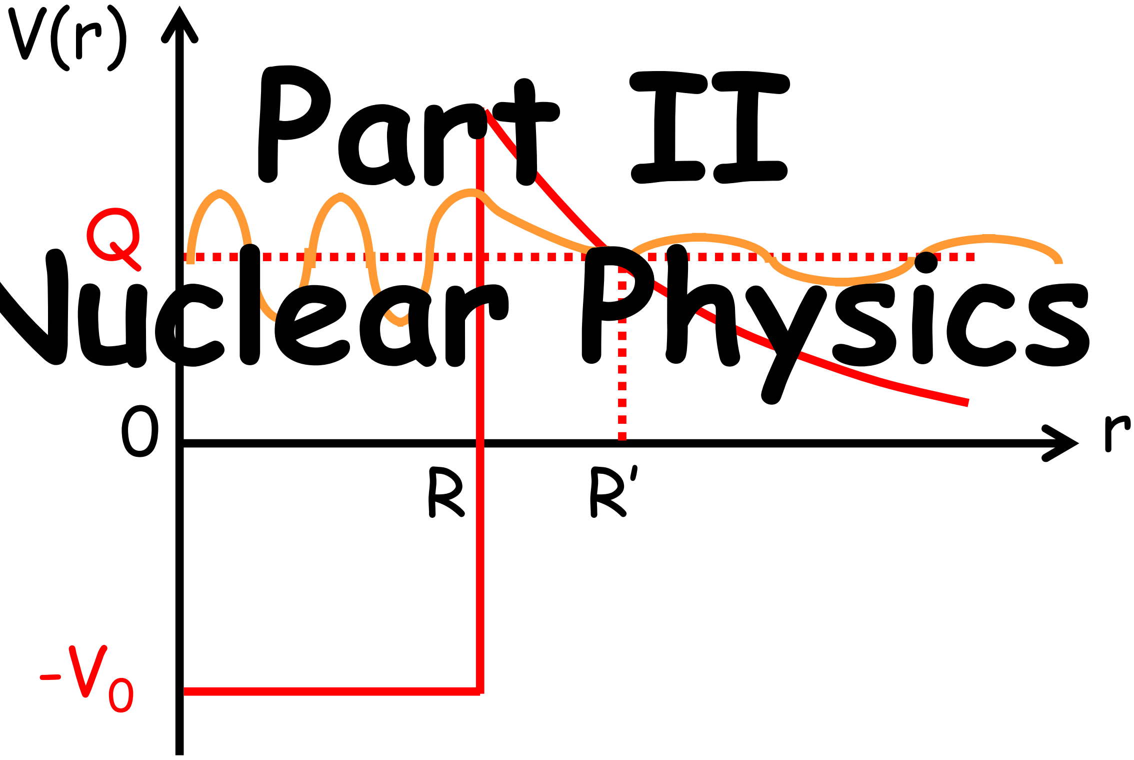
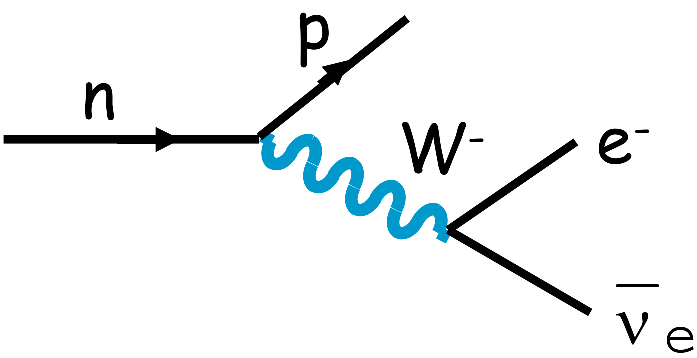
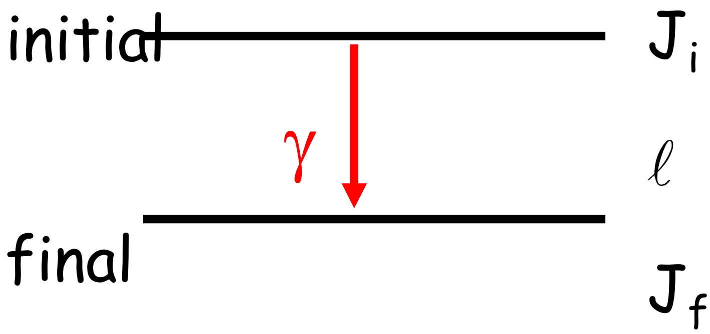


Part II

Nuclear Physics

Handout 4





V.Gibson

Lent Term 2004

Section V
Nuclear Decay

α Decay

α decay is due to the emission of a $^4_2$He nucleus

$^4_2$He is doubly magic and very tightly bound

α decay is energetically favourable for almost all nuclei having $A \geq 190$ and for many $A \geq 150$.

Why α ?

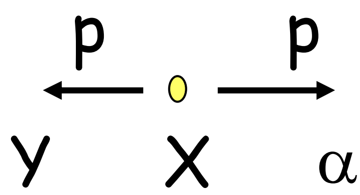
Consider energy release (Q) in various possible decays of ^{232}U

	n	*p*	*^{2}H*	*^{3}H*	*^{3}He*	*^{4}He*	*^{5}He*	*^{6}Li*	*^{7}Li*
Q (MeV)	-7.26	-6.12	-10.70	-10.24	-9.92	+5.41	-2.59	-3.79	-1.94

α easy to form inside nucleus 2p ↑↓+2n ↑↓
(extent to which α exist inside nucleus still unknown)

Kinematics

$$^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + \alpha$$



Conservation of energy:

$$m_X = m_Y + T_Y + m_\alpha + T_\alpha$$

T = kinetic energy = $p^2/2m$ (non-relativistic)

Energy release:

$$Q = T_Y + T_\alpha = m_X - m_Y - m_\alpha$$
$$= B_Y + B_\alpha - B_X$$

If Q > 0, decay energetically possible

$$T_\alpha = \frac{p^2}{2m_\alpha}; \qquad T_Y = \frac{p^2}{2m_Y} = T_\alpha \frac{m_\alpha}{m_Y}$$

$$\Rightarrow T_\alpha = \frac{Q}{(1 + m_\alpha / m_Y)} \approx Q(1 - 4/A) \approx Q$$

Measure T_α (e.g. magnetic spectrometer) $\Rightarrow$ Q
$\Rightarrow$ Info. about nuclear masses/energy levels.

Decay may be to the ground state of Y or to an excited state, in which case γ-decay will follow
$\Rightarrow$ several lines seen in α energy spectrum.

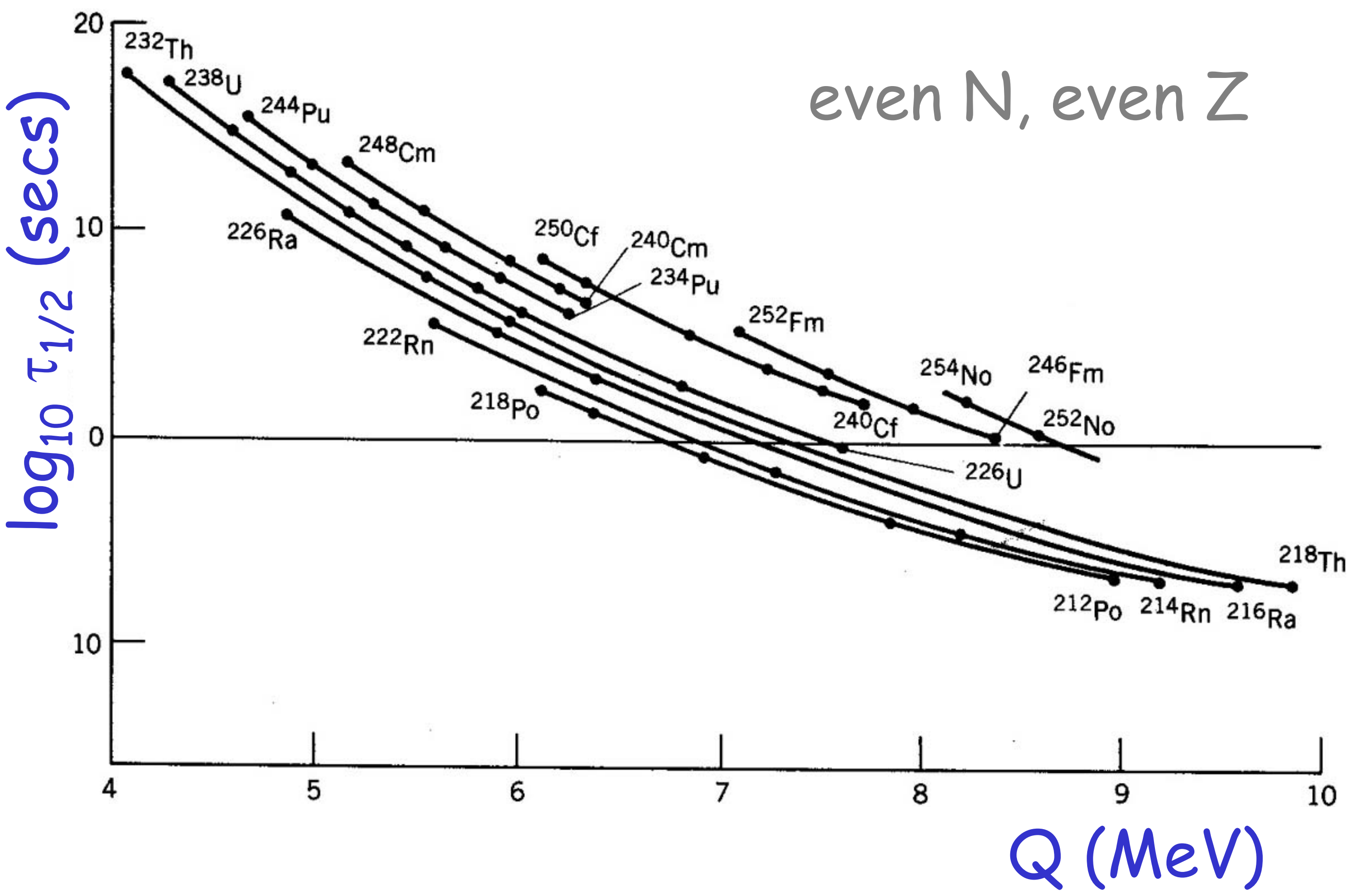
Dependence of $\tau_{1/2}$ on Q (Geiger and Nuttall 1911)

A very striking feature of α decay is the strong dependence of lifetime on Q.

e.g. $^{232}Th \quad Q = 4.08\ \text{MeV} \quad \tau_{1/2} = 1.4 \times 10^{10}\ \text{yrs}$

$\quad\ ^{218}Th \quad Q = 9.85\ \text{MeV} \quad \tau_{1/2} = 1.0 \times 10^{-7}\ \text{secs}$

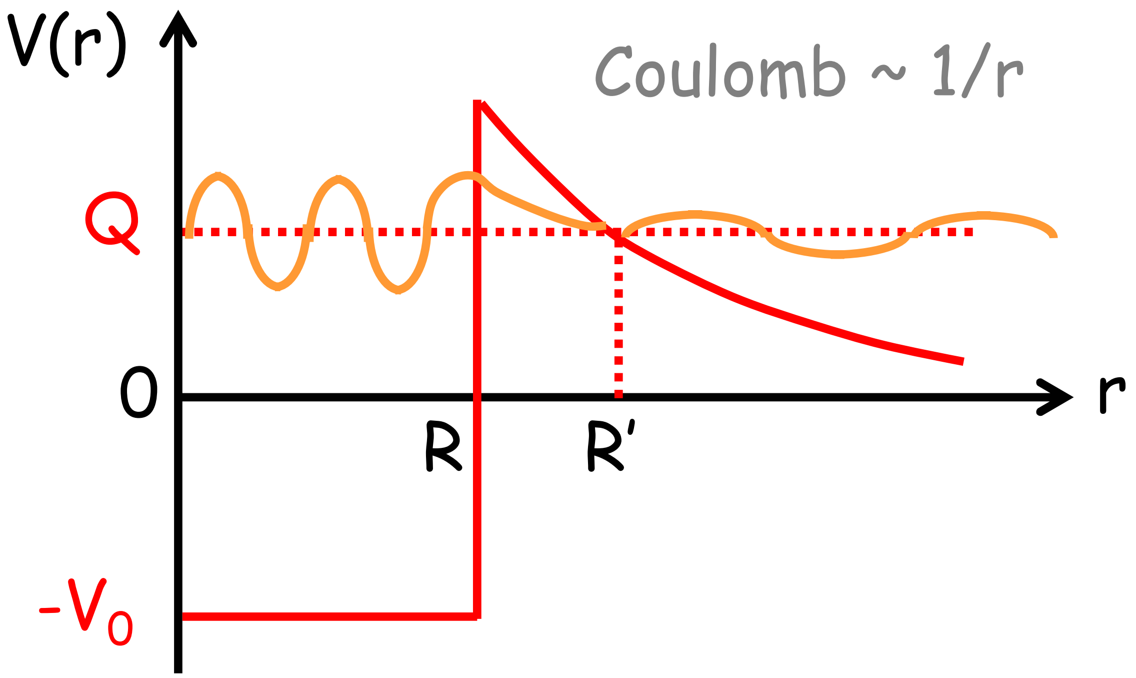
A factor of ~ 2.5 in Q $\Rightarrow$ Factor 10^{24} in $\tau_{1/2}$!



