NNLO event generation for top pair production at the LHC

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In collaboration with
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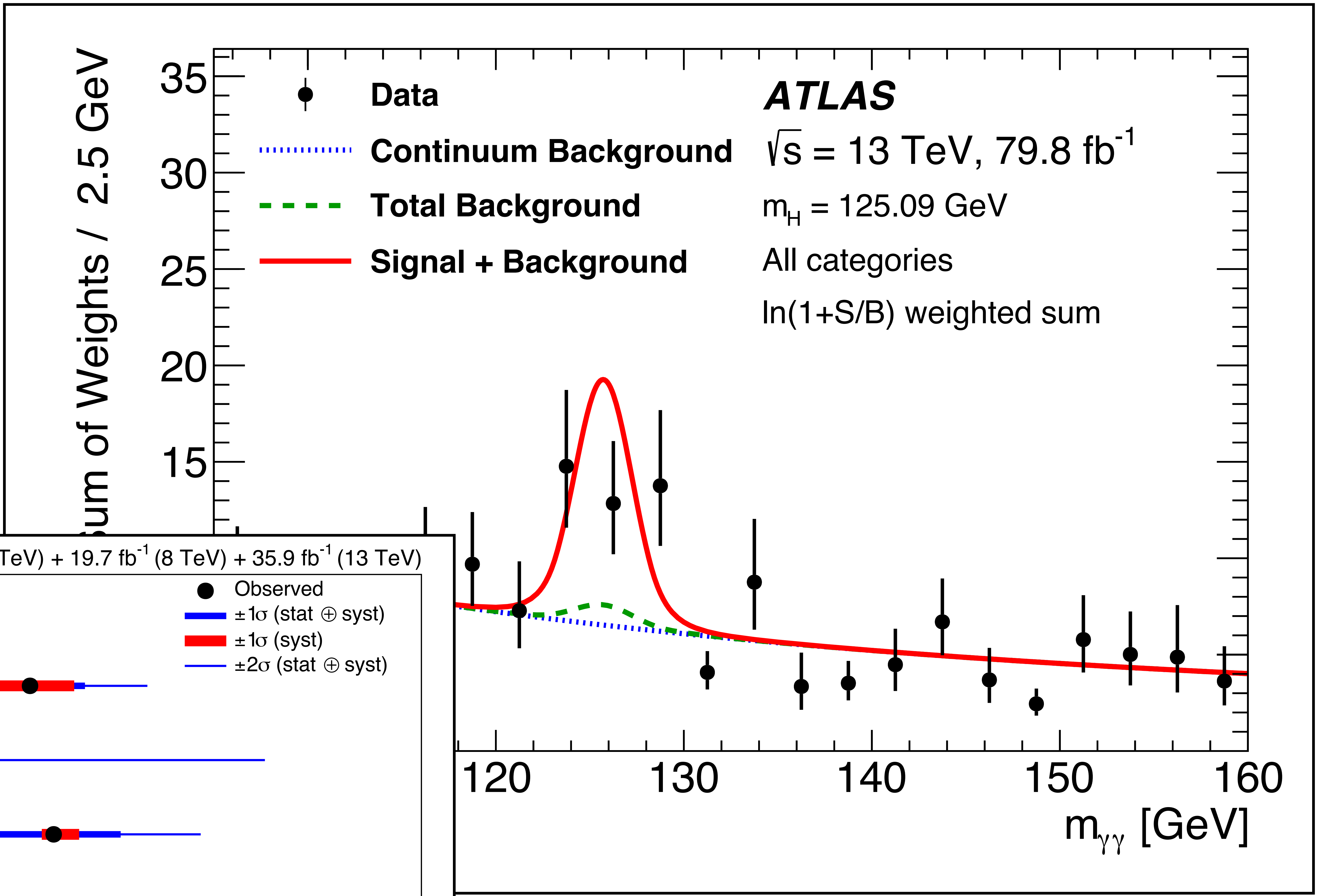
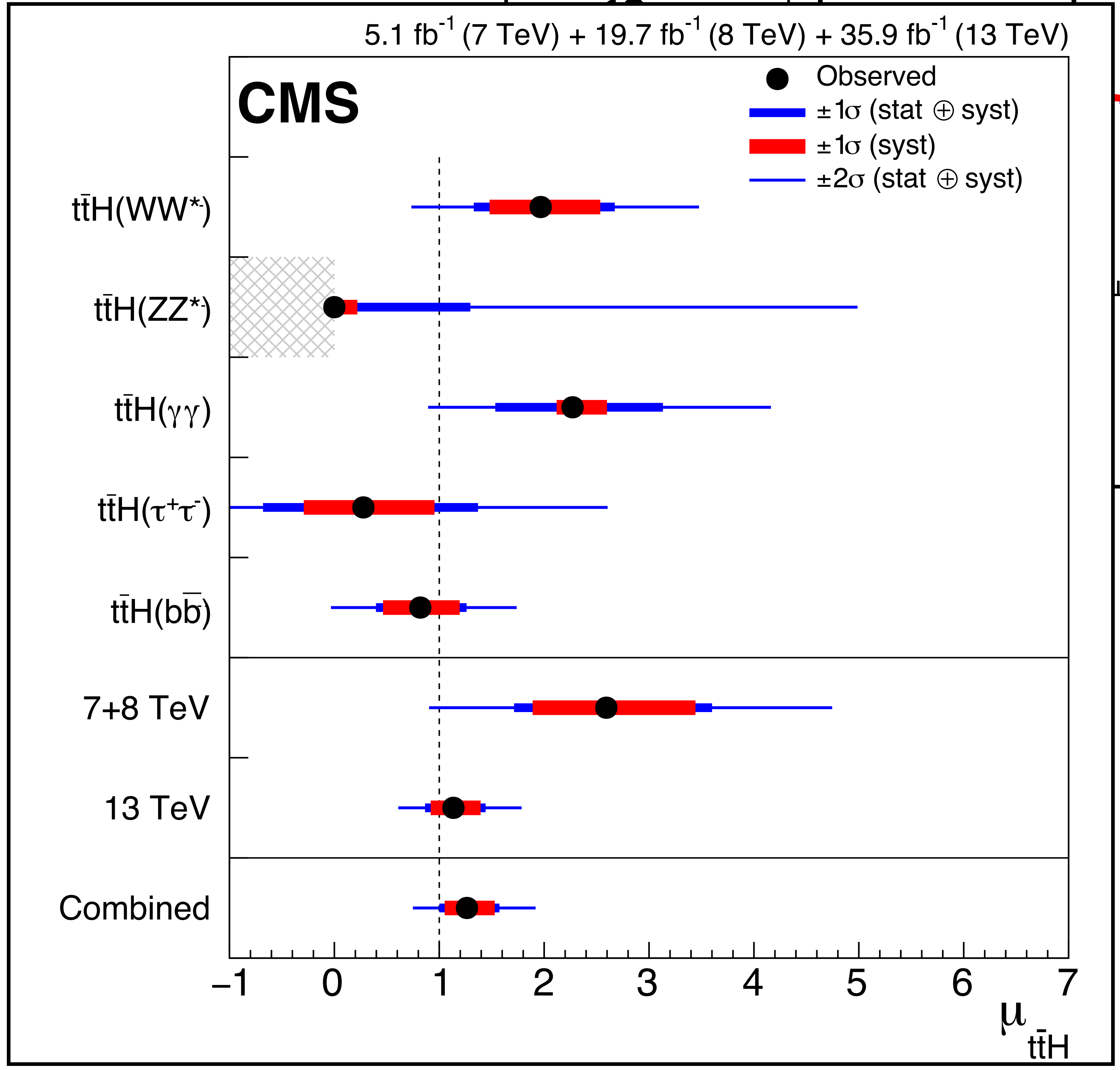
Why study top quarks ?

