APPENDIX A: PHYSICAL CONSTANTS

Summary of the physical constants and conversion factors used in this course:

Electron charge, $e = 1.602 \times 10^{-19}$ C
$\hbar c = 0.197$ GeV fm
$\hbar = 6.6 \times 10^{-25}$ Gev s
Fine structure constant, $\alpha = 1/137$
Bohr magneton, $\mu_{\rm B} = 9.3 \times 10^{-24}$ JT^{-1}
Nuclear magneton, $\mu_{\rm N} = 5.1 \times 10^{-27}$ JT^{-1}

1 eV = 1.602×10^{-19} J, 1 MeV = 10^6 eV, 1 GeV = 10^9 eV
1 fermi(fm) = 10^{-15} m
1 barn(b) = 10^{-28} m^2
1 Curie(Ci) = 3.7×10^{10} decays/s

Atomic masses are often given in unified (or atomic) mass units:
1 unified mass unit(u) = Mass of an atom of ${}^{12}_{6}$C/12
1u = 1g/$N_{\rm A}$ = 1.66×10^{-27} kg = 931.5 MeV/c^2

APPENDIX B: PARTICLE PROPERTIES

From the *Review of Particle Physics*, S.Eidelman *et al.*, Physics Letters **B592** (2004)
http://pdg.lbl.gov/

Quarks (spin 1/2)			
Name	Flavour	Mass (GeV/c^2)	Charge (e)
up	u	≈ 0.35	$+2/3$
down	d	$\mathrm{m_d} \approx \mathrm{m_u}$	$-1/3$
charm	c	1.5	$+2/3$
strange	s	0.5	$-1/3$
top	t	174($\pm$5)	$+2/3$
bottom	b	4.5	$-1/3$

Leptons (spin 1/2)					
Lepton	Charge	Mass (MeV/c^2)	Mean life (s)	Lepton Decay Mode	Branching Fraction (%)
ν_e	0	$< 15\,\mathrm{eV}/c^2$	stable		
ν_μ	0	< 0.17	stable		
ν_τ	0	< 18.2	stable		
e	± 1	0.511^a	stable		
μ	± 1	105.658^b	$2.197 \times 10^{-6^c}$	$e^- \bar{\nu}_e \nu_\mu$	≈ 100
τ	± 1	1777.0($\pm$3)	$290.6(\pm 11) \times 10^{-15}$	$\mu^- \bar{\nu}_\mu \nu_\tau$	17.36($\pm$6)
				$e^- \bar{\nu}_e \nu_\tau$	17.84($\pm$6)
				hadrons $+\nu_\tau$	≈ 65

[a] The error on the e mass is $4 \times 10^{-8}\,\mathrm{MeV}/c^2$.
[b] The error on the μ mass is $9 \times 10^{-6}\,\mathrm{MeV}/c^2$.
[c] The error on the μ lifetime is $4 \times 10^{-11}\,\mathrm{s}$.

N.B. Numbers given in brackets correspond to the error in the last digit.
For example, $\mathrm{m}_\tau = 1777.0(\pm 3)\mathrm{MeV}/c^2 \equiv (1777.0 \pm 0.3)\mathrm{MeV}/c^2$.

Gauge Bosons ($J^P = 1^-$)						
Force	Gauge Boson	Charge (e)	Mass (GeV/c^2)	Full Width (GeV)	Decay Mode	Branching Fraction (%)
E-M	γ	$< 5 \times 10^{-30}$	$< 6 \times 10^{-17}\,\mathrm{eV}/c^2$	stable		
Weak (Charged)	$W^\pm$	± 1	80.43($\pm$4)	2.12($\pm$4)	$e\nu_e$	10.7($\pm$2)
					$\mu\nu_\mu$	10.6($\pm$2)
					$\tau\nu_\tau$	11.7($\pm$3)
					hadrons	68.0($\pm$4)
Weak (Neutral)	Z^0	0	91.188($\pm$2)	2.495($\pm$2)	ee	3.363($\pm$4)
					$\mu\mu$	3.366($\pm$7)
					$\tau\tau$	3.370($\pm$8)
					$\nu\nu$	20.00($\pm$6)
					hadrons	69.91($\pm$6)
Strong	g	0	0	stable		

