Particle and Nuclear Physics

Lent Term 2006

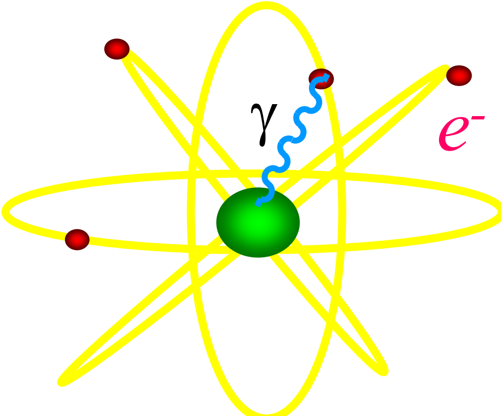
Section I
Matter and Forces

Introduction

These lectures will cover the core topics of Particle and Nuclear physics.

PARTICLE PHYSICS is the study of	NUCLEAR PHYSICS is the study of
MATTER: Elementary particles	MATTER: Complex nuclei (protons and neutrons)
FORCES: Basic forces in nature Electromagnetic Weak Strong	FORCE: Strong "Nuclear" Force (underlying strong force)
Current understanding embodied in THE STANDARD MODEL which successfully describes all current data.	Many-body problem, requires semi-empirical approach. Many models of Nuclear Physics.

Historically, Nuclear Physics studied before Particle Physics. Our discussions will develop from Particle Physics towards Nuclear Physics.



ATOM

Electrons bound to atom by electromagnetic force

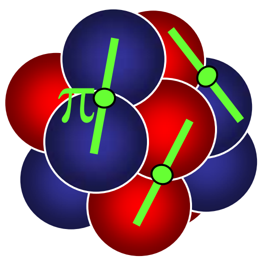
Binding energy 10 eV

Size: Atom ~10^{-10} m, e^- < 10^{-18}m

Charge: Atom is neutral, electron –e

Mass: Atom mass ~ in nucleus, m_e= 0.511 MeV/c^2

Chemical properties depend on Z.



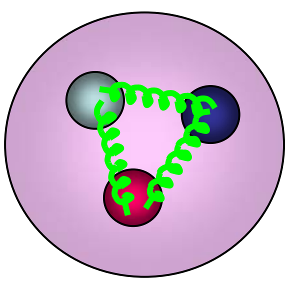
NUCLEUS

Nuclei held together by strong “nuclear” force

Binding energy 0.1 MeV

Size: Nucleus (medium A) ~ 5fm

1fm = 10^{-15}m



NUCLEON

Protons and neutrons held together by the strong force

Binding energy 10 GeV

Size: p, n ~ 1fm

Charge: p +e n 0

Mass : p, n = 939.57 MeV/c^2 ~ 1836 m_e

Matter

We now know that all matter is made of two types of elementary particles (spin ½ fermions):

LEPTONS: e.g. e^-, ν_e

QUARKS: e.g. up quark (u) and down quark (d)
proton (uud)

A consequence of relativity and quantum mechanics is that for every particle there exists an antiparticle which has identical mass, spin, energy, momentum, BUT has the opposite sign interaction.

ANTIPARTICLES: e.g. positron e^+ , antiquarks ($\bar{u}$, $\bar{d}$),
antiproton ($\bar{u}\bar{u}\bar{d}$)

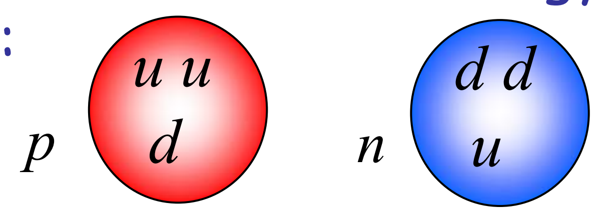
Matter: 1st Generation

Almost all phenomena you will have encountered can be described by the interactions of FOUR spin ½ particles:

THE FIRST GENERATION

Particle	Symbol	Type	Charge Units of e
Electron	e^-	Lepton	-1
Neutrino	ν_e	Lepton	0
Up Quark	u	Quark	+2/3
Down Quark	d	Quark	-1/3

The proton and neutron are the lowest energy states of the combination of 3 quarks:



Matter: 3 Generations

Nature is not quite so simple. There are THREE generations of fundamental fermions:

1st Generation		2nd Generation		3rd Generation	
Electron	e^-	Muon	μ^-	Tau	τ^-
Electron Neutrino	ν_e	Muon Neutrino	ν_μ	Tau Neutrino	ν_τ
Up quark	u	Charm quark	c	Top quark	t
Down quark	d	Strange quark	s	Bottom quark	b

- Each generation e.g. (μ^-, ν_μ, c, s) is an exact copy of (e^-, ν_e, u, d)
- The only difference is the mass of the particles: the 1st generation are the lightest and the 3rd generation are heaviest.
- Clear symmetry – origin of 3 generations is NOT UNDERSTOOD.

Leptons

Particles which DO NO INTERACT via the STRONG interaction.

- Spin ½ fermions
- 6 distinct FLAVOURS
- 3 charged leptons: e^-, μ^-, τ^-

  μ and τ unstable
- 3 neutral leptons: ν_e, ν_μ, ν_τ

  Neutrinos are stable and (almost?) massless

  ν_e mass < 3 eV/c²

  ν_μ mass < 0.17 MeV/c²

  ν_τ mass < 18.2 MeV/c²

Gen.	Flavour	Charge (e)	Approx. Mass (MeV/c²)
1st	e^-	-1	0.511
	ν_e	0	Massless ?
2nd	μ^-	-1	105.7
	ν_μ	0	Massless ?
3rd	τ^-	-1	1777.0
	ν_τ	0	Massless ?

+antimatter partners, e^+, $\overline{\nu}_e$

- Charged leptons only experience the electromagnetic and weak forces
- Neutrinos only experience the weak force

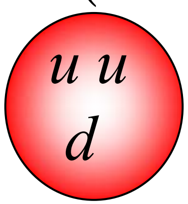
Quarks

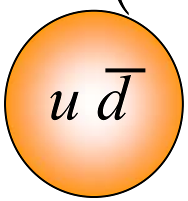
Quarks experience ALL the forces (electromagnetic, strong, weak)

- Spin ½ fermions
- Fractional charge
- 6 distinct flavours
- Quarks come in 3 colours

  Red, Green, Blue
- Quarks are confined within HADRONS

e.g. $p \equiv (uud)$ $\quad \pi^+ \equiv (u\bar{d})$





Gen.	Flavour	Charge (e)	Approx. Mass (GeV/c²)
1st	u	+2/3	0.35
	d	-1/3	0.35
2nd	c	+2/3	1.5
	s	-1/3	0.5
3rd	t	+2/3	174
	b	-1/3	4.5

+antiquarks $\bar{u}, \bar{d}, \ldots$

COLOUR is a label for the charge of the strong interaction. Unlike the electric charge of an electron (-e), the strong charge comes in 3 orthogonal colours RGB.

Hadrons

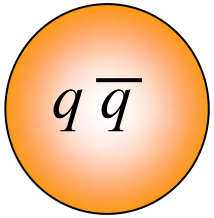
- Single free quarks are NEVER observed, but are always CONFINED in bound states, called HADRONS.
- Macroscopically hadrons behave as point-like COMPOSITE particles.

Hadrons are of two types:

MESONS $(q\bar{q})$

Bound states of a QUARK and an ANTIQUARK

All have INTEGER spin 0, 1, 2,... Bosons



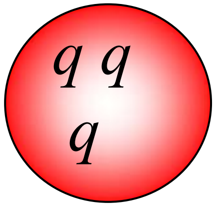
e.g. $\pi^{+} \equiv (u\bar{d})$ charge = +2/3e + 1/3e = +1e

$\pi^{-} \equiv (\bar{u}d)$ charge = -2/3e - 1/3e = -1e

BARYONS (qqq)

Bound states of 3 QUARKS

All have HALF-INTEGER spin 1/2, 3/2,... Fermions

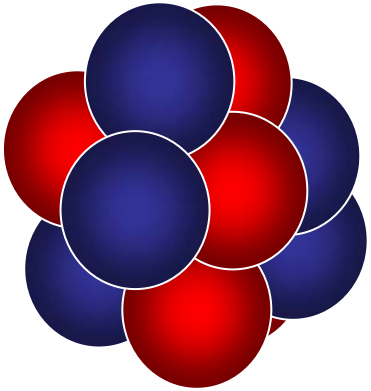


e.g. $p \equiv (uud)$ $n \equiv (udd)$

PLUS ANTIBARYONS $(\overline{q}\overline{q}\overline{q})$ e.g. $\bar{p} \equiv (\bar{u}\bar{u}\bar{d})$ $\bar{n} \equiv (\bar{u}\bar{d}\bar{d})$

Nuclei

- A NUCLEUS is a bound state of Z protons and N neutrons (alternatively bound states of 6, 9, 12 … quarks).



