Towards an all-orders calculation of the electroweak bubble wall velocity

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Outline

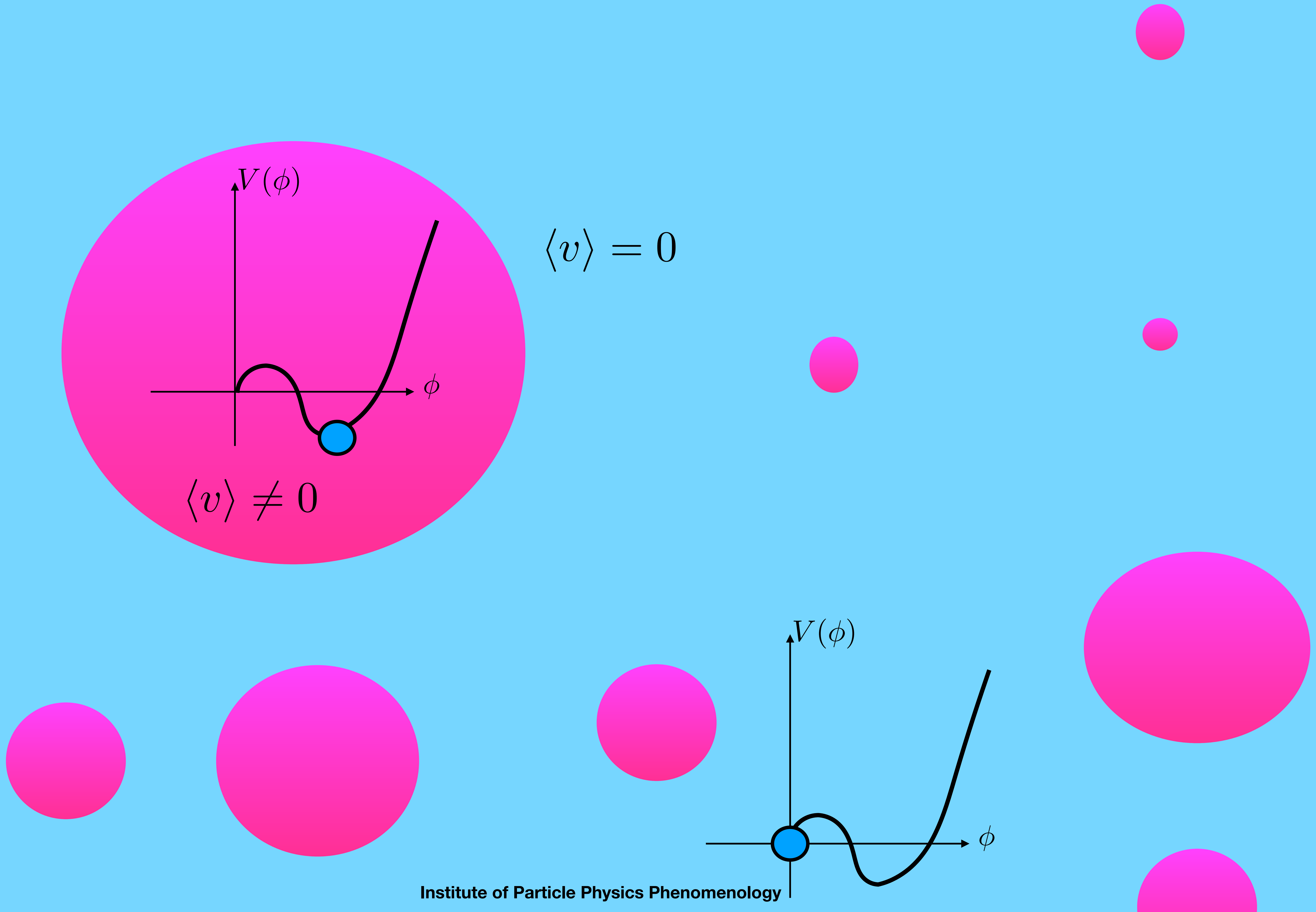
- Motivations for first-order electroweak phase transition
- Relativistic bubble wall velocity calculations: 1-to-1
- Relativistic bubble wall velocity calculations: 1-to-2
- Relativistic bubble wall velocity to all orders
- Summary

First order electroweak phase transition

Within the Standard Model the EWPT is a crossover D'Onofrio & Rummukainen (2015)

Minimal new physics → first order PT Anderson & Hall (1992)

- Matter-antimatter asymmetry Kuzmin, Rubakov & Shaposhnikov (1985)
- Topological defects Achucarro & Vachaspati (2000)
- Primordial magnetic fields Vachaspati (1991)
- Stochastic gravitational wave background Kamionkowski, Kosowsky & Turner (1993)


$V(\phi)$
ϕ
$\langle v \rangle \neq 0$
$\langle v \rangle = 0$
$V(\phi)$
ϕ

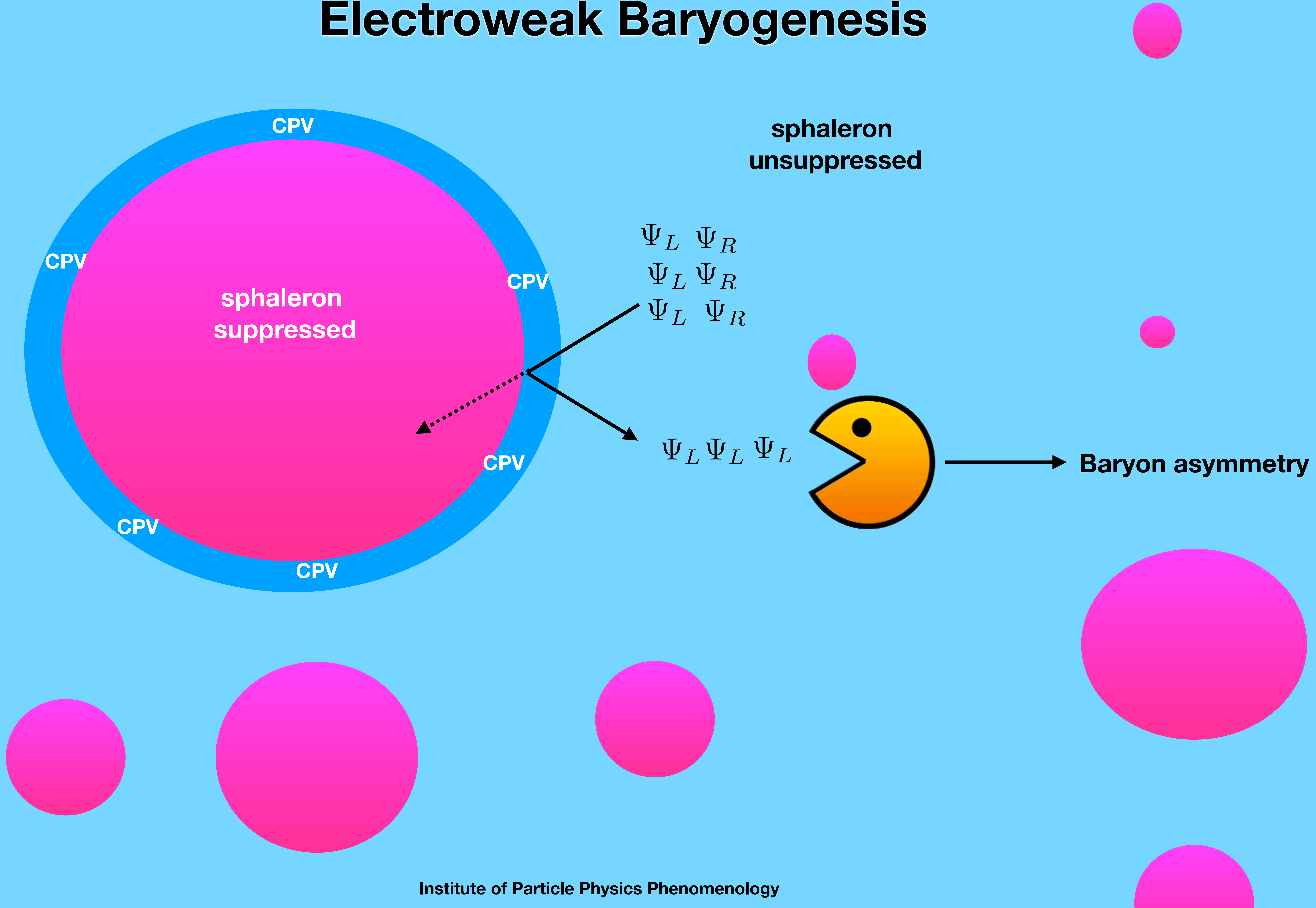
Vacuum pressure from symmetry breaking Higgs potential

Model dependent

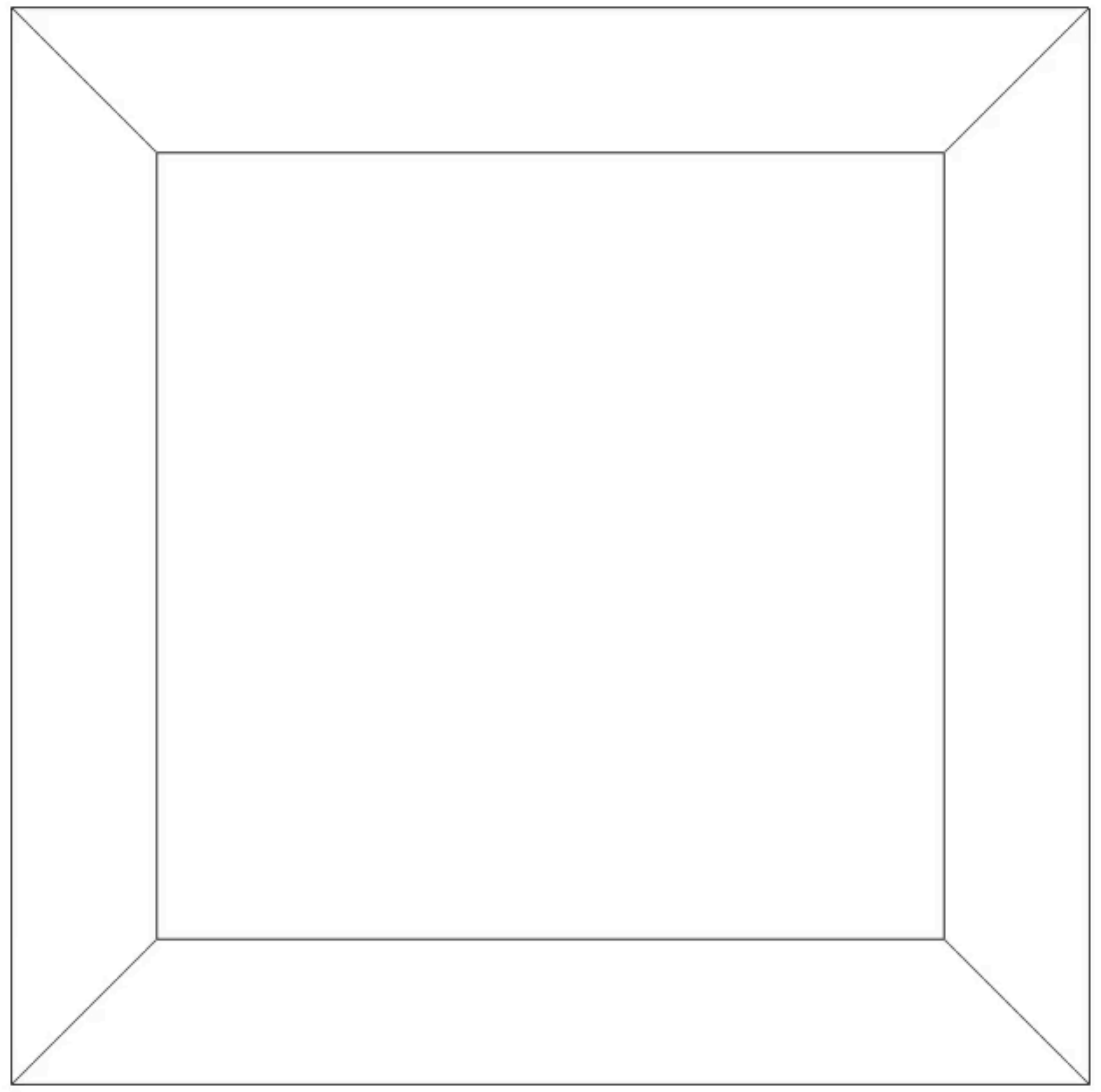
Thermal pressure, resulting from interactions of wall with the plasma particles

Model independent(ish)

