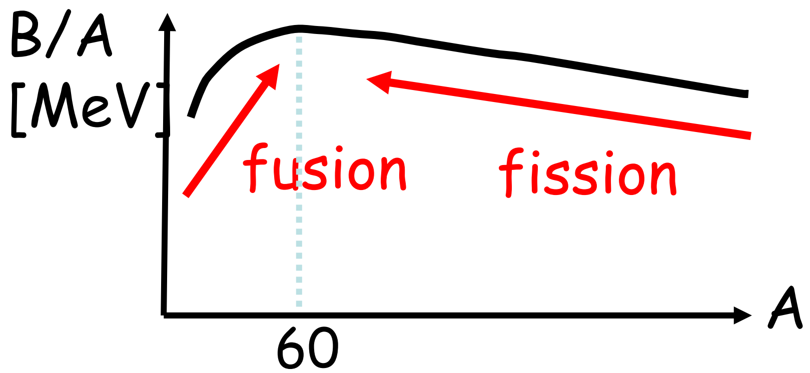
Section XV
Fission and Fusion

Fission and Fusion



Most stable form of nuclear matter is at A~60. Expect a large amount of energy released in the FISSION of a heavy nucleus into two medium nuclei and in the FUSION of two light nuclei into a single medium nucleus.

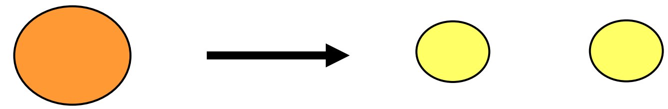
SEMF

$$B(A,Z) = a_V A - a_S A^{2/3} - \frac{a_C Z^2}{A^{1/3}} - a_A \frac{(N-Z)^2}{A} + \delta(A)$$

FISSION occurs because the total Coulomb repulsion energy of p's in a nucleus is reduced if the nucleus splits into two smaller nuclei. The nuclear surface energy increases in the process, but its magnitude is much smaller.

FUSION occurs because the two low A nuclei have too large a surface area for their volume. The surface area decreases when they amalgamate. Coulomb energy increases, but its magnitude is too small.

Spontaneous Fission



(A,Z) $\quad$ (A_1,Z_1) $\quad$ (A_2,Z_2)

- Expect spontaneous fission to occur if energy released

$$E_0 = B(A_1,Z_1) + B(A_2,Z_2) - B(A,Z) > 0$$

- Assume $\frac{A_1}{A} = \frac{Z_1}{Z} = y_1$ and $\frac{A_2}{A} = \frac{Z_2}{Z} = y_2; \quad y_1 + y_2 = 1$
- From SEMF $E_0 = a_S A^{2/3}(1 - y_1^{2/3} - y_2^{2/3}) + a_C \frac{Z^2}{A^{1/3}}(1 - y_1^{5/3} - y_2^{5/3})$
- Maximum energy release when $\frac{\partial E_0}{\partial y_1} = 0 \quad (dy_2 = -dy_1)$

$$\frac{\partial E_0}{\partial y_1} = a_S A^{2/3}\left(-\frac{2}{3}y_1^{-1/3} + \frac{2}{3}y_2^{-1/3}\right) + a_C \frac{Z^2}{A^{1/3}}\left(-\frac{5}{3}y_1^{2/3} + \frac{5}{3}y_2^{2/3}\right) = 0$$ when $y_1 = y_2 = \frac{1}{2}$

SYMMETRIC FISSION

- Maximum $E_0 = 0.37\, a_C \frac{Z^2}{A^{1/3}} - 0.26\, a_S A^{2/3}$

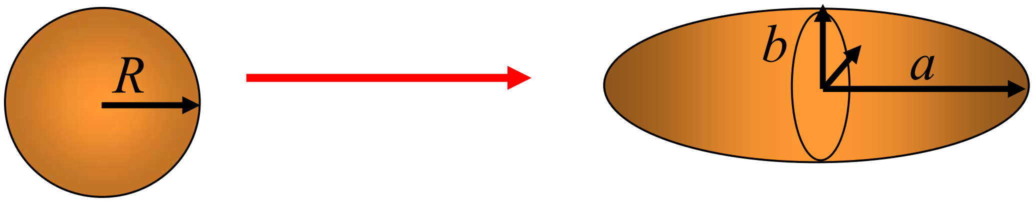
$a_S = 16.8 \quad MeV$

$a_C = 0.72 \quad MeV$

Example: ${}^{238}_{92}U$ Maximum $E_0 \approx$ 200 MeV

~10^6 > energy released in chemical reaction.

- Estimate mass at which nuclei become unstable to fission (i.e. point at which energy change due to deformation gives a change in B.E, $\Delta B > 0$)



$$a = R(1+\varepsilon) \qquad \varepsilon << 1$$
$$b = R(1+\varepsilon)^{-1/2}$$

- SEMF Volume term unchanged: $Volume = \frac{4}{3}\pi ab^2 = \frac{4}{3}\pi R^3 = CONSTANT$

- Change in surface term: $a_S A^{2/3} \longrightarrow a_S A^{2/3}(1+\frac{2}{5}\varepsilon^2)$

- Change in Coulomb term: $a_C \frac{Z^2}{A^{1/3}} \longrightarrow a_C \frac{Z^2}{A^{1/3}}(1-\frac{\varepsilon^2}{5})$

