Section IV
The Standard Model

The Standard Model

<u>Spin ½ Fermions</u>

				Charge (units of e)
LEPTONS	$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix}$	$\begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix}$	$\begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$	-1 0
QUARKS	$\begin{pmatrix} u \\ d \end{pmatrix}$	$\begin{pmatrix} c \\ s \end{pmatrix}$	$\begin{pmatrix} t \\ b \end{pmatrix}$	2/3 -1/3

PLUS antileptons and antiquarks.

<u>Spin 1 Bosons</u>

		Mass (GeV/c²)	
Gluon	g	0	STRONG
Photon	γ	0	EM
W and Z Bosons	$W^{\pm}, Z^0$	91.2/80.3	WEAK

The Standard Model also predicts the existence of a spin 0

HIGGS BOSON

which gives all particles their masses via its interactions.

Theoretical Framework

	Macroscopic →	Microscopic
Slow ↓	Classical Mechanics	Quantum Mechanics
Fast	Special Relativity	Quantum Field Theory

The Standard Model is a collection of related GAUGE THEORIES which are QUANTUM FIELD THEORIES that satisfy LOCAL GAUGE INVARIANCE.

ELECTROMAGNETISM: QUANTUM ELECTRODYNAMICS (QED)
1948 Feynman, Schwinger, Tomonaga (1965 Nobel Prize)

ELECTROMAGNETISM: ELECTROWEAK UNIFICATION
+WEAK
1968 Glashow, Weinberg, Salam (1979 Nobel Prize)

STRONG: QUANTUM CHROMODYNAMICS (QCD)
1974 Politzer, Wilczek, Gross (2004 Nobel Prize)

Klein-Gordon Equation

To describe the fundamental interactions of particles we need a theory of RELATIVISTIC QUANTUM MECHANICS.

Schrodinger Equation:

For a free particle $$\hat{E}\psi = \frac{\hat{p}^2}{2m}\psi$$

with energy and momentum operators: $\hat{E} = i\frac{\partial}{\partial t}$, $\hat{p} = -i\nabla$

$\hbar$ =1 natural units

giving $$i\frac{\partial\psi}{\partial t} = -\frac{1}{2m}\nabla^2\psi$$

which has plane wave solutions $\psi(\vec{r},t) = Ne^{-i(Et-\vec{p}\cdot\vec{r})}$

Schrodinger Equation:

- 1st Order in time derivative
- 2nd Order in space derivatives

} Not Lorentz Invariant!

Schrodinger equation cannot be used to describe the physics of relativistic particles.

From Special Relativity: $E^2 = p^2 + m^2$

From Quantum Mechanics: $\hat{E} = i\frac{\partial}{\partial t} \quad , \quad \hat{p} = -i\nabla$

Combine to give:

$$-\frac{\partial^2 \psi}{\partial t^2} = -\nabla^2 \psi + m^2 \psi$$

$$\boxed{\frac{\partial^2 \psi}{\partial t^2} = \left(\nabla^2 - m^2\right)\psi}$$

KLEIN-GORDON EQUATION

Second order in both space and time derivatives and hence Lorentz invariant.

Plane wave solutions $\psi(\vec{r},t) = Ne^{-i(Et-\vec{p}\cdot\vec{r})}$ give $E^2 = p^2 + m^2$

$$E = \pm\sqrt{|p|^2 + m^2}$$

Negative energy solutions required to form complete set of eigenstates

→ ANTIMATTER

Antimatter

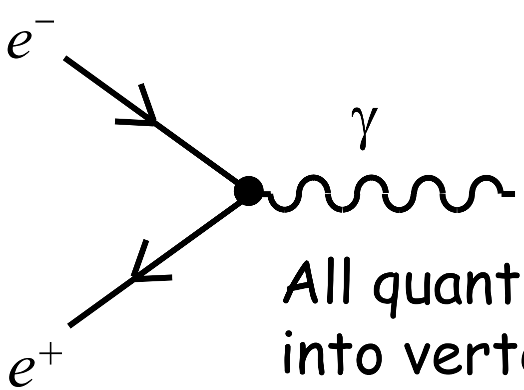
The negative energy solution is equivalent to a negative energy particle state travelling backwards in time

$$e^{-iEt} \equiv e^{-i(-E)(-t)}$$

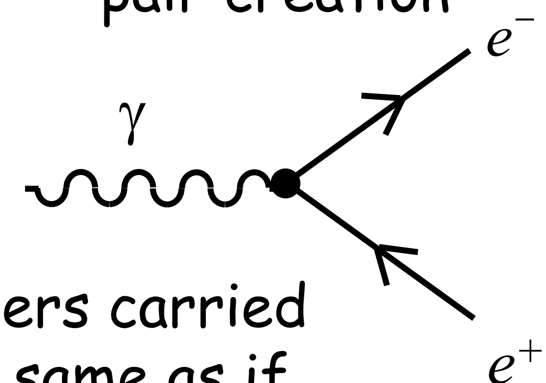
$\Rightarrow$ Interpret as a positive energy antiparticle travelling forwards in time.

Then all solutions describe physical states with positive energy, going forward in time.

<u>Examples:</u> e^+e^- annihilation



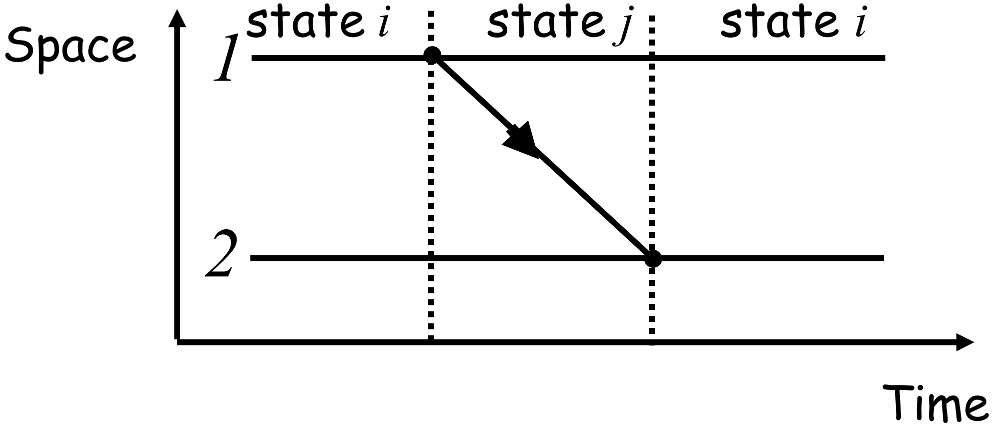
pair creation



All quantum numbers carried into vertex by e^+, same as if viewed as outgoing e^-.

Interaction via Particle Exchange

Consider two particles, fixed at $\vec{r}_1$ and $\vec{r}_2$, which exchange a particle of mass m.



$$q^\mu = (E, \vec{p})$$

$$E = E_j - E_i$$

Calculate shift in energy of state i due to this process, using 2nd order perturbation theory:

$$\Delta E_i = \sum_{j \neq i} \frac{\langle i|H|j\rangle\langle j|H|i\rangle}{E_i - E_j}$$

Sum over all possible states j with different momenta.

