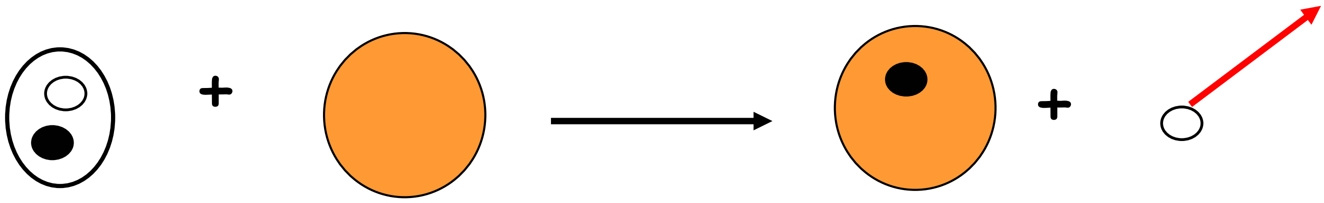
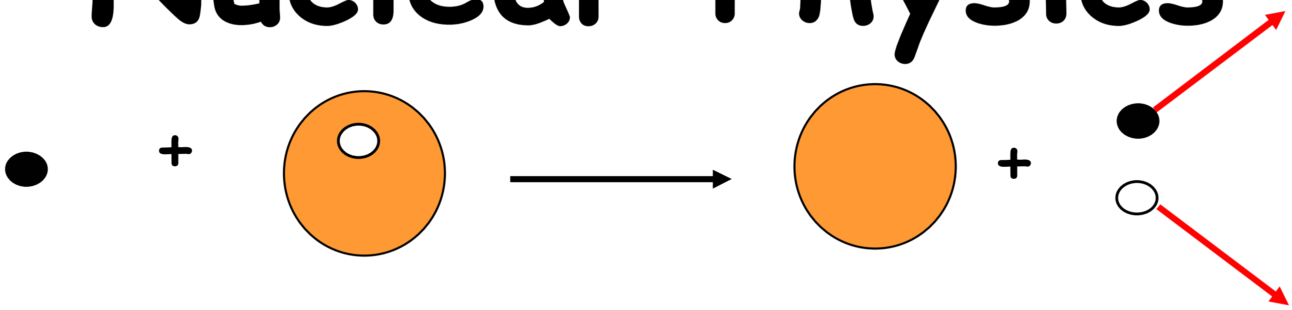
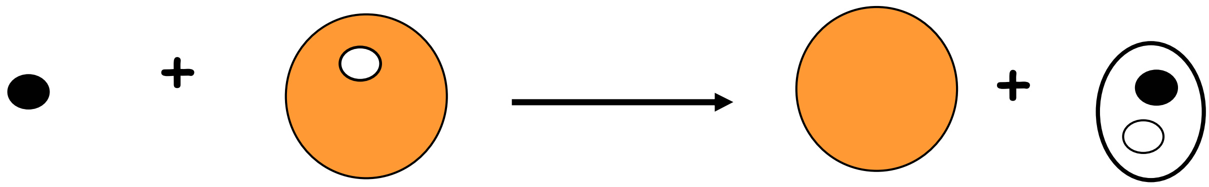


Part II
Nuclear Physics

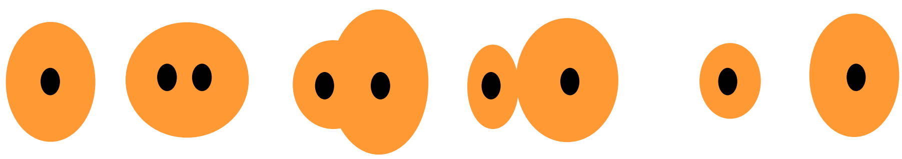


Handout 5



V.Gibson

Lent Term 2004



Section VI
Nuclear Reactions

Kinematics and Conservation Laws

$$p \quad + \quad T \quad \rightarrow \quad R \quad + \quad e$$

Projectile   Target   Recoil   Ejectile

Reaction often written T(p,e)R or (p,e)

Example

$$\alpha + {}^{14}N \rightarrow {}^{17}O + p$$

$${}^{14}N(\alpha,p){}^{17}O \quad \text{or} \quad (\alpha,p)$$

Conserved quantities

- Charge
- Energy
- Linear momentum
- Total angular momentum (linear and spin)
- Parity
- Total number of nucleons

Consider together because orbital a.m. contributes to parity $P=(-1)^{\ell}$ (applies to Total angular momentum and Parity)

In general,

$$1 + 2 \rightarrow 3 + 4$$

$$J_1^{P1} \quad J_2^{P2} \qquad J_3^{P3} \quad J_4^{P4}$$

$$\underbrace{J_1^{P1} \quad J_2^{P2}}_{L} \qquad \underbrace{J_3^{P3} \quad J_4^{P4}}_{\ell}$$

Angular momentum $\quad J_1 \oplus J_2 \oplus L = J_3 \oplus J_4 \oplus \ell$

Parity $\quad P_1 P_2 (-1)^L = P_3 P_4 (-1)^\ell$

<u>Examples</u>

$$\alpha + {}^{16}O \rightarrow {}^{20}Ne^* \rightarrow {}^{20}Ne + \gamma$$

$$0^+ \quad 0^+ \qquad\qquad 0^+ \quad 1^-$$

The only states of $^{20}Ne^*$ which can be reached are $J^P = 0^+, 1^-, 2^+, \ldots$etc.

$$p + {}^7Li \rightarrow \alpha + \alpha$$

$$\underbrace{1/2^+ \quad 3/2^-}_{L} \qquad \underbrace{0^+ \quad 0^+}_{\ell}$$

The α particles are identical bosons

$\therefore$ wavefunction must be symmetric $\rightarrow \ell$ even

$$P(r.h.s) = +1$$

$\rightarrow$ L must be odd to conserve parity.

Q Value (Energy Release)

$$Q = m_p + m_T - m_R - m_e \qquad c=1$$

Total energy conserved

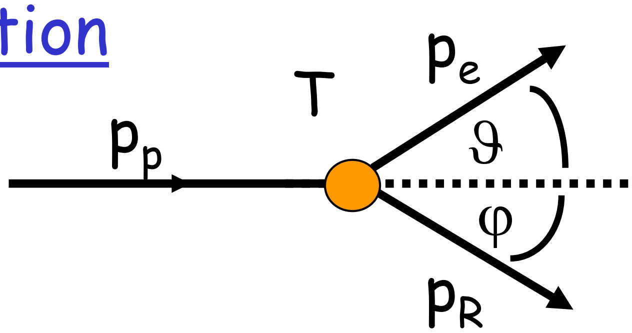
$$Q = T_R + T_e - T_T - T_p \qquad T=\text{kinetic energy}$$

Normally in lab $T_T = 0$

$Q > 0$ Exothermic
$Q < 0$ Endothermic
$Q = 0$ Elastic scattering $a+A \rightarrow a+A$

- For decays $X \rightarrow a + b + \ldots$, $Q > 0$ to be energetically possible
- For endothermic processes to occur, projectile must bring in sufficient energy

Q Equation



Usually measure p_p, p_e, ϑ

Conservation of momentum

→ $$p_p = p_e \cos\vartheta + p_R \cos\varphi$$

↑ $$0 = p_e \sin\vartheta - p_R \sin\varphi$$

Eiminate φ

$$p_R \cos\varphi = p_p - p_e \cos\vartheta$$
$$p_R \sin\varphi = p_e \sin\vartheta$$
$$p_R^2 = p_p^2 + p_e^2 - 2p_p p_e \cos\vartheta$$

$$T = \frac{p^2}{2m} \quad \text{non-rel}$$

$$\begin{aligned} Q &= T_R + T_e - T_p \\ &= \frac{1}{2m_R}\left(p_p^2 + p_e^2 - 2p_p p_e \cos\vartheta\right) + T_e - T_p \\ &= T_e\left(1 + \frac{m_e}{m_R}\right) - T_p\left(1 - \frac{m_p}{m_R}\right) \\ &\quad - \frac{1}{m_R}\left(2m_p T_p\right)^{1/2}\left(2m_e T_e\right)^{1/2}\cos\vartheta \end{aligned}$$

$$Q = T_e\left(1 + \frac{m_e}{m_R}\right) - T_p\left(1 - \frac{m_p}{m_R}\right) - \frac{2\sqrt{m_e T_e m_p T_p}}{m_R}\cos\vartheta$$

The "Q-Equation"

Generally sufficient to replace m by A (atomic mass number).

If Q < 0, then minimum T_p required for process to proceed

$$v'_T = \frac{m_p}{m_T} v'_p$$

Threshold

$$T'_p = \frac{1}{2} m_p v'^2_p + \frac{1}{2} m_T v'^2_T = \frac{1}{2} m_p v'^2_p \left(1 + \frac{m_p}{m_T} \right) = |Q|$$

Transform back to lab

$$v_p = v'_p + v'_T$$

$$= v'_p \left(1 + {}^{m_p}\!/\!_{m_T} \right)$$

Miniumum $$T_p = \frac{1}{2} m_p v_p^2 = \frac{1}{2} m_p v'^2_p \left(1 + {}^{m_p}\!/\!_{m_T} \right)^2$$

$$= |Q| \left(1 + {}^{m_p}\!/\!_{m_T} \right)$$

Principle of Detailed Balance

$$\underset{a}{1 + 2} \underset{\sigma_b}{\overset{\sigma_a}{\rightleftharpoons}} \underset{b}{3 + 4}$$

The cross-sections σ_a and σ_b are related through the Principle of Detailed Balance (based on time reversal invariance $t \rightarrow -t$)

$$\sigma = \frac{\text{Total number of interactions/ sec}}{\text{Flux}} = \frac{\Gamma}{\text{Flux}}$$

$$\Gamma = \frac{2\pi}{\hbar} |M|^2 \rho(E_f)$$

Fermi Golden Rule

- If the hamiltonian, $\hat{H}$, is hermitian

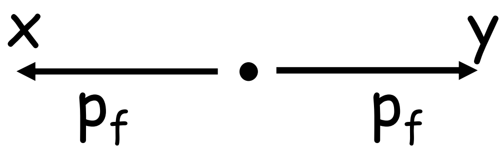
$$M_{if} = \langle f|\hat{H}|i\rangle = \langle i|\hat{H}|f\rangle^*$$

$$|M_{a\rightarrow b}|^2 = |M_{b\rightarrow a}|^2 = |M|^2$$

Detailed Balance

- Density of states

$$\rho(E_f) = \frac{4\pi}{h^3} p_f^2 \frac{dp_f}{dE_f} g_f$$



final state in cms

$$E_f = \frac{p_f^2}{2m_x} + \frac{p_f^2}{2m_y}$$

$$\frac{dE_f}{dp_f} = \frac{p_f}{m_x} + \frac{p_f}{m_y} = v_f$$

relative velocity of final particles

$$g_f = \text{spin degeneracy factor}$$
$$= (2J_x + 1)(2J_y + 1)$$

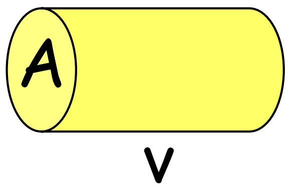
$g_f = 1$ for spinless particles

$\times \frac{1}{2}$ identical particles

$(2J+1) \rightarrow 2$ γ transverse oscillations only

- Flux = number incident particles /unit area/sec

Number through area A/sec = Av
1 particle/unit volume (considering only 1 incident particle)