- p and n are 2 charge states of the NUCLEON
- A (MASS NUMBER) = Z (ATOMIC NUMBER) + N
- A NUCLIDE is a nucleus specified by Z, N

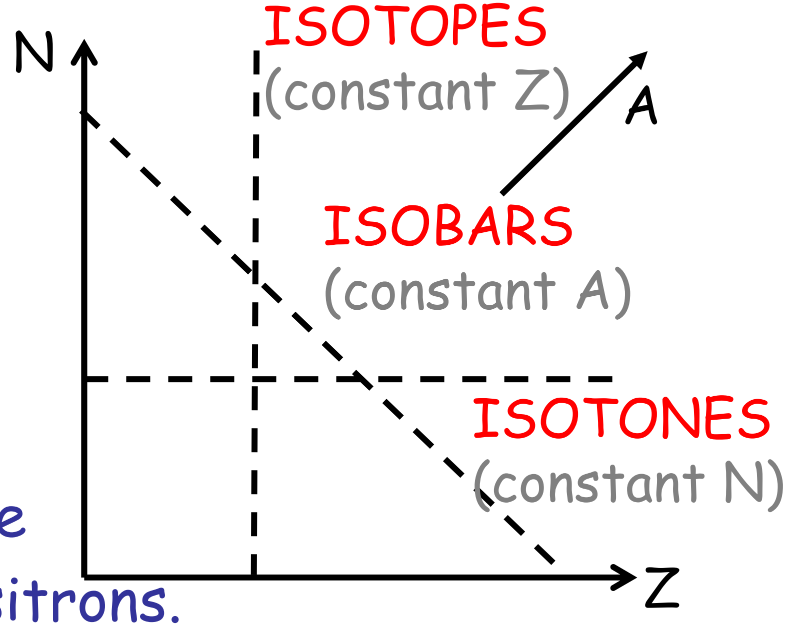
Nuclide $^{A}_{Z}X$

e.g. $^{1}_{1}\mathrm{H}$ or p $\quad Z=1, \quad N=0, \quad A=1$

$^{2}_{1}\mathrm{H}$ or d $\quad Z=1, \quad N=1, \quad A=2$

$^{4}_{2}\mathrm{H}$ or α $\quad Z=2, \quad N=2, \quad A=4$

$^{208}_{82}\mathrm{Pb} \quad Z=82, \quad N=126, \quad A=208$



In principle, ANTINUCLEI can be made from antiprotons, antineutrons and positrons. Experimentally challenging, < 100 antihydrogen atoms made.

The Periodic Table

Only THREE elements are formed in the Big Bang.

ALL other elements are formed in stars.

1 H																	2 He
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba	57 La	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	89 Ac	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Uun								

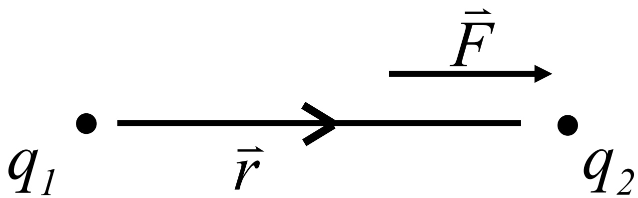
58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr

Natural elements : H (Z=1) to U (Z=92)

Forces

Classical Picture: A force is "something" which pushes matter around and causes objects to change their motion (Newtons II).

e.g. Electromagnetic forces arise via the action at a distance of the $\vec{E}$ and $\vec{B}$ fields.

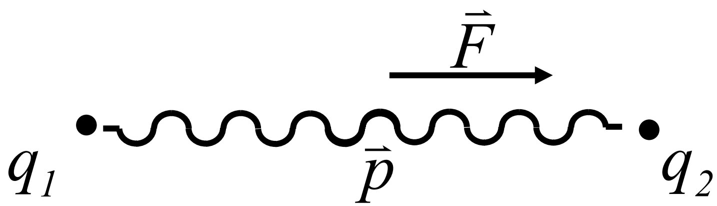


$$\vec{F} = \frac{q_1 q_2 \hat{r}}{r^2}$$

Newton: "... that a body can act upon another at a distance, through a vacuum, without the mediation of anything else,..., is to me a great absurdity"

Forces

Quantum Mechanically: Forces arise due to exchange of VIRTUAL FIELD QUANTA (Gauge Bosons), "second quantization".



Field strength at any point is uncertain

$$pr \sim \hbar \quad t = r/c$$

$\hbar$ =1 natural units

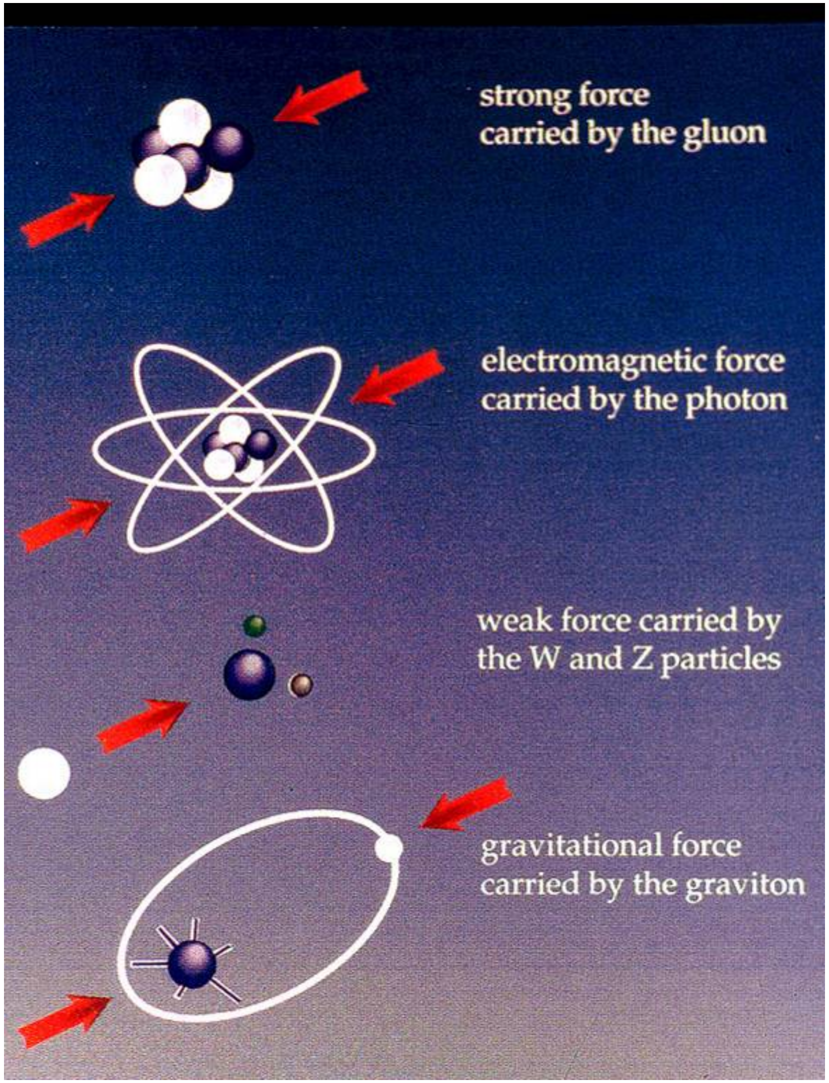
Number of quanta emitted and absorbed $\sim q_1 q_2$

$$\therefore \vec{F} = \frac{d\vec{p}}{dt} = \frac{q_1 q_2}{r^2}\hat{r}$$

Massless particle
e.g. photon

Forces

All (known) particle interactions can be explained by 4 fundamental forces:



STRONG

ELECTROMAGNETIC

WEAK

GRAVITY

Gauge Bosons

GAUGE BOSONS mediate the fundamental forces

- Spin 1 particles (i.e. Vector Bosons)
- No generations
- The manner in which the Gauge Bosons interact with the leptons and quarks determines the nature of the fundamental forces.

Force	Boson		Spin	Strength	Mass (GeV/c^2)
Strong	Gluon	g	1	1	Massless
Electromagnetic	Photon	γ	1	10^{-2}	Massless
Weak	W and Z	$W^{\pm}$, Z^0	1	10^{-7}	80, 91
Gravity	Graviton	?	2	10^{-39}	Massless