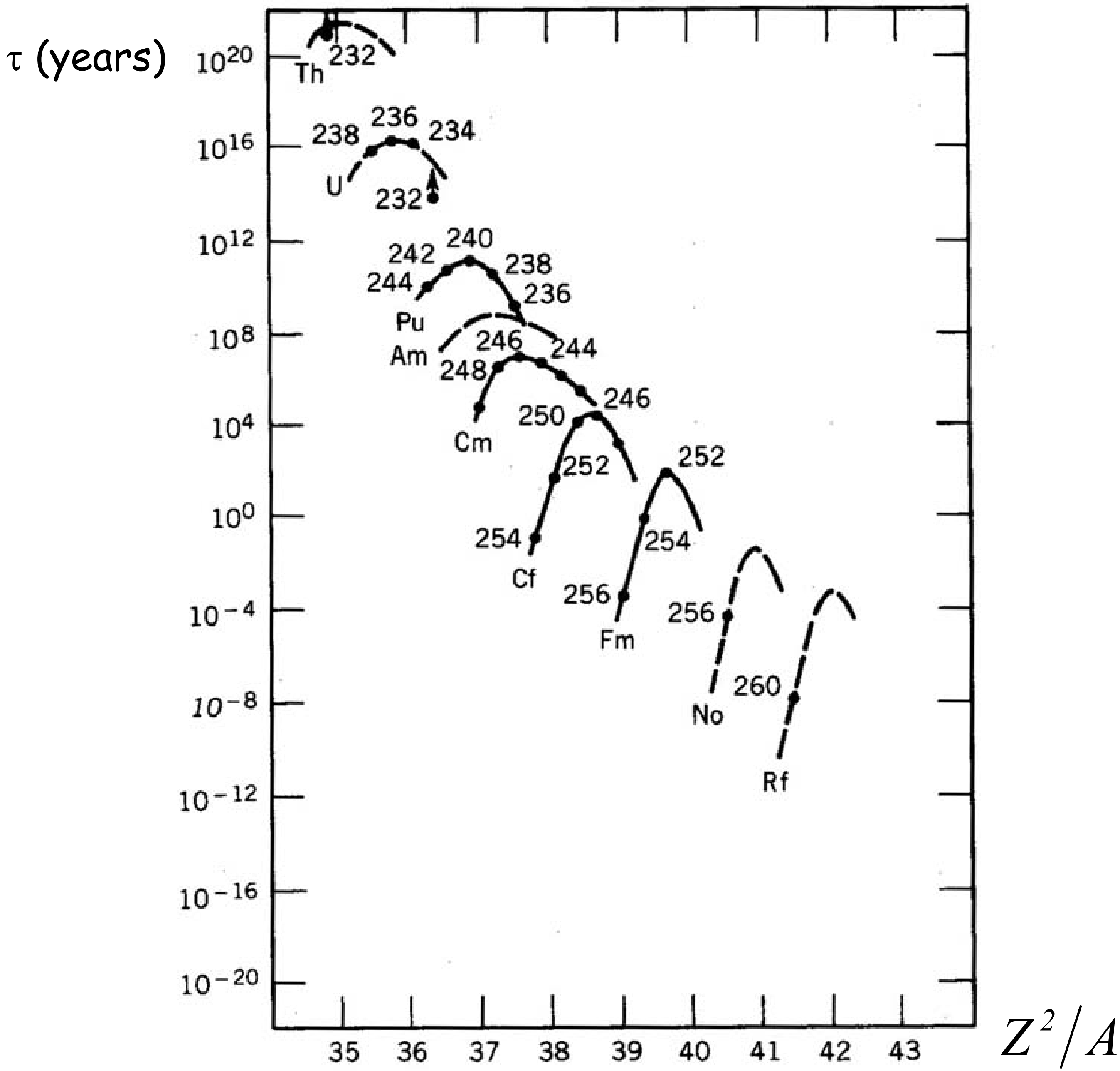
See Segrè Ch 11 11-11

- Change in Binding Energy: $\Delta B = B(\varepsilon) - B(0) = a_C A^{2/3}\left(\frac{Z^2}{A} - \frac{2a_S}{a_C}\right)\frac{\varepsilon^2}{5}$

i.e. if $\frac{Z^2}{A} > \frac{2a_S}{a_C}$ $\Delta B > 0$ and nucleus unstable under deformation

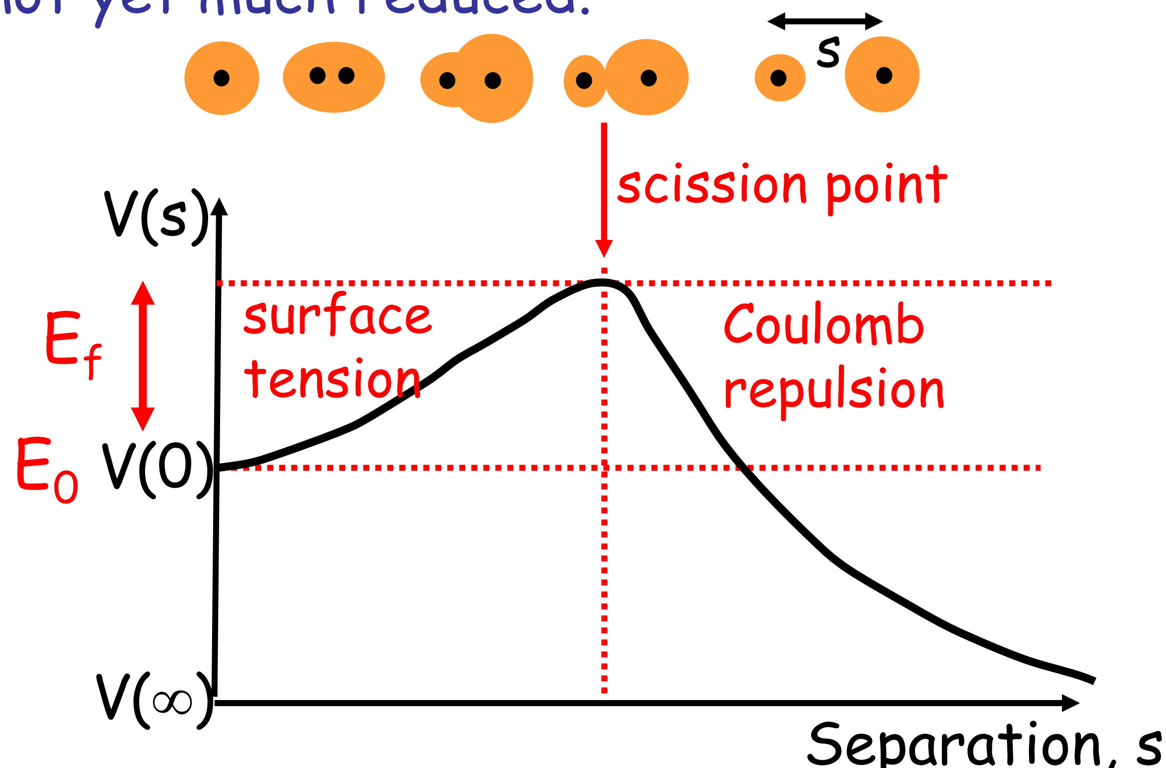
$$\Rightarrow \boxed{\frac{Z^2}{A} > 50}$$

➢ Spontaneous fission lifetimes fall rapidly as Z^2/A approaches 50.



Fission Barrier

In the fission process, nuclei have to pass through an intermediate state where the surface energy is increased, but the Coulomb energy is not yet much reduced.



E_0 = energy released → K.E. of fragments.