Consider $\langle j|H|i\rangle$ (transition from i to j by emission of m at $\vec{r}_1$)

$$\psi_i = \psi_1\psi_2 \qquad \text{Original 2 particles}$$

$$\psi_j = \psi_1\psi_2\psi_3 \qquad \psi_3 = e^{-i(Et-\vec{p}\cdot\vec{r})}$$

ψ_3 represents a free particle with $q^\mu = (E, \vec{p})$

Let g be the probability of emitting m at $\vec{r}_1$

$$\langle j|H|i\rangle = g\int d^3\vec{r}\ \ \psi_1^*\psi_2^*\psi_3^*\,\psi_1\psi_2\ \delta^3(\vec{r}-\vec{r}_1)$$
$$= ge^{i(Et-\vec{p}\cdot\vec{r}_1)}$$

Dirac δ function

$$\int d^3\vec{r}\,\delta^3(\vec{r}-\vec{r}_1)$$
$$=1 \quad \vec{r}=\vec{r}_1$$
$$=0 \quad \vec{r}\neq\vec{r}_1$$

Similarly $\langle i|H|j\rangle$ is the transition from j to i at $\vec{r}_2$

$$\langle i|H|j\rangle = ge^{-i(Et-\vec{p}\cdot\vec{r}_2)}$$

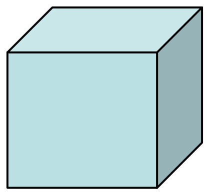
Shift in energy state is

$$\Delta E_i^{1\to 2} = \sum_{j\neq i}\frac{g^2 e^{i\vec{p}\cdot(\vec{r}_2-\vec{r}_1)}}{E_i-E_j} = \sum_{j\neq i}\frac{g^2 e^{i\vec{p}\cdot(\vec{r}_2-\vec{r}_1)}}{-E} \qquad E = E_j - E_i$$

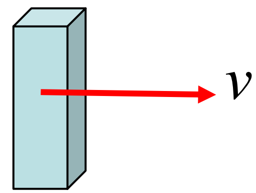
Normalization

Previously normalized wave-functions to 1 particle in a box of side L.
In relativity, the box will be Lorentz contracted by a factor γ

Rest frame
1 particle per V



Lab. frame
1 particle per V/γ



$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} = \frac{E}{m}$$

$$i.e. \quad V' = V\left(\frac{m}{E}\right)$$

i.e. E/m particles per volume V.

Need to adjust normalization volume with energy

Conventional choice:

$$N = \frac{1}{\sqrt{2E}}$$

For initial/final state particles the normalization $1/2E$ is cancelled by corresponding terms in the flux/density of states (like $1/L^3$ before).
For the intermediate particle no such cancellation occurs.

Different states j have different momenta $\vec{p}$. Therefore sum is actually an integral over all momenta:

$$\Delta E_i^{1\to 2} = \int \frac{g^2 e^{i\vec{p}\cdot(\vec{r}_2-\vec{r}_1)}}{-E}\rho(p)dp \qquad \rho(p) = \left(\frac{1}{2\pi}\right)^3 p^2\, d\Omega$$

$$= \int \frac{g^2 e^{i\vec{p}\cdot(\vec{r}_2-\vec{r}_1)}}{-E}\frac{1}{2E}\left(\frac{1}{2\pi}\right)^3 p^2 dp\, d\Omega$$

$$= -g^2\left(\frac{1}{2\pi}\right)^3 \int \frac{e^{i\vec{p}\cdot(\vec{r}_2-\vec{r}_1)}}{2E^2} p^2\, dp d\Omega \qquad E^2 = p^2 + m^2$$

The integral can be done by choosing the z-axis along $\vec{r} \equiv \vec{r}_2 - \vec{r}_1$.

Then $\vec{p}\cdot\vec{r} = prcos\vartheta$ and $d\Omega = 2\pi d(cos\vartheta)$

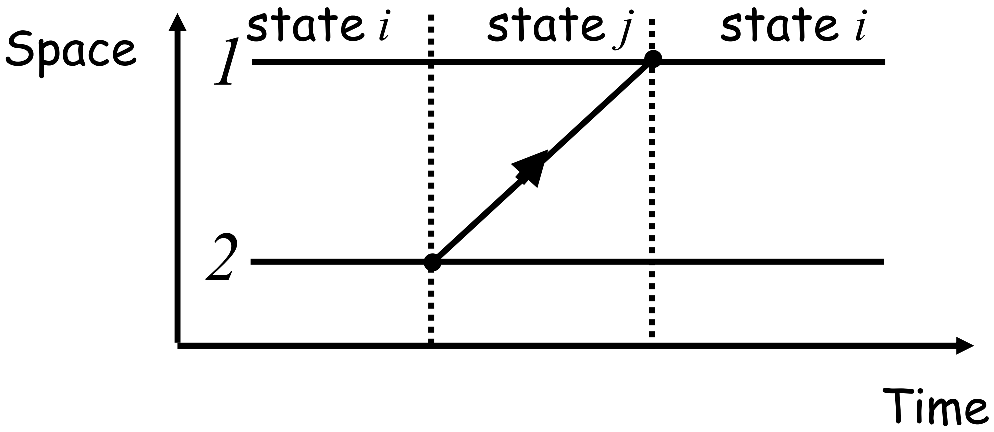
$$\Delta E_i^{1\to 2} = -\frac{g^2}{2(2\pi)^2}\int_0^\infty \frac{p^2}{p^2+m^2}\frac{e^{i\vec{p}\cdot\vec{r}} - e^{-i\vec{p}\cdot\vec{r}}}{ipr}dp$$

Appendix D

Write this integral as one half of the integral from -∞ to +∞, which can be done by residues giving

$$\Delta E_i^{1\to 2} = -\frac{g^2}{8\pi}\frac{e^{-mr}}{r}$$

Can also exchange particle from 2 to 1:



Get the same result: $$\Delta E_i^{2\to 1} = -\frac{g^2}{8\pi}\frac{e^{-mr}}{r}$$

Total shift in energy due to particle exchange is

$$\Delta E_i = -\frac{g^2}{4\pi}\frac{e^{-mr}}{r}$$

ATTRACTIVE force between two particles which decreases exponentially with range r.

Yukawa Potential

YUKAWA POTENTIAL

$$V(r) = -\frac{g^2}{4\pi}\frac{e^{-mr}}{r}$$

Hideki Yukawa
1949 Nobel Prize

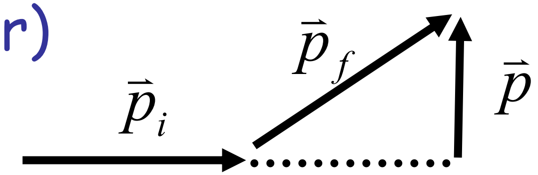
- Characteristic range = 1/m
  (Compton wavelength of exchanged particle)

- For m→0, $$V(r) = -\frac{g^2}{4\pi r}$$ infinite range

Yukawa potential with $m = 139\ MeV/c^2$ gives good description of long range interaction between two nucleons and was the basis for the prediction of the existence of the pion.

Scattering from the Yukawa Potential

Consider elastic scattering (no energy transfer)



Born Approximation $M_{fi} = \int e^{i\vec{p}\cdot\vec{r}} V(r) d^3\vec{r}$

Yukawa Potential $V(r) = -\frac{g^2}{4\pi}\frac{e^{-mr}}{r}$

$$M_{fi} = -\frac{g^2}{4\pi}\int \frac{e^{-mr}}{r} e^{i\vec{p}\cdot\vec{r}} d^3\vec{r} = -\frac{g^2}{p^2+m^2}$$

$q^\mu = \left(E, \vec{p}\right)$

$q^2 = E^2 - |\vec{p}|^2$

q^2 is invariant
"VIRTUAL MASS"

The integral can be done by choosing the z-axis along $\vec{r}$, then $\vec{p}\cdot\vec{r} = pr\cos\vartheta$ and $d^3\vec{r} = 2\pi r^2 dr\, d(\cos\vartheta)$

For elastic scattering, $q^\mu = \left(0, \vec{p}\right)$, $q^2 = -|\vec{p}|^2$ and exchanged massive particle is highly "virtual"

$$M_{fi} = \frac{g^2}{q^2 - m^2}$$

Virtual Particles

Forces arise due to the exchange of unobservable VIRTUAL particles.

- The mass of the virtual particle, q^2, is given by

$$q^2 = E^2 - |\vec{p}|^2$$

and is not the physical mass m, i.e. it is OFF MASS-SHELL.

- The mass of a virtual particle can by +ve, -ve or imaginary.

- A virtual particle which is off-mass shell by amount Δm can only exist for time and range

$$t \sim \frac{\hbar}{\Delta mc^2} = \frac{1}{\Delta m}, \quad range = \frac{\hbar}{\Delta mc} = \frac{1}{\Delta m}$$

$\hbar$ =c=1 natural units

- If $q^2 = m^2$, then the particle is real and can be observed.

For virtual particle exchange, expect a contribution to the matrix element of

$$M_{fi} = \frac{g^2}{q^2 - m^2}$$

where

g	COUPLING CONSTANT
g^2	STRENGTH OF INTERACTION
m^2	PHYSICAL (On-shell) mass
q^2	VIRTUAL (Off-shell) mass
$1/(q^2 - m^2)$	PROPAGATOR

Qualitatively: the propagator is inversely proportional to how far the particle is off-shell. The further off-shell, the smaller the probability of producing such a virtual state.