Electroweak Baryogenesis



Gravitational waves generated from 1st order PT



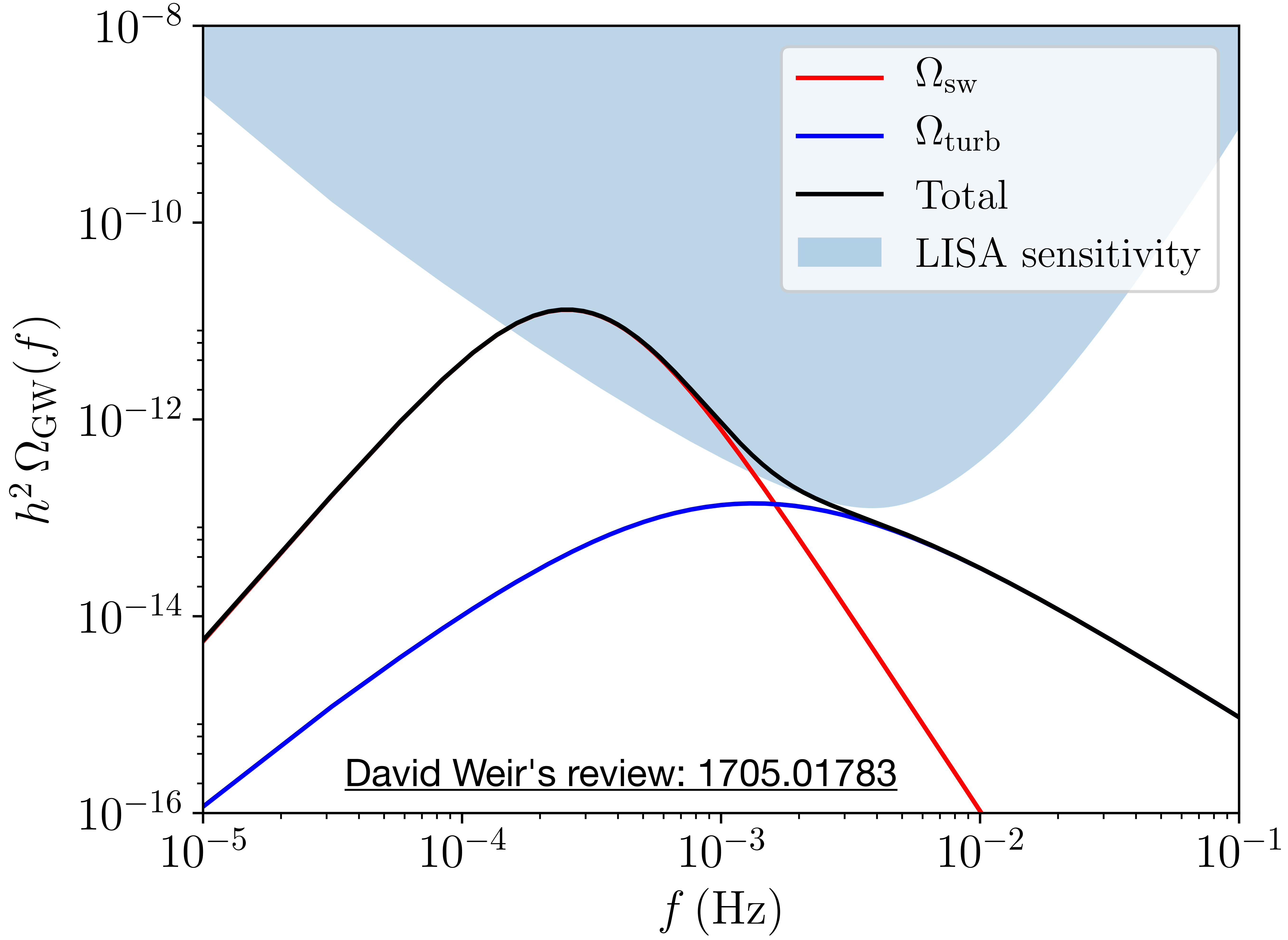
From David Weir's website

GW sourced from **three** contributions:

- Collision bubble walls $\Omega_{\rm env}$
- Sound waves as bubble push through plasma $\Omega_{\rm sw}$
- Turbulence $\Omega_{\rm turb}$

- bubbles "runaway" ($v_w \to c$) latent heat of PT $\to$ KE of the bubble walls
- bubble wall slow $\to$ more energy goes into sound waves and turbulence

Gravitational waves generated from 1st order PT



- Velocity affects SGWB spectrum
- EWBG and SGWB in tension

1-to-1 pressure calculation for relativistic bubble walls

1-to-1 calculation handwaving argument

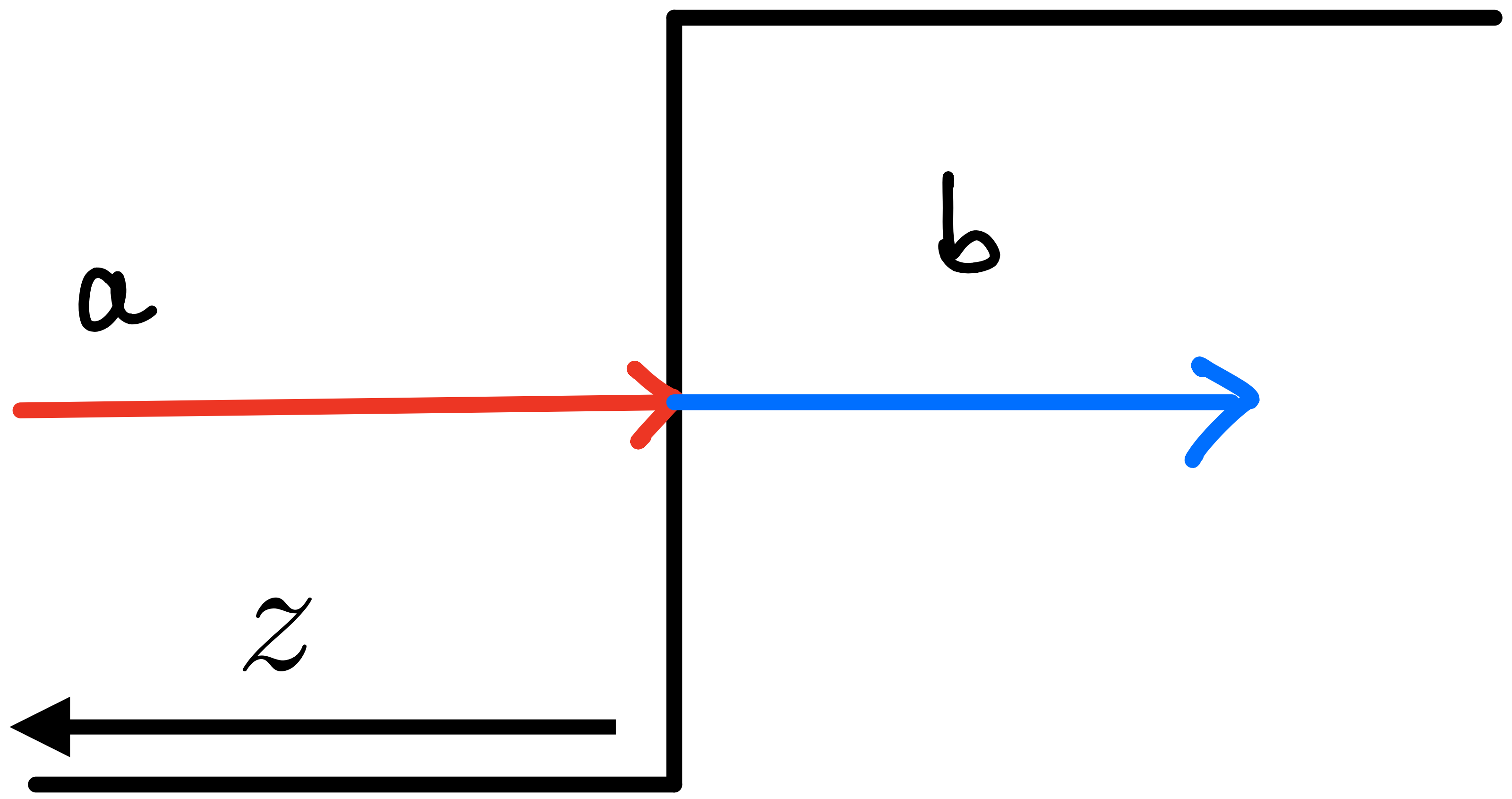
Friction → scattering particles that couple to Higgs condensate

Arnold (1993)

Bodeker & Moore (2009)

$$\gamma = \frac{1}{\sqrt{1-v^2}}$$

Lorentz factor of the wall



$$\Delta p_{\rm wall} \equiv p_{a,z,\ \rm s} - p_{b,z,\ \rm h} = \sqrt{E_a^2 - m_{a,\ \rm s}^2 - \vec{p}_{a,\perp}^{\,2}} - \sqrt{E_b^2 - m_{b,\ \rm h}^2 - \vec{p}_{b,\perp}^{\,2}}$$

$$E_a^2 \sim p_{a,z,\ \rm s}^2 \sim \gamma^2 T^2 \gg m_{a,\ \rm s}^2, m_{b,\ \rm h}^2, \vec{p}_{a,\perp}^{\,2} \Longrightarrow \Delta p_{\rm wall} \approx \frac{m_{b,\ \rm h}^2 - m_{a,\ \rm s}^2}{2E_a} = \frac{m_{b,\ \rm h}^2 - m_{a,\ \rm s}^2}{2\gamma T}$$

$$\mathcal{P}_{1\to 1} \sim \frac{[\text{ force }]}{[\text{ area }]} \sim \frac{\Delta[\text{ momentum }]}{[\text{ area }]\times[\text{ time }]} \sim [\text{ flux }] \times \Delta[\text{ momentum }]$$

$$\sim \gamma T^3 \times \frac{\Delta m^2}{2\gamma T} = \gamma^0 T^2 \Delta m^2$$

1-to-1 calculation

Arnold (1993)

Bodeker & Moore (2009)

- 1-to-1: no flavour change

$$\mathcal{P}_{1\to1} = \sum_a \int \mathrm{d}\mathcal{F}_a \sum_b \int \mathrm{d}\mathbb{P}_{a\to b} \Delta p_z \left(1 \pm f_b\right)$$

$$\mathrm{d}\mathbb{P}_{a\to b} = \frac{\mathrm{d}^3\vec{p}_b}{(2\pi)^3}\frac{1}{2E_b} \times (2\pi)^3 \delta^2\left(\vec{p}_{a,\perp} - \vec{p}_{b,\perp}\right)\delta\left(E_a - E_b\right)\left(2p_{b,z},\ \mathrm{h}\right)\delta_{ab}$$

- Integrate over phase space of “b” noting that $\dfrac{d^3p_b}{(2\pi)^3 2E_b} = \dfrac{d^2\vec{p}_{b,\perp}}{(2\pi)^3}\dfrac{dE_b}{2E_b}\dfrac{E_b}{p_{b,z}}$

$$\mathcal{P}_{1\to1} \approx \sum_a \nu_a \frac{\mathrm{T}^2}{4\pi^2}\left(m^2_{b,h} - m^2_{a,s}\right)$$

- $\mathscr{P} \sim \propto \gamma^0 \Delta m^2 T^2$

$$\mathcal{P}_{\text{vacuum}} > \mathcal{P}_{\text{thermal}} \quad \text{or} \quad -\Delta V_{T=0} \equiv V_{T=0}\big|_{\text{out}} - V_{T=0}\big|_{\text{in}} > \mathcal{P}_{1\to1}$$

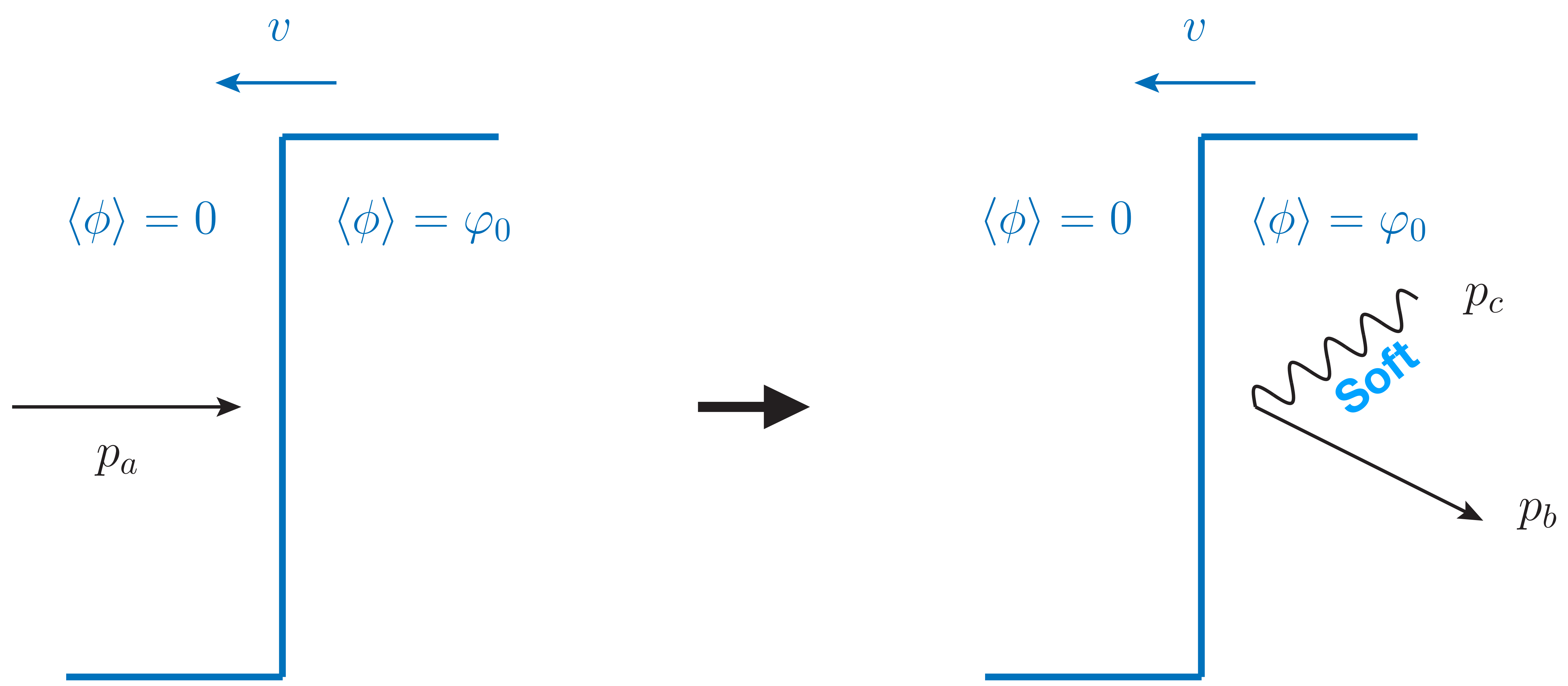
- Bubble can “runaway”

Some eight years later ….