E_f = FISSION ACTIVATION ENERGY

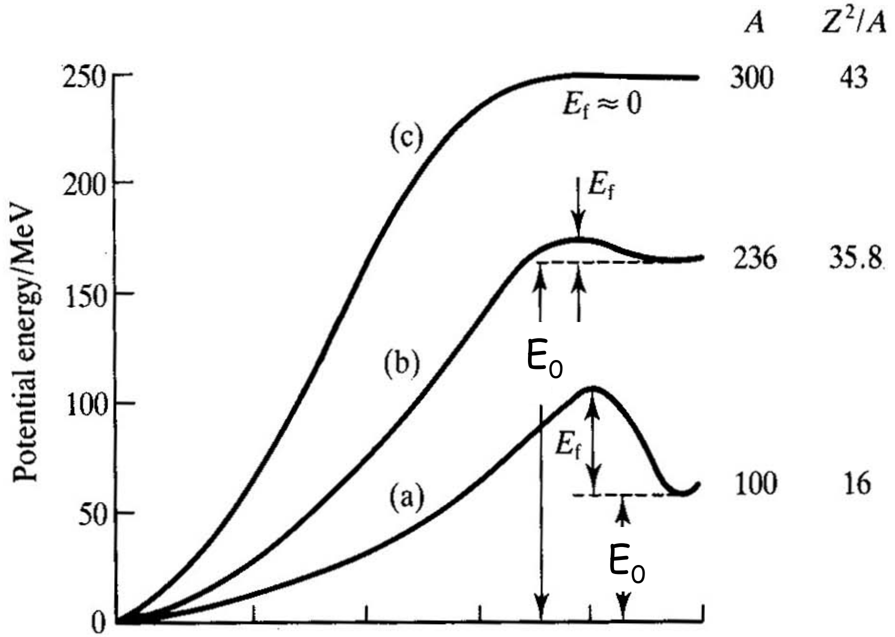
$$E_f = a_C A^{2/3}\left(\frac{Z^2}{A} - \frac{2a_S}{a_C}\right)\frac{\varepsilon^2}{5} \approx \underline{6\ MeV} \qquad {}^{236}_{92}U$$

Spontaneous fission is possible if tunnelling occurs (c.f. α decay).

Tunnelling probability depends on

- Z^2/A.

$$E_f \sim \frac{Z^2}{A}$$



- Mass of fragment.

$$P = e^{-2G} \qquad G \sim m^{1/2}$$

Large mass $\rightarrow$ low probability for tunnelling

e.g. 10^6 less probable than α decay for $^{238}_{92}U$

Induced Fission

LOW ENERGY NEUTRON CAPTURE

- At low energies, neutrons can be absorbed by nuclei (no Coulomb barrier).
- Important for the design of thermonuclear reactors.
- For a low energy excited state, γ decay is most probable.

Breit-Wigner Cross-section

$$\sigma(n,\gamma) = \frac{g\pi \lambda\!\!\!^{-2} \Gamma_n \Gamma_\gamma}{(E-E_0)^2 + \Gamma^2/4}$$

$$\Gamma_n << \Gamma_\gamma \approx \Gamma$$

At resonance

$$\sigma(n,\gamma) = 4\pi \lambda\!\!\!^{-2} g \frac{\Gamma_n \Gamma_\gamma}{\Gamma^2} \approx 4\pi \lambda\!\!\!^{-2} g \frac{\Gamma_n}{\Gamma}$$

Typically, $\Gamma_n \sim 10^{-3}$ eV, $\Gamma \sim 1$ eV, 1eV neutron $\Rightarrow \sigma \sim 10^3$ b (largest $^{135}X_e$ $\sigma \sim 10^6$ b)

Below resonance

$$\sigma(n,\gamma) \approx \lambda\!\!\!^{-2} \Gamma_n \left[\frac{g\pi \Gamma_\gamma}{E_0^2 + \Gamma^2/4} \right]$$

