Parton Evolution Beyond Current Paradigms

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at the
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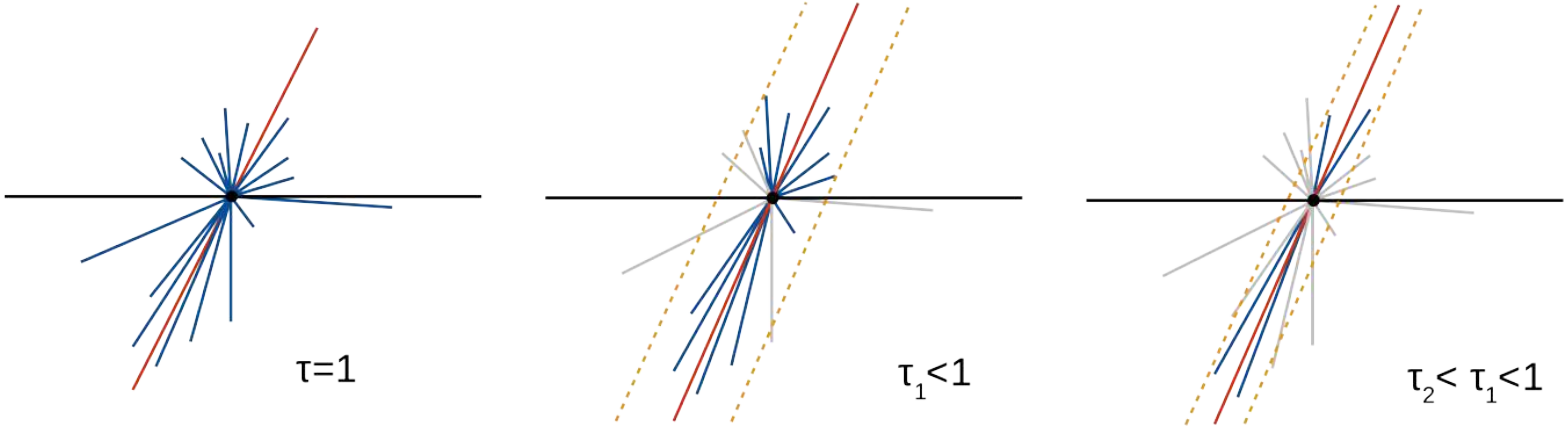
Jets & Event Shapes

Invert the jet evolution to map hadronic configurations to partonic final states.
Observables involve resolution parameter: Limit radiation at certain momentum scales.



$$\sigma(n \text{ jets}, \tau) \sim \sum_k \sum_{l \leq 2k} c_{nkl}\ \alpha_s^k(Q)\ \ln^l \frac{1}{\tau}$$

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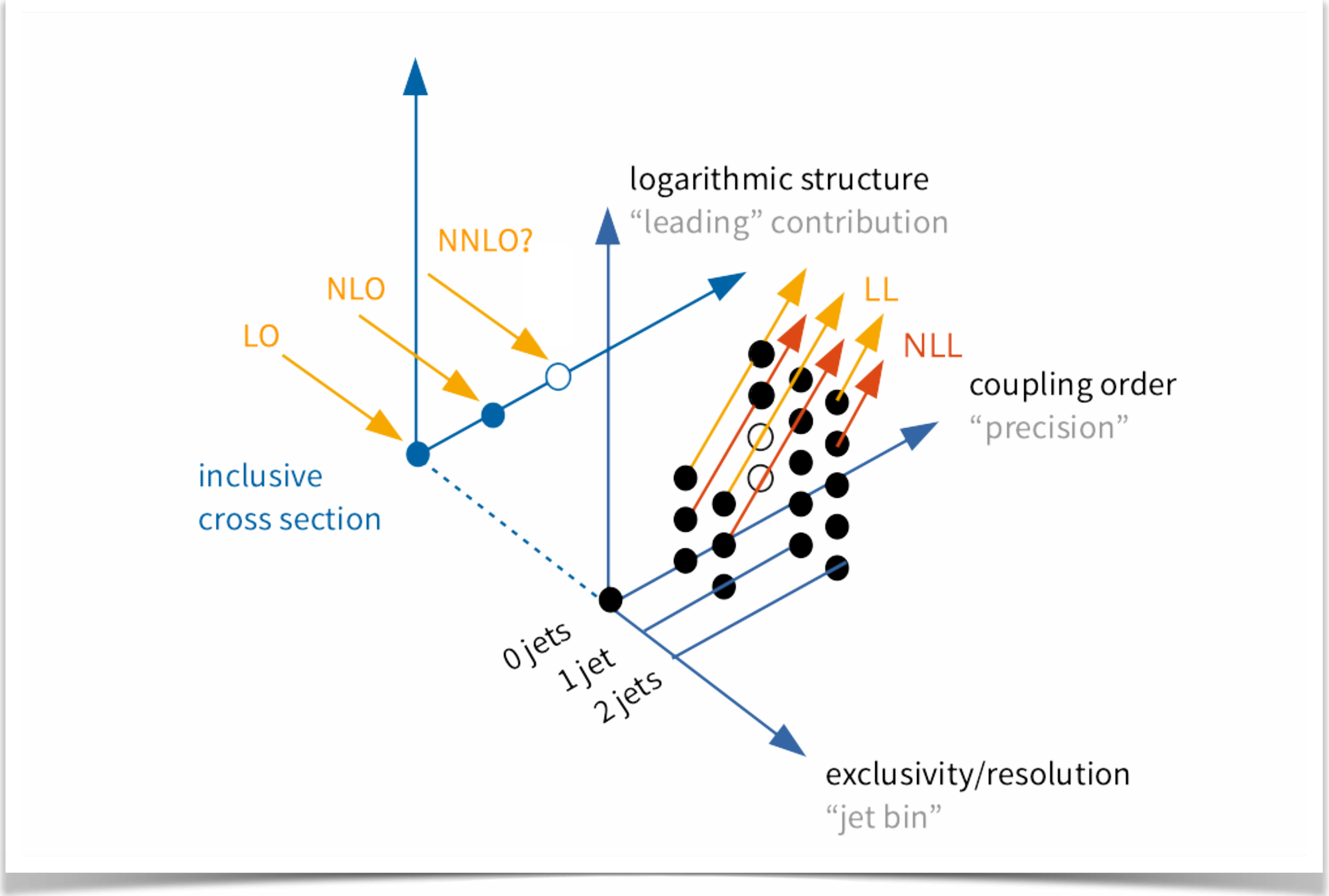
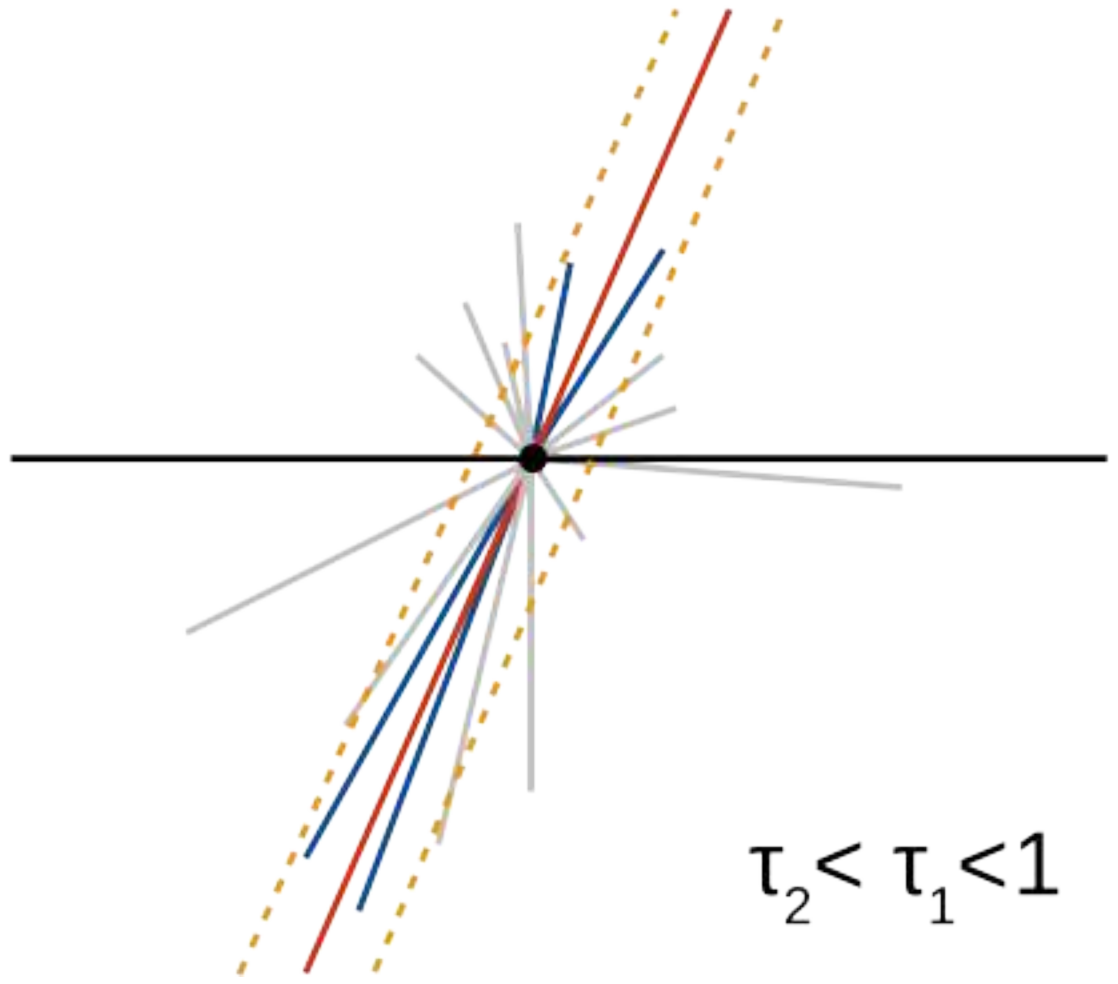




$$\sigma(n \text{ jets}, \tau) \sim \sum_k \sum_{l \leq 2k} c_{nkl}\ \alpha_s^k(Q)\ \ln^l \frac{1}{\tau}$$

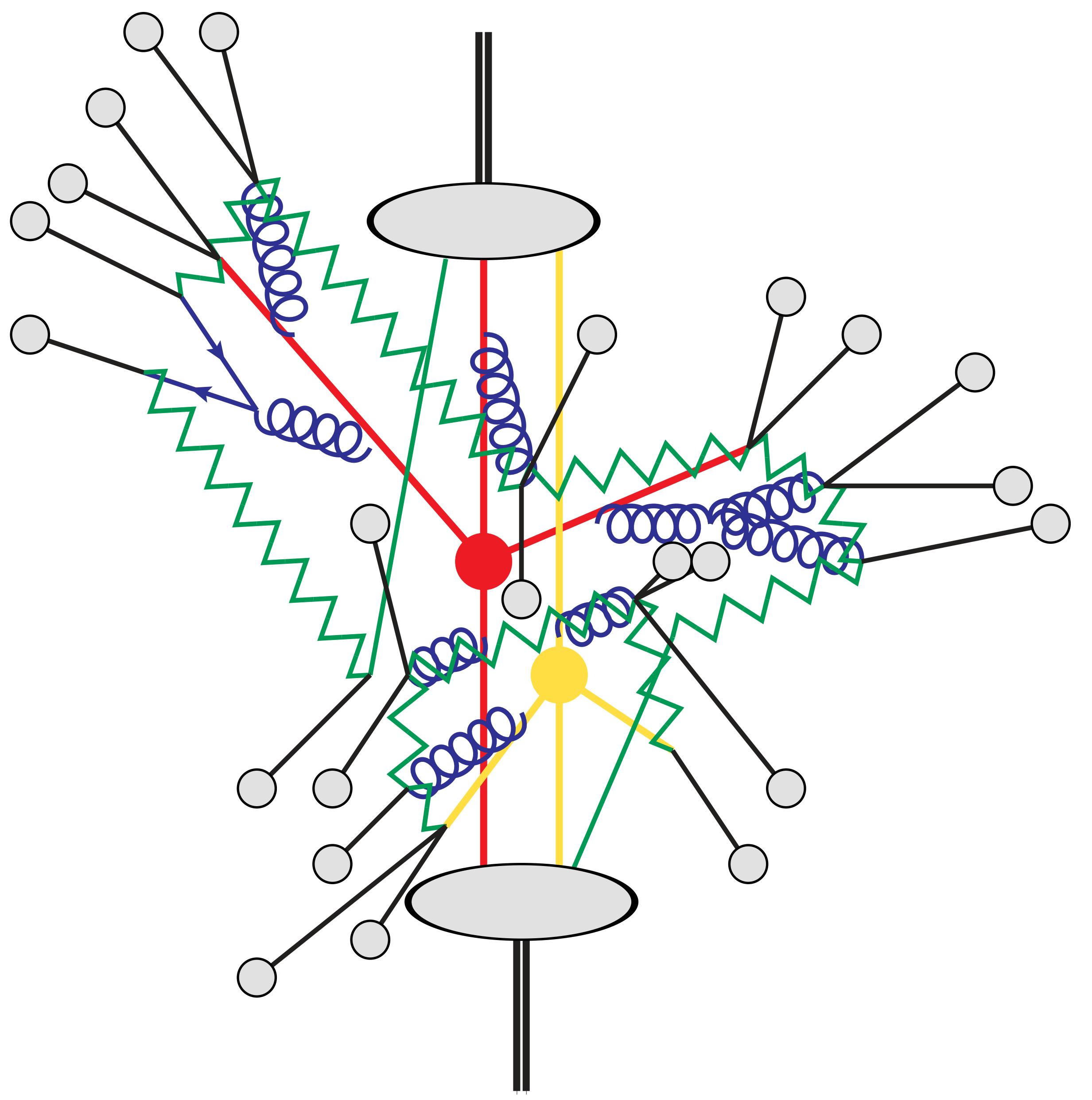
Event Generators

QCD description of collider reactions:
Scale hierarchies and factorisation dictate workflow.

Hard partonic scattering:
NLO QCD routinely

Jet evolution — parton showers:
NLL sometimes, mostly unclear

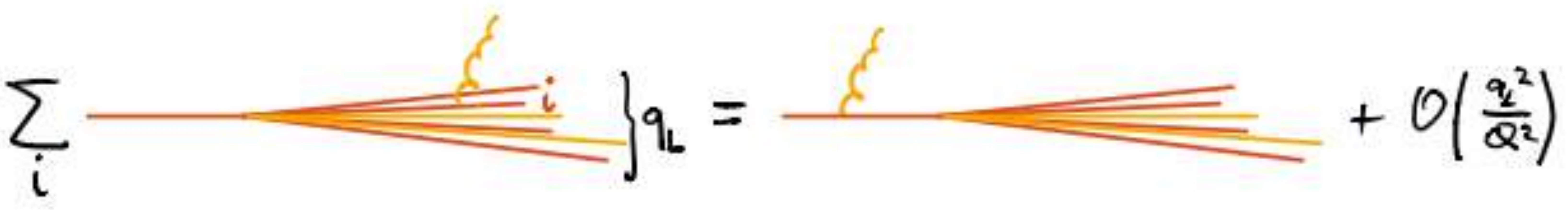
Multi-parton interactions
Hadronization



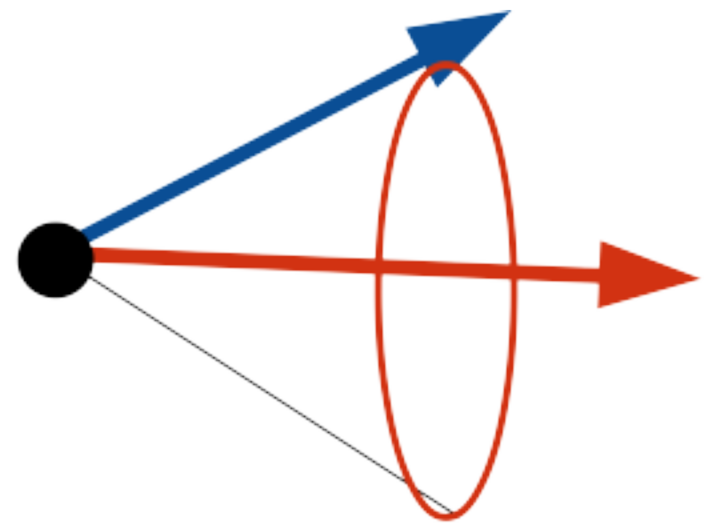
$$d\sigma \sim d\sigma_{\text{hard}}(Q) \times \text{PS}(Q \to \mu) \times \text{Had}(\mu \to \Lambda) \times \ldots$$

QCD Coherence

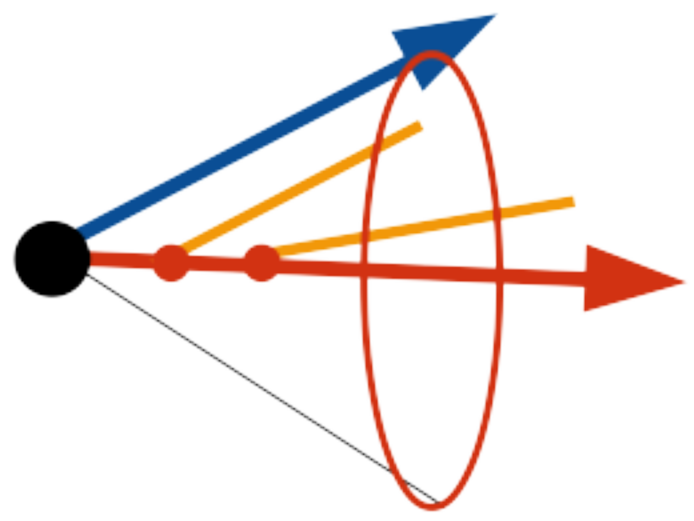
Coherent emission of soft large angle gluons from systems of collinear partons.



Central design criterion behind parton branching algorithms.



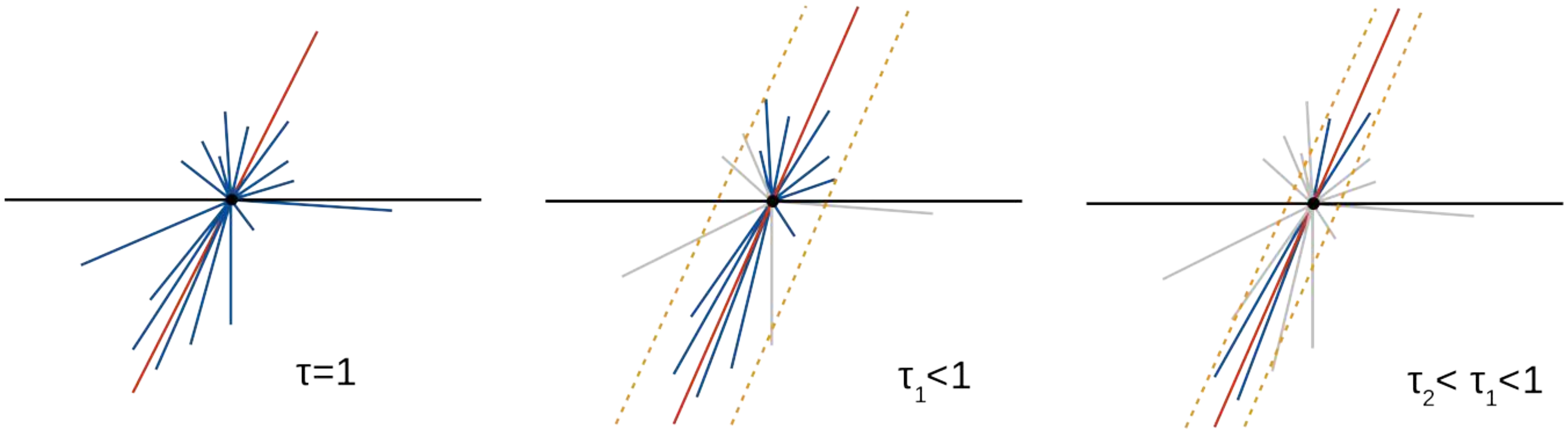
constructive interference
in each collinear region



branchings
order in ~ angle

dipoles
order in ~ p_T

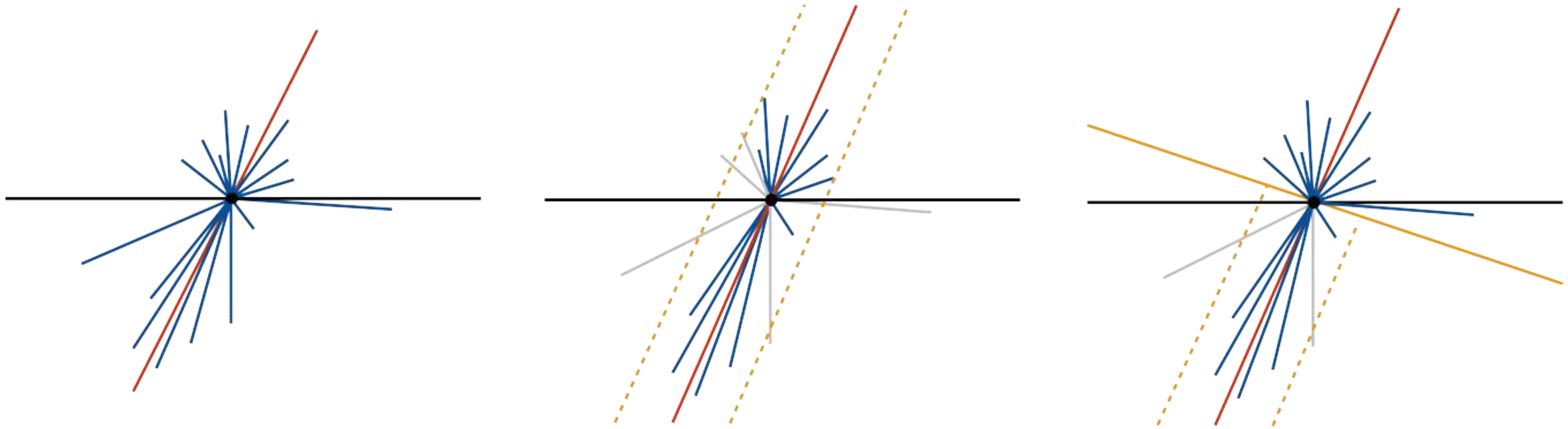
Coherent branching



Resummation using angular ordering.
Initial conditions crucial for large-angle soft radiation.

[Catani, Marchesini, Trentadue, Turnock, Webber ….]