1-to-2 pressure calculation for relativistic bubble walls

1-to-2 calculation

Bodeker & Moore (2017)



Sorry!

$p_c \leftrightarrow k$

Incident particle's energy → mass second particle + transverse momentum

$\Delta p_{1\to1} < \Delta p_{1\to2}$ unless $p_T = 0$ and $m_c = 0$

Kinematics

$$\vec{p}_a = \vec{p}_{a,\perp} + p_{a,z,s}\hat{\boldsymbol{z}} \quad \text{and} \quad E_a = \sqrt{|\vec{p}_{a,\perp}|^2 + p_{a,z,s}^2 + m_a^2}$$
$$\vec{p}_b = \vec{p}_{b,\perp} + p_{b,z,s}\hat{\boldsymbol{z}} \quad \text{and} \quad E_b = \sqrt{|\vec{p}_{b,\perp}|^2 + p_{b,z,s}^2 + m_b^2}$$
$$\vec{p}_c = \vec{p}_{c,\perp} + p_{c,z,s}\hat{\boldsymbol{z}} \quad \text{and} \quad E_c = \sqrt{|\vec{p}_{c,\perp}|^2 + p_{c,z,s}^2 + m_c^2}$$

The transverse component of momentum is conserved, implying

$$\vec{p}_{a,\perp} = \vec{p}_{b,\perp} + \vec{p}_{c,\perp}$$

Energy is also conserved during the scattering

$$E_a = E_b + E_c$$

1-to-2 calculation

Bodeker & Moore (2017)

$$\mathcal{P}_{1\to 2} = \sum_{a,bc} \nu_a \int [dp_a]\,[dp_b]\,[dp_c]\, f\left(p_a, p_b, p_c\right) \Delta p_z (2\pi)^3 \delta^2\left(\vec{p}_{a,\perp} - \vec{p}_{c,\perp} - \vec{p}_{b,\perp}\right) \delta\left(p_a^0 - p_c^0 - p_b^0\right) |\mathcal{M}|^2$$

Integrate over p_b using $\dfrac{d^3 p_b}{(2\pi)^3 2p_b^0} = \dfrac{d^2\overrightarrow{p}_{b,\perp}}{(2\pi)^3} \dfrac{dp_b^0}{2p_b^0} \dfrac{p_b^0}{p_{b,z}}$

Combination of PS + “observable”

$$\mathcal{P}_{1\to 2} = \sum_{abc} \nu_a \int \frac{d^3 p_a}{(2\pi)^3 \left(2p_a^0\right)} \int \frac{d^2 \vec{p}_{c,\perp}}{(2\pi)^2} \frac{dp_c^0}{(2\pi) 2p_c^0} f_{p,a} \left[1 \pm f_{p,c}\right] \left[1 \pm f_{p,b}\right] \left(p_{a,z,s} - p_{b,z,h} - p_{c,z,h}\right) \frac{1}{2p_{b,z}} \frac{p_c^0}{p_{c,z}} |\mathcal{M}|^2$$

B&M region of interest:

Ingoing hard	$\boldsymbol{p}_{a,\perp} \sim T$	$p_{a,z,s} \sim \gamma_w T\,,\ E_a \sim \gamma_w T$	$m_{a,s}, m_{a,h} \ll \gamma_w T$
Outgoing hard	$\boldsymbol{p}_{b,\perp} \sim \max\left[T, m_c\right]$	$p_{b,z,s} \sim \gamma_w T\,,\ E_b \sim \gamma_w T$	$m_{b,s}, m_{b,h} \ll \gamma_w T$
Outgoing soft	$\boldsymbol{p}_{c,\perp} \sim \max\left[T, m_c\right]$	$p_{c,z,s} \sim m_c\,,\ E_c \sim \max\left[T, m_c\right]$	$m_{c,s}, m_{c,h} \ll \gamma_w T$

1-to-2 calculation

Bodeker & Moore (2017)

$$\mathcal{P}_{1\to 2} = \sum_{abc} \nu_a \int \frac{d^3 p_a}{(2\pi)^3\,(2p_a^0)} \int \frac{d^2 \vec{p}_{c,\perp}}{(2\pi)^2} \frac{dp_c^0}{(2\pi) 2p_c^0} f_{p,a}\,[1 \pm f_{p,c}]\,[1 \pm f_{p,b}]\,(p_{a,z,s} - p_{b,z,h} - p_{c,z,h})\,\frac{1}{2p_{b,z}}\,\frac{p_c^0}{p_{c,z}} |\mathcal{M}|^2$$

B&M region of interest:

$$p_{c,z} = \sqrt{1 - \underbrace{2\,\frac{m_c^2(z) + \vec{p}^{\,2}_{c,\perp}}{2\left(p_c^0\right)^2}}_{\epsilon}} \simeq p_c^0(1-\epsilon)$$

ϵ parametrises collinearity "c" also the region of PS $\vec{p}^{\,2}_{c,\perp} \sim m^2_{c,s} \ll p_{c,z,s} m_{c,s}$

$$p_{b,z} = \sqrt{p_b^{0^2} - \vec{p}^{\,2}_{\perp,b} - m_b^2(z)} \approx p_a^0(1-x)$$

x parametrises softness "c"

$$x = p_c^0 / p_a^0$$

$$\left(p_{a,z,s} - p_{c,z,h} - p_{b,z,h}\right) \frac{1}{2p_{b,z}} \frac{p^0_{c,z}}{p_{c,z}} \approx \underbrace{\frac{1}{2p^0_a} \frac{m_c^2(z) + \vec{p}^{\,2}_{c,\perp}}{2p^0_c}}_{\sim x\epsilon + \mathcal{O}(\epsilon^2 x) + \ldots}$$

B&M 1-to-2 master equation:

$$\mathcal{P}_{1\to 2} = \sum \nu_a \int \frac{d^3 p_a}{(2\pi)^3 \left(2p^0_\alpha\right)^2} \int \frac{d^2 \vec{p}_{c,\perp}}{(2\pi)^2} \frac{dp^0_c}{(2\pi) 2p^0_c} f_{p_a} \left[1 \pm f_{p_c}\right] \left[1 \pm f_{p_b}\right] \frac{m^2_{c,h}(z) + \vec{p}^{\,2}_{c,\perp}}{2p^0_c} |\mathcal{M}|^2$$

Still need to determine matrix element squared

Bodeker & Moore (2017)

Differential probability:

$$\mathrm{d}P_{a\to bc} = \frac{\mathrm{d}^3 p_b}{(2\pi)^3}\frac{1}{2E_b}\frac{\mathrm{d}^3 p_c}{(2\pi)^3}\frac{1}{2E_c}\left|\langle p_b p_c|\mathcal{T}|\phi_a\left(p_a\right)\rangle\right|^2$$

The bubble wall is invariant in time and the transverse directions

$$\langle p_c p_b|\mathcal{T}|p_a\rangle = \int d^4x\,\langle p_c p_b\,|\mathcal{H}_{\mathbf{int}}|\,p_a\rangle = (2\pi)^3\delta^2\left(p_{a,\perp} - p_{c,\perp} - p_{b,\perp}\right)\delta\left(p^0 - k^0 - q^0\right)\mathcal{M}$$

$$\mathcal{M} \equiv \int dz\,\chi^*_{p_c}(z)\chi^*_{p_b}(z)V(z)\chi_{p_a}(z)$$

Mode functions are treated in the WKB approximation:

$$\chi_{p_c}(z) \simeq \sqrt{\frac{p_{c,z,s}}{p_{c,z}(z)}}\exp\left(i\int_0^z p_{c,z}\left(z'\right)dz'\right)$$

Mode function quick summary

Bodeker & Moore (2017)

scalar interacting with wall

$$\mathcal{L} = \sum_{f=a,b,c} \left[\frac{1}{2} \left(\partial_\mu \phi_f\right)^2 - \frac{1}{2} m_f^2(z)\phi_f^2 \right]$$

Mass varying in z parametrises spatial inhomegenity

KG field equation

$$\Box\phi_f + m_f^2(z)\phi_f = 0$$

solve with an **homogeneous** mass parameter, solutions can be can labeled by a 3-vector $\overrightarrow{p}$

$$\chi_f(\vec{p}, x) = e^{-iE_f(\vec{p})t} e^{i\vec{p}\cdot\vec{x}} \quad \text{with} \quad E_f(\vec{p}) \equiv \sqrt{|\vec{p}|^2 + m_f^2}$$

$$\phi_f(x) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \tilde{\phi}_f(\vec{p}) \chi_f(\vec{p}, x)$$

Mode function quick summary

Bodeker & Moore (2017)

But we have **inhomogeneous** mass term, make ansatz for solution to KG equation

$$\phi_f(x) = \int \frac{\mathrm{d}^2\vec{p}_\perp}{(2\pi)^2} \frac{\mathrm{d}p_{z,\,\mathrm{s}}}{(2\pi)} \tilde{\phi}_f\left(\vec{p}_\perp, p_{z,\,\mathrm{s}}\right) \chi_f\left(p_{z,\,\mathrm{s}}, z\right) e^{-iE_f(\vec{p}_\perp, p_{z,\,\mathrm{s}})t} e^{i\vec{p}_\perp\cdot\vec{x}_\perp}$$

Sub → KG → WKB solution for a particle with inhomogeneous mass

$$\chi_f\left(p_{z,\,\mathrm{s}}, z\right) \approx \sqrt{\frac{p_{z,\,\mathrm{s}}}{\tilde{p}_z(z)}} \exp\left(i\int_0^z \mathrm{d}z' \tilde{p}_{f,z}\left(z'\right)\right)$$

Amplitude ~ 1 Phase

Analogous 1D scattering off a potential well. Normally there would be a wave function with a negative phase (reflected) Here all particles transmitted

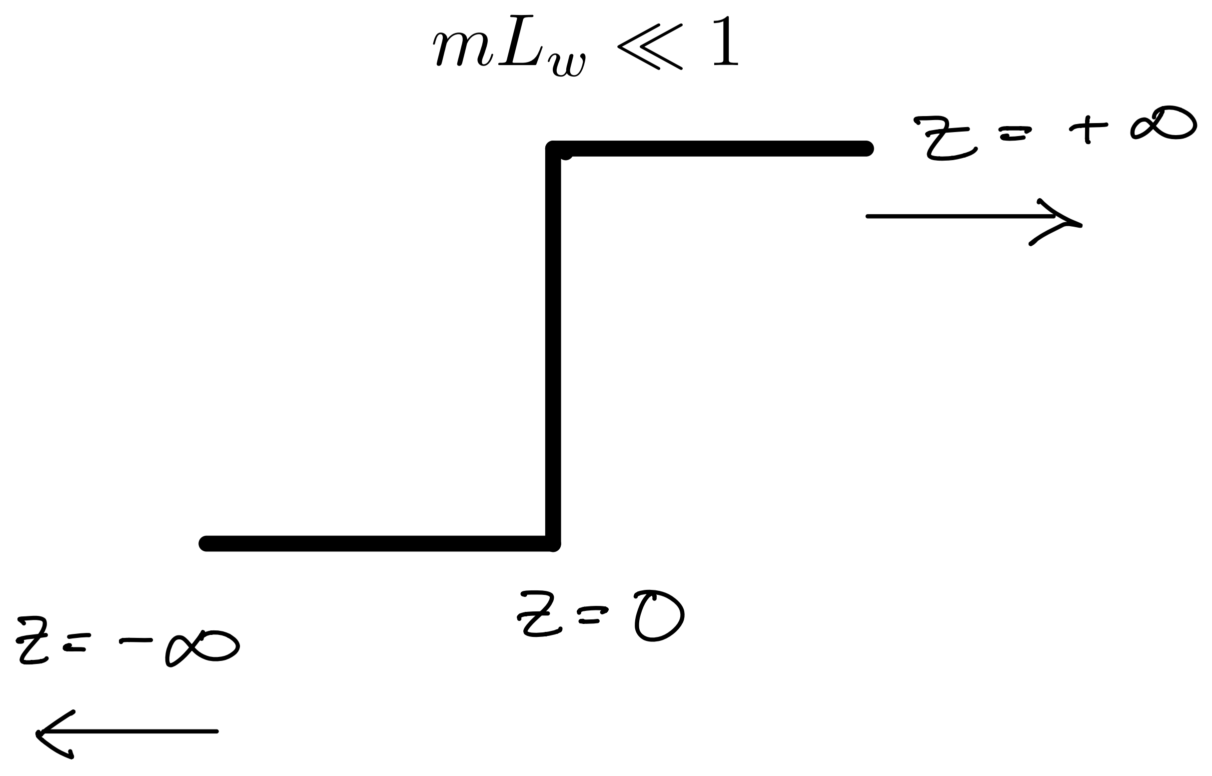
Mode functions don't tell us anything about the nature of the interaction

$V(z)$ the contraction of the interaction Hamiltonian density with all other state information $\equiv$ interaction matrix element if we were considering simple plane wave states.

$$\mathcal{M} \equiv \int dz \chi_k^*(z) \chi_q^*(z) V(z) \chi_p(z)$$

$$\mathcal{M} = V_\mathrm{s} \int_{-\infty}^{0} dz \exp\left[iz \frac{A_\mathrm{s}}{2p^0}\right] + V_\mathrm{h} \int_{0}^{\infty} dz \exp\left[iz \frac{A_\mathrm{h}}{2p^0}\right] = 2ip^0 \left(\frac{V_\mathrm{h}}{A_\mathrm{h}} - \frac{V_\mathrm{s}}{A_\mathrm{s}}\right)$$

$$A_s = E_A \left(p_{a,z,s} - p_{b,z,s} - p_{c,z,s}\right) \quad A_h = E_a \left(p_{a,z,h} - p_{b,z,h} - p_{c,z,h}\right)$$



A's resemble propagators, but they only propagate in the z-direction!