Top as probe of the Higgs sector

- Bridge between QCD and Higgs sectors of SM Lagrangian: study of y_t plays a central role in Higgs physics
  - Hierarchy problem
  - Sensitivity to top partners in tails of distributions (large momentum transfer)
  - background in many Higgs measurements

ttH observation (consistent with SM y_t within unc.)





Top as probe of new physics

- In several NP scenarios, extra states couple dominantly to top quarks
  - rich sensitivity to SMEFT dim. 6 op. Different observables within the same process probe different operators

e.g. in tt

  - contact int. in high energy tails, e.g.

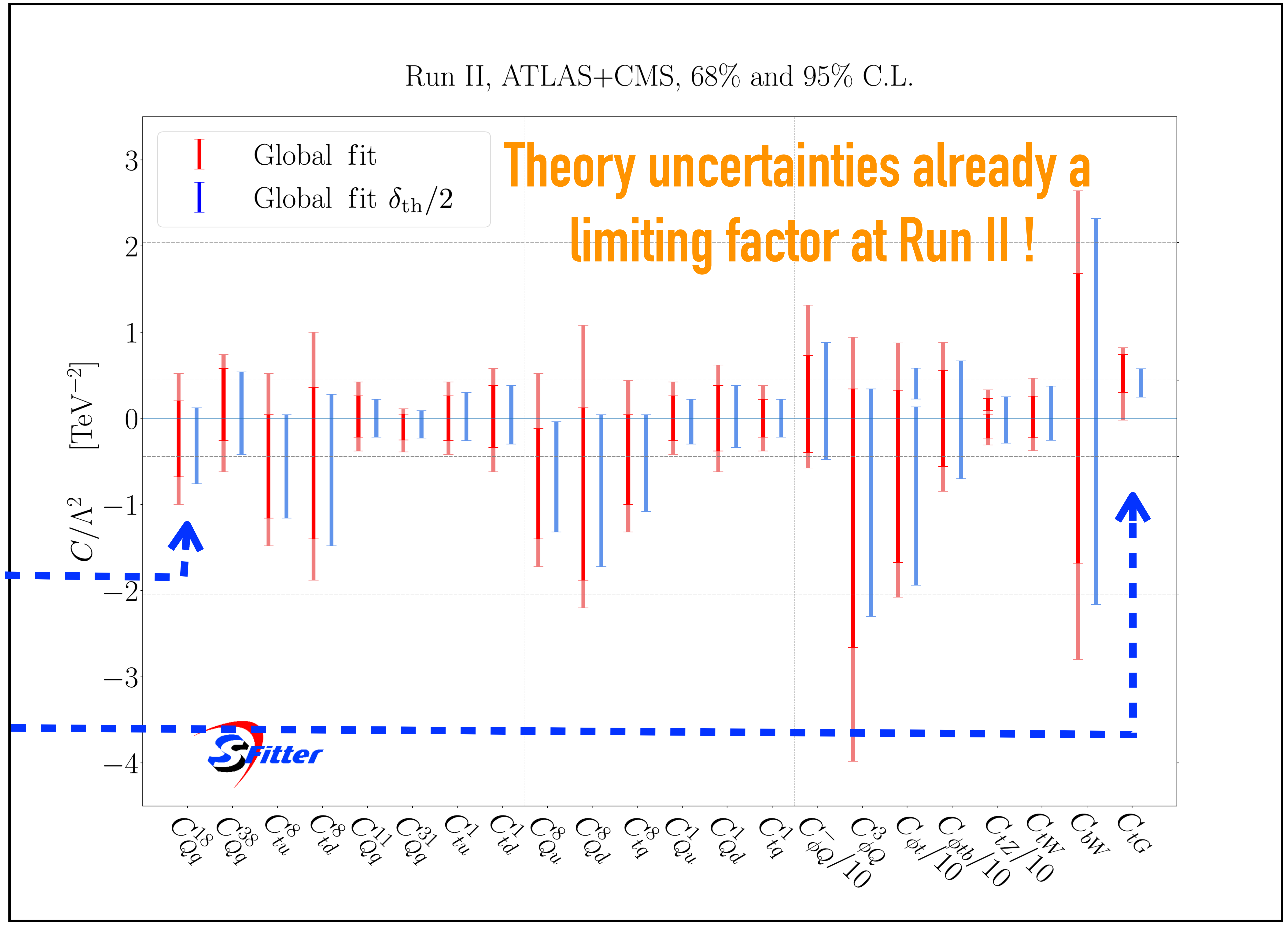
$$(\bar{Q}\gamma_\mu T^A Q)(\bar{q}_i \gamma^\mu T^A q_i) + \dots$$

  - dipole op. in total rates, e.g.

$$(\bar{Q}\sigma^{\mu\nu}T^A t)\,\widetilde{\phi}\,G^A_{\mu\nu} + \text{h.c.}$$

[Brivio, Bruggisser, Maltoni, Moutafis, Plehn, Vryonidou, Westhoff, Zhang '19]