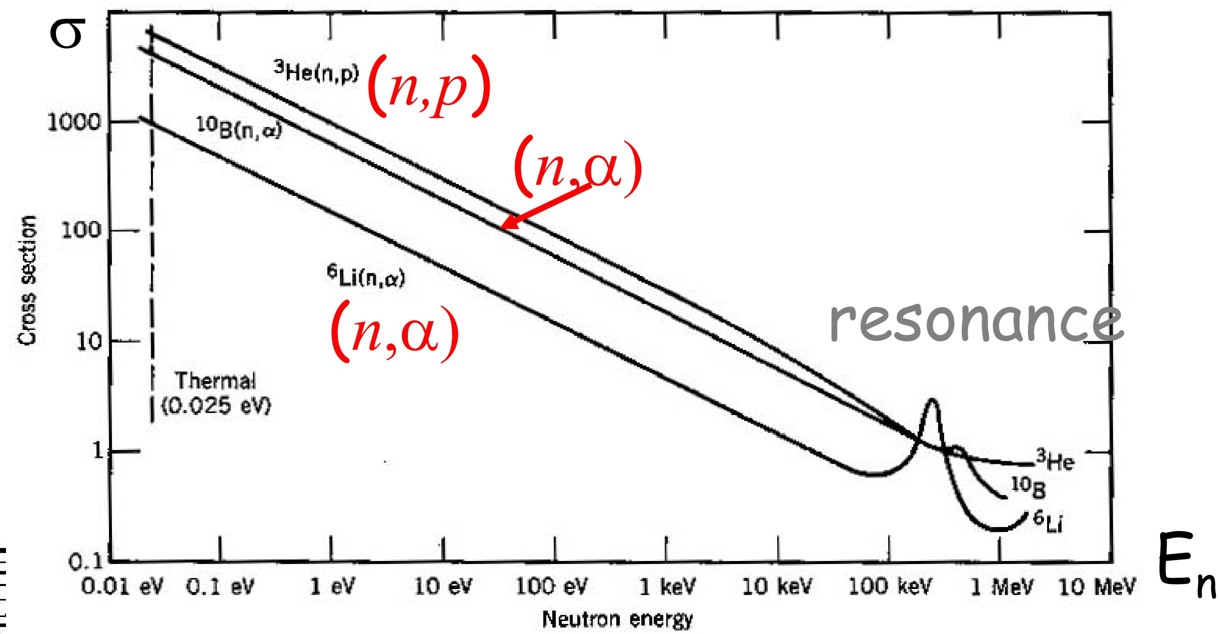
constant

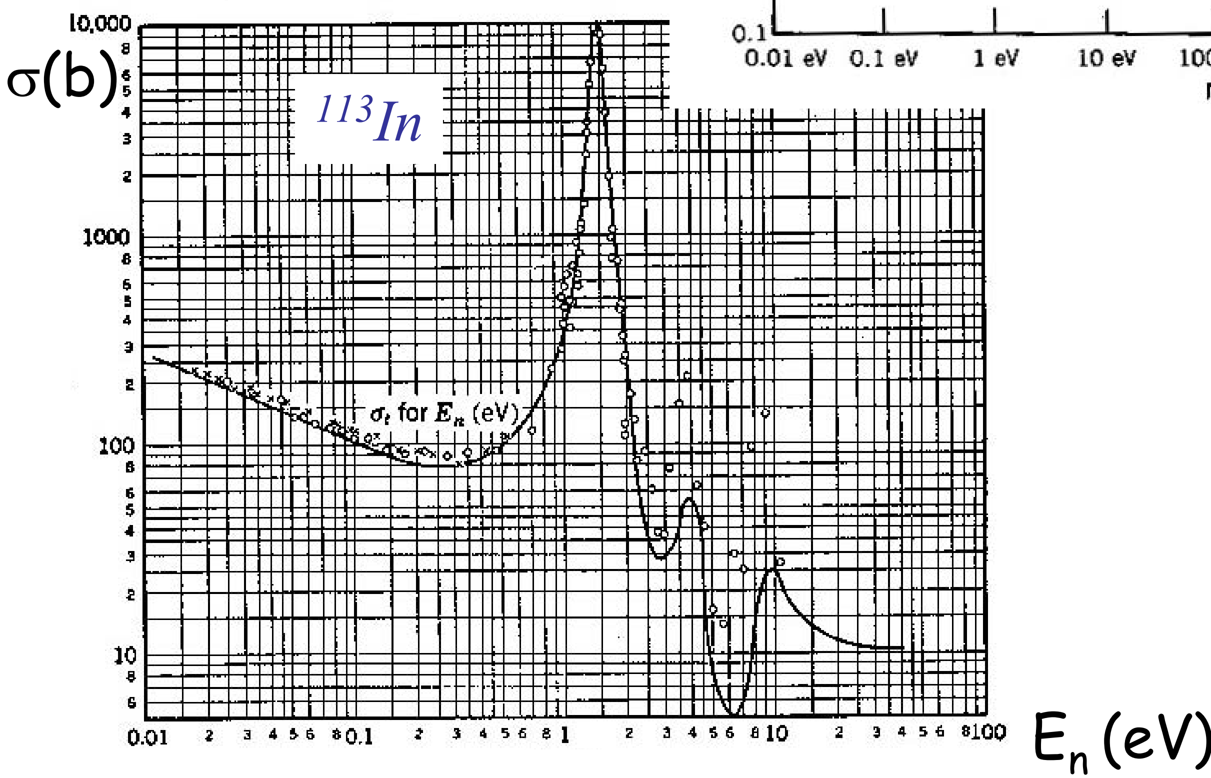
Γ_n dominated by phase space

$$\Gamma_n \sim \frac{p^2}{v} \sim v \,; \qquad \lambda\!\!\!^{-} = \frac{\hbar}{p} \rightarrow \lambda\!\!\!^{-2} \sim \frac{1}{v^2} \longrightarrow \therefore \sigma(n,\gamma) \approx \frac{1}{v}$$

"1/v LAW"

$\sigma \sim 1/v$ dependence far from resonance $\Rightarrow$ Ln σ ~ - Ln E





INDUCED FISSION of nuclei occurs when a nucleus captures a low energy neutron receiving enough energy to climb the fission barrier.

e.g. $n+{}^{235}_{92}U \rightarrow {}^{236}_{92}U \rightarrow X^* + Y^* \rightarrow X + Y + \kappa n$ $\quad \kappa \sim 2.4$

PROMPT NEUTRONS

- If excitation energy > fission activation energy, fission will occur for zero energy neutrons → THERMAL NEUTRONS.

(Available energy from separation energy of n).

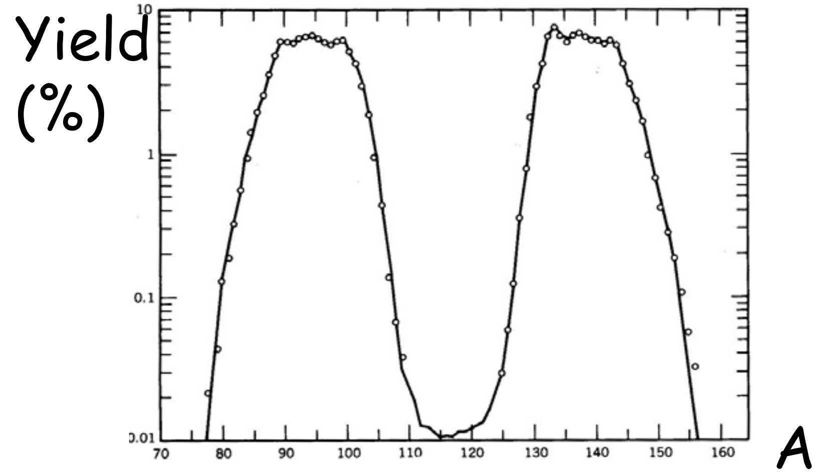
- Otherwise need to supply energy using K.E. of n.

e.g. $n + {}^{238}U \rightarrow {}^{239}U^*$ $\quad E_f \sim 6$ MeV

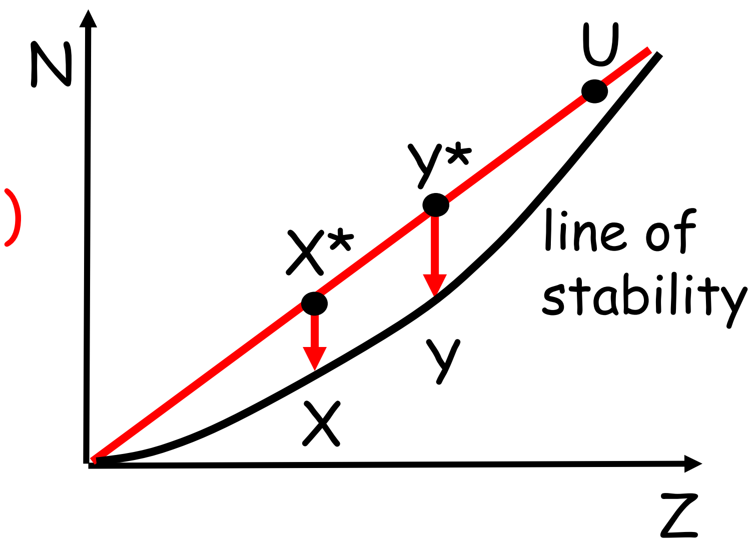
$E_n = 0$ $\quad E \sim 5$ MeV $\quad$ no thermal fission

$E_n = 1.4$ MeV $\quad E \sim 6.4$ MeV $\quad$ fission

- Masses of fragments are unequal (in general). Tend to have Z, N near magic numbers.