Vertex Function

Bodeker & Moore (2017)

$$|\mathcal{M}|^2 \simeq 4p_0^2|V|^2\frac{(A_\mathrm{h}-A_\mathrm{s})^2}{A_\mathrm{h}^2 A_\mathrm{s}^2}$$

$a(p) \to b(k)c(p-k)$	V^2
$S \to V_T S$ $F \to V_T F$ $V \to V_T V$	$4g^2 C_2[R]\frac{1}{x^2}k_\perp^2$
$S \to V_L S$ $F \to V_L F$ $V \to V_L V$	$4g^2 C_2[R]\frac{1}{x^2}m^2$
$F \to F V_T$	$2g^2 C_2[R]\frac{1}{x}\left(k_\perp^2+m_b^2\right)$
$V \to F F$	$2g^2 T[R]\frac{1}{x}\left(k_\perp^2+m_b^2\right)$
$S \to S V_T$	$4g^2 C_2[R]k_\perp^2$
$F \to S F$	$y^2\left(k_\perp^2+4m_a^2\right)$
$S \to S S$	$\lambda^2\varphi^2$

$$k_\perp \equiv p_{c,\perp}$$

These are splitting functions up to the normalisation $P_{b\leftarrow a}(x) = |V|^2 x(1-x)/16\pi^2 k_\perp^2$

Quick Recap on splitting functions

Altarelli & Parisi (1977)

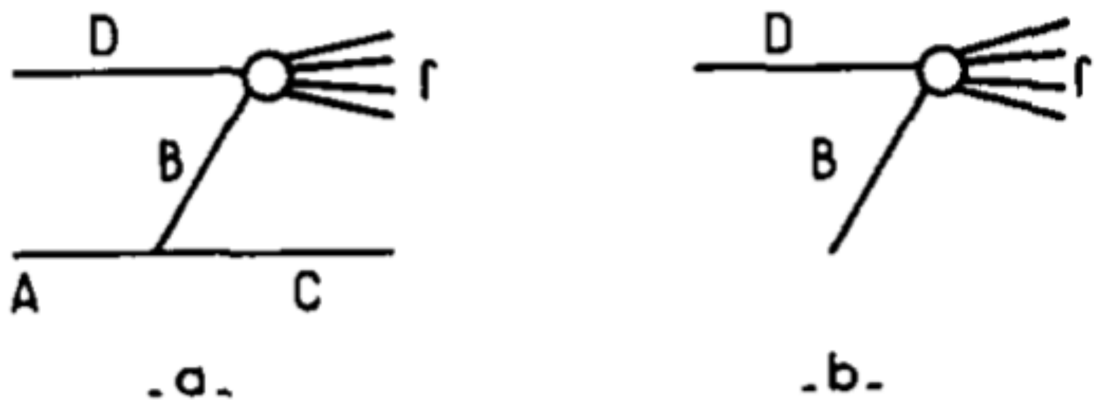


Fig. 1. (a) Contribution of the B intermediate state to the process A + D → C + f. (b) The process B + D → f.

$$\mathrm{d}\sigma_a = \mathrm{d}\mathcal{P}_{\mathrm{BA}}(z)\mathrm{d}z\ \mathrm{d}\sigma_b$$

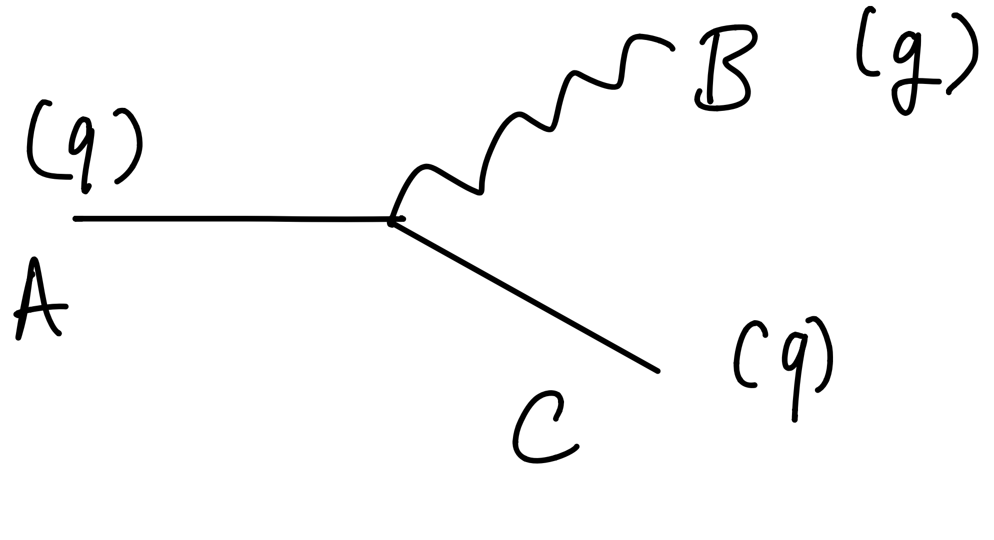
$$k_A = (P, P, \mathbf{0})$$
$$k_B = \left(zP + \tfrac{p_\perp^2}{2zP}, zP, \mathbf{p}_\perp\right)$$
$$k_C = \left((1-z)P + \tfrac{p_\perp^2}{2(1-z)P}, (1-z)P, -\mathbf{p}_\perp\right)$$

$$P_{BA}(z) = \frac{1}{2}z(1-z)\overline{\sum_{\text{spins}}}\frac{|V_{A\to B+C}|^2}{p_\perp^2}$$

$$\overline{\sum_{\text{spin}}}|V_{A\to B+C}|^2 = \frac{1}{2}C_2(R)\mathrm{Tr}(\not{k}_C\gamma_\mu\not{k}_A\gamma_\nu)\overline{\sum_{\text{pol}}}\epsilon^{*\mu}\epsilon_\mu$$

$$\overline{\sum_{\text{spin}}}|V_{A\to B+C}|^2 = \frac{2p_\perp^2}{z(1-z)}\frac{1+(1-z)^2}{z}C_2(R)$$

Treat z as Small parameter Light like Axial gauge



$$p_{f,z,\ \mathrm{s}} \approx E_f - \frac{|\vec{p}_{f,\perp}|^2 + m_{f,\ \mathrm{s}}^2}{2E_f}$$

$$p_{f,z,\ \mathrm{h}} \approx E_f - \frac{|\vec{p}_{f,\perp}|^2 + m_{f,\ \mathrm{h}}^2}{2E_f}$$

$$A_\mathrm{s} \approx 2E_a \times \left(-\frac{|\vec{p}_{a,\perp}|^2 + m_{a,\ \mathrm{s}}^2}{2E_a} + \frac{|\vec{p}_{b,\perp}|^2 + m_{b,\ \mathrm{s}}^2}{2E_b} + \frac{|\vec{p}_{c,\perp}|^2 + m_{c,\ \mathrm{s}}^2}{2E_c} \right) \approx \frac{|\vec{p}_{c,\perp}|^2 + m_{c,\ \mathrm{s}}^2}{E_c/E_a}$$

$$A_\mathrm{h} \approx 2E_a \times \left(-\frac{|\vec{p}_{a,\perp}|^2 + m_{a,\ \mathrm{h}}^2}{2E_a} + \frac{|\vec{p}_{b,\perp}|^2 + m_{b,\ \mathrm{h}}^2}{2E_b} + \frac{|\vec{p}_{c,\perp}|^2 + m_{c,\ \mathrm{h}}^2}{2E_c} \right) \approx \frac{|\vec{p}_{c,\perp}|^2 + m_{c,\ \mathrm{h}}^2}{E_c/E_a}$$

$$|\mathcal{M}|^2 \approx 4E_a^2 \, |V_\mathrm{s}|^2 \, \frac{(A_\mathrm{s} - A_\mathrm{h})^2}{A_\mathrm{s}^2 A_\mathrm{h}^2}$$

$$\approx 4E_a^2 \frac{|V_\mathrm{s}|^2}{(E_c/E_a)^{-2}} \frac{\left(m_{c,\ \mathrm{h}}^2 - m_{c,\ \mathrm{s}}^2\right)^2}{\left(|\vec{p}_{c,\perp}|^2 + m_{c,\ \mathrm{s}}^2\right)^2 \left(|\vec{p}_{c,\perp}|^2 + m_{c,\ \mathrm{h}}^2\right)^2}$$

Lets keep track of what cancels where…

$$|\mathcal{M}|^2 \approx 4E_a^2 \left|V_{\rm s}\right|^2 x^2 \frac{m_{c,\ \rm h}^4}{\left|\vec{p}_{c,\perp}\right|^4 \left(\left|\vec{p}_{c,\perp}\right|^2 + m_{c,\ \rm h}^2\right)^2}$$

$V_{F \to V_T F}$

$$\approx 4E_a^2 4\ {\rm g}^2 C_2[R] \frac{\left|\vec{p}_{c,\perp}\right|^2}{\cancel{x^2}} \cancel{x^2} \frac{m_{c,\ \rm h}^4}{\left|\vec{p}_{c,\perp}\right|^4 \left(\left|\vec{p}_{c,\perp}\right|^2 + m_{c,\ \rm h}^2\right)^2}$$

$$\approx 4E_a^2 4\ {\rm g}^2 C_2[R] \frac{m_{c,\ \rm h}^4}{\left|\vec{p}_{c,\perp}\right|^2 \left(\left|\vec{p}_{c,\perp}\right|^2 + m_{c,\ \rm h}^2\right)^2}$$

In the pressure expression, there is the "observable" which is the momentum transfer (from plasma to wall) in the z-direction:

$$|\mathcal{M}|^2 \times \Delta p_z \approx 16E_a^2\ {\rm g}^2 C_2[R] \frac{m_{c,\ \rm h}^4}{\left|\vec{p}_{c,\perp}\right|^2 \left(\left|\vec{p}_{c,\perp}\right|^2 + m_{c,\ \rm h}^2\right)^2} \times \frac{m_{c,h}^2 + p_{c,\perp}^2}{2p_c^0}$$

$$\approx 8E_a^2\ {\rm g}^2 C_2[R] \frac{m_{c,\ \rm h}^4}{\left|\vec{p}_{c,\perp}\right|^2 \left(\left|\vec{p}_{c,\perp}\right|^2 + m_{c,\ \rm h}^2\right)} \times \frac{1}{p_c^0}$$

Two pieces left: the integration of PS of incoming “a” gives the flux. We also need to integrate over phase space of particle “c” our soft emission

$$\int_{g^2T^2}^{m^2} \frac{dp_{c,\perp}^2}{(2\pi)^2 \left|\vec{p}_{c,\perp}\right|^2 \left(\left|\vec{p}_{c,\perp}\right|^2 + m_{c,\ \mathrm{h}}^2\right)} \approx \frac{1}{24\pi m^2}$$

$$\int \frac{dp_c^0}{{p_c^0}^2} \approx \frac{1}{m}$$

B&M assume $g^2T^2 \ll m^2$ i.e. supercooled PT. Drop this assumption the thermal mass would cut of the integral and you’d get some (possibly large) $\log\left(\frac{m_{c,h}^2}{m_{c,s}^2}\right)$

$$\mathcal{P}_{1\to 2} \sim \gamma T^3 \frac{1}{m^2} \frac{1}{m} m^4$$

Transverse momentum integration → $\frac{1}{m^2}$

From mode function → m^4

Lorentz contracted flux → γT^3

Energy integration → $\frac{1}{m}$

$$\mathcal{P}_{1\to2} \sim m\gamma T^3$$

- $\mathscr{P}_{1\to2} \propto \gamma^2$ while $\mathscr{P}_{1\to1} \propto \gamma$. Since, vacuum pressure does not grow in λ, a terminal velocity will be reached.
- $\mathscr{P} \propto m^2 \implies$ no phase change pressure goes to zero.
- $\mathscr{M}$: WKB and vertex. The vertex part is dominated in the soft regime.
- B&M cut off the $\overrightarrow{k}_\perp$ and k^0 integration by gauge boson mass.
- This interaction looks like a collider/scattering experiment where the collision occurs between the ingoing particle and the wall $\implies$ the centre of mass energy will be large $\implies$ many soft emission.