Smooth dependence at fixed Z

Quantum Mechanical Tunnelling

The nuclear potential for the α particle due to the daughter nucleus includes a Coulomb barrier which inhibits the decay.



Total energy of α = $\underbrace{Q+V_0}_{\text{K.E.}} \underbrace{-V_0}_{\text{P.E.}}$

Classically, α particle cannot enter or escape.

Quantum mechanically, α particle can penetrate the Coulomb barrier

$\Rightarrow$ Quantum Mechanical Tunnelling

Simple Theory (Gamow, Gurney, Condon 1928)

Assume α exists inside the nucleus and hits the barrier.

α decay probability, $\lambda = f\,P$

f = escape trial frequency, P = probability of tunnelling through barrier

semi-classically, $f \sim v/2R$

v = velocity of α particle inside nucleus , R = radius of nucleus

e.g. $V_0 \sim 35$ MeV, Q= 5 MeV $\Rightarrow T_\alpha = 40$ MeV

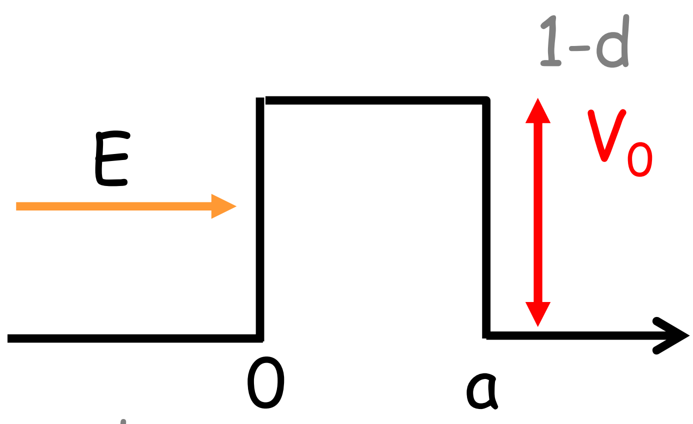
$$f \sim \frac{v}{2R} = \frac{1}{2R}\sqrt{\frac{2T_\alpha}{m_\alpha}} \sim 10^{22}\ s^{-1}$$

$m_\alpha = 3.7$ GeV
$R \sim 2.1$ fm

Obtain tunnelling probability, P, by solving S.E. in 3 regions and using boundary conditions

Transmission probability,

$$P = \left[1 + \frac{V_0^2}{4(V_0 - E)E}\sinh^2 ka\right]^{-1}$$



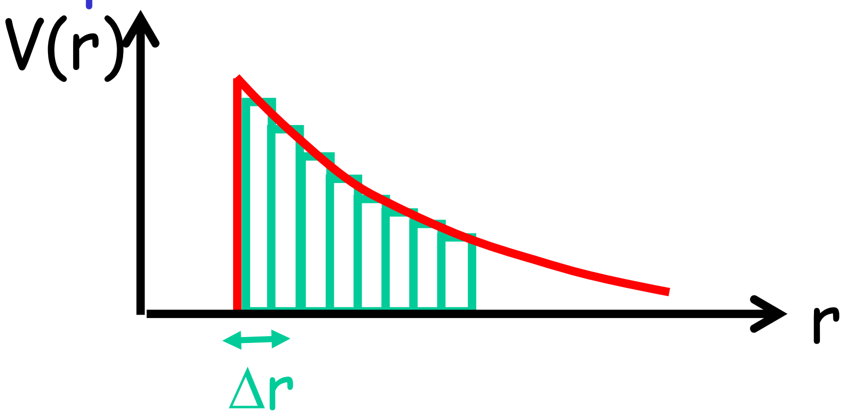
$$\frac{\hbar^2 k^2}{2m} = V_0 - E$$

m = reduced mass

For ka >> 1, P is dominated by exponential decay within barrier

i.e. $$P \sim e^{-2ka}$$

Coulomb potential, V, and thus k varies with r. Divide into rectangular pieces and multiply together exponents.



Probability to tunnel through Coulomb barrier

$$P = \prod_i e^{-2k_i \Delta r} \qquad k = \frac{\left[2m\left(V(r) - Q\right)\right]^{1/2}}{\hbar}$$

$$= \exp\left\{-2\int_R^{R'} \frac{\left[2m\left(V(r) - Q\right)\right]^{1/2}}{\hbar} dr\right\}$$

$$\equiv \exp - 2G$$

where $G = \int_R^{R'} \frac{\left[2m\left(V(r) - Q\right)\right]^{1/2}}{\hbar} dr$

is the Gamow Factor.

For $r > R$ $\quad V(r) = \dfrac{2Z'e^2}{4\pi\varepsilon_0 r} \equiv \dfrac{B}{r}$ $\quad Z' = Z - 2$

α escapes at $r = R'$, $V(R') = Q \Rightarrow R' = B/Q$

$$\therefore G = \int_R^{R'} \left(\frac{2m}{\hbar^2}\right)^{1/2} \left[\frac{B}{r} - Q\right]^{1/2} dr$$

$$= \left(\frac{2mB}{\hbar^2}\right)^{1/2} \int_R^{R'} \left[\frac{1}{r} - \frac{1}{R'}\right]^{1/2} dr$$

Let $r = R'\cos^2\vartheta$

$$\int \left[\frac{1}{r} - \frac{1}{R'}\right]^{1/2} dr = \int \left[\frac{1}{R'\cos^2\vartheta} - \frac{1}{R'}\right]^{1/2} (-2R'\cos\vartheta\sin\vartheta)\, d\vartheta$$

$$= \int -2R'^{1/2} \sin^2\vartheta\, d\vartheta = R'^{1/2}[\sin 2\vartheta - \vartheta]$$

$$= R'^{1/2}\left[2\left(1 - \frac{r}{R'}\right)^{1/2}\left(\frac{r}{R'}\right)^{1/2} - \cos^{-1}\left(\frac{r}{R'}\right)^{1/2}\right]_R^{R'}$$

$$= R'^{1/2}\left[\cos^{-1}\left(\frac{R}{R'}\right)^{1/2} - 2\left\{\left(1 - \frac{R}{R'}\right)\left(\frac{R}{R'}\right)\right\}^{1/2}\right]$$

$$G = \left(\frac{2mB}{\hbar^2}\right)^{1/2} R'^{1/2}\left[\cos^{-1}\left(\frac{R}{R'}\right)^{1/2} - 2\left\{\left(1 - \frac{R}{R'}\right)\left(\frac{R}{R'}\right)\right\}^{1/2}\right]$$

$$G = \left(\frac{2m}{Q}\right)^{1/2} \frac{B}{\hbar}\left[\cos^{-1}\left(\frac{R}{R'}\right)^{1/2} - 2\left\{\left(1 - \frac{R}{R'}\right)\left(\frac{R}{R'}\right)\right\}^{1/2}\right]$$

Most practical cases R << R′,
so term in [...] ~ π/2

$$G \approx \left(\frac{2m}{Q}\right)^{1/2} \frac{B}{\hbar}\frac{\pi}{2} \qquad B = \frac{2Z'e^2}{4\pi\varepsilon_0}$$

e.g. Z = 90, Q ~ 6 MeV $\Rightarrow$ R′ ~ 40 fm >> R

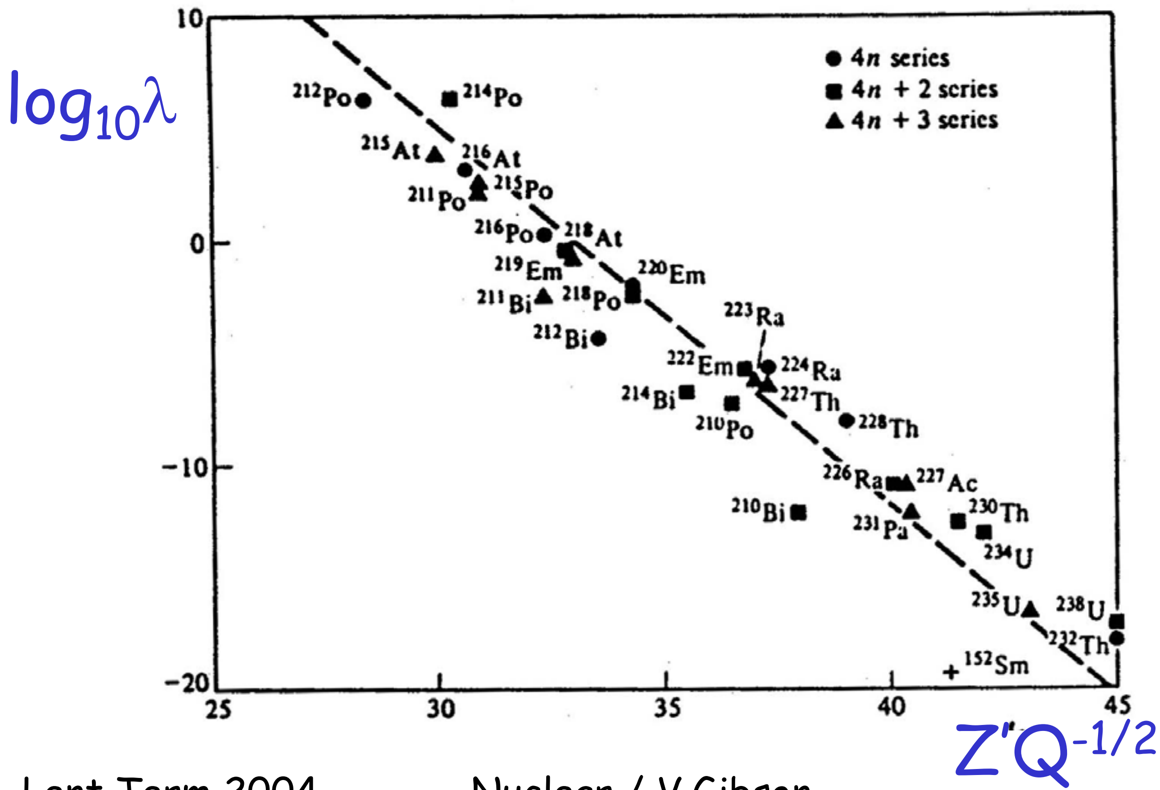
$$G \approx Z'\left(\frac{3.9\ \text{MeV}}{Q}\right)^{1/2}$$

Lifetime $$\tau_{1/2} = \frac{1}{\lambda} = \frac{1}{f\,P} \sim \frac{2R}{v}e^{2G}$$

$$\text{Ln}\,\tau_{1/2} \sim 2G + \text{Ln}\frac{2R}{v}$$

Geiger-Nuttall Law

$$\text{Ln}\,\lambda \sim -\frac{Z'}{Q^{1/2}} + \text{constant}$$



- Simple tunnelling model accounts for strong dependence of $\tau_{1/2}$ on Q.

- Also explains why decay to heavier fragments e.g. ^{12}C disfavoured

$$G \sim m^{1/2}, \quad G \sim \text{charge of fragment}$$

Deficiences

- Assumed existence of one α particle in nucleus and have taken no account of probability of formation.

- Assumed "semi-classical" approach to estimate escape trial frequency, $f \sim v/2R$, and make absolute prediction of decay rate.

- If α emitted with some angular momentum, ℓ, radial wave equation must include a centrifugal barrier term

$$V = \frac{\ell(\ell+1)\hbar^2}{2mr^2}$$

ℓ = relative a.m. of α and daughter nucleus
m = reduced mass

which raises the barrier and hence suppresses emission of α in high ℓ states.

