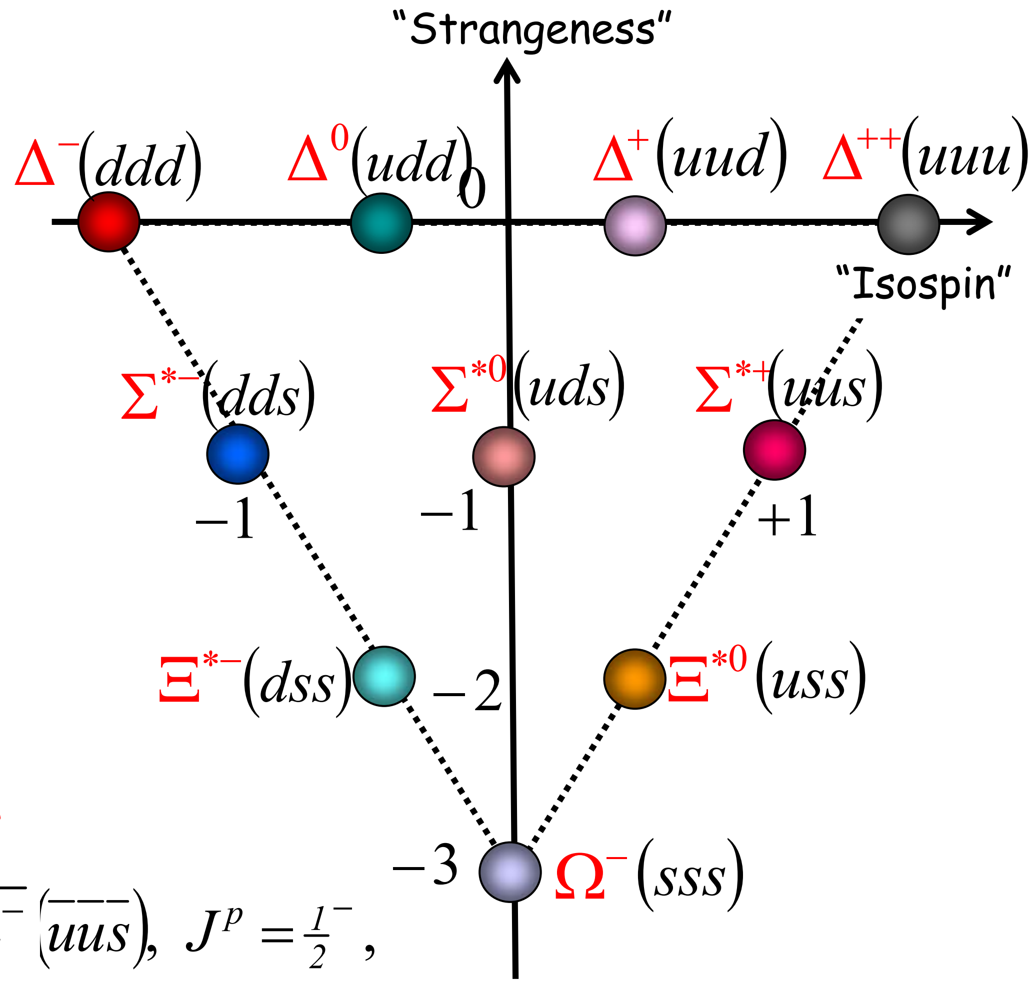
OCTETS (8) of SPIN 1/2 states.

Baryon Multiplets

OCTET $J^P = {}^{1}\!/_{2}^{+}$

DECUPLET $J^P = {}^{3}\!/_{2}^{+}$





Antibaryons are in separate multiplets

<u>Example:</u> Antiparticle of $\Sigma^+(uus)$ is $\overline{\Sigma^-}(\overline{u}\overline{u}\overline{s})$, $J^P = \frac{1}{2}^-$,
and NOT $\Sigma^-(dds)$, $J^P = \frac{1}{2}^+$

Baryon Masses

Baryon Mass Formula ($L=0$)

$$M_{qqq} = m_1 + m_2 + m_3 + A'\left(\frac{\vec{S}_1\cdot\vec{S}_2}{m_1 m_2} + \frac{\vec{S}_1\cdot\vec{S}_3}{m_1 m_3} + \frac{\vec{S}_2\cdot\vec{S}_3}{m_2 m_3}\right)$$

where A' is a constant.

<u>Example:</u> All quarks have same mass, $m_1 = m_2 = m_3 = m_q$

$$M_{qqq} = 3m_q + A'\sum_{i<j}\frac{\vec{S}_i\cdot\vec{S}_j}{m_q^2}$$

$$\vec{S}^2 = \left(\vec{S}_1 + \vec{S}_2 + \vec{S}_3\right)^2 = \vec{S}_1^2 + \vec{S}_2^2 + \vec{S}_3^2 + 2\sum_{i<j}\vec{S}_i\cdot\vec{S}_j$$

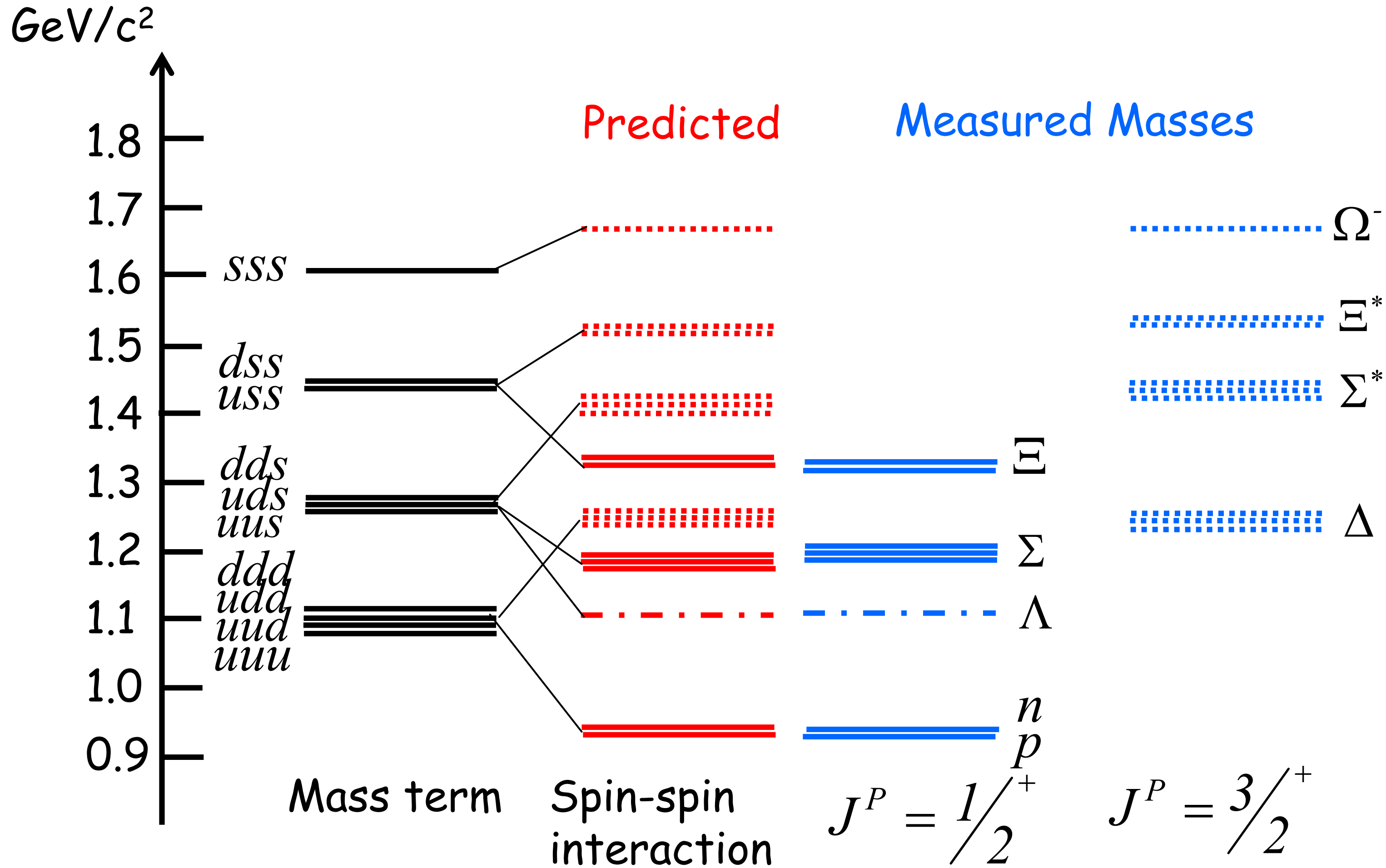
$$2\sum_{i<j}\vec{S}_i\cdot\vec{S}_j = S(S+1) - 3\tfrac{1}{2}(\tfrac{1}{2}+1) = S(S+1) - \tfrac{9}{4}$$

