<u>Flux</u> = number of incident particles passing unit area per second

Consider target of area *A* and incident beam of velocity v=c moving towards target.

Any incident particle within volume c*A* will pass target every second.

$$\text{Flux} = \frac{cA}{A} n_i = c n_i$$

where $n_i$ is the number density of incident particles = 1 per $L^3$

$$\text{Flux} = \frac{c}{L^3} = \frac{1}{L^3} \qquad (c=1)$$

$$d\sigma = \frac{1}{\text{flux}} 2\pi |M|^2 \rho(E_f)$$

$$= L^3 \, 2\pi \left| \frac{1}{L^3} \int e^{i\vec{q}\cdot\vec{r}} V(\vec{r})\, d^3\vec{r} \right|^2 E^2 \left(\frac{L}{2\pi}\right)^3 d\Omega$$

$$\frac{d\sigma}{d\Omega} = \frac{E^2}{(2\pi)^2} \left| \int e^{i\vec{q}\cdot\vec{r}} V(\vec{r})\, d^3\vec{r} \right|^2$$

Born Approximation

$$|\vec{r}-\vec{r}'| = \left[r^2 + r'^2 - 2rr'\cos\vartheta\right]^{1/2}, \quad |\vec{r}-\vec{r}'|^{-1} = r^{-1}\left[1 + \frac{r'^2}{r^2} - 2\frac{r'}{r}\cos\vartheta\right]^{-1/2}$$

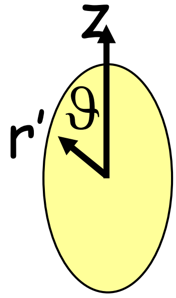
$$|\vec{r}-\vec{r}'|^{-1} = r^{-1}\left[1 - \frac{1}{2}\left(\frac{r'^2}{r^2} - 2\frac{r'}{r}\cos\vartheta\right) + \frac{3}{8}\left(\frac{r'^2}{r^2} - 2\frac{r'}{r}\cos\vartheta\right)^2 + \ldots\right]$$

$$\approx r^{-1}\left[1 + \frac{r'}{r}\cos\vartheta + \frac{1}{2}\frac{r'^2}{r^2}\left(3\cos^2\vartheta - 1\right) + \ldots\right] \qquad r' \ll r$$

$$V(r) = \frac{1}{4\pi\varepsilon_0 r}\left[Ze + \frac{1}{r}\int r'\cos\vartheta\,\rho(r')\,d^3\vec{r}' + \frac{1}{2r^2}\int r'^2\left(3\cos^2\vartheta - 1\right)\rho(r')\,d^3\vec{r}' + \ldots\right]$$

Quantum limit:

$$\rho(\vec{r}') = |\psi(\vec{r}')|^2$$




Let r define z axis

$z = r'\cos\vartheta$

E0 moment = $\int \psi^* \psi \, d^3\vec{r}' = Ze$ — charge

E1 moment = $\int \psi^* z \psi \, d^3\vec{r}'$ — electric dipole

E2 moment = $\frac{1}{e}\int \psi^* \left(3z^2 - r'^2\right)\psi \, d^3\vec{r}'$ — electric quadrupole

Nuclear wavefunctions have definite parity

$$|\psi(r)|^2 = |\psi(-r)|^2$$

<u>Electric Dipole Moment is zero</u>

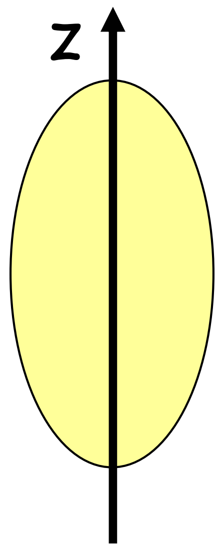
# Electric Quadrupole Moment

$$Q = \frac{1}{e}\int \psi^* \; (3z^2 - r^2)\,\psi \; d^3\vec{r}$$

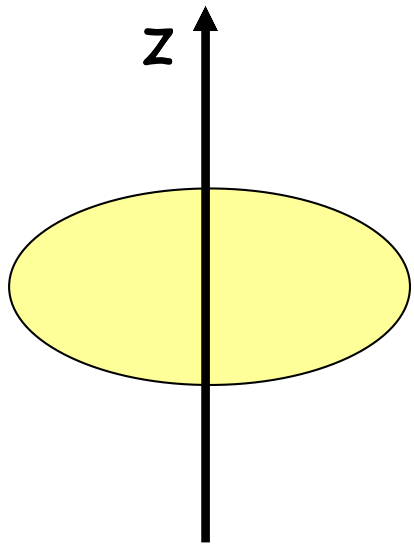
Units: m² or barns   Area

Spherical symmetry, $z^2 = \frac{1}{3}r^2 \quad \Rightarrow$ Q=0

All J=0 nuclei have Q=0

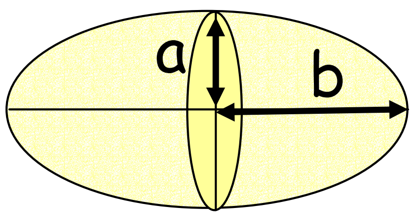



Prolate spheroid
Q +ve




Oblate spheroid
Q -ve

Ellipticity, $\eta = \frac{b-a}{\tfrac{1}{2}(b+a)}$




Experimentally, $\eta$ is typically $\leq$ 10%

Complication when $N_D(0) \neq 0$:

$$N_P(t) + N_D(t) = N_P(0) + N_D(0)$$

2 equations, 3 unknowns

If there is another isotope of D, say D′, for which $N_{D'}(t) = N_{D'}(0) = N_{D'}$ (i.e. D′ stable),

$$\frac{N_P(t) + N_D(t)}{N_{D'}(t)} = \frac{N_P(0) + N_D(0)}{N_{D'}(0)}$$

$$\frac{N_D(t)}{N_{D'}(t)} = \frac{N_P(0) + N_D(0)}{N_{D'}(0)} - \frac{N_P(t)}{N_{D'}(t)}$$

$$\frac{N_D(t)}{N_{D'}(t)} = \frac{N_P(t)}{N_{D'}(t)}\left(e^{\lambda \Delta t} - 1\right) + \frac{N_D(0)}{N_{D'}(0)}$$