Angular Momentum and Parity Selection Rules in α Decay

Nuclear Shell Model α has $J^P=0^+$

Angular momentum

$$X \to Y + \alpha$$

spin J_X J_Y ℓ

$$J_X = J_Y \oplus \ell$$

ℓ ranges from $J_X + J_Y$ to $|J_X - J_Y|$

Parity

Parity is conserved in α decay.
Orbital wavefunction has parity $(-1)^\ell$

X,Y same parity $\Rightarrow \ell$ must be even
X,Y opposite parity $\Rightarrow \ell$ must be odd

e.g. if X,Y both even-even nuclei in their ground states $\Rightarrow$ shell model $J^P=0^+ \Rightarrow \ell=0$

More generally, if X has $J^P=0^+$, the states of Y which can be formed in α decay are

$$J^P = 0^+, 1^-, 2^+, 3^-, 4^+, \ldots$$

β Decay

e^- electron, e^+ positron (antiparticle)

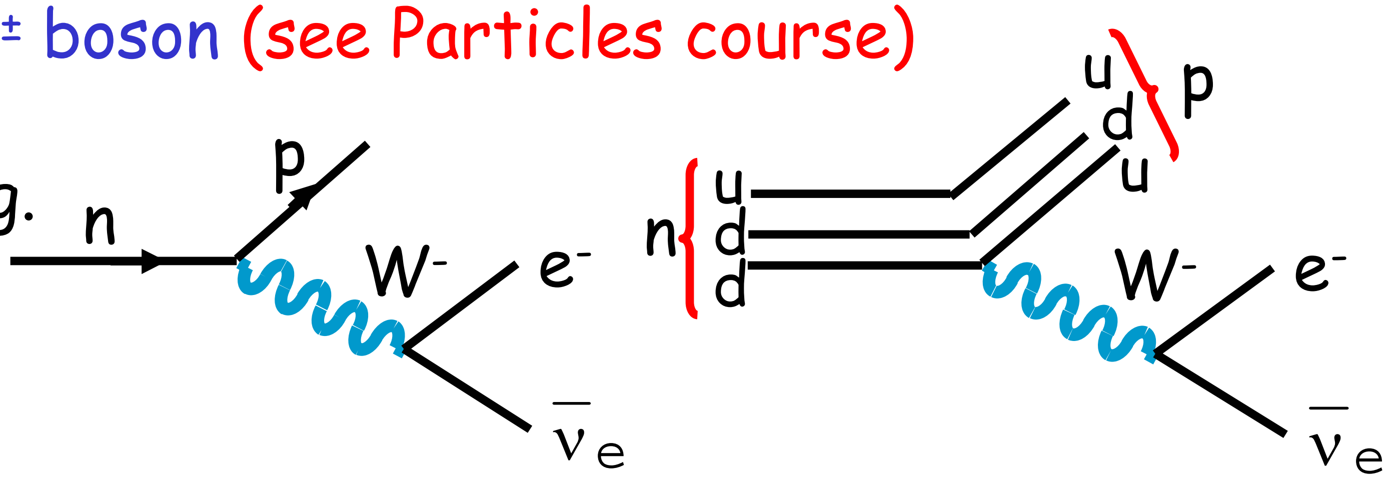
β^- $n \rightarrow p + e^- + \bar{\nu}_e$ $\quad {}^A_Z X \rightarrow {}^A_{Z+1} Y + e^- + \bar{\nu}_e$

β^+ $p \rightarrow n + e^+ + \nu_e$ $\quad {}^A_Z X \rightarrow {}^A_{Z-1} Y + e^+ + \nu_e$

electron capture $p + e^- \rightarrow n + \nu_e$ $\quad {}^A_Z X \rightarrow {}^A_{Z-1} Y + \nu_e$

β decay is a weak interaction mediated by the $W^\pm$ boson (see Particles course)

e.g.



$W^\pm$ bosons are massive $M_W \sim 80$ GeV/c^2

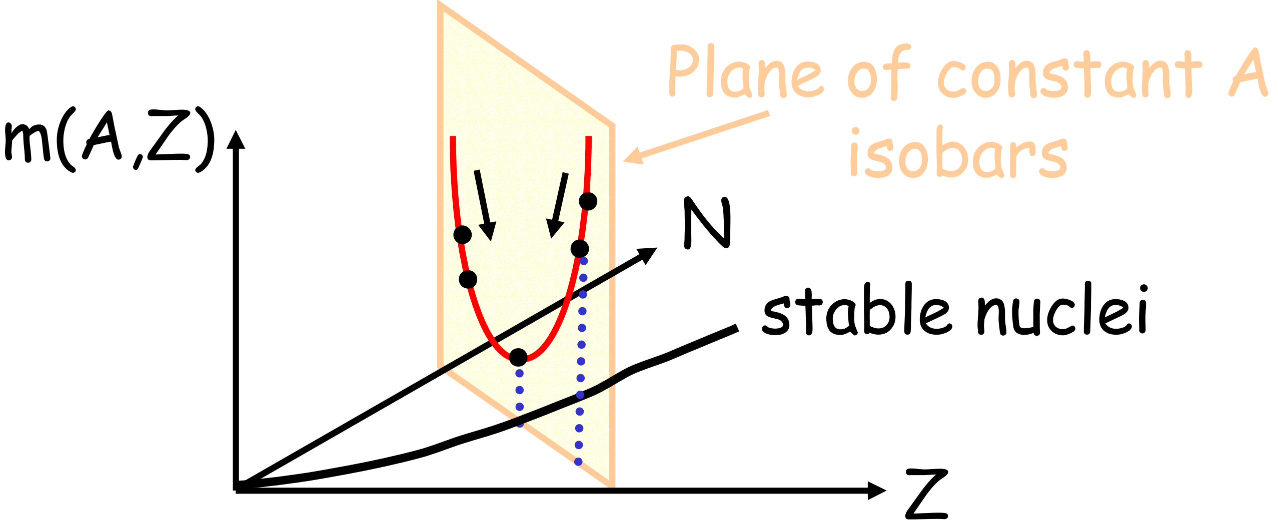
Yukawa Potential $V(r) \sim \dfrac{e^{-M_W r}}{r}$

Range = $1/M_W \sim 2 \times 10^{-18}$ m << nuclear size

Examine nuclear stability against β decay by plotting the nuclear mass on an axis ⊥'r to N-Z plane.

β decay, A is constant, Z changes by ±1

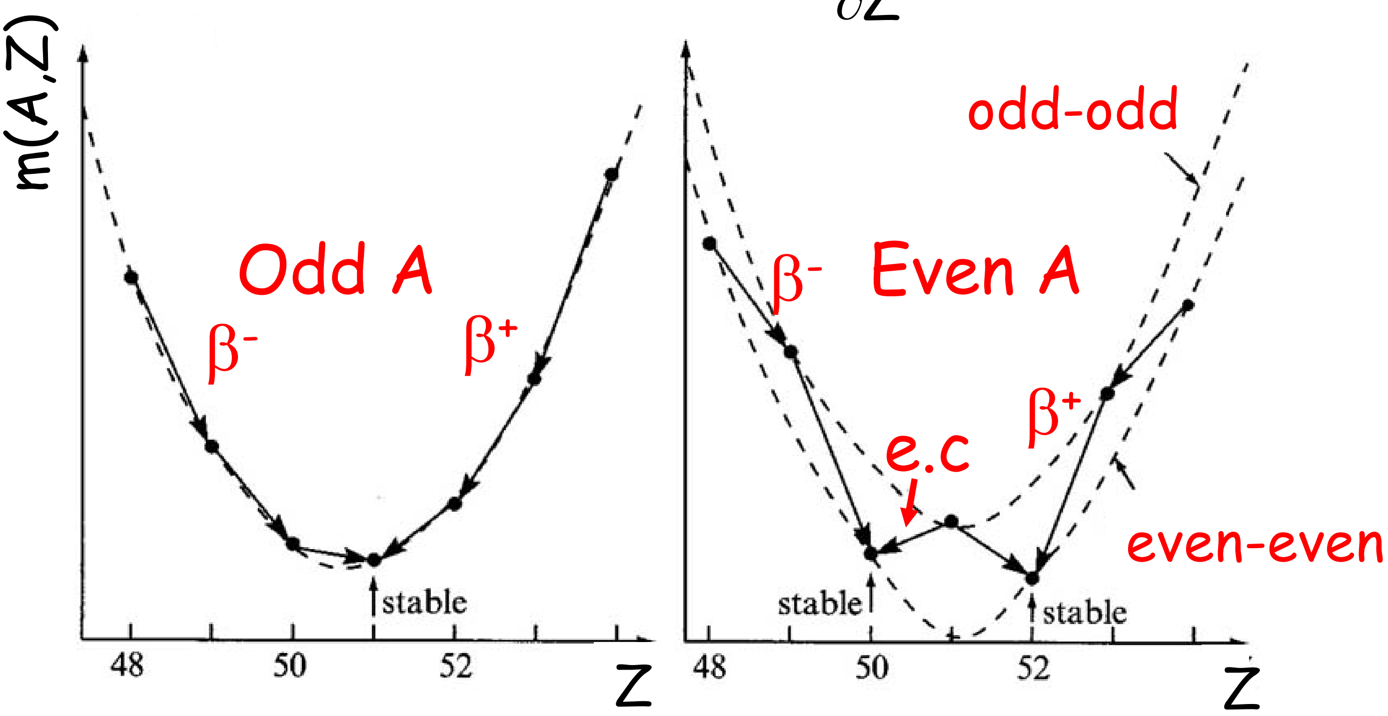
⇒ m(A,Z) is quadratic in Z.



S.E.M.F.

$$m(A,Z) = Z\,m_p + (A-Z)\,m_n - a_v A + a_s A^{2/3} + \frac{a_c Z^2}{A^{1/3}} + a_a \frac{(N-Z)^2}{A} - \delta(A)$$

Most stable nuclei when $\dfrac{\partial m(A,Z)}{\partial Z} = 0$



Kinematics

Q = Energy release in decay (as in α decay)

$\Rightarrow$ **Q > 0, decay possible**

$\beta^{\pm}$ $Q = m_X - m_Y - m_e - m_\nu$ Nuclear masses

e.c. $Q = m_X - m_Y + m_e - m_\nu$ $m_{e-} \approx m_{e+}$

$$M(A,Z) = m(A,Z) + Zm_e$$

Atomic mass Nuclear mass

β^- $Q = M_X - M_Y - m_\nu$ Atomic masses

β^+ $Q = M_X - M_Y - 2m_e - m_\nu$

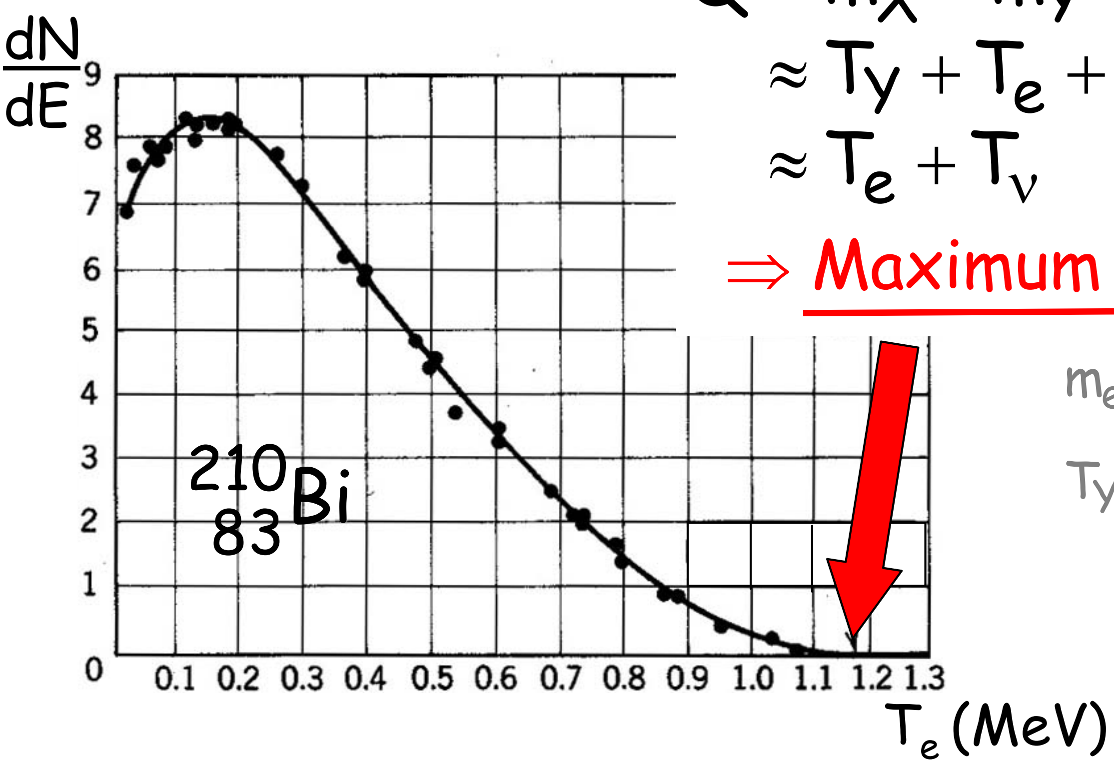
e.c. $Q = M_X - M_Y - m_\nu$

Electron capture is possible if β^+ not allowed (Q<0)

$$Q = m_X - m_Y - m_e - m_\nu$$
$$\approx T_Y + T_e + T_\nu$$
$$\approx T_e + T_\nu$$

$\Rightarrow$ **Maximum T_e = Q**

$m_e << m_Y$, $T_Y = \frac{p^2}{2m}$ small



Fermi Theory of β Decay

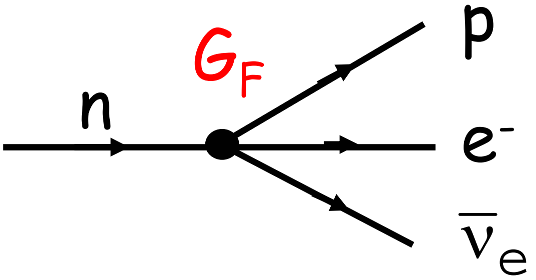
Use perturbation theory (weak interaction)

Transition Rate $\Gamma_{i\to f} = \frac{2\pi}{\hbar}|M|^2 \rho(E_f)$ Fermi Golden Rule

$M = \langle \psi_f | \hat{H} | \psi_i \rangle$ and $\rho(E_f)$= density of final states

$\hat{H}$ = energy operator that describes the weak coupling between initial and final states.