- Fragments tend to have same Z/N ratio as parent → neutron rich nuclei which emit PROMPT NEUTRONS (10^{-16}s)



- X and Y undergo β decay more slowly
  → DELAYED NEUTRON EMISSION
  (~1 delayed n/100 fissions)

Chain Reaction

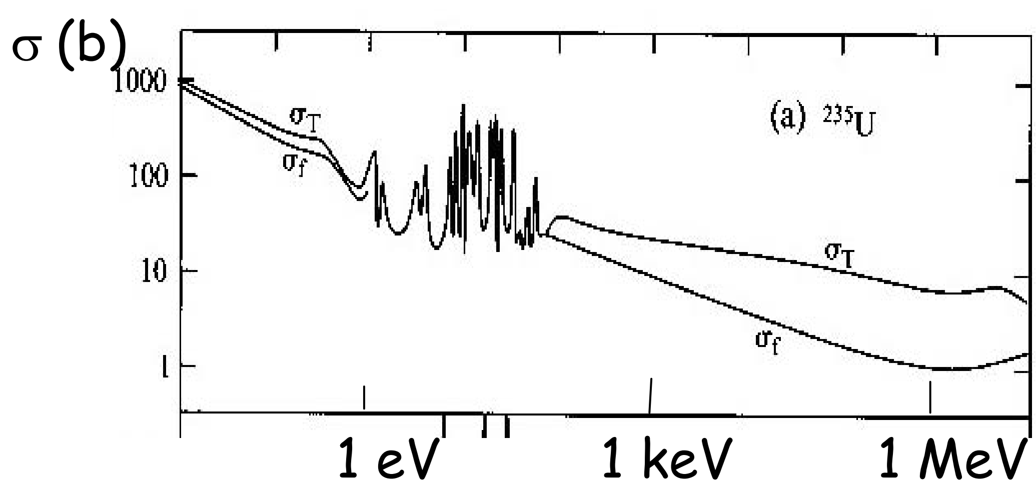
- Neutrons from fission process can be used to induce further fission → CHAIN REACTION

- A chain reaction can be sustained if at least 1 n per fission induces another fission process.

Define k = number of neutrons from one fission which induce another

k = 1 CRITICAL, k < 1 SUBCRITICAL, k > 1 SUPERCRITICAL

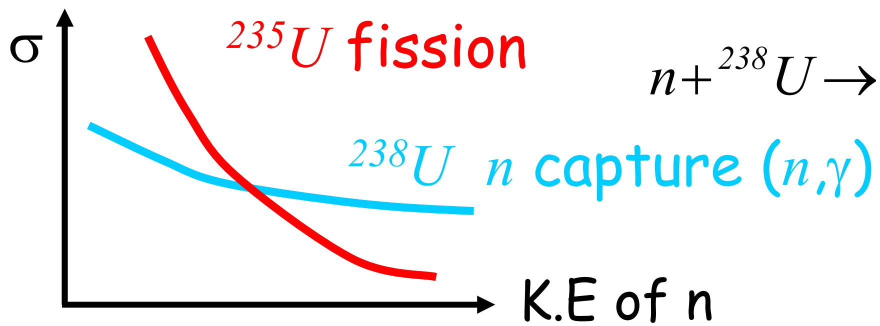
- Prompt neutrons are FAST, $\langle E \rangle \sim 2$ MeV and the fission cross-section is small



- Hence, need to slow down fast neutrons before they escape or absorbed by (n,γ) process → CHAIN REACTION

Fission Reactors

- For reactors want a steady energy release, exactly critical (k = 1).
- A MODERATOR slows neutrons via elastic collisions (large energy transfer). Requires a light nucleus (e.g. ^{12}C).
- PROBLEM: Natural U (99.3% ^{238}U, 0.7% ^{235}U) and n capture cross-section large for ^{238}U



$$n + {}^{238}U \rightarrow {}^{239}U^* \rightarrow {}^{239}U + \gamma$$

- Need to THERMALISE fast neutrons AWAY from ^{238}U to avoid capture (i.e. in rods of ^{12}C).
- CONTROL number of neutrons by absorption (e.g. ^{113}Cd rods).

Typical time between fission and daughter inducing another fission ~10^{-3} s.

→ Mechanical control of rods in times « seconds not possible.

What happens if no control of neutrons ?

$$N(t+dt) = N(t) + (k-1)N(t)\frac{dt}{\tau} \quad \Rightarrow \quad dN = (k-1)N\frac{dt}{\tau}$$

$$\int_{N(0)}^{N(t)} \frac{dN}{N} = \int_0^t (k-1)\frac{dt}{\tau} \quad \Rightarrow \quad \underline{N(t) = N(0)e^{(k-1)t/\tau}}$$

where $N(t)$ is the number of neutrons at time t

$(k\text{-}1)$ is the % change in number of neutrons in 1 cycle

τ mean time for 1 cycle ~10^{-3} s (fission $\rightarrow$ fission)

e.g. k =1.01, τ = 0.001s, t = 1s

$$\frac{N(t)}{N(0)} = e^{0.01/0.001} = \underline{e^{10}} \qquad \text{(22,000 in 1s)}$$

NOTE: U reactor will NOT explode if it goes supercritical. As it heats up, K.E. of neutrons increases and fission cross-section drops. Reactor stabilizes at a very high temperature

$\Rightarrow$ MELTDOWN

- Solution is to make use of delayed n emission. (delay ~ 13 s)
- Design reactor to be subcritical to PROMPT n and use DELAYED n to take it to critical.

THERMAL REACTORS

Require following steps:

fission → fast neutrons

↓

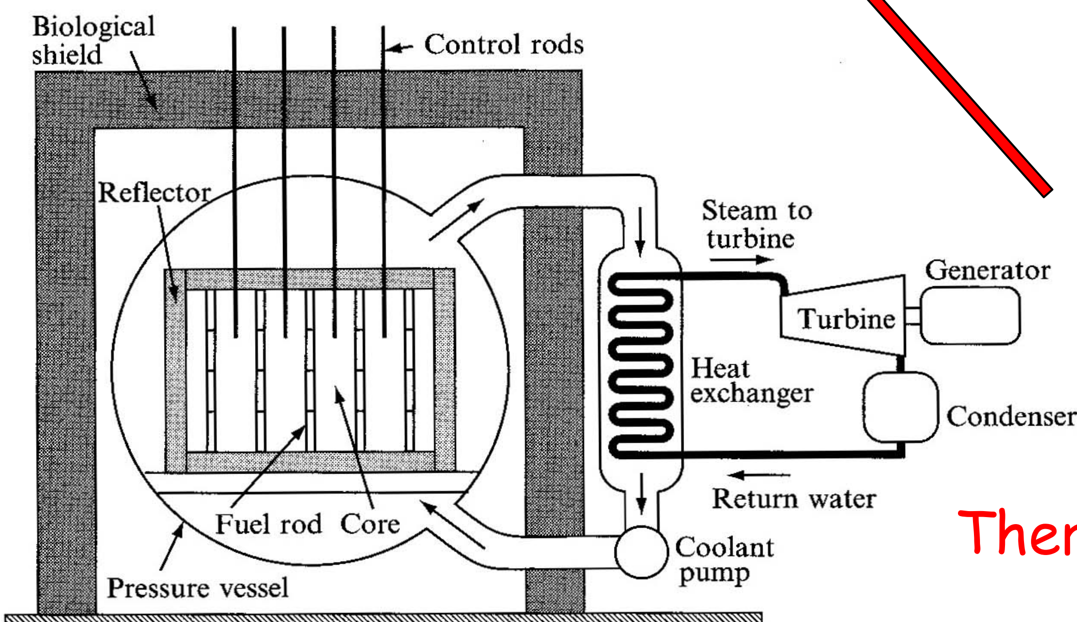
remove from fissile material

↓

thermalize them (gives heat)