Flux = v (relative velocity in initial state)

$$\therefore \frac{\sigma_a}{\sigma_b} = \frac{p_b^2 g_b}{v_a v_b} \frac{v_a v_b}{p_a^2 g_a}$$

$$\frac{\sigma_a}{\sigma_b} = \frac{p_b^2 g_b}{p_a^2 g_a}$$

Types of Reaction

- Elastic Scattering occurs when the projectile and target nucleus scatter with no loss of total Kinetic Energy.

$$a + X \rightarrow X + a \qquad Q = 0$$

e.g. Rutherford scattering

- Inelastic Scattering occurs when the target nucleus is left in an excited state.

$$a + X \rightarrow X^* + a$$

In general, the particles emerging from the collision will not be the same as the initial ones.

$$a + X \rightarrow b + Y + \ldots$$

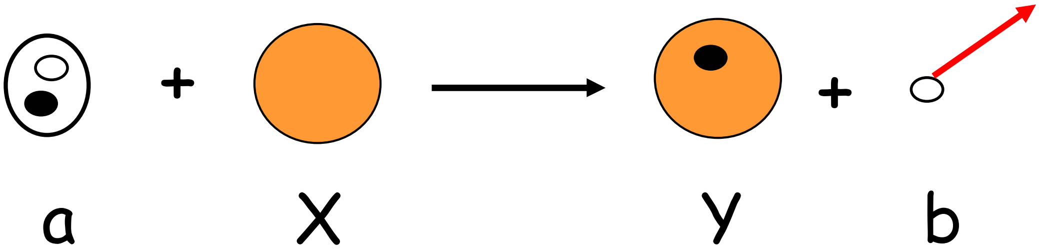
- Direct Reactions occur when the projectile interacts primarily with one or more nucleons in the surface of the target nucleus. Energy and mass transfer are small.

Occurs for high energies- able to resolve individual nucleons.

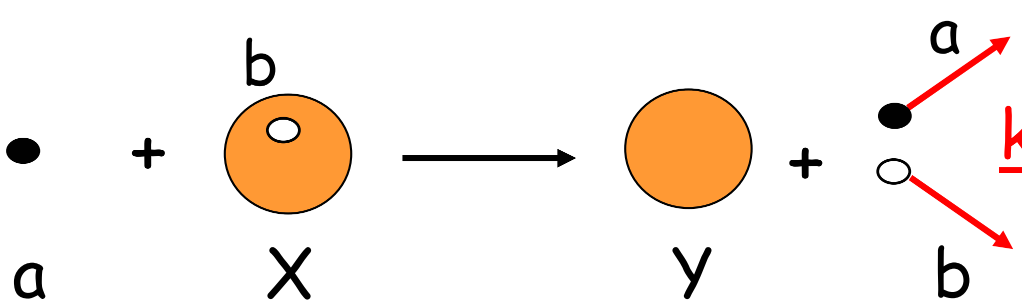
$$\lambda \sim 1\text{fm} \rightarrow E = 20 \text{ MeV}$$

Time for interaction ~time for projectile to travel nuclear diameter (10^{-22} s).

Examples include transfer reactions when one or more nucleons are exchanged as the projectile and target pass each other. The final nuclei may be left in excited states.

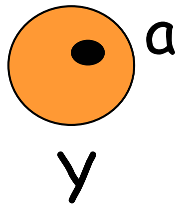


stripping

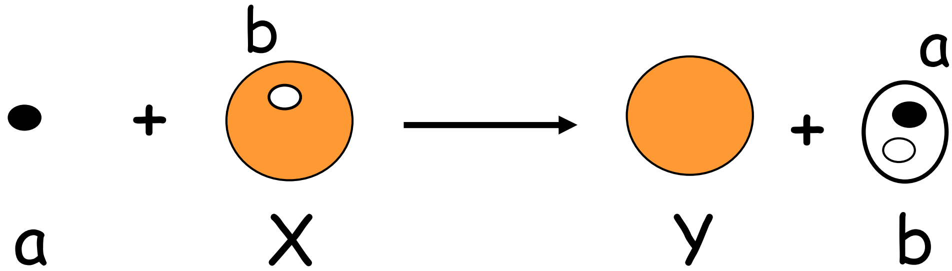


knock-out

$$a + X \rightarrow Y + a + b$$
$$\rightarrow Y' + b$$



a looses energy and is captured



pick-up

Direct reactions tend to be forward peaked (i.e. projectile continues in approx incident direction)

Compound Nuclei

A common reaction is one in which the projectile is absorbed by the nucleus forming an intermediate state (compound-nucleus) which then decays

$$a + X \rightarrow Z^* \rightarrow Y + b$$

Compound nucleus reactions are symmetric in c.m.s. As the projectile energy increases, capture becomes less likely and interactions become more direct in character.

Two stages (Bohr Model)

(1) Formation $a + X \rightarrow Z^*$

When the collision energy ≈ one of the energy levels of the Z nucleus → resonance

(2) Decay $Z^* \rightarrow Y + b$

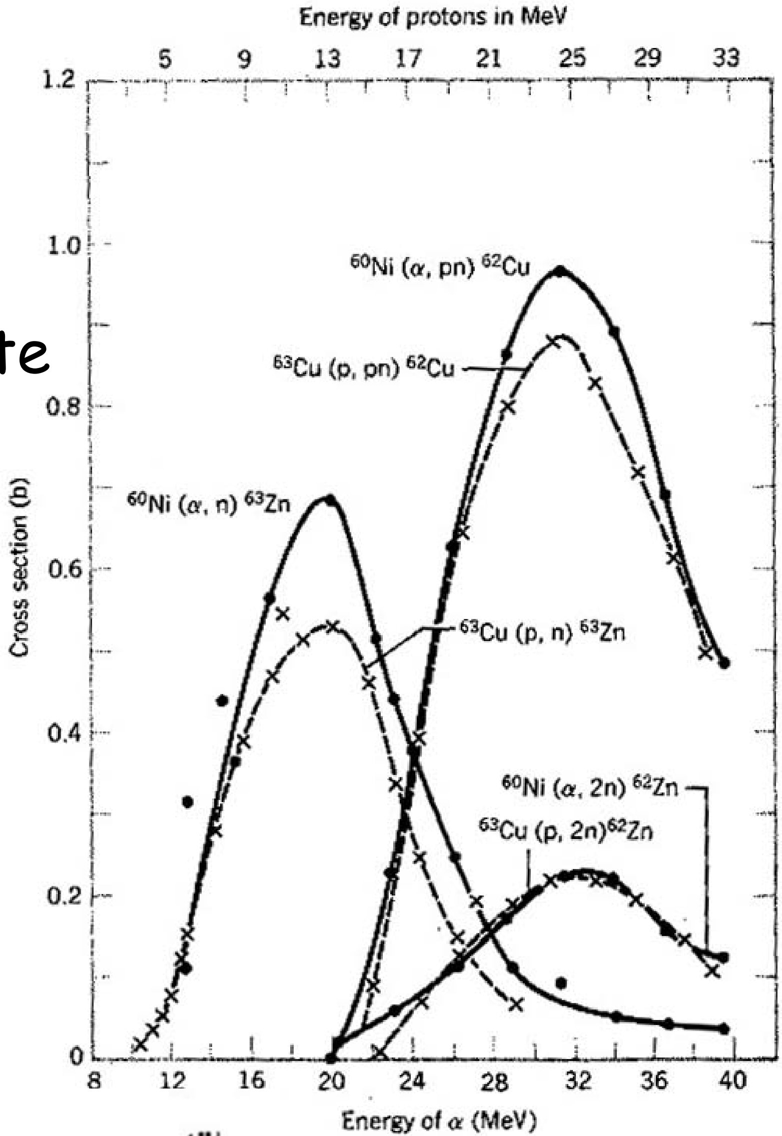
The decay of Z* is independent of the mode of formation and depends only on the properties of the Z*.

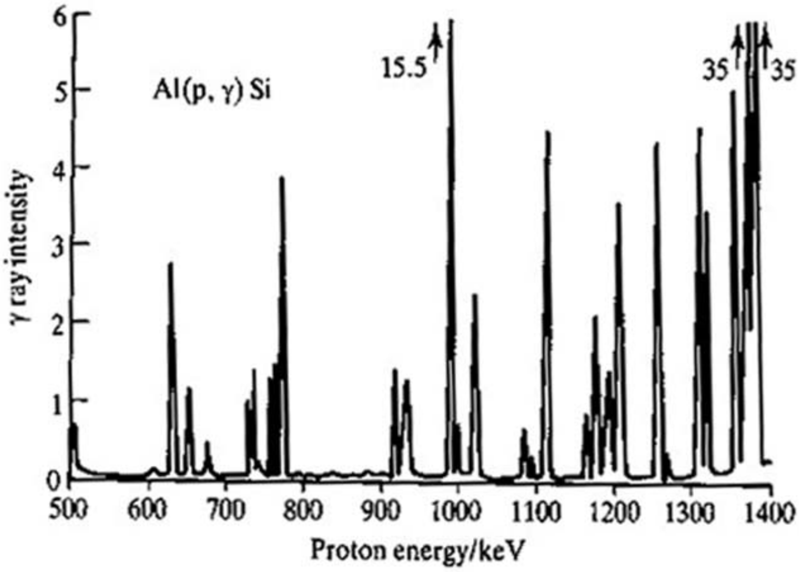
e.g. $p + {}^{63}_{29}Cu \rightarrow {}^{64}_{30}Zn^* \rightarrow {}^{63}Zn + n$

$\alpha + {}^{60}_{28}Ni \rightarrow {}^{64}_{30}Zn^*$; $\quad {}^{64}_{30}Zn^* \rightarrow {}^{62}Cu + n + p$; $\quad {}^{64}_{30}Zn^* \rightarrow {}^{62}Zn + 2n$

$\sigma\ {}^{60}Ni(\alpha,n)Zn \approx \sigma\ {}^{63}Cu(p,n)Zn$

Energy of p selected to give same Zn* state as for α interaction.





