Hawking-like radiation and “temperature” in gravitational scattering beyond the Planck scale

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In collaboration with M. Ciafaloni, F. Coradeschi and G. Veneziano

Cambridge, March 2nd 2018

Introduction

- Gen.Rel. $\leftrightarrow$ Qu.Mech.: One of the main unsolved problems in physics
- Our aim: to investigate processes lying at the interface between QM and GR with theories and tools at our disposal
- In practice: we study formation and evolution of Black Holes in particle collisions to shed light on the *information paradox*
  - GR: concentrating $E = mc^2 \Rightarrow$ BH $\Rightarrow$ information loss
  - QFT or ST: dynamics described by a unitary S-matrix $\Rightarrow$ conservation of information
  - Complication: Hawking radiation: BH emits incoherently

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  - QFT or ST: dynamics described by a unitary S-matrix $\Rightarrow$ conservation of information
  - Complication: Hawking radiation: BH emits incoherently
- Crucial question:

Does a semiclassical picture of collapse
emerge
from a quantum description of gravity?

Outline

- Introduction
  - ACV method for string collisions at transplanckian energies
- Graviton radiation
  - based on emission amplitude unifying
    - central region (Regge limit)
    - fragmentation region
  - resum infinite diagrams
  - energy spectrum
    - small deflection angles (weak coupling)
    - large deflection angles (strong coupling)
- Features:
  - final state radiation is a unitary “pure” state
  - role of gravitational radius: $\langle\omega\rangle \sim R^{-1}$
  - radiation enhancement for $b \sim R$
  - $\mathrm{d}E/\mathrm{d}\omega \sim \mathrm{e}^{-\omega/T}$ with “quasi-temperature” $T \sim T_H$

Historical review

String scattering

[Amati,Ciafaloni,Veneziano'88] considered scattering of 2 massless super-strings at transplanckian CM energies $2E = \sqrt{s} \gg M_P \equiv \sqrt{\hbar/G}$ and impact parameter b in various kinematical regimes.

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Orderings:

- $\lambda_P \ll \lambda_s \quad \Longleftrightarrow \quad$ string loop expansion parameter $g \equiv \lambda_P/\lambda_s \ll 1$
- $\lambda_P \ll R \quad \Longleftrightarrow \quad \sqrt{s} \gg M_P \quad \Longleftrightarrow \quad \alpha \equiv s/M_P^2 = Gs/\hbar \gg 1$
- λ_s, R, b can have any relative ordering, and we distinguish 3 regimes:

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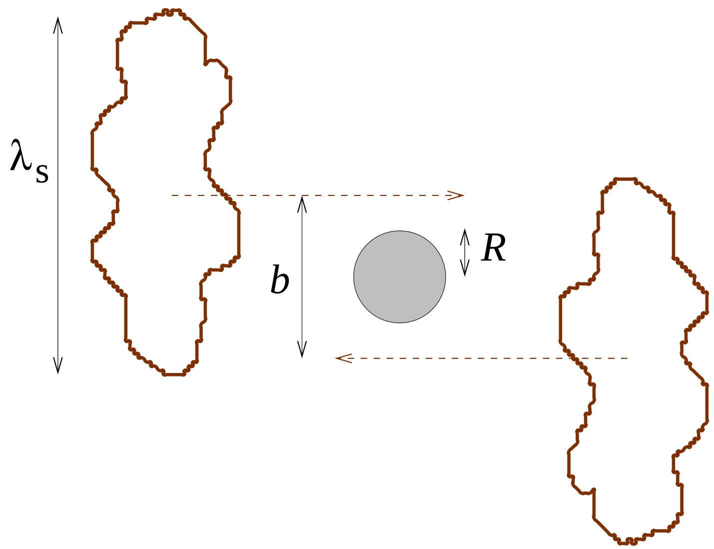
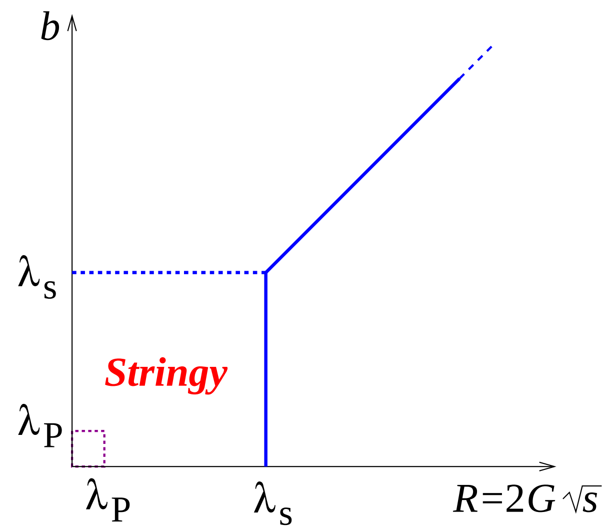


$\lambda_s > R,\ b$

string effects



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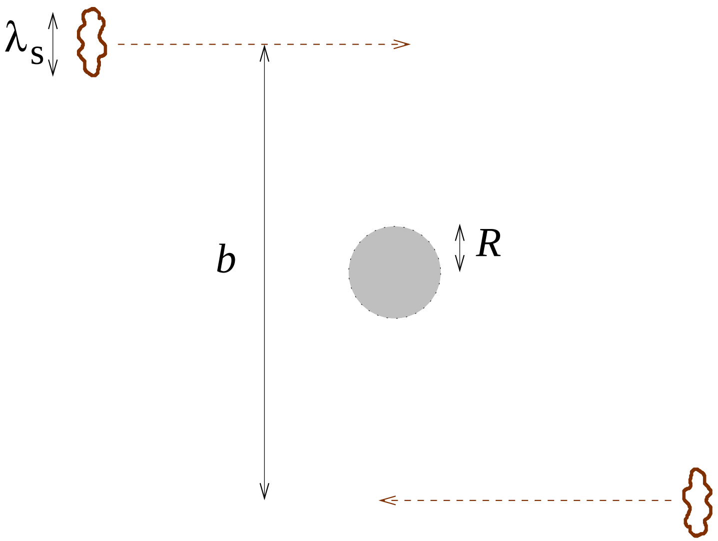
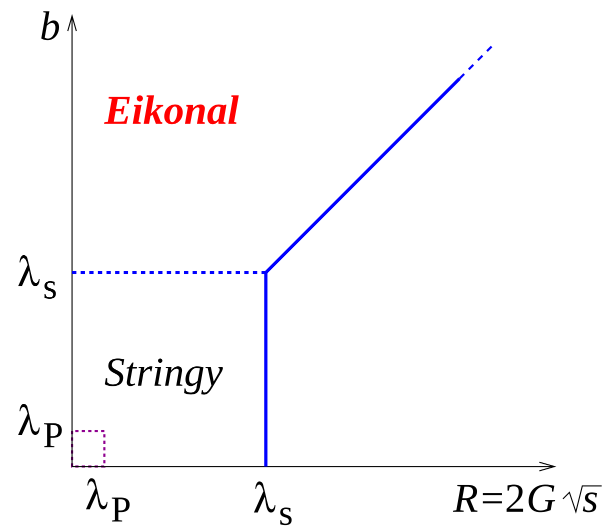


$b > R,\ \lambda_s$

small deflection



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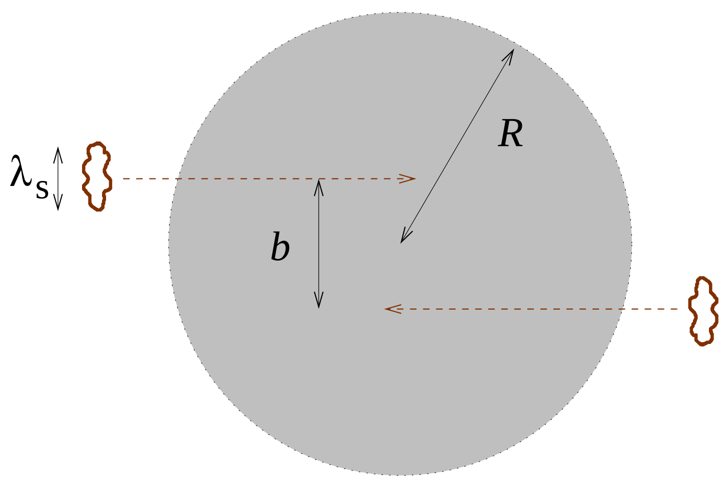
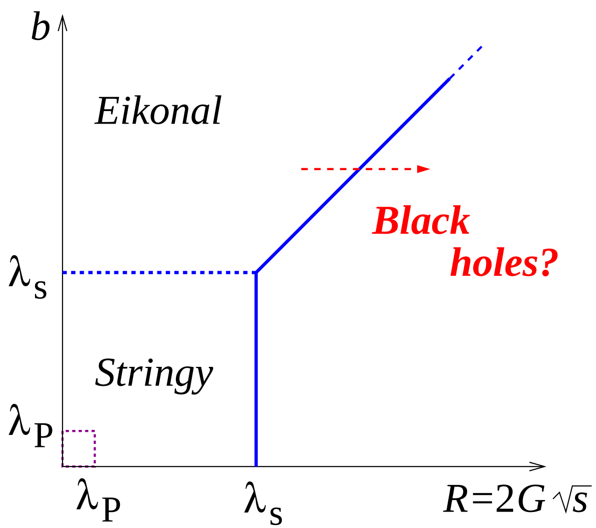
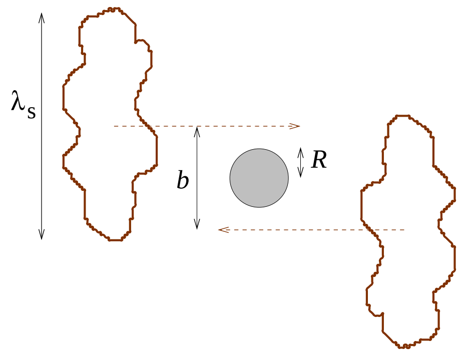
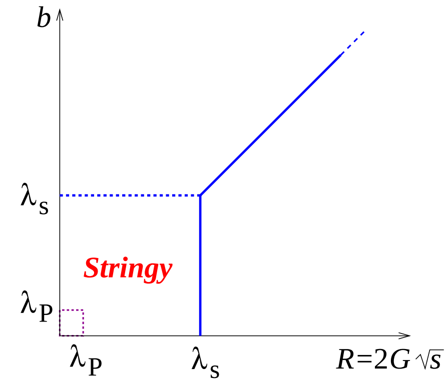