1-to-n pressure calculation for relativistic bubble walls

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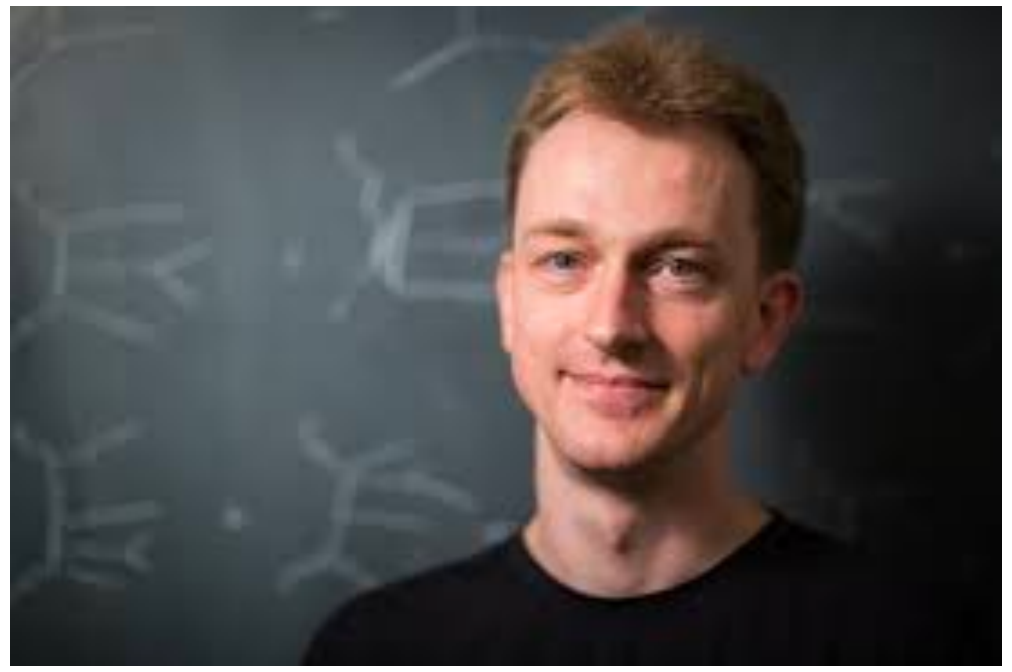
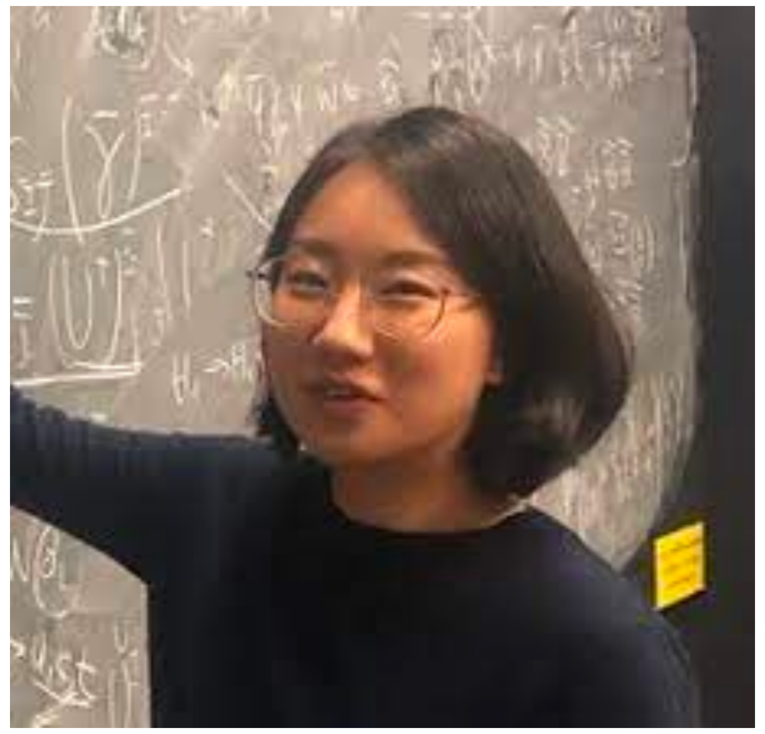


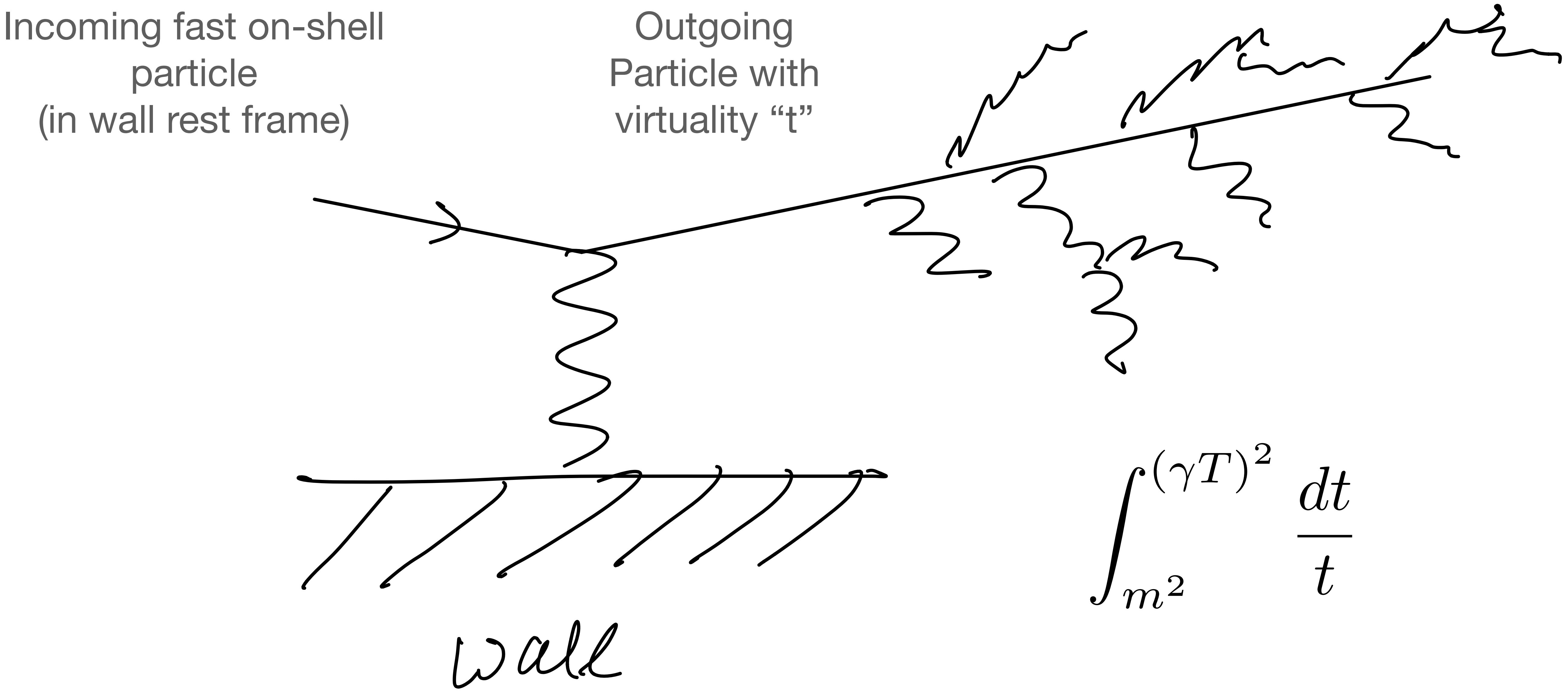
Stefan Höche

Jonathan Kozaczuk

Andrew Long



Yikun Wang



Splitting functions typically used to resum soft radiation.

Only have a few scale in the problem. Incoming particle energy γT (UV scale) and bare mass/thermal mass particles in the bubble (IR scale).

Reformulation of matrix element

Recall B&M used the mode functions from solving KG equation:

$$A_{\rm s} = -2E_a\left(p_{a,z,{\rm s}} - p_{b,z,{\rm s}} - p_{c,z,{\rm s}}\right) \approx \vec{p}^{\,2}_{a,\perp} + m^2_{a,{\rm s}} - \frac{\vec{p}^{\,2}_{b,\perp} + m^2_{b,{\rm s}}}{E_b/E_a} - \frac{\vec{p}^{\,2}_{c,\perp} + m^2_{c,{\rm s}}}{E_c/E_a} \xrightarrow{x\ll 1} -\frac{\vec{k}^{\,2}_{\perp} + m^2_{c,{\rm s}}}{x}$$

$$A_{\rm h} = -2E_a\left(p_{a,z,{\rm h}} - p_{b,z,{\rm h}} - p_{c,z,{\rm h}}\right) \approx \vec{p}^{\,2}_{a,\perp} + m^2_{a,{\rm h}} - \frac{\vec{p}^{\,2}_{b,\perp} + m^2_{b,{\rm h}}}{E_b/E_a} - \frac{\vec{p}^{\,2}_{c,\perp} + m^2_{c,{\rm h}}}{E_c/E_a} \xrightarrow{x\ll 1} -\frac{\vec{k}^{\,2}_{\perp} + m^2_{c,{\rm h}}}{x}\,.$$

$p_c \leftrightarrow k$

We instead use:

$$A_{\rm s} = -2p_{a,{\rm s}}p_{c,{\rm s}} \approx 2\vec{p}_{a,\perp}\cdot\vec{p}_{c,\perp} - \frac{\vec{p}^{\,2}_{a,\perp} + m^2_{a,{\rm s}}}{E_a/E_c} - \frac{\vec{p}^{\,2}_{c,\perp} + m^2_{c,{\rm s}}}{E_c/E_a} \xrightarrow{x\ll 1} -\frac{\vec{k}^{\,2}_{\perp} + m^2_{c,{\rm s}}}{x}$$

$$A_{\rm h} = -2p_{b,{\rm h}}p_{c,{\rm h}} \approx 2\vec{p}_{b,\perp}\cdot\vec{p}_{c,\perp} - \frac{\vec{p}^{\,2}_{b,\perp} + m^2_{b,{\rm h}}}{E_b/E_c} - \frac{\vec{p}^{\,2}_{c,\perp} + m^2_{c,{\rm h}}}{E_c/E_b} \xrightarrow{x\ll 1} -\frac{\vec{k}^{\,2}_{\perp} + m^2_{c,{\rm h}}}{x}$$

Sudakov parametrisation

$$p_a^\mu \approx \left(E_a, \vec{0}, E_a\left(1 - \frac{m_a^2}{2E_a^2}\right)\right)$$

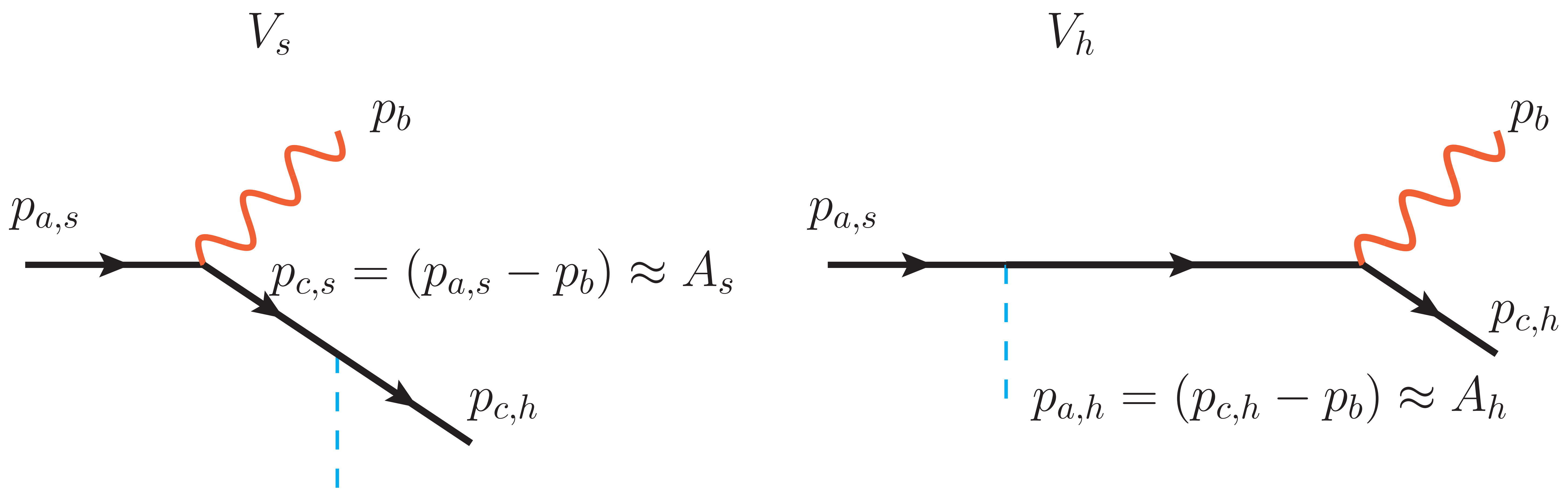
$$p_b^\mu \approx \left((1-x)E_a, \vec{k}_\perp, (1-x)E_a\left(1 - \frac{\vec{k}_\perp^2 + m_b^2}{2(1-x)^2E_a^2}\right)\right)$$

$$p_c^\mu \approx \left(xE_a, \vec{k}_\perp, xE_a\left(1 - \frac{\vec{k}_\perp^2 + m_c^2}{2x^2E_a^2}\right)\right)$$

Note that they are **identical in the soft-limit.**

$$A_{\rm s} = -2p_{a,{\rm s}}p_{c,{\rm s}} \approx 2\vec{p}_{a,\perp}\cdot\vec{p}_{c,\perp} - \frac{\vec{p}_{a,\perp}^{\,2} + m_{a,{\rm s}}^2}{E_a/E_c} - \frac{\vec{p}_{c,\perp}^{\,2} + m_{c,{\rm s}}^2}{E_c/E_a} \xrightarrow{x\ll 1} -\frac{\vec{k}_\perp^{\,2} + m_{c,{\rm s}}^2}{x}$$

$$A_{\rm h} = -2p_{b,{\rm h}}p_{c,{\rm h}} \approx 2\vec{p}_{b,\perp}\cdot\vec{p}_{c,\perp} - \frac{\vec{p}_{b,\perp}^{\,2} + m_{b,{\rm h}}^2}{E_b/E_c} - \frac{\vec{p}_{c,\perp}^{\,2} + m_{c,{\rm h}}^2}{E_c/E_b} \xrightarrow{x\ll 1} -\frac{\vec{k}_\perp^{\,2} + m_{c,{\rm h}}^2}{x}$$



Using the full propagator expression (same as B&M in soft limit) we get a gauge invariant expression.