${}^{27}Al(p,\gamma)\ {}^{28}Si$

Shape of Resonance

Wavefunction for decaying state Z* with mean energy E_0 and lifetime τ

$$\psi(t) = \psi(0)e^{-i\omega_0 t}e^{-t/2\tau} \qquad E = \hbar\omega \quad \hbar = 1$$

$$|\psi(t)|^2 = |\psi(0)|^2 e^{-t/\tau}$$ ← exponential decay

The frequencies present in the wavefunction are given by the Fourier transform of $\psi(t)$

$$f(\omega) = \int_0^\infty \psi(t)e^{i\omega t}dt$$

and so the energies are

$$f(E) = \int_0^\infty \psi(0)e^{-t\left(iE_0 + \frac{1}{2\tau}\right)}e^{iEt}dt$$

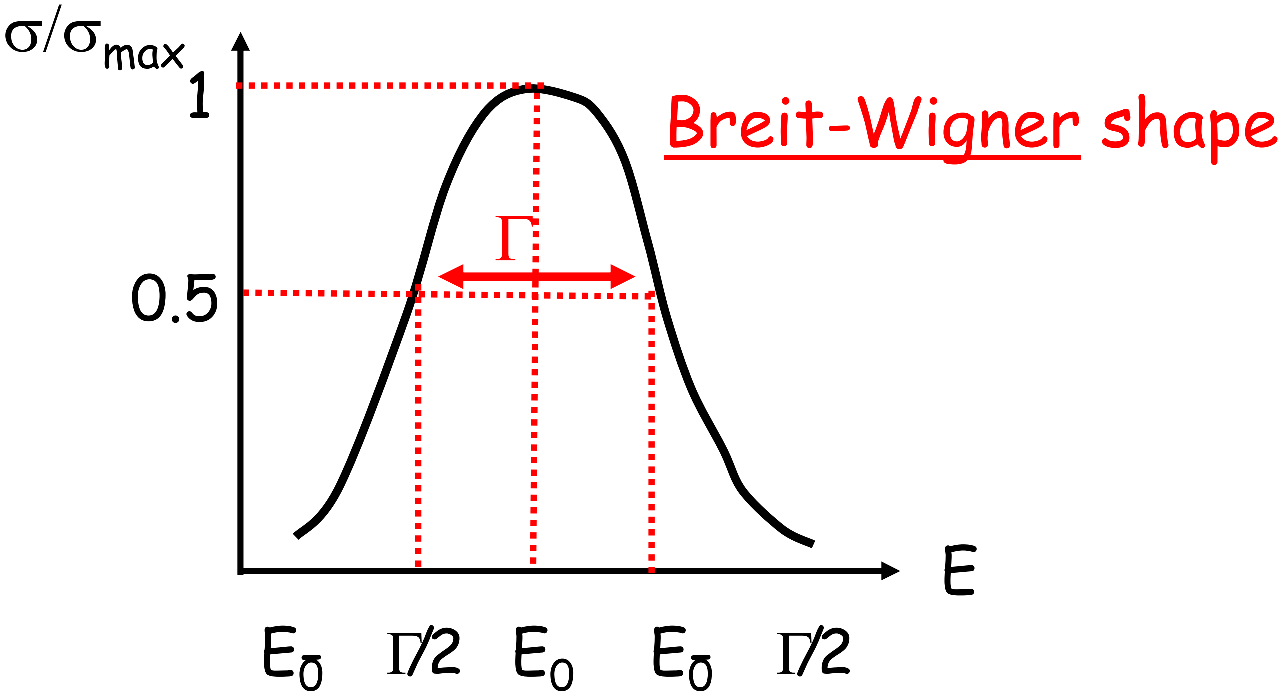
$$= \int_0^\infty \psi(0)e^{-t\left(i(E_0 - E) + \frac{1}{2\tau}\right)}dt$$

$$= \frac{\psi(0)}{(E_0 - E) - i/2\tau}$$

Hence, probability of finding state with energy E=f*f

$$= \frac{|\psi(0)|^2}{(E_0 - E)^2 + 1/4\tau^2}$$

Probability for Z* having energy E



Full width at half maximum = Γ

$$\frac{1}{\left(E_0 - E_0 + \Gamma/2\right)^2 + \frac{1}{4\tau^2}} = \frac{1}{2} 4\tau^2$$

$$\frac{1}{\frac{\Gamma^2}{4} + \frac{1}{4\tau^2}} = 2\tau^2$$

$$\tau = \frac{1}{\Gamma}$$

($\Delta E \Delta t \sim 1 \quad \hbar = 1$)

Finite Lifetime ⇔ Uncertainty in Energy

Cross-section for forming Z* will resonate when $E = E_0$.

Breit-Wigner Cross-section

For the compound nucleus reaction

$$i \to Z \to f$$

Transition rate $\Gamma_{if} = \frac{2\pi}{\hbar} |M_{if}|^2 \rho(E_f)$

$|M_{if}|$ is given by 2nd order perturbation theory

$$M_{if} = \sum_Z \frac{M_{iZ} M_{Zf}}{E - E_Z}$$

Σ over intermediate states

Considering 1 intermediate state described by

$$\psi(t) = \psi(0) e^{-\frac{i}{\hbar}\left(E_0 - i\frac{\Gamma}{2}\right)t}$$

This describes a state with energy = $E_0 - i\Gamma/2$

$$\therefore |M_{if}|^2 = \frac{|M_{iZ}|^2 |M_{Zf}|^2}{(E - E_0)^2 + \Gamma^2/4}$$

$E_Z = E_0 - i\Gamma/2$

Total cross-section

$$\sigma_{if} = \frac{\Gamma_{if}}{flux} = \frac{2\pi}{\hbar} |M_{if}|^2 \left(\frac{4\pi}{h^3} \frac{p_f^2}{v_f}\right) \frac{1}{v_i}$$

see detailed balance

flux

- Decay $Z \rightarrow f$

Define rate for this decay to be $\Gamma_f / \hbar$

Γ_f is called the partial width.

Total decay rate $\Gamma = \sum_i \Gamma_i$ — sum over all decay modes

Full width (Γ), partial widths (Γ_i)

Probability of decay to one particular state $= \Gamma_i / \Gamma$ = Branching ratio

Rate of decay of Z,

$$\Gamma_f = 2\pi \left|M_{Zf}\right|^2 \frac{4\pi}{h^3} \frac{p_f^2}{v_f} \qquad \Gamma(Z \rightarrow f)$$

Rate of formation of Z,

$$\Gamma_i = 2\pi \left|M_{iZ}\right|^2 \frac{4\pi}{h^3} \frac{p_i^2}{v_i} \qquad \Gamma(Z \rightarrow i)$$

$$\therefore \sigma_{if} = \pi\hbar^2 \frac{\Gamma_i \Gamma_f}{p_i^2} \frac{1}{(E - E_0)^2 + \Gamma^2/4}$$

$p_i = \frac{\hbar}{\lambda\!\!\!^{-}}$ $\quad p_i$ = c.m. momentum ~ lab momentum if target heavy

$$\therefore \sigma_{if} = \frac{g\pi\lambda\!\!\!^{-\,2}\Gamma_i \Gamma_f}{(E - E_0)^2 + \Gamma^2/4}$$

Breit-Wigner Cross-section

The factor g takes into account the spin

$$g = \frac{(2J_Z + 1)}{(2J_X + 1)(2J_a + 1)}$$

$a + X \rightarrow Z^* \rightarrow Y + b$

= ratio of number of spin states (m_J values) for the compound nucleus to the total number of spin states for the a+X system.

Notes

- Total scattering cross-section $\sigma_T = \sum_f \sigma(i \rightarrow f)$
  Replace Γ_f by Γ in Breit-Wigner formula
- Elastic scattering $\sigma_{el} = \sigma(i \rightarrow i)$ so $\Gamma_f = \Gamma_i$
- On peak of resonance ($E = E_0$)

$$\sigma_{if} = 4\pi\lambda\!\!\!^{-2} g \frac{\Gamma_i \Gamma_f}{\Gamma^2}$$

Thus,

$$\left.\begin{aligned} \sigma_{el} &= 4\pi\lambda\!\!\!^{-2} g B_i^2 \\ \sigma_T &= 4\pi\lambda\!\!\!^{-2} g B_i \end{aligned}\right\} \quad B_i = \Gamma_i / \Gamma = \sigma_{el} / \sigma_T$$

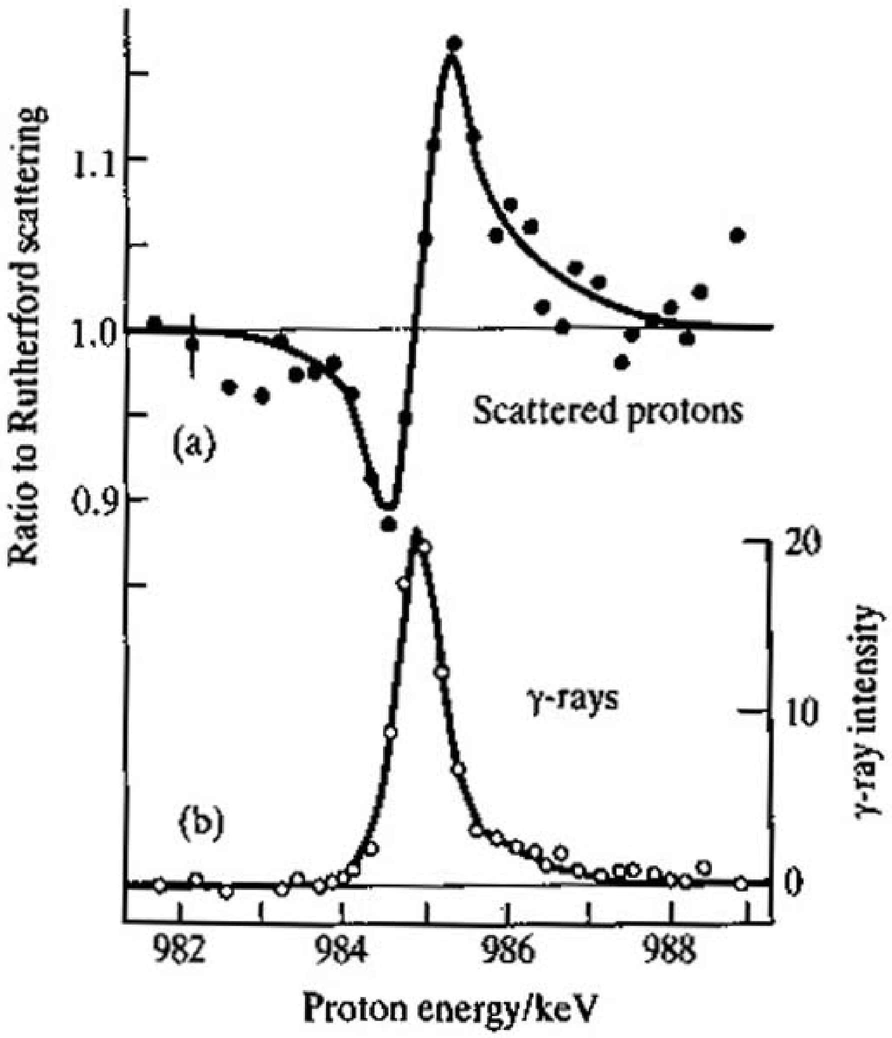
From measurement of σ_T and σ_{el} can infer g and hence spin of compound nucleus.