Range of Forces

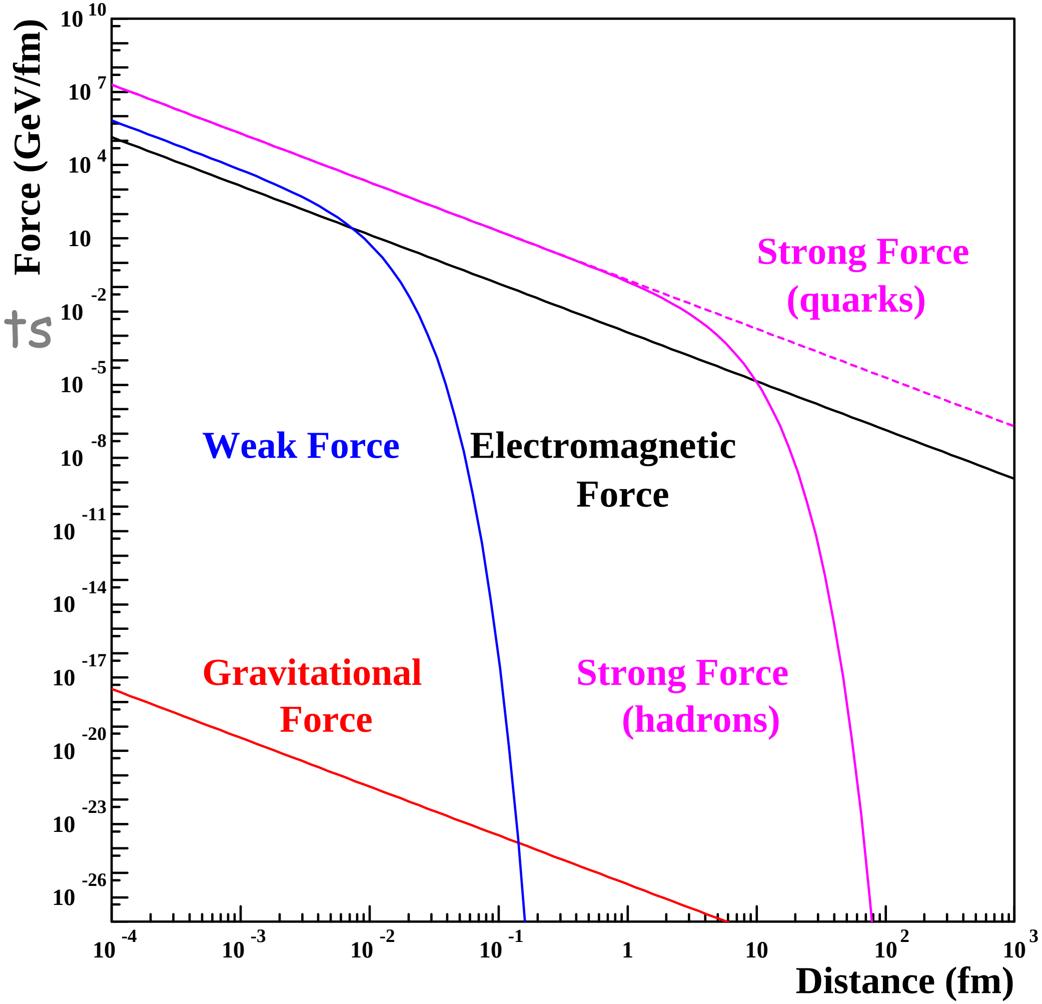
The range of a force is directly related to the mass of the exchanged bosons.

$\Delta E \Delta t \sim \hbar \qquad E = mc^2$

$mc^2 \sim \hbar / \Delta t \sim \hbar c / r$

$r \sim \hbar / mc \qquad \hbar = c = 1$ natural units

Force	Range (m)
Strong Strong (Nuclear)	∞ 10^{-15}
Electromagnetic	∞
Weak	10^{-18}
Gravity	∞



Due to quark confinement, nucleons start to experience the strong interaction at ~ 2 fm

Section II

Relativistic Kinematics

Units

Common practise in particle and nuclear physics NOT to use SI units.

- Energies are measured in units of eV:

  KeV (10^3 eV) MeV (10^6 eV) — Nuclear

  GeV (10^9 eV) TeV (10^{12} eV) — Particle

- Masses quoted in units of MeV/c² or GeV/c² ($m = E/c^2$)

e.g. Electron mass

$$m_e = 9.11\times10^{-31}\ \text{kg} = \left(9.11\times10^{-31}\right)\left(3\times10^{8}\right)^2 / 1.602\times10^{-19}$$
$$= 5.11\times10^{5}\ \text{eV}/\text{c}^2 = 0.511\ \text{MeV}/\text{c}^2$$

- Atomic masses are often given in unified (or atomic) mass units

$$1\ \text{unified mass unit (u)} \equiv \frac{\text{Mass of an atom of } {}^{12}_{6}\text{C}}{12}$$

$1\ \text{u} = 1\text{g}/N_A = 1.66\times10^{-27}\ \text{kg} = 931.5\ \text{MeV}/c^2$

- Cross-sections are usually quoted in barns: $1\ \text{b} = 10^{-28}\ \text{m}^2$

Natural Units

- Choose energy as basic unit of measurement

Energy	GeV	Time	$(\text{GeV}/\hbar)^{-1}$
Momentum	GeV/c	Length	$(\text{GeV}/\hbar c)^{-1}$
Mass	GeV/c^2	Cross-section	$(\text{GeV}/\hbar c)^{-2}$

- Simplify by choosing $\hbar=c=1$

Energy	GeV	Time	GeV^{-1}
Momentum	GeV	Length	GeV^{-1}
Mass	GeV	Cross-section	GeV^{-2}

- Convert back to SI units by reintroducing "missing" factors of ħ and c

$\hbar c = 0.197$ GeV fm $\qquad \hbar = 6.6 \times 10^{-25}$ GeV s

Example:

$$\text{Cross-section (n.u.)} = 1\ \text{GeV}^{-2}$$

$$[L]^2 = [E]^{-2}[\hbar]^n[c]^m$$

$$[L]^2 = [E]^{-2}[E]^n[T]^n[L]^m[T]^{-m}$$

$$\therefore\ n = 2 \quad \text{and} \quad m = 2$$

$$\text{Cross-section(S.I.)} = 1\text{GeV}^{-2} \times \hbar^2 c^2$$

$$= 1\text{GeV}^{-2} \times (0.197\ \text{GeV fm})^2$$

$$= 3.9 \times 10^{-2}\ \text{fm}^2 = 0.39\ \text{mb}$$

- Charge: Use "Heaviside-Lorentz" units: $\varepsilon_0 = \mu_0 = \hbar = c = 1$

Fine structure constant $\alpha = \dfrac{e^2}{4\pi\varepsilon_0\hbar c} \approx \dfrac{1}{137}$

becomes

$$\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}$$

Relativistic Kinematics

- In Special Relativity, the total energy and momentum of a particle of mass m are

$$E = \gamma m \qquad p = \gamma m \beta$$

$$\gamma = E/m \qquad \beta = p/E$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}; \beta = \mathrm{v}; c = 1$$

and are related by $E^2 = p^2 + m^2$

Note: At rest, $E = m$ and for $E >> m, \; E \sim p$

- The K.E. is the extra energy due to motion $T = E - m = (\gamma - 1)m$

In the non-relativistic limit $\beta << 1 \qquad T = \frac{1}{2} m\mathrm{v}^2$

- Low energy nuclear reactions take place with T of $\mathcal{O}$(10 MeV) « nuclear rest energies $\Rightarrow$ non-relativistic formulas.

- Particle physics T is of $\mathcal{O}$(100 GeV) » rest energies $\Rightarrow$ relativistic formulas.

- Always treat β decay relativistically.

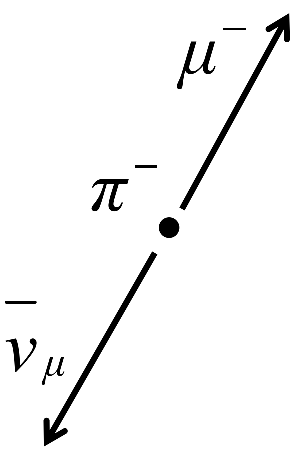
In Special Relativity $(t,\vec{x})$ and $(E,\vec{p})$ transform between frames of reference, BUT

$$d^2 = t^2 - \vec{x}^2 \qquad \text{Invariant interval}$$

$$m^2 = E^2 - \vec{p}^2 \qquad \text{INVARIANT MASS}$$

are CONSTANT.

<u>Example</u>: $\pi^- \rightarrow \mu^- \bar{\nu}_\mu$ decay at rest. (assume $m_\nu=0$)



Conservation of Energy: $E_\pi = E_\mu + E_\nu$

Conservation of Momentum: $0 = \vec{p}_\mu + \vec{p}_\nu$

$$E_\pi = m_\pi, \quad E_\mu^2 = p_\mu^2 + m_\mu^2, \quad E_\nu = |p_\nu|$$

$$E_\pi = E_\mu + E_\nu \Rightarrow m_\pi = E_\mu + p_\mu \Rightarrow (m_\pi - E_\mu)^2 = p_\mu^2$$

$$but \quad E_\mu^2 - m_\mu^2 = p_\mu^2$$

$$\therefore E_\mu = \frac{m_\pi^2 + m_\mu^2}{2m_\pi} = \frac{(140\ \text{MeV})^2 + (106\ \text{MeV})^2}{2 \times 140\ \text{MeV}} = \underline{110\ \text{MeV}}$$

$$|p_\mu| = |p_\nu| = \underline{30\ \text{MeV}}$$

Four-Vectors

- Define FOUR-VECTORS:

$$x^\mu = (t, \vec{x}) \quad p^\mu = (E, \vec{p}) \qquad \mu : 0 \to 3$$
$$x_\mu = (t, -\vec{x}) \quad p_\mu = (E, -\vec{p})$$

where

$$p_\mu = g_{\mu\nu} p^\nu; \quad p^\mu = g^{\mu\nu} p_\nu \qquad g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Scalar product of two four-vectors

$$A^\mu = (A^0, \vec{A}) \quad B^\mu = (B^0, \vec{B})$$

Invariant:

$$A^\mu B_\mu \equiv A \cdot B \equiv A^0 B^0 - \vec{A} \cdot \vec{B}$$

or

$$p^\mu p_\mu \equiv p^\mu g_{\mu\nu} p^\nu = \sum_{\mu=0,3} \sum_{\nu=0,3} p^\mu g_{\mu\nu} p^\nu$$
$$= g_{00} p_0^2 + g_{11} p_1^2 + g_{22} p_2^2 + g_{33} p_3^2$$
$$= E^2 - |\vec{p}|^2 = m^2$$

Colliders and √s

Consider the collision of two particles:

$$p_1^{\mu}\left(E_1, \vec{p}_1\right) \qquad\qquad p_2^{\mu}\left(E_2, \vec{p}_2\right)$$

The invariant quantity $s \equiv \left(p_1^{\mu} + p_2^{\mu}\right)\left(p_{1\mu} + p_{2\mu}\right)$

$$s = \left(p_1^{\mu} p_{1\mu} + p_2^{\mu} p_{2\mu} + 2 p_1^{\mu} p_{2\mu}\right)$$

$$= E_1^2 - \vec{p}_1^{\,2} + E_2^2 - \vec{p}_2^{\,2} + 2\left(E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2\right)$$

$$= m_1^2 + m_2^2 + 2\left(E_1 E_2 - \left|\vec{p}_1\right|\left|\vec{p}_2\right| \cos\vartheta\right)$$

$\sqrt{s}$ is the energy in the zero momentum (centre-of-mass) frame

It is the amount of energy available to interaction e.g. the maximum energy/mass of a particle produced in matter-antimatter annihilation.