Pseudoscalar Mesons ($J^P = 0^-$)

Particle	Quark Content	Mass (MeV/c^2)	Mean Life (s) or Width (keV)	Decay Mode	Branching Fraction (%)
$\pi^\pm$	$u\bar{d}, d\bar{u}$	139.5700(±4)	$2.6033(\pm5)\times10^{-8}$	$\mu^-\bar{\nu}_\mu$	≈ 100
π^0	$(u\bar{u}-d\bar{d})/\sqrt{2}$	134.9766(±6)	$8.4(\pm6)\times10^{-17}$	$\gamma\gamma$	98.80(±3)
η	see note a	547.8(±1)	1.29(±7)	$\gamma\gamma$	39.4(±3)
				$\pi^0\pi^0\pi^0$	32.5(±3)
				$\pi^+\pi^-\pi^0$	22.6(±4)
				$\pi^+\pi^-\gamma$	4.7(±1)
η'	see note a	957.8(±1)	0.20(±2)	$\pi^+\pi^-\eta$	44(±2)
				$\rho^0\gamma$	30(±1)
				$\pi^0\pi^0\eta$	21(±1)
$K^\pm$	$u\bar{s}, s\bar{u}$	493.677(±16)	$1.239(\pm2)\times10^{-8}$	$\mu^-\bar{\nu}_\mu$	63.4(±2)
				$\pi^-\pi^0$	21.1(±1)
				$\pi^+\pi^-\pi^-$	5.58(±3)
				$\pi^0\mu^-\bar{\nu}_\mu$	3.27(±6)
				$\pi^0e^-\bar{\nu}_e$	4.87(±6)
$K^0, \overline{K^0}$	$d\bar{s}, s\bar{d}$	497.65(±2)	K^0_S $0.8953(\pm6)\times10^{-10}$	$\pi^+\pi^-$	69.0(±1)
				$\pi^0\pi^0$	31.1(±1)
			K^0_L $5.18(\pm4)\times10^{-8}$	$\pi^0\pi^0\pi^0$	21.1(±2)
				$\pi^+\pi^-\pi^0$	12.6(±2)
				$\pi^\pm\mu^\mp\nu_\mu$	27.2(±3)
				$\pi^\pm e^\mp\nu_e$	38.8(±3)
$D^\pm$	$c\bar{d}, d\bar{c}$	1869.4(±5)	$1.040(\pm7)\times10^{-12}$	e^- + any[b]	17(±2)
				K^- + any	28(±2)
				K^+ + any	6(±2)
				K^0 + any	
				plus	
				$\overline{K^0}$ + any	61(±8)
$D^0, \overline{D^0}$	$u\bar{c}, c\bar{u}$	1864.6(±5)	$0.410(\pm2)\times10^{-12}$	K^- + any[c]	53(±4)
				K^+ + any	3.4(±5)
				e^+ + any	6.9(±3)
				μ^+ + any	6.5(±8)
				$\overline{K^0}$ + any	
				plus	
				K^0 + any	42(±5)
$D_s^\pm$	$c\bar{s}, s\bar{c}$	1968.3(±5)	$0.490(\pm9)\times10^{-12}$	seen	
$B^\pm$	$u\bar{b}, b\bar{u}$	5279.0(±5)	$1.67(\pm2)\times10^{-12}$	seen	
$B^0, \overline{B^0}$	$d\bar{b}, b\bar{d}$	5279.4(±5)	$1.54(\pm1)\times10^{-12}$	seen	
$B_s^0, \overline{B_s^0}$	$s\bar{b}, b\bar{s}$	5370(±2)	$1.46(\pm6)\times10^{-12}$	seen	
$B_c^\pm$	$c\bar{b}, b\bar{c}$	6400(±400)	$0.46(\pm2)\times10^{-12}$	seen	
η_c	$c\bar{c}$	2980(±2)	13(±4) MeV	hadrons	

[a] η and η' are linear combinations of the quark state $(u\bar{u}+d\bar{d})/\sqrt{2}$ and $s\bar{s}$.
[b] D^- decay modes; [c] D^0 decay modes.