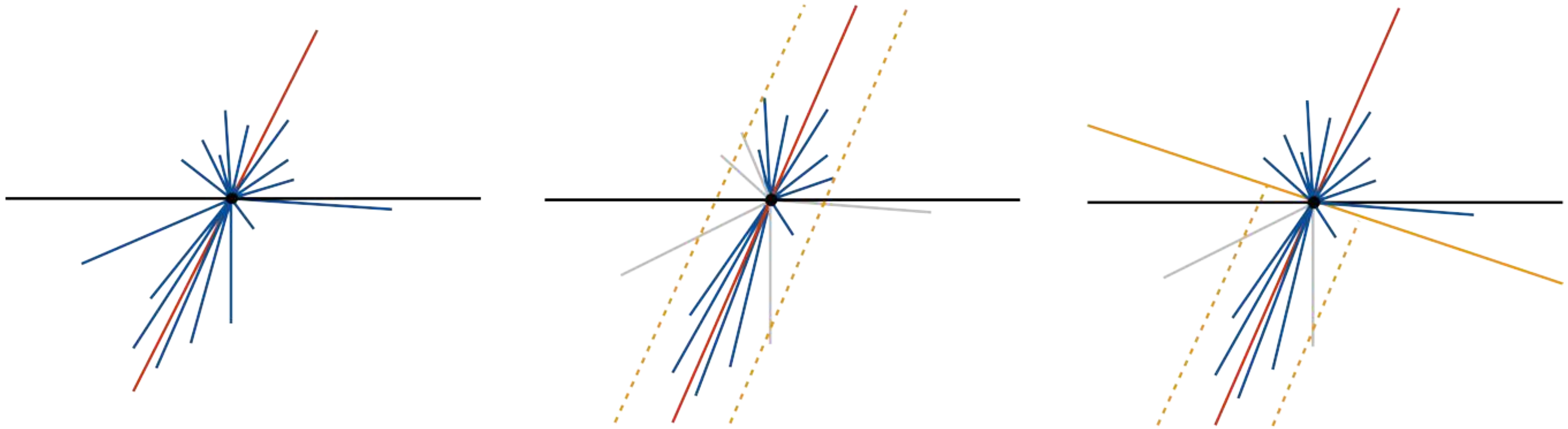
Non-global observables



Unconstrained partonic systems: Full complexity of amplitudes strikes back.
Resummation using dipole-type cascades for large N. [Dasgupta, Salam, Banfi, Marchesini, Smye, Becher et al. …]

Current Event Generators

How accurate are we?



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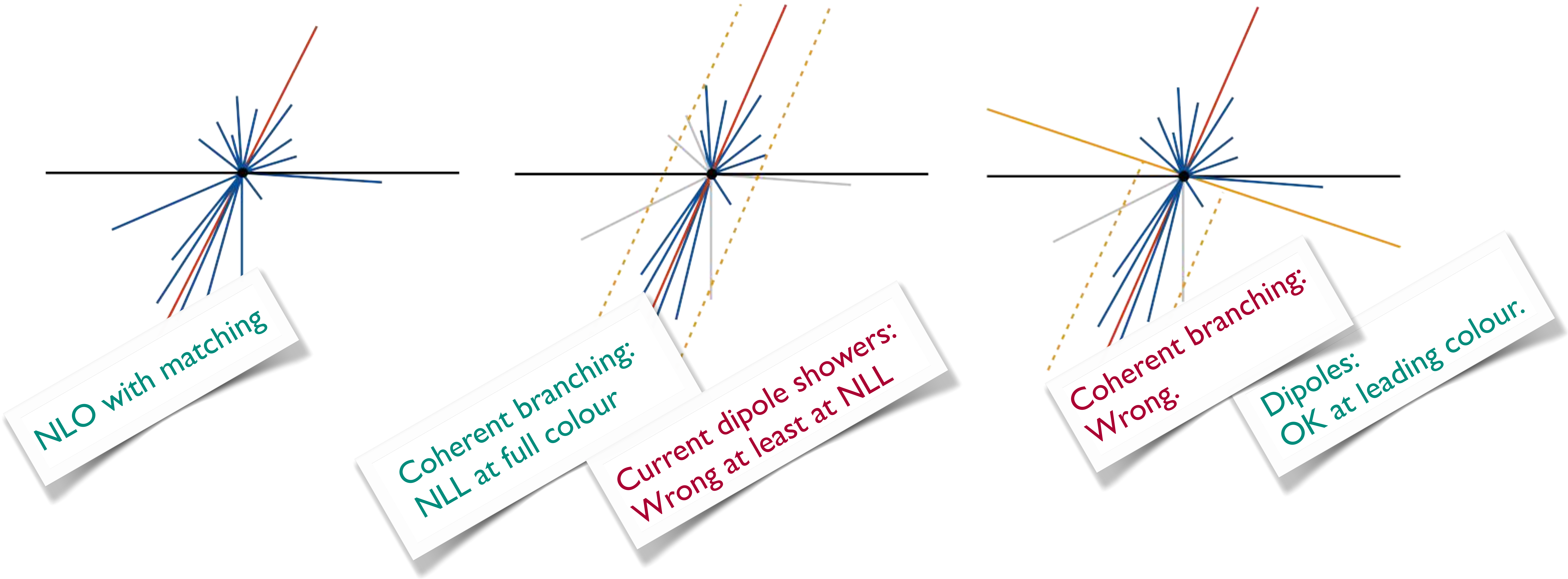

Current Event Generators

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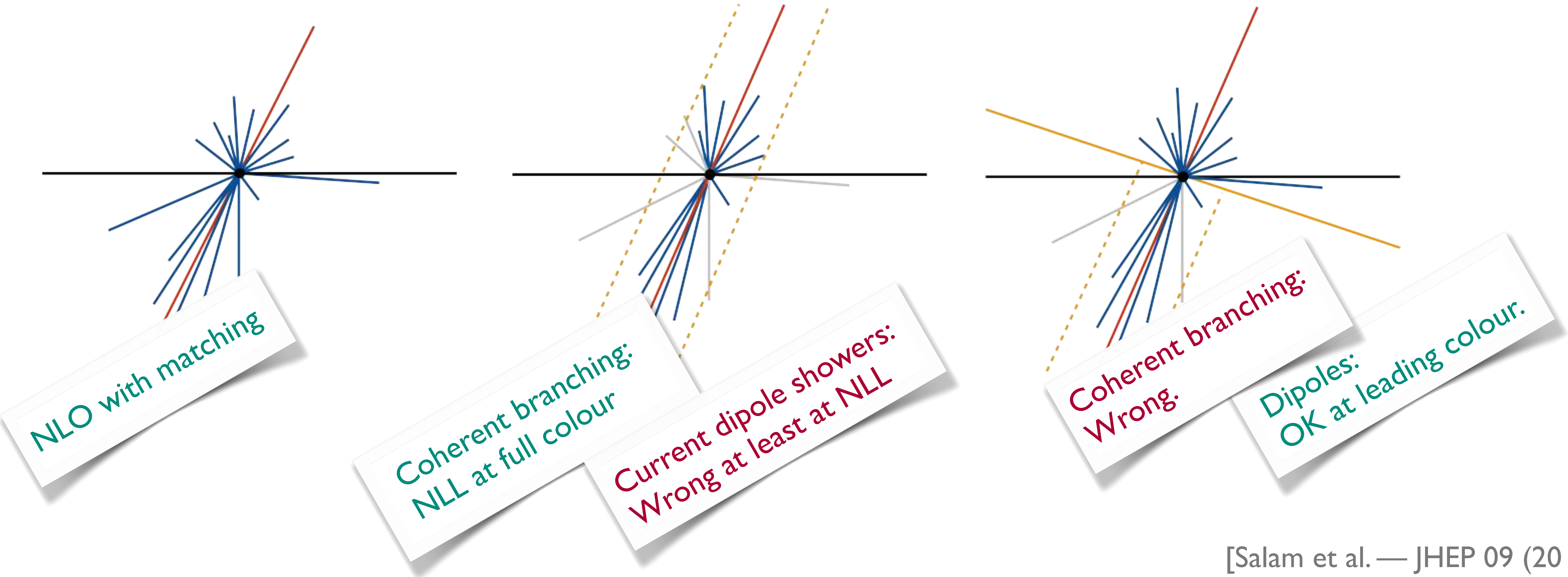


[Salam et al. — JHEP 09 (2018) 033]
solutions also offered in [Forshaw, Holguin, Plätzer – JHEP 09 (2020) 014]

Bottlenecks

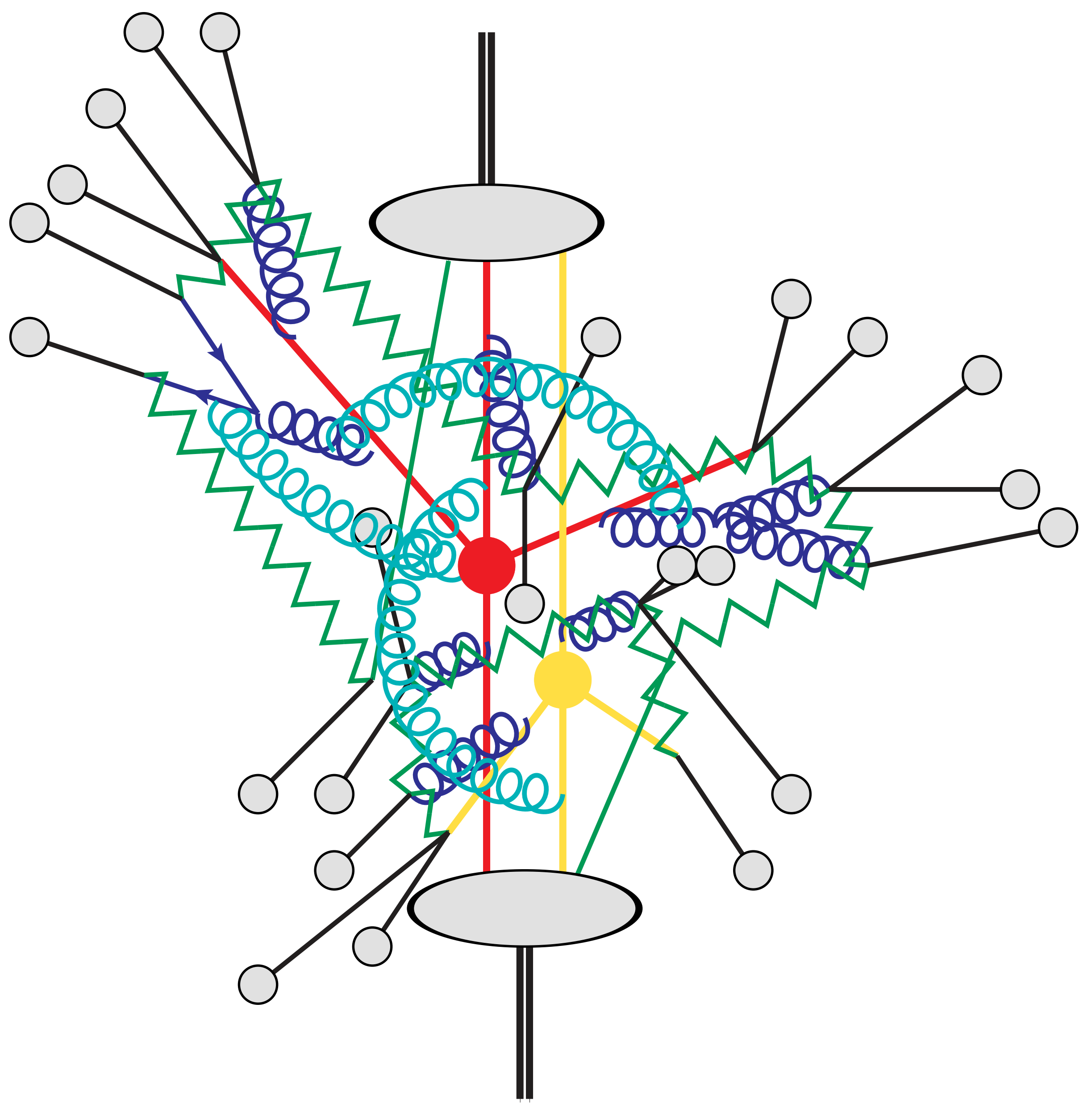
Parton shower algorithms

Lack a systematic expansion, obstruct fully differential NNLO for the hard process, open questions regarding mass effects and unstable particles.

Hadronization models

Lack constraints from perturbative evolution: Hiding perturbative corrections? Genuine uncertainties/constraints?

Rethink foundations of parton showers.



$$\mathrm{d}\sigma \sim \mathrm{Tr}[\mathbf{PS}(Q \to \mu)\mathrm{d}\mathbf{H}(Q)\mathbf{PS}^{\dagger}(Q \to \mu)\mathbf{Had}(\mu \to \Lambda)]$$

Steps towards a novel approach

[Plätzer, Sjödahl – JHEP 1207 (2012) 042]
[Plätzer, Sjödahl, Thoren – JHEP 11 (2018) 009]

Towards an amplitude level formulation:

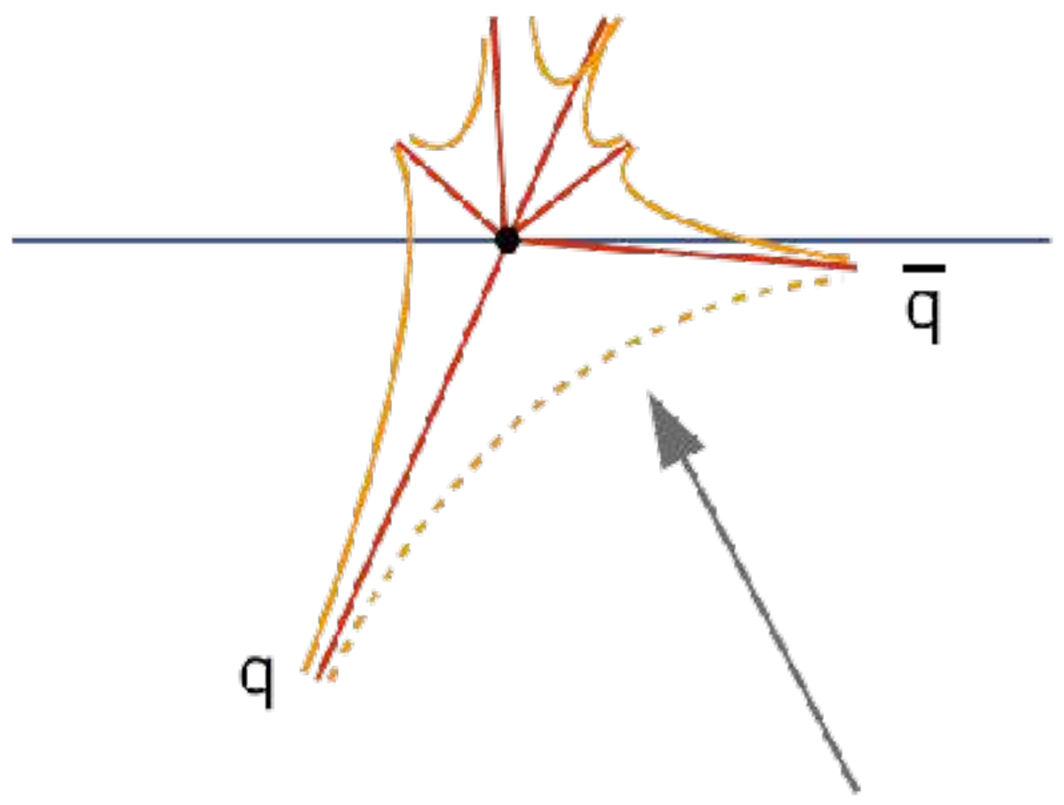
$$\mathrm{d}\sigma_{n+1} \sim |\mathcal{M}_{n+1}|^2 = \langle \mathcal{M}_{n+1} | \mathcal{M}_{n+1} \rangle \sim P \, \mathrm{d}\sigma_n \to \frac{\mathrm{Tr}\left[|\mathcal{M}_n\rangle\langle\mathcal{M}_n| \mathbf{P} \right]}{|\mathcal{M}_n|^2 P} \, P \, \mathrm{d}\sigma_n$$

$$|\mathcal{M}\rangle = \sum_\sigma \mathcal{M}_\sigma |\sigma\rangle$$

Sum of Feynman diagrams, sorted by SU(3) tensor structures → vector space of **colour structures.**

Dipole branching algorithms can be supplemented by correction factors for real emission:

- Cannot take into account colour-mixing virtual corrections
- Not possible to include imaginary parts
- Effects beyond large-angle soft radiation included ad hoc.



Some subleading-N corrections can be restored.

Steps towards a novel approach

[Plätzer, Sjödahl – JHEP 1207 (2012) 042]
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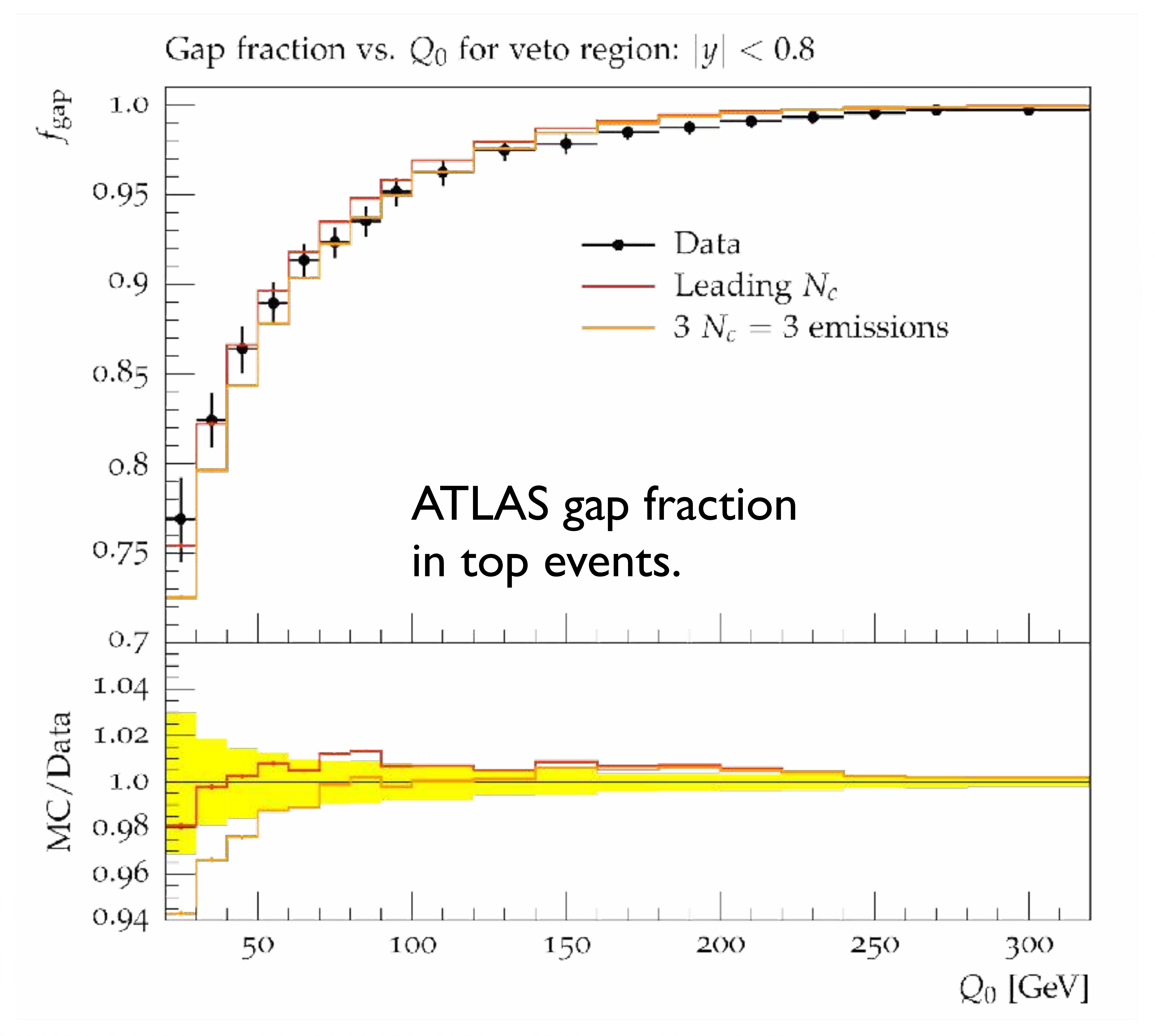
Towards an amplitude lev

$$\mathrm{d}\sigma_{n+1} \sim |\mathcal{M}_{n}$$

$$\rangle\langle\mathcal{M}_n|\mathbf{P}] \over {}_n|^2 P} \; P \; \mathrm{d}\sigma_n$$

$$|\mathcal{M}\rangle = \sum_\sigma \mathcal{M}_\sigma|\sigma\rangle$$

tensor structures → vector



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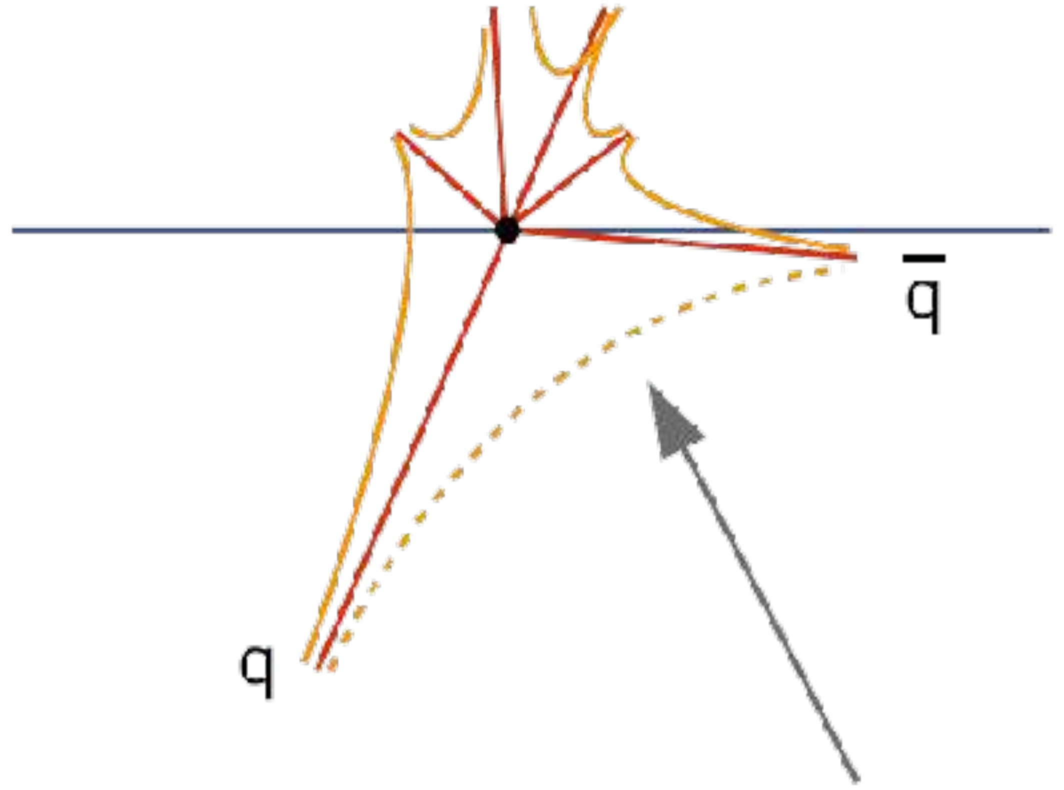

Some subleading-N corrections can be restored.