Fixed Target Collision

$p_1^\mu(E_1, \vec{p}_1)$ $\quad$ $p_2^\mu(m_2, 0)$

$$s = m_1^2 + m_2^2 + 2E_1 m_2$$

For $E_1 >> m_1, m_2$ $\quad s = 2E_1 m_2 \Rightarrow \sqrt{s} = \sqrt{2E_1 m_2}$

e.g. 100 GeV proton hitting a proton at rest:

$$\sqrt{s} = \sqrt{2E_p m_p} \approx \sqrt{2 \times 100 \times 1} \approx 14 \text{ GeV}$$

Collider Experiment

$p_1^\mu(E, \vec{p})$ $\quad$ $p_2^\mu(E, -\vec{p})$

$$s = m_1^2 + m_2^2 + 2\left(E_1 E_2 - |\vec{p}_1||\vec{p}_2| \cos\vartheta\right)$$

For $E_1 >> m_1, m_2$ then $|\vec{p}| = E$ and

$$s = 2\left(E^2 - E^2 \cos\vartheta\right) = 4E^2 \Rightarrow \sqrt{s} = 2E$$

e.g. 100 GeV proton colliding with a 100 GeV proton:

$$\sqrt{\text{s}} = 2 \times 100 = 200 \text{ GeV}$$

In a fixed target experiment most of the proton's energy is wasted- providing momentum to the C.O.M system rather than being available for the interaction.

Section III

Decays and Reactions

How do we study particles and forces ?

- Static Properties

  Mass, spin and parity (J^P), magnetic moments, bound states

- Particle Decays

  Allowed/forbidden decays $\rightarrow$ Conservation Laws

- Particle Scattering

  Direct production of new massive particles in matter/antimatter ANNIHILATION

  Study of particle interaction cross-sections.

Force	Typical Lifetime (s)	Typical Cross-section (mb)
Strong	10^{-23}	10
Electromagnetic	10^{-20}	10^{-2}
Weak	10^{-8}	10^{-13}

Particle Decays

Most particles are transient states – only the privileged few live forever (e^-, u, d, γ, ...)

A decay is the transition from one quantum state (initial state) to another (final or daughter).

The transition rate is given by FERMI'S GOLDEN RULE:

$$\lambda = 2\pi \left| M_{fi} \right|^2 \rho(E_f) \qquad \hbar = 1$$

where λ is the number of transitions per unit time

M_{fi} is the matrix element

$\rho\,(E_f)$ is the density of final states.

$\Rightarrow \lambda\, dt$ is the probability a particle will decay in time dt.

1 particle decay

Let $p(t)$ be the probability that a particle will survive until at least time t, if it is known to exist at $t=0$.

Prob. particle decays in the next time $dt = p(t)\lambda\, dt$

Prob. particle survives in the next time $dt = p(t)(1-\lambda\, dt) = p(t+dt)$

$$p(t)(1-\lambda dt) = p(t+dt) = p(t) + \frac{dp}{dt}dt$$

$$\frac{dp}{dt} = -p(t)\lambda$$

$$\int_1^p \frac{dp}{p} = -\int_0^t \lambda dt$$

$$\Rightarrow \boxed{p(t) = e^{-\lambda t}}$$

EXPONENTIAL DECAY LAW

Probability that a particle lives until time t and then decays in time dt is $p(t)\lambda dt = \lambda e^{-\lambda t} dt$

- The AVERAGE LIFETIME of the particle

$$\tau = \langle t \rangle = \int_0^\infty t\lambda e^{-\lambda t} dt = \left[-te^{-\lambda t}\right]_0^\infty + \int_0^\infty e^{-\lambda t} dt = \left[-\frac{1}{\lambda} e^{-\lambda t}\right]_0^\infty = \frac{1}{\lambda}$$

$$\tau = 1/\lambda \qquad p(t) = e^{-t/\tau}$$

- Finite lifetime $\Rightarrow$ UNCERTAIN energy ΔE $\quad (\Delta E \Delta t \sim \hbar)$
- Decaying states do not correspond to a single energy – they have a width ΔE

$$\Delta E \tau \sim \hbar \quad \Rightarrow \quad \Delta E \sim \hbar/\tau = \hbar\lambda$$

$\hbar$ =1 natural units

- The width, ΔE, of a particle state is
  - Inversely proportional to the lifetime τ
  - Equal to the transition rate λ using natural units.

<u>Many particles</u> (e.g. material containing N nuclei).

- NUMBER of particles at time t,

$$N(t) = N(0)p(t) = N(0)e^{-\lambda t}$$

where $N(0)$ is the number at time $t=0$.

- RATE OF DECAYS $\dfrac{dN}{dt} = -\lambda N(0)e^{-\lambda t} = -\lambda N(t)$

- ACTIVITY $\mathrm{A(t)} = \left|\dfrac{dN}{dt}\right| = \lambda N(0)e^{-\lambda t} = \lambda N(t)$

- Common in nuclear physics to use the HALF-LIFE (i.e. the time over which 50% of the particles decay)

$$N(\tau_{1/2}) = \frac{N(0)}{2} = N(0)e^{-\lambda \tau_{1/2}}$$

$$\boxed{\tau_{1/2} = \frac{Ln2}{\lambda} = 0.693\,\tau}$$

Decay Chain

$$N_1 \xrightarrow{\lambda_1} N_2 \xrightarrow{\lambda_2} N_3 \rightarrow \ldots\ldots$$

Parent Daughter Grandaughter...

e.g. $^{238}\text{U} \rightarrow {}^{231}\text{Th} \rightarrow {}^{231}\text{Pa}$ $\quad \tau_{1/2}(^{238}\text{U}) = 4.5 \times 10^9$ years

$\tau_{1/2}(^{231}\text{Th}) = 26$ hours

Activity of the daughter is $\lambda_2 N_2(t)$

Rate of change of population of the daughter

$$\frac{dN_2(t)}{dt} = \lambda_1 N_1(t) - \lambda_2 N_2(t)$$

Units of radioactivity: are defined as the number of decays per unit time.

Becquerel (Bq) = 1 decay per second

Curie (Ci) 1 Ci = 3.7×10^{10} decays per second.

Decaying States → Resonances

QM description of decaying states

Consider a state with energy E_0 and lifetime τ

$$\psi(t) = \psi(0)e^{-iE_0 t}e^{-t/2\tau} \qquad \mathrm{E} = \hbar\omega \quad \hbar = 1$$

$$|\psi(t)|^2 = |\psi(0)|^2 e^{-t/\tau}$$

i.e. the probability density decays exponentially (as required).

The frequencies present in the wavefunction are given by the Fourier transform of $\psi(t)$

$$f(\omega) = f(E) = \int_0^\infty \psi(t)e^{iEt}dt = \int_0^\infty \psi(0)e^{-t\left(iE_0 + \frac{1}{2\tau}\right)}e^{iEt}dt = \int_0^\infty \psi(0)e^{-t\left(i(E_0 - E) + \frac{1}{2\tau}\right)}dt$$

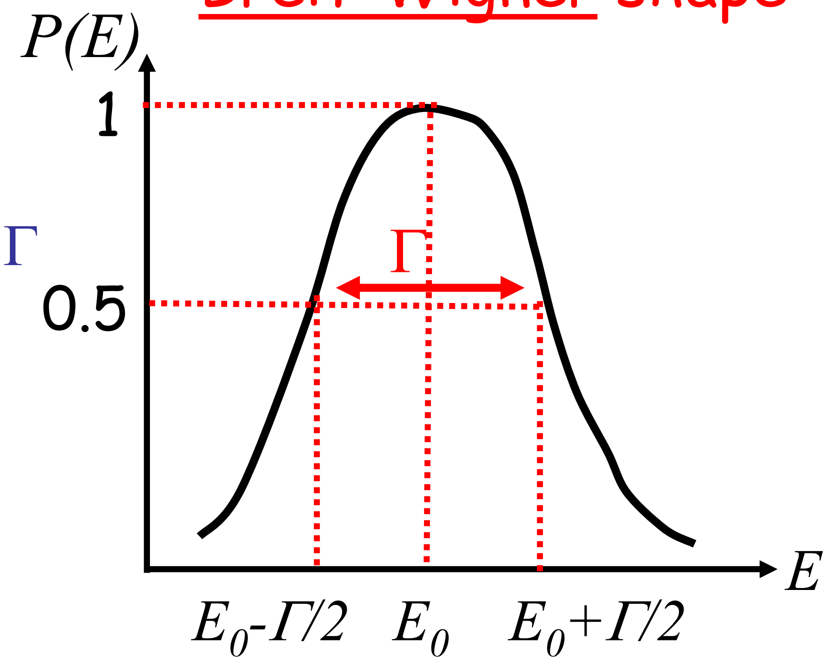
$$= \frac{i\psi(0)}{(E_0 - E) - i/2\tau}$$