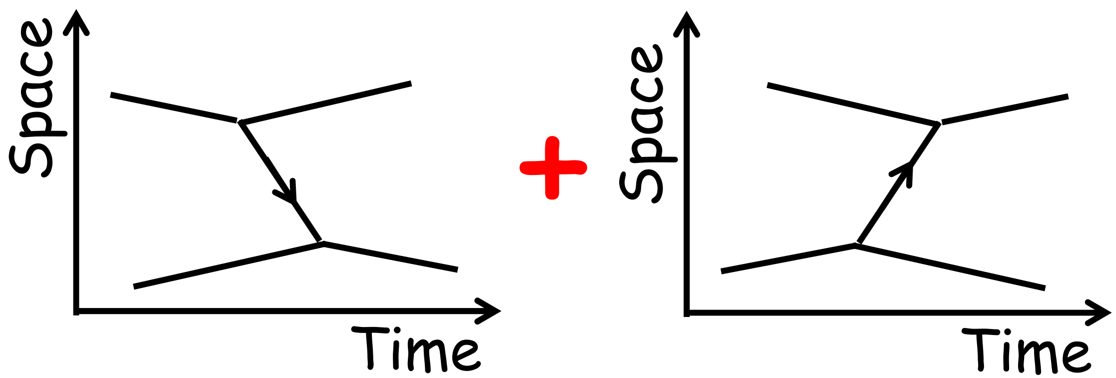
- For $m \to 0$; e.g. single γ exchange $M_{fi} = \frac{g^2}{q^2}$
- $q^2 \to 0$, very low energy transfer EM scattering

Feynman Diagrams

Results of calculations based on a single process in Time-Ordered Perturbation Theory (sometimes called old-fashioned, OFPT) depend on the reference frame.

The sum of all time orderings is not frame dependent and provides the basis for our relativistic theory of Quantum Mechanics.

The sum of all time orderings are represented by FEYNMAN DIAGRAMS.



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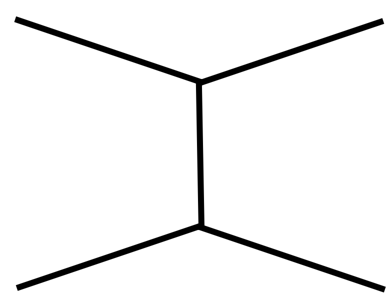
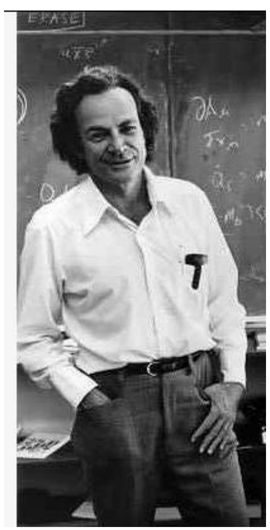


FEYNMAN DIAGRAM



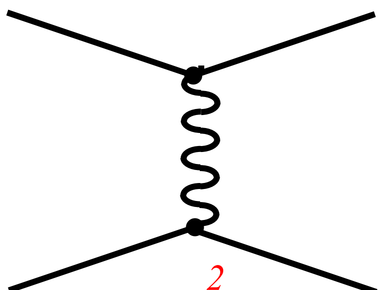
Feynman diagrams represent a term in the perturbation theory expansion of the matrix element for an interaction.

Normally, a matrix element contains an infinite number of Feynman diagrams.

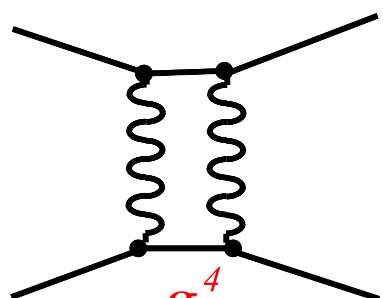
Total amplitude $$M_{fi} = M_1 + M_2 + M_3 + \cdots$$

Total rate $$\Gamma_{fi} = 2\pi \left| M_1 + M_2 + M_3 + \cdots \right|^2 \rho(E)$$ Fermi's Golden Rule

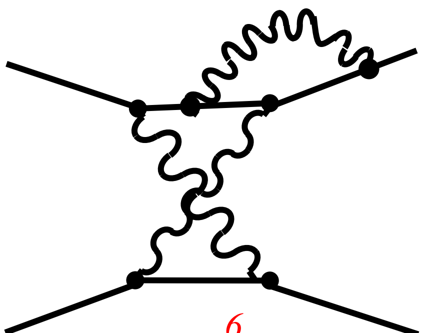
But each vertex gives a factor of g, so if g is small (i.e. the perturbation is small) only need the first few.



g^2



g^4



g^6

Example: QED $$g = e = \sqrt{4\pi\alpha} \qquad \alpha = \frac{e^2}{4\pi} \sim \frac{1}{137}$$

Anatomy of Feynman Diagrams

Feynman devised a pictorial method for evaluating matrix elements for the interactions between fundamental particles in a few simple rules. We shall use Feynman diagrams extensively throughout this course.

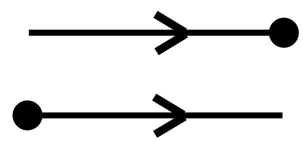
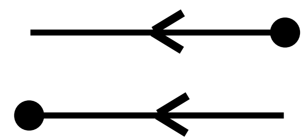
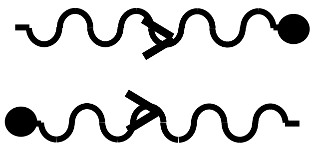
Represent particles (and antiparticles):

Spin $\frac{1}{2}$	Quarks and Leptons	
Spin 1	γ, $W^{\pm}$ and Z^0	
	g	

and their interaction point (vertex) with a "•".

Each vertex gives a factor of the coupling constant, g.

External Lines (visible particles)

Spin ½	Particle		Incoming Outgoing
	Antiparticle		Incoming Outgoing
Spin 1	Particle		Incoming Outgoing

Internal lines (propagators)

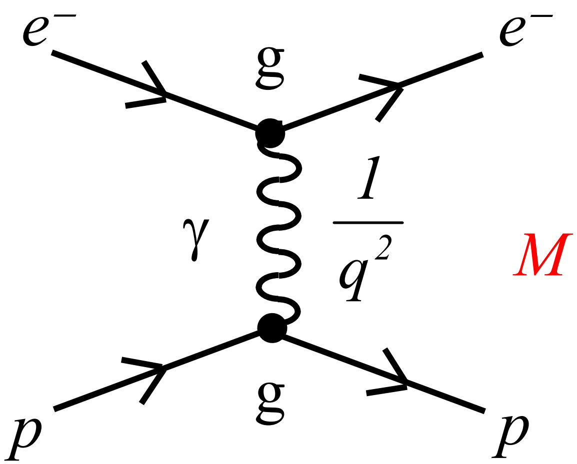
Spin ½	Particle (antiparticle)	
Spin 1	γ, $W^{\pm}$ and Z^0	
	g	

Each propagator gives a factor of

$$\frac{1}{q^2 - m^2}$$

Examples:

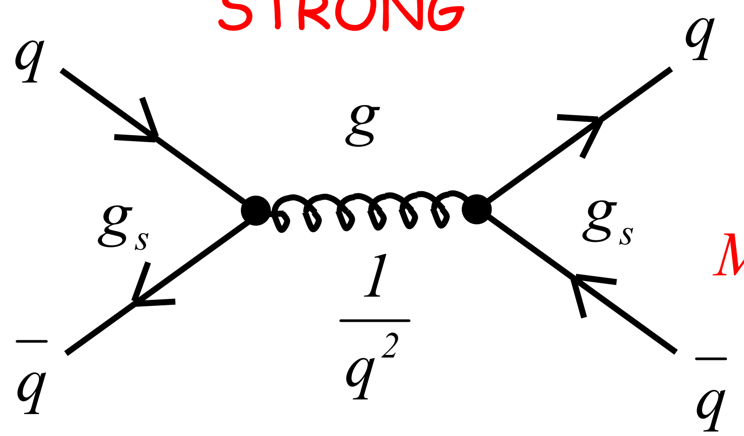
ELECTROMAGNETIC



$$M \sim \frac{g^2}{q^2}$$

Electron-proton scattering

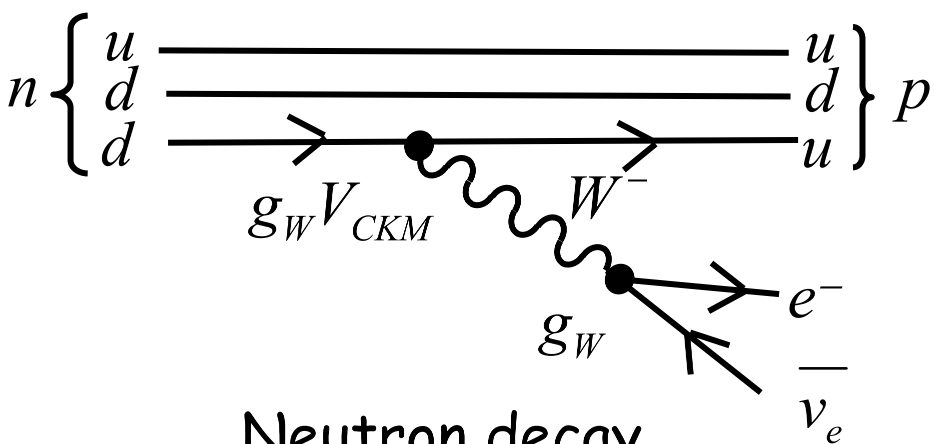
STRONG



$$M \sim \frac{g_s^2}{q^2}$$

Quark-antiquark annihilation

WEAK



$$M \sim \frac{g_W^2 V_{CKM}}{q^2 - M_W^2}$$

Neutron decay

Section V
QED

QED

QUANTUM ELECTRODYNAMICS is the gauge theory of electromagnetic interactions.

Consider a non-relativistic charged particle in an EM field:

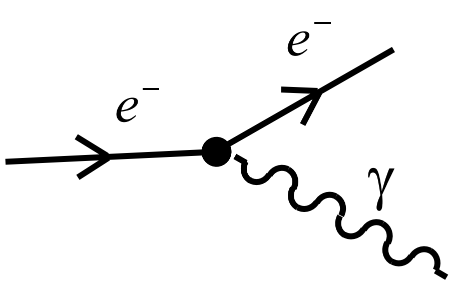
$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$\vec{E}, \vec{B}$ given in term of vector and scalar potentials $\vec{A}, \varphi$

$$\vec{B} = \vec{\nabla} \times \vec{A}\ ; \qquad \vec{E} = -\vec{\nabla}\varphi - \frac{\partial \vec{A}}{\partial t}$$

Maxwells Equations

$$\vec{H} = \frac{1}{2m}\left(\vec{p} - q\vec{A}\ \right)^2 + q\varphi$$



Change in state of e^- requires change in field

$\Rightarrow$ Interaction via virtual γ emission

Schrodinger equation $$\left[\frac{1}{2m}\left(\vec{p}-q\vec{A}\right)^2+q\varphi\right]\psi(\vec{r},t)=i\frac{\partial\psi(\vec{r},t)}{\partial t}$$

is invariant under the gauge transformation

Appendix E

$$\psi \rightarrow \psi' = e^{iq\alpha(\vec{r},t)}\psi$$

$$\vec{A} \rightarrow \vec{A}+\vec{\nabla}\alpha \;\; ; \quad \varphi \rightarrow \varphi - \frac{\partial\alpha}{\partial t}$$

$\Rightarrow$ LOCAL GAUGE INVARIANCE

LOCAL GAUGE INVARIANCE requires a physical GAUGE FIELD (photon) and completely specifies the form of the interaction between the particle and field.

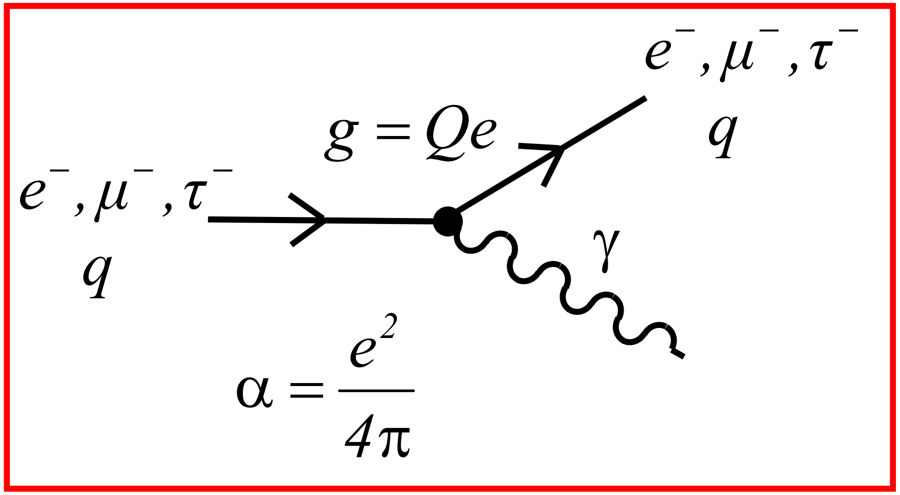
- Photons are massless - in order to cancel phase changes over all space-time, the range of the photon must be infinite.

- Charge is conserved - the charge q which interacts with the field must not change in space or time.

$\Rightarrow$ QED is a GAUGE THEORY

The Electromagnetic Vertex

All electromagnetic interactions can be described by the photon propagator and the EM vertex:



STANDARD MODEL ELECTROMAGNETIC VERTEX

+antiparticles

- The coupling constant, g, is proportional to the fermion charge.
- Energy, momentum, angular momentum and charge always conserved.
- QED vertex NEVER changes particle type or flavour

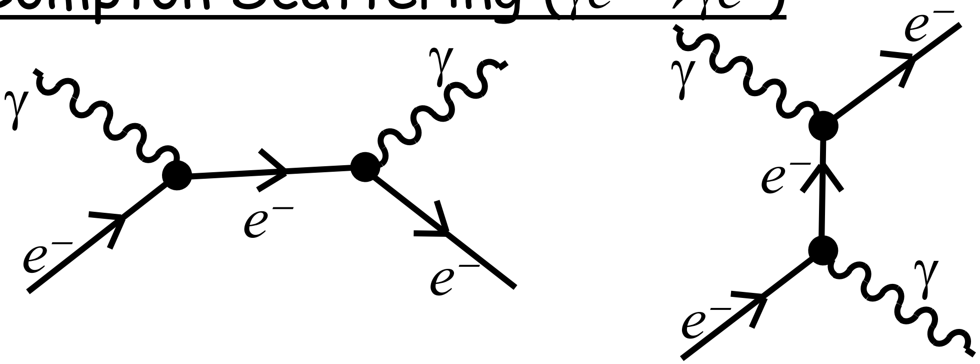
  i.e. $e^- \rightarrow e^- \gamma$ but not $e^- \rightarrow q\gamma$ or $e^- \rightarrow \mu^- \gamma$
- QED vertex always conserves PARITY

Pure QED Processes

$$M \sim \frac{g^2}{q^2}$$

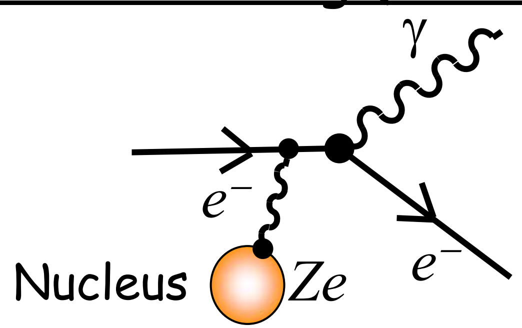
$$\alpha = \frac{e^2}{4\pi}$$

Compton Scattering ($\gamma e^- \to \gamma e^-$)



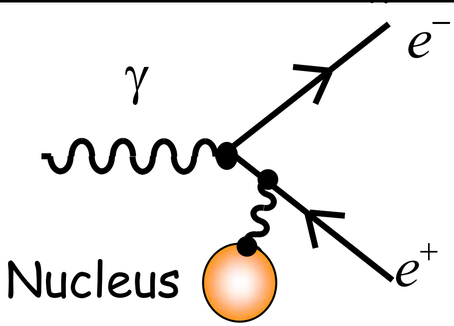
$$M \propto e^2$$
$$\sigma \propto |M|^2 \propto e^4$$
$$\sigma \propto (4\pi)^2 \alpha^2$$