↓

drift back into fuel (→ fission)



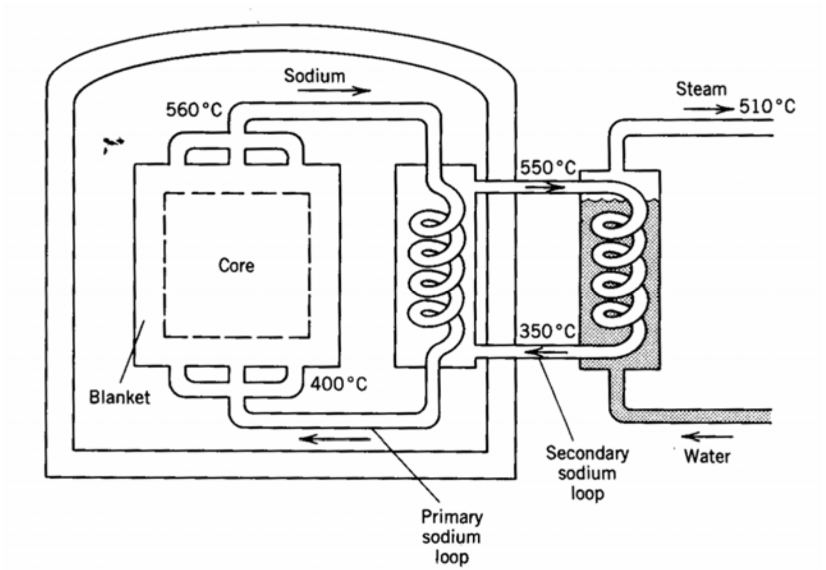
Thermal Reactor

FAST BREEDER REACTORS

~20% Pu + 80% ^{238}U

- Can use fast neutrons to produce chain reaction. Thermal σ higher, but high enrichment compensates. No moderator required.
- n capture by $$n + {}^{238}U \rightarrow {}^{239}U \rightarrow {}^{239}Np \rightarrow {}^{239}Pu$$ fissionable

  Hence, BREEDER. All fuel used.
- Control rods required.



Nuclear Fusion

- Energetically favourable for light nuclei to fuse and release energy. However, light nuclei need energy to overcome Coulomb barrier.

e.g most basic process $p + p \rightarrow d + e^{+} + \nu_e$ $E_0 = 0.42$ MeV

- Coulomb barrier $$V = \frac{e^2}{4\pi\varepsilon_0 R} = \frac{\alpha\hbar c}{R} = \frac{197}{137 \times 1.2} = 1.2\ MeV$$

- ACCELERATORS: Energies above barrier easy to achieve. However, high particle densities for long periods of time very difficult. These are required to get the rate of fusion reactions for desired power.

- STARS: Large proton density 10^{32} m^{-3}. Particle K.E. due to thermal motion.

For kT ~1 MeV require T~10^{10} K
Sun T~10^{7} K Energies ~1 keV

$\Rightarrow$ QUANTUM MECHANICAL TUNNELLING REQUIRED

FUSION RATE IN SUN

- Particles in the sun have Maxwell-Boltzman velocity distribution - very important because tunnelling probability is a strong function of energy.

Reaction rate/unit volume for particles of velocity v $= \sigma(v)\, \varphi\, N$

where $\varphi = Nv$

- σ is modified by tunnelling probability $P = e^{-2G(v)}$

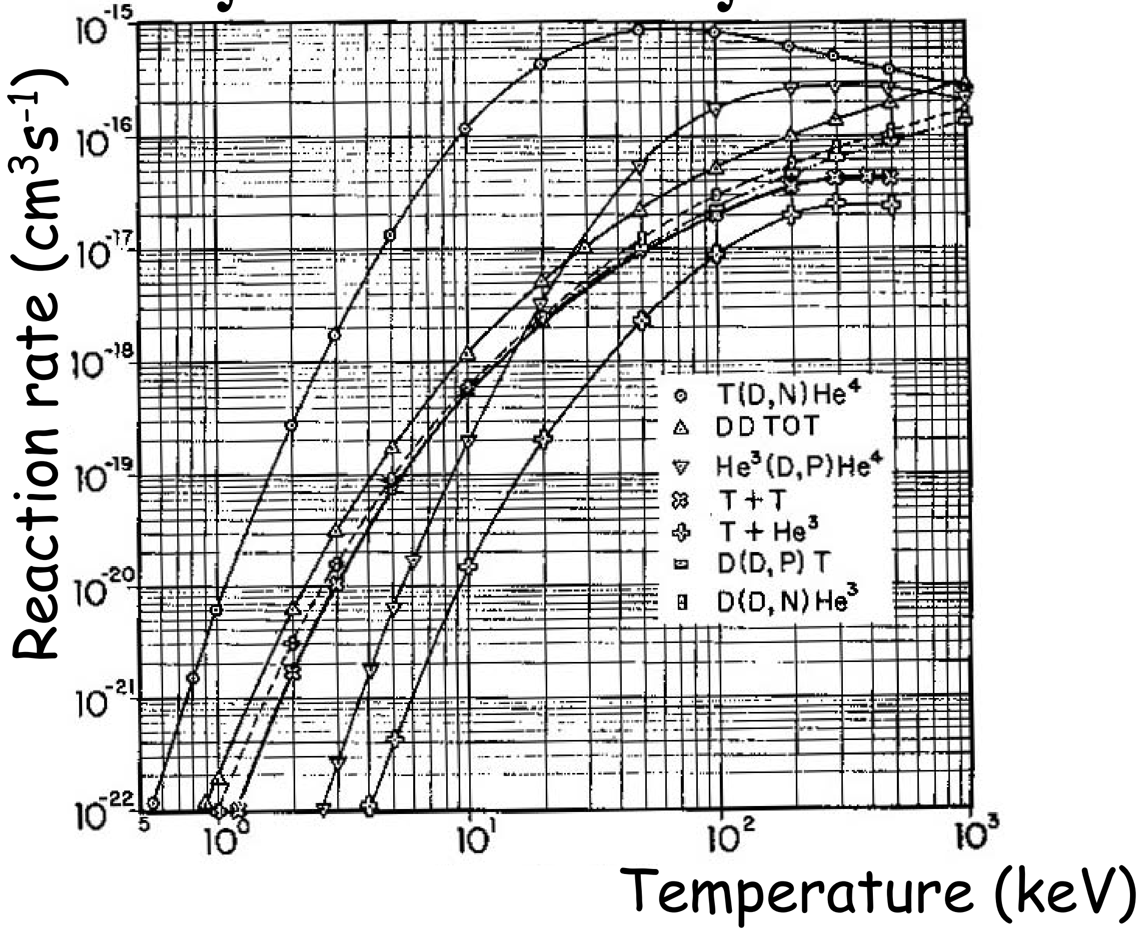
Gamow Factor $G(v) \approx \left(\dfrac{2m}{E_0}\right)^{1/2} \dfrac{e^2}{4\pi\varepsilon_0} \dfrac{Z_1 Z_2}{\hbar} \dfrac{\pi}{2} = \dfrac{e^2}{4\pi\varepsilon_0} \dfrac{\pi Z_1 Z_2}{\hbar v}$