$$\sum_{i<j}\vec{S}_i\cdot\vec{S}_j = -\frac{3}{4} \qquad \mathrm{J} = \tfrac{1}{2}$$

$$\sum_{i<j}\vec{S}_i\cdot\vec{S}_j = +\frac{3}{4} \qquad \mathrm{J} = \tfrac{3}{2}$$

e.g. proton (uud) versus Δ (uud)

$$M_p = 3m_u - \frac{3A'}{4m_u^2}, \qquad M_\Delta = 3m_u + \frac{3A'}{4m_u^2}$$



Excellent agreement using:

$$m_u = 0.362\ GeV, \quad m_d \sim 0.366\ GeV, \quad m_s \sim 0.537\ GeV, \quad A' = 0.026\ GeV^3 \approx A/2$$

Constituent quark mass depends on hadron wave-function and includes cloud of quarks & gluons ⇒ slightly different values for mesons and baryons.

Baryon Magnetic Moments

Magnetic dipole moments arise from

- the orbital motion of charged particles, and
- the intrinsic spin.

Orbital Motion

Classically, current loop

$$\mu = IA = \frac{qv}{2\pi r}\pi r^2 = \frac{qpr}{2m} = \frac{q}{2m}L_z$$

Quantum mechanically, get the same result

$$\hat{\mu} = g_\ell \frac{q}{2m}\hat{L}_z$$

g_l is the "g-factor".
$g_\ell = 1$ charged particles
$g_\ell = 0$ neutral particles

Intrinsic Spin

The magnetic moment operator due to the intrinsic spin of a particle is

$$\hat{\mu} = g_s \frac{q}{2m}\hat{S}_z$$

g_s is the "spin g-factor".
$g_s = 2$ DIRAC spin ½ point-like particles

The magnetic dipole moment is the maximum measurable component of the magnetic dipole moment operator

$$\mu_\ell = \left\langle \psi_{space} \left| g_l \frac{q}{2m} \hat{L}_z \right| \psi_{space} \right\rangle; \qquad \mu_s = \left\langle \psi_{spin} \left| g_s \frac{q}{2m} \hat{S}_z \right| \psi_{spin} \right\rangle$$

Electron

$$\mu_\ell = -g_\ell \frac{e}{2m_e} \hbar \ell = -\mu_B \ell \qquad \mu_s = -g_s \frac{e}{2m_e} \frac{\hbar}{2} = -\mu_B$$

where $\mu_B = e\hbar / 2m_e$ is the Bohr Magneton.

- Observed difference from $g_s = 2$ due to higher order corrections in QED:

$$\mu_s = -\mu_B \left[1 + \frac{\alpha}{2\pi} + O(\alpha^2) + \ldots \right] \qquad \alpha = e^2 / 4\pi \approx 1/137$$

Proton and Neutron

If the proton and neutron were point-like particles,

$$\mu_\ell = g_\ell \frac{e}{2m_p}\hbar\ell; \qquad \mu_s = g_s \frac{e}{2m_p}\frac{\hbar}{2} = \frac{1}{2} g_s \mu_N$$

where $\mu_N = e\hbar/2m_p$ is the Nuclear Magneton.

Expect:

p	spin ½, charge $+e$	$\mu_s = \mu_N$
n	spin ½, charge 0	$\mu_s = 0$

Observe:

p	$\mu_s = +2.793\,\mu_N \rightarrow g_s = +5.586$
n	$\mu_s = -1.913\,\mu_N \rightarrow g_s = -3.826$

Observation that p and n are NOT point-like $\Rightarrow$ evidence for quarks.

$\Rightarrow$ Use QUARK MODEL to estimate baryon magnetic moments.

Baryon Magnetic Moments in the Quark Model

Assume that bound quarks within baryons behave as DIRAC point-like spin ½ particles with fractional charge q_q.

Then quarks will have magnetic dipole moment operator and magnitude:

$$\hat{\mu}_q = \frac{q_q}{m_q}\hat{S}_z \qquad \mu_q = \left\langle \psi^q_{spin} \left| \frac{q_q}{m_q}\hat{S}_z \right| \psi^q_{spin} \right\rangle = \frac{q_q \hbar}{2m_q}$$

where m_q is the quark mass.

Therefore, $$\mu_u = \frac{2}{3}\frac{e\hbar}{2m_u}, \quad \mu_d = -\frac{1}{3}\frac{e\hbar}{2m_u}, \quad \mu_s = -\frac{1}{3}\frac{e\hbar}{2m_s}$$

For quarks bound within an $L=0$ baryon, the baryon magnetic moment is the expectation value of the sum of the individual quark magnetic moment operators:

$$\hat{\mu}_B = \frac{q_1}{m_1}\hat{S}_{1z} + \frac{q_2}{m_2}\hat{S}_{2z} + \frac{q_3}{m_3}\hat{S}_{3z}; \qquad \mu_B = \left\langle \psi^B_{spin} \left| \hat{\mu}_B \right| \psi^B_{spin} \right\rangle$$

where ψ^B_{spin} is the baryon spin wave-function.