Bremsstrahlung ($e^- \to e^- \gamma$)



$$M \propto Ze^3$$
$$|M|^2 \propto Z^2 e^6$$
$$\sigma \propto (4\pi)^3 Z^2 \alpha^3$$

Pair Production ($\gamma \to e^+ e^-$)

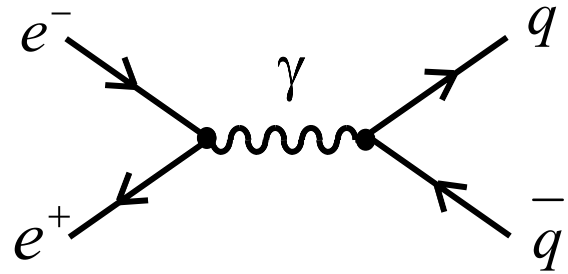


$$M \propto Ze^3$$
$$|M|^2 \propto Z^2 e^6$$
$$\sigma \propto (4\pi)^3 Z^2 \alpha^3$$

The processes $e^- \to e^- \gamma$ and $\gamma \to e^+ e^-$ cannot occur for real $e^\pm$, γ due to energy, momentum conservation.

e^+e^- Annihilation

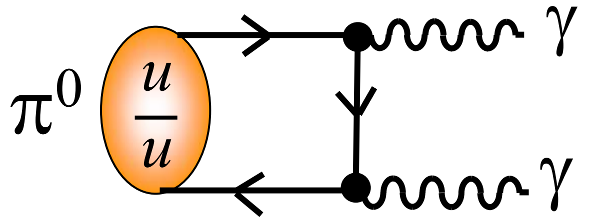
$$e^+e^- \rightarrow q\bar{q}$$



$$M \propto Q_q e^2$$
$$|M|^2 \propto Q_q^2 e^4$$
$$\sigma \propto (4\pi)^2 Q_q^2 \alpha^2$$

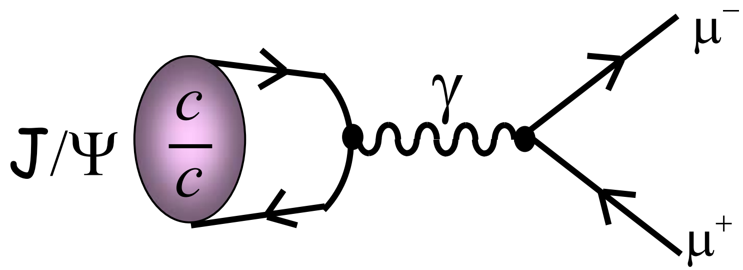
π^0 Decay

$$\pi^0 \rightarrow \gamma\gamma$$



$$M \propto Q_u^2 e^2$$
$$|M|^2 \propto Q_u^4 e^4$$
$$\sigma \propto (4\pi)^2 Q_u^4 \alpha^2$$

$J/\psi \rightarrow \mu^+\mu^-$



$$M \propto Q_c e^2$$
$$|M|^2 \propto Q_c^2 e^4$$
$$\sigma \propto (4\pi)^2 Q_c^2 \alpha^2$$

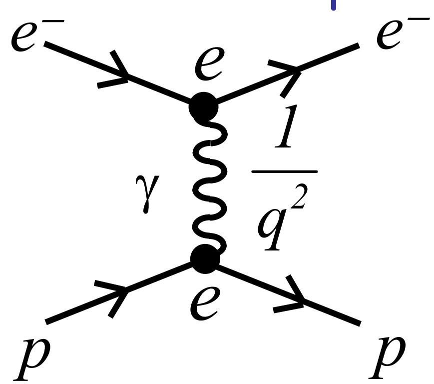
The coupling strength determines "order of magnitude" of the matrix element. For particles interacting/decaying via EM interaction: typical values for cross-sections/lifetimes

$$\sigma_{em} \sim 10^{-2} \text{ mb}$$
$$\tau_{em} \sim 10^{-20} \text{ s}$$

Scattering in QED

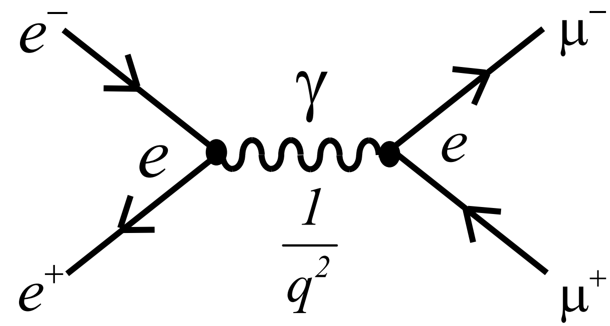
Examples: Calculate the "spin-less" cross-sections for the two processes:

(1)



Electron-proton scattering

(2)



Electron-positron annihilation

Fermi's Golden rule and Born Approximation (see page 48):

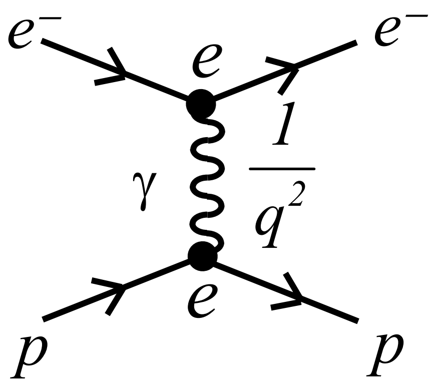
$$\frac{d\sigma}{d\Omega} = \frac{E^2}{(2\pi)^2}|M|^2$$

For both processes write the SAME matrix element

$$M = \frac{e^2}{q^2} = \frac{4\pi\alpha}{q^2}$$

- $e^2 = 4\pi\alpha$ is the strength of the interaction.
- $1/q^2$ measures the probability that the photon carries 4-momentum $q^\mu = (E, \vec{p}); \quad q^2 = E^2 - |\vec{p}|^2$ i.e. smaller probability for higher mass.

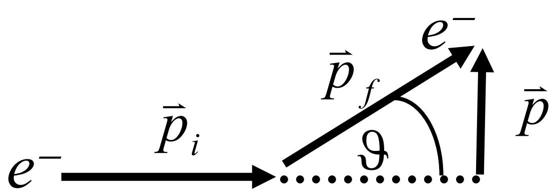
(1) "Spin-less" e-p Scattering



$$M = \frac{e^2}{q^2} = \frac{4\pi\alpha}{q^2}$$

$$\frac{d\sigma}{d\Omega} = \frac{E^2}{(2\pi)^2}|M|^2 = \frac{E^2}{(2\pi)^2}\frac{(4\pi\alpha)^2}{q^4} = \frac{4\alpha^2 E^2}{q^4}$$

q^2 is the four-momentum transfer:



$$q^2 = q^\mu q_\mu = (E_f - E_i)^2 - (\vec{p}_f - \vec{p}_i)^2$$
$$= E_f^2 + E_i^2 - 2E_f E_i - \vec{p}_f^{\,2} - \vec{p}_i^{\,2} + 2\vec{p}_f \cdot \vec{p}_i$$
$$= 2m_e^2 - 2E_f E_i + 2|\vec{p}_f||\vec{p}_i|cos\vartheta$$

Neglecting electron mass: i.e. $m_e^2 = 0$ and $|\vec{p}_f| = E_f$

$$q^2 = -2E_f E_i (1 - cos\vartheta)$$
$$= -4E_f E_i sin^2 \vartheta/2$$

Therefore for ELASTIC scattering $E_i = E_f$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 sin^4 \vartheta/2}$$

RUTHERFORD SCATTERING

Discovery of Quarks

Virtual γ carries 4-momentum $q^{\mu} = (E, \vec{p})$

Large $q \Rightarrow$ Large $\vec{p}$, small λ $\qquad \vec{p} = \hbar / \lambda$

Large E, large ω $\qquad E = \hbar\omega$

High q wave-function oscillates rapidly in space and time $\Rightarrow$ probes short distances and short time.