- Convolve with velocity distribution

Probability velocity between v and $v+dv$ $f(v)dv = v^2 e^{-mv^2/2kT} dv$

$\Rightarrow$ Reaction Rate = $\int N\,(Nv)\, \sigma(v)\, P\, f(v) dv$

Reaction rate $\sim \int v\, e^{-2G} e^{-mv^2/2kT} dv \sim \int e^{-2G} e^{-E/kT} dE$



Typical fusion reactions peak at kT~100 keV $\Rightarrow$ T~10^9K

e.g. $p + p \rightarrow d + e^+ + \nu_e$ Reaction rate/proton/sec ~ 5x10^{-18} s

$\Rightarrow$ Mean life, τ = 10^{10} years

This defines the burning rate in the Sun.

FUSION PROCESSES IN THE SUN

(1) $p + p \rightarrow d + e^{+} + \nu$ $p + p \rightarrow d + e^{+} + \nu$

E_0 = 0.42 MeV E_0 = 0.42 MeV

(2) $p + d \rightarrow {}^{3}He + \gamma$ $p + d \rightarrow {}^{3}He + \gamma$

E_0 = 5.49 MeV E_0 = 5.49 MeV

(3) ${}^{3}He + {}^{3}He \rightarrow {}^{4}He + 2p + \gamma$ E_0 = 12.86 MeV

- Net Reaction
  2e^{+} annihilate with 2e^{-}

$$4p \rightarrow {}^{4}He + 2e^{+} + 2\nu$$

$E_0 = 4m_e$ = 2.04 MeV

- Total energy release in fusion cycle = 26.7 MeV

i.e. energy release per proton in fusion cycle = 26.7/4 = 6.7 MeV

- ν's emerge without further interaction with ~2% of the energy. The rest heats the core.

1 MeV = 1.6 x 10^{-13} J

- Observed luminosity ~ 4 x 10^{26} J/s

Number of protons consumed s^{-1} = $\dfrac{4 \times 10^{26}}{1.6 \times 10^{-13}} \dfrac{1}{6.7}$ = 4 x 10^{38}

Other ^{3}He interactions:

(1) $^{3}He + {}^{4}He \rightarrow {}^{7}Be + \gamma$

(2) $^{7}Be + e^{-} \rightarrow {}^{7}Li + \nu$

$\quad + p \rightarrow 2\,{}^{4}He$

OR

$^{7}Be + p \rightarrow {}^{8}B + \gamma$

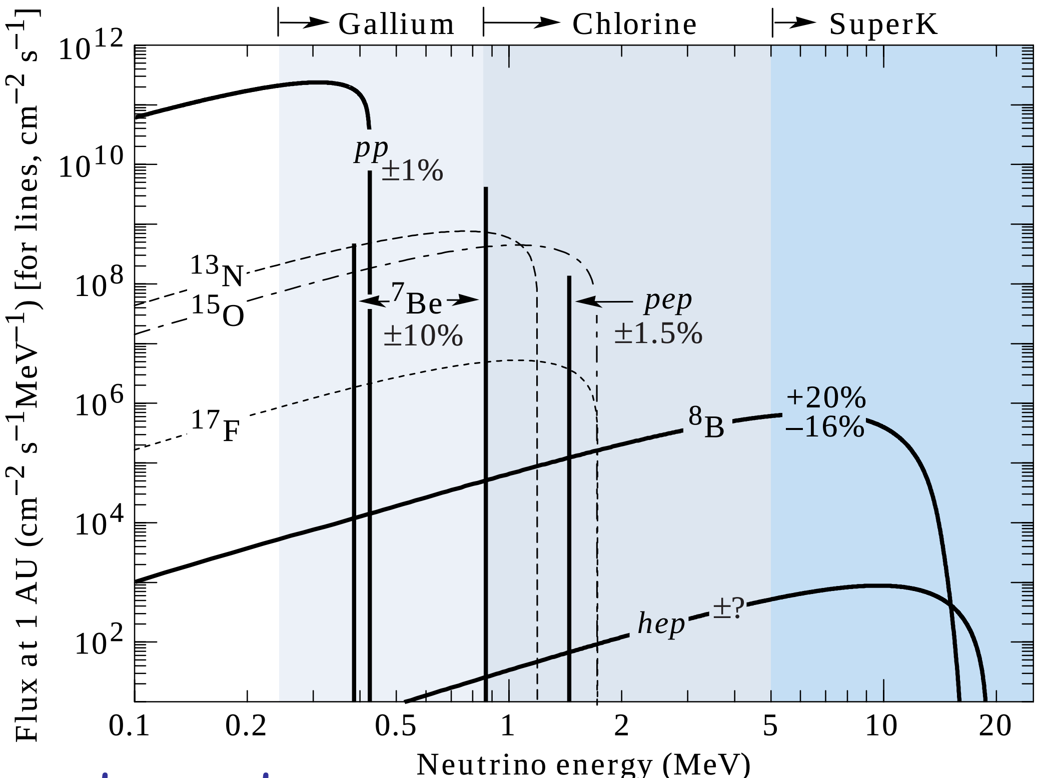
$\quad {}^{8}Be + e^{+} + \nu$

$\quad 2\,{}^{4}He$

- Other fusion cycles also possible e.g. C-N-O
- Observation of solar neutrinos from the various sources directly addresses the theory of stellar structure and evolution (Standard Solar Model).
- The Sun also provides an opportunity to investigate ν properties e.g. mass, oscillations...

