Part II Examples Class, Lent 2004
Nuclear Physics Questions

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Quickies

1. If you were seeking nuclear magnetic resonance in a field of 1 Tesla, in what frequency range would you search ?

2. What does the electric quadrupole moment of the deuteron tell us about the nature of the neutron-proton force ?

3. Why are *delayed neutrons* essential to the working of a fission reactor ?

4. Estimate the size of the Coulomb barrier between two ${}^{16}_{8}\mathrm{O}$ nuclei which needs to be overcome before they can undergo fusion, and thus estimate the temperature needed to bring about fusion.

Longer Questions

1. The following is the semi-empirical mass formula for the binding energy B of a nucleus (A,Z) according to the liquid drop model:

$$\mathrm{B} = \mathrm{a_v A} - \mathrm{a_s A^{2/3}} - \mathrm{a_c}\frac{\mathrm{Z^2}}{\mathrm{A^{1/3}}} - \mathrm{a_a}\frac{(\mathrm{A}-2\mathrm{Z})^2}{\mathrm{A}} \pm \delta(\mathrm{A})$$

where the coefficients have the values in MeV: $\mathrm{a_v} = 15.8, \mathrm{a_s} = 18.0, \mathrm{a_c} = 0.72, \mathrm{a_a} = 23.5$

   (i) Account for the order of magnitude of $\mathrm{a_c}$.

   (ii) Calculate the Z value of the most stable nucleus of mass number A=181. Has it any stable isobars?

   (iii) Estimate values for the minimum mass and radius of a neutron star. You may assume that the radius of a nucleus is given by $1.2\times10^{-15}A^{1/3}$ in metres, the mass of a neutron is 1.67×10^{-27}kg and that the neutron star only contains neutrons. The gravitational constant $\mathrm{G} = 6.67 \times 10^{-11}$ in mks units and there are 1.6×10^{-13} Joules per MeV.

2. According to a particular nuclear shell model calculation, using a square well with rounded edges and strong spin-orbit interaction, the nuclear levels, in increasing order of energy, are

$$1s_{1/2},\ 1p_{3/2},\ 1p_{1/2},\ 1d_{5/2},\ 2s_{1/2},\ 1d_{3/2},\ 1f_{7/2},\ 2p_{3/2},\cdots\cdots\cdots$$

Show that the model predicts $\frac{5}{2}^{+}$ for the ground state spin and parity of ${}^{17}_{9}\mathrm{F}$. What does it predict for the spins and parities of the ground states of ${}^{14}_{7}\mathrm{N}$, ${}^{30}_{14}\mathrm{Si}$ and ${}^{39}_{16}\mathrm{S}$?

3. The $^{180}_{74}$W nucleus has a rotational level sequence with the following energies (in keV) relative to the ground state:

$$0,\ 102,\ 336,\ 690,\ 1147,\ 1670,\ 2250\cdots\cdots$$

What quantum numbers would you expect for these levels ?

Deduce what properties you can from their energies.

Discuss the relationship between your results and the empirical formula $R \approx 1.2A^{1/3}$ fm for the nuclear radius R, in terms of the atomic number, A.

[Moment of inertia of a sphere $= \frac{2}{5}MR^2$.]

4. Consider the following β decays:

(i) $^{11}_{6}\mathrm{C}(\frac{3}{2}^-) \rightarrow ^{11}_{5}\mathrm{B}(\frac{3}{2}^-)$

(ii) $^{80}_{38}\mathrm{Sr}(\frac{5}{2}^+) \rightarrow ^{89}_{39}\mathrm{Y}(\frac{1}{2}^-)$

(iii) $^{60}_{27}\mathrm{Co}(5^+) \rightarrow ^{60}_{28}\mathrm{Ni}(4^+)$

For each decay state the types of matrix element likely to be involved and whether the decay is likely to be forbidden, allowed or super-allowed.

5. A deuteron (spin 1) of speed v interacts with a stationary deuteron to produce a proton and a tritium nucleus (spin 1/2) according to the equation

$$d + d \rightarrow p +^3_1\mathrm{H} \qquad \text{(A)}$$

The intial deuterons are in a relative s-state, and the cross-section for the reaction is denoted by $\sigma(A)$. The cross-section for the inverse reaction

$$p +^3_1 H \rightarrow d + d \qquad (B)$$

is denoted by $\sigma(B)$. Neglecting relativistic effects and the binding energies of the particles, find the speed of the proton in (B) if the centre of mass energies are the same in the two reactions. Show further that, under these conditions, $\sigma(B) = 2\sigma(A)$.

6. The reaction cross-section near a nuclear resonance is given by the Breit-Wigner formula

$$\sigma_{xy} = \frac{\lambda^2}{4\pi} g \frac{\Gamma_x \Gamma_y}{[(E - E_0)^2 + \frac{\Gamma^2}{4}]}$$

where

λ = deBroglie wavelength of the incident particle

g = spin statistics factor = $\frac{2\mathrm{J_z}+1}{(2\mathrm{J_x}+1)(2\mathrm{J_X}+1)}$ where the subscripts Z, x, X refer to the compound nucleus, incident and target particles respectively.

Γ_x = width (proportional to rate) for decay of the resonance into the initial state x.

Γ_y = width for decay of the resonance into the final state y.

$\mathrm{E_0}$ = central energy of the resonance.

E = kinetic energy of incident particle plus binding energy of initial state.

Γ = total width of the resonance = $\sum_\mathrm{n} \Gamma_\mathrm{n}$ over all n models of decay.

(a) Obtain an expression for g in the case of neutrons incident on nuclei of spin 1.

(b) Show that, at a resonance peak, neglecting any background, $\pi\sigma^2 = \lambda^2 g \sigma_s$ where σ and σ_s are the total and elastic scattering cross sections respectively, for neutrons.

(c) Show that the energy dependence in the denominator has the form characteristic of a state decaying exponentially with a mean lifetime $\tau = \hbar/\Gamma$. You may wish to use the facts that the Fourier transform of F(t) is proportional to $\int \mathrm{F(t)e^{i\omega t}dt}$ and that $\int_0^\infty e^{iAt}dt = \frac{-1}{iA}$.

(d) Calculate the peak cross section for the reaction $^{15}\mathrm{N}(\mathrm{p},\gamma)^{16}\mathrm{O}$ proceeding through an excited state $^{16}\mathrm{O}^*$, given that

(i) the same resonance can be excited in the reaction $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$ at an α particle energy of 7.05 MeV;

(ii) the Q values of the reactions $^{15}\mathrm{N}(\mathrm{p},\gamma)^{16}\mathrm{O}$ and $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$ are 12.13 MeV and 7.16 MeV respectively;

(iii) the state $^{16}\mathrm{O}^*$ has J = 1 and a total width made up of the partial widths Γ_α = 93 keV, Γ_γ= 7 eV, and Γ_p = 1.1 keV;

(iv) $^{15}\mathrm{N}$ has J = 1/2.