$R > b,\ \lambda_s$
gravitation strong

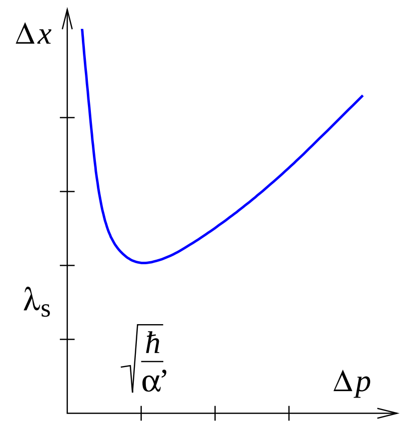


Stringy regime: $\lambda_s \gg b, R$





String effects essential: the extended size soften gravity

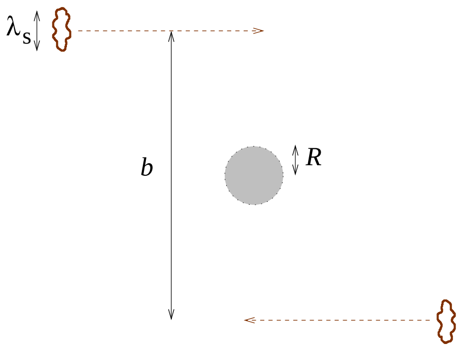


$$\Delta x \gtrsim \frac{\hbar}{\Delta p} + \alpha' \Delta p \gtrsim \sqrt{\hbar \alpha'} = \lambda_s$$

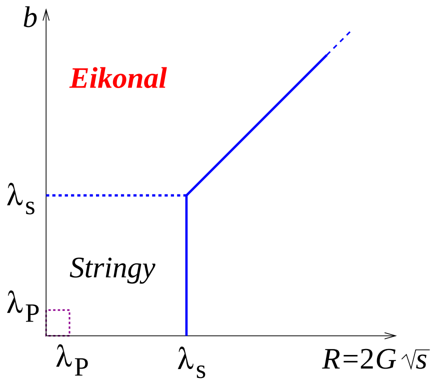
[Gross,Mende]; [ACV]

- $\sim \lambda_s$ minimal observable size $> R$
- Classical gravitational collapse conditions never met
- Energy excites the strings, energy is diluted into objects of larger size

Eikonal regime: $b \gg \lambda_s, R$



Scattering essentially elastic



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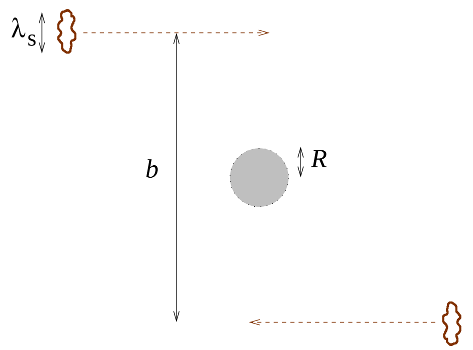

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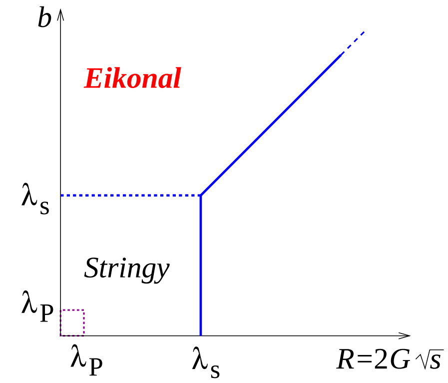
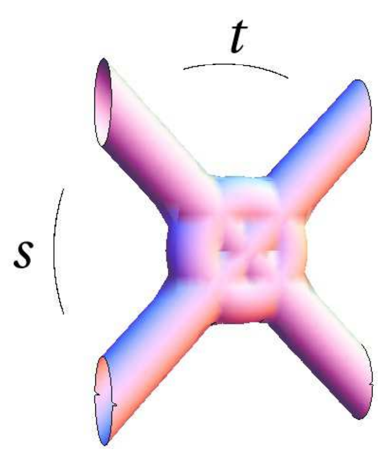
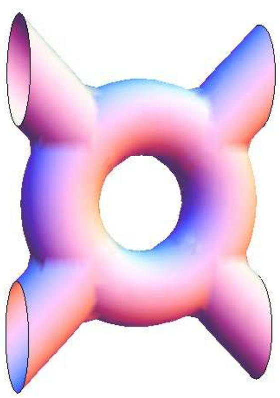
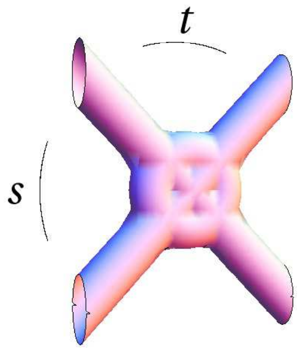


$$\langle p_1' \, p_2' | S | p_1 \, p_2 \rangle = \delta_{fi} + (2\pi)^4 \delta^4 (p_1 + p_2 - p_1' - p_2') \mathrm{i} M(s,t)$$

$M(s,t)$ =  +  + $\cdots$

Eikonal regime: lowest orders



$$M_0(s,t) = g^2 \frac{\Gamma(-\frac{s}{2})\Gamma(-\frac{t}{2})\Gamma(-\frac{u}{2})}{\Gamma(1+\frac{s}{2})\Gamma(1+\frac{t}{2})\Gamma(1+\frac{u}{2})} \qquad (\alpha' = 1)$$

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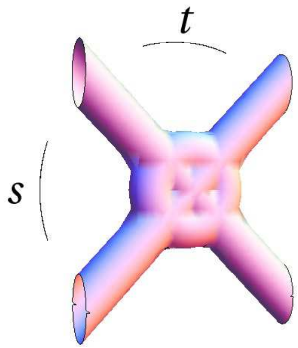

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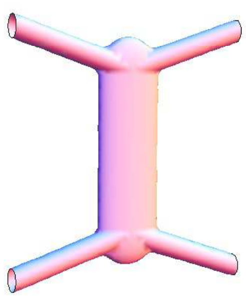
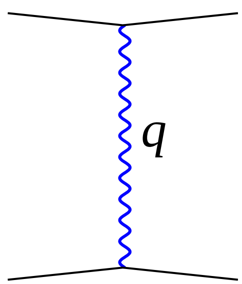
$\xrightarrow{s \gg t}$



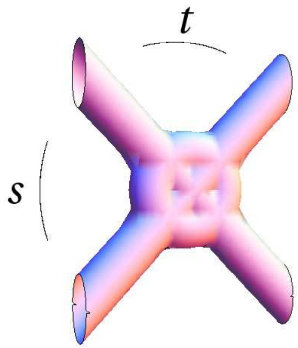
$\equiv$



$$2g^2 \frac{\Gamma(-\frac{t}{2})}{\Gamma(1+\frac{t}{2})} \left(\frac{-\mathrm{i}}{2}\right)^{2+t} s^{2+t} \equiv s\, A_{\mathrm{el}}(s, \boldsymbol{q})$$

$$(t \simeq -\boldsymbol{q}^2)$$

Eikonal regime: lowest orders



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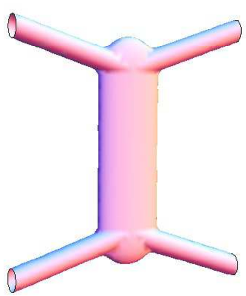
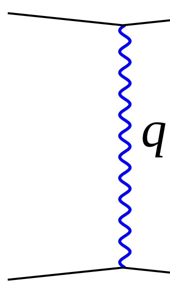
$\xrightarrow{s \gg t}$  $\equiv$ 

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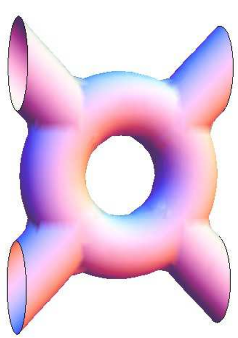
$M_1 =$ 

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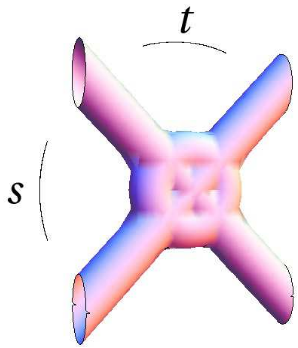

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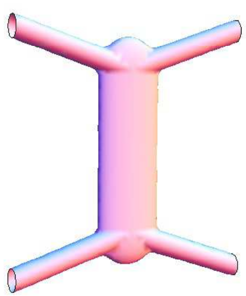

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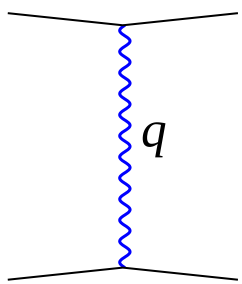