Vector Mesons ($J^P = 1^-$)					
Particle	Quark Content	Mass (MeV/c^2)	Full Width (MeV)	Decay Mode	Branching Fraction (%)
$\rho^\pm$	$u\bar{d}, d\bar{u}$	775.8(±5)	150(±2)	$\pi\pi$	100
ρ^0	$(u\bar{u} - d\bar{d})/\sqrt{2}$				
ω	$(u\bar{u} + d\bar{d})/\sqrt{2}$	782.6(±1)	8.49(±8)	$\pi^+\pi^-\pi^0$	89.1(±7)
				$\pi^0\gamma$	8.9(±3)
				$\pi^+\pi^-$	1.7(±3)
ϕ	$s\bar{s}$	1019.46(±2)	4.26(±5)	K^+K^-	49.1(±6)
				$K^0_L K^0_S$	34.0(±5)
$K^{*\pm}$	$u\bar{s}, s\bar{u}$	891.7(±3)	50.8(±9)	$K\pi$	≈ 100
$K^{*0}, \overline{K^{*0}}$	$d\bar{s}, s\bar{d}$	896.1(±3)	50.7(±6)	$K\pi$	≈ 100
$D^{*\pm}$	$c\bar{d}, d\bar{c}$	2010.0(±5)	0.1(±2)	$D^0\pi^{-a}$	67.7(±5)
				$D^-\pi^0$	30.7(±5)
$D^{*0}, \overline{D^{*0}}$	$u\bar{c}, c\bar{u}$	2006.7(±5)	< 2.1	$D^0\pi^{0b}$	62(±3)
				$D^0\gamma$	38(±3)
$D_s^{*\pm}$	$c\bar{s}, s\bar{c}$	2112.1(±7)	< 1.9	$D_s^\pm\gamma$	94(±3)
				$D_s^\pm\pi^0$	6(±3)
B^*	$u\bar{b}, b\bar{u}, d\bar{b}, b\bar{d}, s\bar{b}, b\bar{s}$	5325.0(±6)		$B\gamma$ seen	
J/ψ	$c\bar{c}$	3096.92(±1)	91(±3) keV	hadrons	87.7(±5)
				e^+e^-	5.9(±1)
				$\mu^+\mu^-$	5.9(±1)
$\Upsilon(1s)$	$b\bar{b}$	9460.3(±3)	53(±2) keV	$\tau^+\tau^-$	2.7(±2)
				e^+e^-	2.4(±1)
				$\mu^+\mu^-$	2.48(±6)

[a] D^{*-} decay modes; [b] D^{*0} decay modes.