Probability of finding state with energy $E = f(E)^*f(E)$

$$P(E) = \frac{|\psi(0)|^2}{(E_0 - E)^2 + 1/4\tau^2}$$

Probability for producing the decaying state has this energy dependence, i.e. RESONANT when $E=E_0$

$$P(E) \propto \frac{1}{(E_0 - E)^2 + \frac{1}{4\tau^2}}$$



Consider full-width at half-maximum Γ

$$P(E = E_0) \propto 4\tau^2$$

$$P(E = E_0 \pm \Gamma/2) \propto \frac{1}{\left(E_0 - E_0 \pm \Gamma/2\right)^2 + \frac{1}{4\tau^2}}$$

$$\frac{1}{\left(E_0 - E_0 \pm \Gamma/2\right)^2 + \frac{1}{4\tau^2}} = \frac{1}{2} 4\tau^2 \Rightarrow \frac{1}{\frac{\Gamma^2}{4} + \frac{1}{4\tau^2}} = 2\tau^2$$

TOTAL WIDTH $\boxed{\Gamma = \frac{1}{\tau} = \lambda}$ (using natural units)

Partial Decay Widths

Particles can often decay with more than one decay mode, each with its own transition rate

$$\lambda_i = 2\pi \left| M_{fi} \right|^2 \rho\left(E_f\right)$$

- The TOTAL DECAY RATE is given by $\lambda = \sum_i \lambda_i$
- This determines the AVERAGE LIFETIME $\tau = \frac{1}{\lambda}$
- The TOTAL WIDTH of a particle state is $\Gamma = \hbar\lambda = \hbar \sum_i \lambda_i$
- DEFINE the PARTIAL WIDTHS $\Gamma_i = \hbar\lambda_i \Rightarrow \Gamma = \sum_i \Gamma_i$
- The proportion of decays to a particular decay mode is called the BRANCHING FRACTION

$$\mathrm{B_i} = \frac{\Gamma_\mathrm{i}}{\Gamma} \qquad \sum_i \mathrm{B_i} = 1$$

Reactions and Cross-sections

The strength of a particular reaction between two particles is specified by the interaction CROSS-SECTION.

A cross-section is an effective target area presented to the incoming particle for it to cause the reaction.

UNITS: σ 1 barn (b) = 10^{-28} m^2 Area

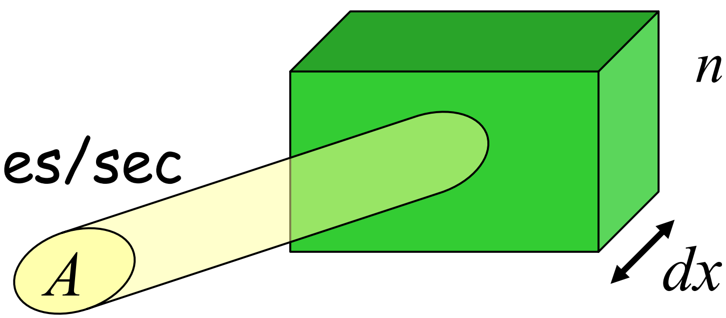
The cross-section, σ, is defined as the reaction rate per target particle, Γ, per unit incident flux, Φ

$$\Gamma = \Phi\sigma$$

where the flux, Φ, is the number of beam particles passing through unit area per second.

Γ is given by Fermi's Golden Rule (NB previously used λ)

Consider a beam of particles incident upon a target:



Number of target particles in area A $= n\, A\, dx$

Effective area for absorption $= \sigma\, n\, A\, dx$

Rate at which particles are removed from beam $= -dN = \dfrac{N}{A}\, \sigma\, n\, A\, dx$

$$-\frac{dN}{N} = \sigma\, n\, dx$$

$$\sigma = \frac{\text{N}^\text{o}\text{ scattered particles /sec}}{N\, n\, dx}$$

Beam attenuation in a target (thickness L)

- thick target (σ n L>>1)

$$\int_{N_i}^{N_f} -\frac{dN}{N} = \int_0^L \sigma \; n \; dx$$

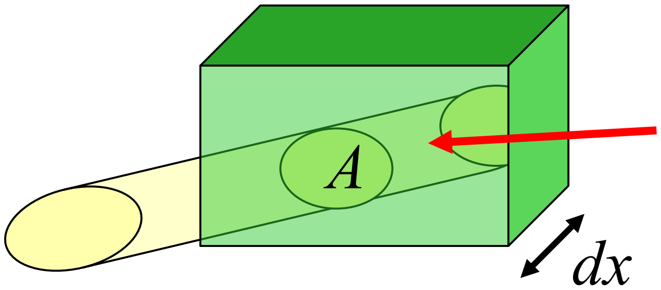
$$N_f = N_i \, e^{-\sigma \, n \, L}$$

- thin target (σ n L<<1, $e^{-\sigma n L} \approx 1-\sigma n L$)

$$N_f = N_i(1 - \sigma \; n \; L)$$

MEAN FREE PATH between interactions = $1/n\sigma$

Rewrite cross-section in terms of the incident flux, $\Phi = N/A$



Number of target particles in cylindrical volume

$$N_T = n * Volume = n\,A\,dx$$

$$\sigma = \frac{N^o \text{ scattered particles /sec}}{N\,n\,dx}$$

$$= \frac{N^o \text{ scattered particles /sec}}{(\Phi A)\,(N_T/A\,dx)\,dx}$$

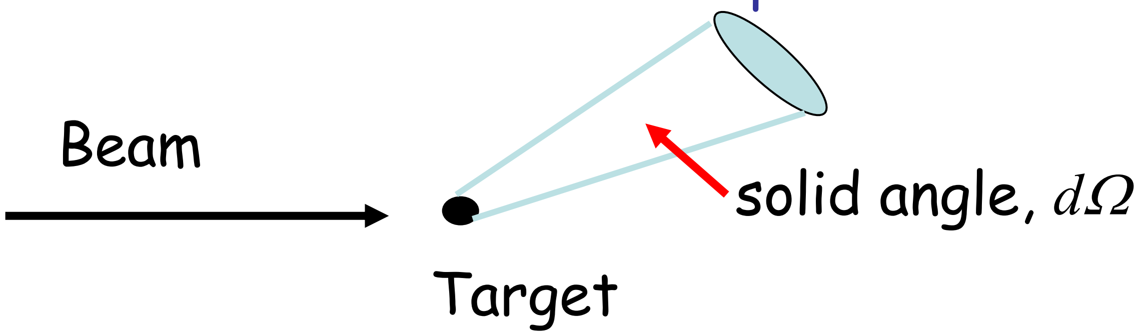
$$\sigma = \frac{N^o \text{ scattered particles /sec}}{\text{Flux} * \text{Number target particles}}$$

Hence,

$$\Gamma = \Phi\sigma$$

Differential Cross-section

The angular distribution of scattered particles is not necessarily uniform



Number of particles scattered into $d\Omega = \Delta N_\Omega$

$$\Delta N_\Omega = d\sigma * \Phi * N_T$$

$$\frac{d\sigma}{d\Omega} = \frac{\Delta N_\Omega}{\Phi * N_T * d\Omega}$$

DIFFERENTIAL CROSS-SECTION

Units: area/steradian

The DIFFERENTIAL CROSS-SECTION is the number of particles scattered per unit time and solid angle divided by the incident flux and by the number of target nuclei defined by the beam area.

- Most experiments do not cover 4π and in general we use $d\sigma/d\Omega$.

- Angular distributions provide more information about the mechanism of the interaction.

- Different types of interaction can occur between particles.

$$\text{Total } \sigma = \sum_i \sigma_i$$

where the σ_i are called PARTIAL CROSS-SECTIONS.

Types of interaction:

Elastic scattering:

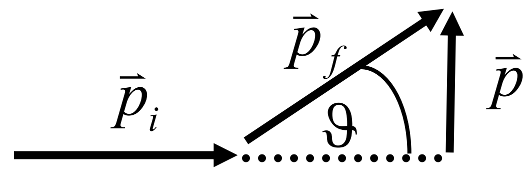
$a + b \rightarrow a + b$ only momenta of a and b change

Inelastic scattering:

$a + b \rightarrow c + d + \ldots\ldots$ final state not the same as initial state

Scattering in QM

Consider a beam of particles scattering in potential $V(r)$:



$$\vec{p} = \vec{p}_i - \vec{p}_f$$

NOTE: natural units $\vec{p} = \hbar\vec{k} \rightarrow \vec{p} = \vec{k} \quad etc$

The scattering rate is characterized by the interaction cross-section

$$\sigma = \frac{\Gamma}{\Phi} = \frac{\text{Number of particles scattered per unit time}}{\text{Incident flux}}$$

Use Fermi's Golden Rule to get the transition rate

$$\Gamma = 2\pi \left| M_{fi} \right|^2 \rho\left(E_f\right)$$

where M_{fi} is the matrix element and $\rho(Ef)$ is the density of final states.