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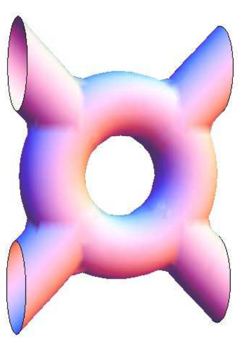

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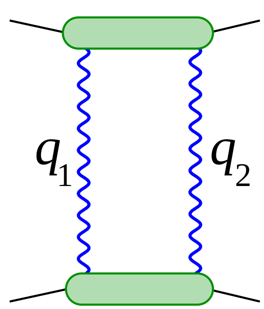
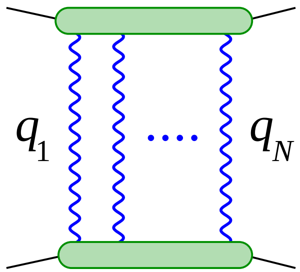


$$\equiv \frac{\mathrm{i}s}{2} \int \begin{matrix} V_2(\boldsymbol{q}_1, \boldsymbol{q}_2) \times \\ A_{\mathrm{el}}(s, \boldsymbol{q}_1) A_{\mathrm{el}}(s, \boldsymbol{q}_2) \delta(\boldsymbol{Q} - \boldsymbol{q}_1 - \boldsymbol{q}_2) \mathrm{d}\boldsymbol{q}_1 \mathrm{d}\boldsymbol{q}_2 \\ \times V_2(\boldsymbol{q}_1, \boldsymbol{q}_2) \end{matrix}$$

Eikonal regime: higher orders

String amplitudes in Regge limit ($s \to \infty$, t fixed):

$$\mathrm{i}M_N(s,t) \xrightarrow{s\gg t}$$



$$\equiv \frac{\mathrm{i}^N s}{N!} \int \begin{matrix} V_N(\boldsymbol{q}_1, \cdots, \boldsymbol{q}_N)\times \\ A_{\mathrm{el}}(s,\boldsymbol{q}_1)\cdots A_{\mathrm{el}}(s,\boldsymbol{q}_N) \\ \times V_N(\boldsymbol{q}_1, \cdots, \boldsymbol{q}_N) \end{matrix} \; \delta(\boldsymbol{q} - \textstyle\sum_n \boldsymbol{q}_n)\mathrm{d}\boldsymbol{q}_1 \cdots \mathrm{d}\boldsymbol{q}_N$$

(reggeized) graviton exchanges between (excited) string states

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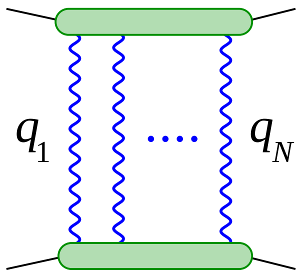

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Strings are extended objs: $\boldsymbol{q} \sim \hbar/b$ are soft, $\langle N\rangle \sim \alpha = Gs/\hbar$ is large
$\Rightarrow \boldsymbol{Q} \sim Gs/b$ can be large $\Rightarrow$ finite $\Theta_E \sim R/b$

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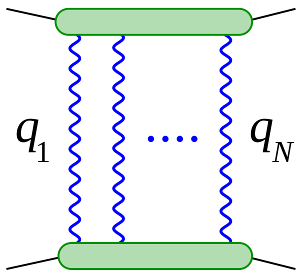

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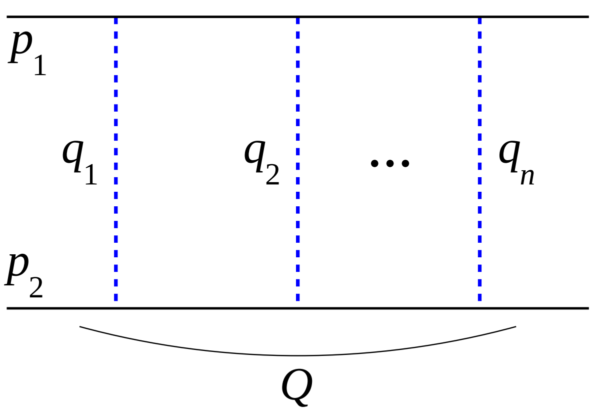
In semiclassical regime $b, R \gg \lambda_s$ strings are not excited $\Rightarrow V_N \to 1$

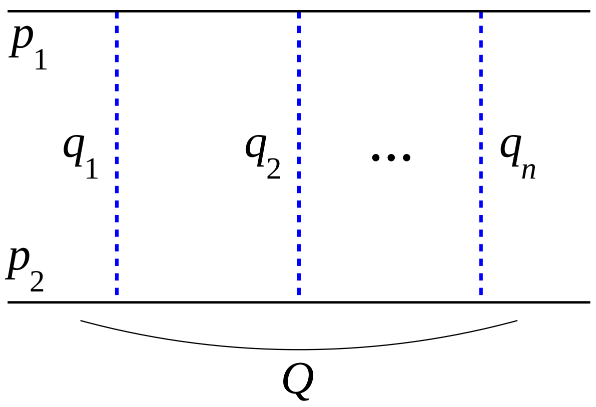
(on-shell point particles)



M_N is a convolution in $\boldsymbol{q}$ space, represented by effective ladder diagrams resumming all powers $(Gs/\hbar)^n$ due to graviton exchanges *[ACV '88]*

Eikonal approximation

$s \gg t \quad \Rightarrow \quad$ effective ladder diagrams *[ACV '88]*



- incoming particles $p_{1,2}$
  (almost) undeflected and on shell
- Amplitude dipends only on transverse components of q_i: $A_{\rm el}(s, \boldsymbol{q}) = \dfrac{Gs}{\boldsymbol{q}^2}$

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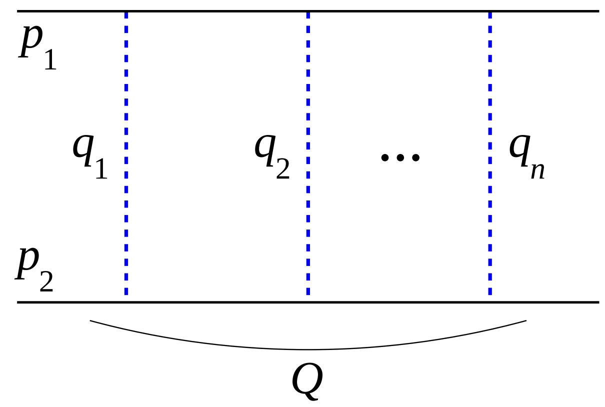

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$$\frac{\mathrm{i}}{s} M_n(s, \boldsymbol{Q}) = \frac{\mathrm{i}^n}{n!} \int \mathrm{d}^2\boldsymbol{q}_1 \cdots \mathrm{d}^2\boldsymbol{q}_n \quad A_{\mathrm{el}}(s, \boldsymbol{q}_1) \cdots A_{\mathrm{el}}(s, \boldsymbol{q}_n) \quad \delta^2(\boldsymbol{q}_1 + \cdots \boldsymbol{q}_n - \boldsymbol{Q})$$

$$\mathrm{i}\mathcal{M}_n(s, \boldsymbol{b}) = \int \mathrm{d}^2\boldsymbol{Q}\ \mathrm{e}^{\mathrm{i}\boldsymbol{Q}\cdot\boldsymbol{b}}\ \mathrm{i}M(s, \boldsymbol{Q}) = \frac{\mathrm{i}^n}{n!}[A_{\mathrm{el}}(s, \boldsymbol{b})]^n$$

IR cutoff

factorization in impact parameter $\boldsymbol{b}$ space: $A_{\mathrm{el}}(s, \boldsymbol{b}) \equiv 2\delta_0 = 2\dfrac{Gs}{\hbar} \log \dfrac{L}{|\boldsymbol{b}|}$

exponentiation: $S_{\mathrm{el}}(s, \boldsymbol{b}) = \sum_{n=0}^{\infty} \mathrm{i}\mathcal{M}_n = \mathrm{e}^{\mathrm{i}A_{\mathrm{el}}(s,\boldsymbol{b})} = \mathrm{e}^{\mathrm{i}2\delta_0(s,\boldsymbol{b})}$

Eikonal approximation

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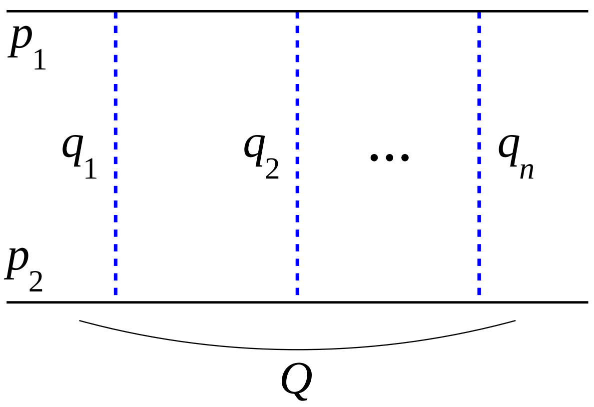

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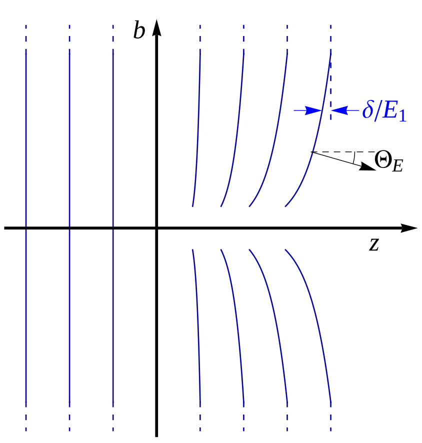
IR cutoff

factorization in impact parameter $\boldsymbol{b}$ space: $\quad A_{\rm el}(s,\boldsymbol{b}) \equiv 2\delta_0 = 2\dfrac{Gs}{\hbar}\log\dfrac{L}{|\boldsymbol{b}|}$