- Only considered one resonance. May have several overlapping resonances. Partial wave analysis uses angular distributions to split data into different ℓ values.

- Resonance amplitude $\sim \dfrac{1}{i(E_0 - E) + \Gamma/2}$

  $\therefore$ phase changes rapidly as resonance is crossed. Typically non-resonant scattering also occurs
  $\rightarrow$ interference effects change shape of peak.

p + ^{27}Al



Low Energy Neutron Capture

At low energies, neutrons can be absorbed by nuclei (no Coulomb barrier). This is important for the design of thermonuclear reactors (see later).

For a low energy excited state, γ decay is most probable.

$$\sigma(n,\gamma) = \frac{g\pi\lambda\!\!\!^{-2}\Gamma_n\Gamma_\gamma}{(E-E_0)^2 + \Gamma^2/4} \qquad \Gamma_n << \Gamma_\gamma \approx \Gamma$$

- At resonance $\sigma(n,\gamma) = 4\pi\lambda\!\!\!^{-2} g\frac{\Gamma_n\Gamma_\gamma}{\Gamma^2} \approx 4\pi\lambda\!\!\!^{-2} g\frac{\Gamma_n}{\Gamma}$

  Typically, $\Gamma_n \sim 10^{-3}$ eV, $\Gamma \sim 1$ eV, 1eV neutron $\Rightarrow \sigma \sim 10^3$ b (largest ^{135}Xe $\sigma \sim 10^6$ b)

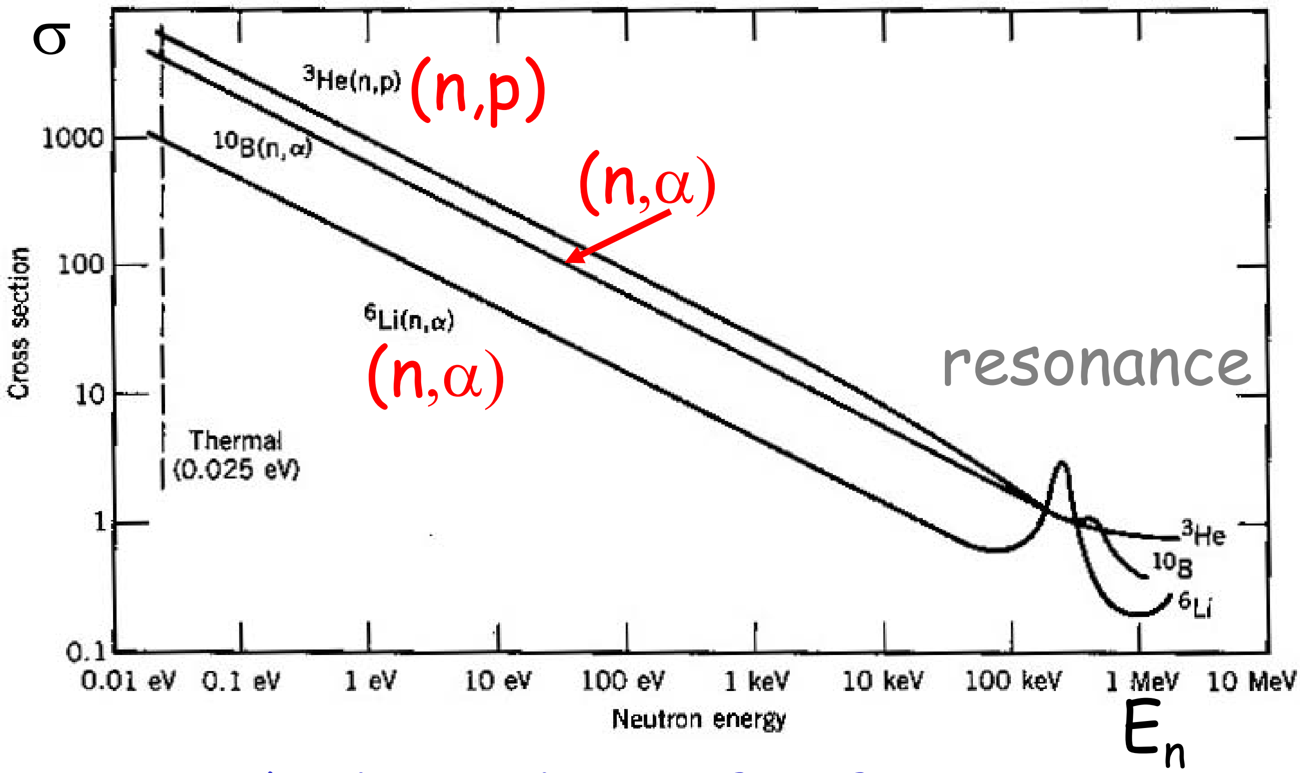
- Below resonance $\sigma(n,\gamma) \approx \lambda\!\!\!^{-2}\Gamma_n\left[\underbrace{\frac{g\pi\Gamma_\gamma}{E_0^2 + \Gamma^2/4}}_{\text{constant}}\right]$

Γ_n dominated by phase space

$$\Gamma_n \sim \frac{p^2}{v} \sim v \qquad \lambda\!\!\!^{-} = \frac{\hbar}{p} \rightarrow \lambda\!\!\!^{-2} \sim \frac{1}{v^2}$$

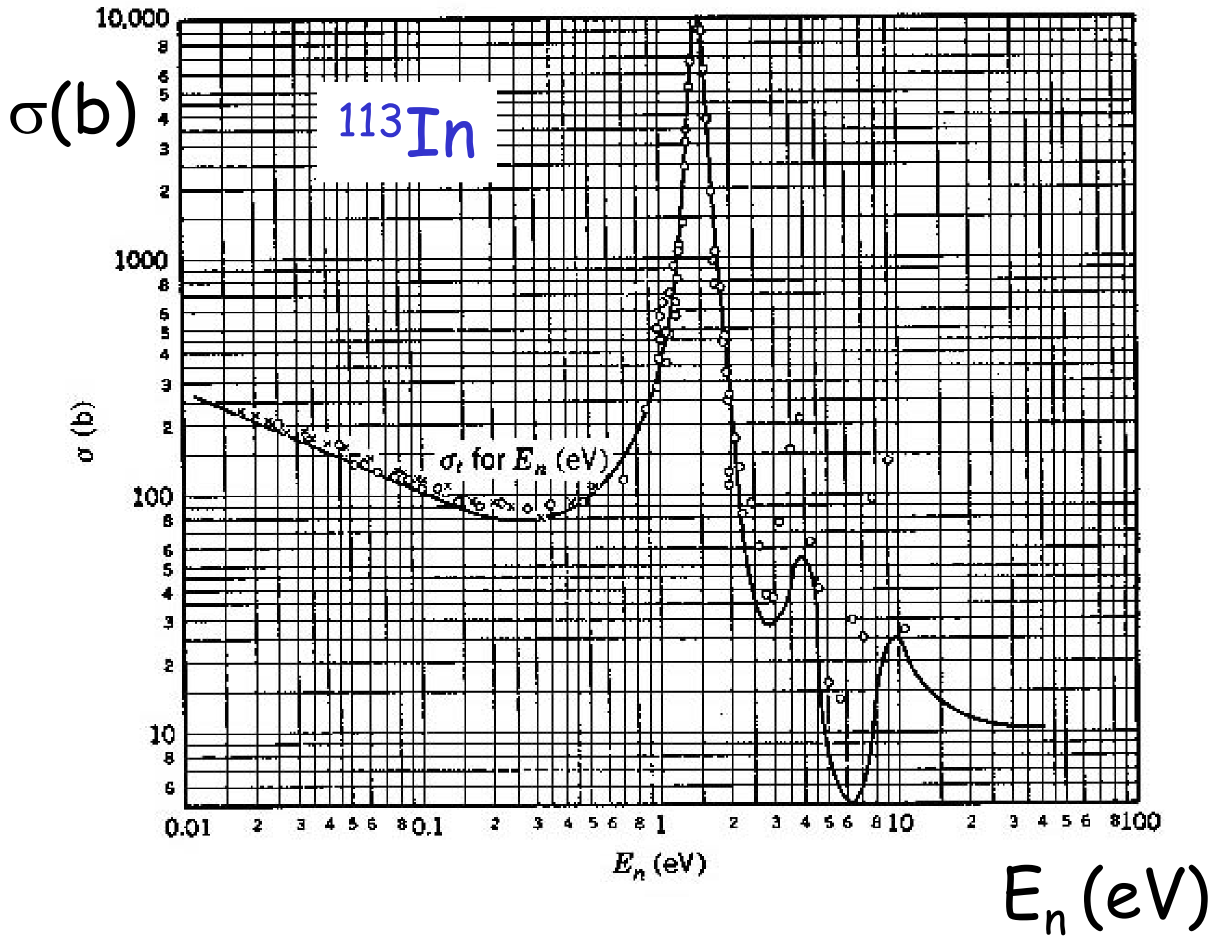
$$\therefore \sigma(n,\gamma) \approx \frac{1}{v}$$

"1/v Law"

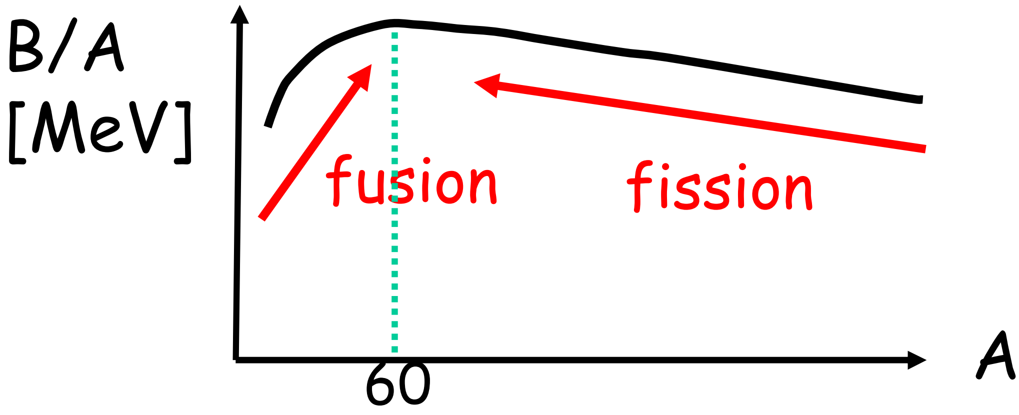


$\sigma \sim 1/v$ dependence far from resonance

$\ln \sigma \sim -\ln E$



Nuclear Fission and Fusion



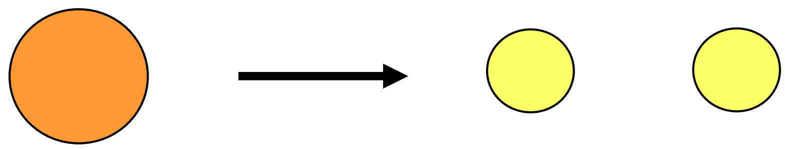
Most stable form of nuclear matter is at $A \sim 60$. Expect a large amount of energy released in the fission of a heavy nucleus into two medium nuclei and in the fusion of two light nuclei into a single medium nucleus.

