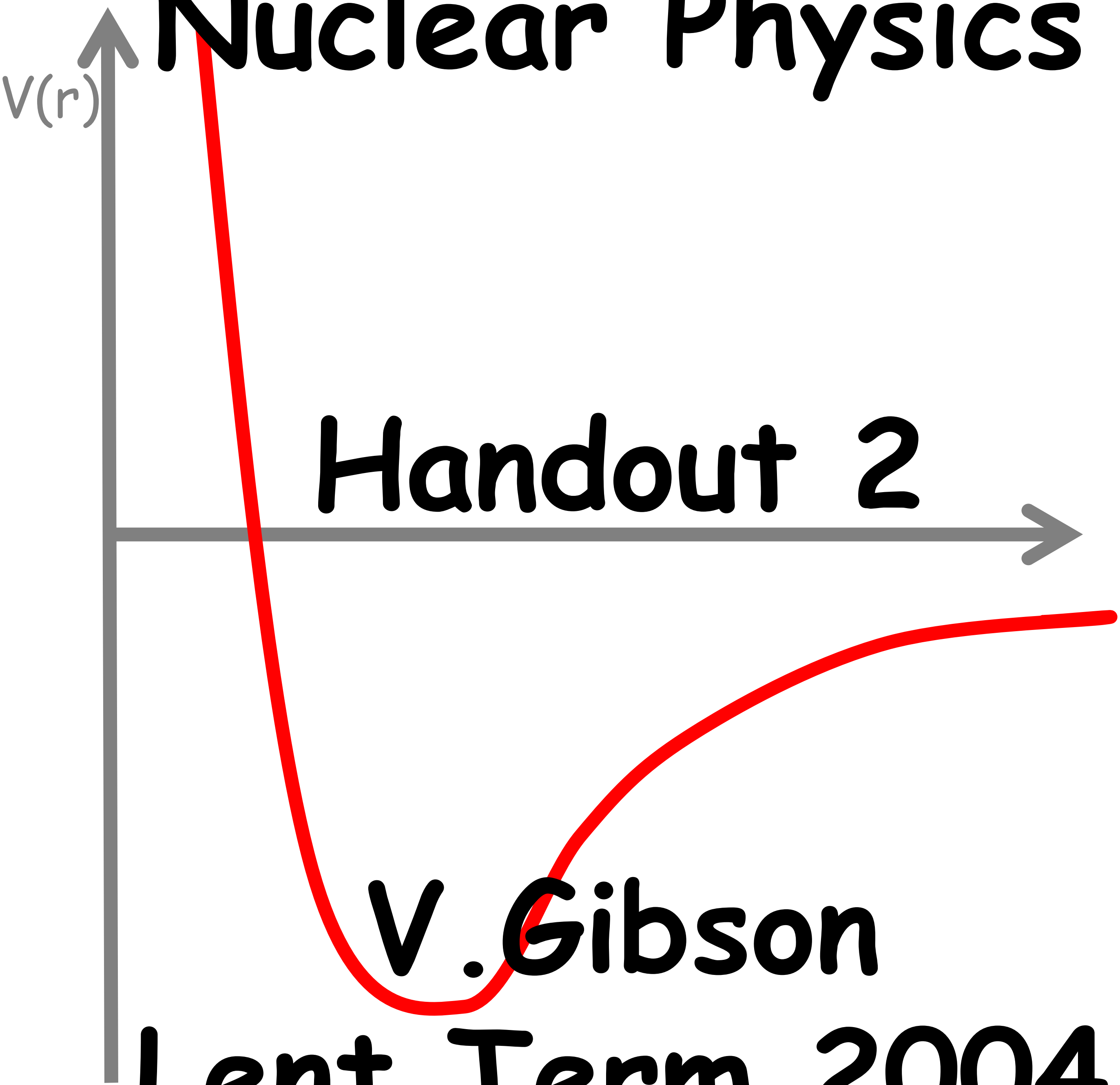
Part II
Nuclear Physics

Handout 2



V.Gibson

Lent Term 2004

Section III
The Nucleon Force

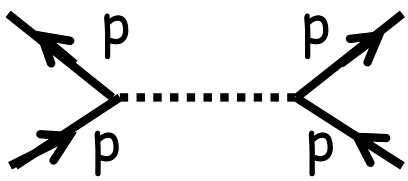
Nucleons are made of spin 1/2 point-like quarks.

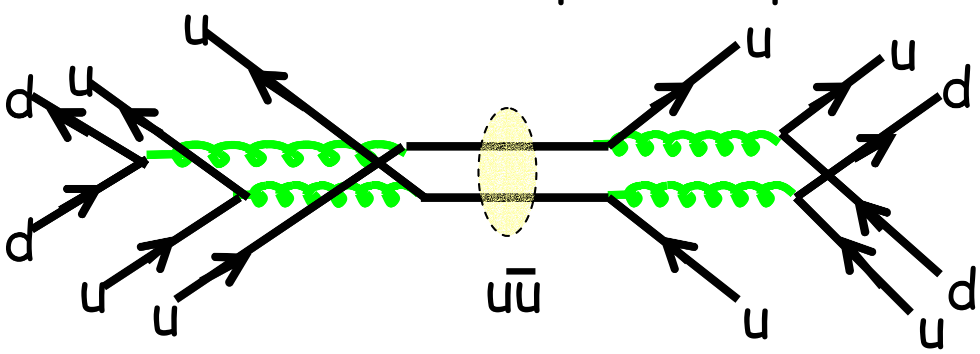
Quarks are held together by the strong interaction arising from the exchange of other quarks and spin 1 gluons (see Particles course).

The force between nucleons (the strong nuclear force) is a many-body problem in which

- quarks do not behave as if they were completely independent inside the nuclear volume
- nor do they behave as if they were completely bound to form protons and neutrons.

e.g. p-p interaction





The nuclear force is therefore not calculable in detail at the quark level and can only be deduced empirically from nuclear data.

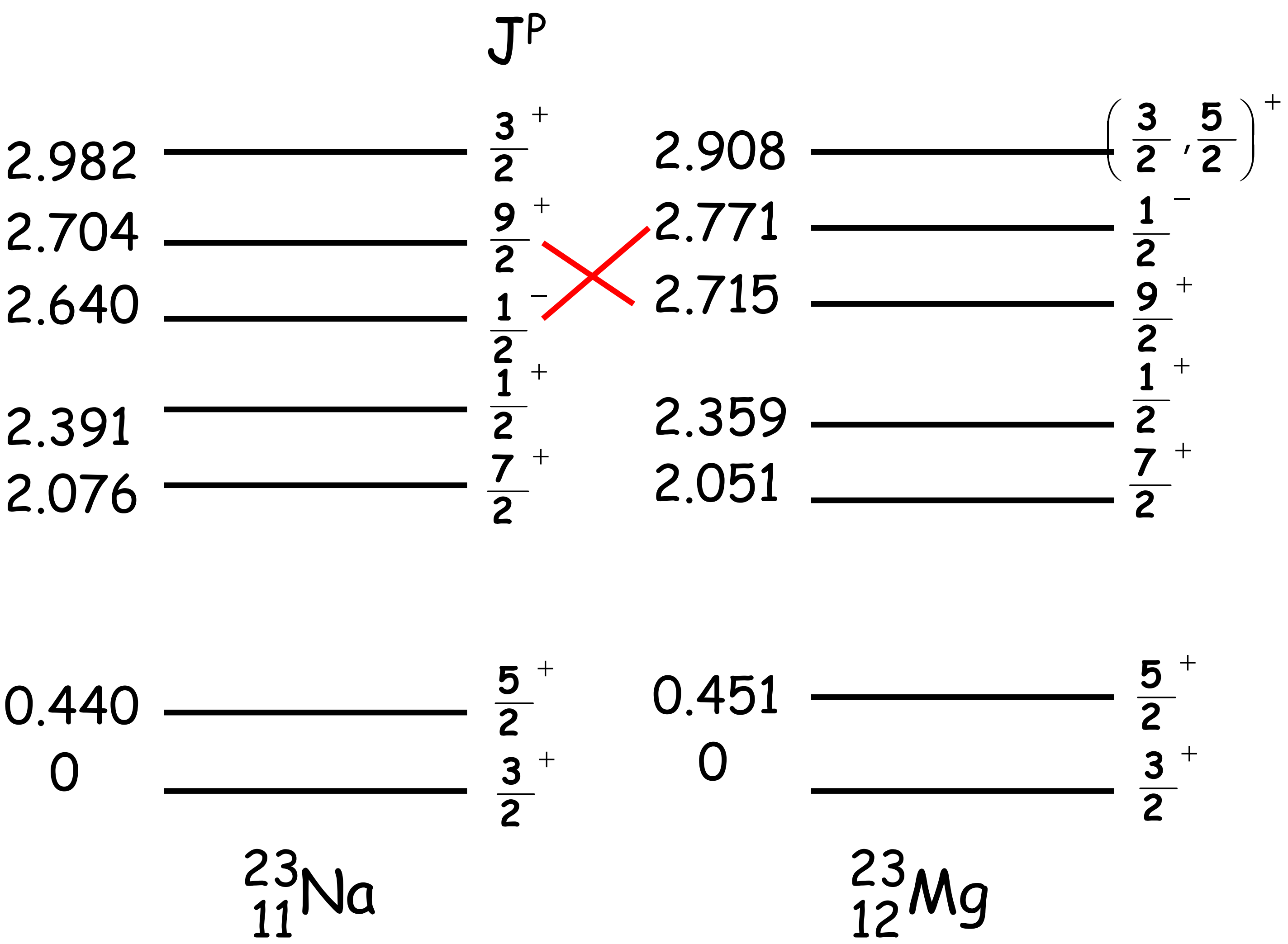
General Features

The fact that a nucleus exists implies that the nuclear force is:

- Strong: stronger than the electromagnetic, weak and gravitational forces.
- Short range: nuclei experience the strong interaction at short distances (~2 fm) as they start to overlap.
- Attractive.
- Repulsive core: Volume~A, nucleus doesn't collapse to ∞ density.
- Saturates: B/A~constant; in a nucleus the nucleons are only attracted by nearby nucleons.
- Charge independent: No distinction between protons and neutrons. Evidence seen from tendency for small nuclei to have N=Z and similarities of the low-lying energy levels of pairs of mirror nuclei.