Global fit of dimension 6 ops. with Run II top measurements



Top for precision physics

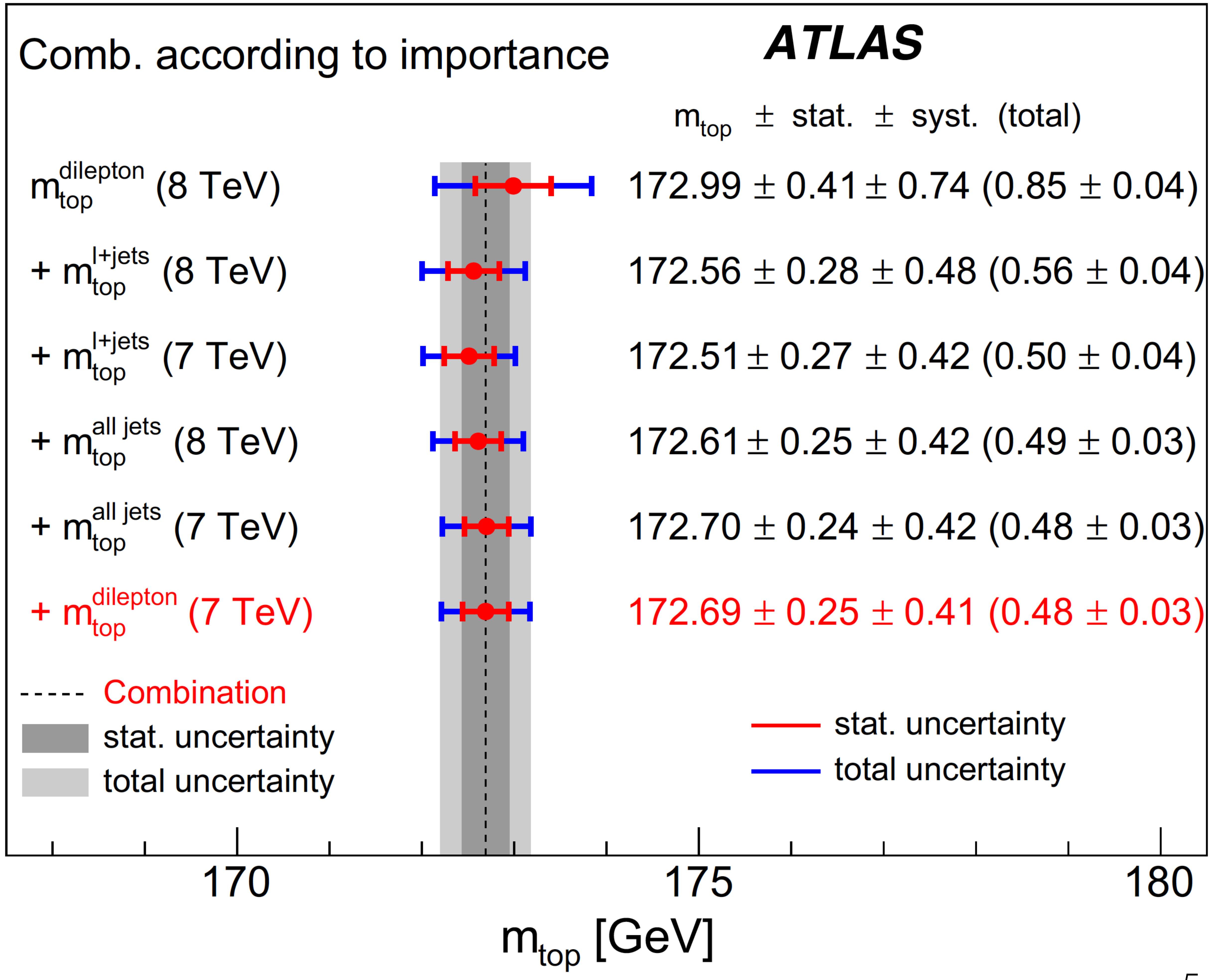
- Precision measurements/theory in top physics: (outstanding performance of LHC)
  - Fast decay allows one to "probe" its pole mass (though linear renormalon ambiguities of $O(\Lambda_{QCD})$ remain)

    [Beneke, Marquard, Nason, Steinhauser (2017)]
    [Hoang, Lepenik, Preisser (2017)]
    - top mass relevant for running of Higgs trilinear coupling (and e.g. stability of the vacuum)
  - Sensitivity of tt to α_S and parton densities

    see e.g. [Klijnsma, Bethke, Dissertori, Salam (2017)]
  - Spin correlations between top quarks