Solar Neutrinos

Many experiments have studied the solar neutrino flux



Expected flux depends on

- ➢ Standard Solar Model (temp,density, composition vs r)
- ➢ Nuclear reaction cross-sections

Observed ν flux ~ 1/3 expected ν flux

"SOLAR ν PROBLEM"

The Solar ν problem has recently been solved by the Sudbury Neutrino Observatory (SNO) collaboration. They have reported evidence for a non-ν_e neutrino component in the solar ν flux

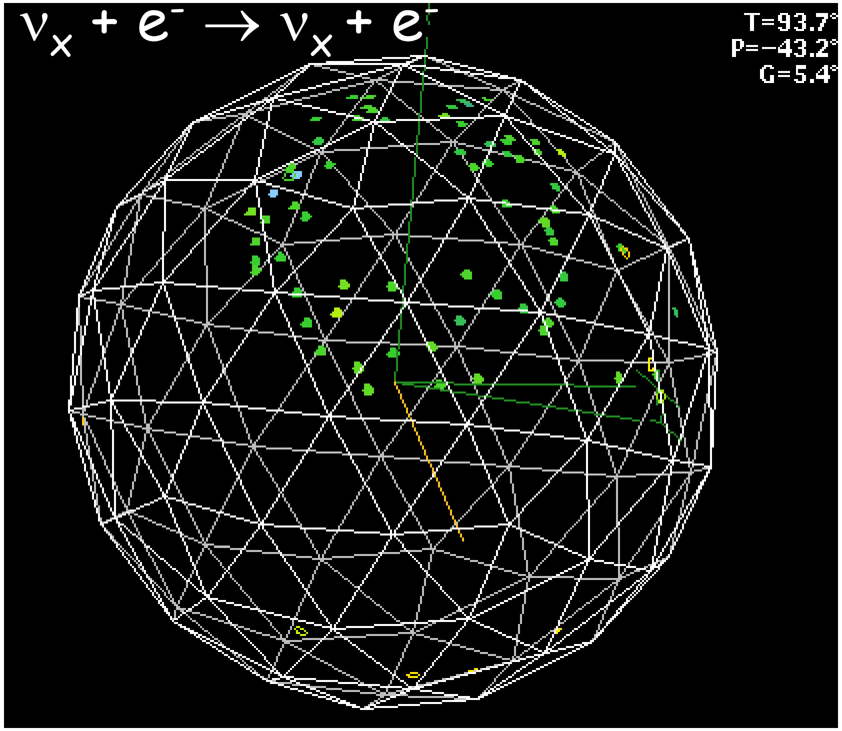
$\rightarrow$ NEUTRINO OSCILLATIONS

SNO measure the ^8B solar ν flux using the reactions

$\nu_e + d \rightarrow e^- + p + p$ — ν_e flux

$\nu_x + d \rightarrow \nu_x + p + n$
$\nu_x + e^- \rightarrow \nu_x + e^-$ } total flux (all ν species)

1000 tons D_2O
in spherical vessel

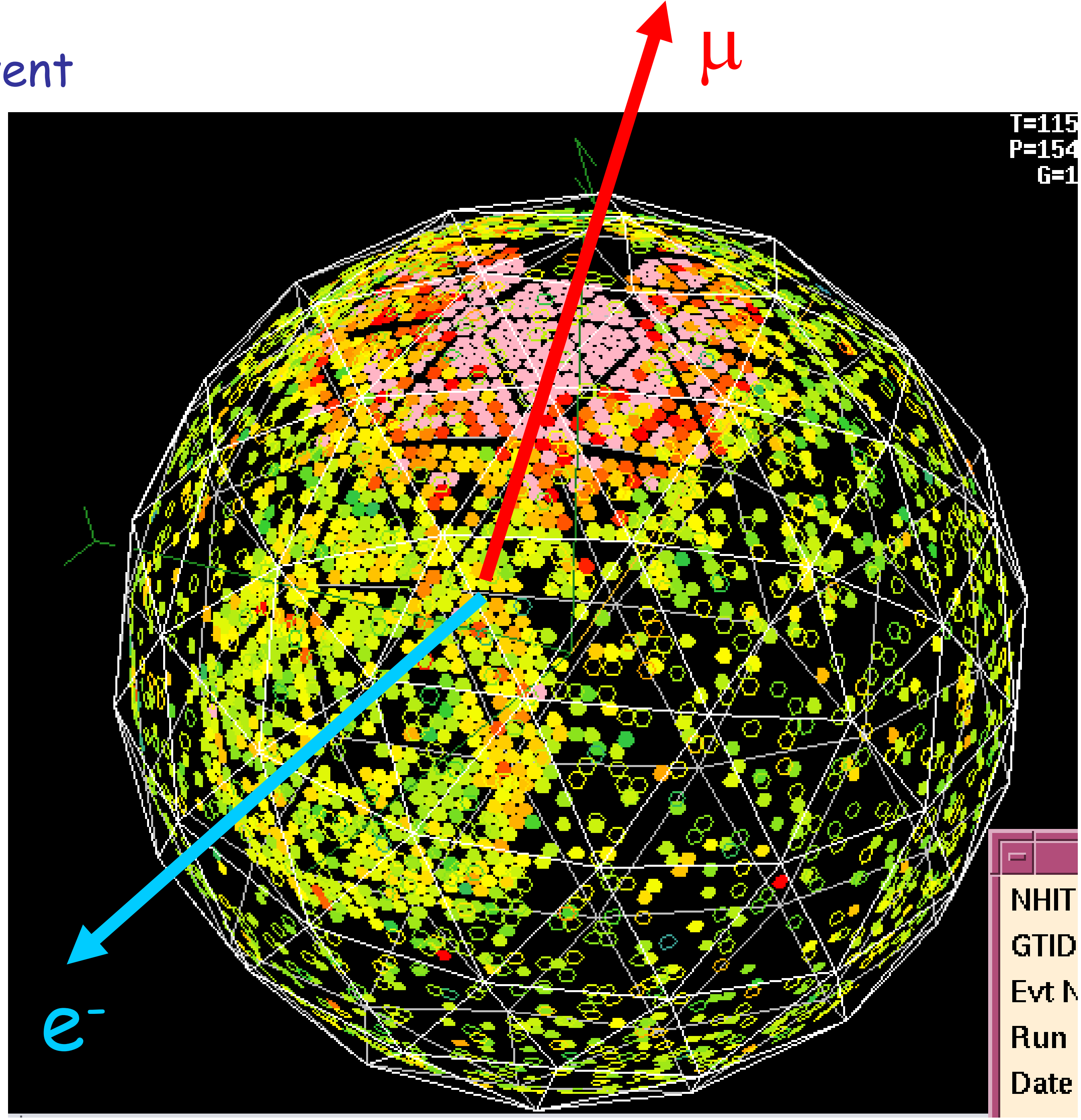


Evidence for $\nu_e \Leftrightarrow \nu_x$ at 5σ

All Solar and reactor ν's $\Rightarrow$

$$\sin^2 2\vartheta \approx 0.81 \quad \Delta m^2 = 7.9 \times 10^{-5}\ eV^2$$

SNO event



The End

Thankyou for being a Great Part II Class !