Parton Branching at Amplitude Level

[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]
[Forshaw, Holguin, Plätzer – JHEP 1908 (2019) 145]

$$\sigma = \sum_n \int \mathrm{Tr}\left[\mathbf{A}_n(\mu)\right] u(p_1, \ldots, p_n)\, \mathrm{d}\phi_n$$

density operator   observable   phase space integration

Density operator is fundamental object, not the amplitude, nor the cross section.

Virtual corrections and colour mixing
in all orders perturbation theory.

$$|\mathcal{M}_n(\mu)\rangle = \mathbf{Z}^{-1}(\mu, \epsilon)|\tilde{\mathcal{M}}_n(\epsilon)\rangle$$

Recursive definition of evolution at amplitude & conjugate amplitude

$$\mathbf{A}_n(E) = \mathbf{V}(E, E_n)\mathbf{D}_n\mathbf{A}_{n-1}(E_n)\mathbf{D}_n^\dagger\mathbf{V}^\dagger(E, E_n)\theta(E - E_n)$$

Parton Branching at Amplitude Level

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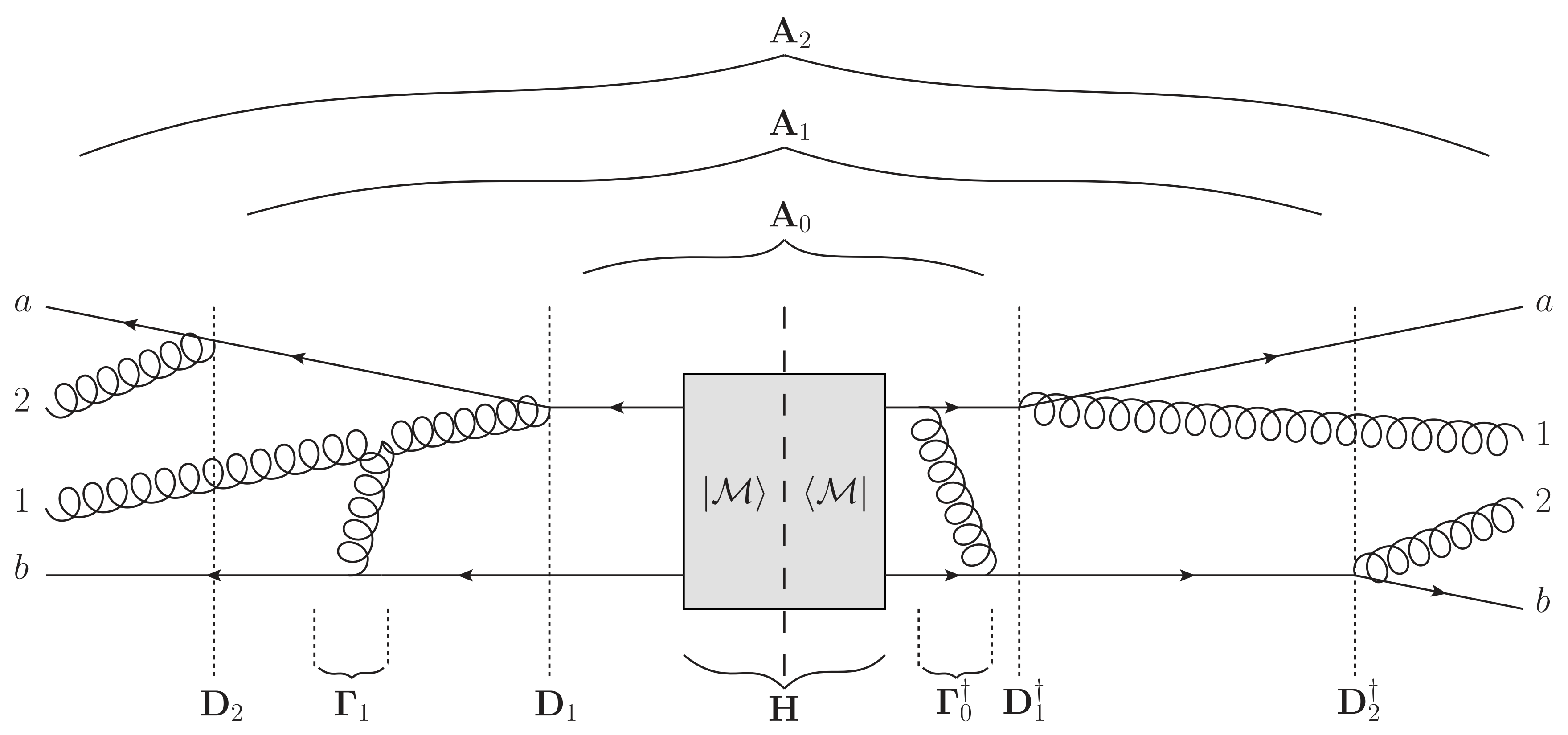
$$\sigma = \sum_n \int \mathrm{Tr}\left[\mathbf{A}_n(\mu)\right] u(p_1, ..., p_n)\, \mathrm{d}\phi_n$$

density operator — observable — phase space integration

$$\mathbf{A}_n(q_\perp; \{p\}_n) = \int \mathrm{d}R_n \mathbf{V}_{q_\perp, q_{n\perp}} \mathbf{D}_n \mathbf{A}_{n-1}(q_{n\perp}; \{p\}_{n-1}) \mathbf{D}_n^\dagger \mathbf{V}^\dagger_{q_\perp, q_{n\perp}} \Theta(q_\perp \leq q_{n\perp})$$

$$\mathbf{V}_{a,b} = \mathrm{Pexp}\left(-\int_a^b \frac{\mathrm{d}q_\perp}{q_\perp} \mathbf{\Gamma}_n(q_\perp)\right)$$

$$\mathbf{D}_n(q_{n\perp}; q_n \cup \{\tilde{p}\}_{n-1})\, \mathbf{O}\, \mathbf{D}_n^\dagger(q_{n\perp}; q_n \cup \{\tilde{p}\}_{n-1}) =$$
$$\sum_{i_n, j_n} \int \delta q_{n\perp}^{(i_n, j_n)}(q_{n\perp})\, \mathbf{S}_n^{i_n}\, \mathbf{O}\, \mathbf{S}_n^{j_n\,\dagger} + \sum_{i_n} \int \delta q_{n\perp}^{(i_n, \vec{n})}(q_{n\perp})\, \mathbf{C}_n^{i_n}\, \mathbf{O}\, \mathbf{C}_n^{i_n\,\dagger}$$

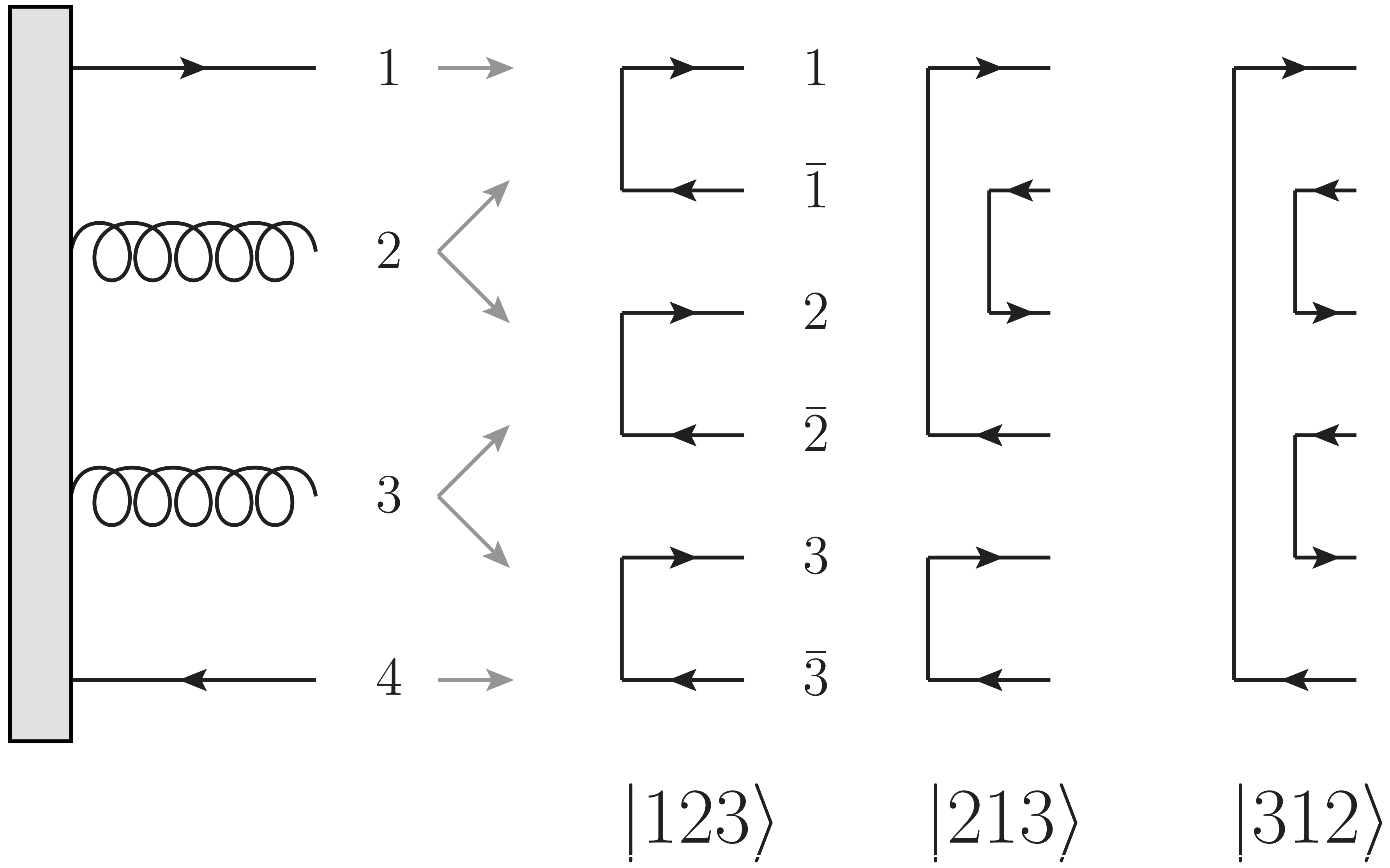


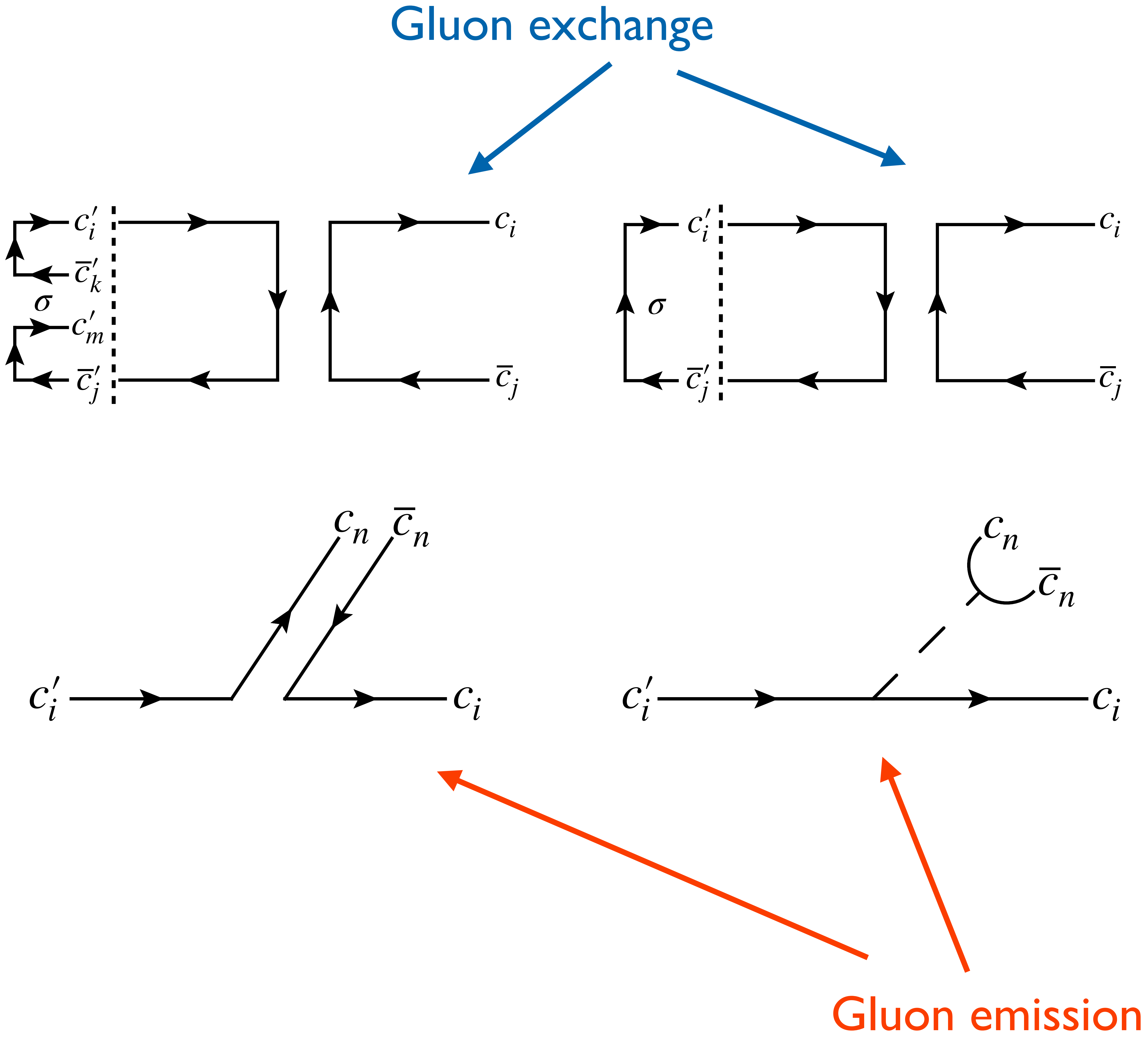
soft contributions

collinear contributions

Tracking colour

Decompose amplitudes in flow of colour charge.





Non-global Observables and Large-N

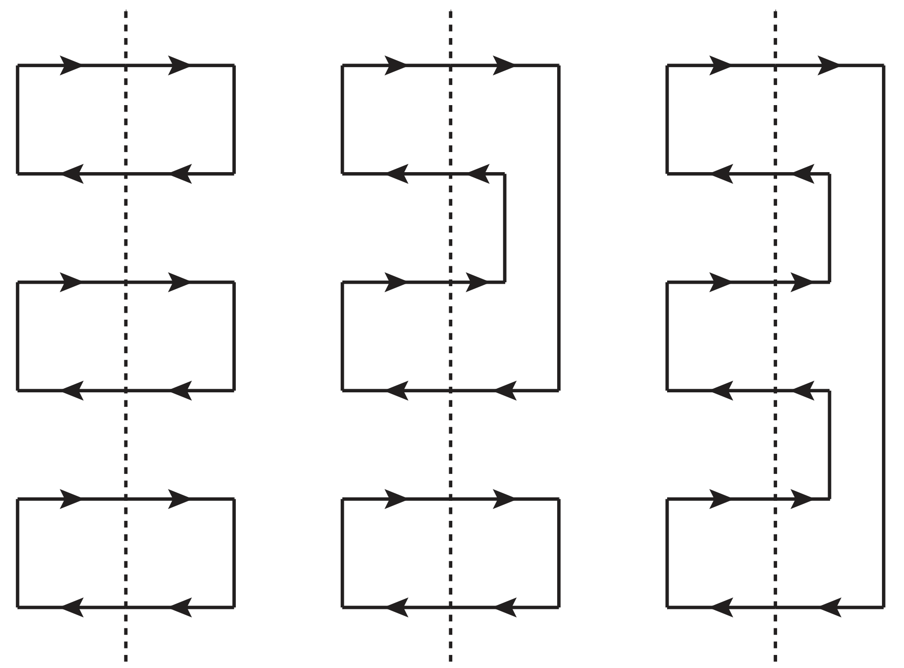
[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]

Primary application: Non-global observables

$$E\frac{\partial \mathbf{G}_n(E)}{\partial E} = -\mathbf{\Gamma}\mathbf{G}_n(E) - \mathbf{G}_n(E)\mathbf{\Gamma}^\dagger + \mathbf{D}_n^\mu \, \mathbf{G}_{n-1}(E) \, \mathbf{D}_{n\mu}^\dagger \, u(E, \hat{k}_n)$$

Utilise colour flow basis, and expand around large-N:

$$\text{Leading}^{(l)}_{\tau\sigma}[\mathbf{A}] = \sum_{k=0}^{l} \mathcal{A}_{\tau\sigma}\Big|_{1/N^k} \; \delta_{\#\text{transpositions}(\tau,\sigma),l-k}$$



$\langle 123|123\rangle$ $\langle 123|213\rangle$ $\langle 123|312\rangle$

Re-derive BMS equation: Prototype of constructing a dipole shower

$$\text{Leading}^{(0)}_{\tau\sigma}\left[\mathbf{V}_n\mathbf{A}_n\mathbf{V}_n^\dagger\right] = \delta_{\tau\sigma}\left|V_\sigma^{(n)}\right|^2 \; \text{Leading}^{(0)}_{\tau\sigma}[\mathbf{A}_n]$$

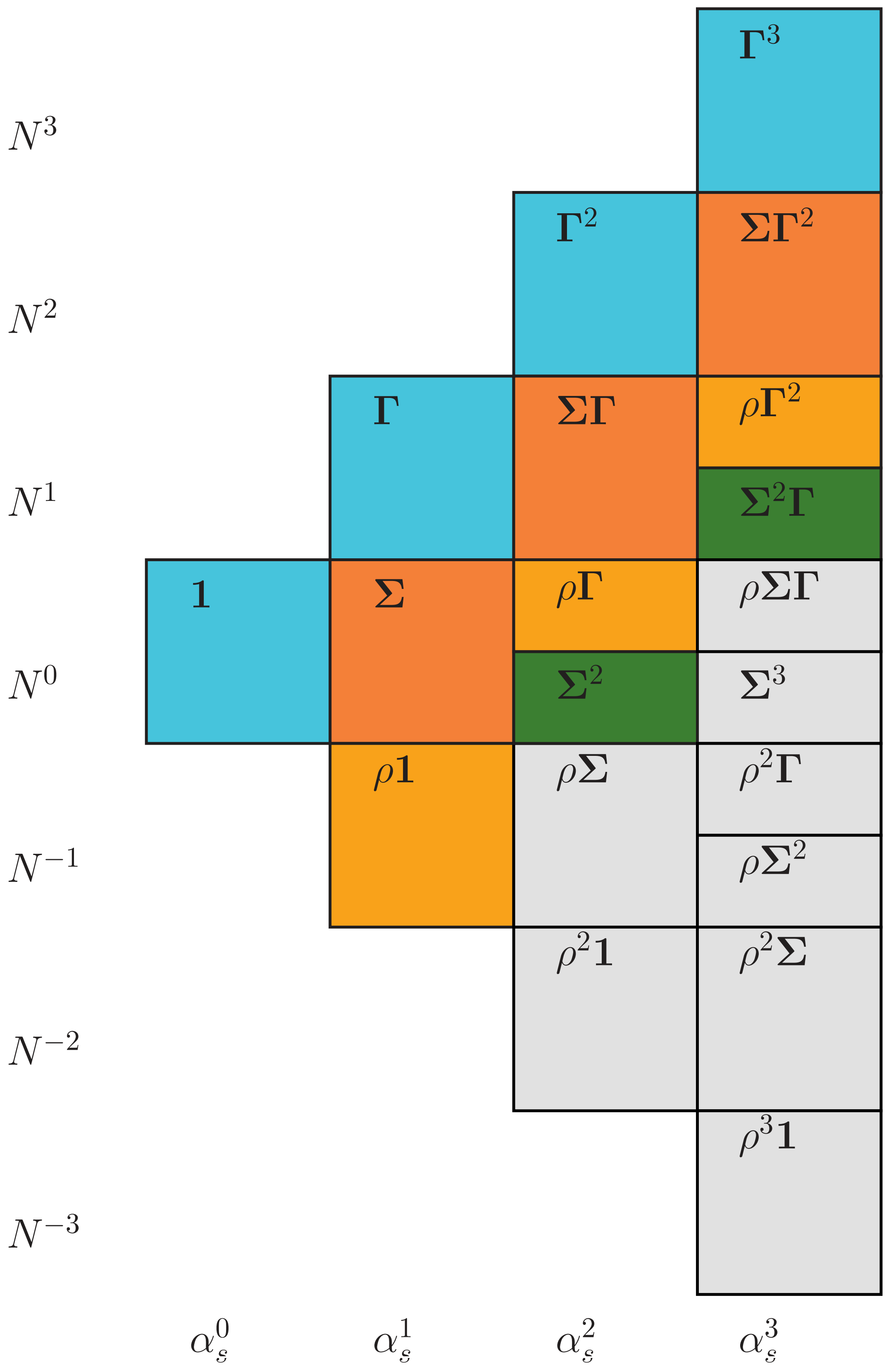
$$V_\sigma^{(n)} = \exp\left(-N\sum_{i,j\ \text{c.c. in}\ \sigma}\lambda_i\bar{\lambda}_j W_{ij}^{(n)}\right)$$

$$\text{Leading}^{(0)}_{\tau\sigma}\left[\mathbf{D}_n\mathbf{A}_{n-1}\mathbf{D}_n^\dagger\right] = \delta_{\tau\sigma}\sum_{i,j\ \text{c.c. in}\ \sigma\backslash n}\lambda_i\bar{\lambda}_j R_{ij}^{(n)} \; \text{Leading}^{(0)}_{\tau\backslash n,\sigma\backslash n}[\mathbf{A}_{n-1}]$$

colour connected dipoles

Beyond Leading Colour

[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]



virtuals	reals
$(0\text{ flips}) \times 1 \times (\alpha_s N)^n$	$(\mathbf{t}[...]\mathbf{t}\vert_{0\text{ flips}})^{r-1}\, \mathbf{t}[...]\mathbf{t}\vert_{2\text{ flips}} \times 1$ $(\mathbf{t}[...]\mathbf{t}\vert_{0\text{ flips}})^{r-1}\, \mathbf{t}[...]\mathbf{s}\vert_{1\text{ flip}} \times N^{-1}$ $(\mathbf{t}[...]\mathbf{t}\vert_{0\text{ flips}})^{r-1}\, \mathbf{s}[...]\mathbf{s}\vert_{0\text{ flips}} \times N^{-2}$
$(1\text{ flip}) \times \alpha_s \times (\alpha_s N)^n$	$(\mathbf{t}[...]\mathbf{t}\vert_{0\text{ flips}})^{r}$ $(\mathbf{t}[...]\mathbf{t}\vert_{0\text{ flips}})^{r-1}\, \mathbf{t}[...]\mathbf{s}\vert_{1\text{ flip}} \times N^{-1}$
$(0\text{ flips}) \times \alpha_s N^{-1} \times (\alpha_s N)^n$	$(\mathbf{t}[...]\mathbf{t}\vert_{0\text{ flips}})^{r}$
$(0\text{ flips}) \times \alpha_s^2 \times (\alpha_s N)^n$ $(2\text{ flips}) \times \alpha_s^2 \times (\alpha_s N)^n$	$(\mathbf{t}[...]\mathbf{t}\vert_{0\text{ flips}})^{r}$ $(\mathbf{t}[...]\mathbf{t}\vert_{0\text{ flips}})^{r-1}\, \mathbf{t}[...]\mathbf{t}\vert_{2\text{ flips}}$

$$[\tau|\mathbf{\Gamma}|\sigma\rangle = N\delta_{\tau\sigma}\Gamma_\sigma + \Sigma_{\tau\sigma} + \frac{1}{N}\delta_{\tau\sigma}\rho$$