1st Order Perturbation Theory using plane wave solutions of form

$$\psi = N e^{-i(Et - \vec{p}\cdot\vec{r})}$$

<u>Require:</u>

- Wave-function normalization
- Matrix element in perturbation theory
- Expression for flux
- Expression for density of states.

<u>Normalization:</u> Normalize wave-functions to one particle in a box of side L:

$$|\psi|^2 = N^2 = 1/L^3$$

$$N = \left(1/L\right)^{\frac{3}{2}}$$

Matrix Element: This contains the physics of the interaction

$$M_{fi} = \langle \psi_f | \hat{H} | \psi_i \rangle = \int \psi_f^* \hat{H} \psi_i \, d^3\vec{r}$$

$$= \int N e^{-i\vec{p}_f \cdot \vec{r}} V(\vec{r}) N e^{i\vec{p}_i \cdot \vec{r}} \, d^3\vec{r}$$

$$= \frac{1}{L^3} \int e^{i\vec{p}\cdot\vec{r}} V(\vec{r}) d^3\vec{r}$$

where $\vec{p} = \vec{p}_i - \vec{p}_f$

Incident Flux: Consider a "target" of area A and a beam of particles travelling at velocity v_i towards the target. Any incident particle within a volume $v_i A$ will cross the target area every second.

$$\Phi = \frac{v_i A}{A} n = v_i n$$

Flux (Φ) = number of incident particles crossing unit area per second:

$$\Phi = {v_i}/{L^3}$$

where n is the number density of incident particles = 1 per L^3

Density of States: for a box of side L states are given by the periodic boundary conditions:

$$\vec{p} = \left(p_x, p_y, p_z\right) = \frac{2\pi}{L}\left(n_x, n_y, n_z\right)$$

Each state occupies a volume $\left(2\pi/L\right)^3$ in p space.

Number of states between in p and $p+dp$ in solid angle $d\Omega$

$$dN = \left(\frac{L}{2\pi}\right)^3 d^3\vec{p} = \left(\frac{L}{2\pi}\right)^3 p^2\, dp\, d\Omega \qquad d^3\vec{p} = p^2\, dp\, d\Omega$$

$$\therefore \rho(p) = \frac{dN}{dp} = \left(\frac{L}{2\pi}\right)^3 p^2\, d\Omega$$

Density of states in energy $E^2 = p^2 + m^2 \Rightarrow 2EdE = 2pdp \Rightarrow \frac{dE}{dp} = \frac{p}{E}$

$$\rho(E) = \frac{dN}{dE} = \frac{dN}{dp}\frac{dp}{dE} = \left(\frac{L}{2\pi}\right)^3 p^2 \frac{E}{p} d\Omega$$

For relativistic scattering: ($E \approx p$)

$$\rho(E) = \left(\frac{L}{2\pi}\right)^3 E^2\, d\Omega$$

Putting all the separate bits together:

$$d\sigma = \frac{1}{\Phi} 2\pi \left|M_{fi}\right|^2 \rho\left(E_f\right)$$

$$d\sigma = \frac{L^3}{v_i} 2\pi \left|\frac{1}{L^3}\int e^{i\vec{p}\cdot\vec{r}} V(\vec{r}) d^3\vec{r}\right|^2 \left(\frac{L}{2\pi}\right)^3 p^2 \frac{E}{p} d\Omega$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{(2\pi)^2 v_i}\left|\int e^{i\vec{p}\cdot\vec{r}} V(\vec{r}) d^3\vec{r}\right|^2 p^2 \frac{E}{p}$$

Relativistic scattering $v_i = c = 1$ and $p \approx E$

$$\frac{d\sigma}{d\Omega} = \frac{E^2}{(2\pi)^2}\left|\int e^{i\vec{p}\cdot\vec{r}} V(\vec{r}) d^3\vec{r}\right|^2$$

BORN APPROXIMATION

Rutherford Scattering

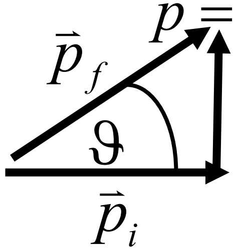
Consider relativistic elastic scattering in a Coulomb potential

$$V(\vec{r}) = -\frac{e^2}{4\pi\varepsilon_0 r} = -\frac{\alpha}{r} \qquad \frac{d\sigma}{d\Omega} = \frac{E^2}{(2\pi)^2}\left|\int e^{i\vec{p}\cdot\vec{r}}V(\vec{r})d^3\vec{r}\right|^2$$

$$\left|M_{fi}\right|^2 = \left|\int e^{i\vec{p}\cdot\vec{r}}V(\vec{r})d^3\vec{r}\right|^2 = \left|-\alpha\int\frac{e^{i\vec{p}\cdot\vec{r}}}{r}d^3\vec{r}\right|^2 = \frac{16\pi^2\alpha^2}{\left|\vec{p}^4\right|}$$

Appendix C

$$\vec{p} = \vec{p}_i - \vec{p}_f$$

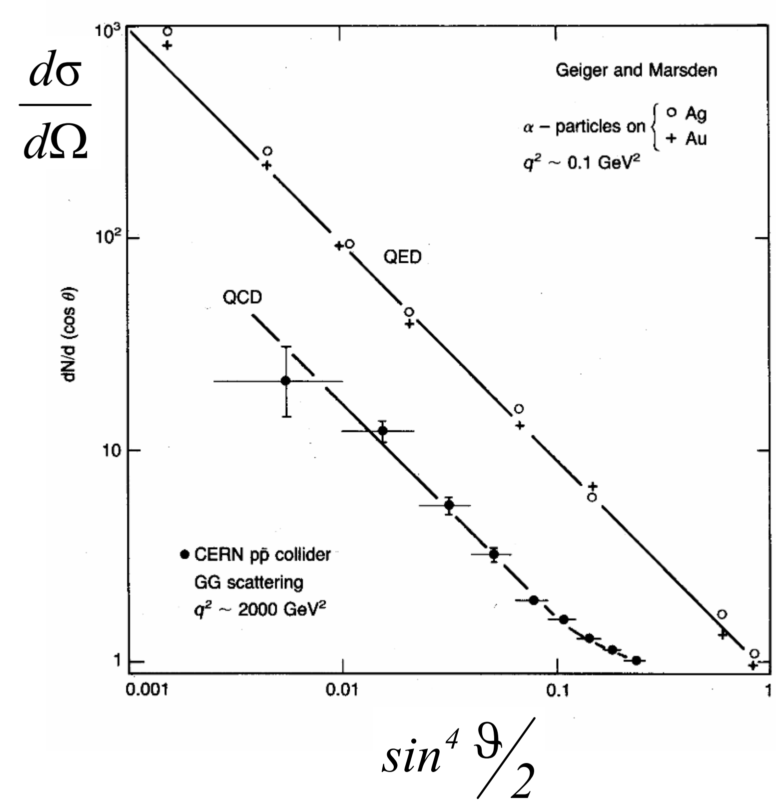


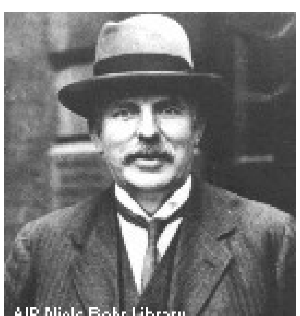
$$\left|\vec{p}\right|^2 = \left|\vec{p}_i\right|^2 + \left|\vec{p}_f\right|^2 - 2\vec{p}_i\cdot\vec{p}_f$$

$$= 2\vec{p}_i^2(1-\cos\vartheta) = 4\vec{p}_i^2\sin^2\vartheta/2$$

$$\frac{d\sigma}{d\Omega} = \frac{E^2}{(2\pi)^2}\frac{16\pi^2\alpha^2}{16E^4\sin^4\vartheta/2}$$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2\sin^4\vartheta/2}$$





Breit-Wigner Cross-section

Some particle interactions proceed via an intermediate RESONANT state which then decays

$$\underbrace{a+b}_{i} \rightarrow Z^{*} \rightarrow \underbrace{c+d}_{f}$$

<u>Two stages:</u> (Bohr Model)

(1) Formation $\underbrace{a+b}_{i} \rightarrow Z^{*}$

Occurs when the collision energy $\sqrt{s} \approx$ the natural frequency of a resonant state.

(2) Decay $Z^{*} \rightarrow \underbrace{c+d}_{f}$

The decay of the resonance Z^{*} is independent of the mode of formation and depends only on the properties of the Z^{*}.