Example: Magnetic moment of a proton

For a spin-up proton:

$$\psi^{p}_{spin} = \tfrac{1}{\sqrt{6}}\left(2u\uparrow u\uparrow d\downarrow - u\uparrow u\downarrow d\uparrow - u\downarrow u\uparrow d\uparrow\right)$$

$$\begin{aligned}
\mu_p &= \tfrac{1}{6}\left\langle 2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow \middle| \hat{\mu}_1 + \hat{\mu}_2 + \hat{\mu}_3 \middle| 2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow \right\rangle \\
&= \tfrac{1}{6}\left\langle 2\uparrow\uparrow\downarrow \middle| \hat{\mu}_1 + \hat{\mu}_2 + \hat{\mu}_3 \middle| 2\uparrow\uparrow\downarrow \right\rangle \\
&+ \tfrac{1}{6}\left\langle -\uparrow\downarrow\uparrow \middle| \hat{\mu}_1 + \hat{\mu}_2 + \hat{\mu}_3 \middle| -\uparrow\downarrow\uparrow \right\rangle \\
&+ \tfrac{1}{6}\left\langle -\downarrow\uparrow\uparrow \middle| \hat{\mu}_1 + \hat{\mu}_2 + \hat{\mu}_3 \middle| -\downarrow\uparrow\uparrow \right\rangle \\
&= \tfrac{1}{6}\left[4(\mu_1 + \mu_2 - \mu_3) + (\mu_1 - \mu_2 + \mu_3) + (-\mu_1 + \mu_2 + \mu_3)\right] \\
&= \tfrac{1}{6}\left[4(\mu_1 + \mu_2) - 2\mu_3\right]
\end{aligned}$$

For a proton $\mu_1 = \mu_2 = \mu_u; \quad \mu_3 = \mu_d = -\tfrac{1}{2}\mu_u$ (assuming $m_u = m_d$)

$$\mu_p = \frac{3}{2}\mu_u = \frac{e\hbar}{2m_u} = \frac{m_p}{m_u}\mu_N$$

where $\mu_N = e\hbar/2m_p$ is the Nuclear Magneton.

Repeat for the other (L=0) Baryons, PREDICT

$$\frac{\mu_n}{\mu_p} = -\frac{2}{3}$$

compared to the experimentally measured value of -0.685.

Baryon	μ_B in Quark Model	Predicted [μ_N]	Observed [μ_N]
p	$\frac{4}{3}\mu_u - \frac{1}{3}\mu_d$	+2.79	+2.793
n	$\frac{4}{3}\mu_d - \frac{1}{3}\mu_u$	-1.86	-1.913
Λ	μ_s	-0.61	-0.614 $\pm$ 0.005
Σ^+	$\frac{4}{3}\mu_u - \frac{1}{3}\mu_s$	+2.68	+2.46 $\pm$ 0.01
Ξ^0	$\frac{4}{3}\mu_s - \frac{1}{3}\mu_u$	-1.44	-1.25 $\pm$ 0.014
Ξ^-	$\frac{4}{3}\mu_s - \frac{1}{3}\mu_d$	-0.51	-0.65 $\pm$ 0.01
Ω^-	$3\mu_s$	-1.84	-2.02 $\pm$ 0.05

Impressive agreement with data using

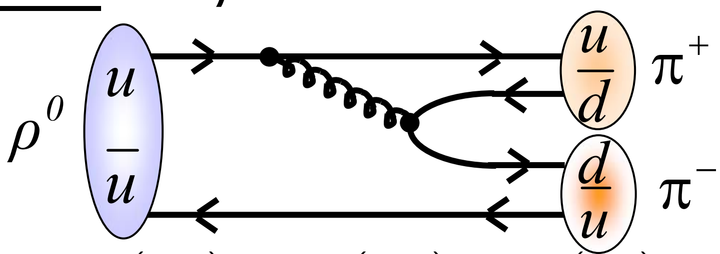
$$m_u = m_d = 0.336\ GeV, \quad m_s \sim 0.509\ GeV$$

Hadron Decays

- Hadrons are eigenstates of the strong force.
- Hadrons will decay via the strong interaction to lighter mass states if energetically feasible.
- Angular momentum and parity MUST be conserved in strong decays.

Examples:

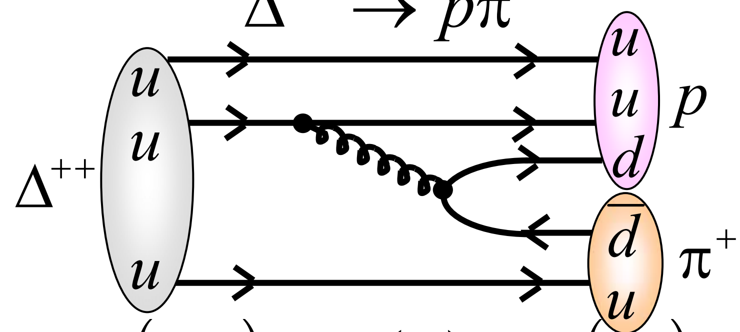
$\rho^0 \to \pi^+\pi^-$



$$m\left(\rho^0\right) > m\left(\pi^+\right) + m\left(\pi^-\right)$$

769 140 140 MeV

$\Delta^{++} \to p\pi^+$



$$m\left(\Delta^{++}\right) > m\left(p\right) + m\left(\pi^+\right)$$

1231 938 140 MeV

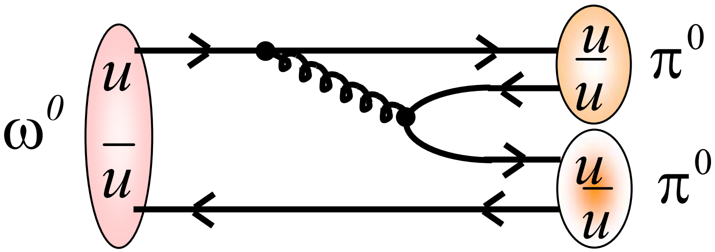
	ρ^0	$\to \pi^+\pi^-$
J^P	1^-	0^-0^-
P	-1	$P(\pi^+)P(\pi^-)(-1)^L$

	Δ^{++}	$\to p\,\pi^+$
J^P	$3/2^+$	$1/2^+\,0^-$
P	$+1$	$P(p)P(\pi^+)(-1)^L$

For $\rho^0 \to \pi^+\pi^-$ and $\Delta^{++} \to p\pi^+$: L = 1 to conserve angular momentum and parity.

- Also need to check for identical particles in the final state.

<u>Example</u> $\omega^0 \to \pi^0\pi^0$



$$m(\omega^0) > m(\pi^0) + m(\pi^0)$$

782 135 135 MeV

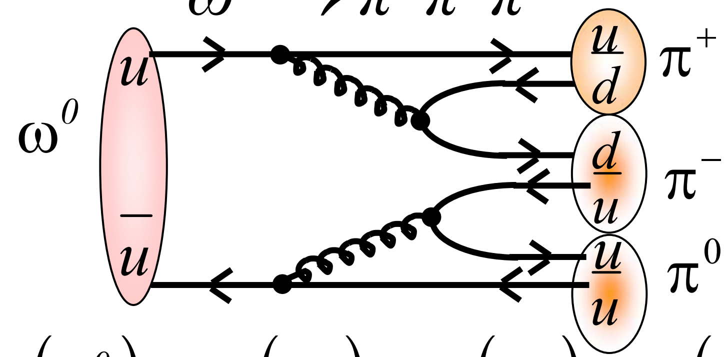
	ω^0	$\to \pi^0\pi^0$
J^P	1^-	0^-0^-
P	-1	$P(\pi^0)P(\pi^0)(-1)^L$

$L=1$ to conserve ang. mom. and parity.

Identical bosons in final state ⇒ wavefunction must be EVEN under exchange.