[Plätzer – EPJ C 74 (2014) 2907]

Systematically sum colour enhanced terms

$$\alpha_s N \sim 1$$

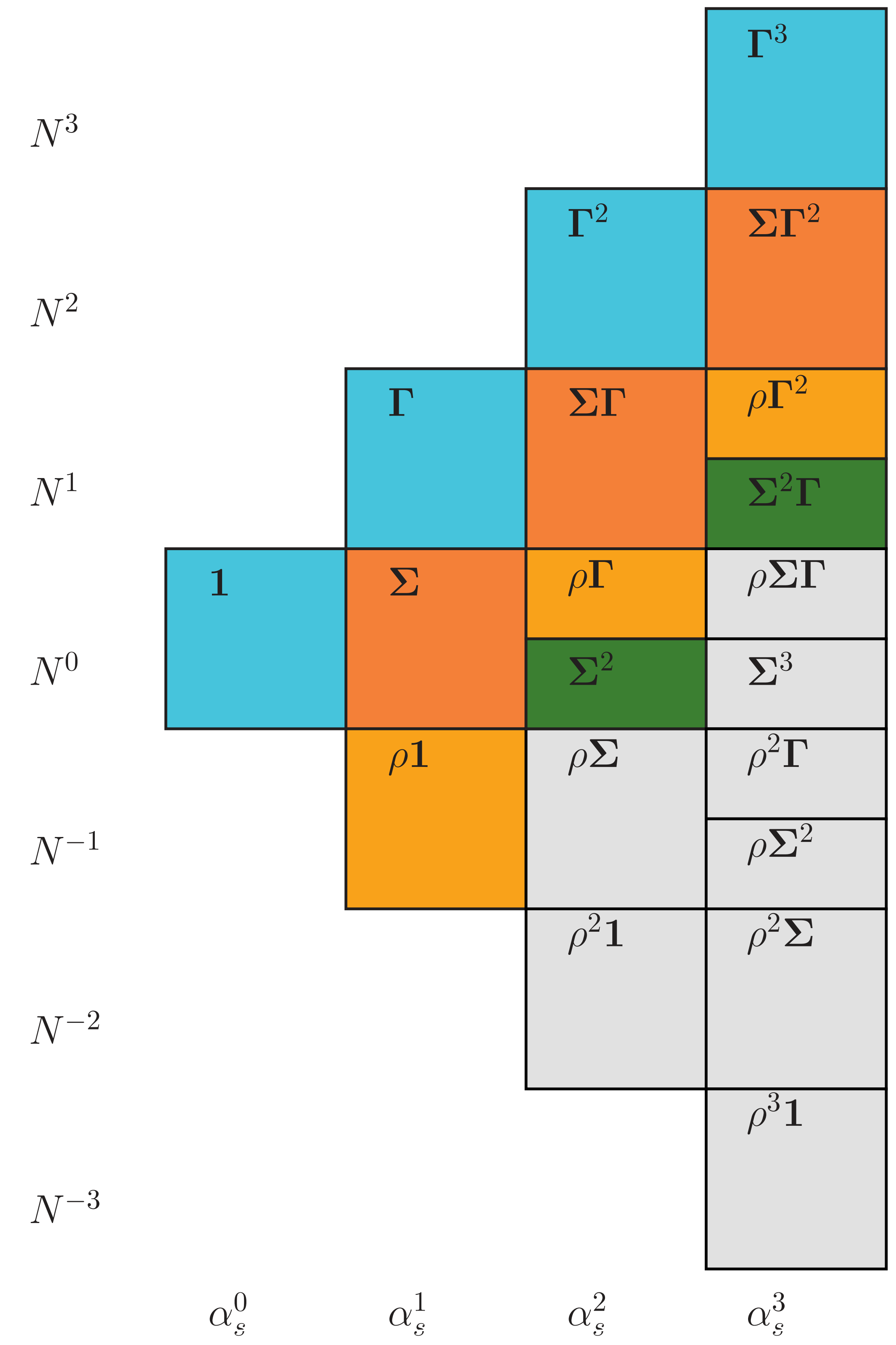
$$\mathbf{V}_n^{\text{LC+NLC}}|\sigma\rangle = V_\sigma^{(n)}|\sigma\rangle - \frac{1}{N}\sum_\tau \delta_{\#\text{transpositions}(\tau,\sigma),1}\Sigma_{\sigma\tau}^{(n)}|\tau\rangle$$

$$\Sigma_{\sigma\tau}^{(n)} = N\frac{e^{-W_\sigma^{(n)}} - e^{-W_\tau^{(n)}}}{W_\sigma^{(n)} - W_\tau^{(n)}} \times \sum_{\substack{i,k \text{ c.c. in } \sigma \\ j,l \text{ c.c. in } \sigma}} \left(\lambda_i\lambda_j W_{ij}^{(n)} + \bar\lambda_k\bar\lambda_l W_{kl}^{(n)} - \lambda_i\bar\lambda_l W_{il}^{(n)} - \bar\lambda_k\lambda_j W_{kj}^{(n)}\right)\delta_{\substack{i,l \text{ c.c. in } \tau \\ k,j \text{ c.c. in } \tau}}$$

dipole flips at next-to-leading colour

Beyond Leading Colour

[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]



virtuals	reals
(0 flips) $\times 1 \times (\alpha_s N)^n$	$(\mathbf{t}[...]\mathbf{t}\|_{0\text{ flips}})^{r-1}\, \mathbf{t}[...]\mathbf{t}\|_{2\text{ flips}} \times 1$ $(\mathbf{t}[...]\mathbf{t}\|_{0\text{ flips}})^{r-1}\, \mathbf{t}[...]\mathbf{s}\|_{1\text{ flip}} \times N^{-1}$
(1 flip) $\times \alpha_s$ ...	
(0 flips) $\times \alpha$ ...	
(0 flips) $\times \alpha$ ... (2 flips) $\times \alpha$ ...	

We expect that the algorithm presented here will not be the last word in algorithms for this purpose. Surely it is possible to do better. Indeed, Ángeles Martínez, De Angelis, Forshaw, Plätzer, and Seymour [16] have provided a formalism for the description of soft gluon emissions that is similar in some ways to the general formalism [4, 7] on which this paper is based. If the approach of Ref. [16] can be extended to include the collinear singularities of QCD, then it will be of great interest to see if there can be a practical implementation of the resulting formalism. Perhaps such an implementation will be able to outperform what this paper provides.

[Nagy, Soper – Phys.Rev. D99 (2019) 054009]

$$\ldots N\delta_{\tau\sigma}\Gamma_\sigma + \Sigma_{\tau\sigma} + \frac{1}{N}\delta_{\tau\sigma}\rho$$

[Plätzer – EPJ C 74 (2014) 2907]

Systematically sum colour enhanced terms

$$= V_\sigma^{(n)}|\sigma\rangle - \frac{1}{N}\sum_\tau \delta_{\#\text{transpositions}(\tau,\sigma),1}\Sigma_{\sigma\tau}^{(n)}|\tau\rangle$$

$$\Sigma_{\sigma\tau}^{(n)} = N\frac{e^{-W_\sigma^{(n)}} - e^{-W_\tau^{(n)}}}{W_\sigma^{(n)} - W_\tau^{(n)}} \times \sum_{\substack{i,k \text{ c.c. in } \sigma \\ j,l \text{ c.c. in } \sigma}} \left(\lambda_i\lambda_j W_{ij}^{(n)} + \bar\lambda_k\bar\lambda_l W_{kl}^{(n)} - \lambda_i\bar\lambda_l W_{il}^{(n)} - \bar\lambda_k\lambda_j W_{kj}^{(n)}\right)\delta_{\substack{i,l \text{ c.c. in } \tau \\ k,j \text{ c.c. in } \tau}}$$

dipole flips at next-to-leading colour

Collinear Subtractions

universität wien

[Forshaw, Holguin, Plätzer – JHEP 1908 (2019) 145]

Identify and subtract collinear singularities in soft evolution

softness

ordering for soft evolution

ordering for collinear evolution

$$\ln \mathbf{W}_{ab} = \frac{\alpha_s}{2\pi} \sum_{i<j} \mathbb{T}_i^g \cdot \mathbb{T}_j^g \int_{a^2}^{b^2} \frac{\mathrm{d}q^2}{q^2} \int \frac{\mathrm{d}^3 k}{2E} \frac{1}{\pi\,(S\cdot k)^2}$$

$$\Bigg(K^2(p_i,p_j;k) \frac{n_i\cdot n_j}{n_i\cdot n\; n\cdot n_j} \delta(q^2 - K^2(p_i,p_j;k))\,\theta_{ij}(k)$$

$$- \frac{K^2(p_i;k)}{n_i\cdot n}\delta(q^2-K^2(p_i;k))\theta_i(k) - \frac{K^2(p_j;k)}{n_j\cdot n}\delta(q^2-K^2(p_j;k))\theta_j(k)\Bigg)$$

Energy ordering

$$\ln \mathbf{W}_{ab}\Big|_{\text{energy}} = \frac{\alpha_s}{\pi}\sum_{i<j}\mathbb{T}_i^g\cdot\mathbb{T}_j^g \int_a^b \frac{\mathrm{d}E}{E}\int\frac{\mathrm{d}\Omega}{4\pi}\left(\frac{n_i\cdot n_j - n_i\cdot n - n_j\cdot n}{n_i\cdot n\; n\cdot n_j}\right)$$

$$= \frac{\alpha_s}{\pi}\sum_{i<j}\mathbb{T}_i^g\cdot\mathbb{T}_j^g \int_a^b \frac{\mathrm{d}E}{E}\;\ln\frac{n_i\cdot n_j}{2}$$

$$\ln \mathbf{K}_{ab}\Big|_{\text{energy}} = \frac{\alpha_s}{\pi}\sum_i (\mathbb{T}_i^g)^2 \int_a^b \frac{\mathrm{d}E}{E}\int\frac{\mathrm{d}\Omega}{4\pi}\frac{2}{n_i\cdot n}$$

(Dipole) pt ordering

$$\ln \mathbf{W}_{ab}\Big|_{k_T} = \frac{\alpha_s}{\pi}\sum_{i<j}\mathbb{T}_i^g\cdot\mathbb{T}_j^g\int_a^b\frac{\mathrm{d}k_\perp}{k_\perp}\int\frac{\mathrm{d}y\,\mathrm{d}\phi}{2\pi}\,(\theta_{ij}(k)-\theta_i(k))$$

$$\ln \mathbf{K}_{ab}\Big|_{k_T} = \frac{\alpha_s}{2\pi}\sum_i(\mathbb{T}_j^g)^2\int_a^b\frac{\mathrm{d}k_\perp}{k_\perp}\int_\alpha^1\frac{\mathrm{d}z}{1-z+\alpha}\int\frac{\mathrm{d}\phi}{2\pi}$$

$$= \frac{\alpha_s}{2\pi}\sum_i(\mathbb{T}_j^g)^2\int_a^b\frac{\mathrm{d}k_\perp}{k_\perp}\int_0^{1-\alpha}\frac{\mathrm{d}z}{1-z}\int\frac{\mathrm{d}\phi}{2\pi}$$

Collinear Subtractions

[Forshaw, Holguin, Plätzer – JHEP 1908 (2019) 145]

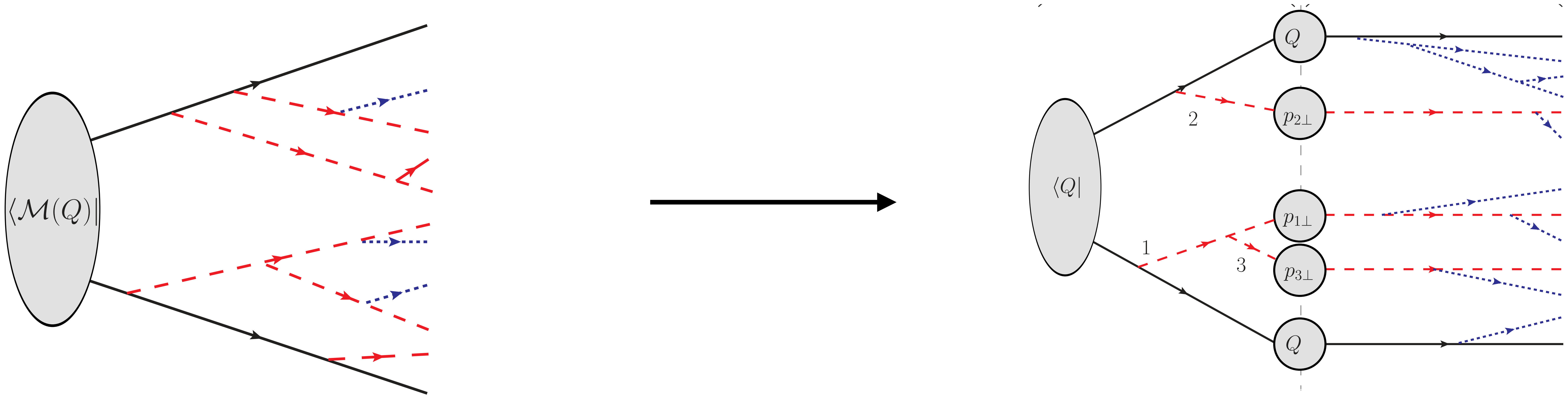
Identify and subtract collinear singularities in soft evolution

$$\ln \mathbf{W}_{ab} = \frac{\alpha_s}{2\pi} \sum_{i<j} \mathbb{T}_i^g \cdot \mathbb{T}_j^g \int_{a^2}^{b^2} \frac{\mathrm{d}q^2}{q^2} \int \frac{\mathrm{d}^3 k}{2E} \frac{1}{\pi\,(S\cdot k)^2}$$
$$\Bigg(K^2(p_i,p_j;k)\frac{n_i\cdot n_j}{n_i\cdot n\; n\cdot n_j}\delta(q^2 - K^2(p_i,p_j;k))\,\theta_{ij}(k)$$
$$- \frac{K^2(p_i;k)}{n_i\cdot n}\delta(q^2 - K^2(p_i;k))\theta_i(k) - \frac{K^2(p_j;k)}{n_j\cdot n}\delta(q^2 - K^2(p_j;k))\theta_j(k)\Bigg)$$

softness

ordering for soft evolution

ordering for collinear evolution



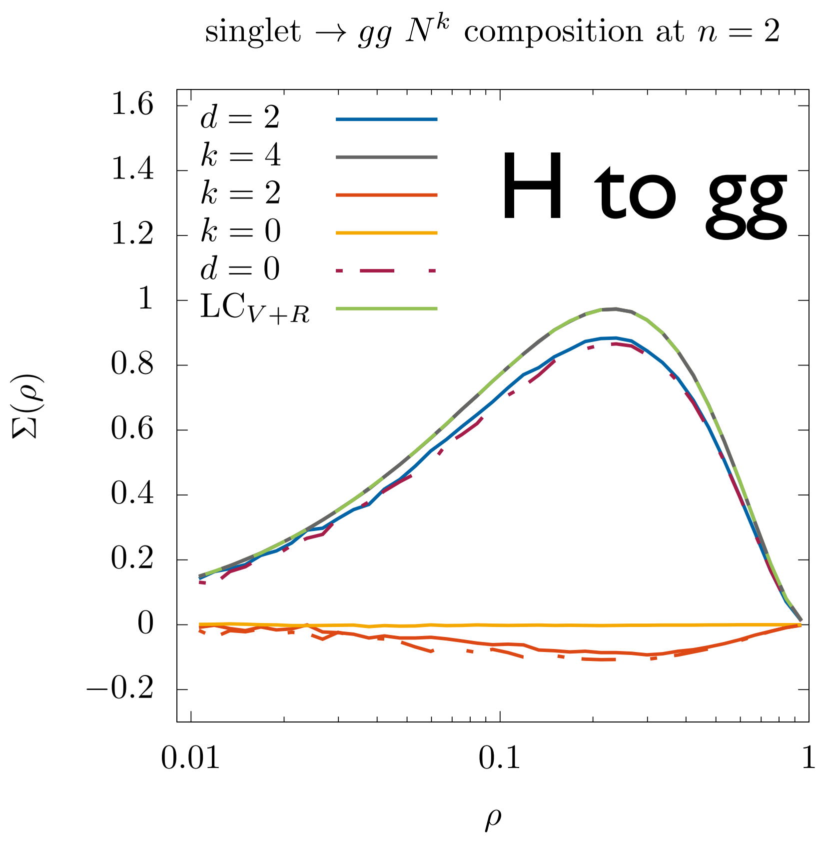
Beyond Leading Colour

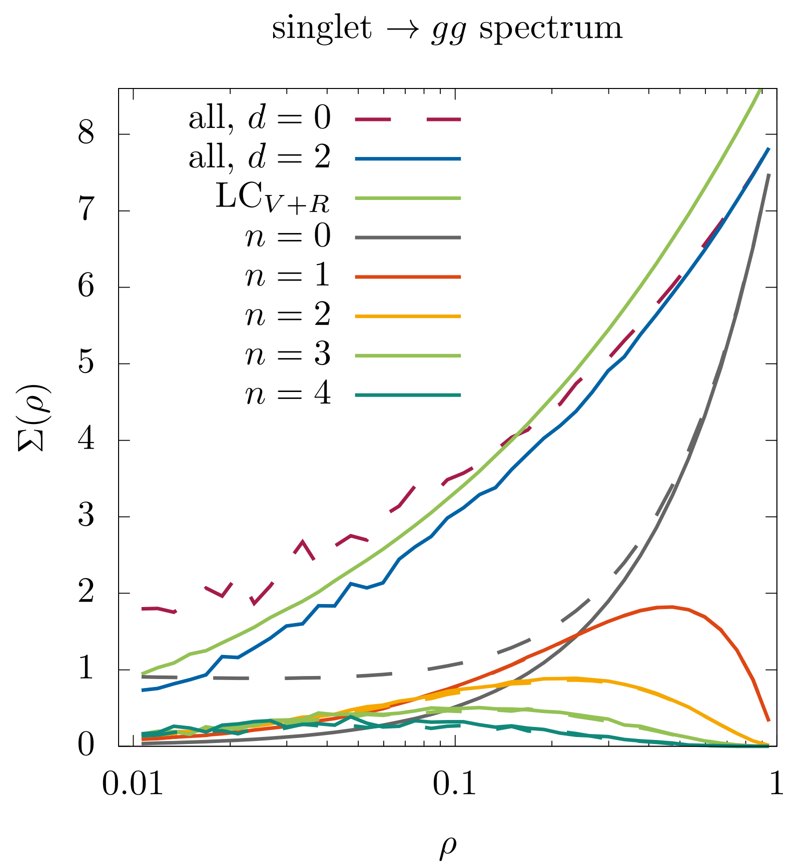
CVolver library implements numerical evolution in colour space.

