Shape of Resonance

Wavefunction for decaying state Z* with mean energy E_0 and lifetime τ

$$\psi(t) = \psi(0)e^{-i\omega_0 t}e^{-t/2\tau} \qquad E = \hbar\omega \quad \hbar = 1$$

$$|\psi(t)|^2 = |\psi(0)|^2 e^{-t/\tau}$$

← exponential decay

The frequencies present in the wavefunction are given by the Fourier transform of $\psi(t)$

$$f(\omega) = \int_0^\infty \psi(t)e^{i\omega t}dt$$

and so the energies are

$$f(E) = \int_0^\infty \psi(0)e^{-t\left(iE_0 + \frac{1}{2\tau}\right)}e^{iEt}dt$$

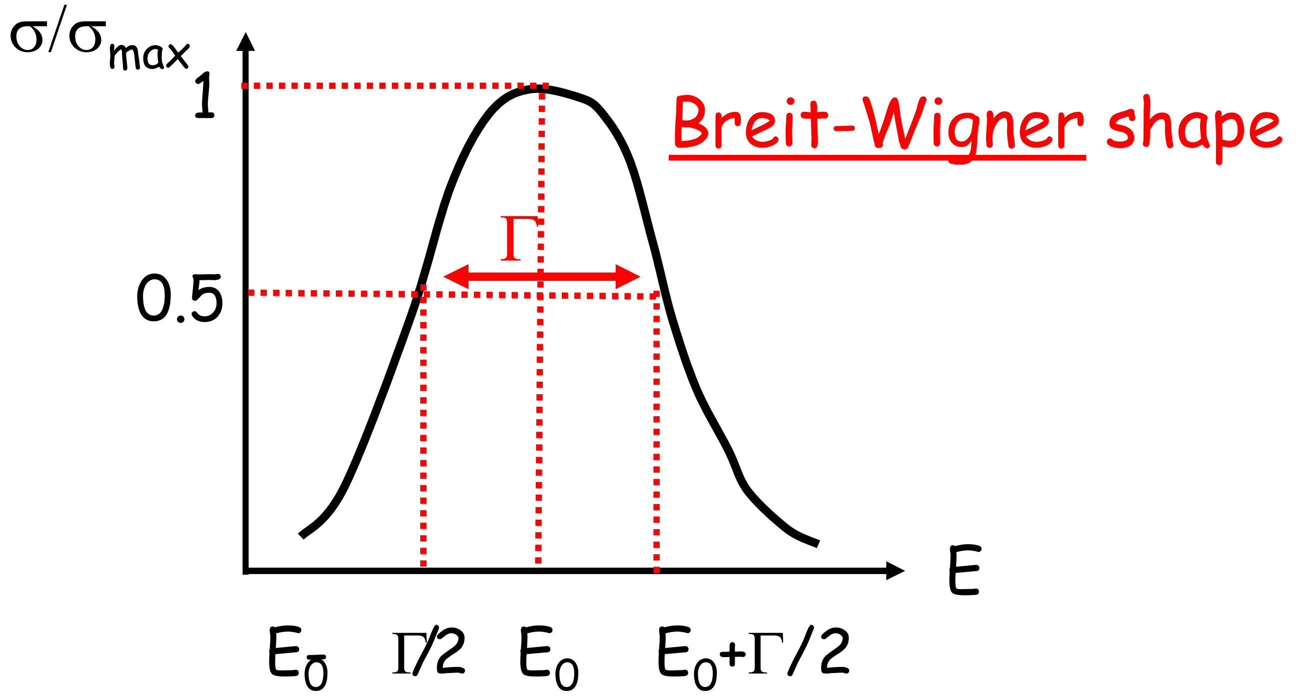
$$= \int_0^\infty \psi(0)e^{-t\left(i(E_0 - E) + \frac{1}{2\tau}\right)}dt$$

$$= \frac{\psi(0)}{-i(E_0 - E) - 1/2\tau}$$

Hence, probability of finding state with energy E=f*f

$$= \frac{|\psi(0)|^2}{(E_0 - E)^2 + 1/4\tau^2}$$

Probability for Z* having energy E



Full width at half maximum = Γ

$$\frac{1}{\left(E_0 - E_0 + \Gamma/2\right)^2 + \frac{1}{4\tau^2}} = \frac{1}{2} 4\tau^2$$

$$\frac{1}{\frac{\Gamma^2}{4} + \frac{1}{4\tau^2}} = 2\tau^2$$

$$\boxed{\tau = \frac{1}{\Gamma}}$$

($\Delta E \Delta t \sim 1 \quad \hbar = 1$)

Finite Lifetime ⇔ Uncertainty in Energy

Cross-section for forming Z* will resonate when $E = E_0$.