SEMF $$B(A,Z) = a_V A - a_S A^{2/3} - \frac{a_C Z^2}{A^{1/3}} - a_a \frac{(N-Z)^2}{A} + \delta(A)$$

Fission occurs because the total Coulomb repulsion energy of p's in a nucleus is reduced if the nucleus splits into two smaller nuclei. The nuclear surface energy increases in the process, but its magnitude is much smaller.

Fusion occurs because the two low A nuclei have too large a surface area for their volume. The surface area decreases when they amalgamate. Coulomb energy increases, but its magnitude is too small.

Spontaneous Fission



Expect spontaneous fission to occur if

$$Q = B(A_1,Z_1) + B(A_2,Z_2) - B(A,Z) > 0$$

Assume $\frac{A_1}{A} = \frac{Z_1}{Z} = y_1$ and $\frac{A_2}{A} = \frac{Z_2}{Z} = y_2$; $y_1 + y_2 = 1$

From SEMF

$$Q = a_s A^{2/3}(1 - y_1^{2/3} - y_2^{2/3}) + a_c \frac{Z^2}{A^{1/3}}(1 - y_1^{5/3} - y_2^{5/3})$$

Maximum energy release when

$$\frac{\partial Q}{\partial y_1} = 0 \quad (dy_2 = -dy_1)$$

$$\frac{\partial Q}{\partial y_1} = a_s A^{2/3}(-\frac{2}{3}y_1^{-1/3} + \frac{2}{3}y_2^{-1/3}) + a_c \frac{Z^2}{A^{1/3}}(-\frac{5}{3}y_1^{5/3} - y_2^{5/3})$$

$= 0$ when $y_1 = y_2 = 1/2$ symmetric fission

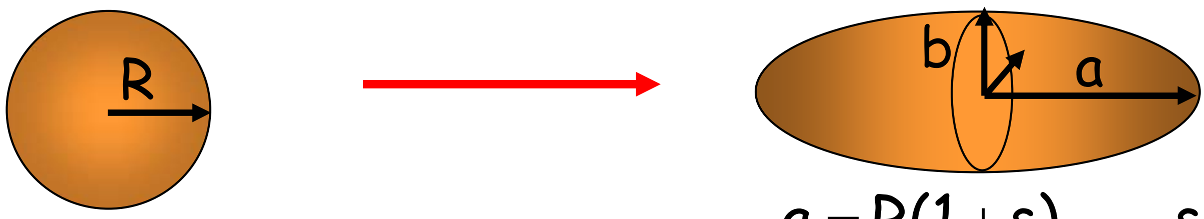
Maximum $Q = 0.37\, a_c \frac{Z^2}{A^{1/3}} - 0.26\, a_s A^{2/3}$

$a_s = 16.8$ MeV
$a_c = 0.72$ MeV

e.g. ${}^{238}_{92}U$ Maximum $Q \approx 200$ MeV

~10^6 > energy released in chemical reaction.

Estimate mass at which nuclei become unstable to fission (i.e. point at which energy change due to deformation gives a change in B.E, $\Delta B > 0$)



$$a = R(1+\varepsilon) \qquad \varepsilon << 1$$
$$b = R(1+\varepsilon)^{-1/2}$$

$$\text{Volume} = \frac{4}{3}\pi a b^2 = \frac{4}{3}\pi R^3 = \text{CONSTANT}$$

SEMF volume term unchanged

Change in surface term

$$a_s A^{2/3} \longrightarrow a_s A^{2/3}\left(1+\frac{2}{5}\varepsilon^2\right)$$

See Segrè Ch 11 11-11

Change in Coulomb term

$$a_c \frac{Z^2}{A^{1/3}} \longrightarrow a_c \frac{Z^2}{A^{1/3}}\left(1-\frac{\varepsilon^2}{5}\right)$$

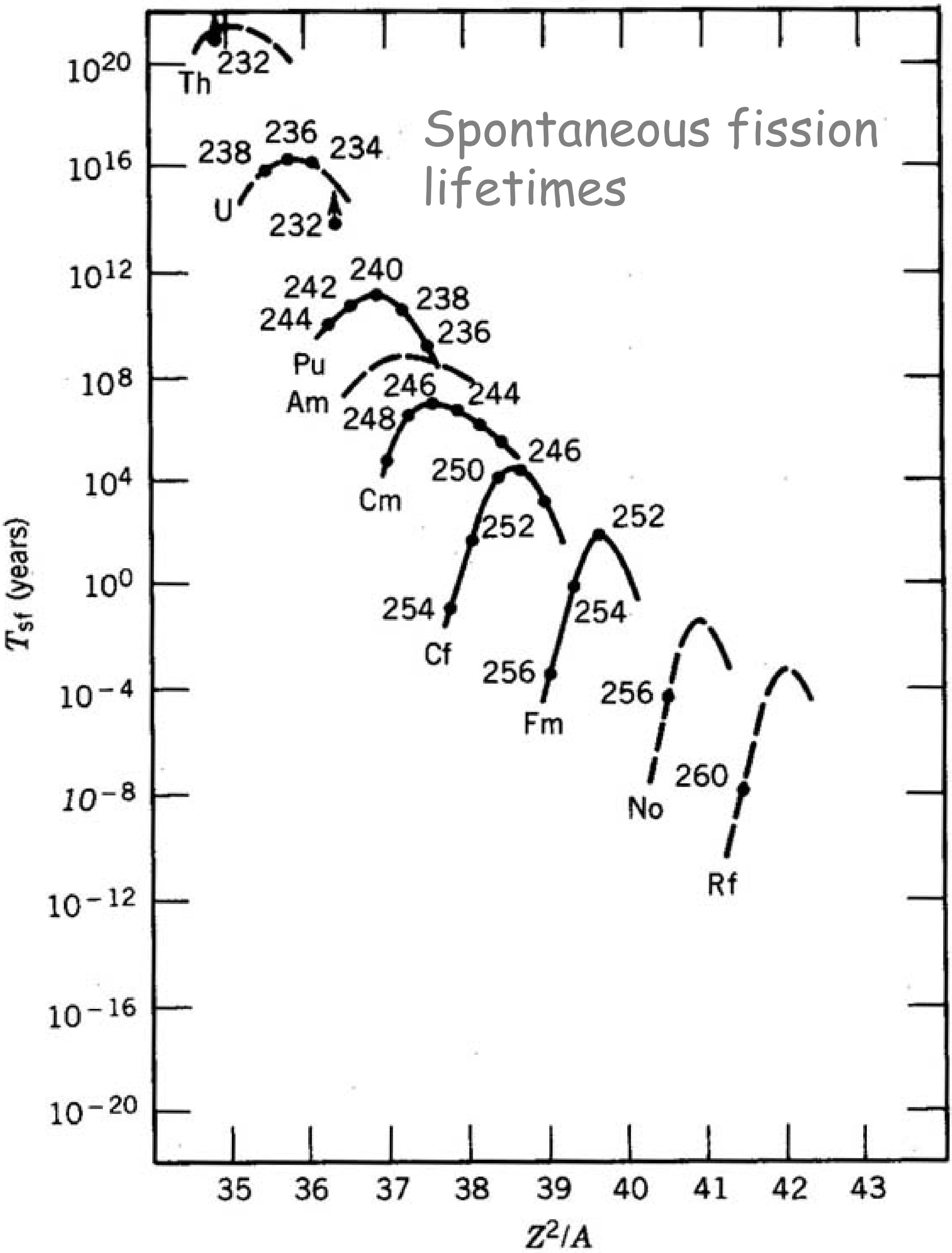
Change in Binding energy

$$\Delta B = B(\varepsilon) - B(0) = a_c A^{2/3}\left(\frac{Z^2}{A} - \frac{2a_s}{a_c}\right)\frac{\varepsilon^2}{5}$$

i.e. if $\frac{Z^2}{A} > \frac{2a_s}{a_c}$ $\Delta B > 0$ and nucleus unstable under deformation

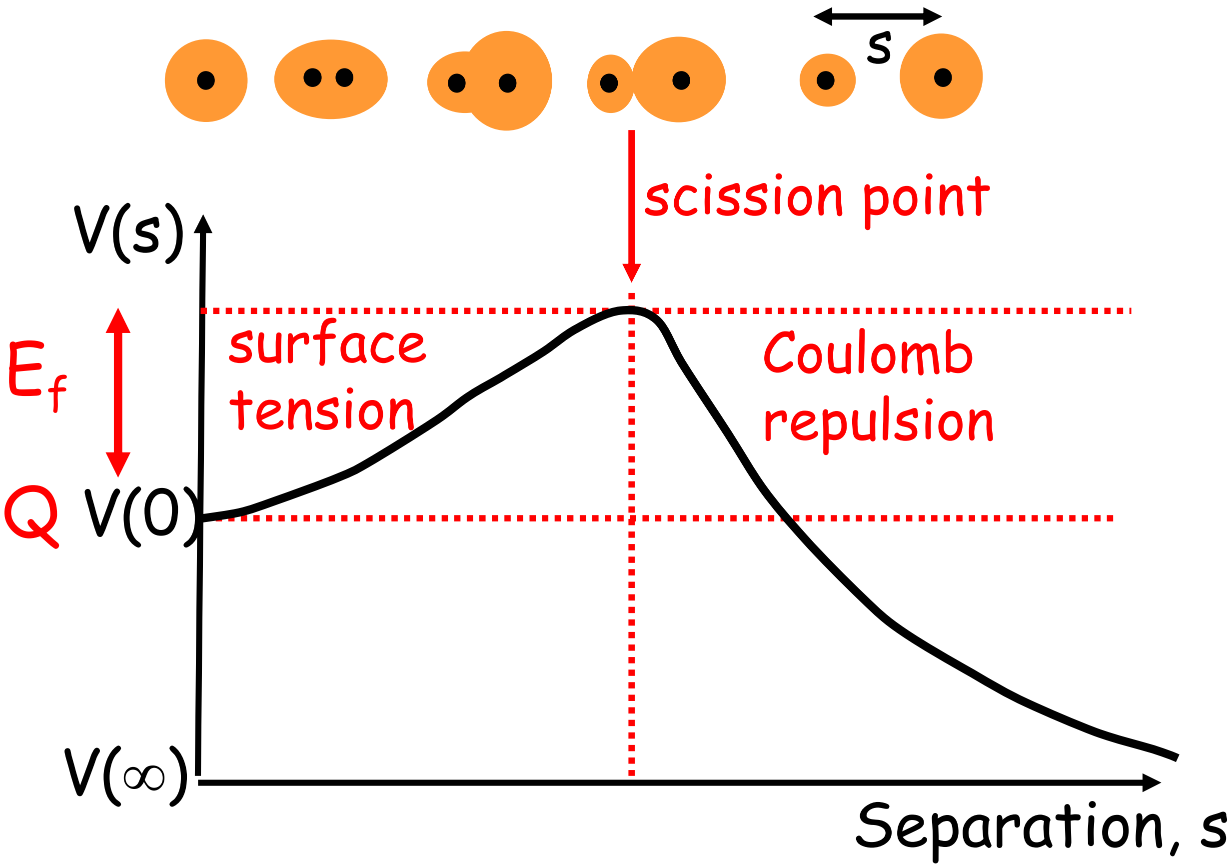
$$\longrightarrow \frac{Z^2}{A} > 50$$

Spontaneous fission lifetimes fall rapidly as Z^2/A approaches 50.