Mirror Nuclei

Example $({}^{23}_{11}Na,\ {}^{23}_{12}Mg)$

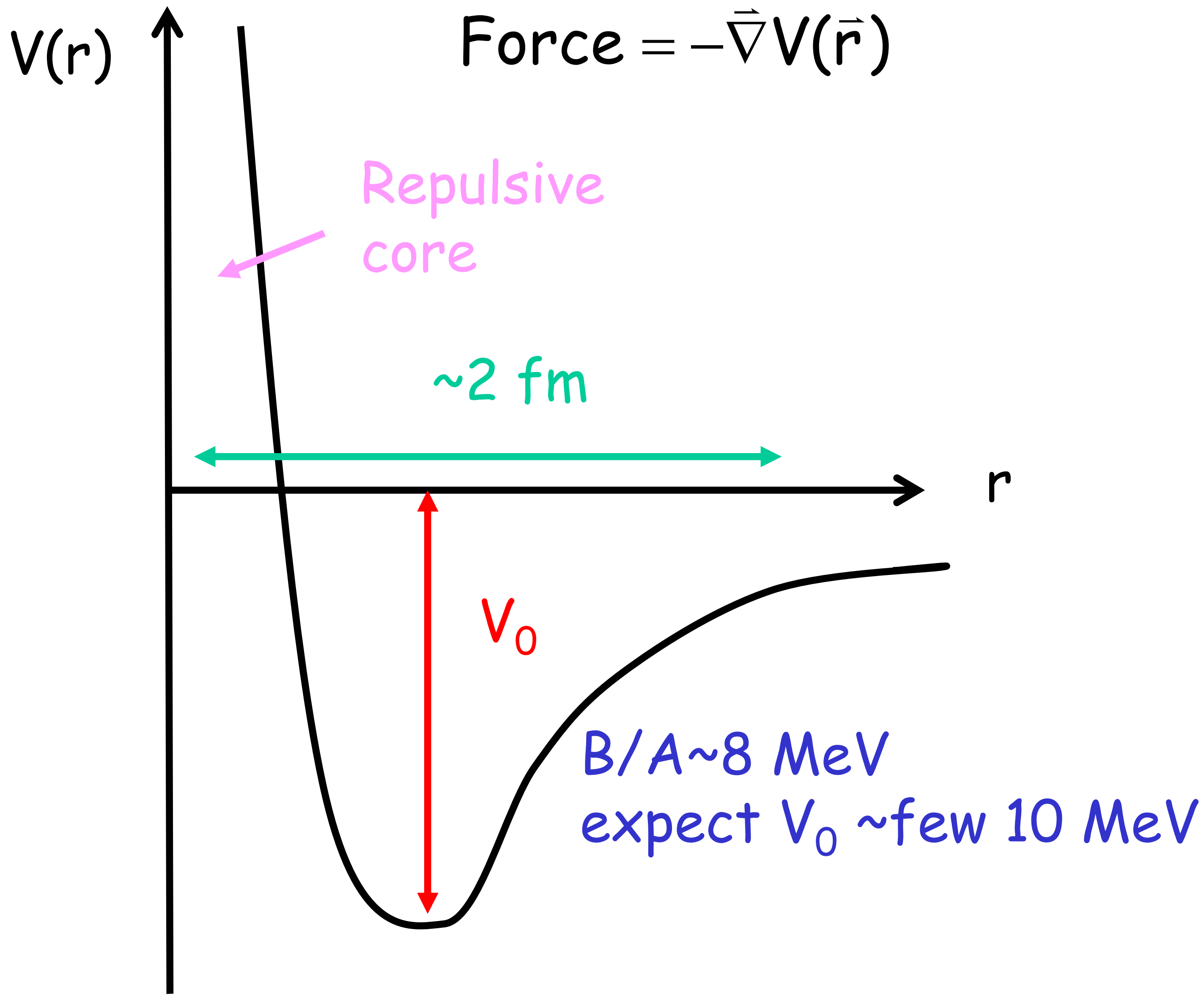


Illustrates charge symmetry,

p-p interaction ≡ n-n interaction

Does not imply p-n = p-p or n-n because the number of p-n pairs is the same in both nuclei.

The Nucleon-Nucleon Potential



Study detailed features using interactions between two nucleons:

The deuteron and nucleon-nucleon scattering.

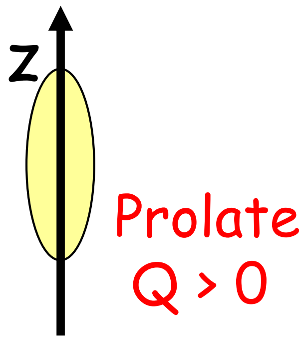
The Deuteron

The deuteron is the only two nucleon (n-p) bound state (no p-p or n-n bound states).

^{2}H or ^{2}D

Property summary:

$B = 2.23$ MeV
$J^P = 1^+$
$\mu = +0.857\ \mu_N$
$Q = +2.82 \times 10^{-31}\ m^2$
$R = 2.1$ fm
No excited states observed



Deductions:

n-p state

$^3S_1$ (ℓ=0, S=1 $\uparrow\uparrow$) : $\mu = \mu_p + \mu_n = 0.88\ \mu_N$

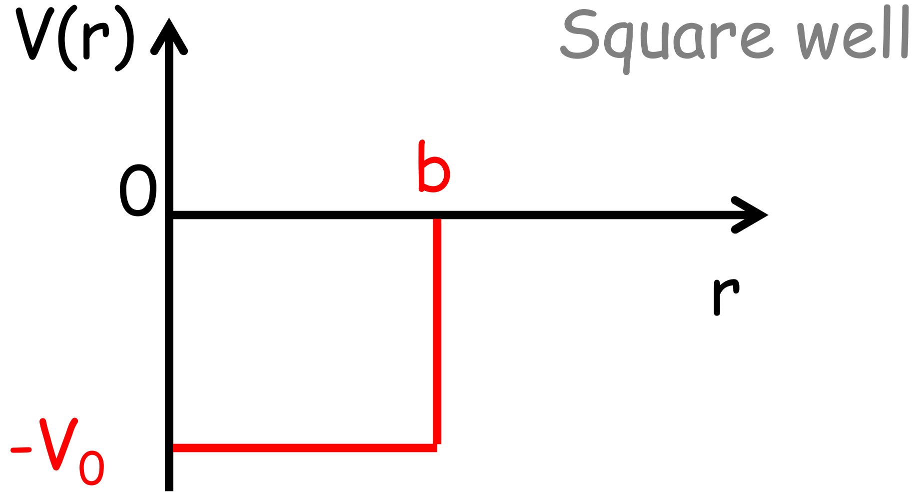
$^1S_0$ (ℓ=0, S=0 $\uparrow\downarrow$) : $\mu = \mu_p - \mu_n = 4.71\ \mu_N$

Experimental value $\mu = +0.857\ \mu_N$

$\Rightarrow$ n=1, no orbital contributions to μ (ℓ=0)

Deuteron is a $^3S_1$ state.

Assume simple V(r)



Consider radial Schrödingers equation (ℓ=0):

$$\left[\frac{-\hbar^2}{2M}\frac{d^2}{dr^2}+V(r)\right]R(r)=ER(r)$$

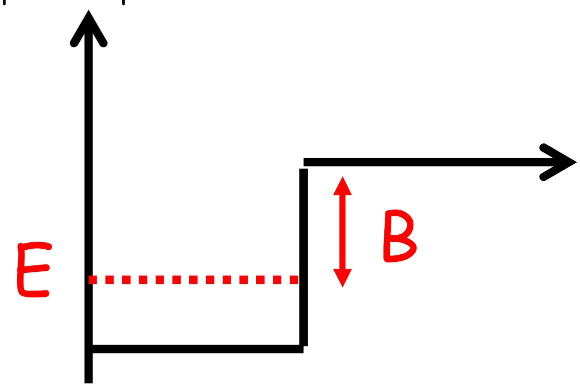
where $M = \textbf{reduced mass} = \dfrac{m_p m_n}{m_p+m_n}$

Let $u(r) = rR(r)$ r = internucleon distance

Probability particle between r and r+dr=

$$r^2|R(r)|^2dr = |u(r)|^2 dr$$

For bound state E < 0
= -Binding energy



Two regions:

1) $r < b$ $V = -V_0$, $E = -B$

2) $r > b$ $V = 0$, $E = -B$

1) $r < b$

$$\frac{-\hbar^2}{2M}\frac{d^2u(r)}{dr^2} + (B - V_0)u(r) = 0$$

General solution $u(r) = A\sin\alpha r + C\cos\alpha r$

$$\alpha^2 = \frac{2M}{\hbar^2}(V_0 - B)$$

Require $R(r) = \frac{u(r)}{r}$ finite for $r \to 0 \Rightarrow C=0$

(i.e. don't want infinite density $|R(r)|^2$ at centre of nucleus.)

$$\therefore u(r) = A\sin\alpha r \qquad r < b$$

2) $r > b$

$$\frac{-\hbar^2}{2M}\frac{d^2u(r)}{dr^2} + Bu(r) = 0$$

General solution $u(r) = De^{-\beta r} + Fe^{+\beta r}$

$$\beta^2 = \frac{2MB}{\hbar^2}$$

$r \to \infty$, $e^{+\beta r} \to \infty \Rightarrow F=0$

$$\therefore u(r) = De^{-\beta r} \qquad r > b$$

r=b u(r) and du(r)/dr continuous

u(r) $$A\sin\alpha b = D e^{-\beta b}$$

du(r)/dr $$\alpha A\cos\alpha b = -\beta D e^{-\beta b}$$

Ratio $$\cot\alpha b = -\frac{\beta}{\alpha} = -\left(\frac{B}{V_0 - B}\right)^{1/2}$$

Assume $V_0 > B$: 2 unknowns b, V_0

$$\cot\alpha b \approx 0 \qquad \alpha b = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