⇒ FORBIDDEN DECAY

$\omega^0 \to \pi^+\pi^-\pi^0$



$$m(\omega^0) > m(\pi^+) + m(\pi^-) + m(\pi^0)$$

782 140 140 135 MeV

	ω^0	$\to \pi^+\pi^-\pi^0$
J^P	1^-	$0^-0^-0^-$
P	-1	$P(\pi^+)P(\pi^-)P(\pi^0)(-1)^{L_1}(-1)^{L_2}$

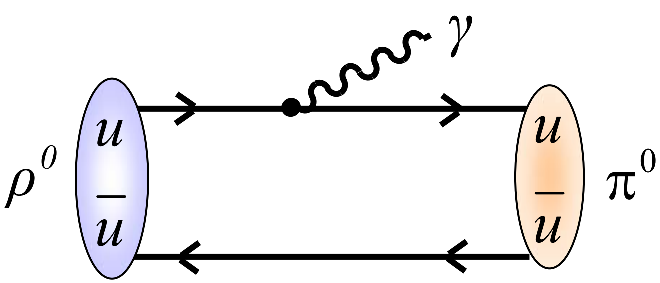
$L_1 = L_2 = 1$ to conserve ang. mom. and parity.

⇒ ALLOWED DECAY

Branching Fraction ~ 90%

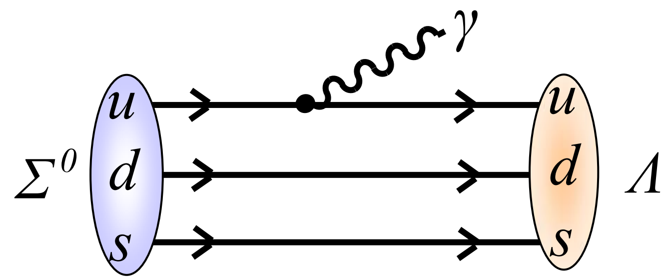
- Hadrons can also decay via the electromagnetic interaction.

$$\rho^0 \to \pi^0 \gamma$$



$$B \sim 8 \times 10^{-4}$$

$$\Sigma^0 \to \Lambda \gamma$$



$$B \sim 100\%$$

	ρ^0	$\to \pi^0\gamma$
J^P	1^-	$0^- 1^-$
P	-1	$P(\pi^0)P(\gamma)(-1)^L$

$\Rightarrow L = 1$

	Σ^0	$\to \Lambda\gamma$
J^P	$\frac{1}{2}^+$	$\frac{1}{2}^+ 1^-$
P	$+1$	$P(\Lambda)P(\gamma)(-1)^L$

$\Rightarrow L = 1$

- The lightest mass states ($p, K^{\pm}, K^0, \overline{K^0}, \Lambda, n$) require a change of quark flavour in the decay and therefore decay via the weak interaction (see later).

Summary

- Baryons and mesons are composite particles (complicated).
- However, the Quark Model can be used to make predictions for masses/magnetic moments.
- The predictions give reasonably consistent values for the constituent quark masses:

	$m_{u/d}$	m_s
Meson Masses	307 MeV	487 MeV
Baryon Masses	364 MeV	537 MeV
Baryon Mag. Moms.	336 MeV	509 MeV

$$m_u \approx m_d \approx 335\ MeV, \quad m_s \approx 510\ MeV$$

- Hadrons will decay via the STRONG interaction to lighter mass states if energetically feasible.
- Hadrons can also decay via the EM interaction.
- The lightest mass states require a change of quark flavour to decay and therefore decay via the WEAK interaction (see later).

Discovery of the J/ψ ($c\bar{c}$)

- 1974: Discovery of a NARROW RESONANCE in e^+e^- collisions at $\sqrt{s} \approx 3.1 GeV$

J/ψ (3097)

Observed width ~ 3MeV, all due to experimental resolution.

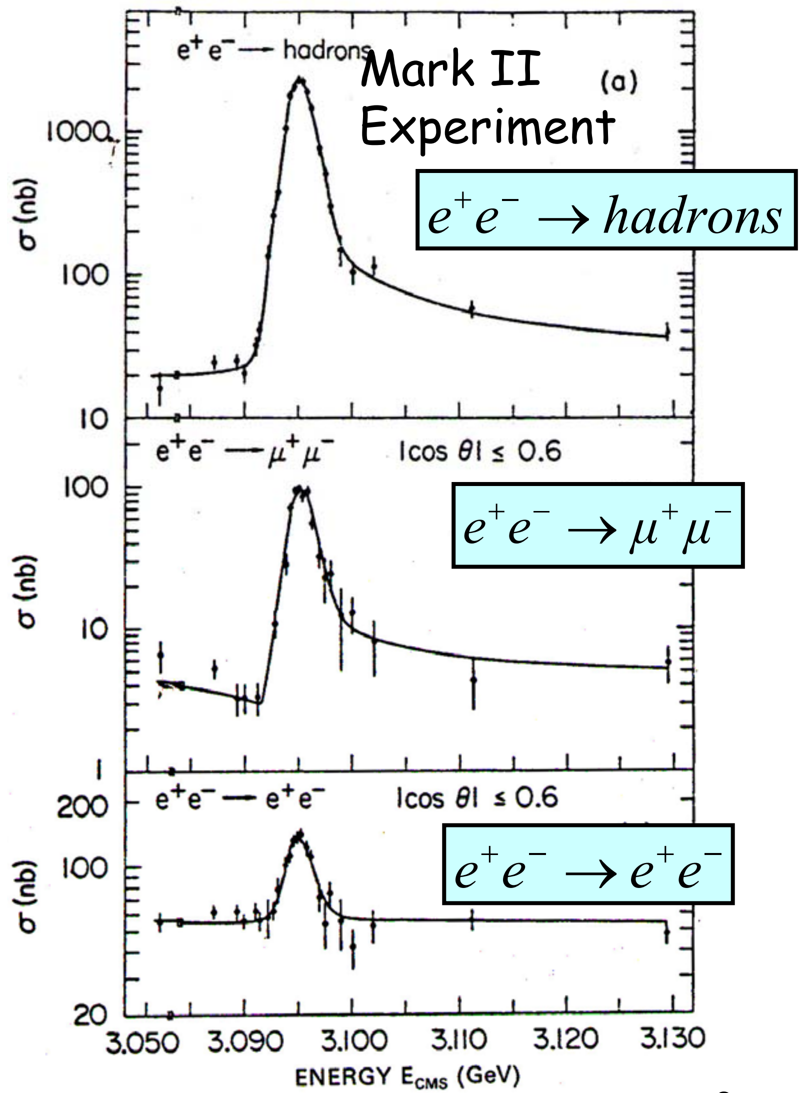
Actual TOTAL WIDTH, $\Gamma_{J/\psi}$ ~ 87 keV.