Assume 4 particles in β decay interact at a single point.



G_F = Fermi constant
= strength of the weak interaction

$G_F/(\hbar c)^3 = 1.166 \times 10^{-5}\ \text{GeV}^{-2}$

In Fermi theory,

$$M = G_F \int \psi_n \psi_p^* \psi_e^* \psi_{\bar{\nu}}^* \, d\tau$$

Free particle $\psi_e = e^{i\vec{p}\cdot\vec{r}}$ $\psi_\nu = e^{i\vec{q}\cdot\vec{r}}$

$$\therefore M = G_F \int \psi_p^* \, e^{-i(\vec{p}+\vec{q})\cdot\vec{r}} \psi_n d\tau$$

- Typically, e and ν have energies ~MeV, so $\vec{p}\cdot\vec{r} \approx 10^{-2}$ << size of nucleus.

  $\therefore$ Approximately, $\psi_e, \psi_\nu = 1$ corresponding to ℓ=0 states for e and ν

  [c.f. Partial waves kr<<1 $\Rightarrow$ ℓ=0]

  Allowed Transitions

$$\therefore M \approx G_F \int \psi_p^* \psi_n d\tau$$

where $\int \psi_p^* \psi_n d\tau$ is the nuclear matrix element and depends on the degree of overlap of the initial state n and final state p wavefunctions (not predicted by Fermi theory)

- If p,n wavefunctions are very similar, the nuclear matrix element is $\approx$1
  $\Rightarrow$ large M
  $\Rightarrow$ β decay favoured.

  Superallowed Transitions

e.g. $n \rightarrow p$, ${}^3_1H \rightarrow {}^3_2He$, ${}^{13}_7N \rightarrow {}^{13}_6C$

Electron Energy Spectrum

Consider density of final states $\rho(E_f)$

$$X \to Y + e + \nu$$

Number of e states in range $p_e \to p_e + dp_e$

$$dN_e = 4\pi \frac{p_e^2}{h^3} dp_e$$

Number of ν states in range $p_\nu \to p_\nu + dp_\nu$

$$dN_\nu = 4\pi \frac{p_\nu^2}{h^3} dp_\nu$$

Total number of states

$$dN = 4\pi \frac{p_e^2}{h^3} dp_e \, 4\pi \frac{p_\nu^2}{h^3} dp_\nu$$

Energy of final state

$$E_f = Q \approx T_e + p_\nu c$$

nuclear recoil negligible

$$\therefore \rho(E_f) = \frac{dN}{dE_f} = \left(\frac{4\pi}{h^3}\right)^2 \frac{1}{c^3} p_e^2 (E_f - T_e)^2 dp_e$$

Probability of β decay producing an e with energy in the range $T_e \to T_e + dT_e$ and momentum in the range $p_e \to p_e + dp_e$

$$d\Gamma = \frac{2\pi}{\hbar} |M|^2 \rho(E_f)$$

$$= \frac{2\pi}{\hbar} |M|^2 \left(\frac{4\pi}{h^3}\right)^2 \frac{1}{c^3} p_e^2 (E_f - T_e)^2 dp_e$$

$$N(p_e) = \frac{d\Gamma}{dp_e} \sim p_e^2 (Q - T_e)^2$$

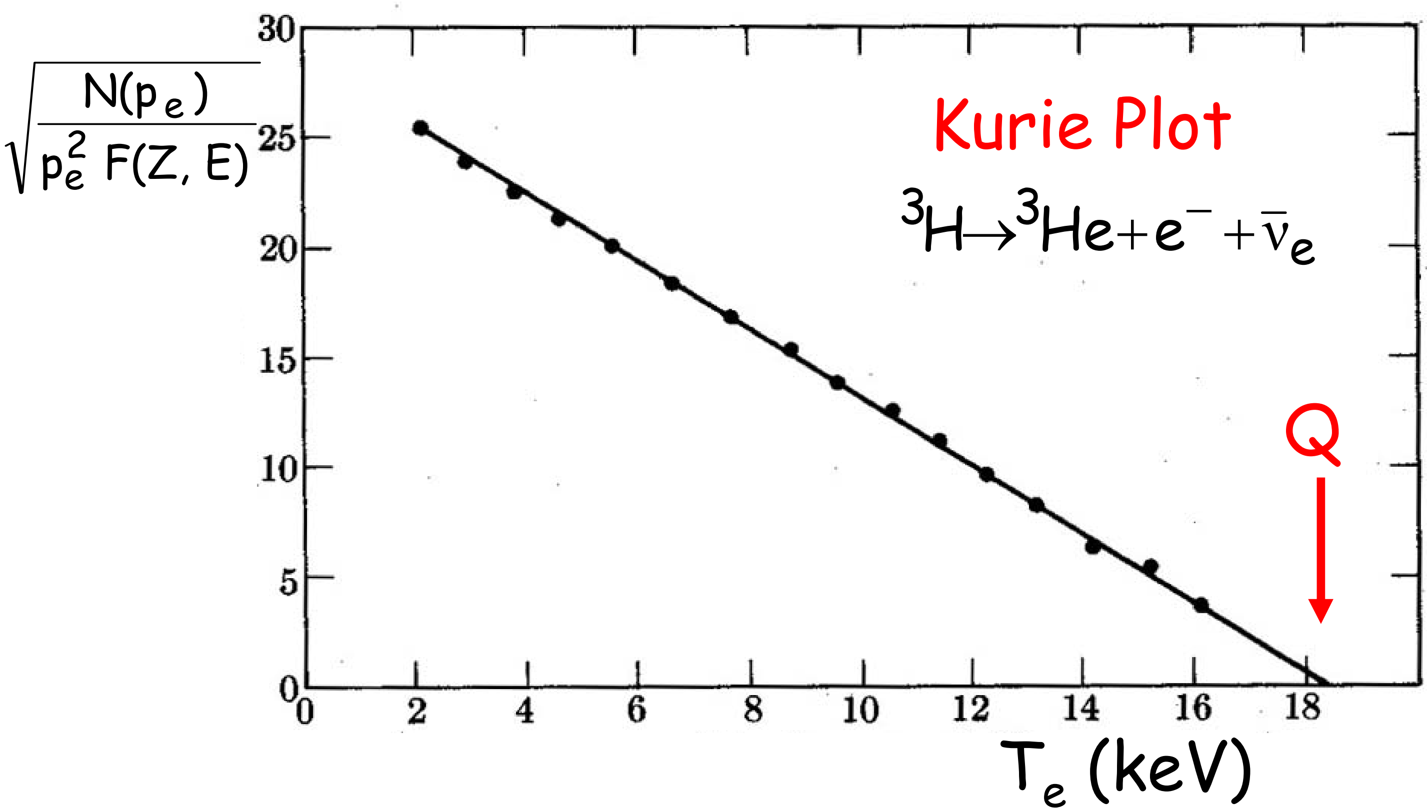
The momentum of the e is modified by the Coulomb interaction as it moves away from the nucleus (different for e^- and e^+).

$\Rightarrow$ Multiply spectrum by Fermi function F(Z,E)

$$N(p_e) = \frac{d\Gamma}{dp_e} = \frac{1}{2\pi^3 \hbar^7 c^3} |M|^2 \, p_e^2 (Q - T_e)^2 \, F(Z,E)$$

Kurie Plot

$$\sqrt{\frac{N(p_e)}{p_e^2 F(Z,E)}} = \textbf{constant} \times (Q - T_e)$$



ν Mass

$m_\nu = 0$ End point of electron energy spectrum = Q

$m_\nu \neq 0$ $$E_f = Q \approx T_e + \left(p_\nu^2 + m_\nu^2\right)^{1/2}$$ c = 1

$$p_\nu^2 = (Q - T_e)^2 - m_\nu^2, \qquad p_\nu dp_\nu = (Q - T_e)$$

$$p_\nu^2 dp_\nu = \left[(Q - T_e)^2 - \underline{m_\nu^2}\right]^{1/2}(Q - T_e)$$

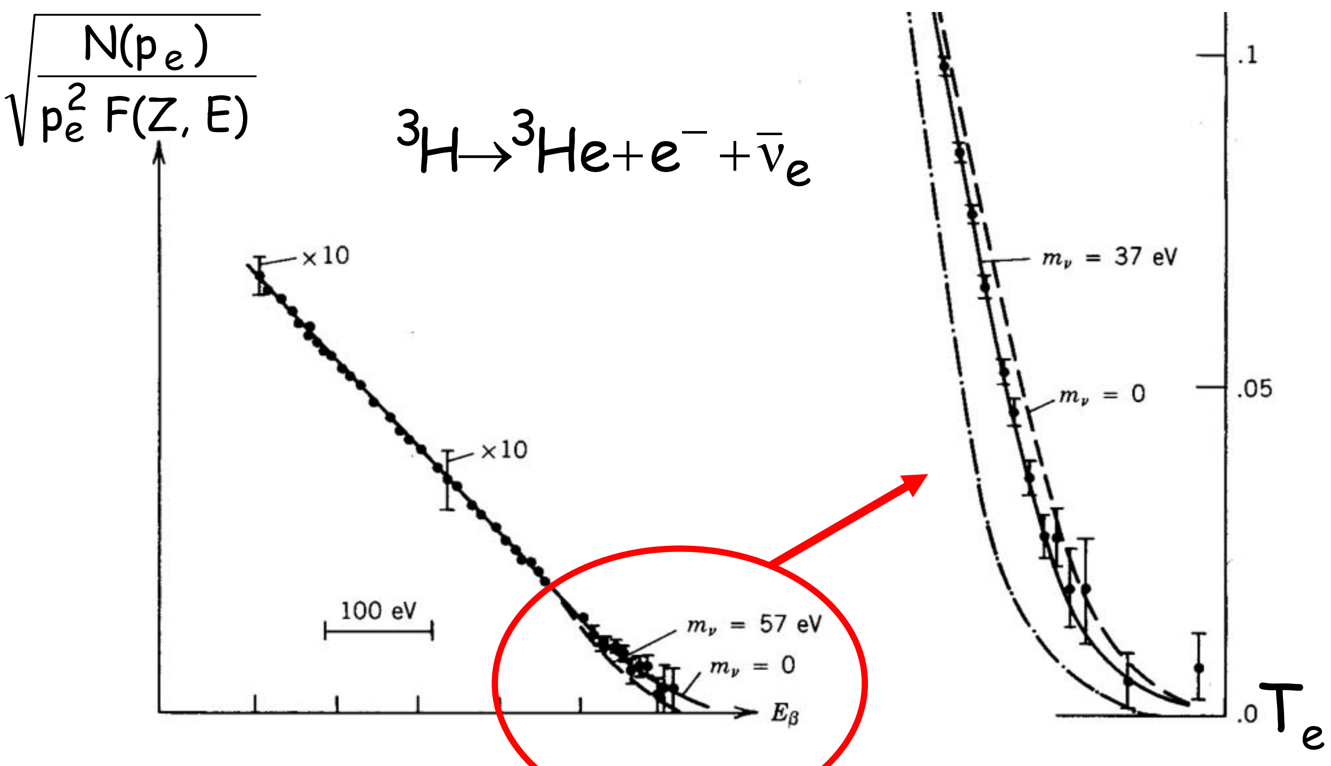
Use electron energy spectrum to determine m_ν.

m_ν small $\Rightarrow$ only significant effect where Te $\approx$ Q

Experimentally very difficult due to low intensity.