Fission Barrier

In the fission process, nuclei have to pass through an intermediate state where the surface energy is increased, but the Coulomb energy is not yet much reduced.



Q = energy released → K.E. of fragments.

E_f = fission activation energy

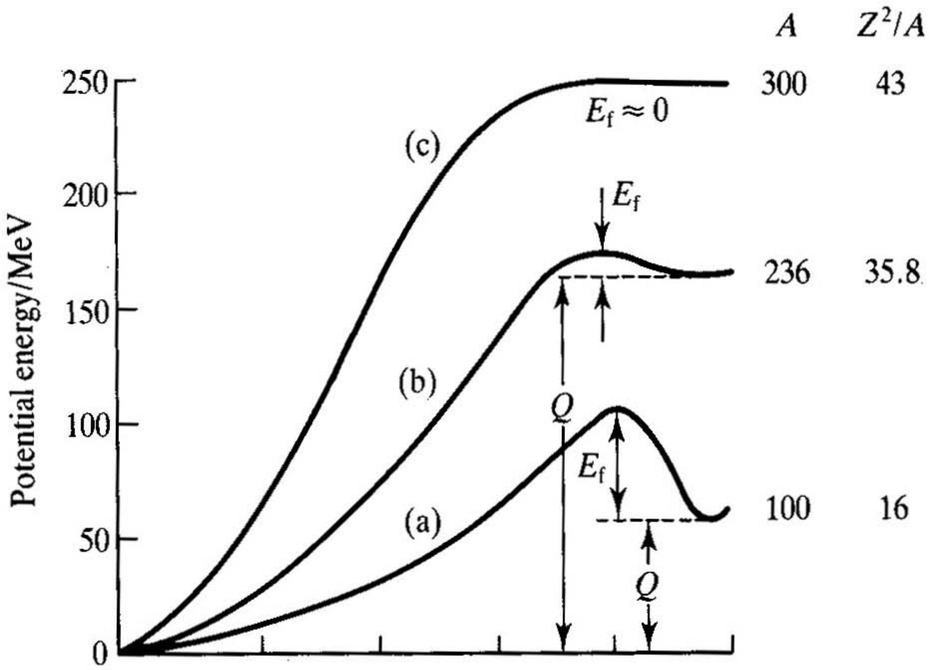
$$E_f = a_c A^{2/3}\left(\frac{Z^2}{A} - \frac{2a_s}{a_c}\right)\frac{\varepsilon^2}{5} \approx 6\ \text{MeV} \qquad {}^{236}_{92}U$$

Spontaneous fission is possible if tunnelling occurs (c.f. α decay).

Tunnelling probability depends on

- Z^2/A.

$$E_f \sim \frac{Z^2}{A}$$



- Mass of fragment.

$$P = e^{-2G} \quad G \sim m^{1/2}$$

Large mass $\rightarrow$ low probability for tunnelling e.g. 10^6 less probable than α decay for $^{238}_{92}U$

Induced Fission

Induced fission of nuclei occurs when a nucleus captures a low energy neutron receiving enough energy to climb the fission barrier.

e.g. $n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U \rightarrow X^* + Y^* \rightarrow X + Y + \kappa n$

$\kappa \sim 2.4$

prompt neutrons

If excitation energy > fission activation energy, fission will occur for zero energy neutrons

$\rightarrow$ thermal neutrons.

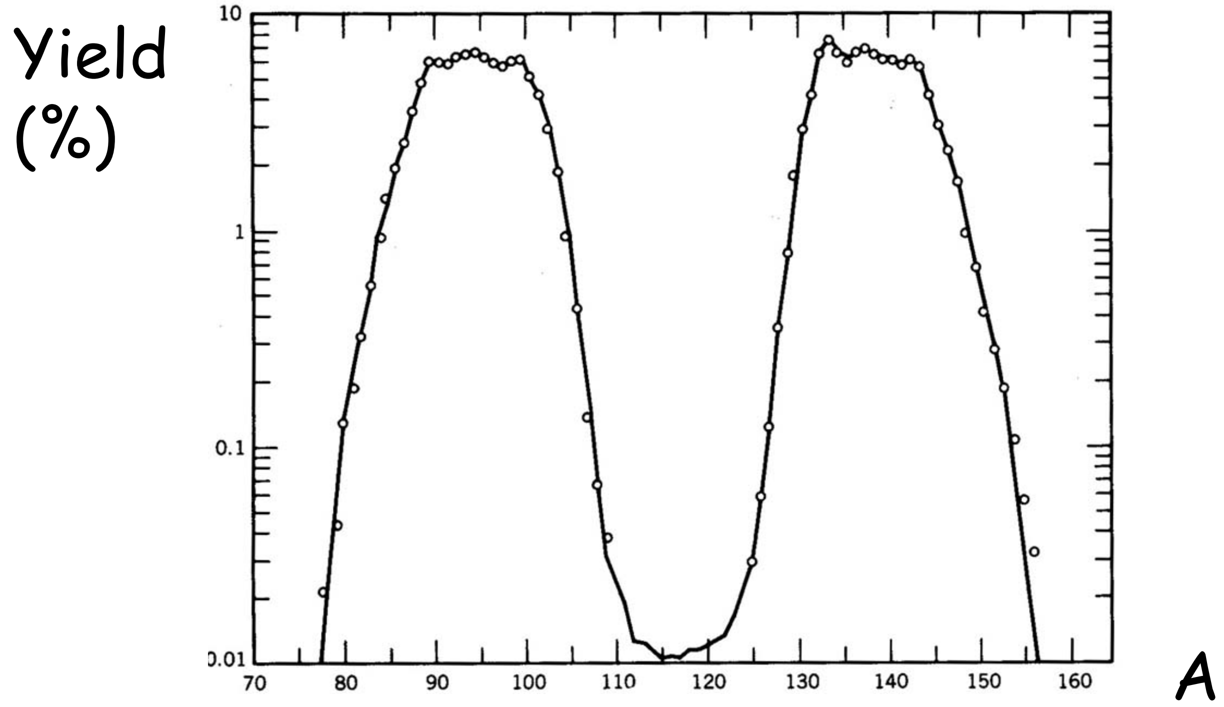
(Available energy from separation energy of n).

Otherwise need to supply energy using K.E. of n.

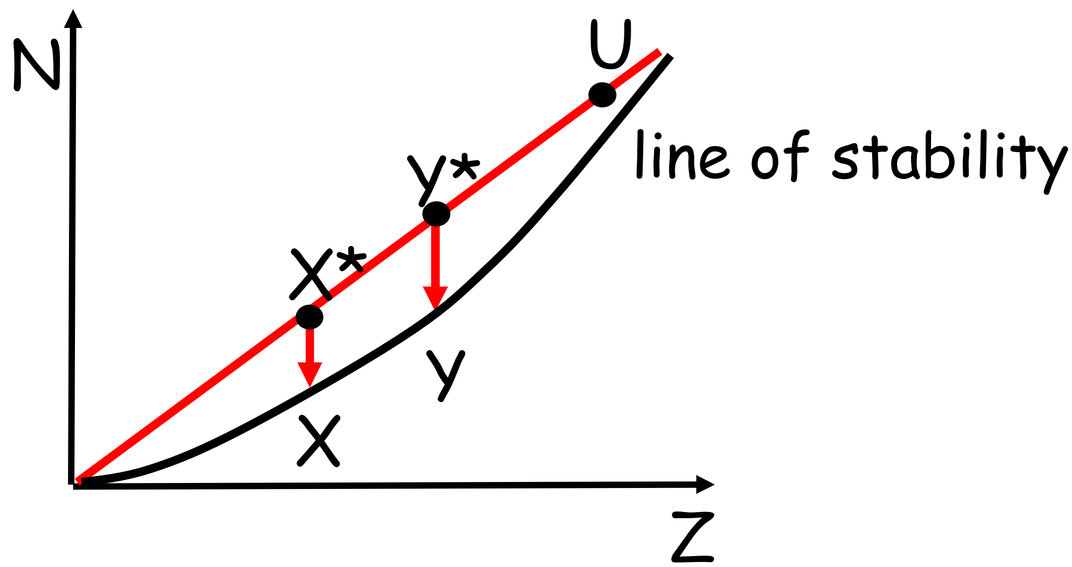
e.g. $n + {}^{238}U \rightarrow {}^{239}U^*$ $\qquad E_f \sim 6$ MeV

$E_n = 0$	$E \sim 5$ MeV	no thermal fission
$E_n = 1.4$ MeV	$E \sim 6.4$ MeV	fission

- Masses of fragments are unequal, in general. Tend to have Z, N near magic numbers.