If you choose a different gauge (such as Feynman gauge $\zeta = 0$) in the previous "A" expressions you will find that the matrix element doesn't match axial gauge. The gauge choice comes in the polarisation sum of the gauge boson.

$$\sum_{\kappa=\pm} \epsilon(p)^{\kappa}_{\mu} \epsilon^{\kappa}_{\nu}(p)^* = -g_{\mu\nu} + \zeta g_{\mu\rho} g_{\nu\sigma} \left(\frac{n^\rho p_b^\sigma + n^\sigma p_b^\rho}{p_b \cdot n} - n \cdot n \frac{p_b^\rho p_b^\sigma}{(p_b \cdot n)^2} \right)$$

In our expressions, lightlike axial gauge ($\zeta = 1$) with $n = p_{b,h}$ simplifies the matrix element*

$$|V_\mathrm{h}|^2 = V_\mathrm{h}^* V_\mathrm{s} = V_\mathrm{s}^* V_\mathrm{h} = 0$$

$$|V_\mathrm{s}|^2 = 4|g|^2 \left(\frac{2(p_{a,\mathrm{s}} p_{b,\mathrm{h}})(p_{a,\mathrm{s}} p_c)}{p_{b,\mathrm{h}} p_c} - \frac{p_{b,\mathrm{h}}^2 (p_{a,\mathrm{s}} p_c)^2}{(p_{b,\mathrm{h}} p_c)^2} - p_{a,\mathrm{s}}^2 \right)$$

$$\approx 4|g|^2 \, \vec{k}_\perp^{\,2} \left(\frac{\vec{k}_\perp^{\,2} + x(m_{b,\mathrm{h}}^2 - (1-x) m_{a,\mathrm{s}}^2)}{\vec{k}_\perp^{\,2} + x^2 m_{b,\mathrm{h}}^2} \right)^2 \xrightarrow[\vec{k}_\perp^2 \ll x^2 m_{b,\mathrm{h}}^2]{m_{a,\mathrm{s}}^2 \ll m_{b,\mathrm{h}}^2} 4|g|^2 \frac{\vec{k}_\perp^{\,2}}{x^2} \,.$$

Ultracollinear limit

*analogue of $\gamma^* \to q\bar{q}g$ LLA gauge switches off interference term. Total amplitude GI but sub amplitude need not be

$$\left|\mathcal{M}^{(0)}_{a\to bc}\right|^2 = 4E_a^2 \frac{|V_s|^2}{A_s^2}\bigg|_{n^\mu = p^\mu_{b,h}} \xrightarrow{m^2_{a,\ \mathrm{s}} \ll \vec{k}^2_\perp, m^2_{b,\ \mathrm{h}}} 8E_a^2 |g|^2 \frac{2x^2}{\vec{k}^2_\perp}\left(\frac{\vec{k}^2_\perp + x m^2_{b,\ \mathrm{h}}}{\vec{k}^2_\perp + x^2 m^2_{b,\ \mathrm{h}}}\right)^2$$

$$\left|\mathcal{M}^{(0)}_{a\to bc}\right|^2 = 4E_a^2 \frac{|V_s|^2}{A_s^2}\bigg|_{n^\mu = p^\mu_{b,h}} \xrightarrow{m^2_{a,\ \mathrm{s}}, m^2_{b,\ \mathrm{h}} \ll k^2_\perp} 8E_a^2 |g|^2 \frac{2x^2}{\vec{k}^2_\perp}$$

$$\frac{1}{\left|\mathcal{M}^{(0)}_{a\to b}\right|^2}\int \frac{\mathrm{d}^3\vec{p}_c}{(2\pi)^3 2E_c}\left|\mathcal{M}^{(0)}_{a\to bc}\right|^2 \approx \begin{cases} \dfrac{\alpha}{2\pi}\displaystyle\int \frac{\mathrm{d}\vec{k}^2_\perp}{\vec{k}^2_\perp} C_{abc} \log \frac{m^2_{b,\ \mathrm{h}}}{\vec{k}^2_\perp} \ \text{if} \quad m^2_{a,\ \mathrm{s}} \ll k^2_\perp \ll m^2_{b,\ \mathrm{h}} \\ \dfrac{\alpha}{2\pi}\displaystyle\int \frac{\mathrm{d}\vec{k}^2_\perp}{\vec{k}^2_\perp} C_{abc} \ \text{if} \quad m^2_{a,\ \mathrm{s}}, m^2_{b,\ \mathrm{h}} \ll k^2_\perp \end{cases}$$

Small $k_\perp$ →

Large $k_\perp$ →

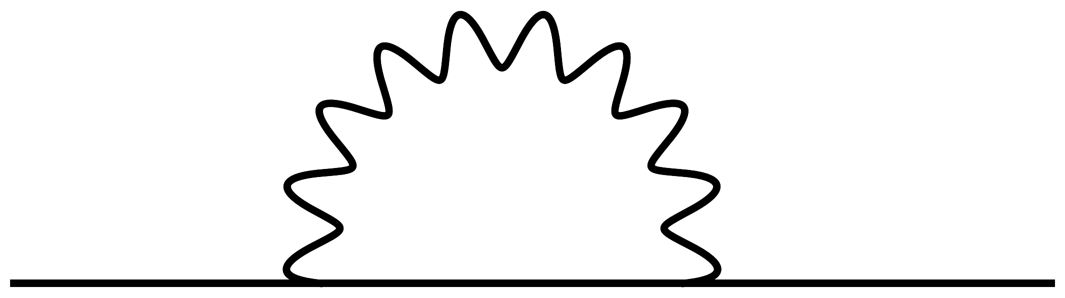
- In IR regime, logarithms can be large → invalidate fixed order calculation
- One thing to note, our ME squared does not have the Δm^2 suppression factor. As phase change is required but this can come from the hard outgoing fermion leg.

Our gauge invariant matrix element:

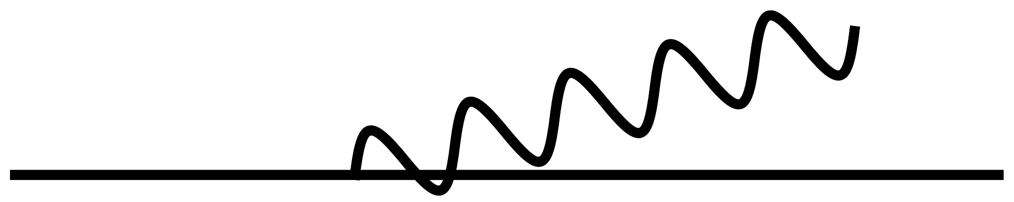
$$\left|\mathcal{M}^{(0)}_{a\to bc}\right|^2 = 4E_a^2|g|^2\left(\frac{2p_{a,s}p_{b,h}}{p_{a,s}p_c p_{b,h}p_c} - \frac{m_{a,s}^2}{\left(p_{a,s}p_c\right)^2} - \frac{m_{b,h}^2}{\left(p_{b,h}p_c\right)^2}\right)$$

First need to cancel poles: allows for the meaningful resummation. IR divergent parts in the real and virtual diagrams computed using dim reg

$$W^{(2)\mathrm{IR}}_{a\to b} = -\frac{\alpha}{\pi}C_{abc}\left(\frac{1}{2\varepsilon^2} - \frac{1}{2\varepsilon}\left(1+\log\frac{\left(2p_ap_b\right)^2}{\mu^2p_b^2}\right) + \frac{1}{4}\log^2\frac{\left(2p_ap_b\right)^2}{\mu^2p_b^2} - \frac{1}{2}\log^2\frac{2p_ap_b}{p_b^2} + \dots\right)$$

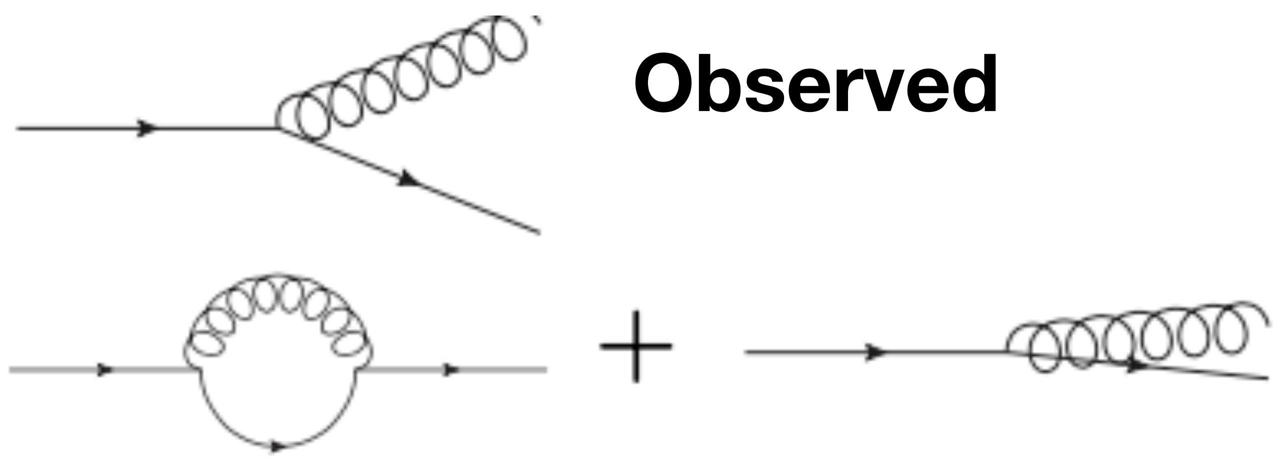


$$\int \mathrm{d}W^{2(1)\mathrm{IR}}_{a\to bc} = +\frac{\alpha}{\pi}C_{abc}\left(\frac{1}{2\varepsilon^2} - \frac{1}{2\varepsilon}\left(1+\log\frac{\left(2p_ap_b\right)^2}{\mu^2p_b^2}\right) + \frac{1}{4}\log^2\frac{\left(2p_ap_b\right)^2}{\mu^2p_b^2} - \frac{1}{2}\log^2\frac{2p_ap_b}{p_b^2} + \dots\right)$$



Radiative corrections as branching processes

Marchesini & Webber (1983)
Sjöstrand (1985)



Observed

Unobserved

Virtual correction + **Unresolved Emission**

$$P_{\text{no em}} + P_{em} = 1$$

Sudakov factor → no-emission probability

$$dN = -\lambda N dt$$

$N = \text{population}$

$\lambda = \text{decay constant}$

Survival probability at time "t": $e^{-\lambda t}$ where $t \leftrightarrow$ energy scale ($\log(1/\mathrm{v})$)

Change in population analogous to boson emission probability

$$\lambda N dt = \int [dk] M^2(k) \Theta(v - V(\{\tilde{p}\}, k))$$

$$[\text{ virt. } + \text{ unres. }] = e^{-\int [dk] M^2(k) \Theta(V(\{\tilde{p}\},k) - v)}$$

Probability of not emitting bosons above v

Analytic Resummation

Banfi, Salam & Zanderighi (2005)

$R(v)$ probability for decay $a \to bc$. For this splitting to produce momentum transfer of v we require that it did not produce a large momentum transfer before.

$$R(v) = \int [dk] \left|M^2(k)\right| \Theta[V(\{p\},k) - v]$$

$$V(p_a, p_b, p_c) = \frac{\Delta p_z}{\gamma T} \approx \frac{\vec{k}_\perp^2 / (2E_a^2)}{x(1-x)}$$

$$R_{abc}(V) = C_{abc}|g|^2 \int \frac{\mathrm{d}^3\vec{p}_c}{(2\pi)^3 2E_c} \left(\frac{2p_{b,\ \mathrm{h}} p_{a,\ \mathrm{s}}}{p_{a,\ \mathrm{s}} p_c p_{b,\ \mathrm{h}} p_c} + \mathcal{O}\left(\frac{m_{a,\ \mathrm{s}}^2}{\vec{k}_\perp^2}, \frac{m_{b,\ \mathrm{h}}^2}{\vec{k}_\perp^2} \right) \right) \Theta\left(V(p_a, p_b, p_c) - V\right) \Theta\left(p_{b,z,\ \mathrm{h}}\right) \Theta\left(p_{c,z,\ \mathrm{h}}\right)$$

Additional constraints on kinematics

Rewrite phase space and matrix element squared in terms of observable V

$$R_{abc}(V) = C_{abc} \frac{\alpha}{2\pi} \int_V^1 \frac{\mathrm{d}V'}{V'} \int_0^1 \mathrm{d}x 2x \Theta\left(\frac{1}{1+V'} - x \right) \Theta\left(x - \frac{V'}{1+V'} \right)$$

$$R_{abc}(V) = \frac{\alpha}{2\pi} C_{abc} \left(L + 2\log\left(1 + e^{-L}\right)\right) \text{ where } L = \log \frac{1}{V}$$

Single log as we focus on $\vec{k}_\perp^2 \gg m_{b,h}^2$ region

$$\Delta_a(V) = \exp\left\{-\sum_b R_{ab}(V)\right\}, \quad \text{where} \quad R_{ab}(V) = \sum_c R_{abc}(V)$$

$$\left\langle \frac{\Delta p_z}{\gamma T} \right\rangle = \int_0^1 \mathrm{d}V V \frac{\mathrm{d}}{\mathrm{d}V} \prod_{a \in \mathcal{S}} \Delta_a(V)$$