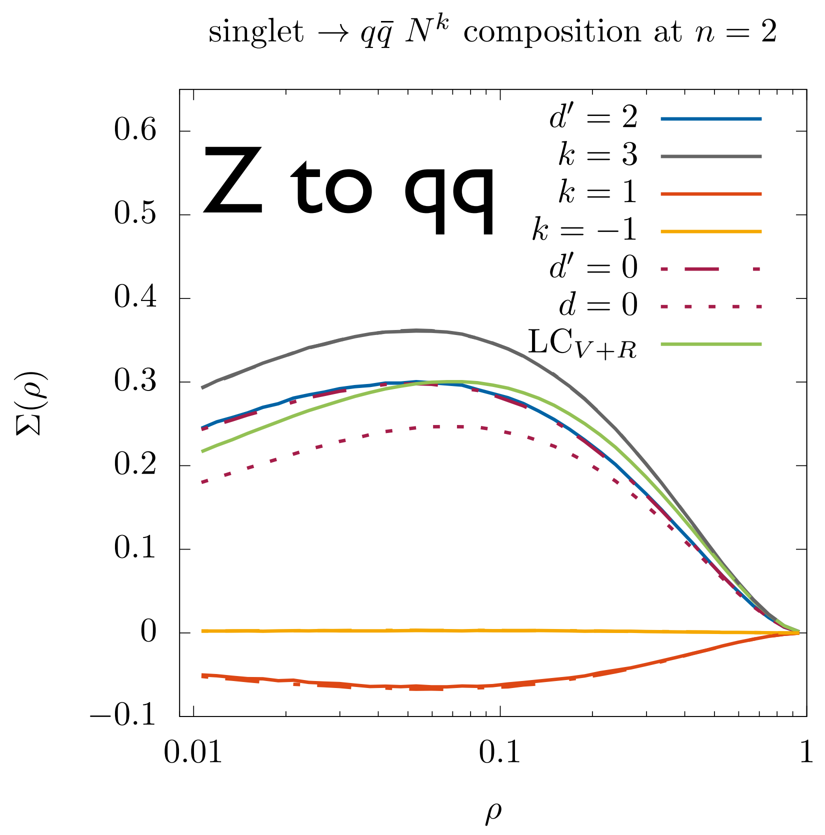
origins in [Plätzer – EPJ C 74 (2014) 2907]

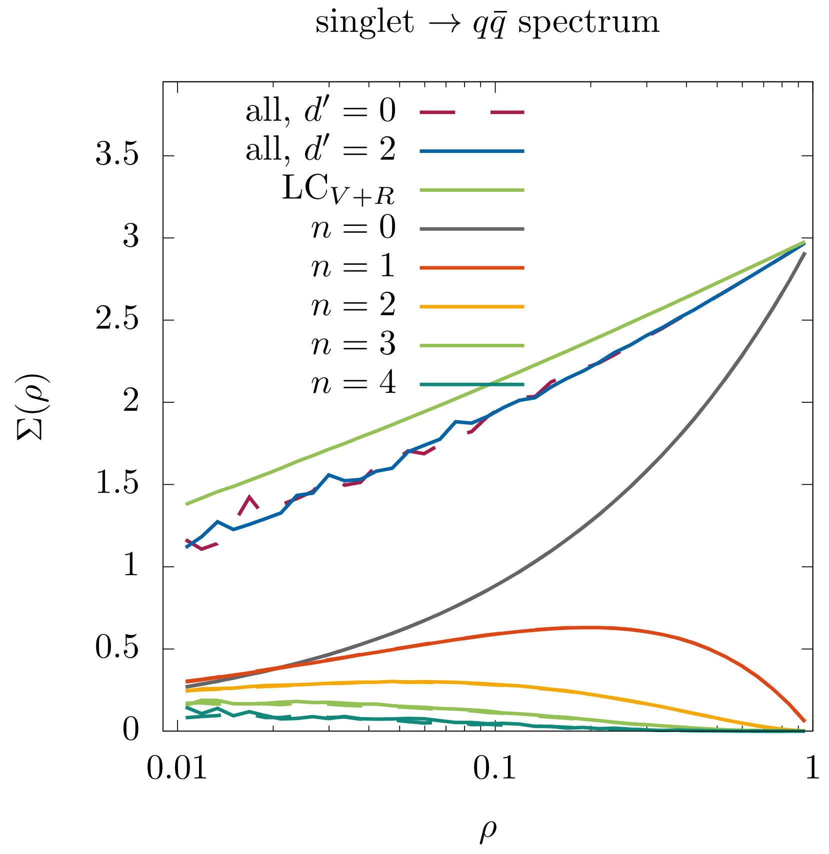
[De Angelis, Forshaw, Plätzer — arXiv:2007.09648]

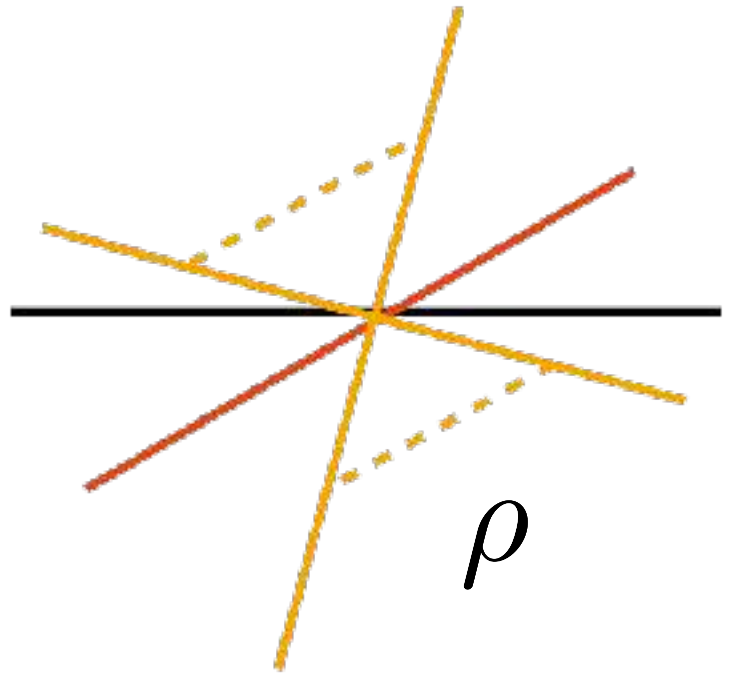
Resummation of non-global logarithms at full colour:



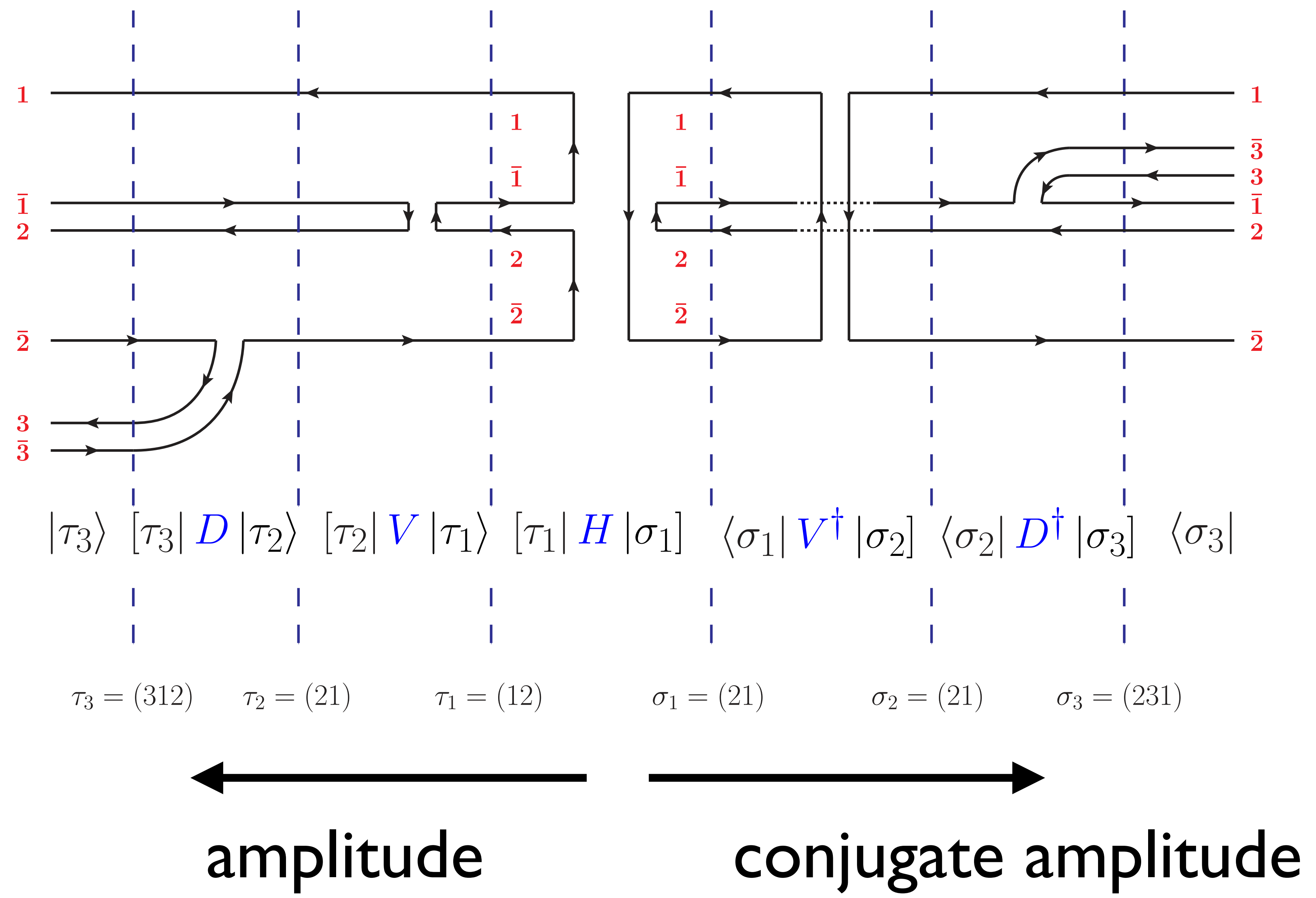








$$\Sigma(\rho) = \sum_n \int \mathrm{d}\sigma(\{p_i\}) \prod_i \theta_{\mathrm{in}}(\rho - E_i)$$



Avoid complexity which grows with colour space dimensionality:

- Monte Carlo over colour flows,
- events at intermediate steps carry complex weights.

Beyond Leading Order

universität wien

[Plätzer, Ruffa — arXiv:2012.15215 & in progress]

Include simultaneously unresolved emissions and higher loop structures

$$E\frac{\partial}{\partial E}\mathbf{A}_n(E) = \mathbf{\Gamma}_n(E)\mathbf{A}_n(E) + \mathbf{A}_n(E)\mathbf{\Gamma}_n^\dagger(E) - \sum_k \mathbf{R}_n^{(k)}(E)\mathbf{A}_{n-k}(E)\mathbf{R}_n^{(k),\dagger}(E)$$

$$[\tau|\mathbf{\Gamma}|\sigma\rangle = (\alpha_s N)[\tau|\mathbf{\Gamma}^{(1)}|\sigma\rangle + (\alpha_s N)^2[\tau|\mathbf{\Gamma}^{(2)}|\sigma\rangle + \ldots$$

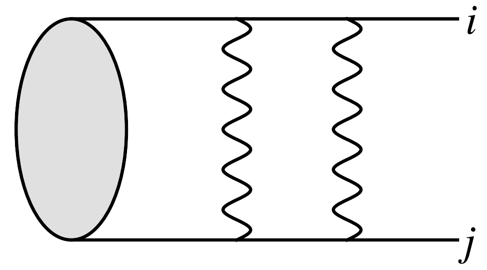
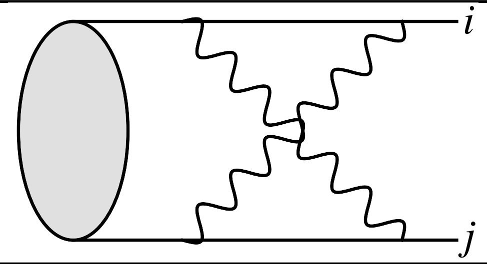
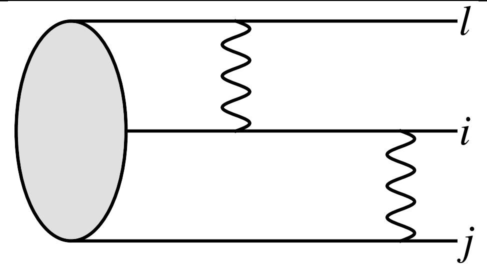
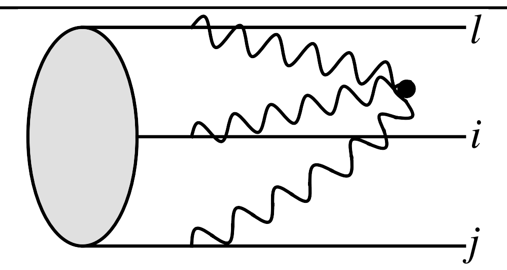
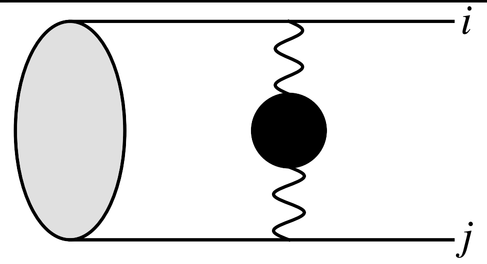
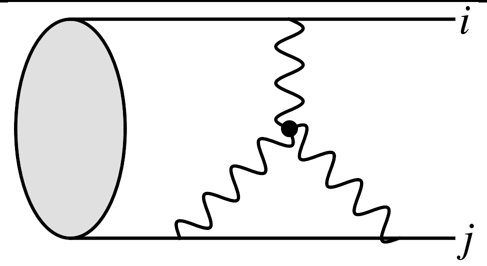
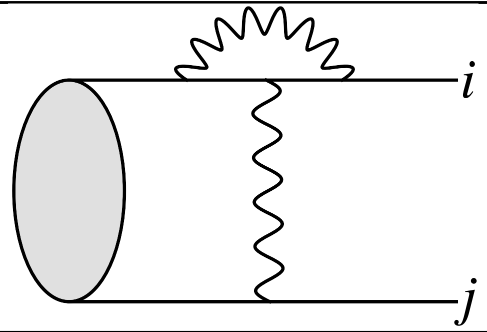
Express kinematic dependence as phase space type integrals — e.g. one-loop case:

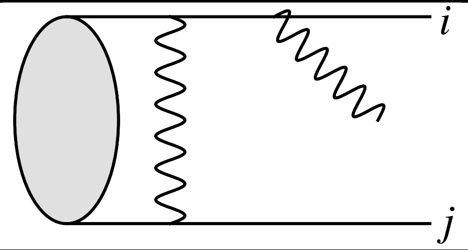
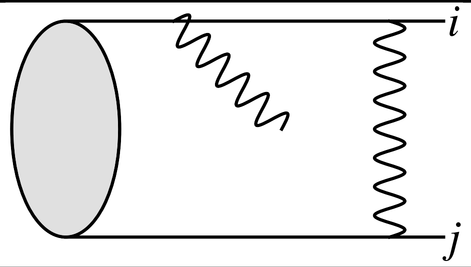
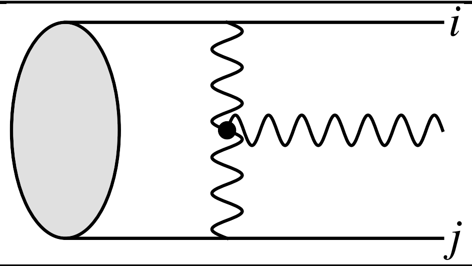
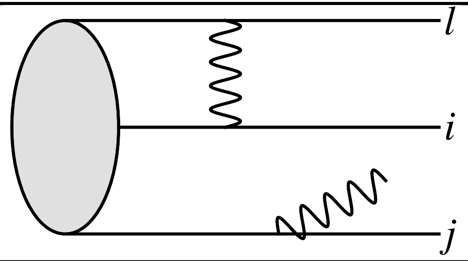
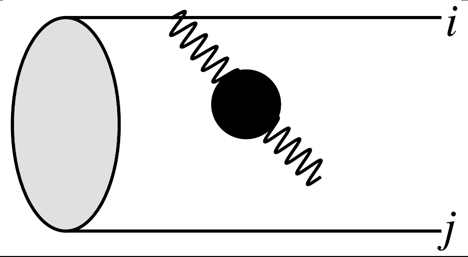
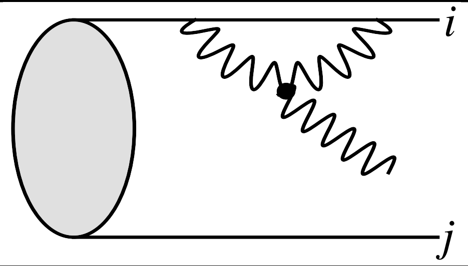
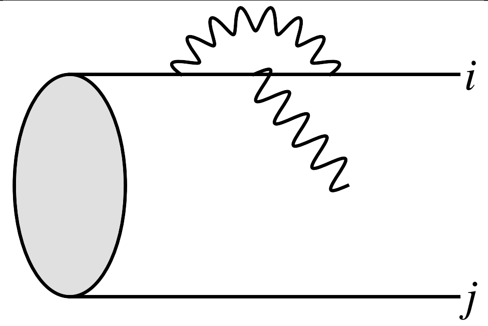
$$\mathbf{\Gamma}^{(1)} = \frac{1}{2}\sum_{i,j}\Omega_{ij}^{(1)}\ \frac{1}{N}\mathbf{T}_i\cdot\mathbf{T}_j \qquad \Omega_{ij}^{(1)} = i\mu^{2\epsilon}\int\frac{\mathrm{d}\!\!{}^-{}^d k}{i\pi^{d/2}}\frac{p_i\cdot p_j}{(k^2+i0)(p_i\cdot k+i0)(p_j\cdot k-i0)} = \int_0^\infty\frac{\mathrm{d}E}{E}\left(\frac{\mu^2}{E^2}\right)^\epsilon\omega^{(ij)}$$

Contour integration reveals well-known formula

$$\omega^{(ij)} = \frac{(2\pi)^{2\epsilon}}{\pi}\left[\int\frac{\mathrm{d}\Omega^{(d-2)}}{4\pi}\frac{n_i\cdot n_j}{n_i\cdot n\ n\cdot n_j} - i\pi\int\frac{\mathrm{d}\Omega^{(d-3)}}{2\pi}\right]$$

Two Loops & One loop one emission

Coefficient	Diagram	Colour-factor
$\Omega_{ij}^{(2)}$		$(\mathbf{T}_i \cdot \mathbf{T}_j)(\mathbf{T}_i \cdot \mathbf{T}_j)$
$\tilde{\Omega}_{ij}^{(2)}$		$\mathbf{T}_i^a \mathbf{T}_i^b \mathbf{T}_j^b \mathbf{T}_j^a$
$\Omega_{ijl}^{(2)}$		$(\mathbf{T}_i \cdot \mathbf{T}_l)(\mathbf{T}_i \cdot \mathbf{T}_j)$
$\hat{\Omega}_{ijl}^{(2)}$		$i f^{abc} \mathbf{T}_i^a \mathbf{T}_j^b \mathbf{T}_l^c$
$\Omega_{ij,\mathrm{self-en.}}^{(2)}$		$T_R (\mathbf{T}_i \cdot \mathbf{T}_j)$
$\Omega_{ij,\mathrm{vertex-corr.}}^{(2)}$		$T_R (\mathbf{T}_i \cdot \mathbf{T}_j)$
$\hat{\Omega}_{ij}^{(2)}$		$\mathbf{T}_i^b \mathbf{T}_i^a \mathbf{T}_i^b \mathbf{T}_j^a$

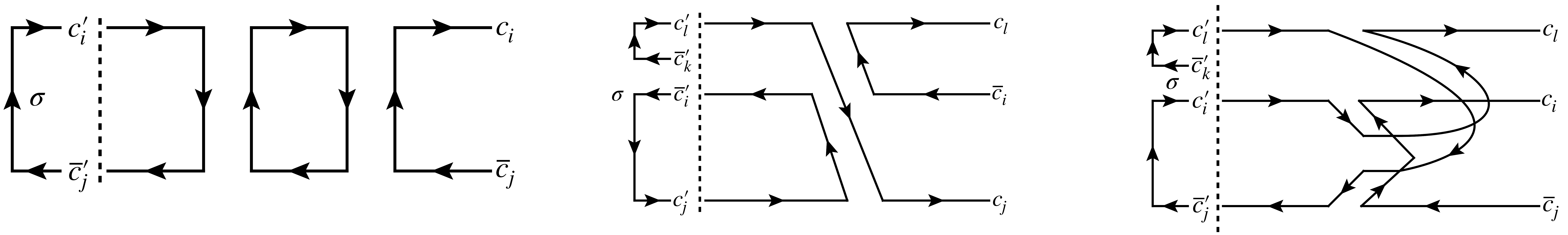
Coefficient	Diagram	Colour-factor
$\Omega_{ij}^{(1,1)}$		$\mathbf{T}_i^a (\mathbf{T}_i \cdot \mathbf{T}_j)$
$\tilde{\Omega}_{ij}^{(1,1)}$		$(\mathbf{T}_i \cdot \mathbf{T}_j) \mathbf{T}_i^a$
$\overline{\Omega}_{ij}^{(1,1)}$		$i f^{abc} \mathbf{T}_i^b \mathbf{T}_j^c$
$\Omega_{ijl}^{(1,1)}$		$\mathbf{T}_l^a (\mathbf{T}_i \cdot \mathbf{T}_j)$
$\Omega_{i,\mathrm{self-en.}}^{(1,1)}$		$T_R \mathbf{T}_i^a$
$\Omega_{i,\mathrm{vertex-corr.}}^{(1,1)}$		$T_R \mathbf{T}_i^a$
$\hat{\Omega}_{ij}^{(1,1)}$		$\mathbf{T}_i^b \mathbf{T}_i^a \mathbf{T}_i^b$

Two Loops & One loop one emission

[Plätzer, Ruffa — arXiv:2012.15215]

Anomalous dimension at two loops:

$$[\tau|\mathbf{\Gamma}|\sigma\rangle = (\alpha_s N)[\tau|\mathbf{\Gamma}^{(1)}|\sigma\rangle + (\alpha_s N)^2[\tau|\mathbf{\Gamma}^{(2)}|\sigma\rangle + \ldots$$



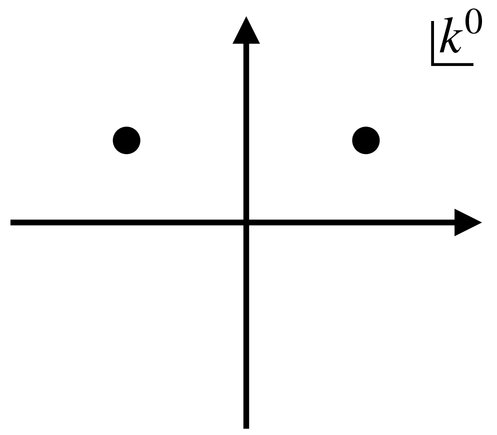
Colour structures imply colour-diagonal three parton correlations: Dipoles are not enough!

$$[\tau|\mathbf{\Gamma}^{(2)}|\sigma\rangle = \left(\Gamma^{(2)}_\sigma + \frac{1}{N^2}\left(\rho_\sigma + \tilde{\rho}\right) + \frac{1}{N^4}\rho^{(2)}\right)\delta_{\sigma\tau} + \frac{1}{N}\left(\Sigma^{(2)}_{\sigma\tau} + \hat{\Sigma}^{(2)}_{\sigma\tau}\right) + \frac{1}{N^3}\tilde{\Sigma}^{(2)}_{\sigma\tau} + \frac{1}{N^2}\left(\Sigma'^{(2)}_{\sigma\tau} + \Sigma''^{(2)}_{\sigma\tau}\right)$$

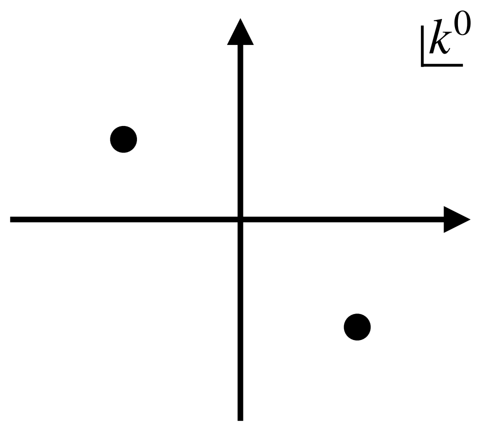
Cutting rules

[Plätzer, Ruffa — arXiv:2012.15215]

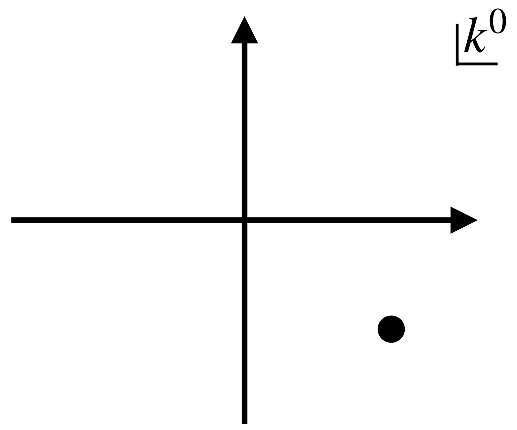
Algorithmic treatment of virtual corrections needed