The RESONANCE CROSS-SECTION is given by

$$\sigma = \Gamma / \Phi \qquad \text{with} \qquad \Gamma = 2\pi |M_{fi}|^2 \rho(E_f)$$

$$d\sigma = \frac{1}{\Phi} 2\pi |M_{fi}|^2 \rho(E_f)$$

$$d\sigma = \frac{1}{v_i} 2\pi |M_{fi}|^2 \frac{p_f^2}{v_f (2\pi)^3} d\Omega$$

(Factors of L^3 cancel as before)

$$\frac{d\sigma}{d\Omega} = \frac{p_f^2}{(2\pi)^2 v_i v_f} |M_{fi}|^2$$

$$\rho(p_f) = \frac{dN}{dp} = \frac{p_f^{\,2}}{(2\pi)^3} d\Omega$$

$$\rho(E_f) = \frac{dN}{dp}\frac{dp}{dE} = \frac{p_f^{\,2}}{(2\pi)^3}\frac{E_f}{p_f} d\Omega$$

$$= \frac{p_f^{\,2}}{v_f (2\pi)^3} d\Omega$$

$$v_f = p_f / E_f$$

The matrix element M_{if} is given by 2nd Order Perturbation Theory

$$M_{fi} = \sum_Z \frac{M_{iZ} M_{zf}}{E - E_z}$$

where the sum is over all intermediate states.

Consider 1 intermediate state described by

$$\psi(t) = \psi(0)e^{-iE_0 t}e^{-t/2\tau} = \psi(0)e^{-i\left(E_0 - i\frac{\Gamma}{2}\right)t}$$

This describes a state with energy = $E_0 - i\,\Gamma/2$

$$|M_{fi}|^2 = \frac{|M_{iZ}|^2 |M_{zf}|^2}{(E - E_0)^2 + \Gamma^2/4}$$

Rate of decay of Z:

$$\Gamma_{Z\to f} = 2\pi |M_{Zf}|^2 \rho(E_f) = 2\pi |M_{Zf}|^2 \frac{4\pi p_f^2}{(2\pi)^3 v_f} = |M_{Zf}|^2 \frac{p_f^2}{\pi v_f}$$

Rate of formation of Z:

$$\Gamma_{i\to Z} = 2\pi |M_{iZ}|^2 \rho(E_i) = 2\pi |M_{iZ}|^2 \frac{4\pi p_i^2}{(2\pi)^3 v_i} = |M_{iZ}|^2 \frac{p_i^2}{\pi v_i}$$

Hence,

$$\frac{d\sigma}{d\Omega} = \frac{p_f^2}{(2\pi)^2 v_i v_f} \left| M_{fi} \right|^2$$

$$\sigma = \frac{4\pi p_f^2}{(2\pi)^2 v_i v_f} \frac{\pi v_f}{p_f^2} \frac{\pi v_i}{p_i^2} \frac{\Gamma_{i\to Z}\Gamma_{Z\to f}}{(E-E_0)^2 + \Gamma^2/4}$$

$$= \frac{\pi}{p_i^2} \frac{\Gamma_{i\to Z}\Gamma_{Z\to f}}{(E-E_0)^2 + \Gamma^2/4}$$

p_i is the C.O.M. momentum ~ lab momentum if the target is heavy.

$$p_i = \frac{\hbar}{\lambda\!\!\!^{-}} = \frac{1}{\lambda\!\!\!^{-}}$$ natural units

$$\sigma = \frac{g\pi\lambda\!\!\!^{-2}\,\Gamma_{i\to Z}\Gamma_{Z\to f}}{(E-E_0)^2 + \Gamma^2/4}$$

BREIT-WIGNER CROSS-SECTION

The factor g takes into account the SPIN

$$g = \frac{(2J_Z + 1)}{(2J_a + 1)(2J_b + 1)} \qquad \underbrace{a + b}_{i} \rightarrow Z^* \rightarrow \underbrace{c + d}_{f}$$

and is the ratio of the number of spin states for the resonant state to the total number of spin states for the $a+b$ system.

Notes

- Total cross-section $\sigma_{tot} = \sum_f \sigma(i \rightarrow f)$

  Replace Γ_f by Γ in the Breit-Wigner formula

- Elastic cross-section $\sigma_{el} = \sigma(i \rightarrow i)$ so $\Gamma_f = \Gamma_i$

- On peak of resonance ($E=E_0$) $\sigma_{peak} = \dfrac{4\pi\lambda\!\!\!^{-2} g\Gamma_i\Gamma_f}{\Gamma^2}$

  Thus,

$$\sigma_{el} = 4\pi\lambda\!\!\!^{-2} g B_i^2 \qquad \sigma_{tot} = 4\pi\lambda\!\!\!^{-2} g B_i \qquad B_i = \Gamma_i / \Gamma = \sigma_{el} / \sigma_{tot}$$

From the measurement of σ_{tot} and σ_{el} can infer g and hence spin of resonant state.

Example: Nuclear Physics

$$p + {}^{63}_{29}Cu \rightarrow {}^{64}_{30}Zn^{*} \qquad \alpha + {}^{60}_{28}Ni \rightarrow {}^{64}_{30}Zn^{*}$$

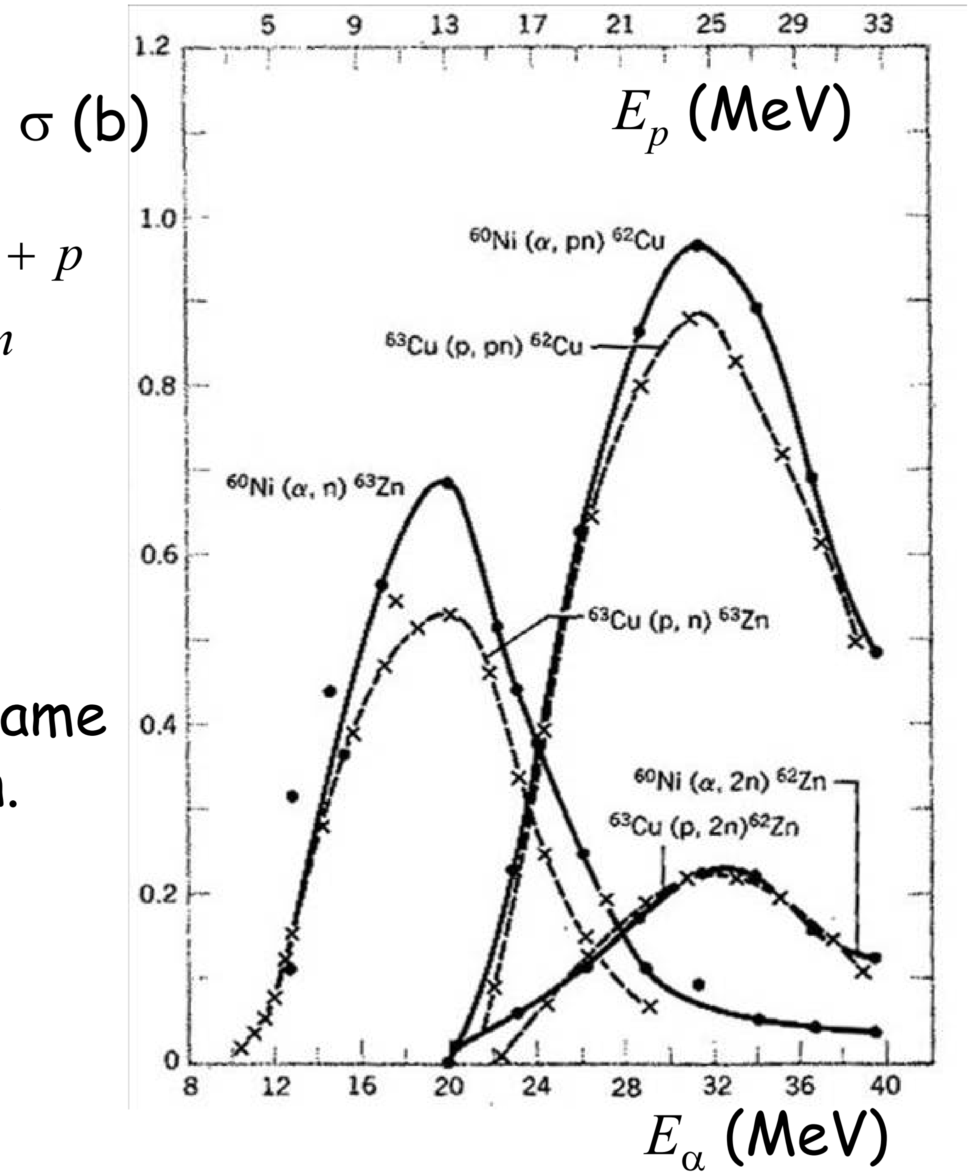
$${}^{64}_{30}Zn^{*} \rightarrow {}^{63}Zn + n$$

$${}^{64}_{30}Zn^{*} \rightarrow {}^{62}Cu + n + p$$

$${}^{64}_{30}Zn^{*} \rightarrow {}^{62}Zn + 2n$$

$$\sigma\ {}^{60}Ni(\alpha,n)Zn \approx \sigma\ {}^{63}Cu(p,n)Zn$$

Energy of p selected to give same Zn* state as for α interaction.