Baryons ($J^P = 1/2^+$)					
Particle	Quark Content	Mass (MeV/c^2)	Mean Life (s) or Full Width (MeV)	Decay Mode	Branching Fraction (%)
p	uud	938.27203(±8)	$> 2.1 \times 10^{29}$ years		
n	udd	939.56536(±8)	885.7(±8)	$pe^-\bar{\nu}_e$	100
Λ^0	uds	1115.683(±6)	$2.63(\pm 2) \times 10^{-10}$	$p\pi^-$	63.9(±5)
				$n\pi^0$	35.8(±5)
Σ^+	uus	1189.37(±7)	$0.802(\pm 3) \times 10^{-10}$	$p\pi^0$	51.6(±3)
				$n\pi^+$	48.3(±3)
Σ^0	uds	1192.64(±2)	$7.4(\pm 7) \times 10^{-20}$	$\Lambda^0\gamma$	100
Σ^-	dds	1197.45(±3)	$1.48(\pm 1) \times 10^{-10}$	$n\pi^-$	99.848(±5)
Ξ^0	uss	1314.8(±2)	$2.90(\pm 9) \times 10^{-10}$	$\Lambda^0\pi^0$	99.52(±3)
Ξ^-	dss	1321.3(±1)	$1.64(\pm 2) \times 10^{-10}$	$\Lambda^0\pi^-$	99.89(±4)
Λ_c^+	udc	2284.9(±6)	$2.00(\pm 6) \times 10^{-13}$	seen	
Λ_b	udb	5624(±9)	$1.23(\pm 8) \times 10^{-12}$	seen	
Baryons ($J^P = 3/2^+$)					
Δ	uuu, uud udd, ddd	≈ 1232	≈ 120	$N\pi$	> 99
Σ^*	uus, uds, dds	≈ 1385	≈ 36	$\Lambda^0\pi$	88(±2)
				$\Sigma\pi$	12(±2)
Ξ^*	uss, dss	≈ 1530	≈ 9	$\Xi\pi$	100
Ω^-	sss	1672.5(±3)	$0.82(\pm 1) \times 10^{-10}$	$\Lambda^0 K^-$	67.8(±7)
				$\Xi^0\pi^-$	23.6(±7)
				$\Xi^-\pi^0$	8.6(±4)

APPENDIX C: Rutherford Scattering from a Coulomb potential

Consider relativistic elastic scattering from a Coulomb potential,

$$V(\vec{r}) = -\frac{\alpha}{r}.$$

The matrix element is given by

$$|M_{if}|^2 = \left| \int \exp(\imath\vec{p}\cdot\vec{r}) V(\vec{r})\ d^3\vec{r} \right|^2 .$$

In order to perform the integral, choose the z axis to lie along $\vec{r}$. Then $\vec{p}\cdot\vec{r} = pr\cos\theta$ and

$$\begin{aligned}
\int \exp(\imath\vec{p}\cdot\vec{r}) V(\vec{r})\ d^3\vec{r} &= \int_0^\infty \int_0^{2\pi} \int_0^\pi V(r) \exp(ipr\cos\theta)\ r^2 \sin\theta\ d\theta\ d\phi\ dr \\
&= \int_0^\infty \int_{-1}^1 2\pi V(r) \exp(\imath pr\cos\theta)\ r^2\ d(\cos\theta)\ dr \\
&= \int_0^\infty 2\pi V(r)\ r^2 \left(\frac{\exp(\imath pr) - \exp(-\imath pr)}{\imath pr} \right)\ dr \\
&= \int_0^\infty 2\pi V(r)\ r^2 \frac{2\sin(pr)}{pr}\ dr.
\end{aligned}$$

The potential, $V(\vec{r}) = -\alpha/r$, gives an ill-defined integral, $\int_0^\infty \sin(pr)dr$ (keeps oscillating). Therefore, a screening term $\exp(-r/a)$ where a is the screening parameter, is introduced to perform the integral:

$$\begin{aligned}
&\int_0^\infty 2\pi r^2 \left(\frac{\exp(\imath pr) - \exp(-\imath pr)}{\imath pr} \right) \left(-\frac{\alpha}{r} \exp(r/a) \right) dr \\
&= \int_0^\infty \frac{2\pi\alpha}{ip} \left(\exp(-\imath(p+1/a)r) - \exp(\imath(p-1/a)r) \right) dr \\
&= \frac{2\pi\alpha}{ip} \left[\frac{\exp(-\imath(p+1/a)r)}{(p+1/a)} + \frac{\exp(-\imath(p-1/a)r)}{(p-1/a)} \right]_0^\infty \\
&= \frac{2\pi\alpha}{ip} \left[\frac{1}{(p+1/a)} + \frac{1}{p-1/a} \right] \\
&= \frac{4\pi\alpha}{p^2 - 1/a^2}.
\end{aligned}$$

When $a \to \infty$, there is zero screening and the matrix element becomes

$$\underline{|M_{if}|^2 = \frac{16\pi^2\alpha^2}{p^4}}$$