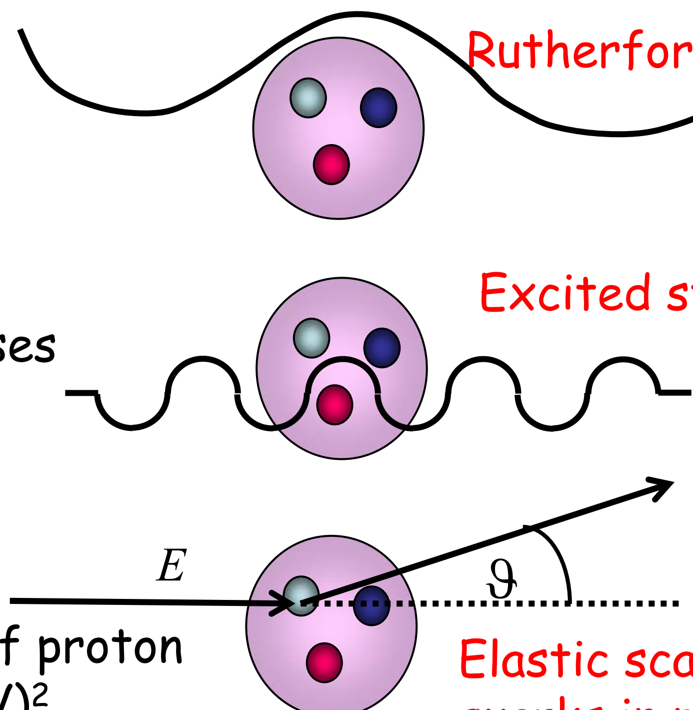
q^2 small — Rutherford Scattering

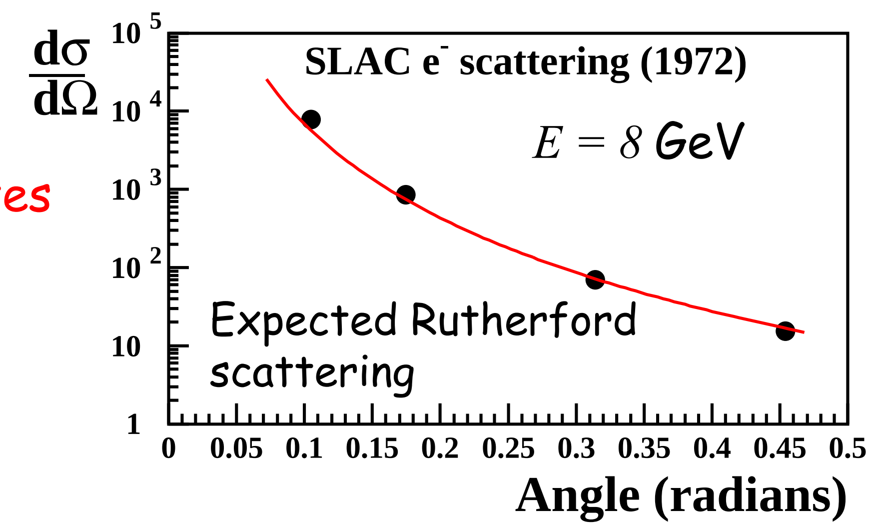
q^2 increases — Excited states

q^2 large — Elastic scattering from quarks in proton

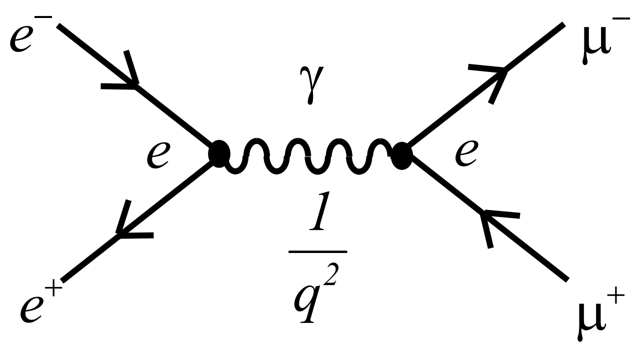
$\lambda <<$ size of proton

$q^2 > 1\ (\text{GeV})^2$





(2) "Spin-less" e^+e^- Annihilation



$$M = \frac{e^2}{q^2} = \frac{4\pi\alpha}{q^2}$$

$$\frac{d\sigma}{d\Omega} = \frac{E^2}{(2\pi)^2}|M|^2 = \frac{E^2}{(2\pi)^2}\frac{(4\pi\alpha)^2}{q^4} = \frac{4\alpha^2E^2}{q^4}$$

Same formula, but different 4-momentum transfer:

$$q^2 = q^\mu q_\mu = (E_{e+} + E_{e-})^2 - (\vec{p}_{e+} + \vec{p}_{e-})^2$$

Assuming we are in the centre-of-mass system

$$E_{e+} = E_{e-} = E$$
$$\vec{p}_{e-} = -\vec{p}_{e+}$$
$$q^2 = q^\mu q_\mu = (2E)^2 = s$$

$$\frac{d\sigma}{d\Omega} = \frac{4\alpha^2E^2}{q^4} = \frac{4\alpha^2E^2}{16E^4} = \frac{\alpha^2}{s}$$

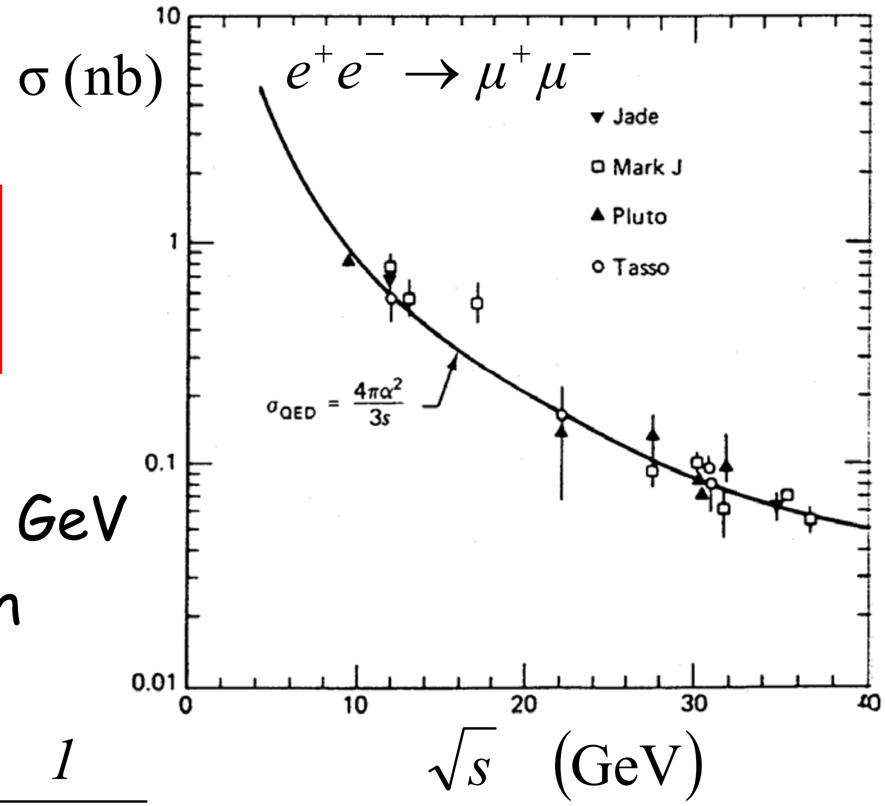
Integrating gives total cross-section:

$$\boxed{\sigma = \frac{4\pi\alpha^2}{s}}$$

This is not quite correct, because we have neglected spin. The actual cross-section (using the Dirac equation) is

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\left(1+\cos^2\vartheta\right)$$

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}$$



Example: Cross-section at $\sqrt{s} = 22$ GeV (i.e. 11 GeV electrons colliding with 11 GeV positrons)