Current limit,

$\underline{m_\nu < 3\ \text{eV}/c^2}$ (c.f. $m_e = 0.511\ \text{MeV}/c^2$)



Total Decay Rate

Total Decay Rate, $\Gamma = \int_0^{p_{max}} \frac{d\Gamma}{dp_e} dp_e$

$$\Gamma = \frac{1}{2\pi^3\hbar^7c^3} |M|^2 \int_0^{p_{max}} p_e^2 (Q - T_e)^2 F(Z,E)\, dp_e$$

$m_\nu = 0$

- If e^- is highly relativistic then $T_e \sim p_e - m_e$ and

$$\Gamma \sim p_{max}^5 \sim Q^5$$

"Sargents Rule"

- All the information about the nuclear wavefunctions is contained in the matrix element. Values for the complicated Fermi integral,

$$f(Z,E) \equiv \frac{1}{m_e^5 c^7} \int_0^{p_{max}} p_e^2 (Q - T_e)^2 F(Z,E)\, dp_e$$

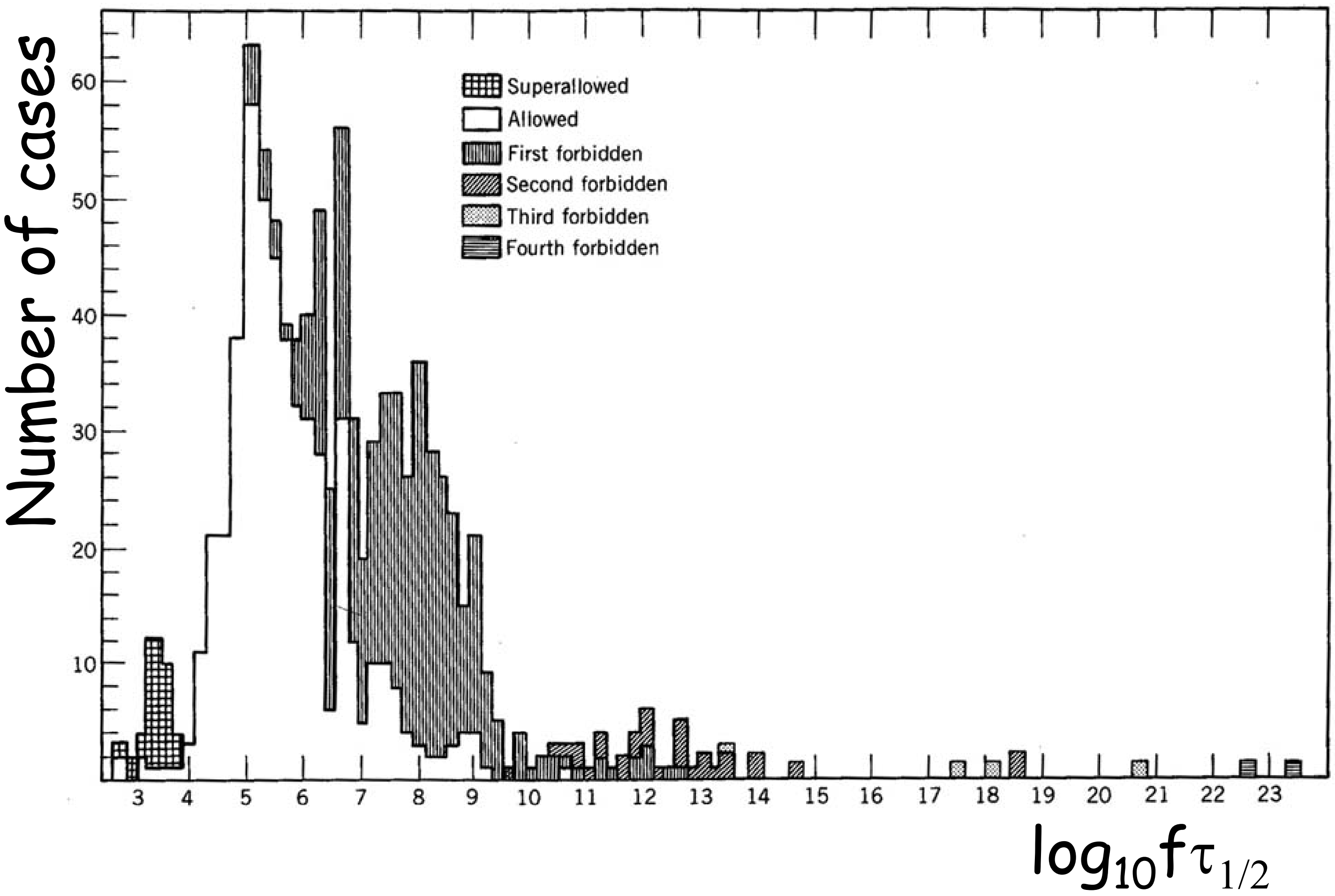
are tabulated. ← make f dimensionless

Mean Lifetime, $\tau = \frac{1}{\Gamma}$ Half-Life, $\tau_{1/2} = \frac{\text{Ln}2}{\Gamma}$

$$\Rightarrow \quad f\tau_{1/2} = \text{Ln}2 \frac{2\pi^3\hbar^7}{m_e^5 c^4 |M|^2}$$

Comparative Half-life

Comparative Half-lives



Decays with

$\log_{10} f\tau_{1/2}$ ~3-4 Superallowed

~4-7 Allowed

≥ 6 Forbidden

(i.e. suppressed, small M)

Angular Momentum and Parity Selection Rules in β Decay

Fermi theory, $$M = G_F \int \psi_p^* \underbrace{e^{-i(\vec{p}+\vec{q})\cdot\vec{r}}}_{e,\,\nu} \psi_n d\tau$$

So far have ignored angular momentum of e and ν.

- **Superallowed Transitions**

$$M \sim \int \psi_p^* \psi_n d\tau \approx 1$$ $\log_{10} f\tau_{1/2} \sim 3\text{-}4$

- **Allowed Transitions**

  Angular momentum of eν pair relative to nucleus, $\ell = 0$.

$$e^{-i(\vec{p}+\vec{q})\cdot\vec{r}} \sim 1$$ $\log_{10} f\tau_{1/2} \sim 4\text{-}7$

There are two types of allowed transitions depending on the relative spin states of the emitted e and ν.

e, ν both have spin 1/2

Total spin of eν system, $S_{e\nu}$ = 0 or 1

$$X \rightarrow Y + e + \nu \qquad J_X = J_Y \oplus S_{e\nu}$$

Fermi transitions $S_{e\nu} = 0$

$$n\uparrow \rightarrow p\uparrow + \underbrace{e^-\uparrow + \bar{\nu}_e\downarrow}_{S_{e\nu}=0} \qquad \Delta J = 0$$

$J_X = J_Y$ $S_{e\nu} = 0$

Gamow-Teller transitions $S_{e\nu} = 1$

$$n\uparrow \rightarrow p\uparrow + \underbrace{e^-\uparrow + \bar{\nu}_e\uparrow}_{S_{e\nu}=\pm 1} \qquad \Delta J = \pm 1$$

$J_X = J_Y$ $S_{e\nu} = \pm 1$

$$n\uparrow \rightarrow p\downarrow + \underbrace{e^-\uparrow + \bar{\nu}_e\uparrow}_{S_{e\nu}=\pm 1} \qquad \Delta J = 0$$

$J_X = J_Y \pm 1$ $S_{e\nu} = \pm 1$

$0 \rightarrow 0$ Forbidden

Total number of spin states of $e\nu$ = 4

(3 G-T, 1 Fermi)

No change in angular momentum of the $e\nu$ pair relative to the nucleus $\Rightarrow$ Parity unchanged

▸ Forbidden Transitions $\log_{10} f\tau_{1/2} \geq 6$

Angular momentum of the eν pair relative to the nucleus $\ell > 0$.

$$e^{-i(\vec{p}+\vec{q})\cdot\vec{r}} = 1 - i(\vec{p}+\vec{q})\cdot\vec{r} - \tfrac{1}{2}\left[(\vec{p}+\vec{q})\cdot\vec{r}\right]^2 - \ldots$$

ℓ	0	1	2	...
$P=(-1)^{\ell}$	even	odd	even	...
	Allowed	1st forbidden	2nd forbidden	

Transition probabilities for large ℓ are small
$\Rightarrow$ forbidden transitions.

Forbidden transitions are only competitive if an allowed transition cannot occur (selection rules). The lowest permitted order of "forbiddeness" will dominate.

In general,

n^{th} forbidden $\Rightarrow$ eν system carries orbital angular momentum $\ell = n$, and $S_{e\nu}$ 0 (Fermi) or 1 (G-T).

Examples

$$^{34}Cl(0^+) \rightarrow {}^{34}S(0^+)$$

$$^{14}C(0^+) \rightarrow {}^{14}N(1^+)$$

$$n(1/2^+) \rightarrow p(1/2^+)$$

$$^{39}Ar(7/2^-) \rightarrow {}^{39}K(3/2^+)$$

$$^{87}Rb(3/2^-) \rightarrow {}^{87}Sr(9/2^+)$$

Parity Violation in β Decay

Parity is conserved in nuclear interactions.

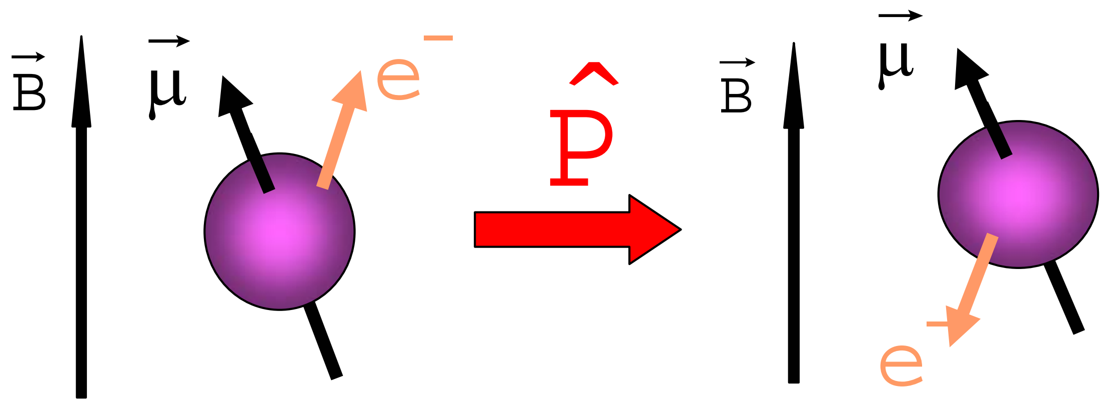
β decay Co^{60} Wu et. al. Phys. Rev. 105 (1957) 1413

$$Co^{60} \rightarrow Ni^{60} + e^- + \bar{\nu}_e$$

$J^P=5^+$ $J^P=4^+$

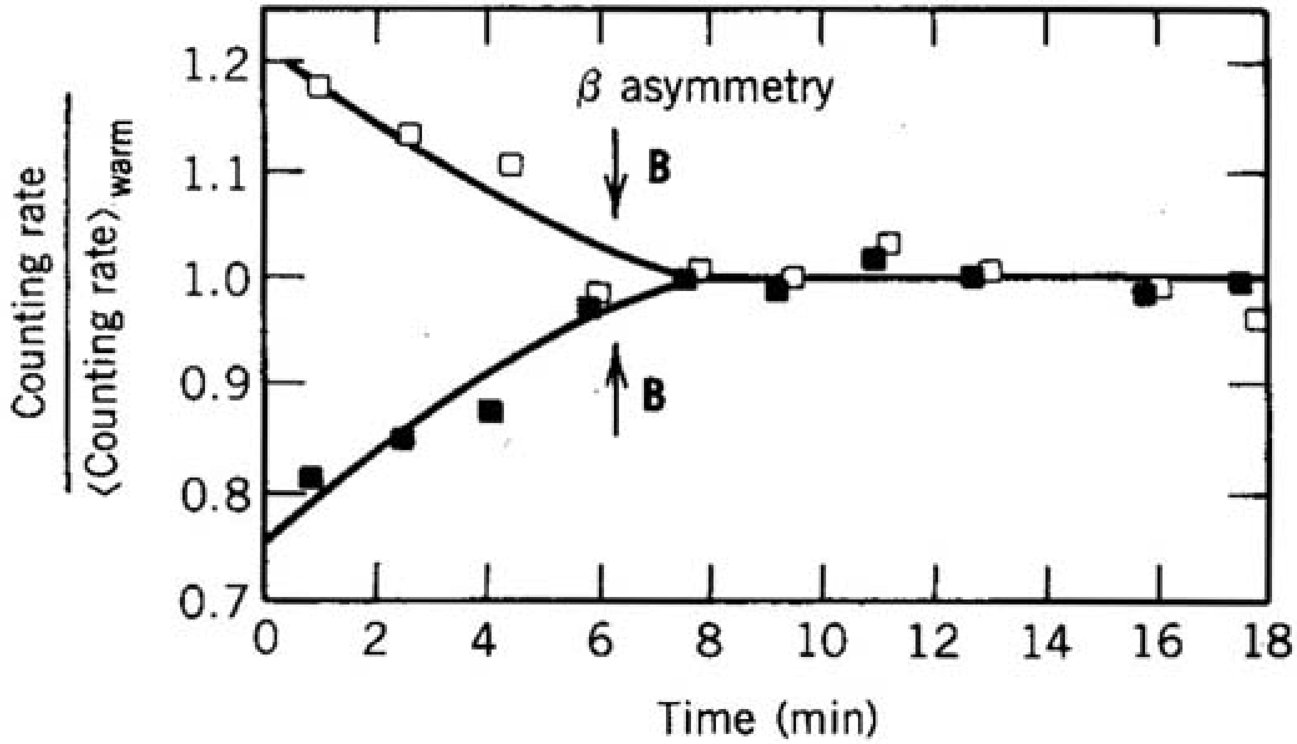
$\Delta J=1$ no parity
Pure G-T transition