Average mom transfer per
incoming particle @ fixed coupling

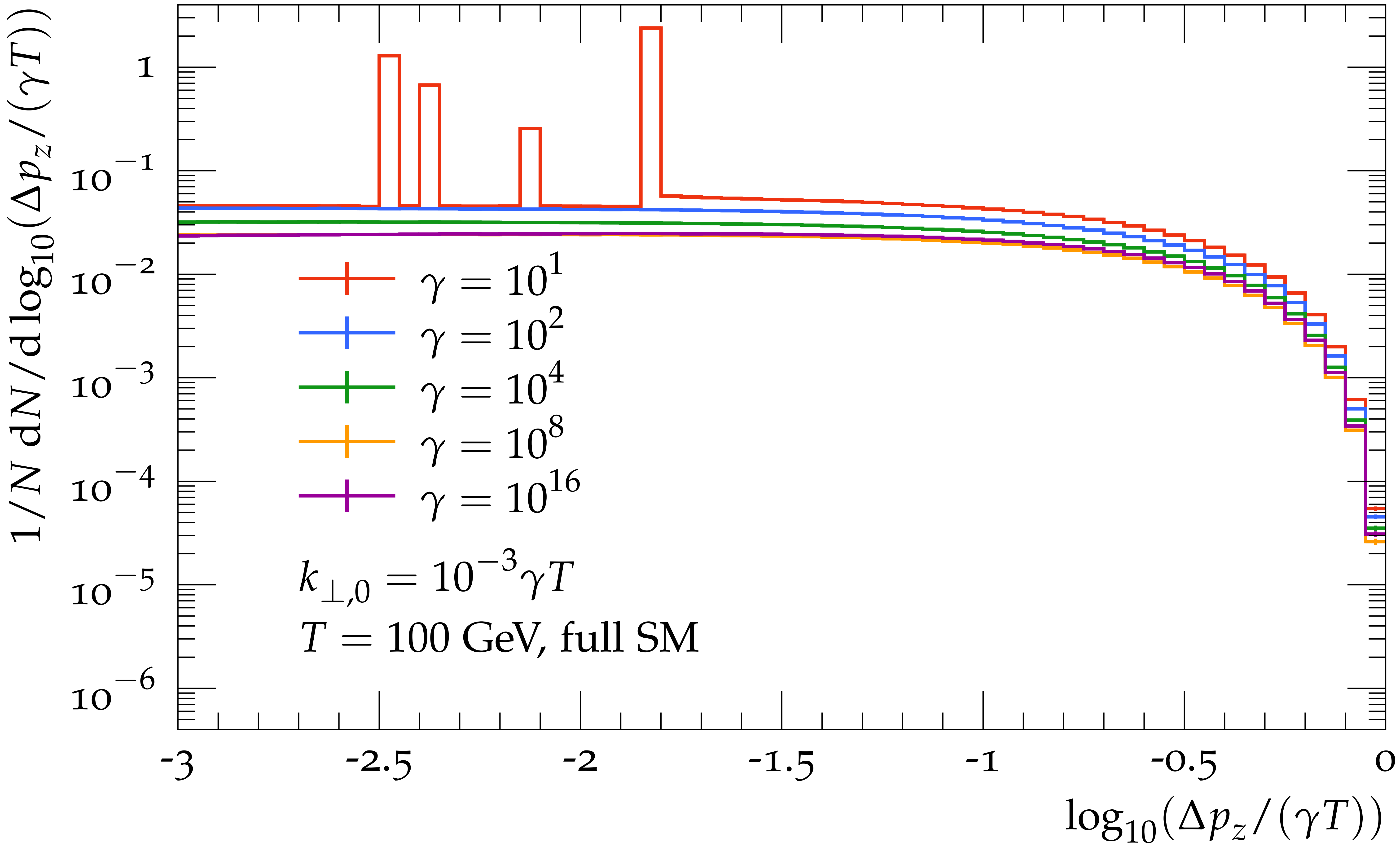
$$\zeta = \frac{(\alpha C)_\Sigma}{2\pi}$$

$$\left\langle \frac{\Delta p_z}{\gamma T} \right\rangle_{\mathrm{FC}} = \int_0^\infty \mathrm{d}L e^{-L} \frac{(\alpha C)_\Sigma}{2\pi} \frac{e^L - 1}{e^L + 1} \exp\left\{-\frac{(\alpha C)_\Sigma}{2\pi}\left(L + 2\log\left(1 + e^{-L}\right)\right)\right\} \approx \zeta(\log 4 - 1)$$

Importantly:

$$\langle \Delta p_z \rangle \sim \gamma T$$

Numerical Resummation



$$\left\langle \frac{\Delta p_z}{\gamma T} \right\rangle = 0.89(17)\,\% - 0.14(3)\,\%\log_{10}\gamma \implies P \propto \gamma^2 T^4$$

Summary

- 1st order EWPT is plausible and has many interesting physical consequences such as baryogenesis & GW production. Both quantitatively depend on the velocity of the bubble wall. Faster walls $\implies$ bigger waves!

- Bubble wall velocity is a force balancing exercise: pressure from Higgs potential versus frictional pressure from plasma.

- We reformulated the calculation of the latter in a GI way and calculated the average pressure to all orders.

- Pressure $\propto \gamma^2$ also massless GB contribute the largest pressure of all SM. Numerical and analytic resumption agree to 10% level.

- Prokopec et al found the same scaling using an entropy argument, it would be interest to connect parton shower $\leftrightarrow$ entropy

Thank you for your time!

Backup slides

Backup slide: kinematics

$$|M|^2 = \frac{2p_{a,s}p_{c,h}}{(kp_{a,s})(kp_{c,h})} - \frac{{m_{a,s}}^2}{(kp_{a,s})^2} - \frac{{m_{c,h}}^2}{(kp_{c,h})^2}$$

$$R(v) = g^2 \int \frac{d^3k}{(2\pi)^3 2k_0} \left(\frac{2\left(p_{a,s}p_{c,h}\right)}{(kp_{a,s})} (kp_{c,h}) - \frac{m_{a,s}}{(kp_{a,s})^2} - \frac{m_{c,h}}{(kp_{c,h})^2} \right) \Theta\left(k_z\right) \Theta\left(p_{c,z}\right) \Theta(V - v)$$

$$= \frac{g^2}{16\pi^2} \int dk_t^2 \int \frac{dx}{x} \left(\frac{2\left(p_{a,s}p_{c,h}\right)}{(kp_{a,s})(kp_{c,h})} - \frac{m_{a,s}}{(kp_{a,s})^2} - \frac{m_{c,h}}{(kp_{c,h})^2} \right) \Theta\left(k_z\right) \Theta\left(p_{c,z}\right) \Theta(V - v)$$

$$= \frac{g^2}{16\pi^2} E_a^2 \int 2x(1-x)dV \int \frac{dx}{x} \left(\frac{2\left(p_{a,s}p_{c,h}\right)}{(kp_{a,s})(kp_{c,h})} - \frac{m_{a,s}}{(kp_{a,s})^2} - \frac{m_{c,h}}{(kp_{c,h})^2} \right) \Theta\left(k_z\right) \Theta\left(p_{c,z}\right) \Theta(V - v)$$

$$= \frac{g^2}{16\pi^2} E_a^2 \int_v^1 dV \int_0^1 2(1-x)dx \left(\frac{2\left(p_{a,s}p_{c,h}\right)}{(kp_{a,s})(kp_{c,h})} - \frac{m_{a,s}}{(kp_{a,s})^2} - \frac{m_{c,h}}{(kp_{c,h})^2} \right) \Theta\left(x - \frac{V}{1+V}\right) \Theta\left(-x + \frac{1}{1+V}\right.$$

$$V = \frac{\Delta \mathrm{p}_z}{E_a} \approx \frac{{k_t}^2}{2\times(1-x)E_a^2} \implies \mathrm{dk}_t^2 = 2x(1-x)E_a^2\mathrm{dV}$$

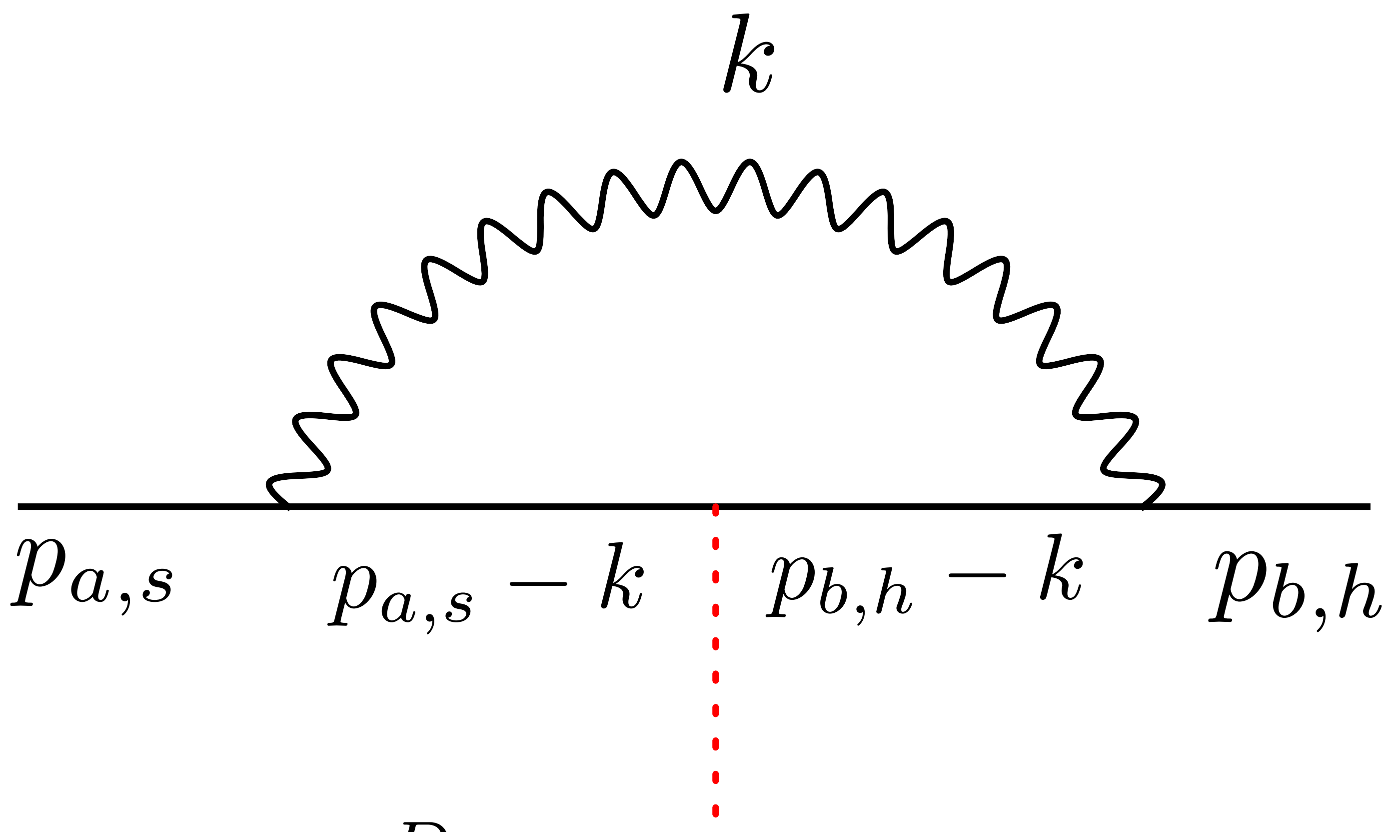
$$\eta_k = \log\left(\frac{x}{k_t/(\gamma T)}\right) = \frac{1}{x}\log\left(\frac{x}{V}\right)$$

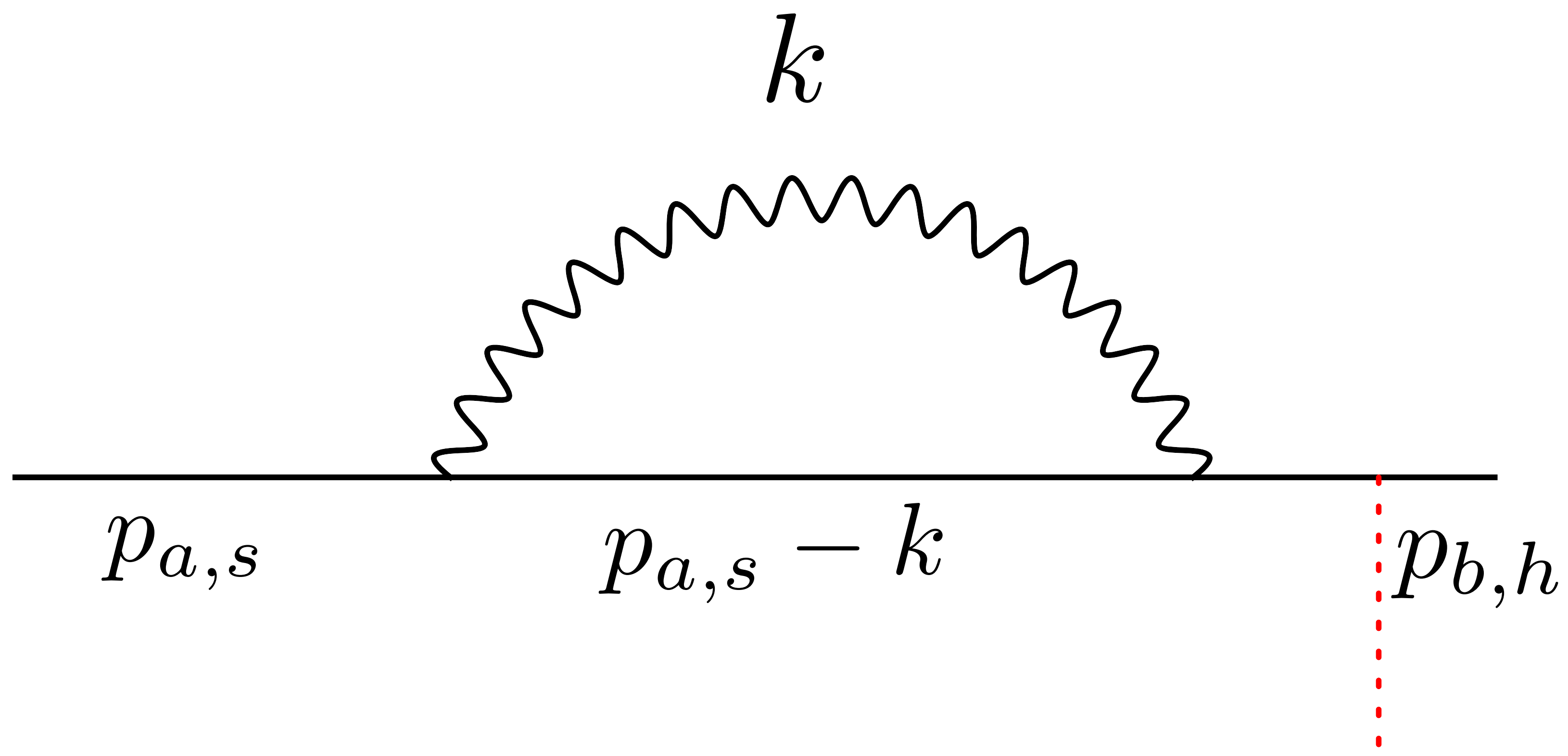
$$k_z = xE_a\left(1 - \frac{k_t^2 + m_b^2}{2x^2E_a^2}\right) \geq 0 \implies x - \frac{v}{1+V} \geq 0$$