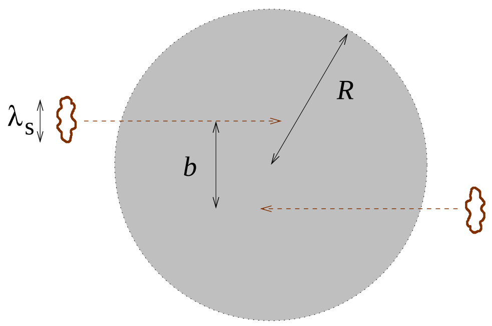
exponentiation: $\quad S_{\rm el}(s,\boldsymbol{b}) = \displaystyle\sum_{n=0}^{\infty}\mathrm{i}\mathcal{M}_n = \mathrm{e}^{\mathrm{i}A_{\rm el}(s,\boldsymbol{b})} = \mathrm{e}^{\mathrm{i}2\delta_0(s,\boldsymbol{b})}$

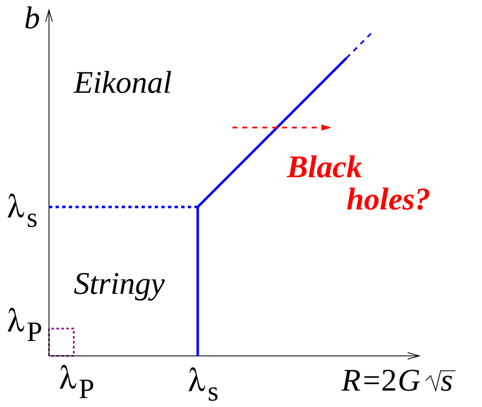


Einstein deflection: $\Theta_E = \dfrac{2}{\sqrt{s}}|\nabla_{\boldsymbol{b}}\delta_0| = \dfrac{2R}{|\boldsymbol{b}|}$

in agreement with GR

Strong gravity regime: $R \gtrsim b \gg \lambda_s$





can be approached by computing subleading contributions

Corrections $(\lambda_s/b)^n$ are negligible

Corrections $(R/b)^n$ are essential:

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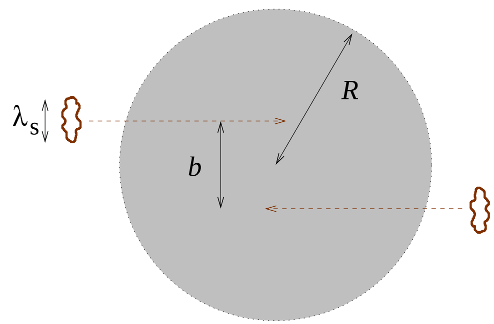

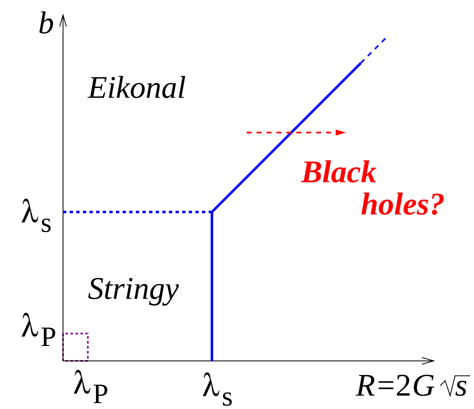

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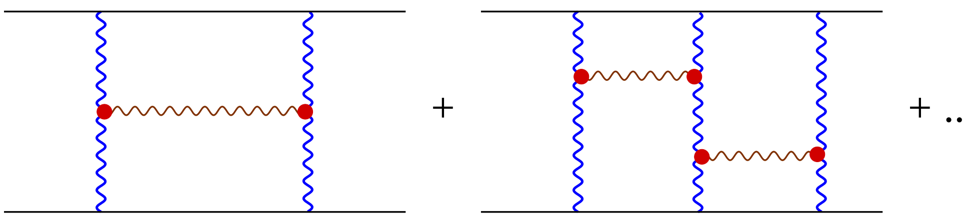
[ACV] (multi) H-diagrams



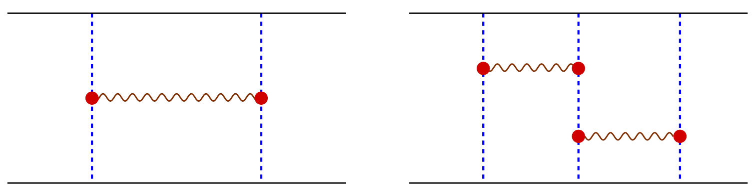
First correction: H diagram

● = Lipatov's vertex

$$\delta = \frac{Gs}{\hbar}\left[\log\frac{L}{b} + \frac{1}{2}\left(\frac{R}{b}\right)^2 + \cdots\right] \qquad \sin\frac{\Theta_s}{2} = \frac{R}{b}\left[1 + \left(\frac{R}{b}\right)^2 + \cdots\right]$$

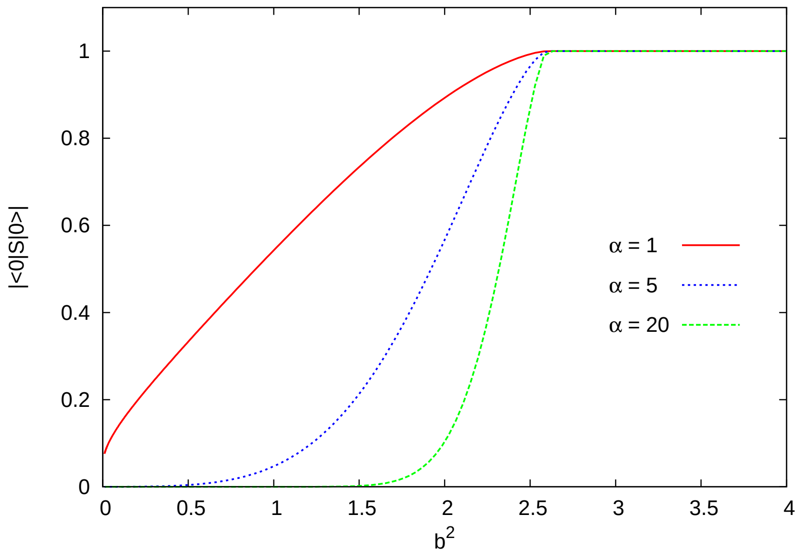
Strong gravity regime: $b \to R$

One can reach the collapse regime *[ACV '07]*
by resumming sub-leading "multi H" diagrams



- Phase shift $\delta(s, \boldsymbol{b})$ acquires an imaginary part for $b < b_c = 1.6R$
- $|S_{\rm el}(s, \boldsymbol{b})| = \mathrm{e}^{2\mathrm{i}\delta} < 1$
- $|S_{\rm el}(s, 0)| = \mathrm{e}^{-\pi\alpha}$

$\alpha \equiv \frac{Gs}{\hbar} \gg 1$



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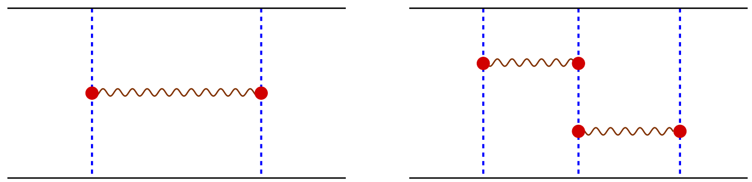

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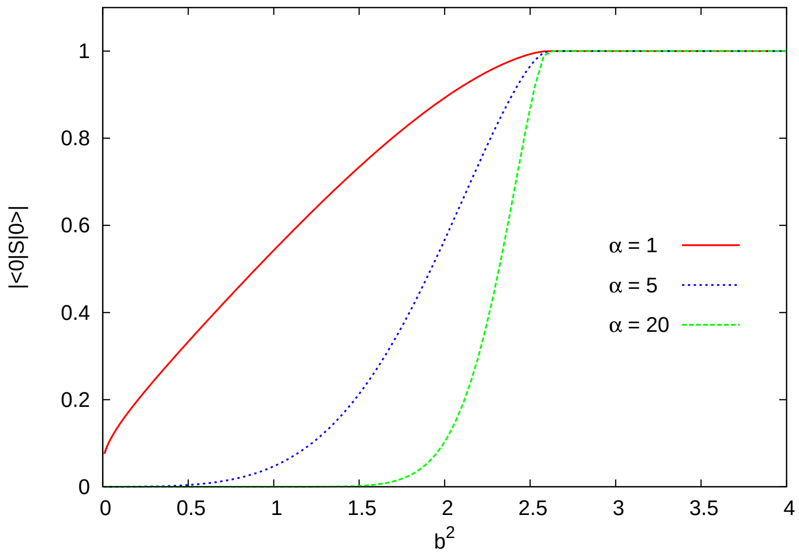
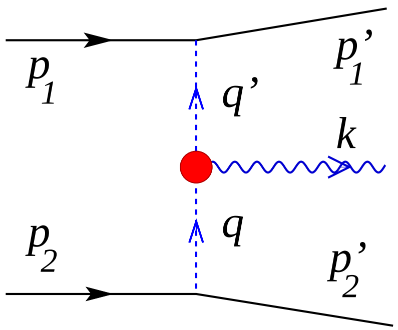


- Unitarity deficit in elastic ch.: compensated by inelastic production of gravitons?
- Is this the signal of gravitational collapse? $1 - |S_{\rm el}| \sim (b_c - b)^{3/2}$

Graviton radiation

Bremsstrahlung: Regge amplitude



- *[ACV 2007]* used Lipatov's vertex for graviton radiation
- emitted energy larger than $\sqrt{s}$ if $R < b < \sqrt{\alpha}\,R$
  energy crisis!