Look at direction of emission of electrons



Under parity $\vec{r} \rightarrow -\vec{r}$ and $\vec{p} \rightarrow -\vec{p}$ but $\vec{\mu}$ does not change $\left(\vec{L} = \vec{r} \times \vec{p} \rightarrow -\vec{r} \times -\vec{p} = \vec{L}\right)$

If parity is conserved, expect equal numbers of electrons parallel and antiparallel to $\vec{B}$

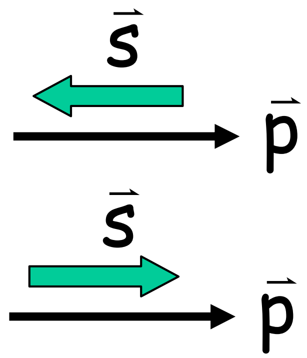


➡ Parity violation in the weak interaction

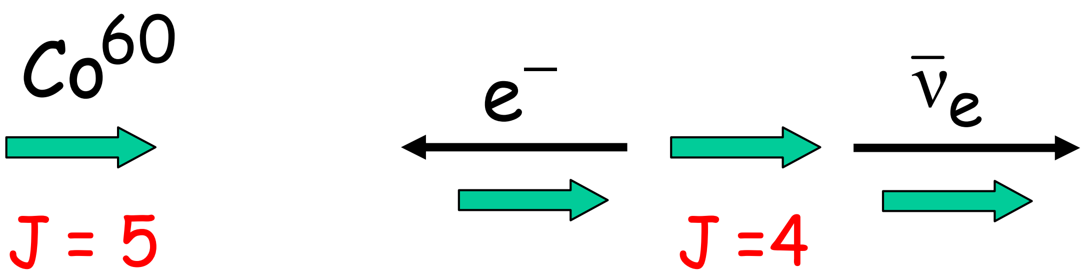
The weak interaction couples preferentially to

left-handed particles

right-handed antiparticles



Hence,



is preferred in order to conserve a.m.

γ Decay

Emission of γ-rays (electromagnetic radiation) occur when a nucleus is formed in an excited state (e.g. after α, β decay or collision).

The treatment of radiative transitions in nuclei is generally the same as for atoms, except

Atom $E\gamma \sim$ eV $\lambda \sim 10^8$ fm $\Gamma \sim 10^9\ s^{-1}$

Only dipole transitions are important

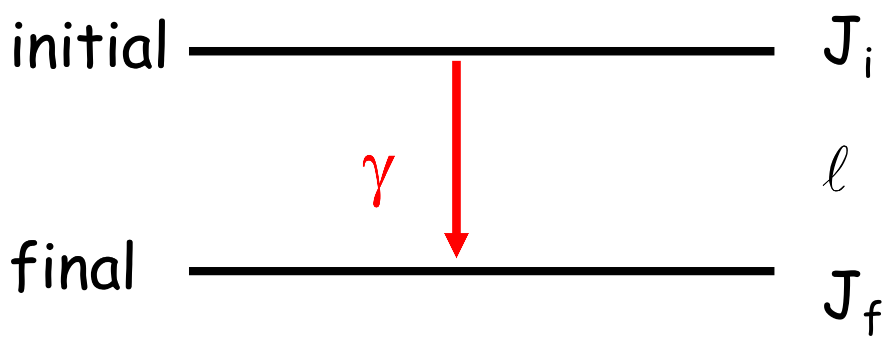
Nuclei $E\gamma \sim$ MeV $\lambda \sim 10^2$ fm $\Gamma \sim 10^{16}\ s^{-1}$

Higher order transitions also important. Collective motion of many p's lead to higher transition rates.

Two types of transitions:

Electric (E) transitions arise from an oscillating charge which causes an oscillation in the external electric field.

Magnetic (M) transitions arise from a varying current or magnetic moment which sets up a varying magnetic field.



- In the simplest case, the photon carries away angular momentum ℓ when a proton in the nucleus makes a transition from its initial a.m. state J_i to its final a.m. state J_f.

$$\vec{J}_i = \vec{\ell} \oplus \vec{J}_f$$

and
$$|J_i - J_f| \leq \ell \leq |\vec{J}_i + \vec{J}_f|$$

- The photon has intrinsic $J^P = 1^- \Rightarrow \ell \geq 1$

$\therefore$ Single γ emission is forbidden for a transition between two J=0 states.

$0 \rightarrow 0$ transitions can only occur via internal conversion or emission of more than 1 γ.

- The transition probabilities obtained using

Transition Rate
$$\Gamma_{i \rightarrow f} = \frac{2\pi}{\hbar} |M|^2 \rho(E_f)$$
Fermi Golden Rule

$M = \langle \psi_f | \hat{H} | \psi_i \rangle$ and $\rho(E_f)$ = density of final states

Electric dipole radiation (E1) $\ell = 1$

Insert dipole matrix element into FGR

$$\Gamma_{i\to f} = \frac{\omega^3}{3\pi\varepsilon_0 c^3 \hbar}\left|\langle \psi_f | e\vec{r} | \psi_i \rangle\right|^2$$

see QM II after averaging over initial and summing over final states

For an order of magnitude estimate of this rate,

$$\left|\langle \psi_f | e\vec{r} | \psi_i \rangle\right|^2 \approx |eR|^2$$

R = radius of nucleus

$$\Gamma = \frac{4}{3}\alpha E_\gamma^3 R^2$$

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \qquad E_\gamma = \hbar\omega$$

$$\hbar = c = 1$$

E_γ = 1 MeV, R = 5 fm

$$\Gamma(E1) = 0.24 \ \text{MeV}^3\text{fm}^2$$

$$= \frac{0.24}{(197)^2\, 6.6\times 10^{-22}} \ s^{-1}$$

$$= 10^{16} \ s^{-1}$$

$\hbar c$ = 197 MeVfm

$\hbar$ = 6.6 × 10^{-22} MeVs

(c.f. atoms $\Gamma \sim 10^9\ s^{-1}$)

As nuclear wavefunctions have definite parity, the matrix element can only be non-zero if the initial and final states have opposite parity

$$e\vec{r} \xrightarrow{\hat{P}} -e\vec{r}$$

$\therefore$ E1 transition $\Rightarrow$ parity change of nucleus

Magnetic dipole radiation (M1) $\ell = 1$

Matrix element

$$\left|\langle \psi_f | \mu\sigma | \psi_i \rangle\right|^2$$

μ = magnetic moment, σ = Pauli spin matrix

Typically, $\langle \mu\sigma \rangle \sim \dfrac{e\hbar}{2m_p} = \mu_N$ Nuclear magneton

Proton

$$\lambda = \frac{\hbar}{m_p} \sim 0.2\,\text{fm} \qquad \sim \frac{R}{25} \quad \text{for } R = 5\,\text{fm}$$

$$\therefore \frac{\Gamma(M1)}{\Gamma(E1)} = \left(\frac{e\hbar}{2m_p}\right)^2 \frac{1}{e^2 R^2} = \underline{10^{-3}}$$

Magnetic moment transforms as angular momentum

$$e\vec{r} \times \vec{v} \xrightarrow{\hat{P}} e(-\vec{r}) \times (-\vec{v}) = e\vec{r} \times \vec{v} \qquad \text{even}$$

$\therefore$ <u>M1 transition $\Rightarrow$ no parity change of nucleus</u>

Higher order decays (Eℓ, Mℓ where ℓ > 1)

If the initial and final nuclear states differ by more than 1 unit of angular momentum

$\Rightarrow$ Higher multipole radiation

The perturbing Hamiltonian is a function of electric and magnetic fields and hence of the vector potential $\vec{A}$

$$\langle \psi_f | H'(\vec{A}) | \psi_i \rangle$$

$\vec{A}$ for a photon is taken to have the form of a plane wave

$$A e^{-i\vec{k}\cdot\vec{r}} = 1 - i\vec{k}\cdot\vec{r} - \frac{1}{2}(\vec{k}\cdot\vec{r})^2 + \ldots \frac{(-i\vec{k}\cdot\vec{r})^n}{n!}$$

Dipole Quadrupole Octopole…

ℓ	1	2	3
	E1	E2	E3
	M1	M2	M3

Each successive term in $\vec{A}$ is reduced from the previous one approx by a factor kR.

For k ~ 1 MeV, R~5 fm kR~5 MeVfm ~ 0.025

$|kR|^2 \sim 10^{-3}$

$$\therefore \frac{\Gamma(E2)}{\Gamma(E1)} \sim 10^{-3} \sim \frac{\Gamma(M1)}{\Gamma(E1)}$$

The matrix element for E2 transitions ~ r^2
i.e. even under a parity transformation

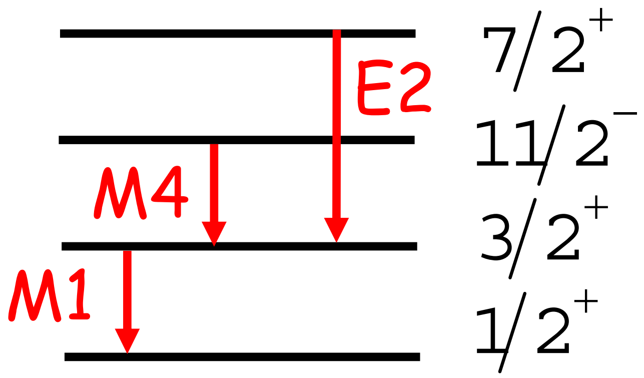
In general, $E\ell$ transitions Parity = $(-1)^{\ell}$
$M\ell$ transitions Parity = $(-1)^{\ell+1}$

Rate	1	10^{-3}	10^{-6}	10^{-9}...
	E1	E2	E3	E4 ...
		M1	M2	M3 ...
Parity change	✓	✗	✓	✗

In general, a decay will proceed dominantly by the highest order process permitted by angular momentum and parity.

e.g. if a process has $\Delta J = 2$, no parity change, it will go by E2, even though M3, E4 are also allowed.

Example $^{117}_{50}\mathrm{Sn}$



$3/2^+ \rightarrow 1/2^+$ M1 (E2 also allowed)

$11/2^- \rightarrow 3/2^+$ M4

more likely than $11/2^- \rightarrow 1/2^+$ (E5)

$7/2^+ \rightarrow 3/2^+$ E2

M2 } less likely $7/2^+ \rightarrow 11/2^-$

M3 } less likely $7/2^+ \rightarrow 1/2^+$

Information about nature of transitions is useful in inferring J^P values of states.

This discussion of rates is very naïve. More complete formulae can be found in books.

Also collective effects may be important

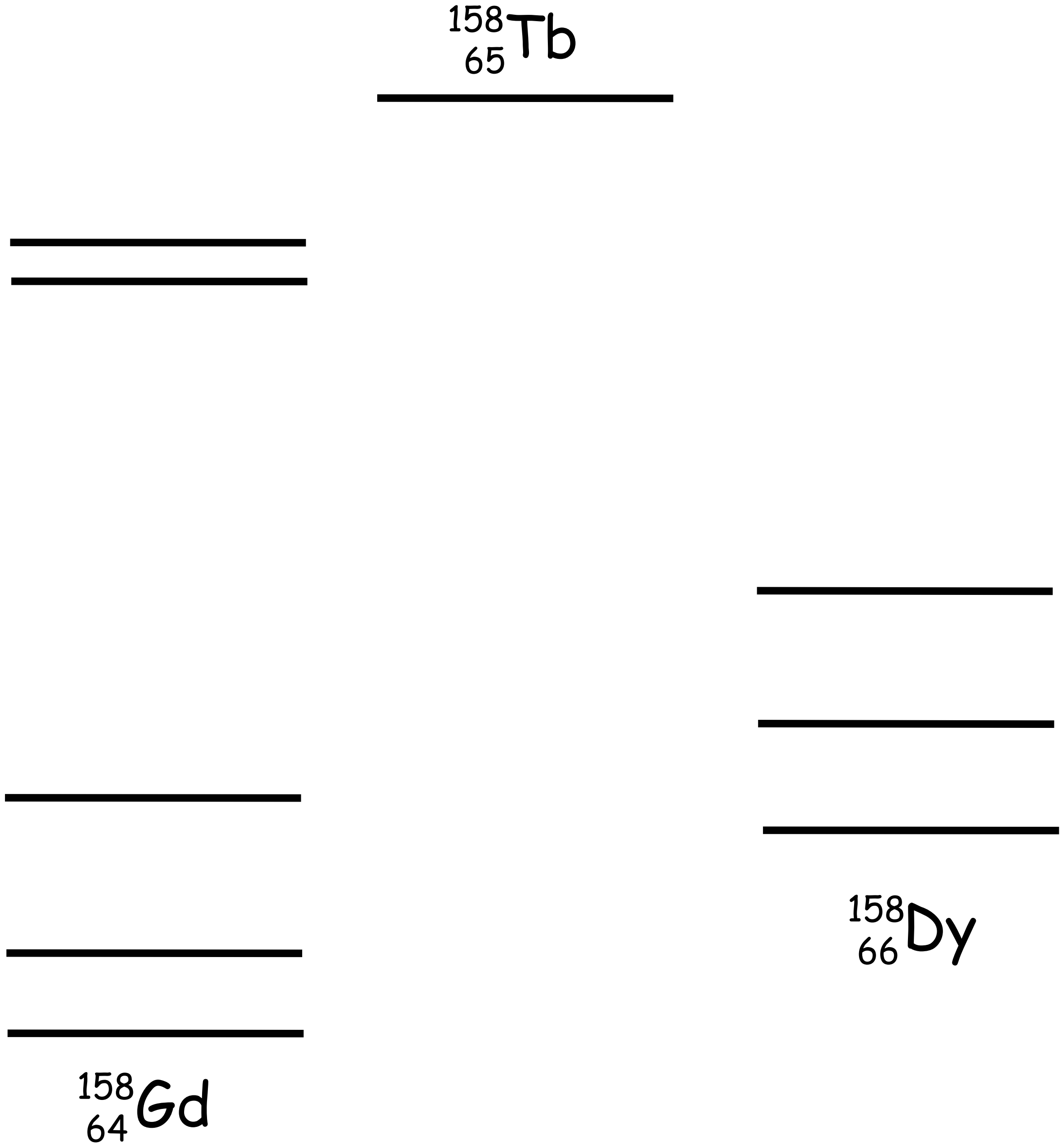
- many nucleons participate in transitions.
- If nucleus has a large Q $\rightarrow$ rotational excited states enhance E2 transitions