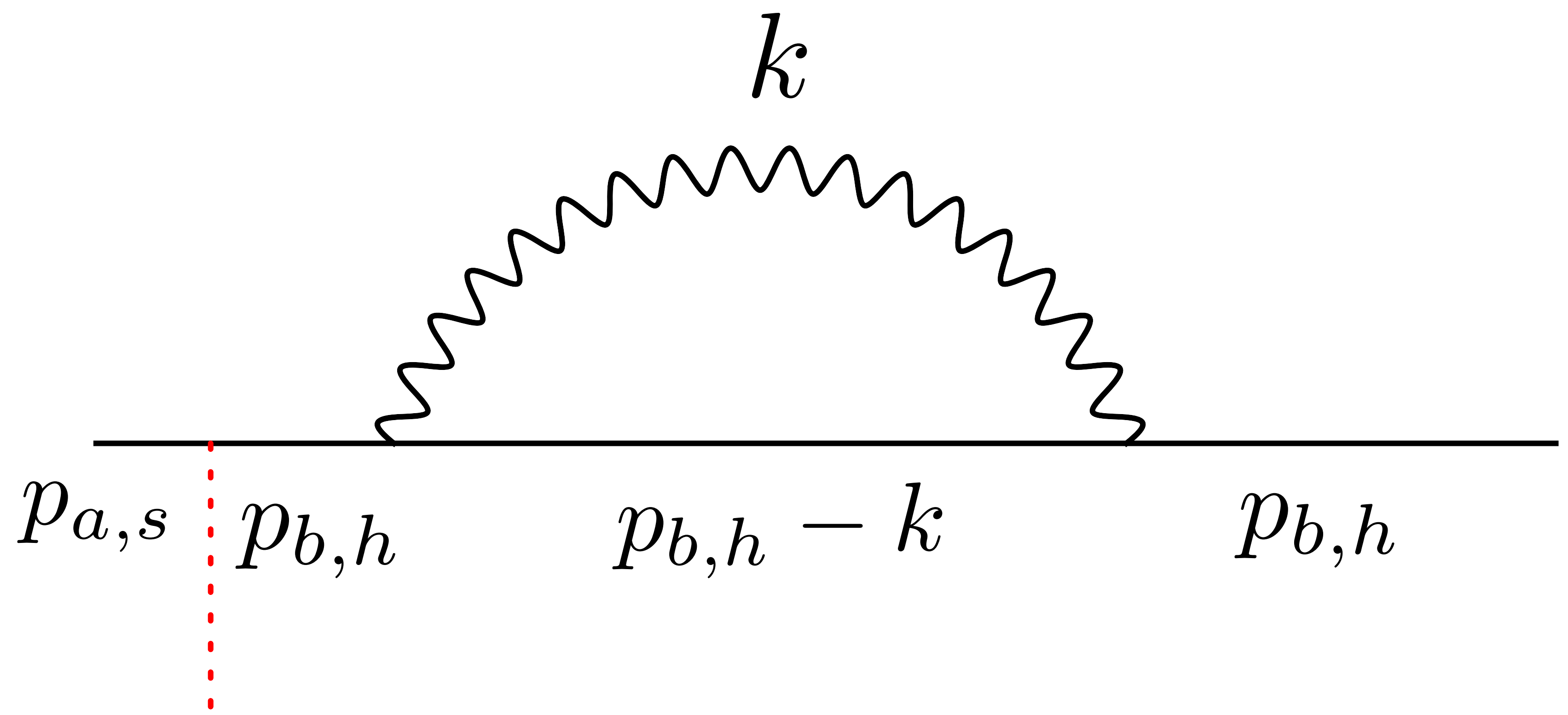
$$V = \frac{\Delta \mathrm{p}_z}{E_a} \approx \frac{{k_t}^2}{2\times(1-x){E_a}^2} \implies \mathrm{dk}_t^2 = 2x(1-x)E_a^2\mathrm{dV}$$

$$k_z = xE_a\left(1 - \frac{{k_t}^2}{2x^2E_a^2}\right) \geq 0 \implies x - \frac{v}{1+V} \geq 0$$

$$p_{c,z} = (1-x)E_a\left(1 - \frac{{k_t}^2}{2(1-x)^2E_a^2}\right) \geq 0 \implies x + \frac{1}{1+V} \geq 0$$



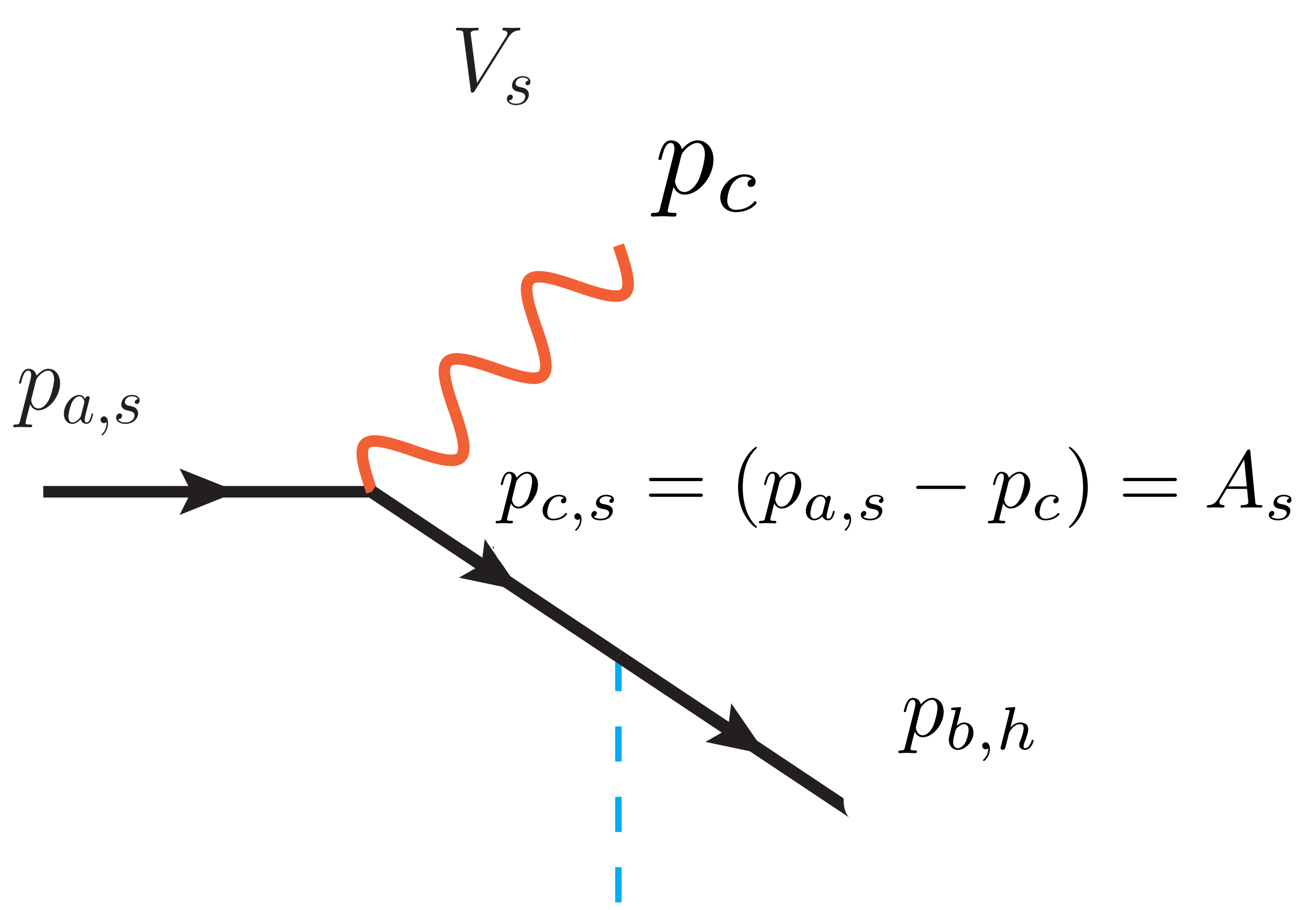


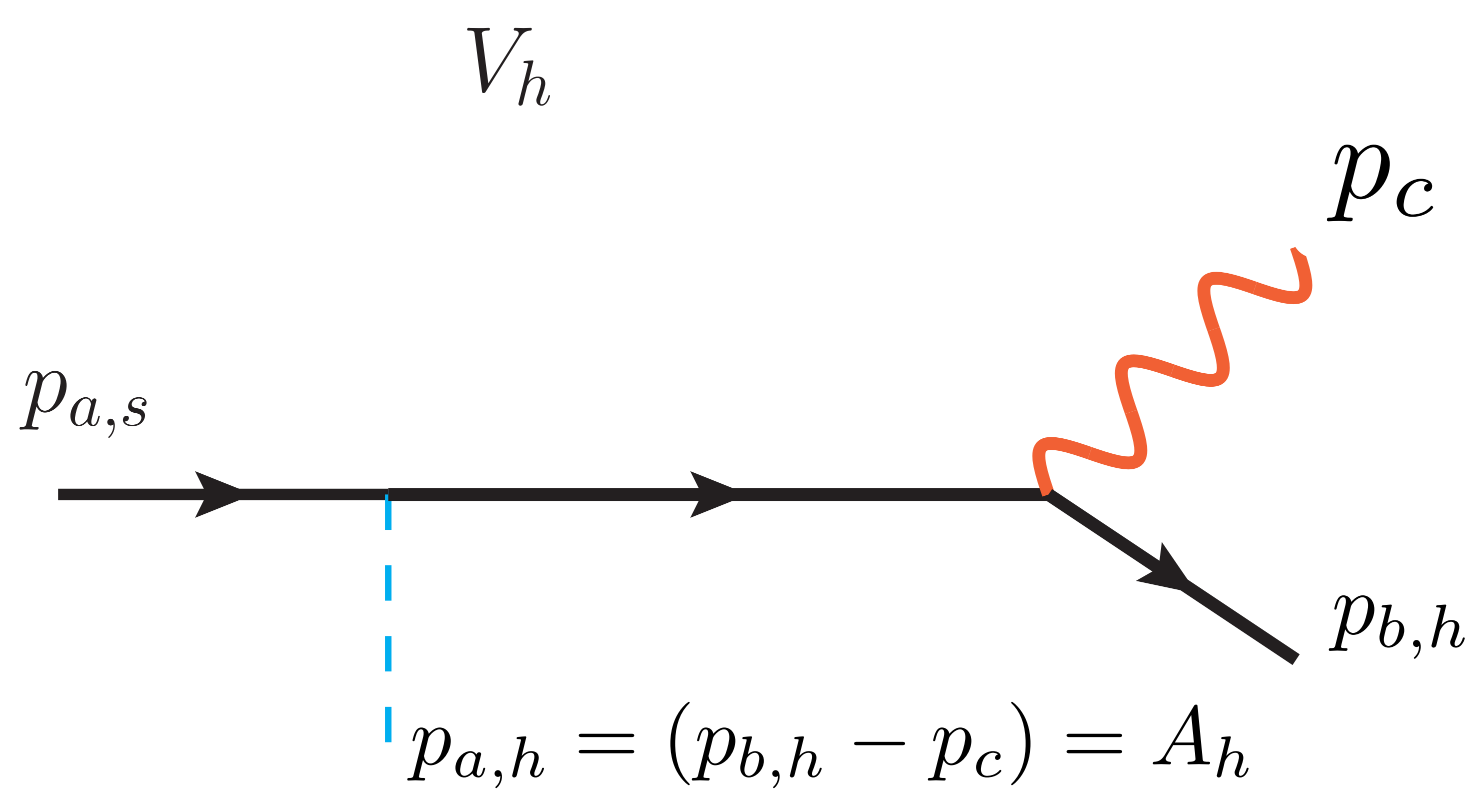


$$\int \frac{d^D k}{(2\pi)^D} \frac{p_{a,s} p_{b,h}}{k^2 (p_{a,s} - k)^2 (p_{b,h} - k)^2}$$

$$\int \frac{d^D k}{(2\pi)^D} \frac{p_{a,s}^2}{k^2 (p_{a,s} - k)^2}$$

$$\int \frac{d^D k}{(2\pi)^D} \frac{p_{b,h}^2}{k^2 (p_{b,h} - k)^2}$$





$$\int \frac{d^D k}{(2\pi)^{D-1}} \frac{2 p_{a,s} p_{b,h}}{(p_{a,s} - k)^2 (p_{b,h} - k)^2}$$

$$\int \frac{d^D k}{(2\pi)^{D-1}} \frac{p_{a,s}^2}{(p_{a,s} - k)^2 (p_{a,s} - k)^2}$$

$$\int \frac{d^D k}{(2\pi)^{D-1}} \frac{p_{b,h}^2}{(p_{b,h} - k)^2 (p_{b,h} - k)^2}$$

$D \to 4 - 2\epsilon$ then $\epsilon \to 0$ Infrared divergences cancel amongst VC and RE