- Fragments tend to have same Z/N ratio as parent $\rightarrow$ neutron rich nuclei which emit <u>prompt neutrons</u> (10^{-16}s)



X,Y β decay more slowly $\rightarrow$ <u>delayed n emission</u> (~1 delayed n/100 fissions)

Chain Reaction

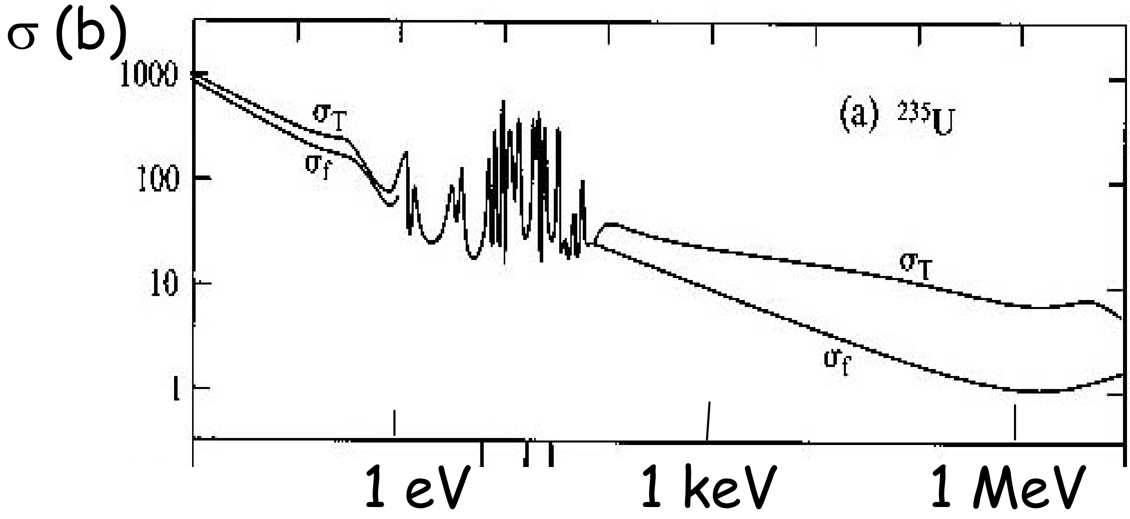
Neutrons from fission process can be used to induce further fission → chain reaction

A chain reaction can be sustained is at least 1 n/fission induces another fission process.

Define k = number of neutrons from one fission which induce another

Reactors → $k = 1$ critical
$k < 1$ subcritical
$k > 1$ supercritical

Prompt neutrons are fast $\langle E \rangle \sim 2$ MeV and the fission cross-section is small

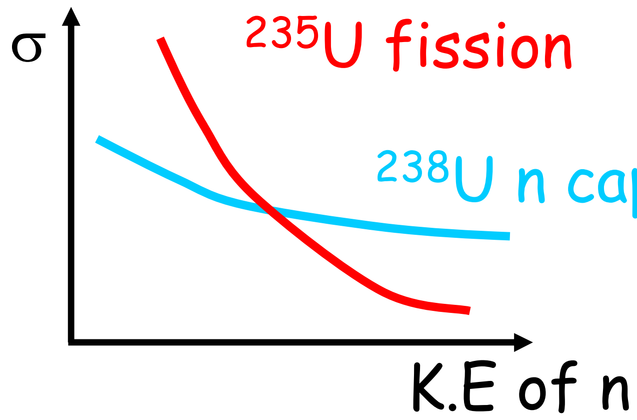


Hence, need to slow down fast neutrons before they escape or absorbed by (n,γ) process
→ chain reaction

Fission Reactors

For reactors want a steady energy release, exactly critical ($k = 1$).

- A moderator slows neutrons via elastic collisions (large energy transfer). Requires a light nucleus (e.g. ^{12}C).
- Problem : Natural U (99.3% ^{238}U, 0.7% ^{235}U) and n capture cross-section large for ^{238}U



$$n + {}^{238}U \rightarrow {}^{239}U^* \rightarrow {}^{239}U + \gamma$$

Need to thermalise fast neutrons away from ^{238}U to avoid capture (i.e. in rods of ^{12}C).

- Control of reaction rate

  Control number of neutrons by absorption (e.g. ^{113}Cd rods).

  Typical time between fission and daughter inducing another fission ~10^{-3} s.

  → Mechanical control of rods in times « seconds not possible.

What happens if no control of neutrons ?

$$N(t+dt) = N(t) + (k-1)N(t)\frac{dt}{\tau}$$

$$dN = (k-1)N\frac{dt}{\tau}$$

$$\int_{N(0)}^{N(t)} \frac{dN}{N} = \int_0^t (k-1)\frac{dt}{\tau} \quad \rightarrow \quad \underline{N(t) = N(0)e^{(k-1)t/\tau}}$$

where N(t) is the number of neutrons at time t

(k – 1) is the % change in number of neutrons in 1 cycle

τ mean time for 1 cycle ~10^{-3} s (fission $\rightarrow$ fission)

e.g. k =1.01, τ = 0.001s, t = 1s

$$\frac{N(t)}{N(0)} = e^{0.01/0.001} = \underline{e^{10}}$$ (22,000 in 1s)

N.B. U reactor will <u>not</u> explode if it goes supercritical. As it heats up, K.E. of neutrons increases and fission cross section drops. Reactor stabilizes at a very high temperature

$\rightarrow$ meltdown

Solution is to make use of delayed n emission.

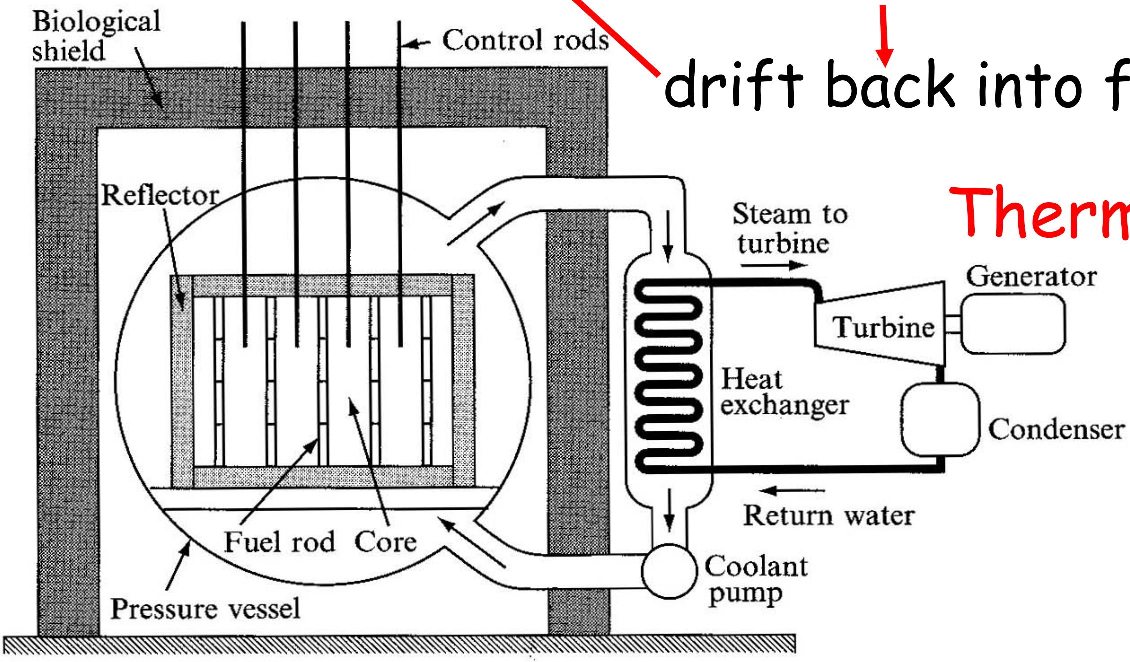
(delay ~ 13 s)

Design reactor to be subcritical to <u>prompt n</u> and use <u>delayed n</u> to take it to critical.

Thermal Reactors

Require following steps

fission → fast neutrons → remove from fissile material → thermalize them (gives heat) → drift back into fuel → fission



Thermal Reactor

Fast Breeder Reactors ~20% Pu + 80% ^{238}U

- Can use fast neutrons to produce chain reaction. Thermal σ higher, but high enrichment compensates. No moderator required.
- n capture by $n + {}^{238}U \rightarrow {}^{239}U \rightarrow {}^{239}Np \rightarrow {}^{239}Pu$ (fissionable)
  Hence, breeder. All fuel used.
- Control rods required.

Nuclear Fusion

Energetically favourable for light nuclei to fuse and release energy. However, light nuclei need energy to overcome Coulomb barrier.

e.g most basic process

$$p + p \rightarrow d + e^+ + \nu_e \qquad Q = 0.42\ \text{MeV}$$

Coulomb barrier

$$V = \frac{e^2}{4\pi\varepsilon_0 R} = \frac{\alpha\hbar c}{R} = \frac{197}{137 \times 1.2} = \underline{1.2\ \text{MeV}}$$

- **Accelerators** - Energies above barrier easy to achieve. However, high particle densities for long periods of time very difficult. These are required to get the rate of fusion reactions for desired power.

- **Stars** - Large proton density $10^{32}\ \text{m}^{-3}$. Particle K.E. due to thermal motion.

For kT ~1 MeV require $T \sim 10^{10}$ K
Sun $T \sim 10^7$ K Energies ~1 keV

$\Rightarrow$ Quantum Mechanical Tunnelling required

Fusion Rate in Sun

Particles in the sun have Maxwell-Boltzman velocity distribution - very important because tunnelling probability is a strong function of energy.

Reaction rate/unit volume for particles of velocity v = $\sigma(v)\, F\, N$ $\qquad F = N v$

Flux (F), Proton density (N)

- σ is modified by tunnelling probability

$$P = e^{-2G(v)}$$

Gamow Factor

$$G(v) \approx \left(\frac{2m}{Q}\right)^{1/2} \frac{e^2}{4\pi\varepsilon_0} \frac{Z_1 Z_2}{\hbar} \frac{\pi}{2} = \frac{e^2}{4\pi\varepsilon_0} \frac{\pi Z_1 Z_2}{\hbar v}$$