Branching Fractions

$$B(J/\psi \to hadrons) \sim 88\%$$
$$B(J/\psi \to \mu^+\mu^-) \approx B(J/\psi \to e^+e^-) \sim 6\%$$

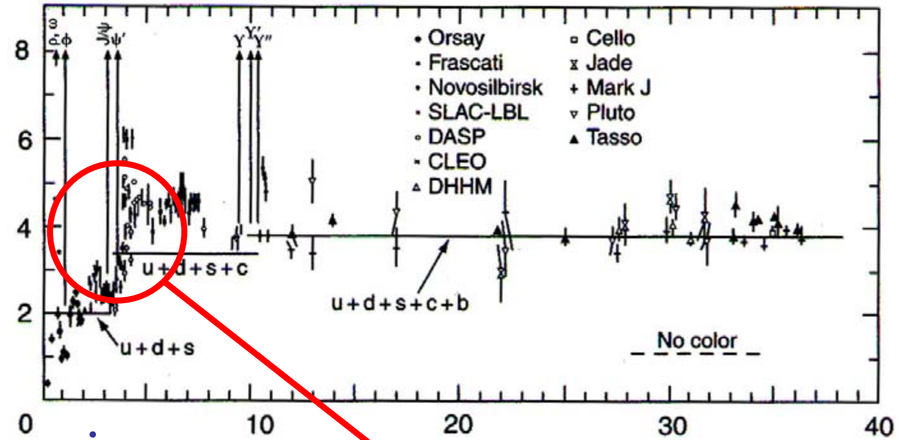
Partial widths

$$\Gamma_{J/\psi \to hadrons} \sim 77 \text{ keV}$$
$$\Gamma_{J/\psi \to \mu^+\mu^-} \approx \Gamma_{J/\psi \to e^+e^-} \sim 5 \text{ keV}$$



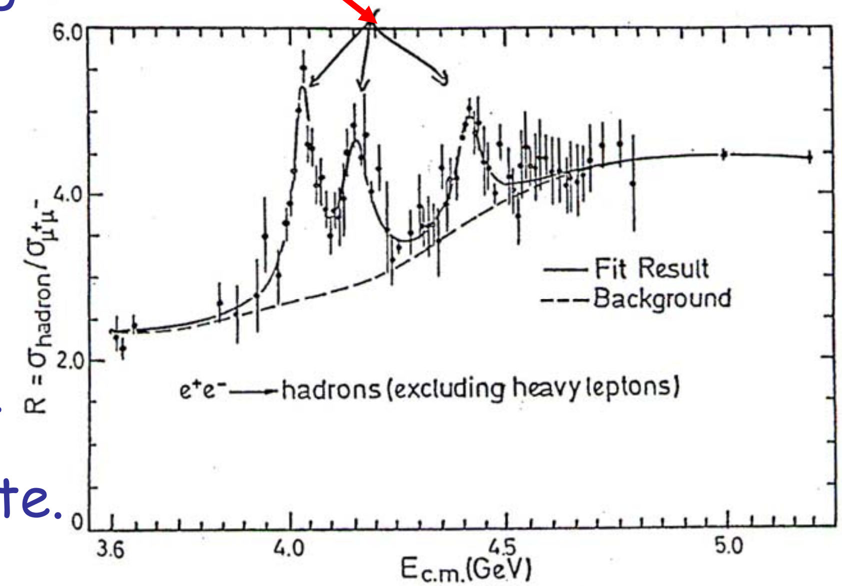
Resonance seen in

$$R = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



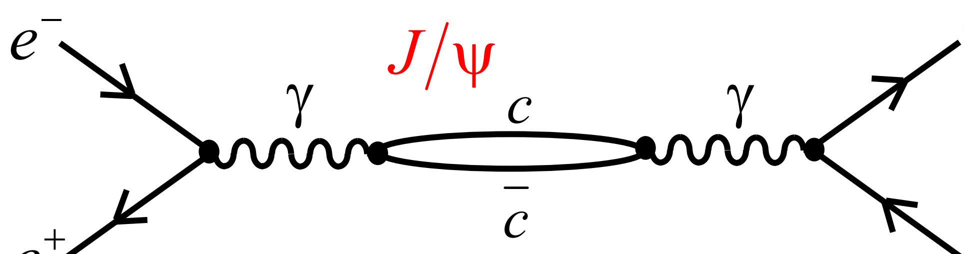
Zoom in to the CHARMONIUM ($c\overline{c}$) region:

$$\sqrt{s} \sim 2m_c$$



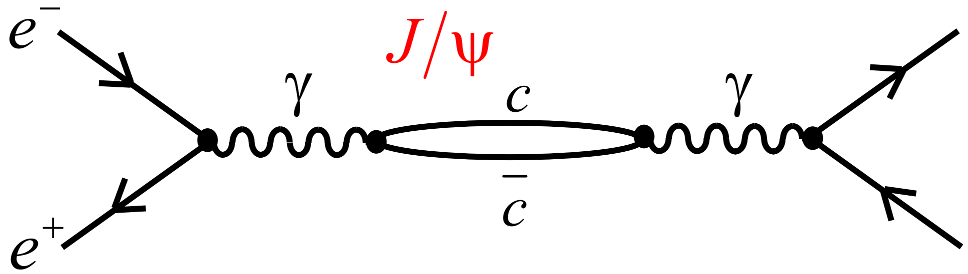
mass of charm quark, m_c~1.5 GeV.

Resonances due to formation of BOUND unstable $c\overline{c}$ states. The lowest energy of these is the narrow J/ψ state.



Charmonium

- $c\bar{c}$ bound states produced directly in e^+e^- collisions have the same spin and parity as the photon



$J^P = 1^-$

- However, expect to see a spectrum of bound $c\bar{c}$ states (analagous to e^+e^- bound states, positronium)