Example: Part II Paper 3 Question C12 (1999)

The nucleus $^{158}_{65}Tb$ undergoes β^- decay to $^{158}_{66}Dy$. Careful study of the β^- spectrum reveals the presence of two components, with endpoint energies E_0. $^{158}_{65}Tb$ also undergoes electron capture (EC) to four excited states of $^{158}_{64}Gd$. The relative strength f of each component, and the energies $E\gamma$ of the corresponding associated γ ray(s) are:

Decay	f	E_0/MeV	$E\gamma$/MeV
β^-	1%	0.628	0.218, 0.099
β^-	13%	0.845	0.099
EC	38%		0.963,0.781,0.182,0.079
EC	41%		0.945, 0.079
EC	3%		0.182, 0.079
EC	4%		0.079

1) Indicate the observed β and γ transitions.
2) What is the likely nature of the lowest lying excitations of the Dy and Gd nuclei, and hence what are their likely spin parity values?
3) Using the relevant selection rules, make spin parity assignments to all the levels involved.
4) Explain your reasoning, specifying the nature of each of the γ transitions (i.e. E1, M1 etc) and the β transitions (i.e. allowed, forbidden).


$^{158}_{65}Tb$
$^{158}_{66}Dy$
$^{158}_{64}Gd$

Internal Conversion

The direct transfer of nuclear excitation energy to atomic electrons (internal conversion) competes with γ decay.

The atomic electron is emitted with energy

$$T_e = Q - B_e$$

Q = excitation energy
B_e = atomic B.E. of e^-

Observe X-rays or Auger electrons as vacancy is filled. Electrons emitted from different atomic shells appear at different energies. The effect on the nucleus is same as for γ emission.

Internal Conversion Coefficient

$$\alpha = \frac{\text{Rate } (A^* \to A + e^-)}{\text{Rate } (A^* \to A + \gamma)} = \frac{\Gamma_e}{\Gamma_\gamma}$$

$\alpha \sim 10^{-2}$-10^{-4} for few 100 keV < Q < few MeV

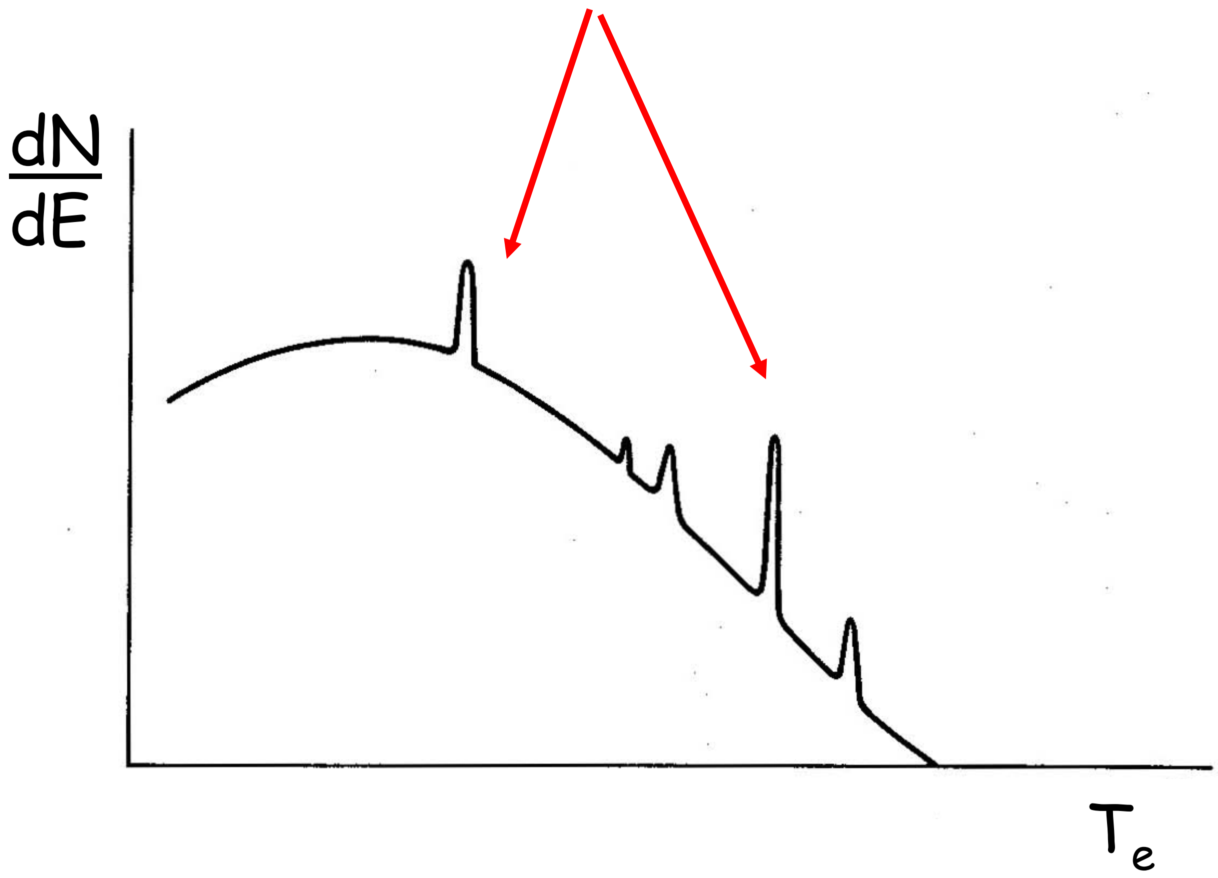
Main characteristics: $\alpha \sim Z^3$

α decreases rapidly with n
α decreases with Q
α increases with multipolarity

$J^P = 0^+ \to 0^+$ transitions (e.g. ^{16}O, ^{90}Zr) even-even nuclei are allowed by internal conversion.

β decay spectrum:

See internal conversion lines superimposed on continuous spectrum.



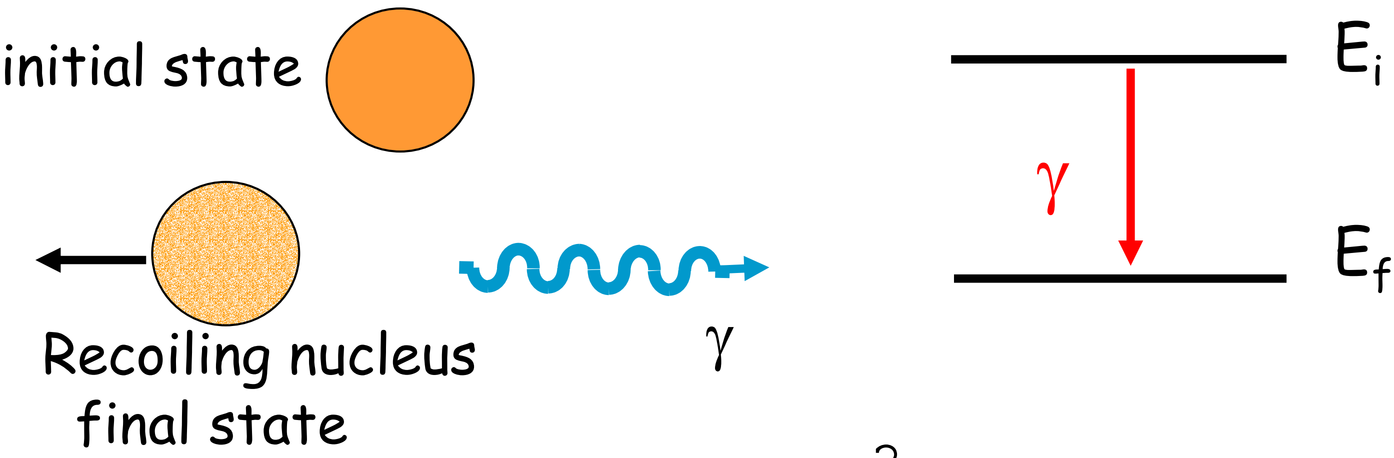
$$n \rightarrow p + e^- + \bar{\nu}_e$$

The Mössbauer Effect

Measurement of small energy differences with extremely high precision using the natural width of nuclear states.

Basic idea:

Nucleus in excited state emits γ to g.s.



$$E_i = E_f + E_\gamma + \frac{p^2}{2m}$$

p = nucleus recoil momentum = $p_\gamma = E_\gamma$

$$E_\gamma = \Delta E - \frac{E_\gamma^2}{2m}$$

c = 1

$\Delta E = E_i - E_f$

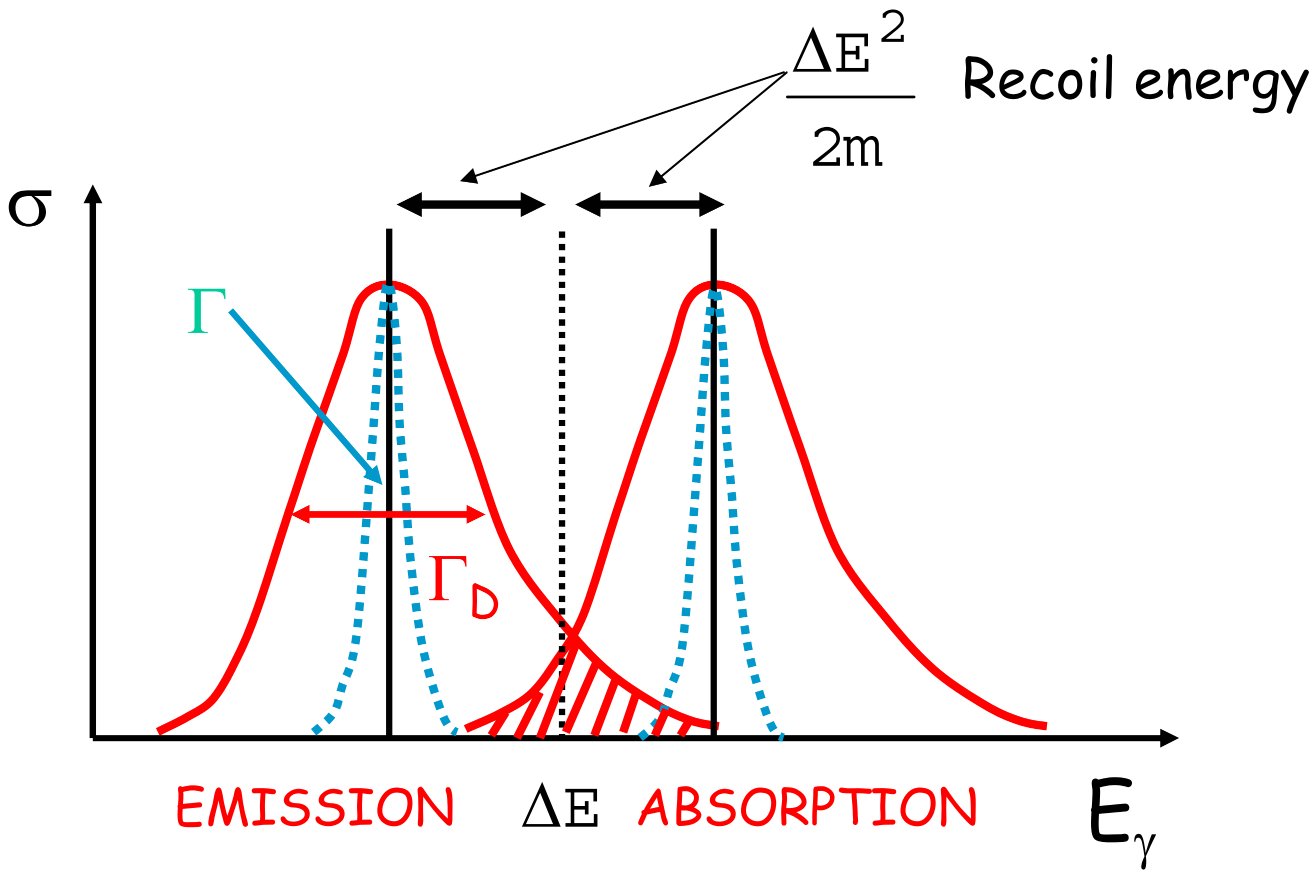
$E_\gamma \approx \Delta E$

$$E_\gamma \approx \Delta E - \frac{\Delta E^2}{2m}$$

Emission

Similarly, for γ absorption

$$E_\gamma \approx \Delta E + \frac{\Delta E^2}{2m}$$



- Eγ varies due to natural width of energy levels

e.g. $^{191}_{77}\mathrm{Ir}$ $\tau \sim 1.4 \times 10^{-10}$ s, $E_i = 0.13$ MeV