Advanced



Feynman



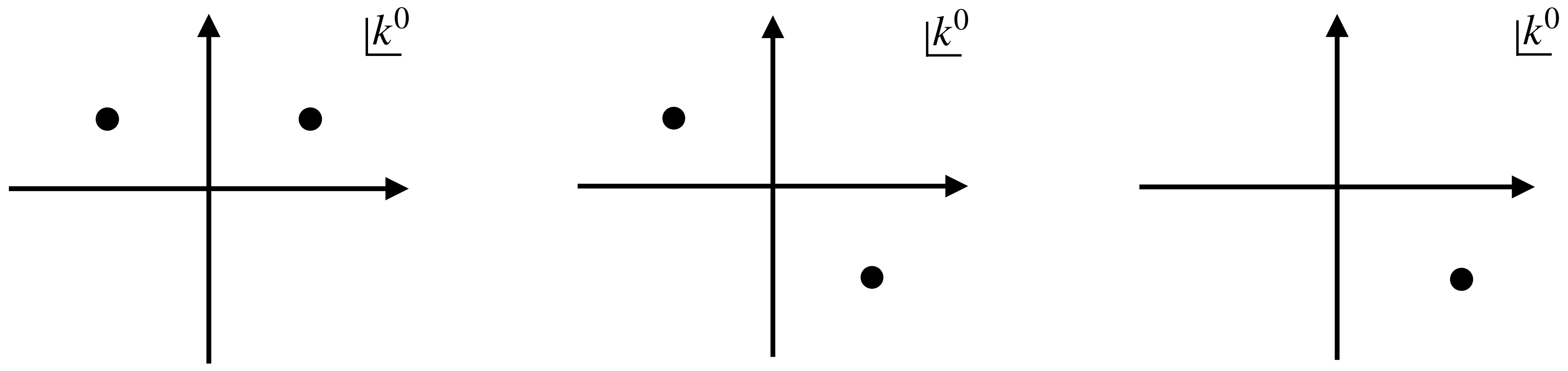
Feynman tree theorem:

$$\frac{1}{[q^2 - i0(T\cdot q)|T\cdot q|]} = \frac{1}{[q^2 + i0(T\cdot q)^2]} + 2\pi i\delta(q^2)\theta(T\cdot q)$$

Cutting rules

[Plätzer, Ruffa — arXiv:2012.15215]

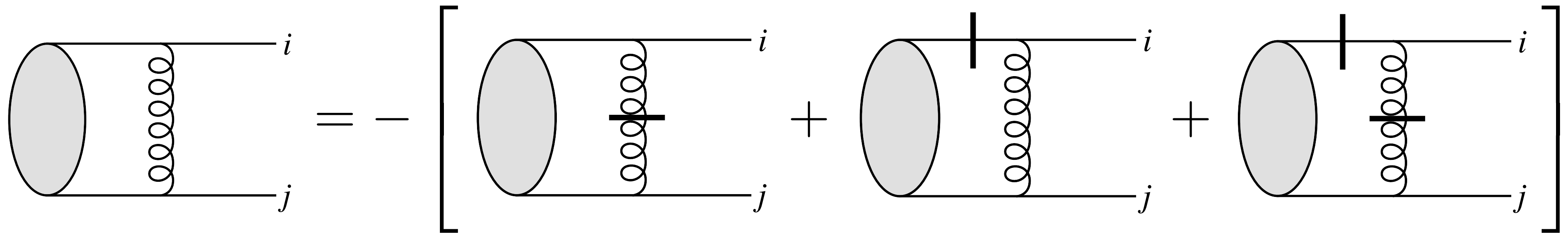
Algorithmic treatment of virtual corrections needed



Advanced Feynman Eikonal

Feynman tree theorem:

$$\frac{1}{[q^2 - i0(T\cdot q)|T\cdot q|]} = \frac{1}{[q^2 + i0(T\cdot q)^2]} + 2\pi i \delta(q^2)\theta(T\cdot q)$$



$$\omega^{(ij)} = \frac{(2\pi)^{2\epsilon}}{\pi}\left[\int \frac{\mathrm{d}\Omega^{(d-2)}}{4\pi}\frac{n_i\cdot n_j}{n_i\cdot n\ n\cdot n_j} - i\pi\int\frac{\mathrm{d}\Omega^{(d-3)}}{2\pi}\right]$$

Extend to Eikonal and higher-power propagators:

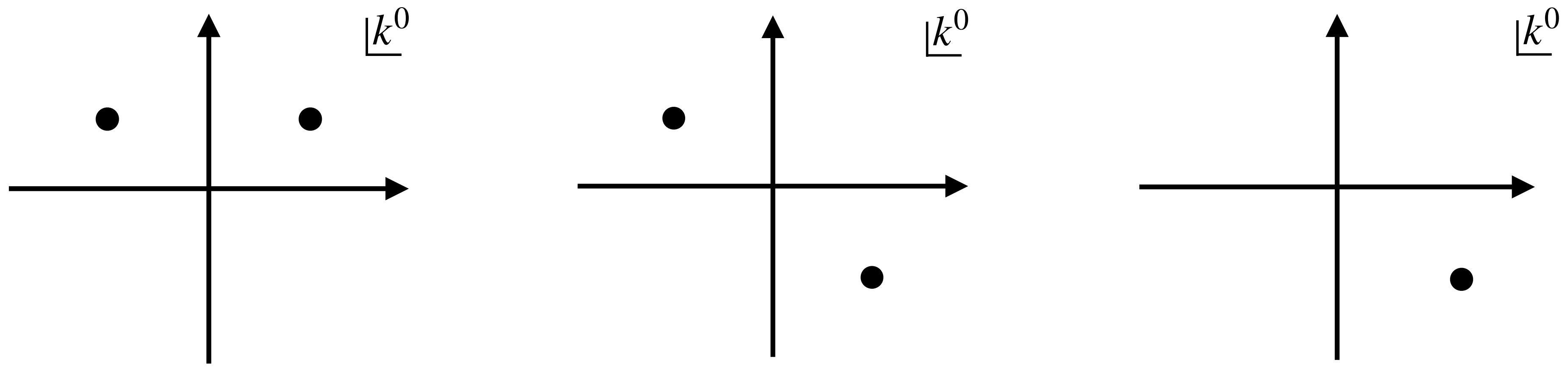
$$\frac{1}{2p_i\cdot k - i0(T\cdot p_i)^2} = \frac{1}{2p_i\cdot k + i0(T\cdot p_i)^2} + 2\pi i\ \delta(2p_i\cdot k)$$

$$\frac{1}{[q^2 - i0(T\cdot q)|T\cdot q|]^2} - \frac{1}{[q^2 + i0(T\cdot q)^2]^2} = -2i\pi\theta(T\cdot q)\delta'(q^2)$$

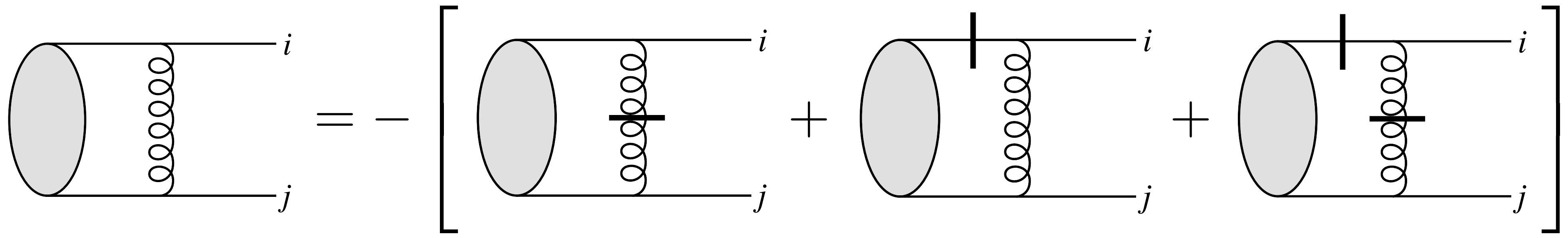
Cutting rules

[Plätzer, Ruffa — arXiv:2012.15215]

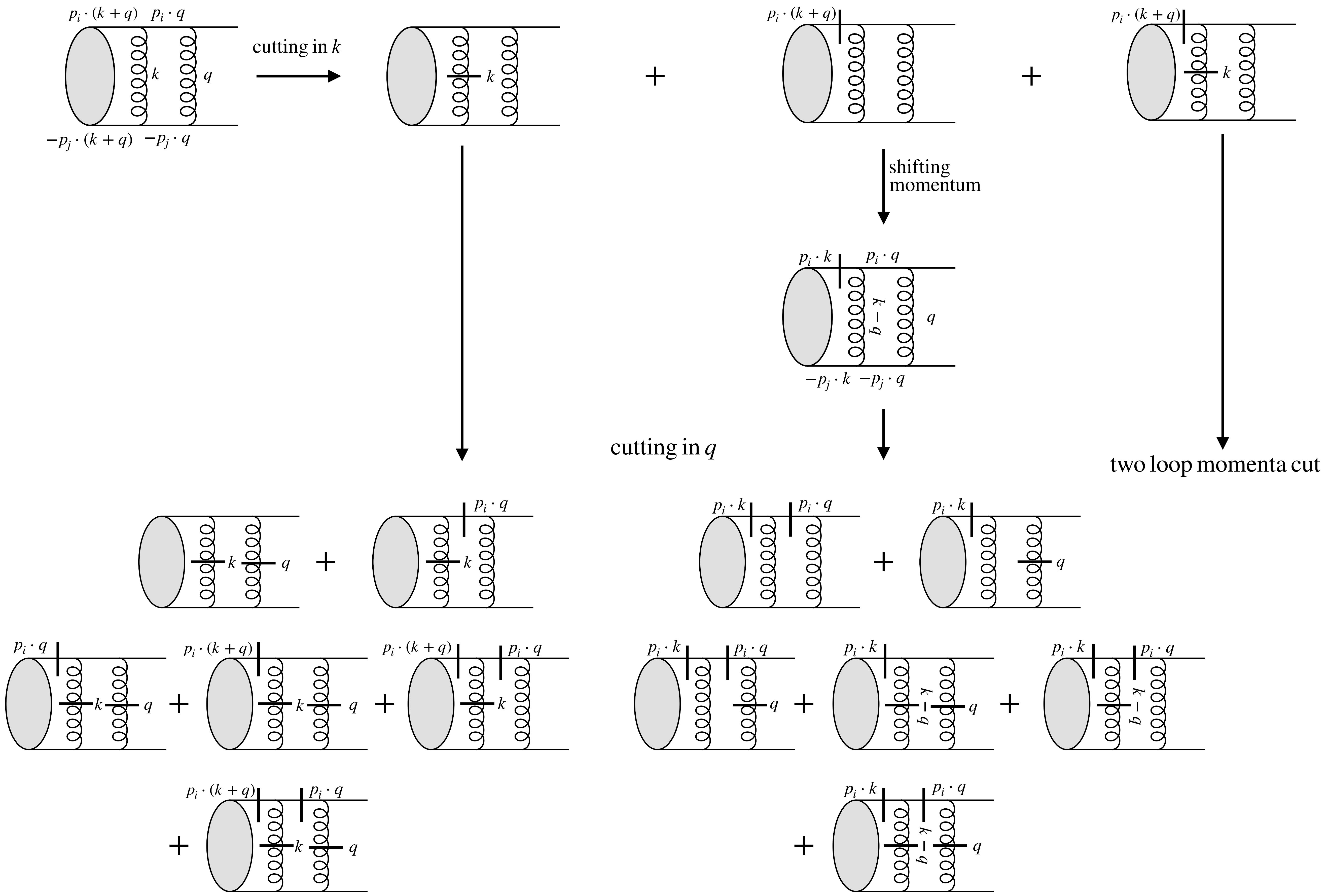
Algorithmic treatment of virtual corrections needed



Advanced Feynman Eikonal

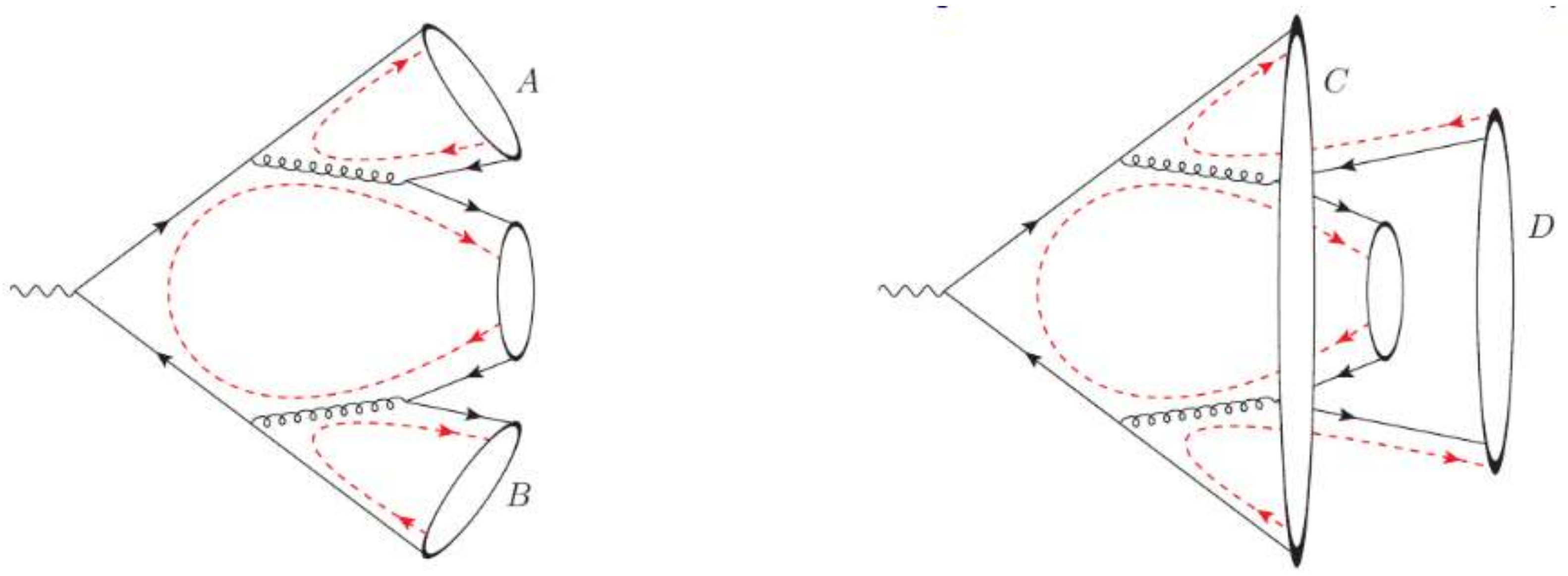


$$\omega^{(ij)} = \frac{(2\pi)^{2\epsilon}}{\pi}\left[\int \frac{\mathrm{d}\Omega^{(d-2)}}{4\pi}\,\frac{n_i\cdot n_j}{n_i\cdot n\; n\cdot n_j} - i\pi\int\frac{\mathrm{d}\Omega^{(d-3)}}{2\pi}\right]$$

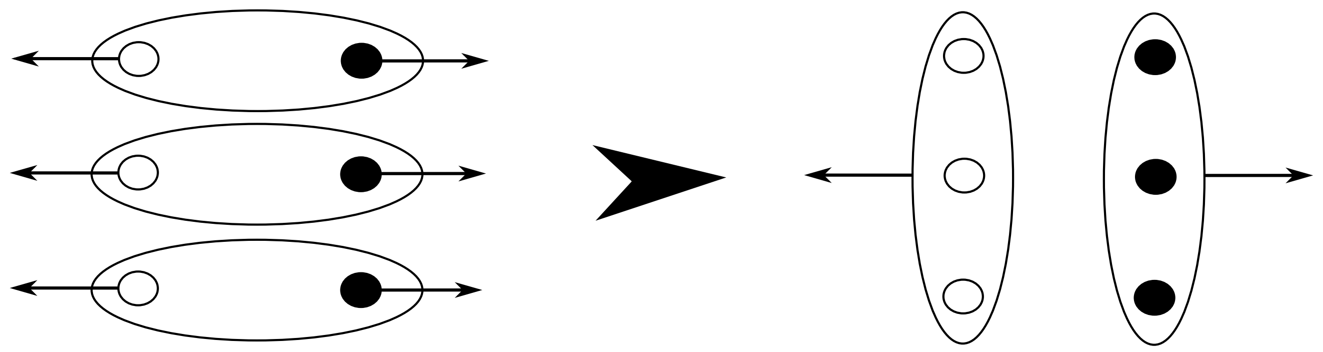


Colour Reconnection & Hadronization

Cluster re-wiring based on geometric criterion including Baryonic reconnection.



$$R_{q,qq} + R_{\bar{q},\bar{q}\bar{q}} < R_{q,\bar{q}} + R_{qq,\bar{q}\bar{q}}$$



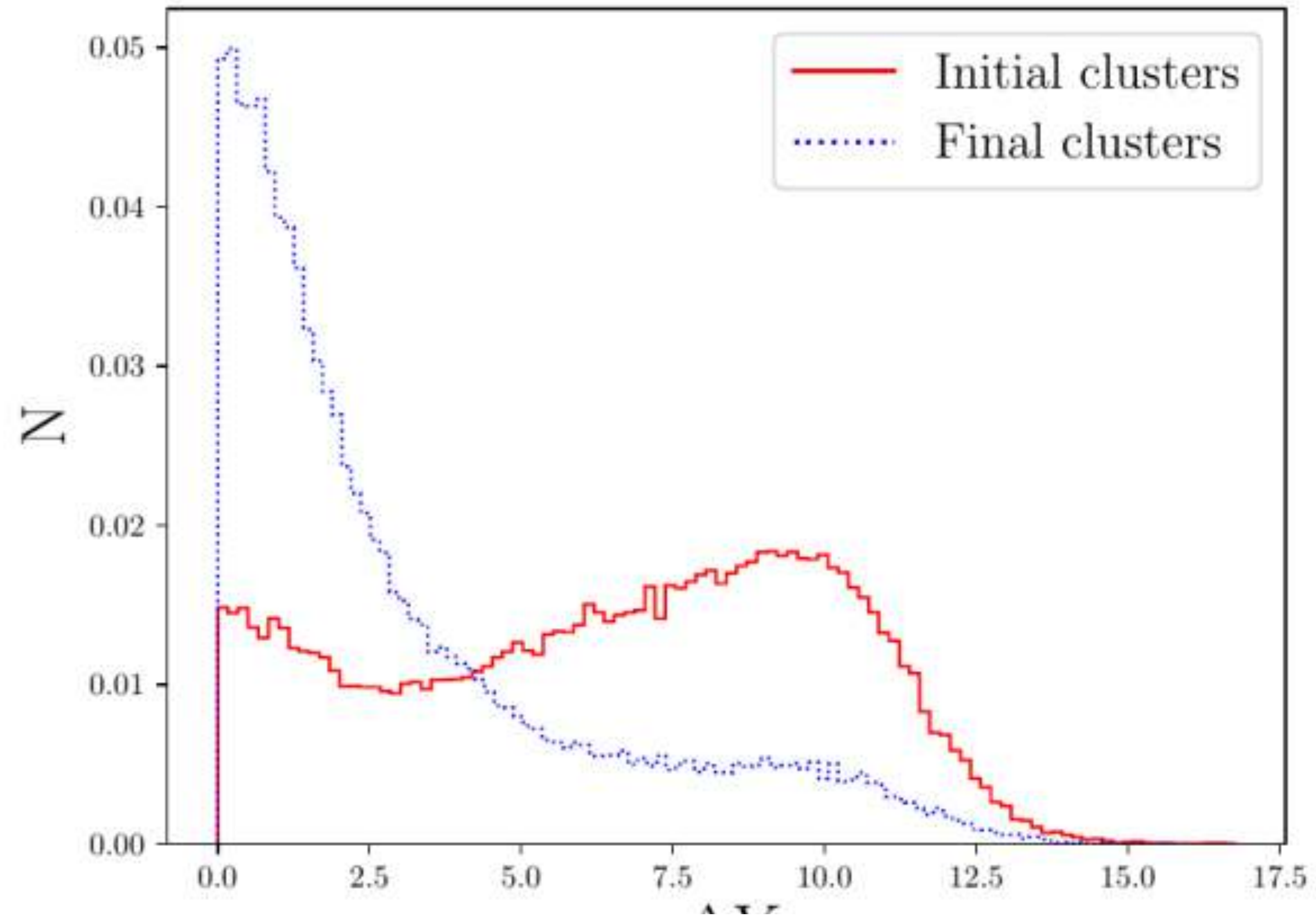
[Gieseke, Kirchgaesser, Plätzer – EPJ C 78 (2018) 99]

Approach colour reconnection from colour evolution: perturbative component?

Reconnection amplitude

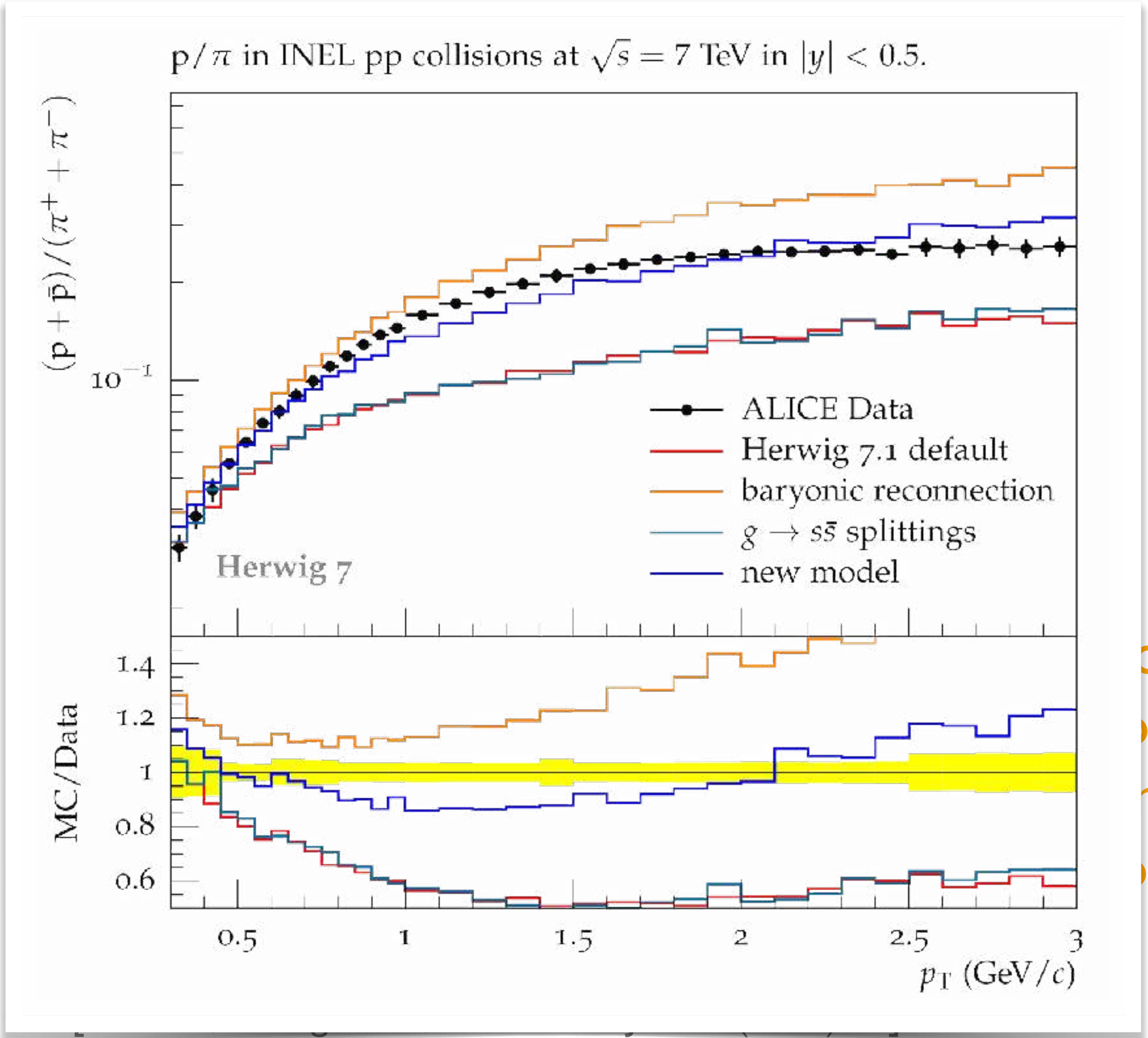
$$\mathcal{A}_{\tau\to\sigma} = \langle\sigma|\mathbf{U}\left(\{p\},\mu^2,\{M_{ij}^2\}\right)|\tau\rangle$$

Strong support for geometric models from perturbative evolution.