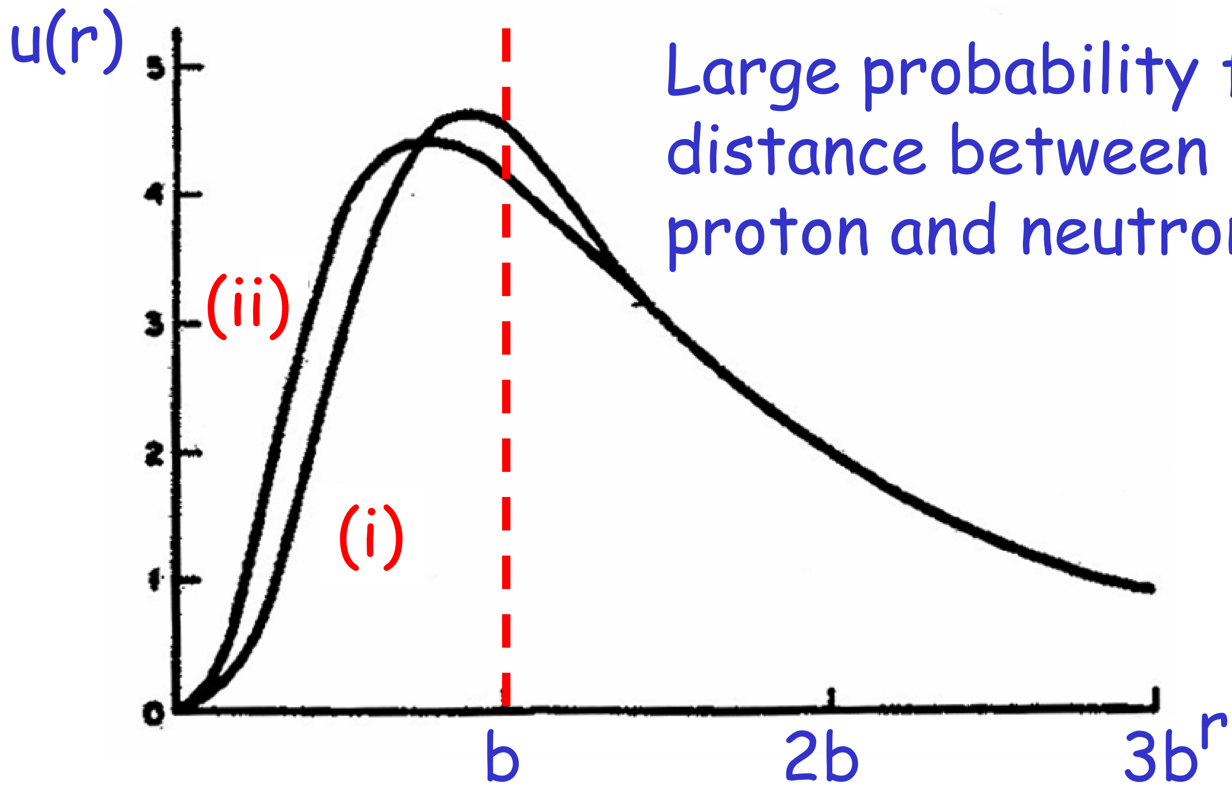
lowest energy

$$\frac{2M}{\hbar^2} V_0 b^2 = \left(\frac{\pi}{2}\right)^2$$

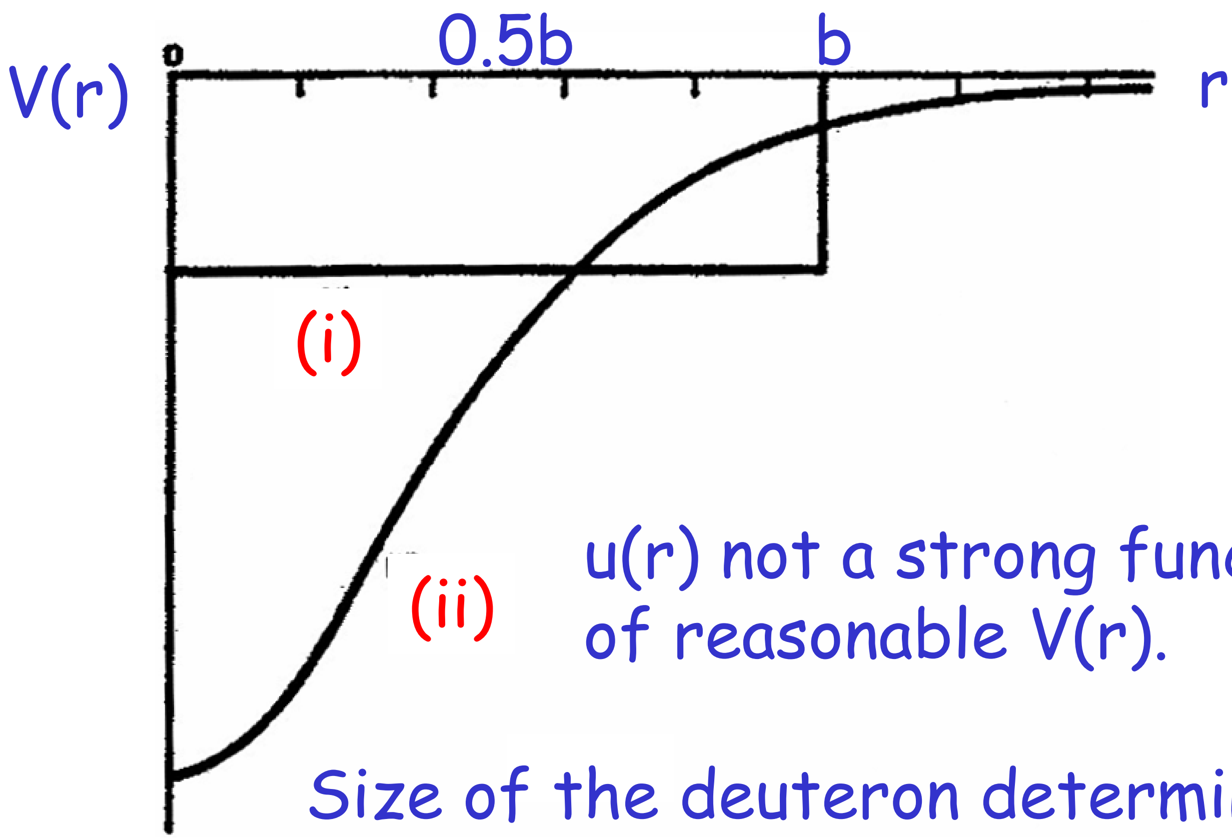
$$V_0 b^2 \approx \frac{\pi^2\hbar^2}{8M} \approx 10^{-28} \text{ MeVm}^2$$

For b = 2 fm $\Rightarrow V_0 \sim 25$ MeV

(c.f. B/A~8 MeV)



Large probability for distance between proton and neutron > b.



u(r) not a strong function of reasonable V(r).

Size of the deuteron determined by Binding Energy not range of force.

Spin Dependence

If there were no spin dependence in the deuteron potential, expect to observe both J=0 and J=1 bound states with same energy.

Only J=1 states is observed (↑↑)

Also no n-n and p-p bound states are observed which would require spins to be ↑↓ due to exclusion principle.

Require angular momentum and parity to be conserved

$\Rightarrow$ scalar potential, simplest $\sim \vec{s}_1 \cdot \vec{s}_2$

Deuteron spin

$$\vec{J} = \vec{s}_1 + \vec{s}_2$$

$$\vec{J}^2 = (\vec{s}_1 + \vec{s}_2)^2$$

$$J(J+1)\hbar^2 = s_1(s_1+1)\hbar^2 + s_2(s_2+1)\hbar^2 + 2\vec{s}_1 \cdot \vec{s}_2$$

$$\langle \vec{s}_1 \cdot \vec{s}_2 \rangle = \frac{1}{2}\left[J(J+1) - s_1(s_1+1) - s_2(s_2+1)\right]\hbar^2$$

J=1, $s_1=s_2=1/2$ $\quad \langle \vec{s}_1 \cdot \vec{s}_2 \rangle = \frac{\hbar^2}{4}$

J=0, $\quad \langle \vec{s}_1 \cdot \vec{s}_2 \rangle = -\frac{3\hbar^2}{4}$

Different potentials for singlet and triplet states

Non-Central Term

The deuteron has a small electric quadrupole moment, $Q = +2.82\times10^{-31}$ m^2. Hence, the nucleon potential is not spherically symmetric.

However, $^3S_1$ $\ell=0$, $J=1$ $\uparrow\uparrow$ is symmetric

Require some angular dependence in the deuteron wavefunction, ψ.

For J=1, other possible states:

L	S	
1	0	$^1P_1$
1	1	$^3P_1$
2	1	$^3D_1$

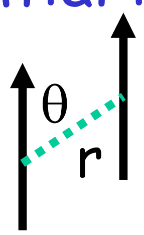
$P = (-1)^{\ell}$

Overall state must have definite parity (same as $^3S_1$ $J^P=1^+$).

ψ has 5% $^3D_1$ state.

($\psi=a_1\psi_S+a_2\psi_D$, $|a_1|^2=0.95$, $|a_2|^2=0.05$)

i.e. 5% of time L switches $0\rightarrow2$ and hence nuclear force must apply a torque. Potential is a function of θ as well as r.



$\Rightarrow$ non-central force (tensor force).

Also explains small difference in expected $\mu = 0.88\ \mu_N$ and measured $\mu = +0.857\ \mu_N$.

Deuteron Summary

In addition to the general features of the nucleon-nucleon interaction, the properties of the deuteron imply:

- Depth of nucleon potential,
  V_0 ~25 MeV for nuclear radius (b) =2 fm.

- Nuclear force is spin dependent

- Non-central terms in potential

Two nucleon potential

$$V = V_C(r) + V_S(\vec{s}_1 \cdot \vec{s}_2) + V_T(\vec{r}, \vec{s}) + \ldots$$

central spin tensor

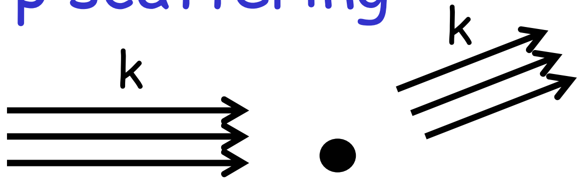
Limitations:

- only one state to study
- not enough information to determine all parameters required
- no information for $\ell \neq 0$ and excited states

$\Rightarrow$ Study nucleon-nucleon scattering

Nucleon-Nucleon Scattering

Consider n-p scattering



plane wave

E≤10 MeV Nucleus

Elastic scattering from near centre of nucleus (ℓ=0)

$\longrightarrow$ z

$$\psi_{IN} = e^{ikz} = e^{ikr\cos\vartheta}, \quad k = \frac{1}{\hbar}\sqrt{2mE}$$

Expand ψ_{IN} in spherical harmonics

where $\psi_{IN} = e^{ikr\cos\vartheta} = \sum_{\ell=0}^{\infty} B_\ell(r) Y_{\ell,\phi}(\vartheta)$

Spherical harmonic

$$B_\ell(r) = i^\ell (4\pi(2\ell+1))^{1/2} j_\ell(kr)$$

Spherical Bessel function

$$j_0 = \frac{\sin kr}{kr}, \quad j_1 = \frac{\sin kr}{(kr)^2} - \frac{\cos kr}{kr}, \ldots$$

Each term is a solution of the Schrödinger equation in spherical coordinates for a constant potential energy and with scattering centre as the origin.

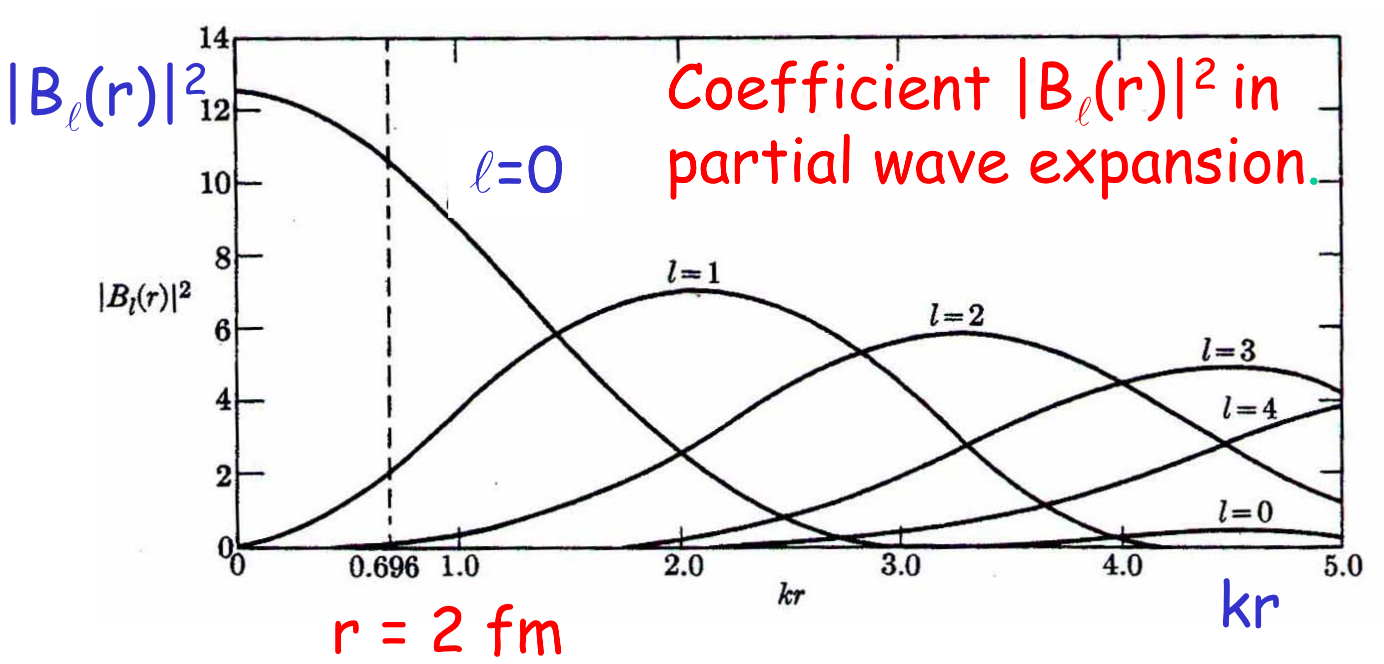
Terms are called Partial Waves

For low energies (E ≤ 10 MeV) only need consider ℓ=0 term.