$^{2S+1}L_J$

$$n = 1 \qquad L = 0 \qquad S = 0,1 \qquad {}^1S_0,\ {}^3S_1$$

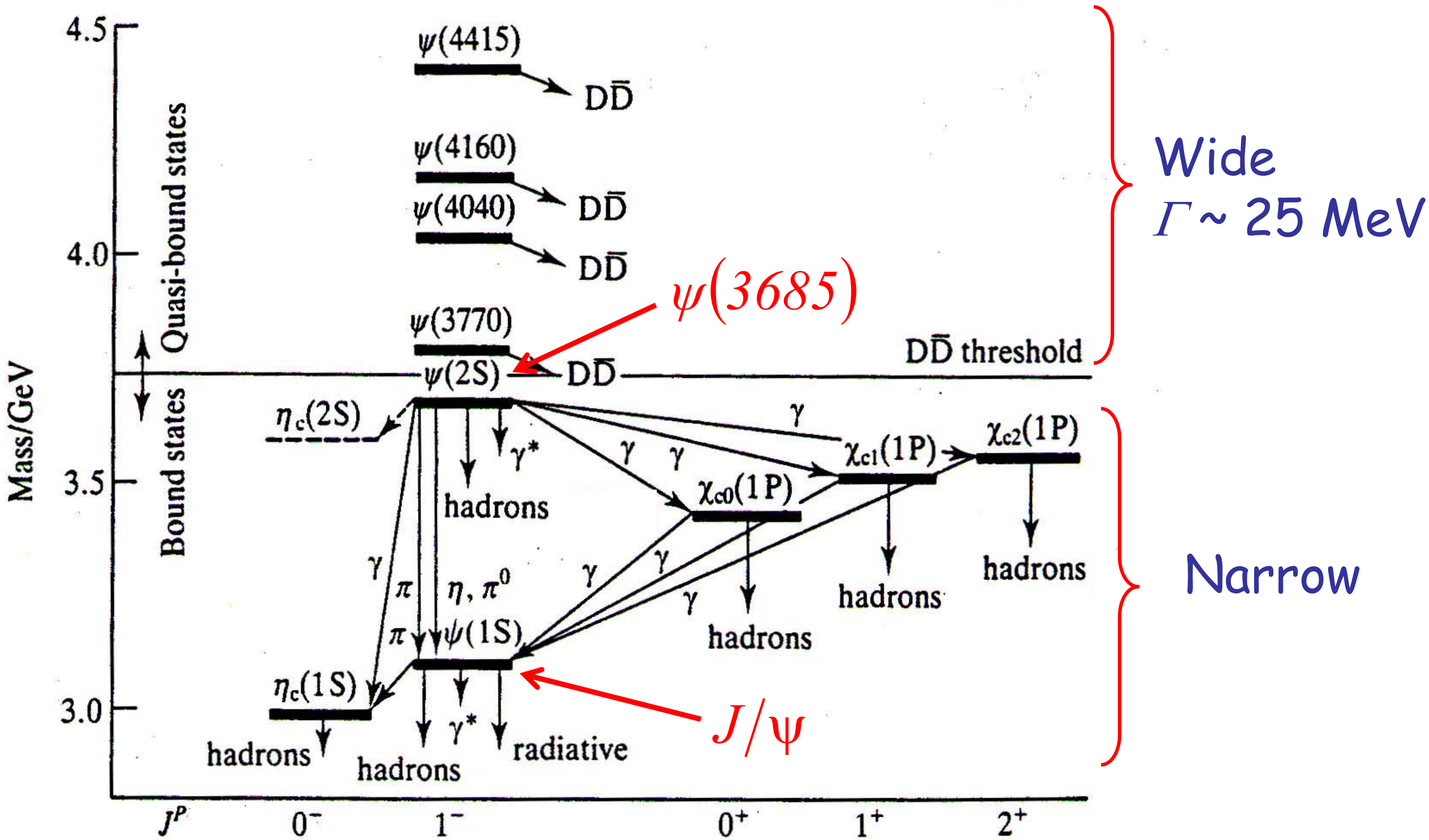
$$n = 2 \qquad L = 0,1 \qquad S = 0,1 \qquad {}^1S_0,\ {}^3S_1,\ {}^1P_1,\ {}^3P_{012}$$

$$\vdots$$

$$etc$$

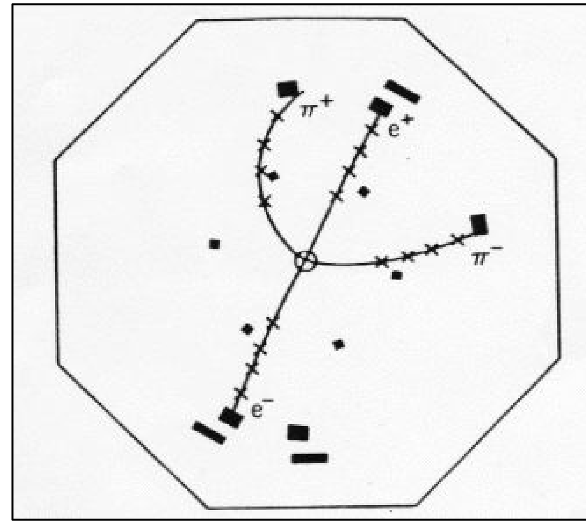
Parity = $(-1)(-1)^L$

The Charmonium System



$^1S_0$ $^3S_1$ $^3P_0$ $^3P_1$ $^3P_2$

➢ All $c\bar{c}$ bound states observed via their DECAY:



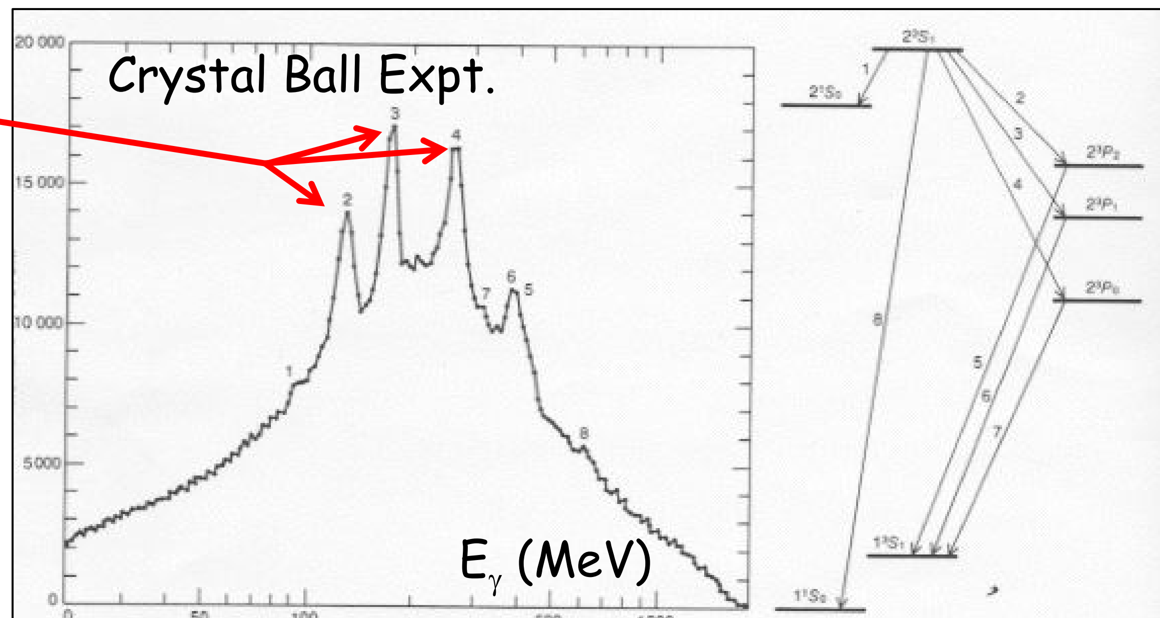
Example: Hadronic decay

$$\psi(3685) \to J/\psi\, \pi^+\pi^-$$

Example: Photonic decays

$$\psi(3685) \to \chi + \gamma$$
$$\hookrightarrow J/\psi + \gamma$$

Peaks in γ spectrum



- Knowing the $c\bar{c}$ energy levels provides a probe of the QCD potential.

  - Because QCD is a theory of a strong confining force (self-interacting gluons), it is VERY difficult to calculate the exact form of the QCD potential from first principles.

  - However, it is possible to experimentally "determine" the QCD potential by finding an appropriate form which gives the observed charmonium states.

  - In practise, the QCD potential

$$V_{QCD} = -\frac{4}{3}\frac{\alpha_s}{r} + kr$$

with $\alpha_s = 0.2$ and $k = 1$ GeV fm^{-1} provides a good description of the EXPERIMENATLLY OBSERVED levels in the charmonium system.

Why is the J/ψ so Narrow ?