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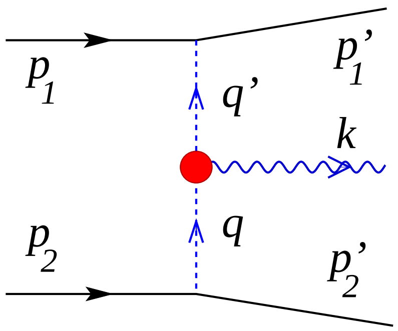

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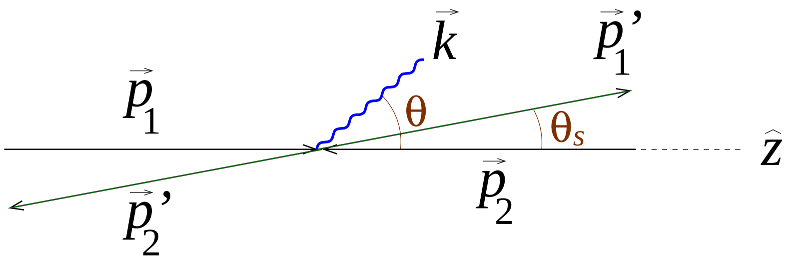
Problem: Lipatov's vertex accurate only in central region $\theta \gg \theta_s$ (Regge kinematics)



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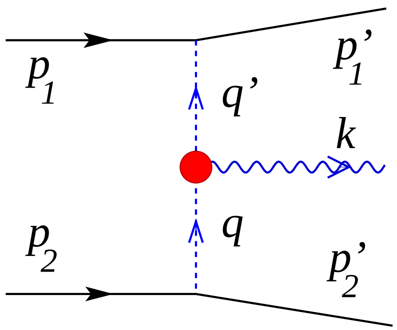

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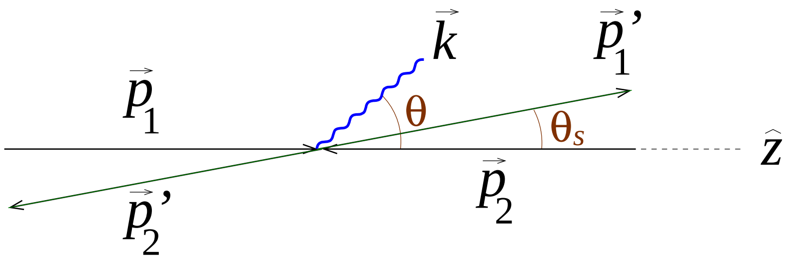


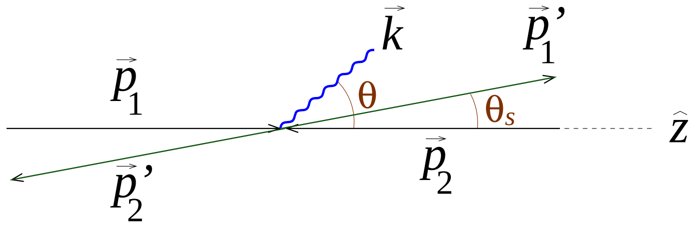
$$M_{\text{Regge}} \simeq M_{\text{el}} \times J_L^{\mu\nu}(\boldsymbol{q}, \boldsymbol{q}')\epsilon_{\mu\nu}^{(+)} = \kappa^3 s \frac{\mathrm{e}^{2\mathrm{i}(\phi_{\boldsymbol{q}} - \phi_{\boldsymbol{q}'})} - 1}{\boldsymbol{k}^2}$$

$$= \kappa^3 s \frac{\boldsymbol{k}^* \boldsymbol{q} - \boldsymbol{k}\boldsymbol{q}^*}{\boldsymbol{k}\boldsymbol{k}^* \boldsymbol{q}\boldsymbol{q}'^*} \quad (\boldsymbol{k} \equiv k_x + \mathrm{i}k_y \in \mathbb{C})$$

Helicity amplitude has an unphysical collinear singularity at $\boldsymbol{k} = 0$ i.e. $\theta = 0$

Bremsstrahlung: Unified amplitude

For $\omega \ll E$ *[Weinberg '65]* theorem holds:



$$M_{\rm soft} \simeq M_{\rm el} \times J_W^{\mu\nu}(\boldsymbol{k})\epsilon_{\mu\nu}^{(+)} = \kappa^3 s \frac{1 - e^{2i(\phi_{\boldsymbol{\theta}} - \phi_{\boldsymbol{\theta}-\boldsymbol{\theta}_s})}}{E^2 \boldsymbol{\theta}_s^2}$$

$$= \kappa^3 s \frac{\boldsymbol{k}^* \boldsymbol{q} - \boldsymbol{k}\boldsymbol{q}^*}{(\boldsymbol{k} - \frac{\omega}{E}\boldsymbol{q})\boldsymbol{k}^*\boldsymbol{q}\boldsymbol{q}^*} \qquad \begin{cases} \boldsymbol{k} & = \omega\boldsymbol{\theta} \\ \boldsymbol{q} & \simeq \boldsymbol{p}_1' = E\boldsymbol{\theta}_s \end{cases}$$

No collinear singularity at $\boldsymbol{\theta} = 0$ ($\boldsymbol{k} = 0$) nor at $\boldsymbol{\theta} = \boldsymbol{\theta}_s$ ($\boldsymbol{k} = \frac{\omega}{E}\boldsymbol{q}$)

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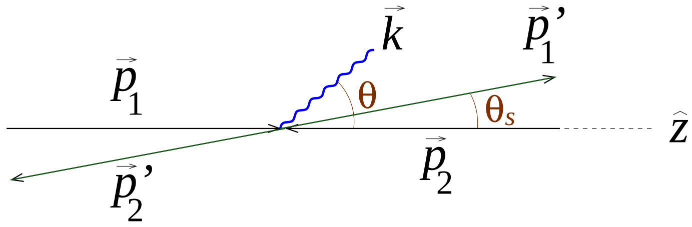

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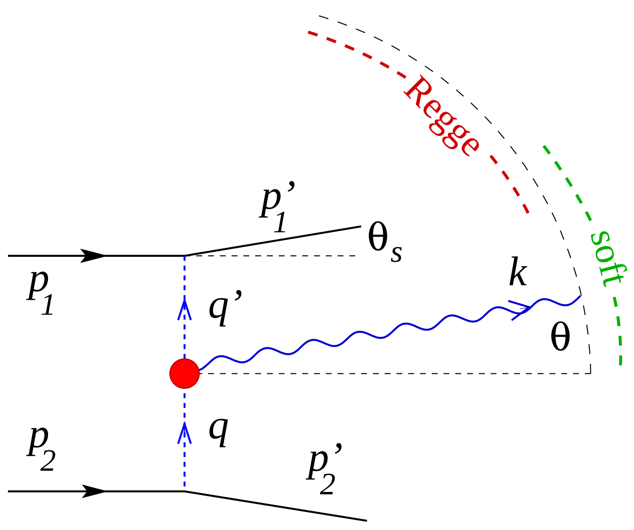

$$\frac{M_{\rm Regge}}{\kappa^3 s} = \frac{\boldsymbol{k}^*\boldsymbol{q} - \boldsymbol{k}\boldsymbol{q}^*}{\boldsymbol{k}\boldsymbol{k}^*\boldsymbol{q}\boldsymbol{q}'^*}$$

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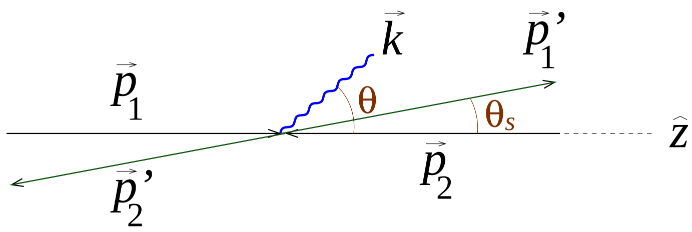

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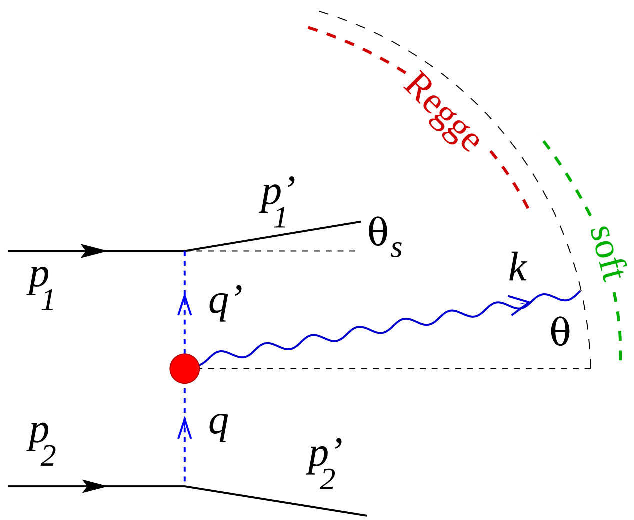

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$$\frac{\boldsymbol{k}^*\boldsymbol{q} - \boldsymbol{k}\boldsymbol{q}^*}{(\boldsymbol{k} - \frac{\omega}{E}\boldsymbol{q})\boldsymbol{k}^*\boldsymbol{q}\boldsymbol{q}'^*} = \frac{M_{\rm unified}}{\kappa^3 s}$$

$$\frac{M_{\rm soft}}{\kappa^3 s} = \frac{\boldsymbol{k}^*\boldsymbol{q} - \boldsymbol{k}\boldsymbol{q}^*}{(\boldsymbol{k} - \frac{\omega}{E}\boldsymbol{q})\boldsymbol{k}^*\boldsymbol{q}\boldsymbol{q}^*}$$