With several mineral sources from the same source expect

- same age $\Delta t$
- same $N_D(0)/N_{D'}(0)$
- different $N_P(0)$

Plot $\dfrac{N_D(t)}{N_{D'}(t)}$ versus $\dfrac{N_P(t)}{N_{D'}(t)}$

slope $e^{\lambda \Delta t} - 1$ intercept $\dfrac{N_D(0)}{N_{D'}(0)}$

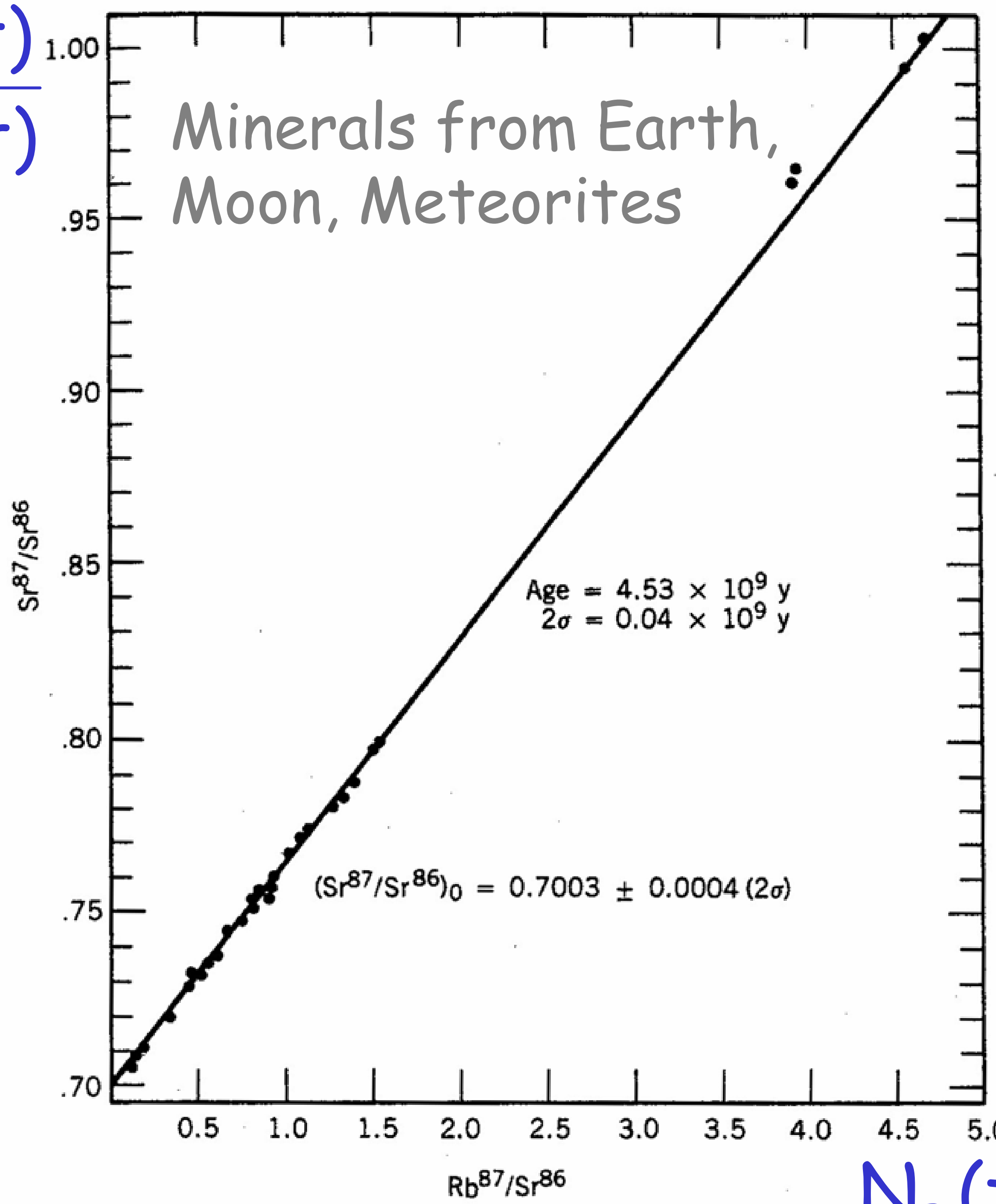
## Example: Use $\beta^-$ decay

$^{87}Rb \rightarrow {}^{87}Sr \quad (\tau_{1/2} = 4.8 \times 10^{10}$ years$)$

$D' = {}^{86}Sr$ (stable)

**Age of Earth from slope = $4.5 \times 10^9$ years**

$\dfrac{N_D(t)}{N_{D'}(t)}$




$\dfrac{N_P(t)}{N_{D'}(t)}$