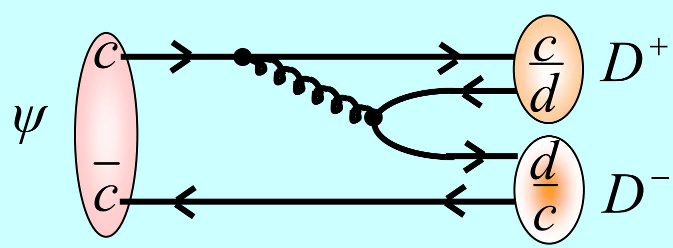
Consider the charmonium $^3S_1$ states:

1^3S_1	$\psi(3097)$	$\Gamma \approx 0.09$ MeV
2^3S_1	$\psi(3685)$	$\Gamma \approx 0.24$ MeV
3^3S_1	$\psi(3767)$	$\Gamma \approx 25$ MeV
4^3S_1	$\psi(4040)$	$\Gamma \approx 50$ MeV

- Width depends on whether the decay to lightest mesons containing c quarks, $D^-(d\bar{c})$, $D^+(c\bar{d})$, is kinematically possible:

$$m_{D^\pm} = 1869.4 \pm 0.5 \ \text{MeV}$$

$m(\psi) > 2m(D)$

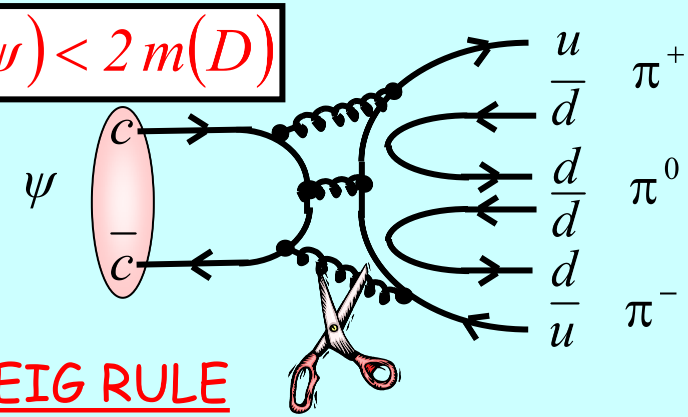


$\psi \to D^+D^-$ ALLOWED

"ordinary" STRONG decay

$\Rightarrow$ LARGE WIDTH

$m(\psi) < 2m(D)$



ZWEIG RULE

Unconnected lines in the Feynman diagram lead to SUPPRESSION of the decay amplitude

$\Rightarrow$ NARROW WIDTH

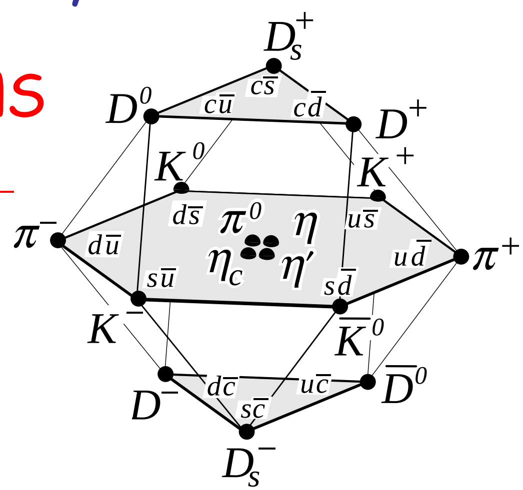
Charmed Hadrons

The existence of the c quark $\Rightarrow$ expect to see CHARMED mesons and baryons (i.e. containing a c quark).

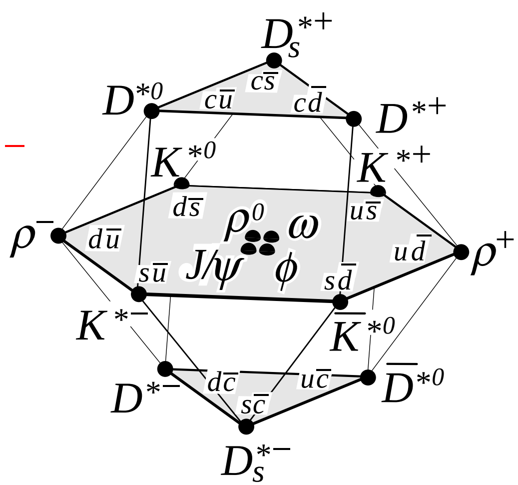
Extend quark symmetries to 3 dimensions:

MESONS

$J^P = 0^-$

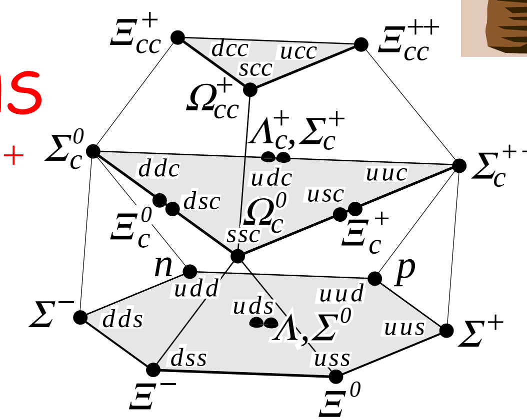


$J^P = 1^-$

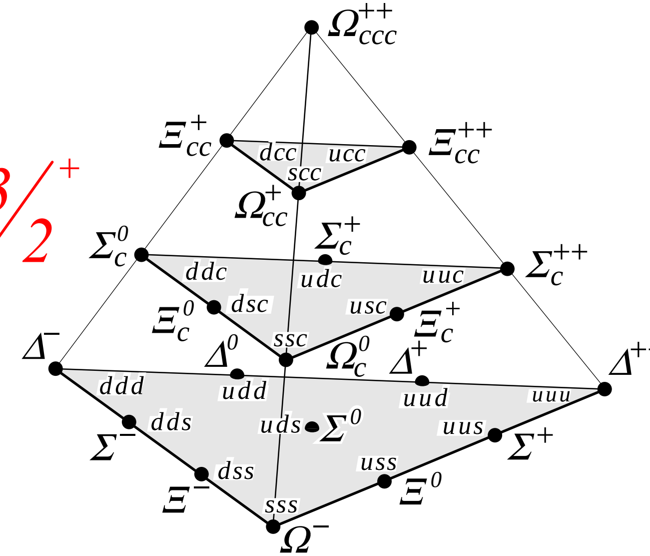


BARYONS

$J^P = 1/2^+$



$J^P = 3/2^+$

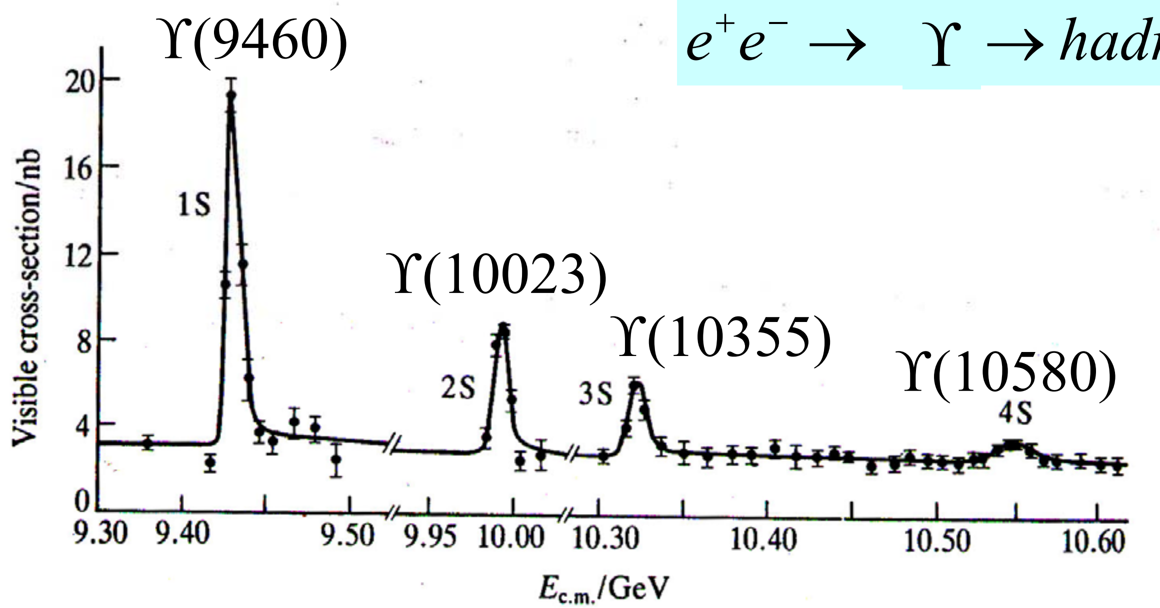


Discovery of the Υ ($b\bar{b}$)