Bremsstrahlung: Unified amplitude

In coordinate (impact parameter) space: “Soft-based” representation

$$M_{\text{unified}}(\boldsymbol{b}, \boldsymbol{x}) = \frac{1}{\boldsymbol{x}^2} \left[\frac{E}{\omega} \ln \left| \frac{\boldsymbol{b} - \frac{\omega}{E}\boldsymbol{x}}{\boldsymbol{b}} \right| - (E \to \omega) \right] = M_{\text{soft}}(E) - M_{\text{soft}}(\omega)$$

interpretation: graviton insertions on external + internal legs

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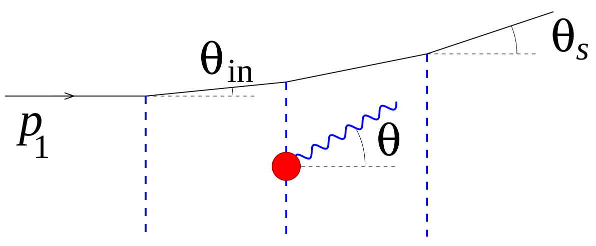
- The denominator $1/\boldsymbol{x}^2$ corresponds to Riemann tensor of the *[Aichelburg-Sexl '71]* gravitational shock wave generated by one of the incoming (massless) particle
- $M_{\text{unified}} \leftrightsquigarrow$ metric field $h_{11} - h_{22} + 2ih_{12}$ generated by the collision of the 2 *[AS]* shock waves travelling with the incoming particles

Resummation

Total single-graviton emission amplitude resums emissions from the whole ladder



- "Local" incoming particle has direction $\boldsymbol{\theta}_{\rm in} \neq 0$
- Corresponding amplitude related by a rotation

$$M(\boldsymbol{\theta}_{\rm in}, \boldsymbol{\theta}) = {\rm e}^{-{\rm i}\lambda\phi_{\boldsymbol{\theta}}} M(\mathbf{0}, \boldsymbol{\theta} - \boldsymbol{\theta}_{\rm in}) {\rm e}^{{\rm i}\lambda\phi_{\boldsymbol{\theta}-\boldsymbol{\theta}_{\rm in}}}$$

- Amplitudes interfere constructively only if $\theta \gg \theta_{\rm in} \sim \theta_s$

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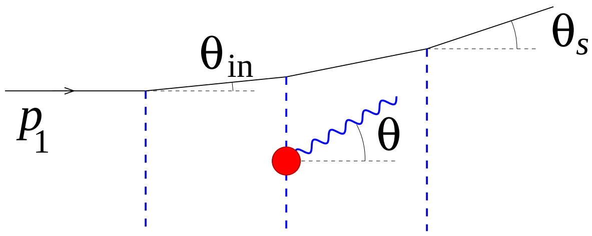

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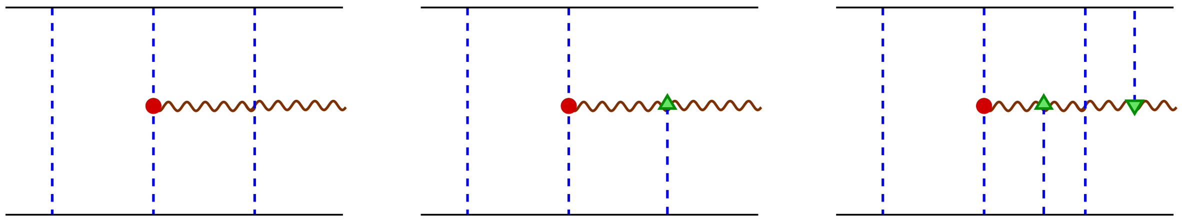

$\boldsymbol{x}$ = graviton transv. position
$\boldsymbol{b}$ = opposite particle " "
interaction = $G\omega\sqrt{s}\Delta(\boldsymbol{b} - \boldsymbol{x})$

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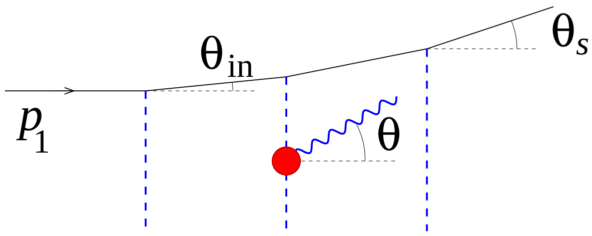

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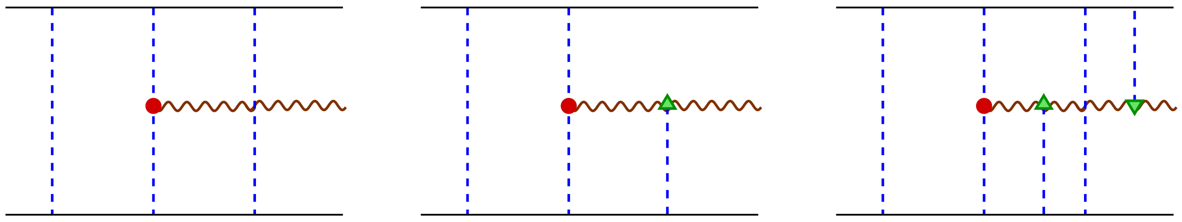

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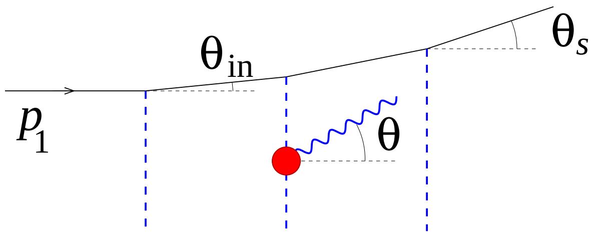

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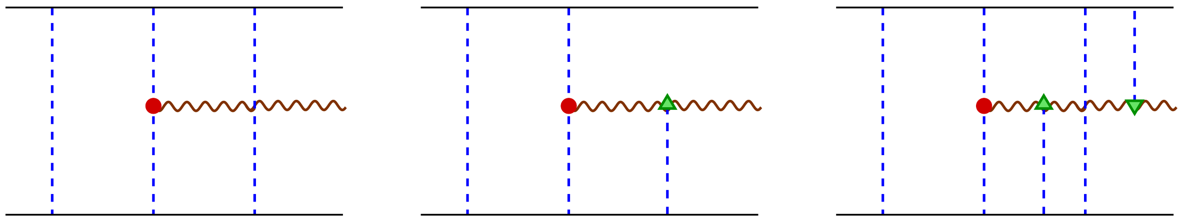

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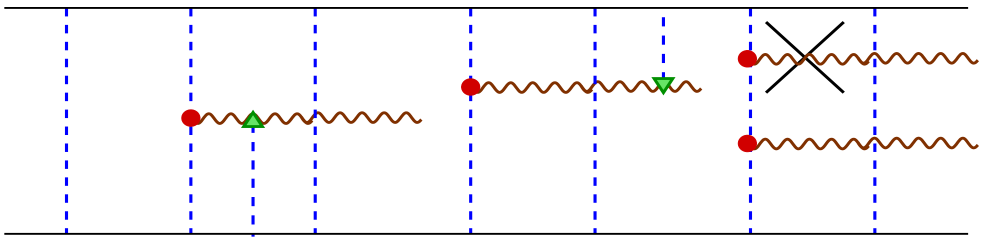
Multiple Emissions

In eikonal approximation $b \gg R$: multi-graviton emissions are (almost) independent (small correlations) and amplitude can be resummed in close form



$$M_{\mathrm{tot}}(2 \to 2 + \boldsymbol{k}_1 \cdots \boldsymbol{k}_n) \equiv \mathrm{e}^{\mathrm{i}2\delta_0(b)} \frac{1}{n!} \mathfrak{M}(\boldsymbol{k}_1) \cdots \mathfrak{M}(\boldsymbol{k}_n)$$

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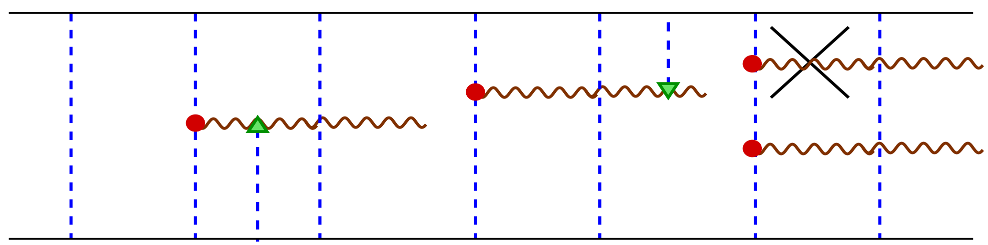