Example: A nucleon of 10 MeV (5 MeV cms)

$$k = \sqrt{2ME} = \sqrt{2 \cdot \frac{940}{2} \cdot 5}$$
$$= 68.6 \text{ MeV}$$
$$= 0.348 \text{ fm}^{-1}$$

M = reduced mass $\approx m_n/2$
m_n = 940 MeV/c^2
$\hbar$=1, $\hbar c$=197 MeV fm



For a range of 2 fm for the nuclear force only the ℓ=0 partial wave important for beam energies $\leq$ 10 MeV.

Angular momentum conserved, ℓ does not change in scattering processes.

Consider ℓ=0:

Free particle wavefunction,

V=0 $$\psi_{IN} \rightarrow \psi_0 = \frac{e^{ikr} - e^{-ikr}}{2ikr}$$

e^{-ikr} represents a spherical wave going towards the origin.

e^{+ikr} represents a spherical wave going away from the origin.

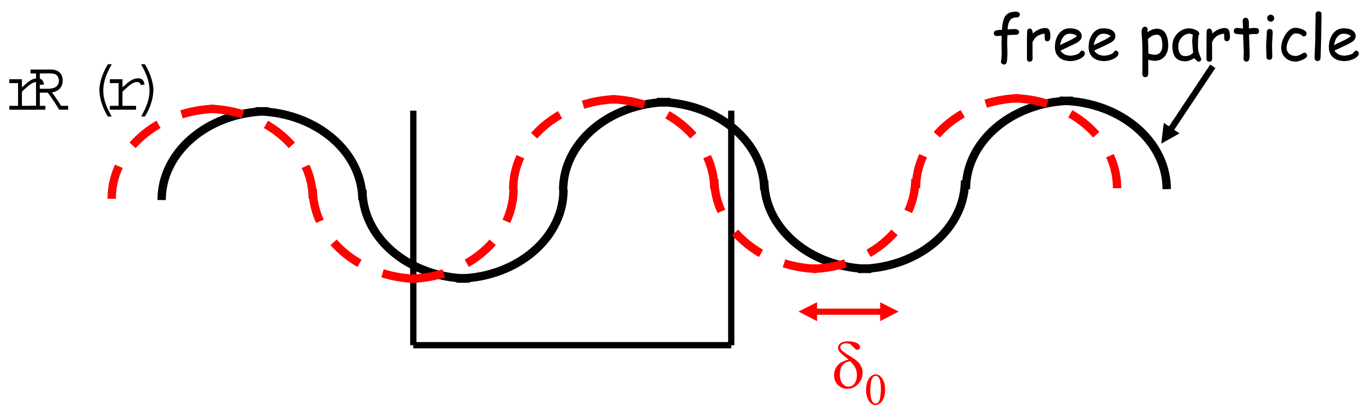
In presence of nucleon potential,

e^{-ikr} is not affected for r > range of potential (i.e. before particle gets to scattering centre)

e^{+ikr} for elastic scattering, amplitude must be same as e^{-ikr} part i.e. no particles created or destroyed.

$\Rightarrow$ Phase change only

Attractive potential raises K.E. within range of force $\Rightarrow \lambda$ decreases.



Attractive potential $\Rightarrow$ +ve phase shift $\delta_0 > 0$
Repulsive potential $\Rightarrow$ -ve phase shift $\delta_0 < 0$

Introducing $V \neq 0$ changes the phase of the outgoing wave:

$$V \neq 0 \quad \psi' = \frac{e^{i(kr+2\delta_0)} - e^{-ikr}}{2ikr} = \frac{e^{i\delta_0}\sin(kr+\delta_0)}{kr}$$

Convention:

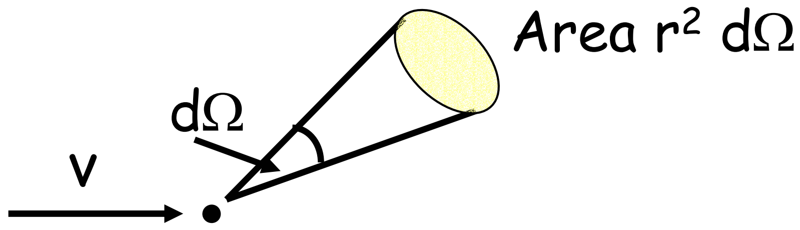
$2\delta_0$ phase shift in outgoing partial wave
δ_0 " " in ℓ=0 scattered wave

Probability of scattering given by the amplitude of the scattered wave:

$$\psi_{scat} = \psi' - \psi_0 = \frac{e^{i(kr+\delta_0)}}{kr}\sin\delta_0$$

Differential cross section:

$$\frac{d\sigma}{d\Omega} = \frac{\text{Number of particles/sec scattered into } d\Omega}{\text{Incident flux . } d\Omega}$$



Number particles/sec through area $r^2\, d\Omega$

$= |\psi_{scat}|^2\, r^2\, d\Omega\, v$ v=velocity of particles

Flux= Number of particles through unit area/sec=v

$$\frac{d\sigma}{d\Omega} = \frac{|\psi_{scat}|^2 r^2\, d\Omega\, v}{v\, d\Omega} = |\psi_{scat}|^2 r^2 = \frac{\sin^2\delta_0}{k^2 r^2} r^2$$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{\sin^2\delta_0}{k^2}}$$

In centre of mass, ℓ=0, isotropic $\quad \underline{\sigma = \frac{4\pi \sin^2\delta_0}{k^2}}$

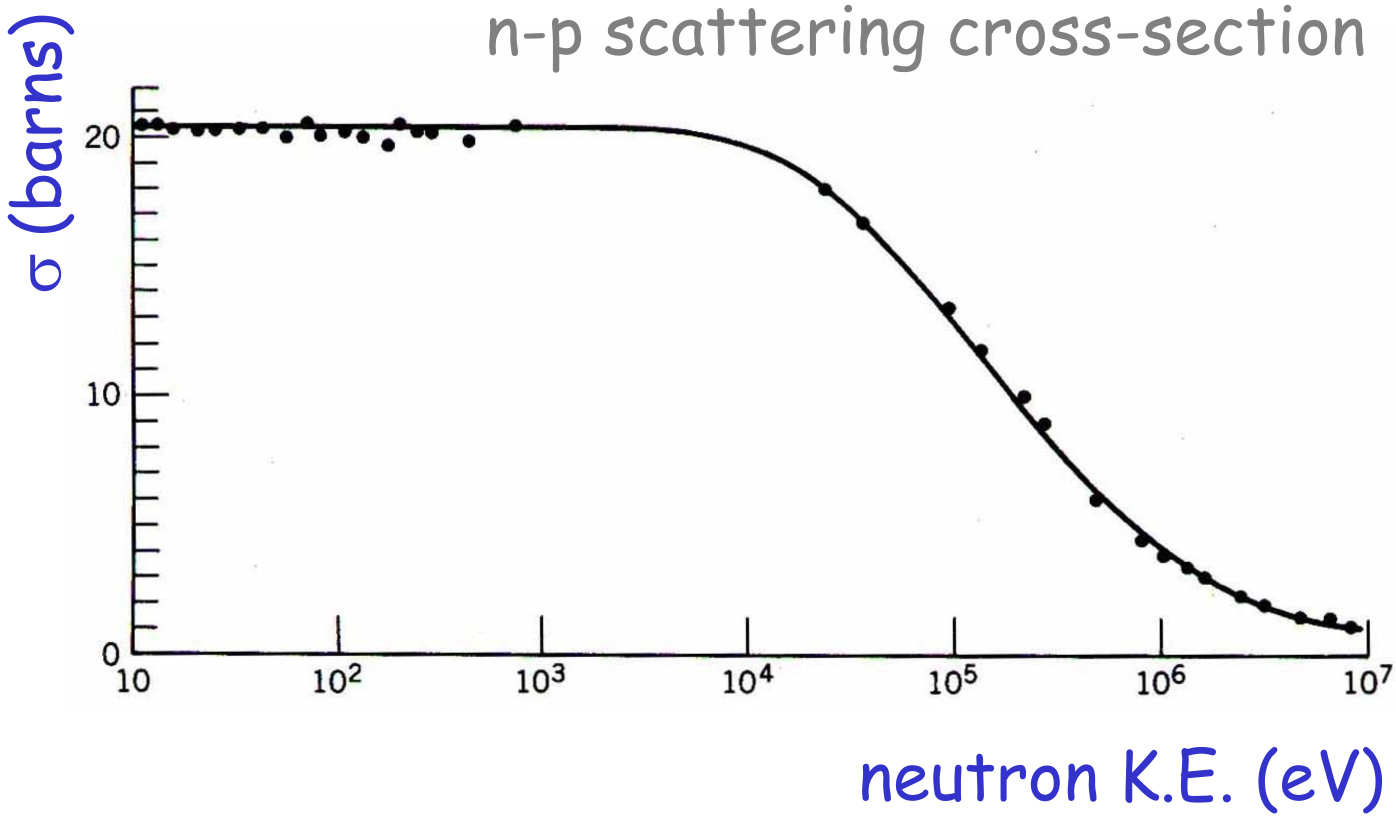
Low E, $k \to 0,\ \delta_0 \to 0$

$$\psi_{scat} = \frac{e^{i(kr+\delta_0)}}{kr} \sin\delta_0 = \frac{(e^{2i\delta_0}-1)}{2ik}\frac{e^{ikr}}{r}$$

$$\xrightarrow[k \to 0,\ \delta_0 \to 0]{} \frac{\delta_0}{k}\frac{e^{ikr}}{r} = \frac{a e^{ikr}}{r}$$

$$\boxed{\sigma = 4\pi a^2}$$

a is the <u>amplitude</u> of ψ_{scat}, often called <u>the "scattering length".</u>



Low energy $\sigma \sim$ constant $\approx$ 20 barns

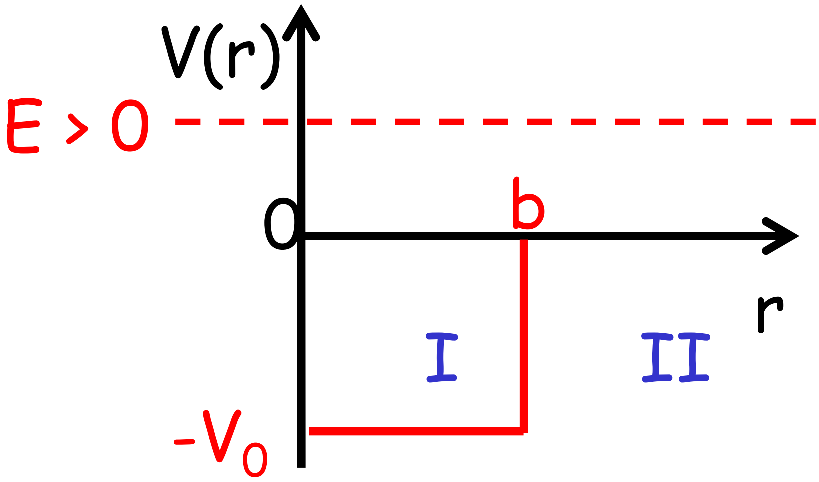
Extract phase shifts, δ_0 , from experimental measurements of the differential cross-section and compare to predicted phase shifts to determine the nucleon potential.