- 1977: Discovery of the Υ(9460) resonance state.
- Lowest energy $^3S_1$ bound $b\bar{b}$ state (bottomonium).
- $\Rightarrow m_b \sim 4.7$ GeV

Similar properties to the ψ

$e^+e^- \rightarrow \Upsilon \rightarrow hadrons$

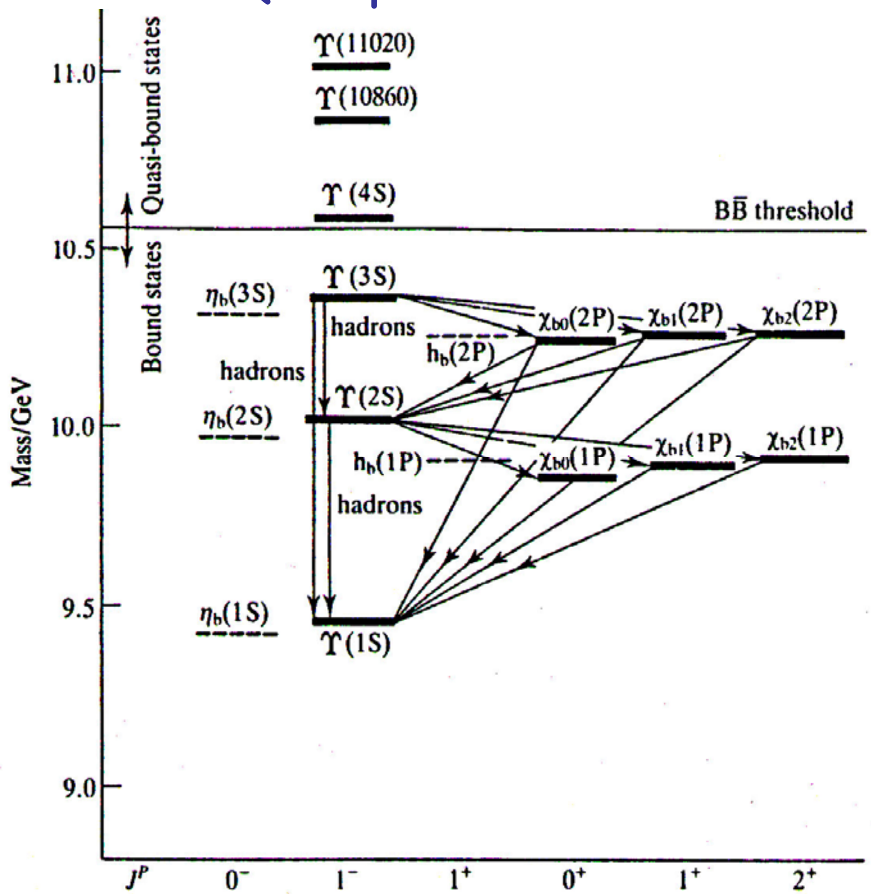


Full Width ~53 keV 44 keV 26 keV 14 MeV

NARROW WIDE

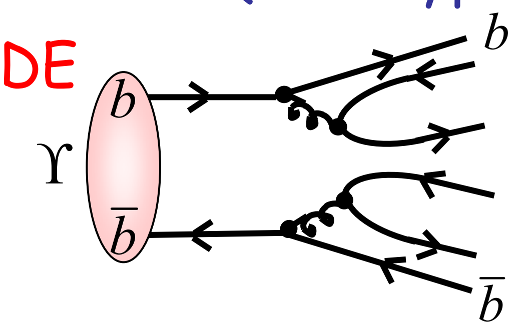
Bottomonium

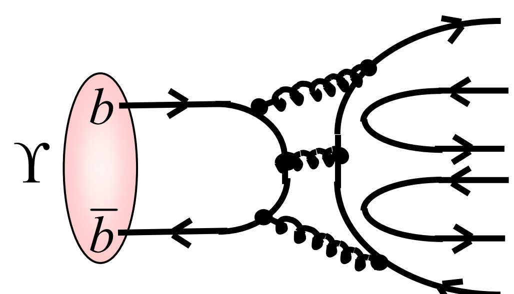
- Bottomonium is the analogue of charmonium for b quark.
- Bottomonium spectrum well described by same QCD potential as used for charmonium.
- Evidence that QCD potential does not depend on Quark type.



WIDE

NARROW





Zweig suppressed

Bottom Hadrons

Extend quark symmetries to 4 dimensions (difficult to draw!)

Examples:

Mesons ($J^P = 0^-$): $B^-(b\bar{u})$; $B^0(\bar{b}d)$; $B_s^0(\bar{b}s)$; $B_c^-(b\bar{c})$

The B_c^- is the heaviest meson discovered so far: $m_{B_c^-} = 6.4 \pm 0.4$ GeV

Mesons ($J^P = 1^-$): $B^{*-}(b\bar{u})$; $B^{*0}(\bar{b}d)$; $B_s^{*0}(\bar{b}s)$

The mass of the B^* mesons is ONLY 50 MeV above the B meson mass. Expect ONLY electromagnetic decays $B^* \to B\gamma$

Baryons ($J^P = 1/2^+$): $\Lambda_b(bud)$; $\Sigma_b(buu)$; $\Xi_b(bus)$

Summary

- The c and b quarks were first observed in bound state resonances.

- Consequences of the existence of c and b quarks are
  - Spectra of $c\bar{c}$ (charmonium) and $b\bar{b}$ (bottomonium) bound states
  - Increase in $R = \sigma(e^+e^- \to hadrons) / \sigma(e^+e^- \to \mu^+\mu^-)$
  - Existence of mesons and baryons containing c and b quarks

- The majority of charm and bottom hadrons decay via the WEAK interaction (strong and electromagnetic decays are forbidden by energy conservation).

- The t quark is VERY HEAVY and decays via the WEAK interaction before a $t\bar{t}$ bound state can be formed.

$$\begin{pmatrix} m_u \approx 335\ MeV \\ m_d \approx 335\ MeV \end{pmatrix} \begin{pmatrix} m_c \approx 1.5\ GeV \\ m_s \approx 510\ MeV \end{pmatrix} \begin{pmatrix} m_t \approx 175\ GeV \\ m_b \approx 4.5\ GeV \end{pmatrix}$$