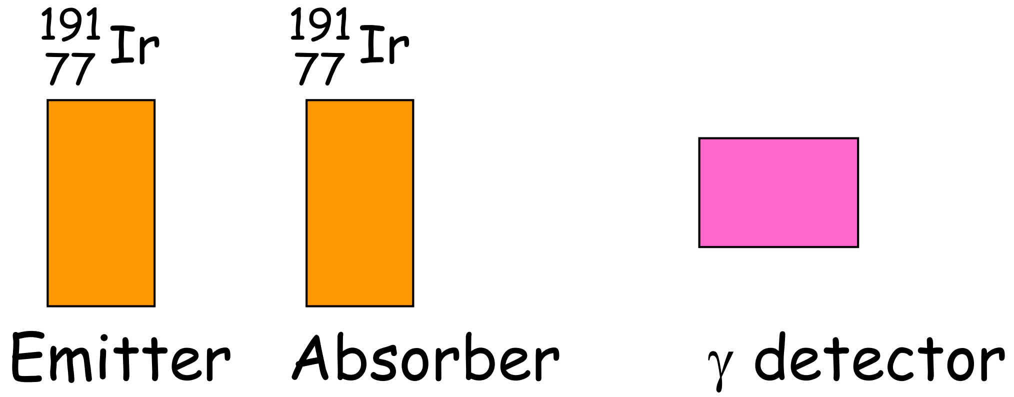
$$\Gamma = \frac{\hbar}{\tau} = \underline{5\times10^{-6}\ \mathrm{eV}} \qquad \frac{\Gamma}{E_i} \sim 10^{-11}$$

- Initial state is in thermal motion ⇒ Doppler shift

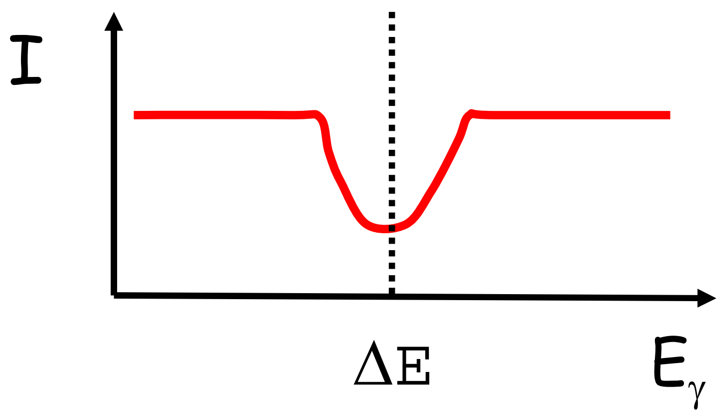
$$\Gamma_D \sim \underline{\text{few } 10^{-1}\ \mathrm{eV}} \gg \Gamma$$

room temperature

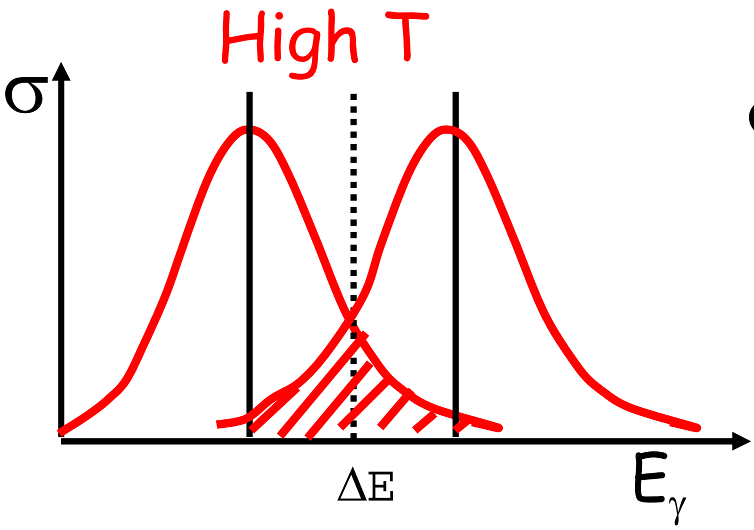
Mössbauer Experiment (1958)

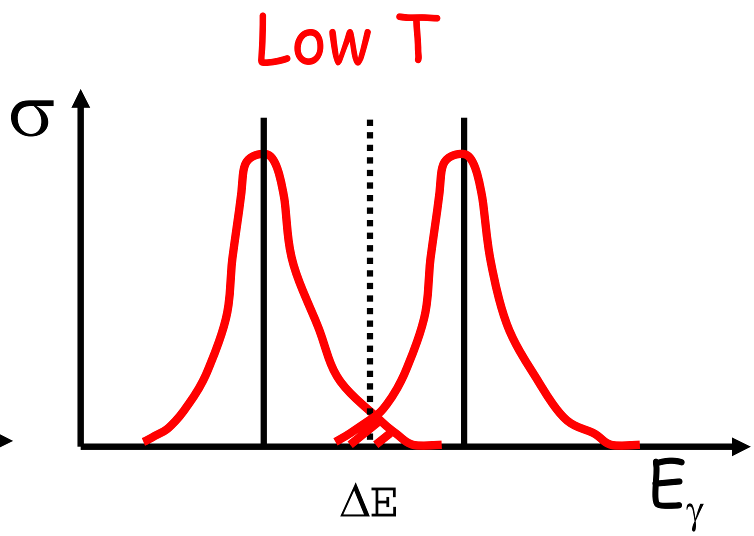


Absorption of γ's can only occur for energies in the overlap region. If absorption occurs, re-emission is isotropic $\therefore$ expect reduced intensity



Expect



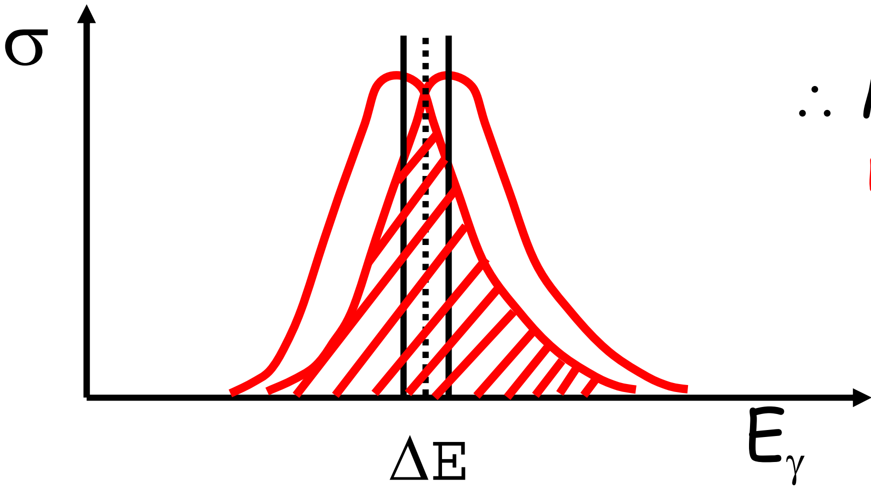


High T $\rightarrow \Gamma_D$ increases
Larger overlap
More absorption

Low T $\rightarrow \Gamma_D$ decreases
Smaller overlap
Less absorption

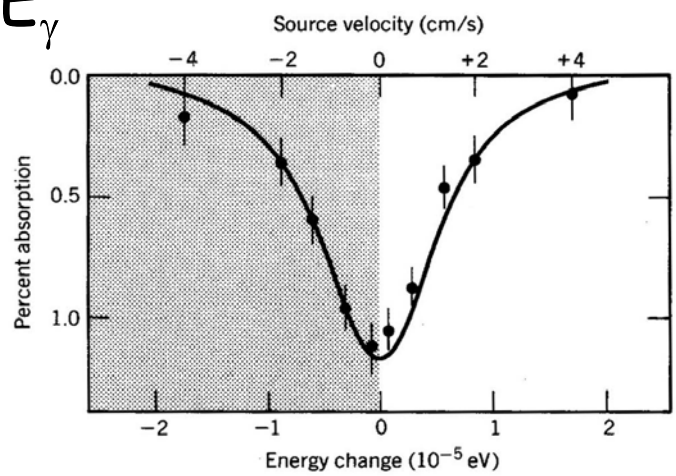
However, Mössbauer found that at low temp the absorption increased.

Γ_D reduced BUT nucleus bound in crystal lattice



$\therefore M_{nucleus} \rightarrow M_{crystal}$
negligible recoil

Mössbauer effect



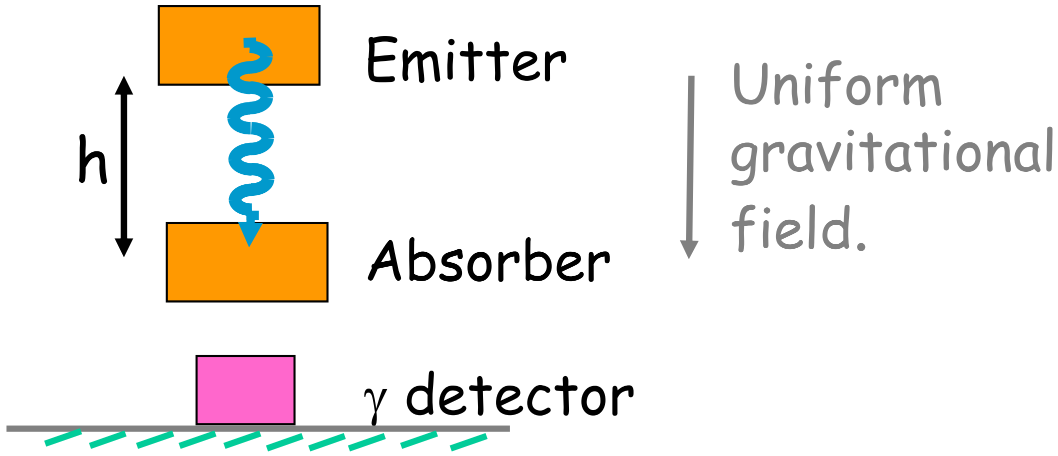
The Mössbauer effect can be used to measure energy differences ~ same order as width of the resonance (e.g. 10^{-5} eV Ir, 10^{-8} eV Fe).

Most applications determine the properties of the physical or chemical environment of a nucleus. When the emitter and absorber are in different environments, the emission and absorption peaks do not occur at precisely the same energy. The relative velocity required to obtain maximum absorption is measured.

Gravitational Red Shift

Principle of Equivalence:

Effects of a local uniform gravitational field cannot be distinguished from those of a uniformly accelerated reference frame.



Nucleus in emitter has additional

P.E. = mgh = Egh/c²

$$E_i\left(1+\frac{gh}{c^2}\right)-E_f\left(1+\frac{gh}{c^2}\right)=\Delta E\left(1+\frac{gh}{c^2}\right)$$

Radiated photons are Doppler red-shifted

$$\frac{\Delta E_\gamma}{E_\gamma}=\frac{gh}{c^2}\sim 10^{-16}\ \mathrm{m}^{-1}$$

Experiment (Harvard-Tower: 1960 Pound and Rebka)

^{57}Fe sensitivity $\Gamma/E\gamma \sim 3\times10^{-13}$

$$\frac{\Delta E_\gamma}{E_\gamma}=(4.902\pm0.041)\times10^{-15}$$

(c.f. 4.905×10^{-15} expected)

One of the most precise tests of GR