Need to relate the phase shift, δ_0, to the parameters of nucleon potential.

Phase Shift δ_0

Solve Schrödingers equation in interaction region. Particles can collide in $\underset{n\,p}{\uparrow\downarrow}$ or $\underset{n\,p}{\uparrow\uparrow}$

Consider a square well potential:



Radial wave equation: u(r)=rR(r)

$$\frac{-\hbar^2}{2M}\frac{d^2u(r)}{dr^2} + V(r)\,u(r) = E\,u(r)$$

M=reduced mass

Region I
r < b

$$\frac{d^2u(r)}{dr^2} + \frac{2M}{\hbar^2}(V_0 + E)\,u(r) = 0$$

$$u_I(r) = A\sin kr \qquad k = \sqrt{2M(E + V_0)}/\hbar$$

Region II
r > b

$$\frac{d^2u(r)}{dr^2} + \frac{2M}{\hbar^2}E\,u(r) = 0$$

$$u_{II}(r) = B\sin(k'r + \delta_0) \qquad k' = \sqrt{2ME}/\hbar$$

Boundary conditions: u(r), du(r)/dr continuous

u(r) $A\sin kb = B\sin(k'b+\delta_0)$

du(r)/dr $kA\cos kb = k'B\cos(k'b+\delta_0)$

ratio $k\cot kb = k'\cot(k'b+\delta_0)$

$$\underline{\delta_0 = \cot^{-1}\left(\frac{k}{k'}\cot kb\right) - k'b}$$

$$k = \sqrt{2M(E+V_0)}/\hbar \qquad k' = \sqrt{2ME}/\hbar$$

Given a set of potential well parameters V_0, b, δ_0 can be compared to the measured value extracted from $\sigma = 4\pi\sin^2\delta_0/k'^2$ as a function of energy.

Example: Using triplet parameters for deuteron
$V_0 b^2 \approx 10^{-28}$ MeV m^2
b = 2.1 fm and V_0 = 25 MeV

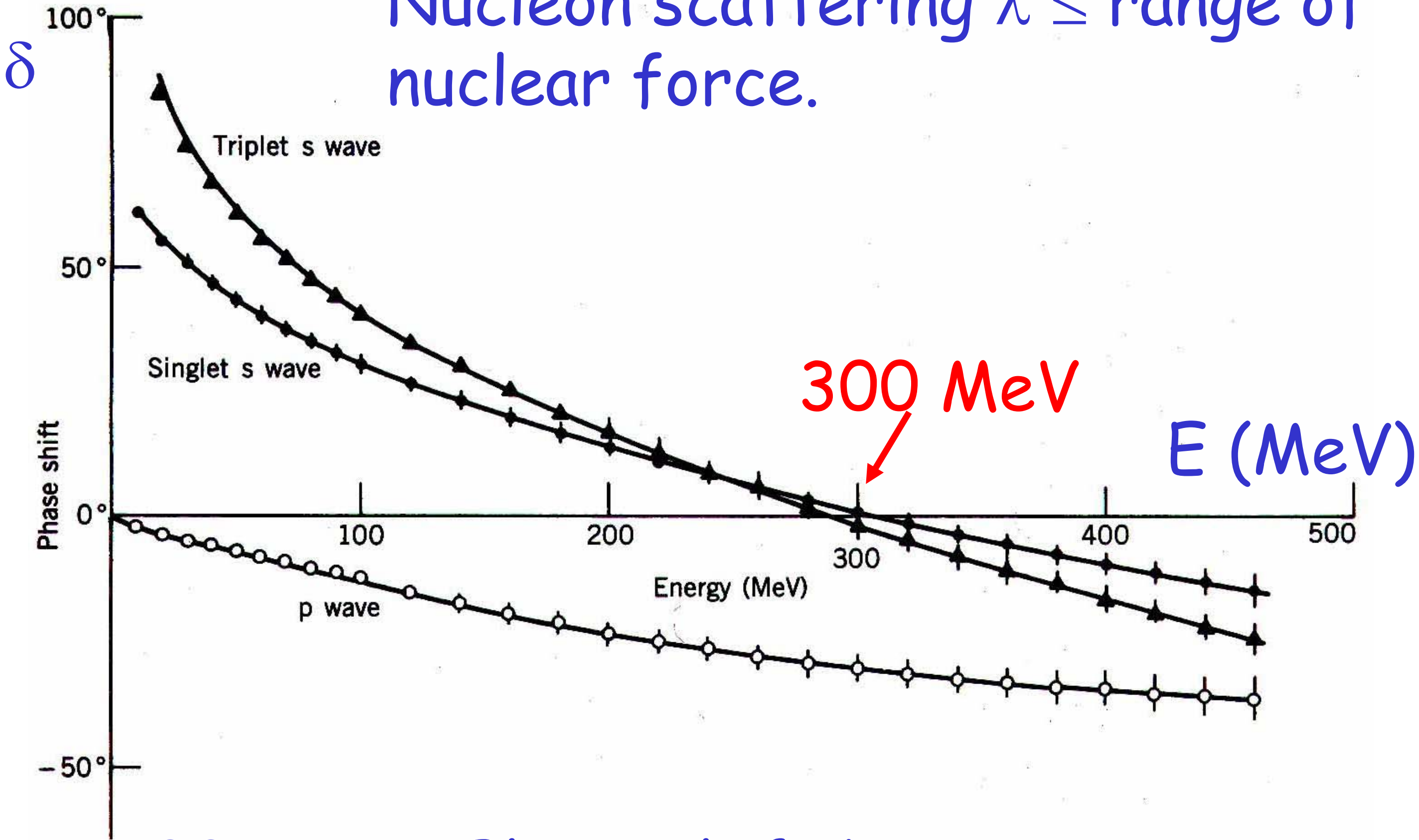
$$\Rightarrow \underline{\sigma \sim 5 \text{ barns}}$$

(c.f. 20 barns experimentally)

Need to consider spin dependence

Repulsive Core

Nucleon scattering $\lambda \leq$ range of nuclear force.



At 300 MeV: Phase shift becomes negative

$\Rightarrow$ repulsive force

$$
\begin{aligned}
V &= +\infty & r &< R_{core} \\
&= -V_0 & R_{core} &< r < R \\
&= 0 & r &> R
\end{aligned}
$$

$$\lambda\!\!\!^{-} = \frac{1}{p} = \frac{1}{\sqrt{2mE}} = \frac{1}{\sqrt{2 \times 940 \times 300}}$$

$$= 1.3 \times 10^{-3}\ \mathrm{MeV}^{-1}$$

$m = 940\ \mathrm{MeV}/c^2$

$\hbar = 1$, $\hbar c = 197$ MeV fm

$$\Rightarrow R_{core} \approx 0.5\ \mathrm{fm}$$

Spin Dependence

The total cross-section for n-p scattering is made up of a fixed mixture of n-p interactions in the n-p states:

S=0 $^1S_0$ ↑↓-↓↑

and S=1 $^3S_1$ ↑↑, ↓↓, ↑↓+↓↑

If the orientations of the neutrons in the incident beam and protons in the target are random, then

$$\sigma = \frac{3}{4}\sigma_t + \frac{1}{4}\sigma_s$$

t=triplet
s=singlet

To separate the contributions of σ_t and σ_s, scatter very low-energy neutrons (E<1KeV) from ortho- and para-hydrogen (H_2):

ortho-H_2	↑ ↑ p p	$S_{H2} = 1$
para-H_2	↑ ↓ p p	$S_{H2} = 0$

Low neutron E: $\lambda \gg$ separation of protons in H_2

Get coherent scattering $\sigma = \left(\sum \text{amplitudes}\right)^2$

(c.f. incoherent $\sigma = \sum (\text{amplitudes})^2$)

Total amplitude for scattering neutron from one proton

$$\hat{a}_p = a_s\hat{\pi}_s + a_t\hat{\pi}_t$$

where $\hat{\pi}_s$ and $\hat{\pi}_t$ are operators that project singlet and triplet parts of the n-p wavefn:

$\uparrow p \uparrow n$ $\quad \hat{\pi}_s|\psi_{np}\rangle = 0, \quad \hat{\pi}_t|\psi_{np}\rangle = 1, \quad a_p = a_t$

$\uparrow p \downarrow n$ $\quad \hat{\pi}_s|\psi_{np}\rangle = 1, \quad \hat{\pi}_t|\psi_{np}\rangle = 0, \quad a_p = a_s$

For coherent scattering of neutron from two protons in H_2

$$\hat{a} = \hat{a}_{p_1} + \hat{a}_{p_2} = a_s(\hat{\pi}_{s_1} + \hat{\pi}_{s_2}) + a_t(\hat{\pi}_{t_1} + \hat{\pi}_{t_2})$$

Total cross-section $\sigma = 4\pi a^2$

para-H_2 $\uparrow\downarrow$ $\quad \sigma_{para} = 4\pi\left(\frac{1}{2}a_s + \frac{3}{2}a_t\right)^2$ see question sheet

ortho-H_2 $\uparrow\uparrow$ $\quad \sigma_{ortho} = 4\pi\left(\underbrace{\frac{2}{3}(2a_t)^2}_{\substack{\uparrow\ n \quad \uparrow\uparrow\ o\text{-}H_2 \\ S=3/2}} + \underbrace{\frac{1}{3}\left(\frac{3}{2}a_s + \frac{1}{2}a_t\right)^2}_{\substack{\uparrow\ n \quad \downarrow\downarrow\ o\text{-}H_2 \\ S=1/2}}\right)$

σ_{para} can be measured using H_2 at 20K. H_2 at 20K is all para-hydrogen.

If the nuclear force were independent of spin, $\sigma_t = \sigma_s$ and $a_t = a_s$, thus σ_{para} and σ_{ortho} would be the same.