Example: Particle Physics

$$e^+e^- \rightarrow Z^0 \rightarrow q\bar{q}$$

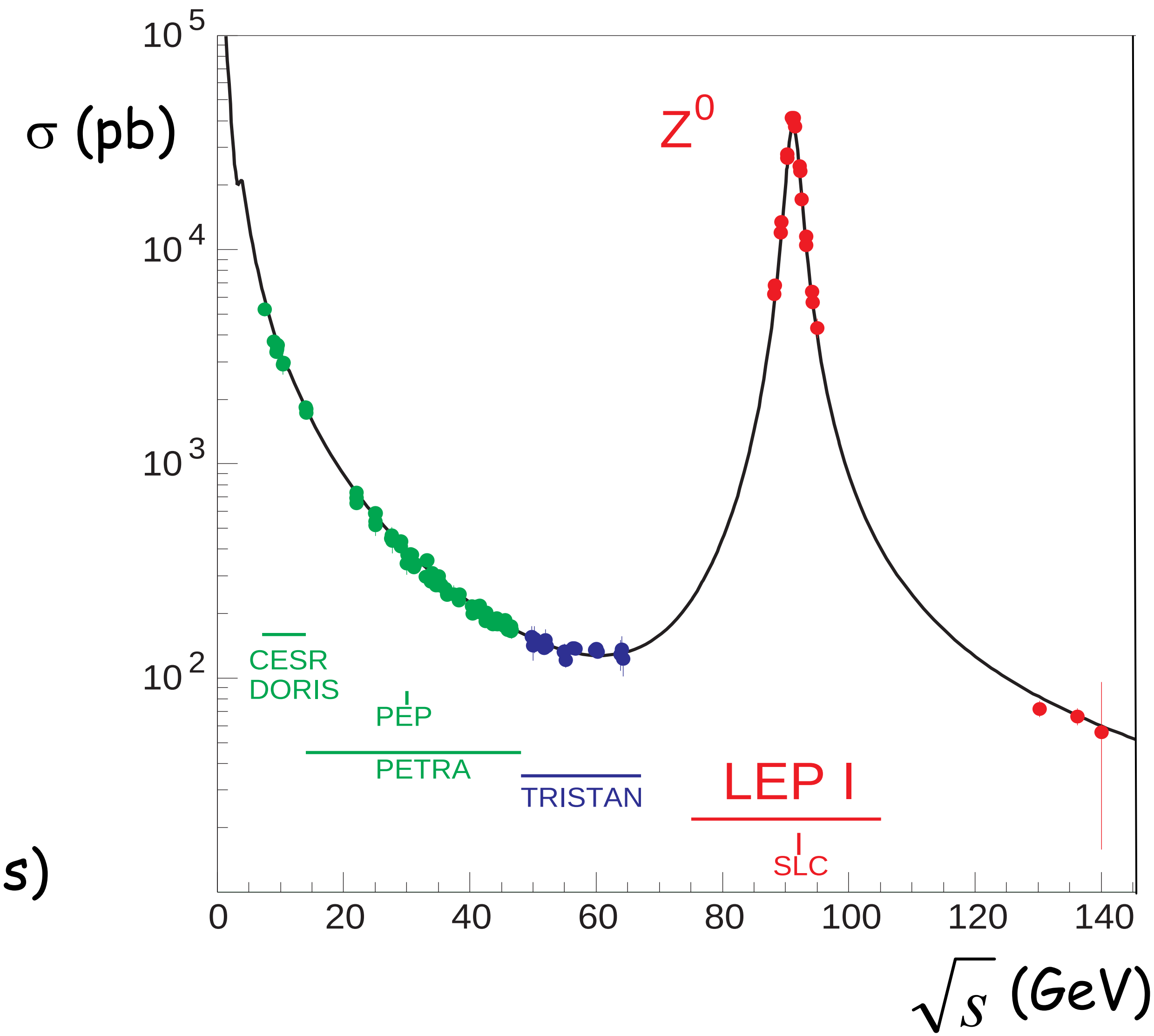
$$\Gamma_Z \approx 2.5 \text{ GeV}$$

$$\tau = \frac{1}{\Gamma_Z} = 0.4 \text{ GeV}^{-1}$$

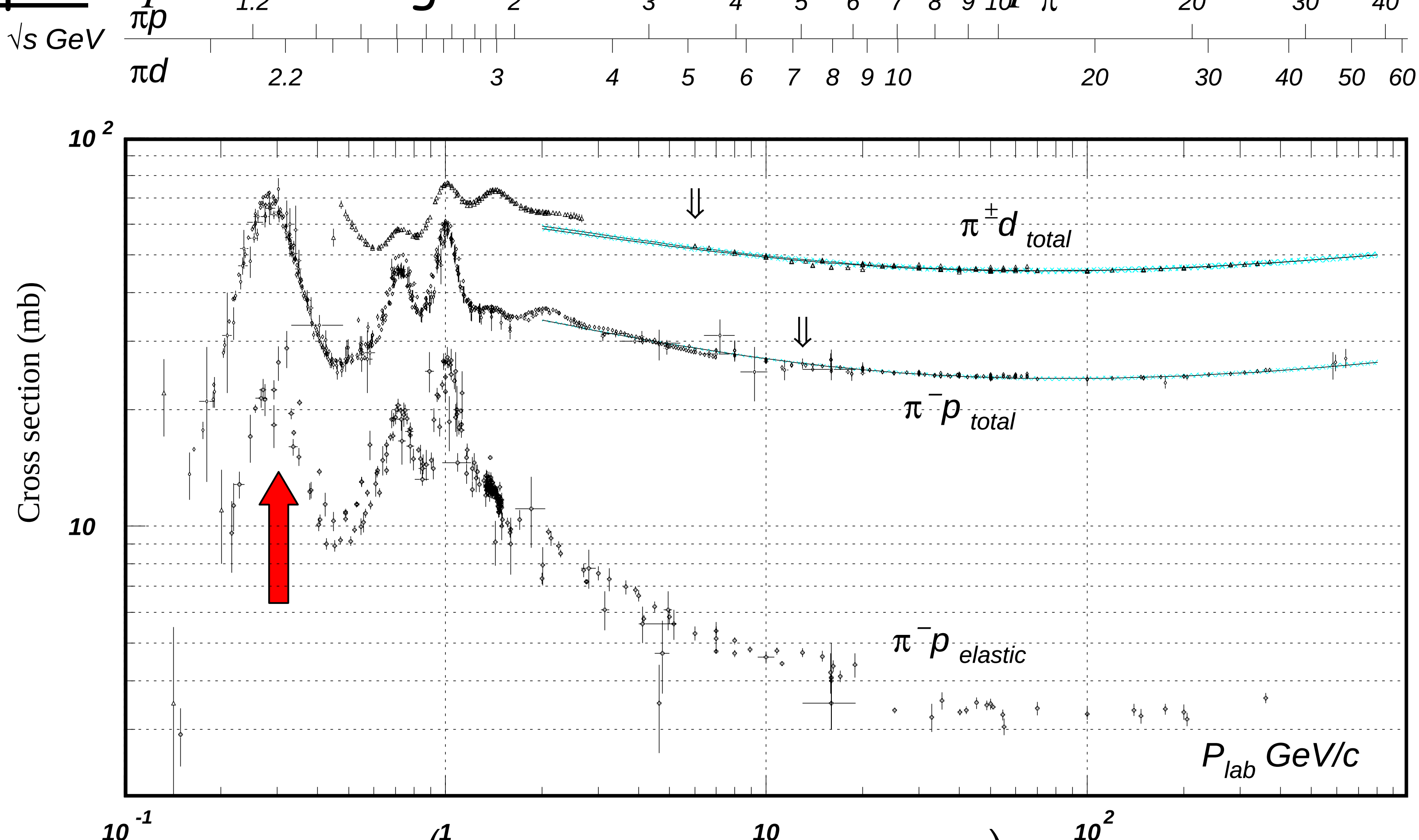
$$= 0.4 \times \hbar$$

$$= 2.6 \times 10^{-25} \text{ s}$$

$$(\hbar = 6.6 \times 10^{-25} \text{GeV s})$$



Example: $\pi^- p$ scattering. Observe resonance at $p_\pi \sim 0.3$ GeV/c



$$\sigma_{\text{total}} = \sigma\left(\pi^- p \to R \to anything\right) \approx 72\,\text{mb}$$

$$\sigma_{\text{elastic}} = \sigma\left(\pi^- p \to R \to \pi^- p\right) \approx 28\,\text{mb}$$

$$\sigma = \frac{g\pi\bar{\lambda}^2 \Gamma_{i\to Z}\Gamma_{Z\to f}}{(E-E_0)^2 + \Gamma^2/4}$$

At resonance: $\sigma_{total} = \frac{4\pi\bar{\lambda}^2 g\Gamma_{\pi p}}{\Gamma}; \quad \sigma_{elastic} = \frac{4\pi\bar{\lambda}^2 g\Gamma_{\pi p}^2}{\Gamma^2} \Rightarrow \frac{\sigma_{elastic}}{\sigma_{total}} = \frac{\Gamma_{\pi p}}{\Gamma}$

$$\sigma_{total} = \frac{4\pi\bar{\lambda}^2 g\,\sigma_{elastic}}{\sigma_{total}} \Rightarrow g = \frac{1}{4\pi\bar{\lambda}^2}\frac{\sigma_{total}^2}{\sigma_{elastic}}$$

$$p_{lab} = 0.3\,\mathrm{GeV/c} \quad \Rightarrow p_{cm} = \frac{1}{\bar{\lambda}} = 0.23\,\mathrm{GeV/c}$$

$$\sigma_{total} \approx 72\,\mathrm{mb}; \quad \sigma_{elastic} \approx 28\,\mathrm{mb}$$

$$g \approx 2 = \frac{(2J+1)}{(2J_\pi+1)(2J_p+1)} \qquad J_\pi = 0; \quad J_p = 1/2$$

$$J = 3/2$$

The resonance is a *udd* state (see Quark Model) with J=3/2.