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s} = \frac{4\pi}{(137)^2}\frac{1}{3\times 22^2}$$

$$= 4.6\times 10^{-7}\ GeV^{-2}$$

$$= 4.6\times 10^{-7} \times (0.197)^2\ fm^2$$

$$= 1.8\times 10^{-8}\ fm^2$$

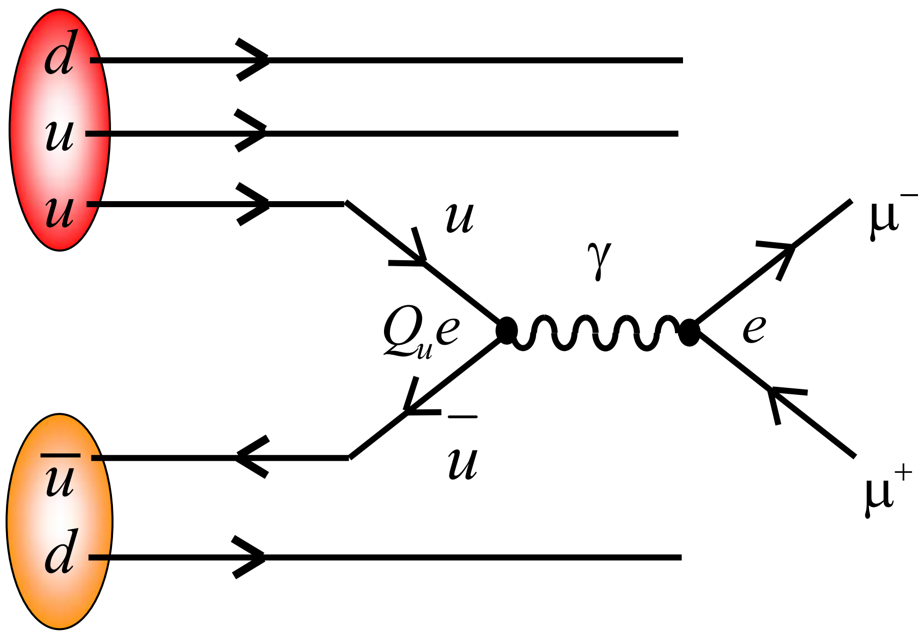
$$= 0.18\ nb$$

$\hbar c = 0.197\ GeVfm$

The Drell-Yan Process

Can also annihilate $q\bar{q}$ as in the Drell-Yan process.

<u>Example:</u> $\pi^- p \rightarrow \mu^+\mu^- + hadrons$ See example sheet 1 (Question 13)



$$\sigma\left(\pi^- p \rightarrow \mu^+\mu^- + hadrons\right) \propto Q_u^2 e^4 \propto Q_u^2 \alpha^2$$

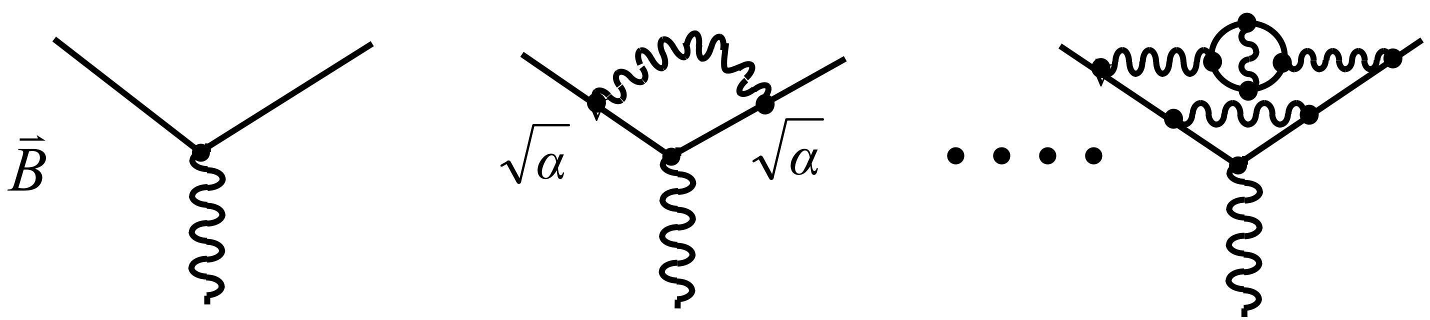
Experimental Tests of QED

QED is an extremely successful theory tested to very high precision.

Example:

- Magnetic moments of $e^{\pm}$, $\mu^{\pm}$: $\vec{\mu} = g \frac{e}{2m} \vec{s}$
- For a point-like spin ½ particle: $g = 2$ Dirac Equation

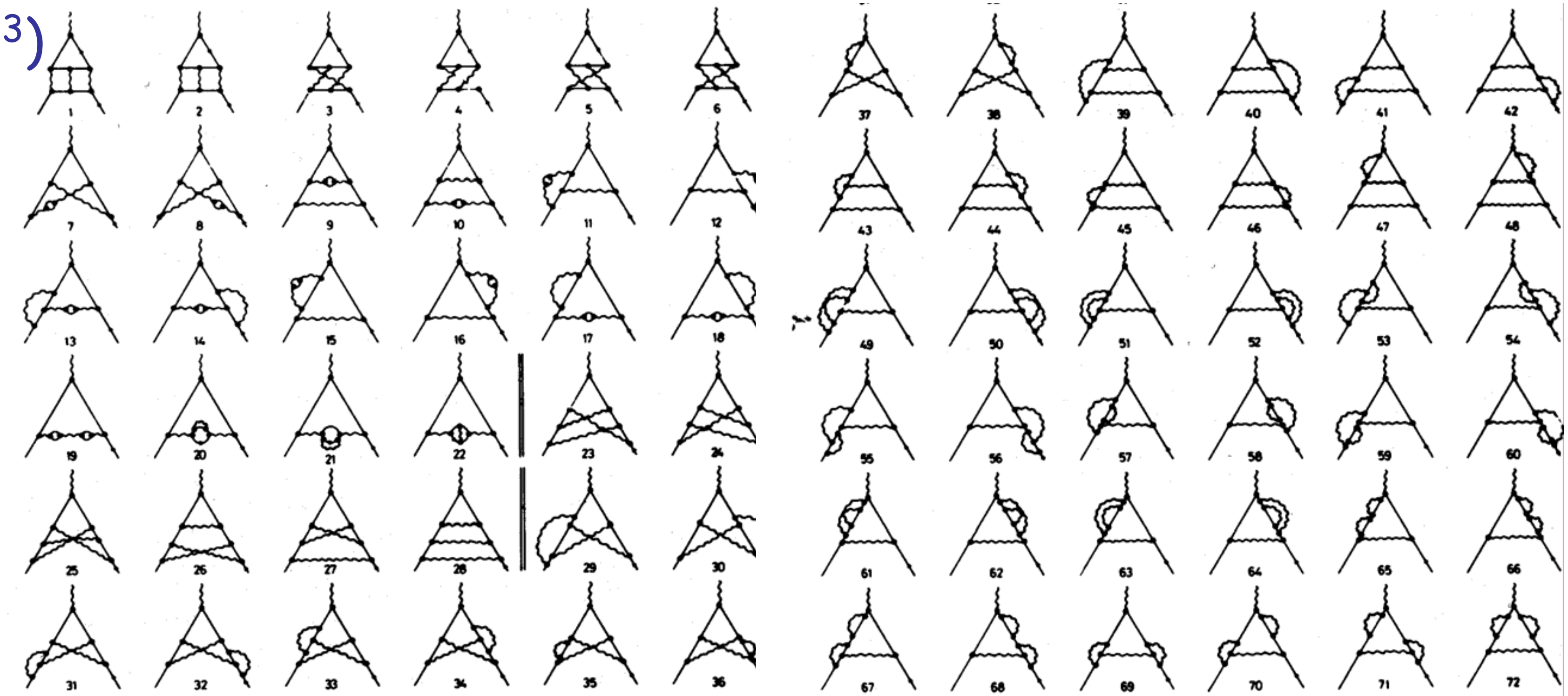
However, higher order terms introduce an anomalous magnetic moment i.e. g not quite 2.



O(1) O(α) O(α^4)

12672 diagrams

O(α^3)



$$\frac{g_e - 2}{2} = 11596521.869 \pm 0.041 \times 10^{-10}$$ Experiment

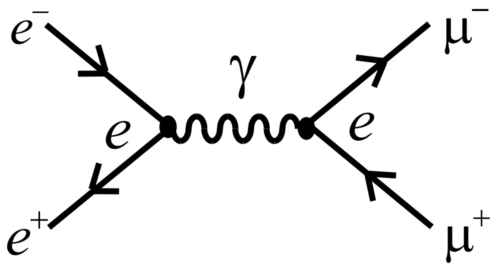
$$\frac{g_e - 2}{2} = 11596521.3 \pm 0.3 \times 10^{-10}$$ Theory

- Agreement at the level of 1 in 10^8.
- QED provides a remarkable precise description of the electromagnetic interaction !