[Gieseke, Kirchgaesser, Plätzer, Siodmok – JHEP 11 (2018) 149]

Colour Reconnection & Hadronization



Approach colour reconnection from colour evolution: perturbative component?

Reconnection amplitude

$$\mathcal{A}_{\tau \to \sigma} = \langle \sigma | \mathbf{U} \left(\{p\}, \mu^2, \{M_{ij}^2\} \right) | \tau \rangle$$

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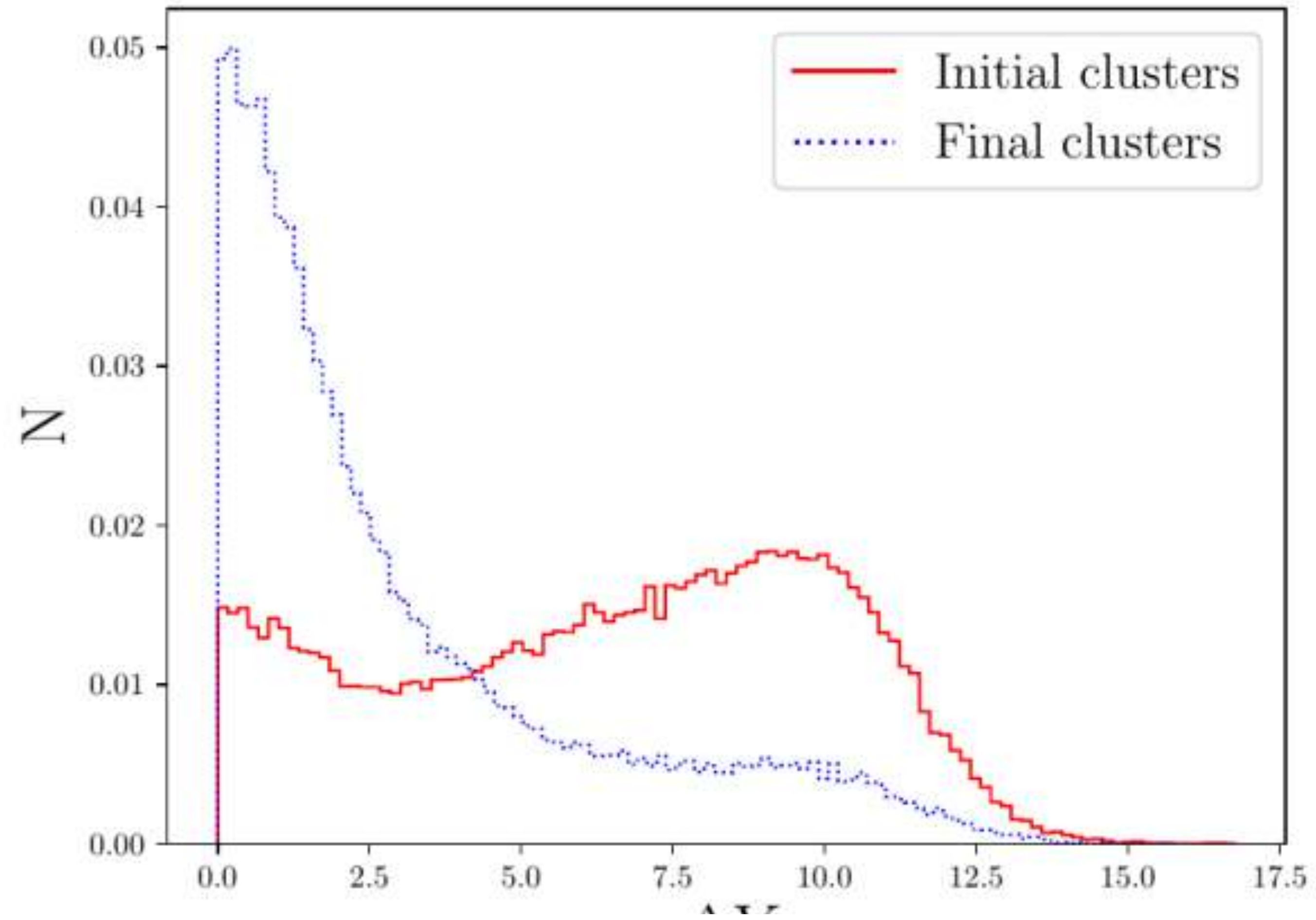
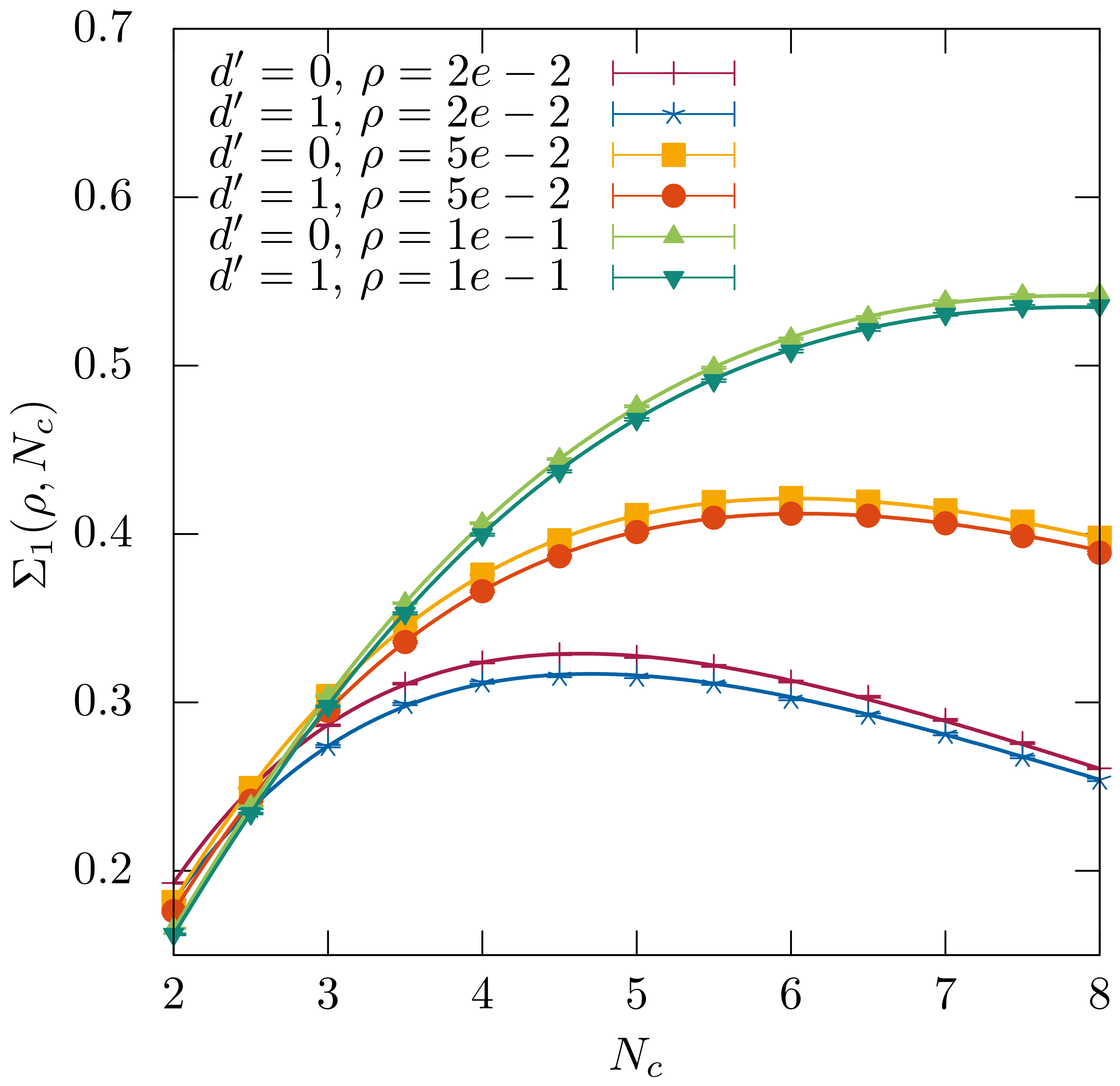


[Gieseke, Kirchgaesser, Plätzer, Siodmok – JHEP 11 (2018) 149]

Thank you!



0.7
0.6
0.5
0.4
0.3
0.2
2
3
4
5
6
7
8
N_c
$\Sigma_1(\rho, N_c)$
$d' = 0,\ \rho = 2e-2$
$d' = 1,\ \rho = 2e-2$
$d' = 0,\ \rho = 5e-2$
$d' = 1,\ \rho = 5e-2$
$d' = 0,\ \rho = 1e-1$
$d' = 1,\ \rho = 1e-1$

Colour Matrix Element Corrections vs Full Colour

Colour matrix element corrections reconsidered.

[Holguin, Forshaw, Plätzer – arXiv:2003:06399]

[Hoeche, Reichelt – arXiv:2001.11492v1]

$$\frac{\mathrm{d}\sigma_{n+k+1}}{\sigma_{n+k}} = \mathrm{d}\Phi_{+1} 8\pi\alpha_\mathrm{s} \frac{\langle m_{n+k} | \, \mathbf{\Gamma}_{n+k}(\mathbf{1}) \, | m_{n+k} \rangle}{\langle m_{n+k} | m_{n+k} \rangle}$$

$$\mathbf{\Gamma}_n(\mathbf{\Gamma}) = -\sum_{\substack{i,j=1 \\ i \neq j}}^{n} \mathbf{T}_i \, \mathbf{\Gamma} \, \mathbf{T}_j \, \omega_{ij}$$

Cross-section unitarity is not sufficient to produce correct subleading-N virtual evolution.

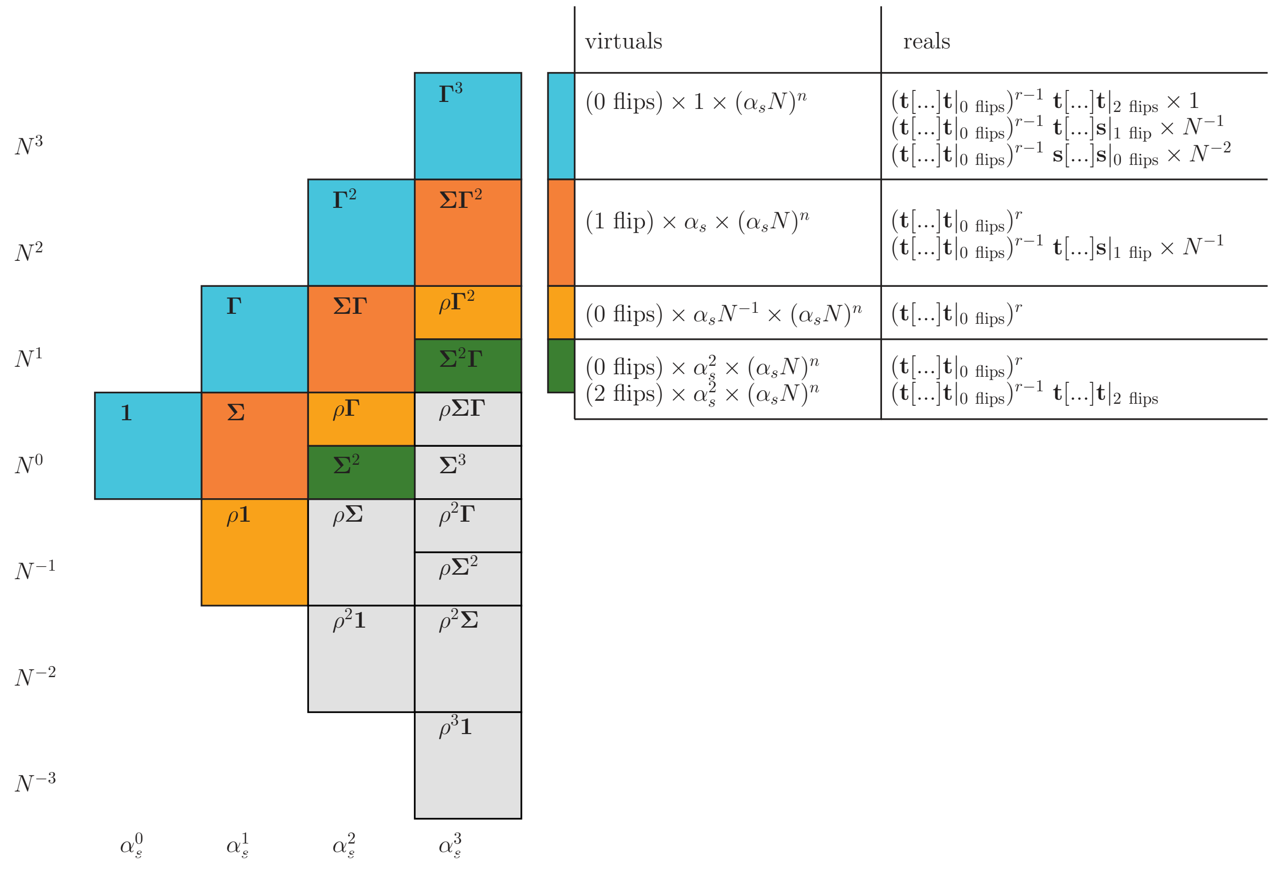
$$\mathrm{Tr}_\mathrm{norm}\left(e^{\mathbf{V}}\right) = e^{\mathrm{Tr}_\mathrm{norm}(\mathbf{V})} + \sum_{n \geq 2} \mathcal{O}\left(\alpha_\mathrm{s}^n N_\mathrm{c}^{n-2} (\mathrm{Tr}_\mathrm{norm} \delta \mathbf{V}^2 - (\mathrm{Tr}_\mathrm{norm} \delta \mathbf{V})^2)\right)$$

Compare to "exponent counting"

$$\Sigma(L) = \sum_{n=0}^{\infty} (N_\mathrm{c}\alpha_\mathrm{s})^n \sum_{m=0}^{n+1} C_{n,m}(L)$$

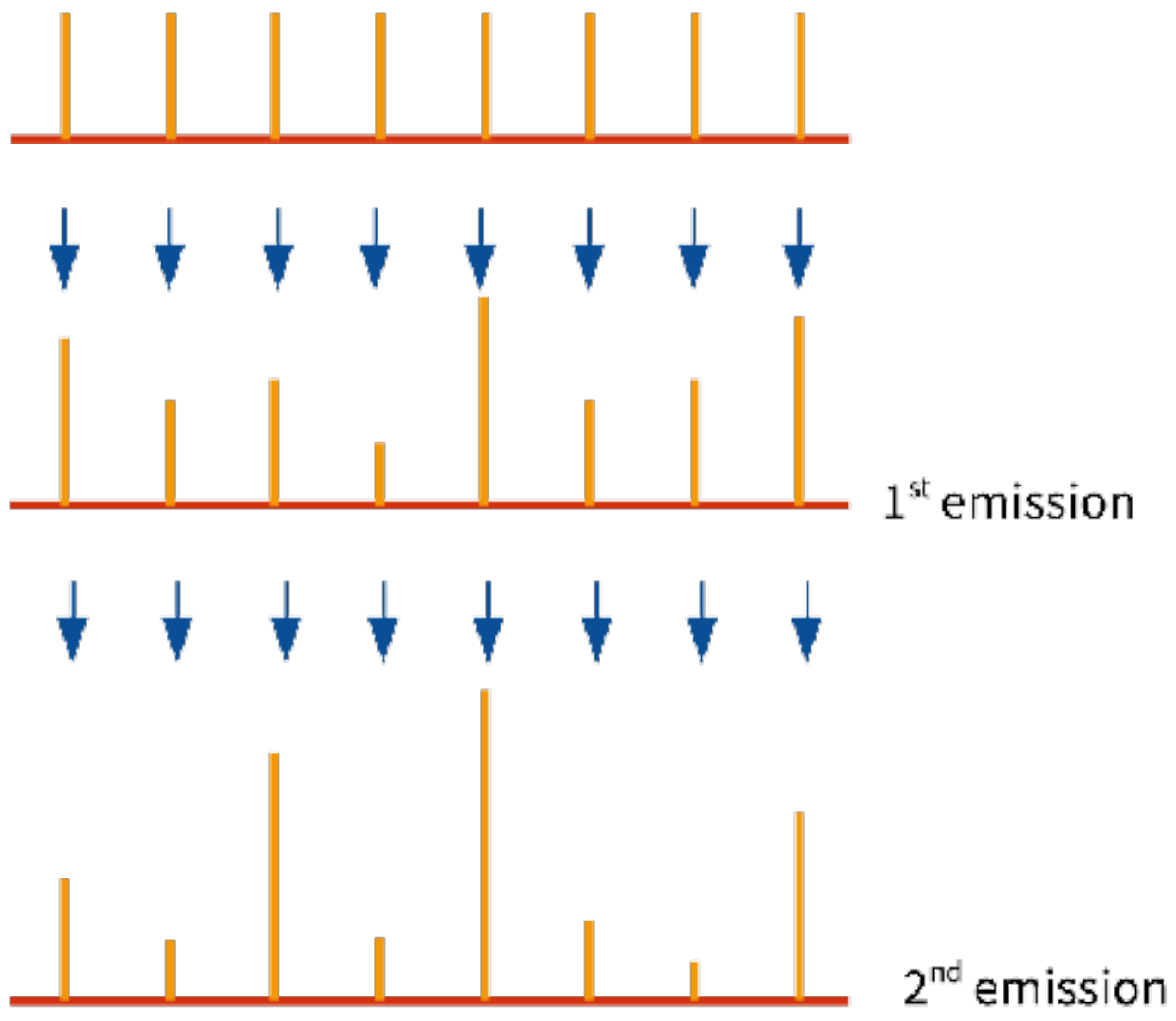
breaks for weak-counting NNLC effects

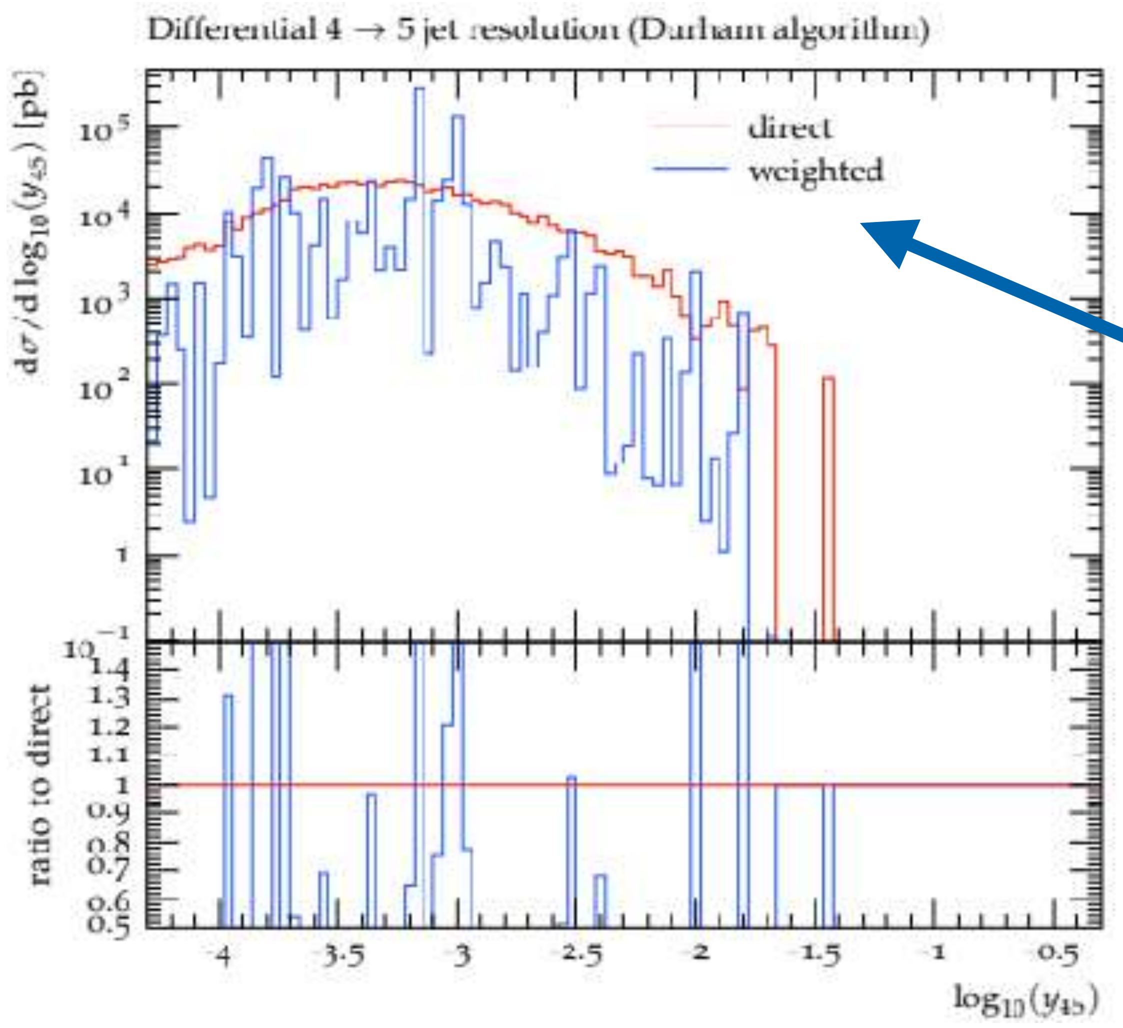
$$C_{n,m} = \underbrace{C_{n,m}^{(0)}}_{\mathrm{LC}_\Sigma} + \underbrace{\frac{1}{N_\mathrm{c}} C_{n,m}^{(1)}}_{\mathrm{NLC}_\Sigma} + \underbrace{\frac{1}{N_\mathrm{c}^2} C_{n,m}^{(2)}}_{\mathrm{NNLC}_\Sigma} + \ldots$$