Experimentally, $\sigma_{para} = 4$ b and $\sigma_{ortho} = 130$ b

$\Rightarrow$ Nuclear force is spin-dependent

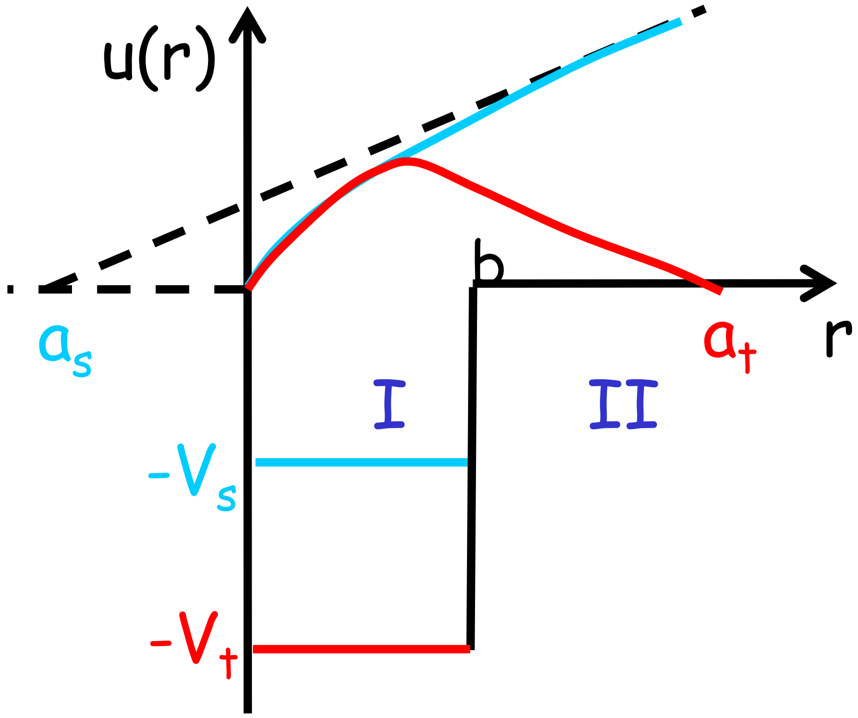
The large difference between the measured values shows that $a_t \neq a_s$ and that a_t and a_s must have different signs to make σ_{para} small.

$$a_t = 5.4 \text{ fm} \quad \text{and} \quad a_s = -23.7 \text{ fm}$$

$\Rightarrow \uparrow\downarrow$ singlet state is unbound

$\uparrow\uparrow$ triplet state is bound

Sketch of the radial wavefunction solution to Schrödingers equation



r < b $\quad u_I(r) = A \sin kr \qquad k = \sqrt{2M(E+V_0)}/\hbar$

r > b $\quad u_{II}(r) = B \sin(k'r + \delta_0) \qquad k' = \sqrt{2ME}/\hbar$

a is where $u_{II}(r)$ crosses r axis for $k' \to 0$

$$u_{II}(r) = 0 \quad \text{at} \quad r = -\frac{\delta_0}{k'} = a$$

Convention: $a = -\lim_{k' \to 0} \dfrac{\sin \delta_0}{k'}$

a negative $\to$ unbound state
a positive $\to$ bound state

Charge Dependence

Study charge dependence of nuclear force by comparing p-p and n-n scattering.

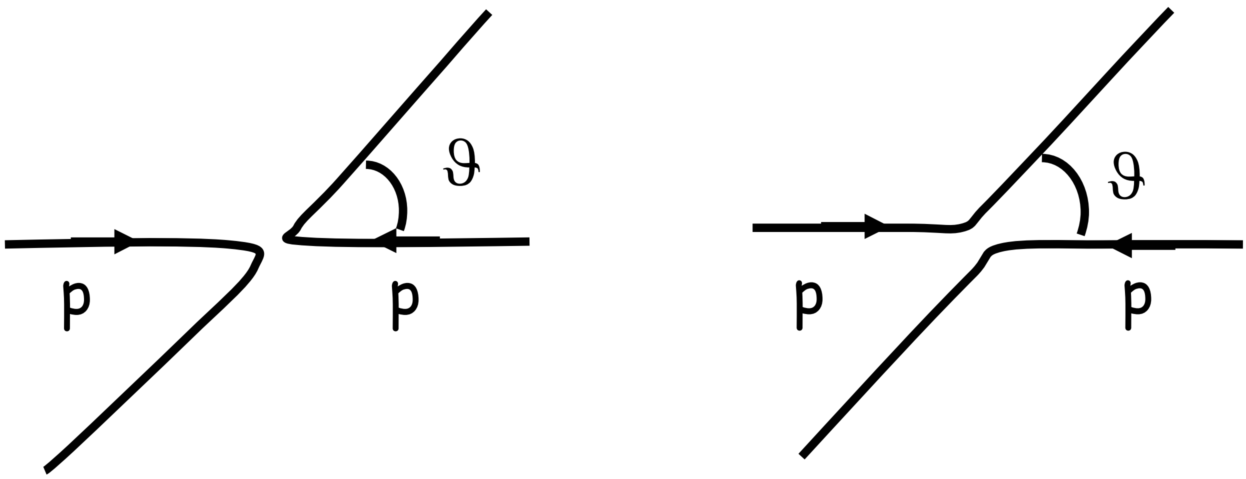
Important difference to n-p scattering:

$\rightarrow$ Identical particles

- Total wavefunction antisymmetric

$\therefore$ ℓ=0 (i.e. low energy) scattering only possible in singlet state $\uparrow\downarrow$

- Cannot distinguish



Must include interference between 2 possibilities.

p-p scattering

Exclusive study of singlet interaction. However, both Coulomb and nuclear interactions are present.

Theoretical expression for $d\sigma/d\Omega$ for p-p scattering:

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \frac{1}{4T^2}\left\{\frac{1}{\sin^4(\vartheta/2)}\right.$$

Rutherford scattering

$$+\frac{1}{\cos^4(\vartheta/2)} - \frac{\cos\left[\eta \mathrm{Ln}\tan^2(\vartheta/2)\right]}{\sin^2(\vartheta/2)\cos^2(\vartheta/2)}$$

Rutherford classical term

Wave-mechanical interference term

Mott Scattering

Corrections for two identical particles

$$-\frac{2}{\eta}\sin\delta_0\left(\frac{\cos\left[\delta_0+\eta \mathrm{Ln}\sin^2(\vartheta/2)\right]}{\sin^2(\vartheta/2)} + \frac{\cos\left[\delta_0+\eta \mathrm{Ln}\cos^2(\vartheta/2)\right]}{\cos^2(\vartheta/2)}\right)$$

Wave intereference cross-terms between Coulomb and nuclear potential scattering

$$\left.+\frac{4}{\eta^2}\sin^2\delta_0\right\}$$

Pure nuclear potential scattering

T = laboratory K.E.

ϑ = scattering angle in c.m.s system

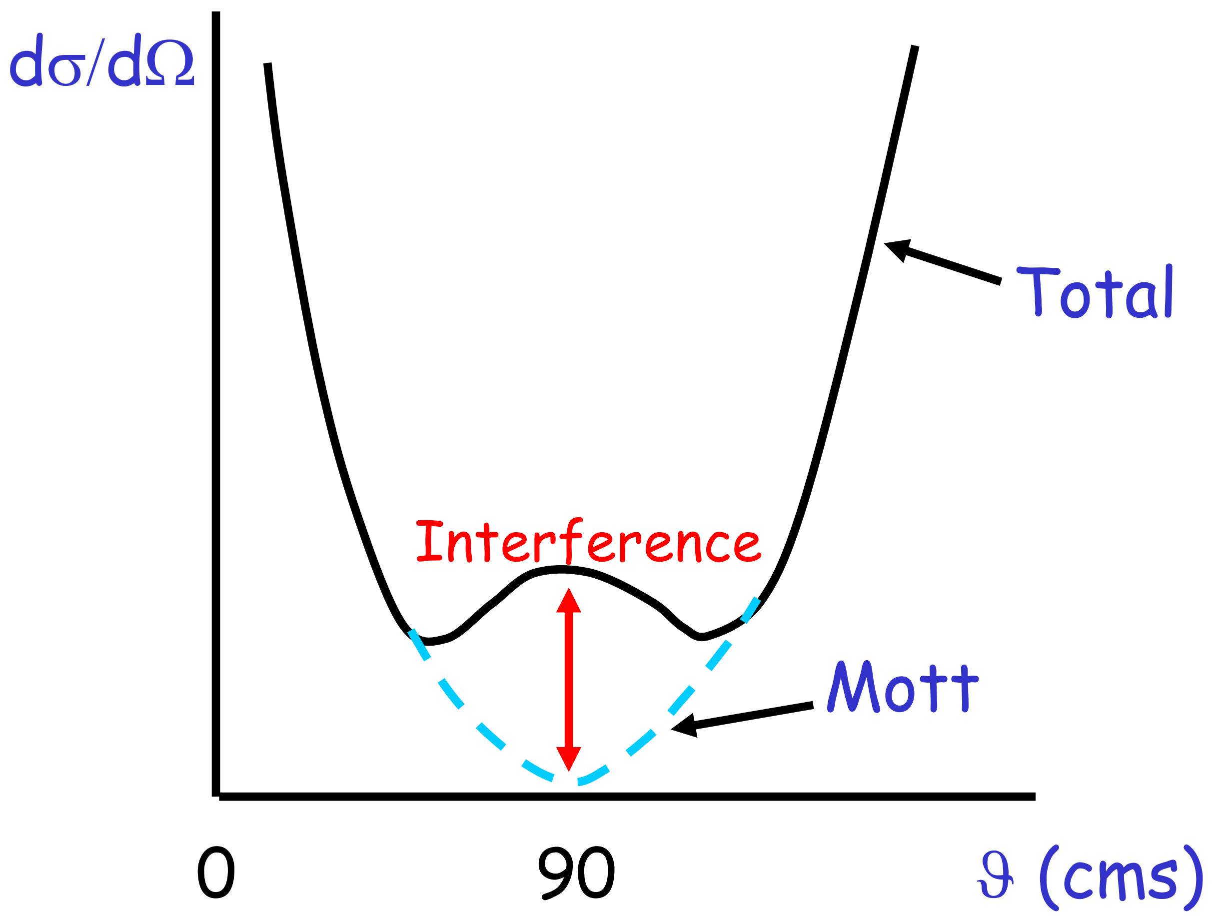
$\eta = (e^2/4\pi\varepsilon_0\hbar c)\beta^{-1}$ $\quad\beta = v/c$

δ_0 = ℓ=0 phase shift

δ_0 only unknown

From $d\sigma/d\Omega$ find sign and magnitude of δ_0

Interference allows sign determination:



Find a -ve $\rightarrow$ no pp bound states

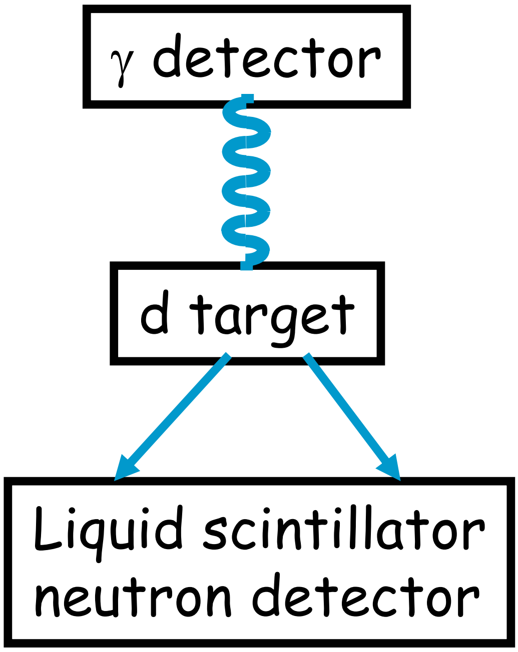
$\sigma_{pp} = 36.7 \pm 0.1$ b

n-n scattering

Difficult as no neutron only targets.