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$$|\text{gravitons}\rangle = \mathrm{e}^{\mathrm{i}2\delta_0(b)} \exp\left\{ \mathrm{i} \sum_{\lambda=\pm} \int \frac{\mathrm{d}^3 k}{2\omega_k} \mathfrak{M}^{(\lambda)}(k) a^{(\lambda)\dagger}(k) + \text{h.c.} \right\} |0\rangle$$

$$= S|0\rangle$$

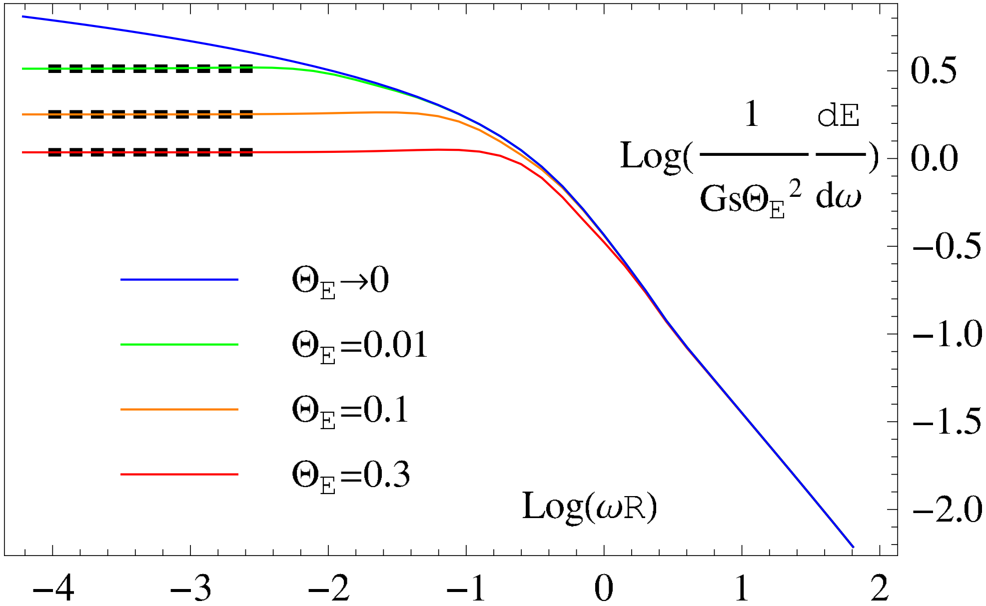
virtual contributions à la Weinberg

- Unitary S-matrix describing transplanckian scattering and bremsstrahlung

Results

Spectrum: subplanckian from transplanckian

$$\frac{\mathrm{d}E^{\mathrm{GW}}}{\mathrm{d}\omega} = \omega \int \frac{\mathrm{d}^2\boldsymbol{\theta}}{(2\pi)^3} \sum_{\lambda=\pm 2} |\mathfrak{M}_\lambda(\boldsymbol{b}, \omega\boldsymbol{\theta})|^2 \xrightarrow[\omega R \gg 1]{} 0.2\frac{Gs\,\boldsymbol{\Theta}_E^2}{\omega R}$$



$$\mathfrak{M}_\lambda(\boldsymbol{b}, \boldsymbol{k}) = \frac{\mathrm{e}^{\mathrm{i}\lambda\phi_{\boldsymbol{\theta}}}}{(2\pi)^2}\frac{\sqrt{\alpha}}{\mathrm{i}\omega}\int \frac{\mathrm{d}^2\boldsymbol{x}}{\boldsymbol{x}^2}\frac{\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}}{\mathrm{e}^{\mathrm{i}\lambda\phi_{\boldsymbol{x}}}}$$

$$\times \left\{\mathrm{e}^{\mathrm{i}2\alpha\left[\Delta(\boldsymbol{b}-\frac{\omega}{E}\boldsymbol{x})-\Delta(\boldsymbol{b})\right]} - \mathrm{e}^{-\mathrm{i}2\omega R[\Delta(\boldsymbol{b})-\Delta(\boldsymbol{b}-\boldsymbol{x})]}\right\}, \qquad \Delta_0(\boldsymbol{b}) = \ln\frac{L}{|\boldsymbol{b}|}$$

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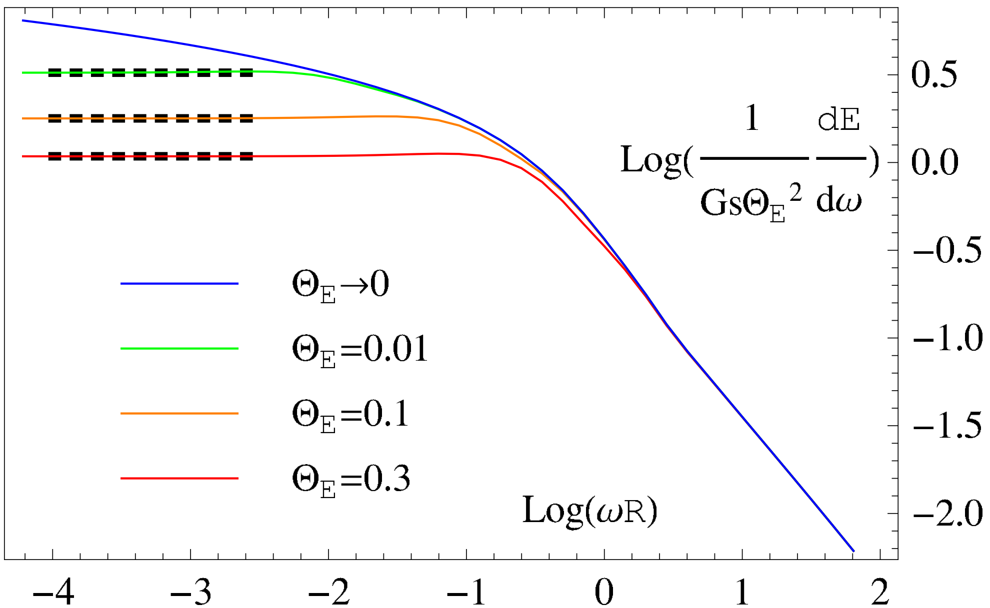
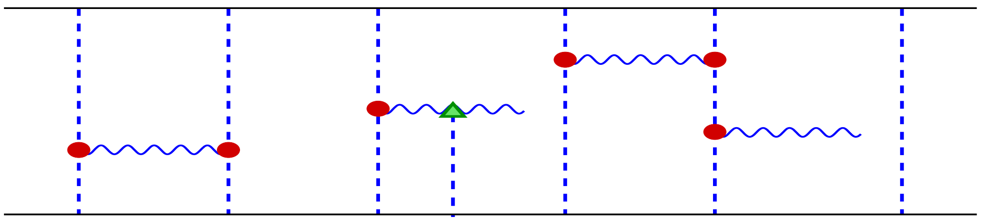


- Agrees with Zero Frequency Limit ($\omega \to 0$)
- Almost universal (b-independent) shape, Characteristic frequency $\langle\omega\rangle \sim 1/R$
- $\langle\omega\rangle$ decreases as $\sqrt{s}$ increases, like Hawking radiation
- Agrees with *[Gruzinov,Veneziano 2015]* for $\hbar \to 0$

Towards $b \to R$



We have to take into account

- H diagrams along the ladder
- emissions from H diagrams

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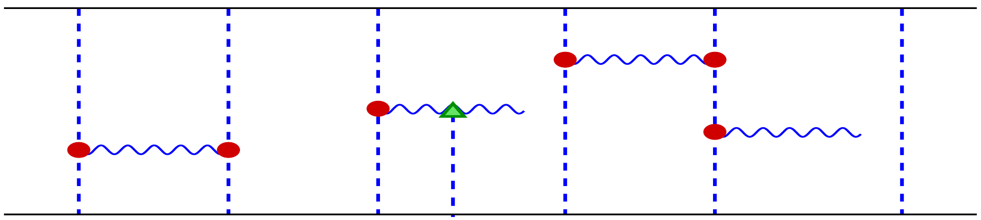
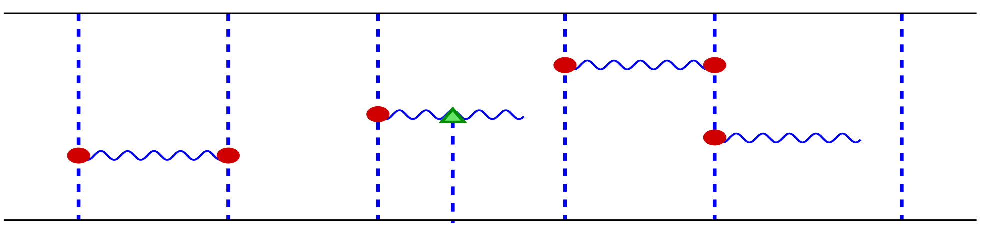


- H diagrams along the ladder

$\mathrm{e}^{\mathrm{i}2\alpha\Delta_0} \to \mathrm{e}^{\mathrm{i}2\alpha\Delta}$

- emissions from H diagrams

Towards $b \to R$



We have to take into account

- H diagrams along the ladder — $e^{i2\alpha\Delta_0} \to e^{i2\alpha\Delta}$
- emissions from H diagrams — evaluated using "soft-based" representation

$$M_H = M_{H,\mathrm{soft}} - M_{H,\mathrm{soft}}\big|_{E\to\omega}$$

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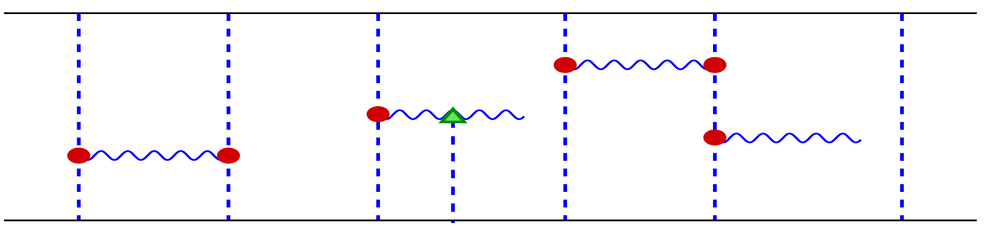