Novel Algorithmic Frameworks

[Olsson, Plätzer, Sjödahl — EPJ C80 (2020) 10, 934]





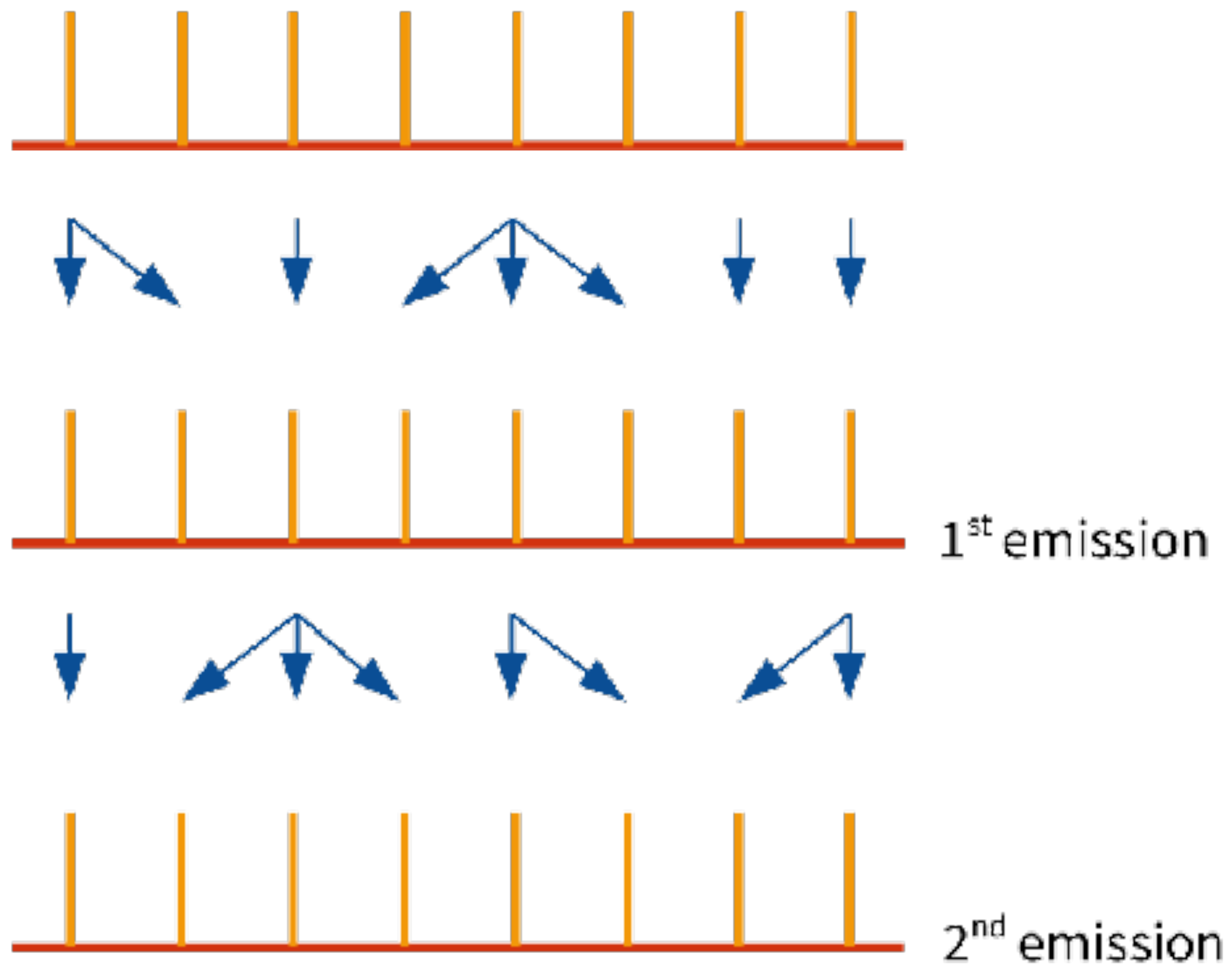
Weighted branching algorithms exhibit prohibitive weight distributions & convergence issues.

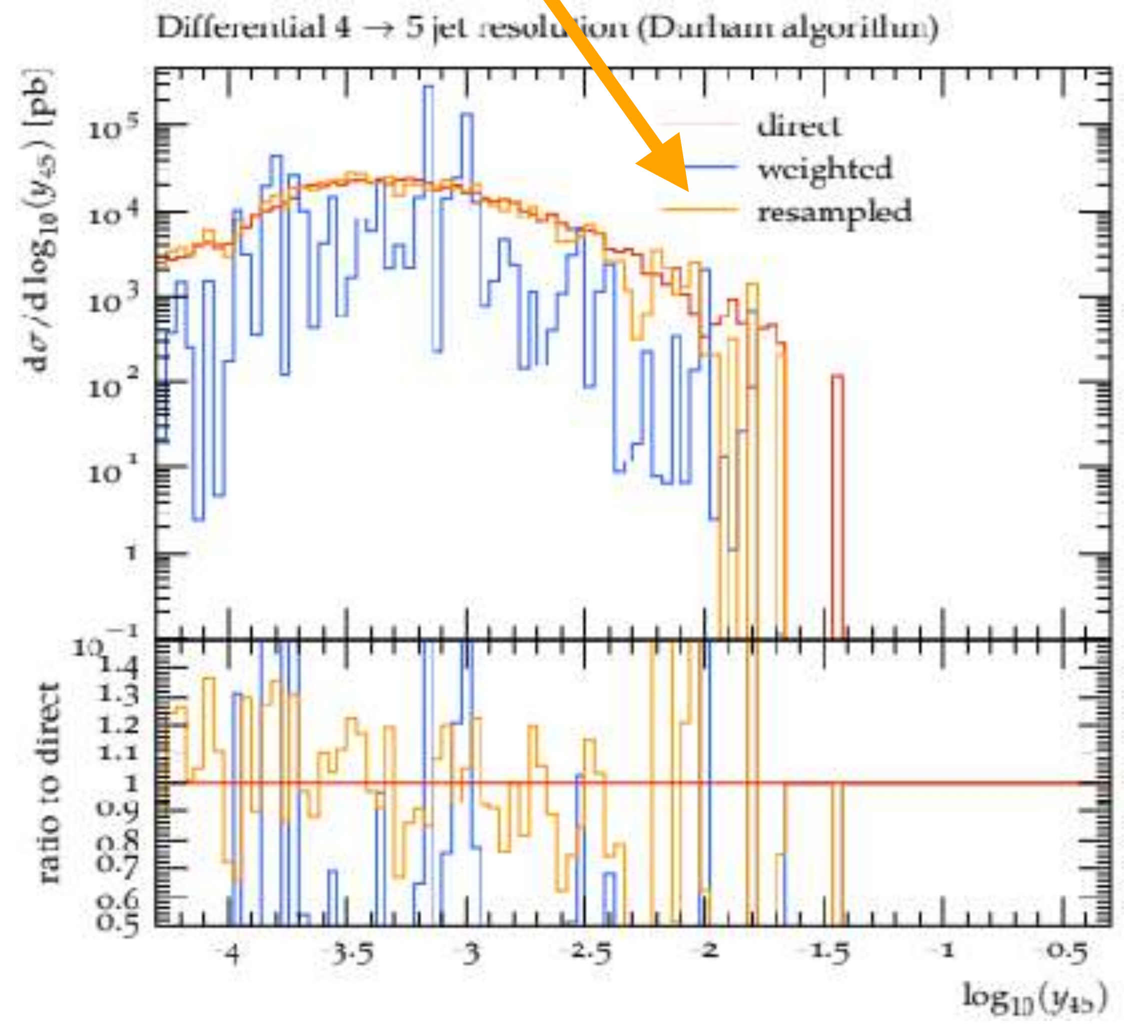
Result without resampling

Result with resampling

Resampling algorithms can compress weight distributions at intermediate steps.

Interdisciplinary exchange!





Parton Showers from Amplitude Evolution

[Forshaw, Holguin, Plätzer – arXiv:2003:06400]

Start from amplitude evolution equations

$$q_\perp \frac{\partial \mathbf{A}_n(q_\perp;\{p\}_n)}{\partial q_\perp} = -\,\mathbf{\Gamma}_n(q_\perp)\,\mathbf{A}_n(q_\perp;\{p\}_n) - \mathbf{A}_n(q_\perp;\{p\}_n)\,\mathbf{\Gamma}_n^\dagger(q_\perp) + \int \mathrm{d}R_n\,\mathbf{D}_n(q_{n\,\perp})\,\mathbf{A}_{n-1}(q_{n\,\perp};\{p\}_{n-1})\,\mathbf{D}_n^\dagger(q_{n\,\perp})\,q_\perp\,\delta(q_\perp - q_{n\,\perp})$$

$$\mathrm{d}\sigma_n(\mu) = \left(\prod_{i=1}^{n} \mathrm{d}\Pi_i\right) \mathrm{Tr}\,\mathbf{A}_n(\mu) \qquad \Sigma(\mu;\{p\}_0,\{v\}) = \int \sum_n \mathrm{d}\sigma_n(\mu)\,u(\{p\}_n,\{v\})$$

Combine insight from soft evolution, large-N expansions and collinear subtractions:

- Can we reproduce existing algorithms as well-defined limits of amplitude evolution?
- Can we use this to obtain an ideal combination of coherent and dipole branching?

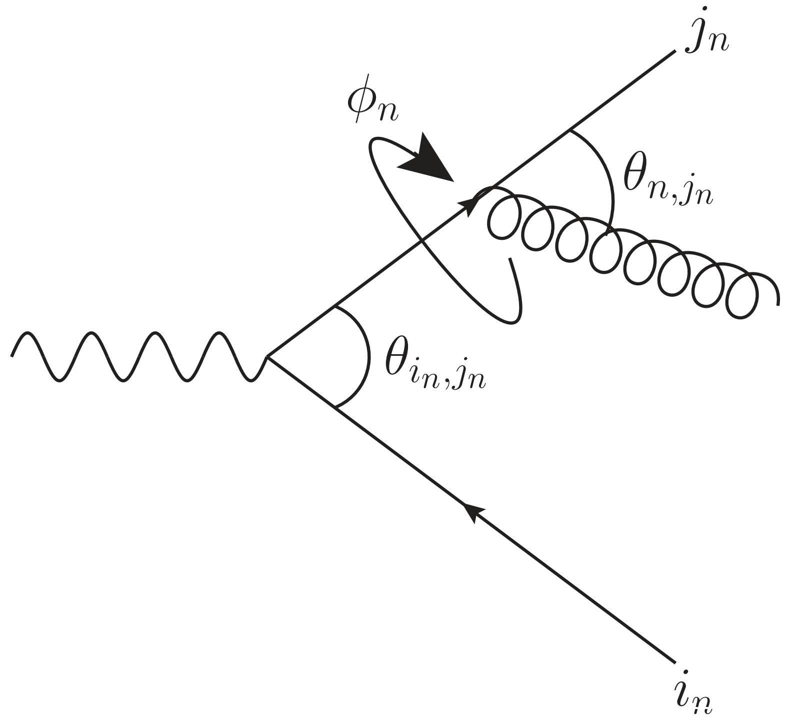
Collinear Subtractions & Angular Ordering

[Forshaw, Holguin, Plätzer – arXiv:2003:06400]

Collinear subtractions within a dipole?
Recall angular ordering and coherent branching:

$$\frac{n_{i_n} \cdot n_{j_n}}{n_{i_n} \cdot n \; n_{j_n} \cdot n} = P_{i_n j_n} + P_{j_n i_n}, \quad \text{where} \quad 2P_{i_n j_n} = \frac{n_{i_n} \cdot n_{j_n} - n_{i_n} \cdot n}{n_{i_n} \cdot n \; n_{j_n} \cdot n} + \frac{1}{n_{i_n} \cdot n}$$

Azimuthal average will result in angular ordering and simplify colour structures.



$$\begin{aligned}\left\langle |\mathcal{M}_n|^2 \, u(\{p\}_n) \right\rangle_{1,\ldots,n} &= \left\langle |\mathcal{M}_n|^2 \right\rangle_{1,\ldots,n} \left\langle u(\{p\}_n) \right\rangle_{1,\ldots,n} \\ &+ \sum_{m=1}^{n} \sigma_m(\langle |\mathcal{M}_n|^2 \rangle_{1,\ldots,n}) \, \sigma_m(\langle u(\{p\}_n) \rangle_{1,\ldots,n}) \, \mathrm{Cor}_m(\langle |\mathcal{M}_n|^2 \rangle_{1,\ldots,n}, \langle u(\{p\}_n) \rangle_{1,\ldots,n}) \\ &+ \text{higher order correlations}\end{aligned}$$

irrelevant for global observables at NLL

Collinear Subtractions & Angular Ordering

[Forshaw, Holguin, Plätzer – arXiv:2003:06400]

Collinear subtractions within a dipole?
Recall angular ordering and coherent branching:

$$\frac{n_{i_n}\cdot n_{j_n}}{n_{i_n}\cdot n\; n_{j_n}\cdot n} = P_{i_nj_n} + P_{j_ni_n}, \quad \text{where} \quad 2P_{i_nj_n} = \frac{n_{i_n}\cdot n_{j_n} - n_{i_n}\cdot n}{n_{i_n}\cdot n\; n_{j_n}\cdot n} + \frac{1}{n_{i_n}\cdot n}$$

Azimuthal average will result in angular ordering and simplify colour structures.

$$\zeta\frac{\partial \left\langle|\mathcal{M}_n(\zeta)|^2\right\rangle_{1,\dots,n}}{\partial\zeta} \approx$$

$$-\sum_{j_{n+1}}\sum_{\upsilon}\frac{\alpha_s}{\pi}\int \mathrm{d}z\,\mathcal{P}_{\upsilon\upsilon_{j_{n+1}}}(z)\,\langle\Theta_{\text{on shell}}\rangle_{n+1}\,\left\langle|\mathcal{M}_n(\zeta)|^2\right\rangle_{1,\dots,n} + \sum_{\upsilon}\frac{\alpha_s}{\pi}\mathcal{P}_{\upsilon\upsilon_{j_n}}(z_n)$$

$$\times\,\langle\Theta_{\text{on shell}}\rangle_n\int \mathrm{d}^4p_{j_n}\;\delta^4(p_{j_n} - z_n^{-1}\tilde{p}_{j_n})\;\left\langle|\mathcal{M}_{n-1}(\zeta_{n,j_n})|^2\right\rangle_{1,\dots,n-1}\;\zeta_{n,j_n}\,\delta(\zeta-\zeta_{n,j_n})$$

includes momentum mapping and physical phase space boundaries for on-shell partons

Dipoles, Recoil & Partitioning

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[Forshaw, Holguin, Plätzer – arXiv:2003:06400]

Colour in dipole shower evolution follows the BMS derivation.

$$q_\perp \text{Leading}^{(0)}_{\tau\sigma}\left[\frac{\partial \hat{\mathbf{A}}_n(q_\perp)}{\partial q_\perp}\right] \approx -\frac{\alpha_s}{\pi}\int \frac{\mathrm{d}S_2^{(q_{n+1})}}{4\pi} \sum_{i_{n+1},j_{n+1}\ \text{c.c. in}\ \sigma}$$

$$\times\, 4\lambda_{i_{n+1}}\bar{\lambda}_{j_{n+1}} N_c \int \delta q^{(i_{n+1},j_{n+1})}_{n+1\,\perp}(q_\perp)\,\Theta_{\text{on shell}}\,\delta_{\tau\sigma}\,\text{Leading}^{(0)}_{\tau\sigma}\left[\hat{\mathbf{A}}_n(q_\perp)\right]$$

$$+\int\left(\prod_{i_n}\mathrm{d}^4 p_{i_n}\right)\sum_{i_n,j_n\ \text{c.c. in}\ \sigma}\lambda_i\bar{\lambda}_j N_c\int \delta q^{(i_n,j_n)}_{n\,\perp}(q_{n\,\perp})\,\mathfrak{R}^{\text{soft}}_{i_n j_n}$$

$$\times\,\delta_{\tau\sigma}\,\text{Leading}^{(0)}_{\tau\backslash n\,\sigma\backslash n}\left[\hat{\mathbf{A}}_{n-1}(q_{n\,\perp})\right]\; q_\perp\,\delta(q_\perp - q_{n\,\perp}).$$

Combination with collinear contributions: partition using coherent branching logic

$$\frac{p_{i_n}\cdot p_{j_n}}{p_{i_n}\cdot q_n\; p_{j_n}\cdot q_n} \longrightarrow \frac{p_{i_n}\cdot p_{j_n}}{p_{i_n}\cdot q_n\; p_{j_n}\cdot q_n} - \frac{T\cdot p_{j_n}}{T\cdot q_n}\frac{1}{p_{j_n}\cdot q_n} + \frac{T\cdot p_{i_n}}{T\cdot q_n}\frac{1}{p_{i_n}\cdot q_n}$$

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Evolution now per colour flow matrix element:

$$q_\perp \frac{\partial |\mathcal{M}_n^{(\sigma)}(q_\perp)|^2}{\partial q_\perp}$$

$$\approx -\frac{\alpha_s}{\pi} \sum_{i^c_{n+1}} \int \mathrm{d}q_\perp^{(i^c_{n+1},\overline{i^c}_{n+1})} \delta(q_\perp^{(i^c_{n+1},\overline{i^c}_{n+1})} - q_\perp) \int \mathrm{d}z\, \Theta_{\text{on shell}}\, P_{v_{in} v_{in}}(z)\, |\mathcal{M}_n^{(\sigma)}(q_\perp)|^2$$

$$+\frac{\alpha_s}{\pi} \int \left(\prod_{j_n} \mathrm{d}^4 p_{j_n}\right) \mathfrak{R}^{\text{dipole}}_{i^c_n} P_{v_{in} v_{in}}(z_n)\, q_\perp \delta(q_{n\perp}^{(i^c_n,\overline{i^c}_n)} - q_\perp) |\mathcal{M}_{n-1}^{(\sigma/n)}(q_{n\perp}^{(i^c_n,\overline{i^c}_n)})|^2$$

New dipole shower evolution, reduces to coherent branching upon azimuthal average and BMS evolution for large-angle soft.

$$\mathfrak{R}^{\text{dipole}}_{i^c_n} = \left(\frac{1}{2} + \text{Asym}_{i^c_n \overline{i^c}_n}(q_n)\right) \mathfrak{R}_{i^c_n}$$

$$(q_{n\perp}^{(i^c_n,\overline{i^c}_n)})^2 = \frac{2(p_{i^c_n}\cdot q_n)(p_{\overline{i^c}_n}\cdot q_n)}{p_{i^c_n}\cdot p_{\overline{i^c}_n}}$$

$$\text{Asym}_{i^c_n \overline{i^c}_n}(q_n) = \left[\frac{T\cdot p_{i^c_n}}{4T\cdot q_n} \frac{(q_{n\perp}^{(i^c_n \overline{i^c}_n)})^2}{p_{i^c_n}\cdot q_n} - \frac{T\cdot p_{\overline{i^c}_n}}{4T\cdot q_n} \frac{(q_{n\perp}^{(i^c_n \overline{i^c}_n)})^2}{p_{\overline{i^c}_n}\cdot q_n}\right]$$

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[Forshaw, Holguin, Plätzer – arXiv:2003:06400]

Recoil needs to be addressed separately.

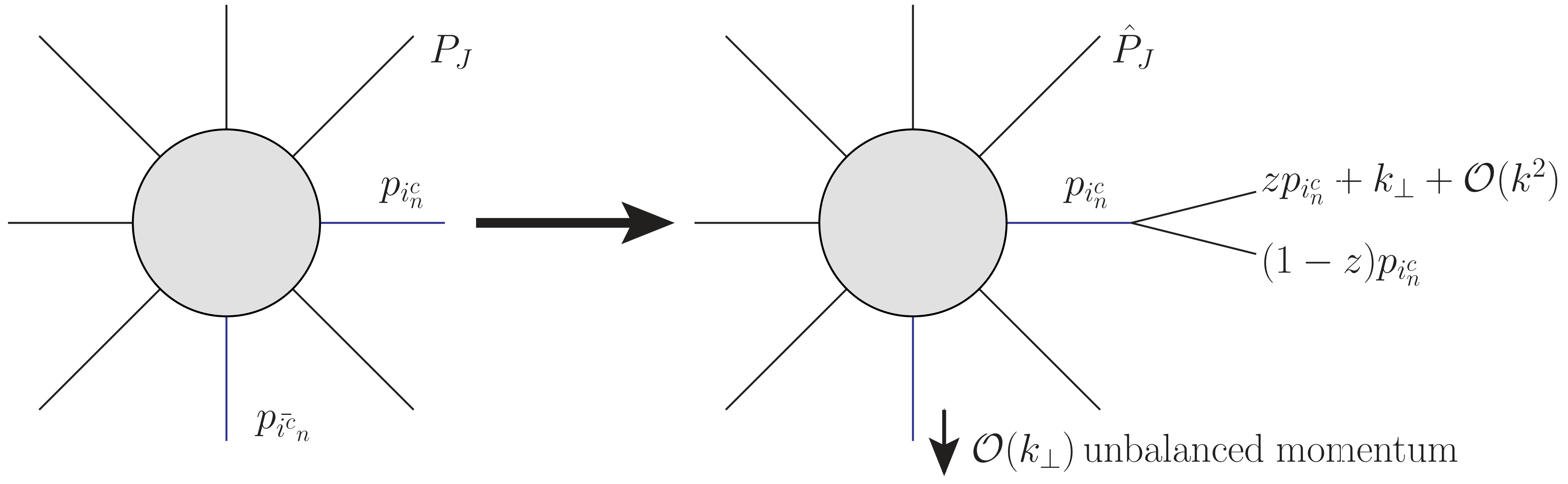
$$\hat{q}_n = (1-z_n)p_{i_n^c} + k_\perp + \frac{\left(q_{n\perp}^{(i_n^c\,\overline{i^c}_n)}\right)^2}{1-z_n}\,\frac{p_{\overline{i^c}_n}}{2p_{i_n^c}\cdot p_{\overline{i^c}_n}},$$

$$\hat{p}_{i_n^c} = z_n p_{i_n^c}, \qquad \left(q_{n\perp}^{(i_n^c\,\overline{i^c}_n)}\right)^2 = -k_\perp^2, \qquad k_\perp\cdot p_{i_n^c} = k_\perp\cdot p_{\overline{i^c}_n} = 0$$

balance only longitudinal fractions

Redistribute recoil globally, but per emission, inspired by Herwig's kinematic reconstruction.

recent discussion in:
[Bewick, Ferrario, Richardson, Seymour — arXiv:1904:11866]



This scheme is free of the issues encountered for local dipole recoils.

[Dasgupta, Dreyer, Hamilton, Monni, Salam — JHEP 09 (2018) 033]