Use reactions that create 2 neutrons within nuclear range as separate (comparable to a scattering experiment)

e.g. $\pi^- + d \rightarrow 2n + \gamma$



- π stop in target
- Look for coincidences of 2n and γ

If 2n bound $\Rightarrow$ γ monochromatic, 2 body final state

If no n-n interaction $\Rightarrow$ energy shared between 3 particles

$$\sigma_{nn} = 33.8 \pm 1.8 \text{ b}$$

$$(\text{c.f. } \sigma_{pp} = 36.7 \pm 0.1 \text{ b})$$

$\Rightarrow$ Nuclear force is charge independent

Spin-Orbit Potential

A momentum dependent force can be represented by a spin-orbit term in the potential

$$\sim V_{so}(r)\vec{L}\cdot\vec{S}$$

Atomic electrons experience a spin-orbit coupling arising from the interaction of the electrons spin and the internal magnetic field of the atom.

Nucleons experience a spin-orbit coupling arising from the interaction of the nucleon spin and the strong nuclear force.

20x strength and opposite sign
c.f. atomic spin-orbit term

Evidence for a nucleon spin-orbit term from the polarization of scattered nucleons.

Polarized: magnetic substates not equally populated

$$\text{Polarization} = \frac{N(\uparrow) - N(\downarrow)}{N(\uparrow) + N(\downarrow)}$$

$P = \pm 1$ 100% polarization, $P=0$ unpolarized

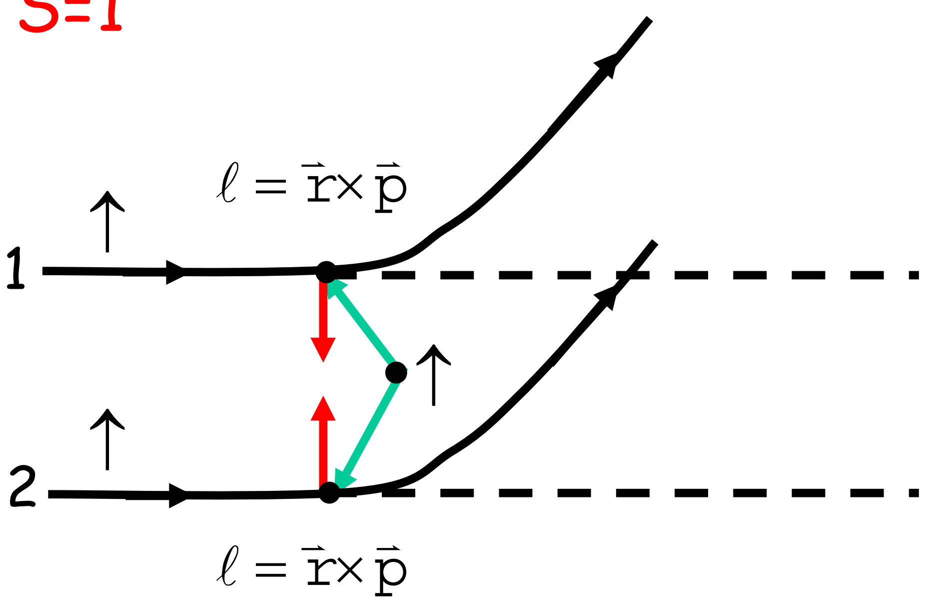
Observe polarization of scattered nucleon when beam and target <u>unpolarized.</u>

Assume $V \sim -V_{so}(r)\vec{L}\cdot\vec{S}$

<u>3 possibilities:</u>

(i) Beam nucleon spin ↑, target nucleon spin ↑

Total S=1



Nucleon 1: $\ell = \vec{r}\times\vec{p}$ into plane $\therefore \vec{\ell}\cdot\vec{s}$ -ve

$\Rightarrow$ V is +ve i.e. <u>repulsive.</u>

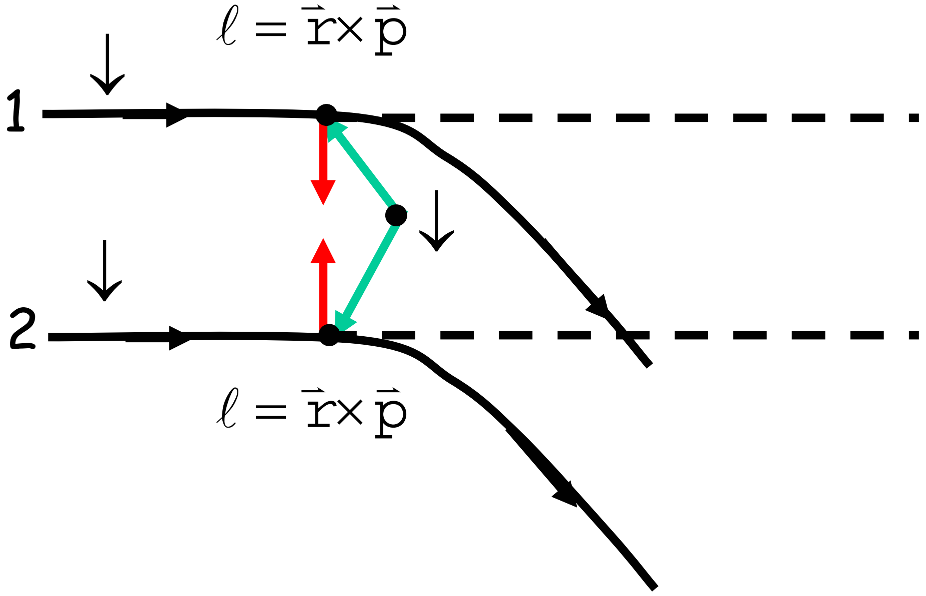
Nucleon 2: $\ell = \vec{r}\times\vec{p}$ out of plane $\therefore \vec{\ell}\cdot\vec{s}$ +ve

$\Rightarrow$ V is -ve i.e. <u>attractive.</u>

All spin ↑ incident on spin ↑ (target) deflected in same direction due to spin-orbit potential.

(ii) Beam nucleon spin ↓, target nucleon spin ↓

Total S=1



Nucleon 1: $\ell = \vec{r}\times\vec{p}$ into plane $\therefore \vec{\ell}\cdot\vec{s}$ +ve

$\Rightarrow$ V is -ve i.e. attractive.

Nucleon 2: $\ell = \vec{r}\times\vec{p}$ out of plane $\therefore \vec{\ell}\cdot\vec{s}$ - ve

$\Rightarrow$ V is +ve i.e. repulsive.

(iii) Beam nucleon spin ↓ or ↑, target nucleon spin ↓ or ↑. Total S=0

$\vec{\ell}\cdot\vec{s} = 0$ $\therefore$ No deflection due to spin orbit

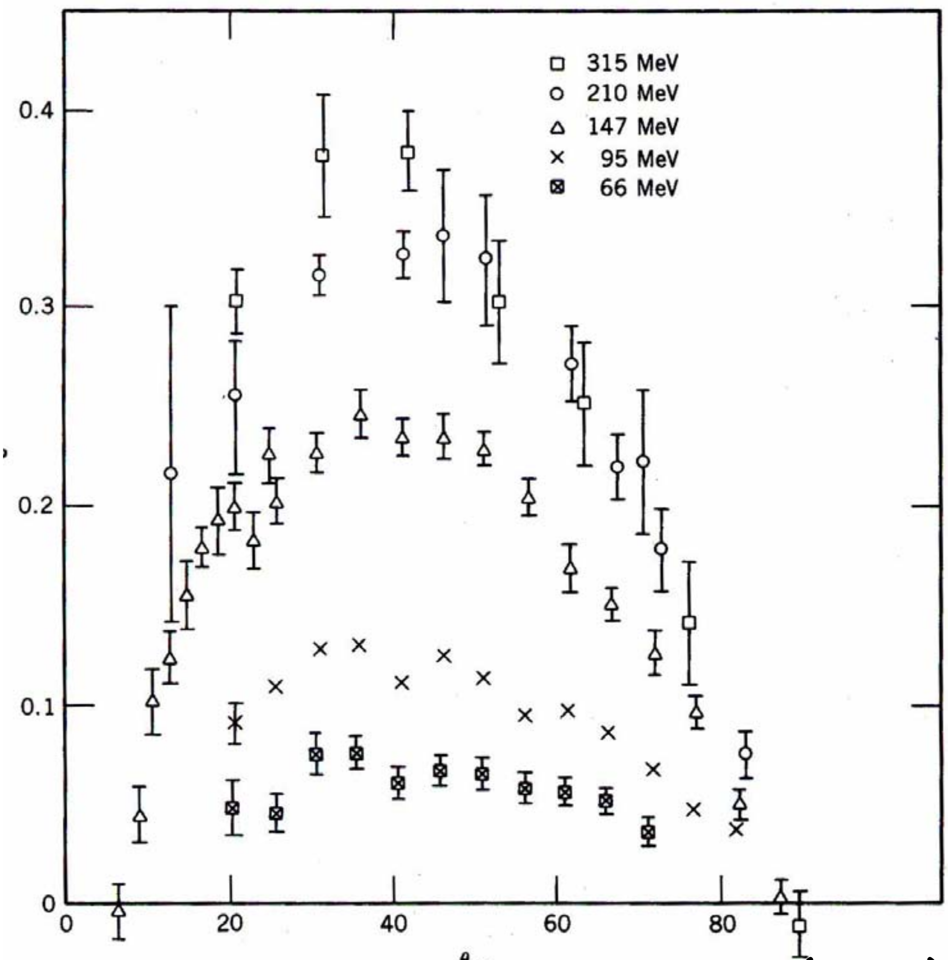
The spin-orbit interaction deflects the spin ↑ component of the incident beam left and the spin ↓ component of the incident beam right if total S=1.

Any individual nucleon passing through the interior of a nucleus will on average pass an equal number of nucleons with spin ↑ and spin ↓, hence there will be no net spin-orbit interaction.

A net spin-orbit interaction is obtained from those nucleons passing near the surface of the nucleus.

p-p scattering

$$P = \frac{N(\uparrow) - N(\downarrow)}{N(\uparrow) + N(\downarrow)}$$



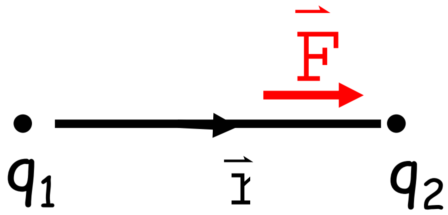
ϑ (cms)

Only see effect when incident beam energy is high enough for $\ell > 0$.
Polarization increases with energy.

Yukawa Potential

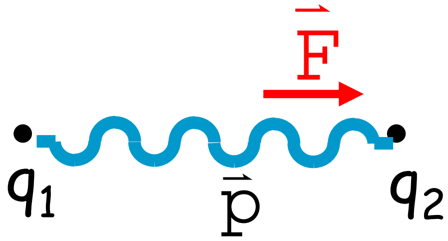
Consider the electromagnetic interaction:

- Classically: E-M forces arise from action at a distance of the $\vec{E}$ and $\vec{B}$ fields.



$$\vec{F} = \frac{q_1 q_2}{r^2}\hat{r}$$

- Quantum mechanically: Forces arise due to exchange of virtual field quanta (second quantization)



The field strength at any point is uncertain

$$\Delta p \Delta r \sim \hbar \qquad \Delta t = \Delta r / c$$

Number of quanta emitted and absorbed $\sim q_1 q_2$

$$\Rightarrow \vec{F} = \frac{dp}{dt} = \frac{q_1 q_2}{r^2}\hat{r}$$

Massless particle e.g. photon, force has infinite range.