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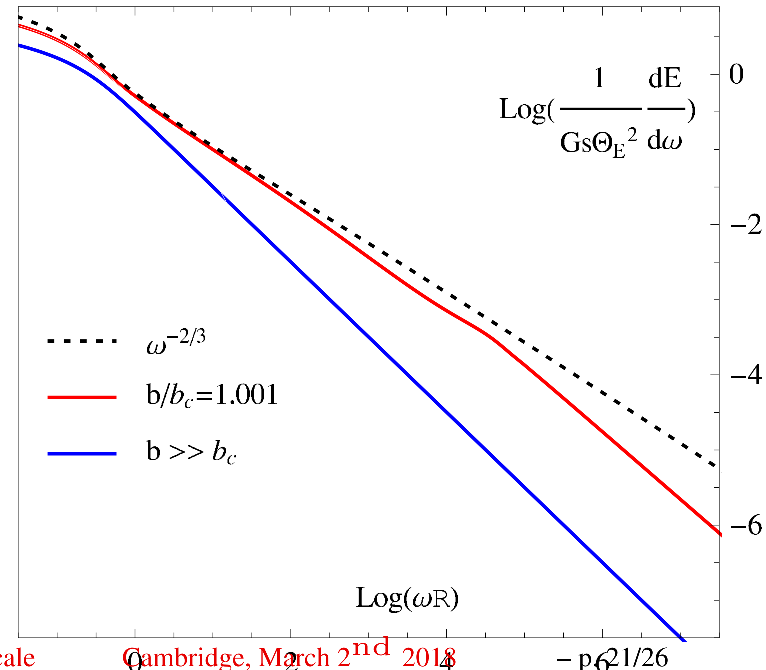
Neglecting correlations, we end up with the same formulas, if $\Delta_0(b) \to \Delta_0(b) + \Delta_H(b) \equiv \Delta(b)$
Δ has the branch-cut at $b_c = 1.6R$

- For $b \to b_c$ significant enhancement of radiation $\sim \omega^{-2/3}$ instead of ω^{-1} due to large tidal forces
- increasing fraction of energy is radiated off $\longrightarrow$ include energy conservation!!



Energy conservation

Standard independent emissions:

$$|\text{gravitons}\rangle = e^{2i\delta_0(b)} \exp\left\{ i \sum_k \left(\mathfrak{M}_k a_k^\dagger + \mathfrak{M}_k^* a_k \right) \right\} |0\rangle$$

$$= e^{2i\delta_0(b)} e^{-\frac{1}{2}\sum_k |\mathfrak{M}|^2} \exp\left\{ i \sum_k \mathfrak{M}_k a_k^\dagger \right\} |0\rangle$$

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Projectors on n-graviton subspaces and probabilities

$$\Pi_0 = |0\rangle\langle 0| \qquad P_0 = \langle \text{grav}|\Pi_0|\text{grav}\rangle = e^{-\sum_k |\mathfrak{M}_k|^2}$$

$$\Pi_1 = \sum_k |k\rangle\langle k| \qquad P_1 = \langle \text{grav}|\Pi_1|\text{grav}\rangle = P_0 \sum_k |\mathfrak{M}_k|^2$$

$$\Pi_2 = \frac{1}{2!} \sum_{k_1,k_2} |k_1 k_2\rangle\langle k_1 k_2| \qquad P_2 = \langle \text{grav}|\Pi_2|\text{grav}\rangle = P_0 \frac{1}{2!} \left(\sum_k |\mathfrak{M}_k|^2 \right)^2$$

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Unitarity:

$$\sum_{n=0}^{\infty} P_n = \sum_n e^{-\sum_k |\mathfrak{M}_k|^2} \frac{1}{n!}\left(\sum_k |\mathfrak{M}_k|^2 \right)^n = 1$$

One-particle incl. energy distr. $\dfrac{dE^{GW}}{d^3k} = \langle \text{grav}|\omega_k a_k^\dagger a_k|\text{grav}\rangle = \omega_k |\mathfrak{M}_k|^2$

Energy conservation

Keep coherent independent emissions:

$$|\text{gravitons}\rangle = \mathrm{e}^{2\mathrm{i}\delta_0(b)} \mathrm{e}^{-\frac{1}{2}\sum_k |\mathfrak{M}|^2} \exp\left\{ \mathrm{i} \sum_k \mathfrak{M}_k a_k^\dagger \right\} |0\rangle$$

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Include energy conservation on projectors

$$\Pi_n = \frac{1}{n!}\sum_{k_1\cdots k_n} |k_1\cdots k_n\rangle\langle k_1\cdots k_n|\,\Theta\big(E-(\omega_1+\cdots+\omega_n)\big)$$

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$$N(E) \equiv \sum_{n=0}^{\infty} P_n = \int_{-\mathrm{i}\infty}^{+\mathrm{i}\infty}\frac{\mathrm{d}\lambda}{2\pi\mathrm{i}}\,\frac{\mathrm{e}^{\lambda E}}{\lambda+\varepsilon}\exp\left\{\sum_k |\mathfrak{M}_k|^2[\mathrm{e}^{\omega\lambda}-1]\right\} \quad < 1$$

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Define renormalized projectors and probabilities *[Veneziano'05]* $\leftarrow$ *[AGK'72 rules]*

$$\tilde{P}_n \equiv \frac{1}{N(E)} P_n \qquad \text{such that} \qquad \sum_n \tilde{P}_n = 1$$

Energy conservation and "temperature"

$$\left.\frac{\mathrm{d}E^{\mathrm{GW}}}{\mathrm{d}^3k}\right|_{\substack{\text{ener}\\\text{cons}}} = \omega_k |\mathfrak{M}_k|^2 \times \frac{N(E-\omega_k)}{N(E)}$$

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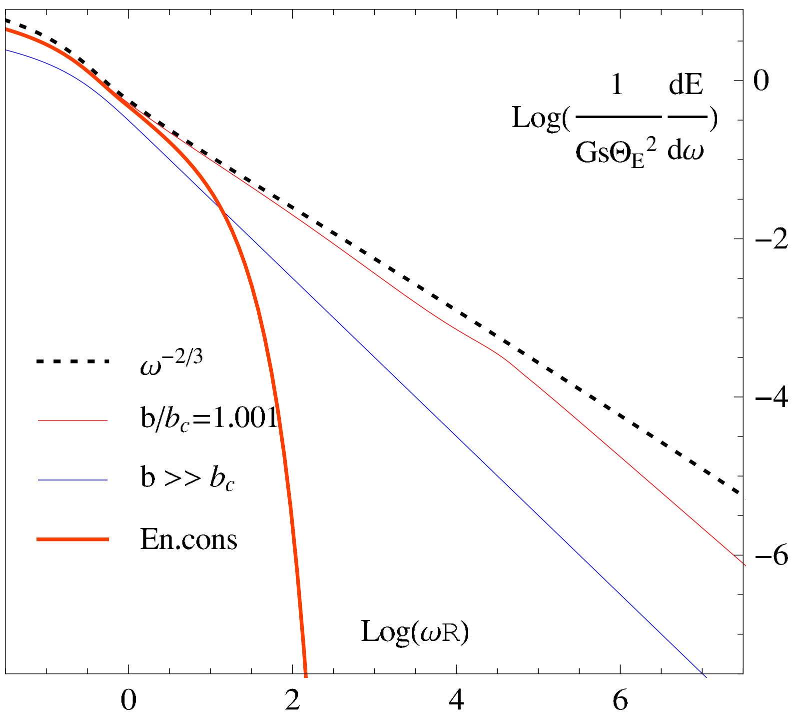
$$\left.\frac{\mathrm{d}E^{\mathrm{GW}}}{\mathrm{d}\omega}\right|_{\substack{\text{ener}\\\text{cons}}} \simeq \left.\frac{\mathrm{d}E^{\mathrm{GW}}}{\mathrm{d}\omega}\right|_0 \times \mathrm{e}^{-\hbar\omega/\tau} \qquad \left(\tau \overset{b\to b_c}{\sim} \frac{\hbar}{R} = k_B T_{\mathrm{Hawking}}\right)$$

Coherent radiation

with thermal-like shape



Collapse region: $b < b_c$

Finally we cross the critical impact parameter and go towards $b \ll R$

- Elastic amplitude provides exponential suppression: $\Delta(\boldsymbol{b}=0) = \mathrm{i}\pi/2$

$$M \sim \exp(-\pi\alpha) = \exp(-\pi\, E\, R)$$

- Rescattering term of emission factor is enhanced $\sim \exp(+\pi\, \omega\, R)$

$$M_n(\boldsymbol{b}, \boldsymbol{k}_1, \cdots, \boldsymbol{k}_n) = \mathrm{e}^{\mathrm{i}2ER\Delta(\boldsymbol{b})} \prod_{j=1}^{n} \mathfrak{M}_{\lambda_j}(\boldsymbol{b}, \boldsymbol{k}_j)$$

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- If $\sum_j \omega_j = E$ enhacement compensates suppression: unitarity is possible
- First preliminary estimate of quasi-temperature yields ($k_B = 1$)
$T \simeq 0.7\frac{\hbar}{R} \simeq T_H(\sqrt{s}/9)$ similar to T at $b \simeq b_c$

Conclusions

- We can compute graviton radiation in transplanckian collisions
  - Determined unified limiting form of graviton emission amplitudes
  - Resolution of energy crisis: $\sim 1/\omega$ spectrum for $\omega > R^{-1}$
- We see the role of $R^{-1} = \langle\omega\rangle$ in spectrum like in Hawking radiation $\forall s\,,\; b$
- For $b \lesssim R$ non-linear effects (tidal forces) provide enhanced emission; all energy is radiated off
- Requiring energy conservation:
  - coherent radiation sample $\Rightarrow$ no information loss
  - Spectrum is exponentially suppressed, like thermal radiation
  - "quasi-temperature" $\simeq$ Hawking's T_H for a BH somewhat lighter than $\sqrt{s}$
- Suggests a possible mechanism of solving the information paradox (to be confirmed for $b < b_c$)