Higher Orders

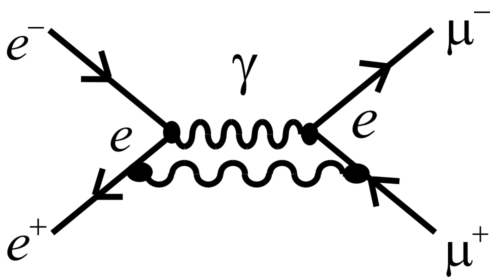
So far only considered lowest order term in the perturbation series. Higher order terms also contribute

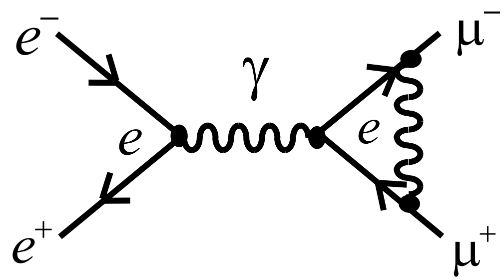
Lowest Order



$$|M|^2 \propto e^4 \propto \alpha^2 \sim \left(1/137\right)^2$$

Second Order

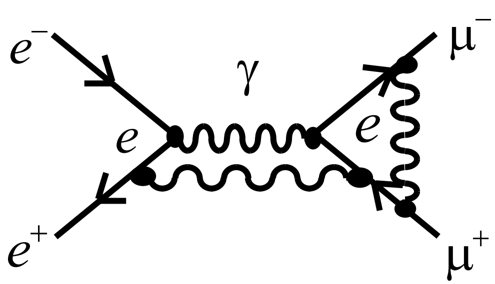


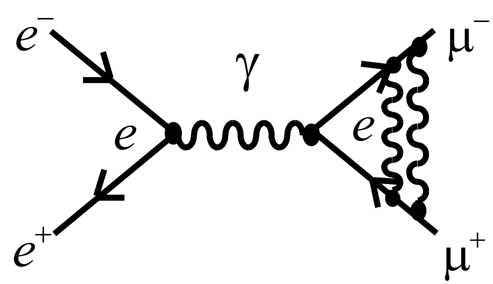


$$|M|^2 \propto \alpha^4 \sim \left(1/137\right)^4$$

+ ....

Third Order





$$|M|^2 \propto \alpha^6 \sim \left(1/137\right)^6$$

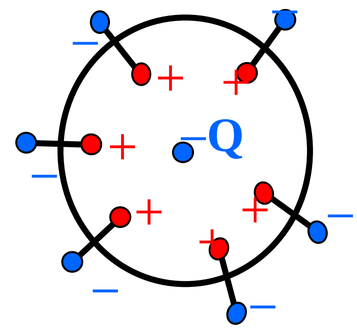
+ ....

$$\alpha = \frac{e^2}{4\pi}$$

Second order suppressed by α^2 relative to first order. Provided α is small, i.e. perturbation is small, lowest order dominates.

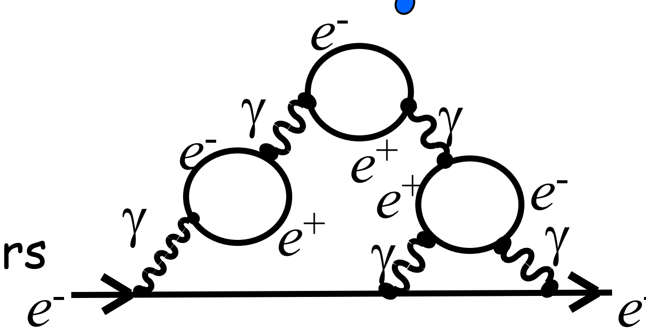
Running of α

- $\alpha = e^2/4\pi$ specifies the strength of the interaction between an electron and a photon.
- BUT α is NOT a constant.



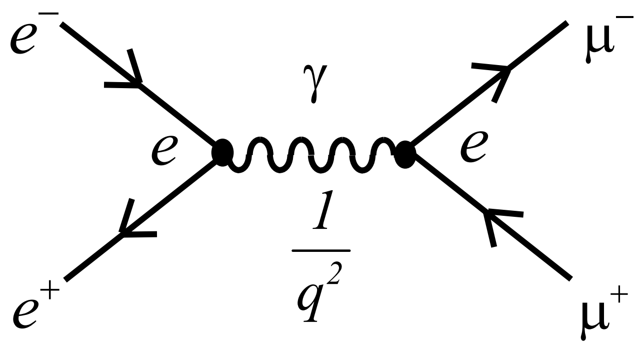
Consider an electric charge in a dielectric medium.
Charge Q appears screened by a halo of +ve charges.
Only see full value of charge Q at small distance.

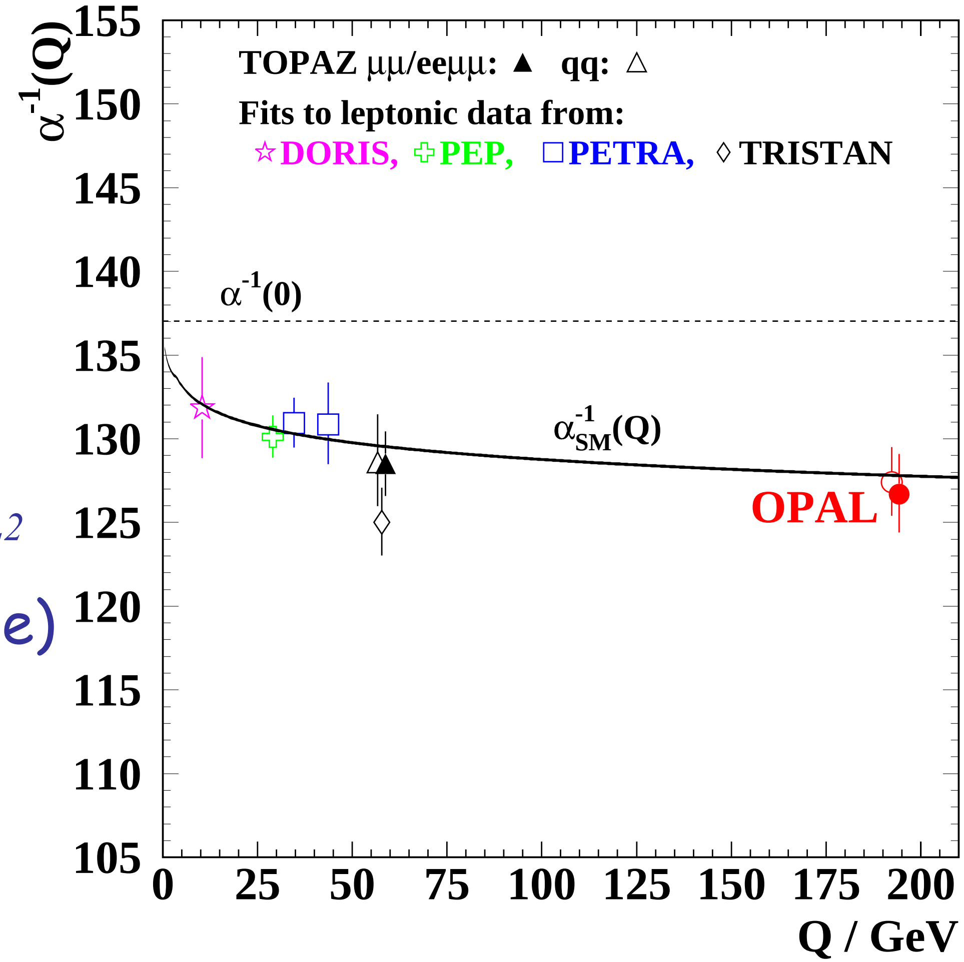
Consider a free electron.
The same effect can happen due to quantum fluctuations that lead to a cloud of virtual e^+e^- pairs



- The vacuum acts like a dielectric medium
- The virtual e^+e^- pairs are polarised
- At large distances the bare electron charge is screened.
- At shorter distances, screening effect reduced and see a larger effective charge i.e. α.

Measure $\alpha(q^2)$ from $e^+e^- \to \mu^+\mu^-$ etc





- α increases with increasing q^2 (i.e. closer to the bare charge)
- At $q^2=0$: $\alpha=1/137$
- At $q^2=(100\ GeV)^2$: $\alpha=1/128$