Nucleons start to experience the strong interaction at a distance of ~ 2fm.

$$\Delta E \Delta t \sim \hbar \qquad E = mc^2$$

$$\Rightarrow mc^2 \sim \frac{\hbar}{\Delta t} \sim \frac{\hbar c}{r}$$

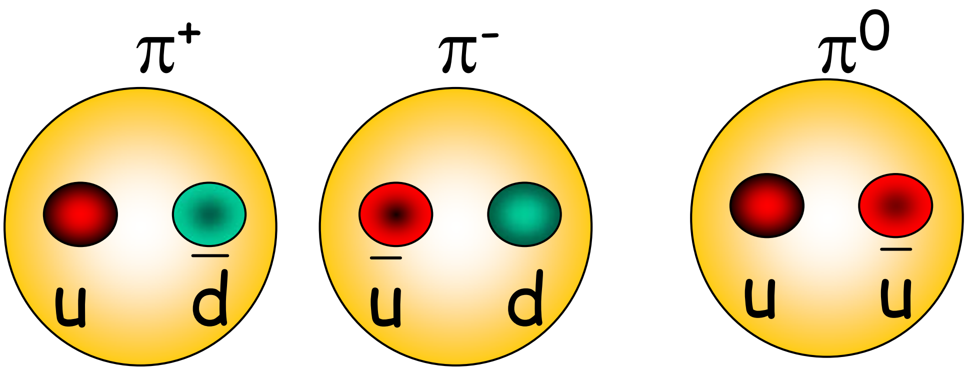
$$\underline{m \sim \frac{\hbar}{rc} = \frac{1}{r}} \qquad \hbar = c = 1$$

For r=2 fm, predict existence of a particle of mass ≈ 100 MeV/c^2 (i.e. m ≈ 200 me)

⇒ <u>Meson (e.g. pion)</u>

The pion (π) was discovered in cosmic ray photographic plates in 1947.

3 types:



mass(π^+)=mass(π^-)=139.6 MeV/c^2
mass(π^0)=135 MeV/c^2
$J^P = 0^+$

Relativistic wave equation:

$$E^2 = p^2 + m^2 \qquad c = 1$$

Operator $E \to i\frac{\partial}{\partial t} \qquad p \to -i\vec{\nabla}$

$$-\frac{\partial^2 \psi}{\partial t^2} = -\nabla^2 \psi + m^2 \psi$$

$$\nabla^2 \psi - m^2 \psi - \frac{\partial^2 \psi}{\partial t^2} = 0$$

Klein-Gordon Equation

Static solution

$$\psi = -\frac{g^2}{4\pi}\frac{e^{-mr}}{r}$$

Assume the pion wavefunction can be represented equivalently by a potential in the vicinity of the nucleon:

$$\boxed{V(r) = -\frac{g^2}{4\pi}\frac{e^{-mr}}{r}}$$

Yukawa Potential

g= coupling constant (dimensionless)

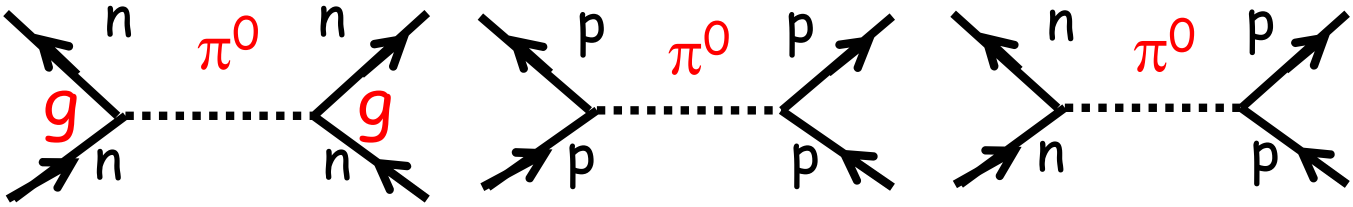
Note: $m \to 0$, $V(r) \to 1/r$

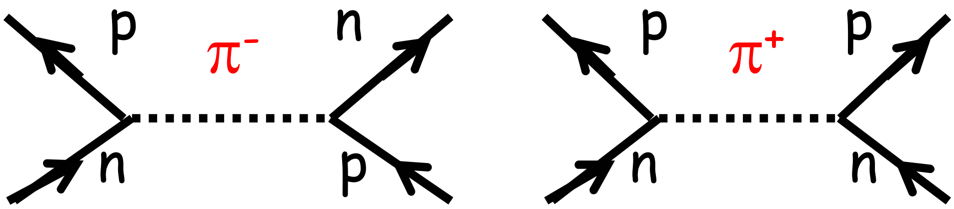
$$\alpha_s = \frac{g^2}{4\pi} \sim O(1) \qquad \text{c.f.} \quad \alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}$$

Strong coupling constant

Fine structure constant (EM)

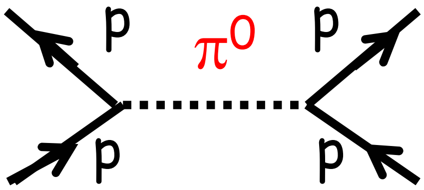
Yukawa interaction between 2 nucleons:

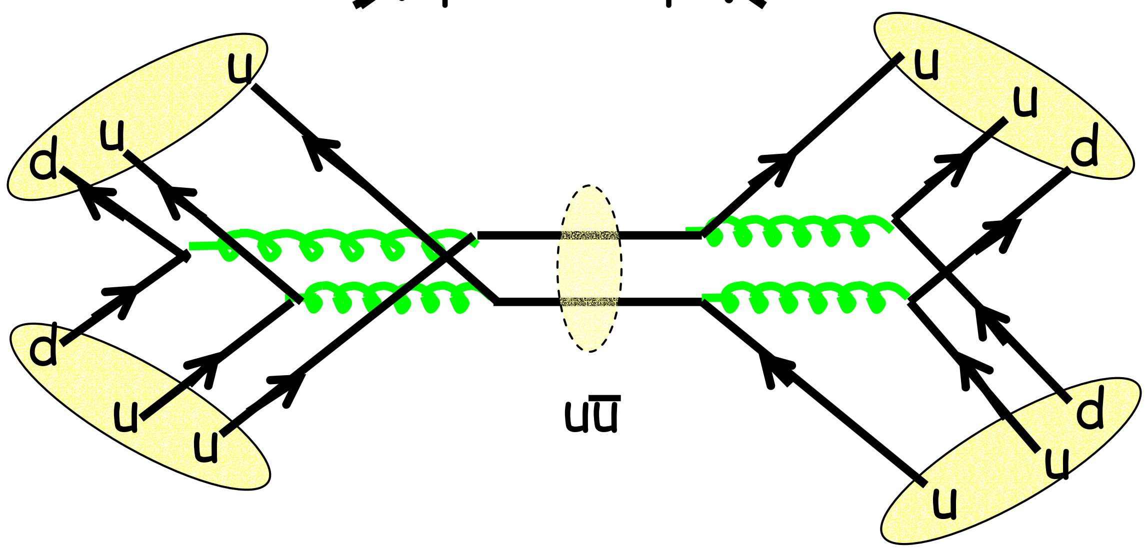




Quark model:

(see Particles course)

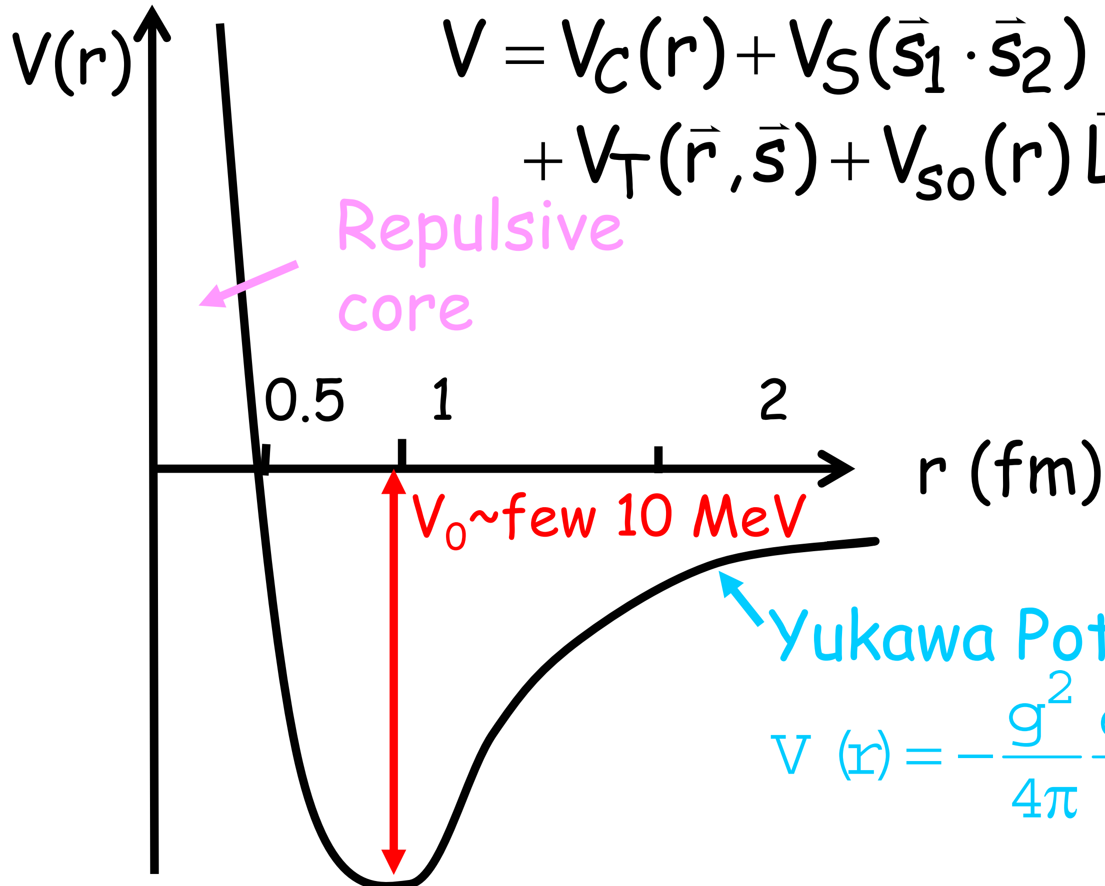




Summary

Nucleon-Nucleon Potential

$$V = V_C(r) + V_S(\vec{s}_1 \cdot \vec{s}_2) + V_T(\vec{r}, \vec{s}) + V_{so}(r)\vec{L}\cdot\vec{S} + \ldots$$



$$V(r) = -\frac{g^2}{4\pi}\frac{e^{-mr}}{r}$$

The nucleon force is

- strong, (existence of nuclei, nucleon scattering, strength α_s~O(1))
- short range, (nuclei size → range ~ 2 fm)
- attractive, (existence of nuclei)
- has a repulsive core, (nucleon scattering r < 0.5 fm)
- saturates, (B/A~constant)
- charge independent, (mirror nuclei, pp vs nn vs np scattering)
- spin dependent, (deuteron J=1, np and nH_2 scattering)
- spin-orbit interaction (polarization in nucleon scattering at high E)
- non-central terms (deuteron Q)