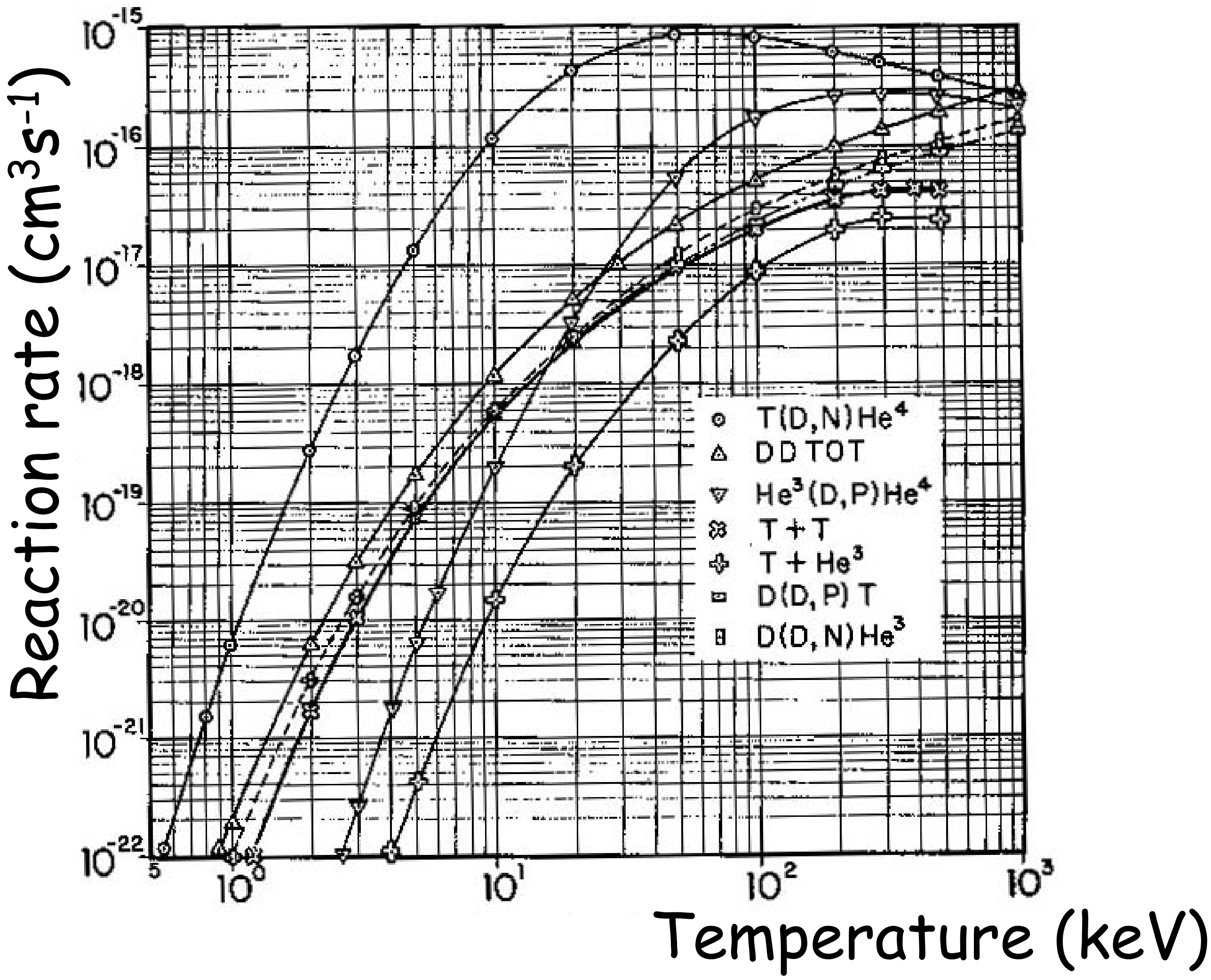
- Convolve with velocity distribution

Probability velocity, between v and v+dv $\quad f(v)dv = v^2 e^{-mv^2/2kT} dv$

$$\text{Reaction Rate} = \int N\,(Nv)\,\sigma(v)\,P\,f(v)dv$$

Reaction rate

$$\sim \int v\, e^{-2G} e^{-mv^2/2kT} dv \sim \int e^{-2G} e^{-E/kT} dE$$



Typical fusion reactions peak at kT~100 keV

$\rightarrow$ T~10^9K

e.g. $p + p \rightarrow d + e^+ + \nu_e$

Reaction rate/proton/sec ~ 5×10^{-18} s

$\rightarrow$ Mean life, $\tau = 10^{10}$ years

This defines the burning rate in the Sun.

Fusion Processes in the Sun

(1) $p + p \rightarrow d + e^+ + \nu$ $p + p \rightarrow d + e^+ + \nu$

Q=0.42 MeV Q=0.42 MeV

(2) $p + d \rightarrow {}^3He + \gamma$ $p + d \rightarrow {}^3He + \gamma$

Q=5.49 MeV Q=5.49 MeV

(3) $^3He + {}^3He \rightarrow {}^4He + 2p + \gamma$

Q=12.86 MeV

Net Reaction

$$4p \rightarrow {}^4He + 2e^+ + 2\nu$$

$2e^+$ annihilate with $2e^-$ $Q=4m_e=2.04$ MeV

Total energy release in fusion cycle = 26.7 MeV

i.e. energy release per proton in fusion cycle
= 26.7/4 = 6.7 MeV

ν's emerge without further interaction with ~2% of the energy. The rest heats the core.

Observed luminosity ~4×10^{26} J/s

Number of protons consumed s^{-1}

$$= \frac{4 \times 10^{26}}{1.6 \times 10^{-13}} \; \frac{1}{6.7} = 4 \times 10^{38}$$

$1 \text{ MeV} = 1.6 \times 10^{-13}$ J

Other ^3He interactions:

(1) $$^3He + {}^4He \rightarrow {}^7Be + \gamma$$

(2) $$^7Be + e^- \rightarrow {}^7Li + \nu$$

$$\hookrightarrow + p \rightarrow 2\,{}^4He$$

OR

$$^7Be + p \rightarrow {}^8B + \gamma$$

$$\hookrightarrow {}^8Be + e^+ + \nu$$

$$\hookrightarrow 2\,{}^4He$$

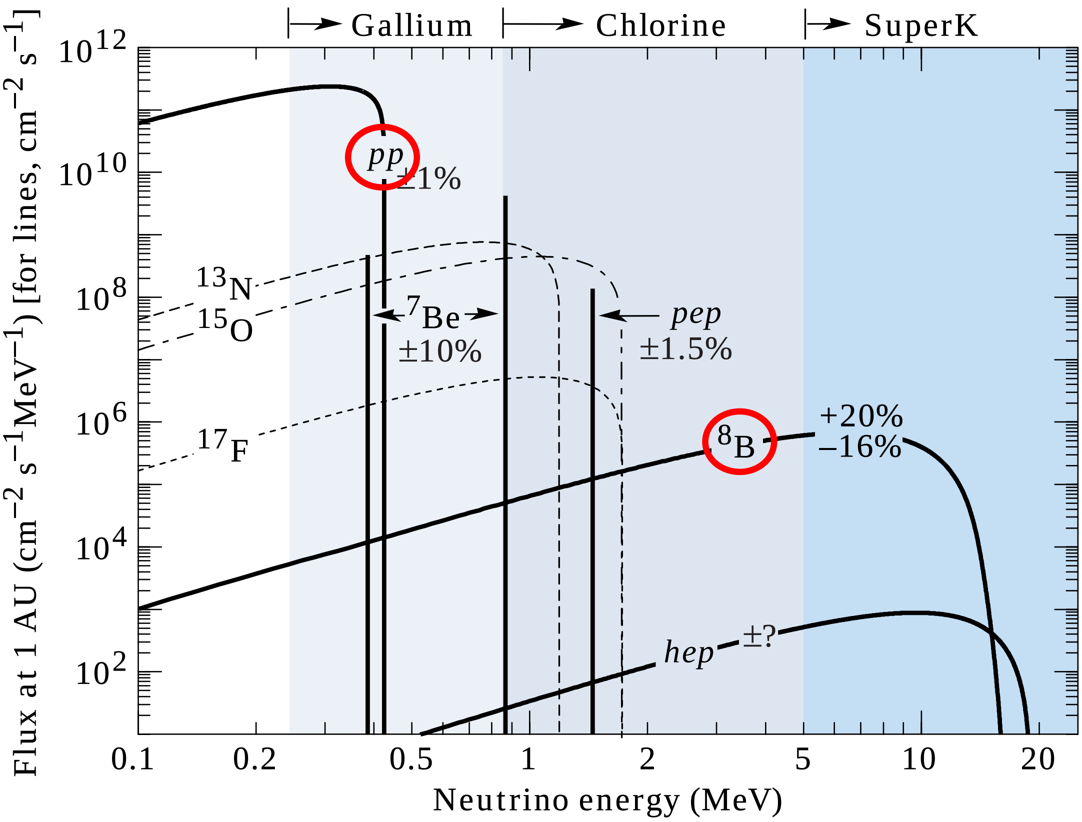
Other fusion cycles also possible
e.g. C-N-O

Observation of solar neutrinos from the various sources directly addresses the theory of stellar structure and evolution (Standard Solar Model).

The Sun also provides an opportunity to investigate ν properties e.g. mass, oscillations..

Solar Neutrinos

Many experiments have studied the solar neutrino flux



Expected flux depends on

- Standard Solar Model
  (temp,density, composition vs r)
- Nuclear reaction cross-sections

Observed ν flux ~ 1/3 expected ν flux

"Solar ν problem"

Neutrino Oscillations

Recently, the Sudbury Neutrino Observatory (SNO) collaboration has reported evidence for a non-ν_e neutrino component in the solar ν flux

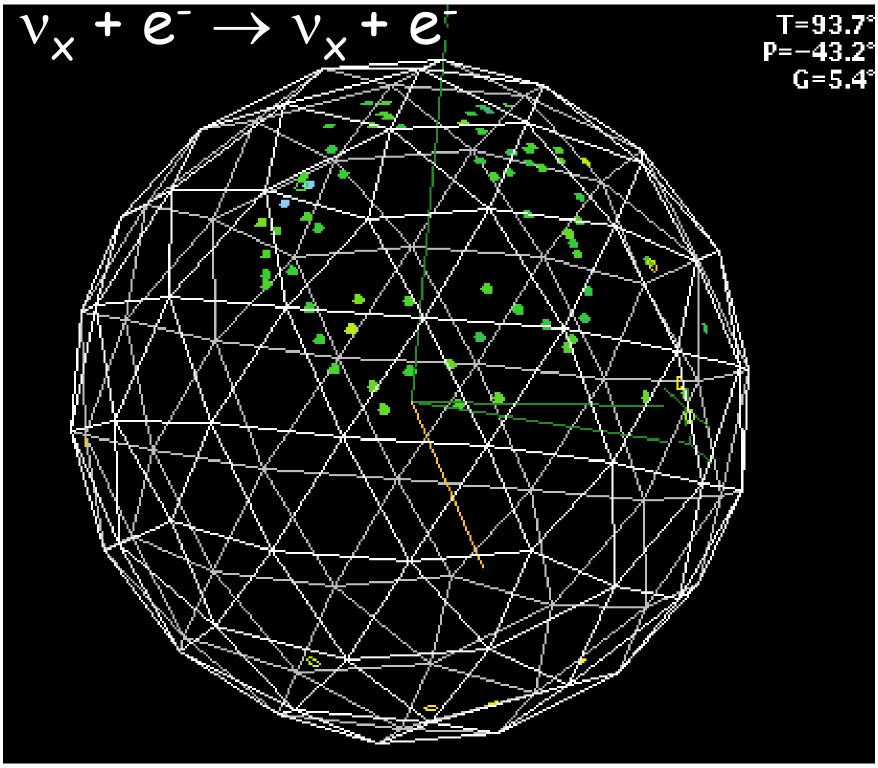
→ **Neutrino Oscillations**

SNO measure the ^8B solar ν flux using the reactions

$$\nu_e + d \rightarrow e^- + p + p \qquad \nu_e \text{ flux}$$

$$\left.\begin{array}{l} \nu_x + d \rightarrow \nu_x + p + n \\ \nu_x + e^- \rightarrow \nu_x + e^- \end{array}\right\} \quad \text{total flux (all } \nu \text{ species)}$$

1000 tons D_2O
in spherical vessel



Evidence for $\nu_e \Leftrightarrow \nu_x$ at 5σ

SNO event