[ATLAS EPJC (2019) 290]

e.g. top pole mass from template fits



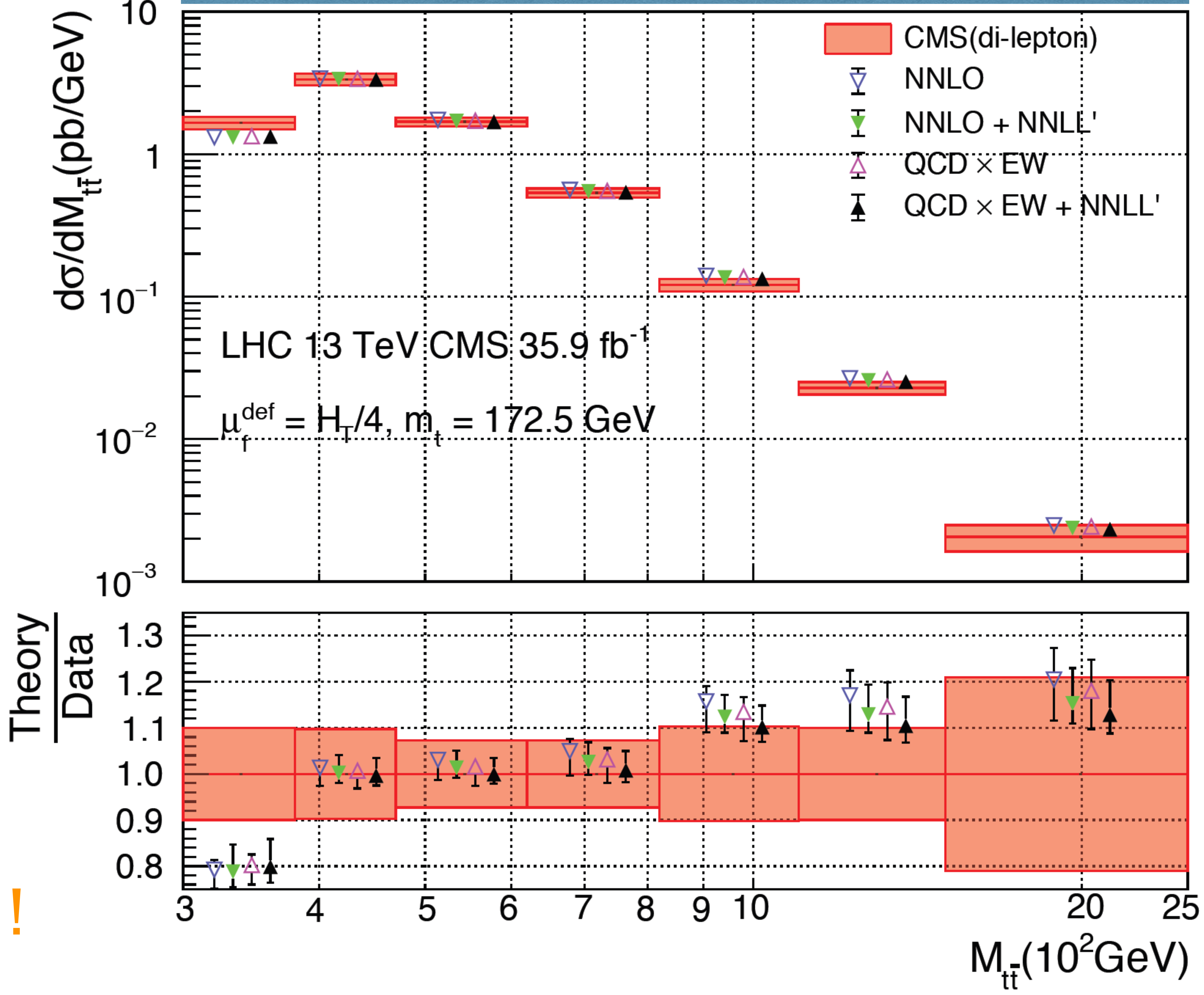
Top pair production at the LHC

- Great advances in perturbative calculations (fixed/all orders) led to remarkable theory accuracy for tt observables
  - NNLO QCD (production & decay in NWA, + spin correlations)
  - Full off-shell effects @ NLO
  - NLO EW
  - Resummations ($q_\perp$, threshold, Coulomb corrections)
  - bottom quark fragmentation @ NNLO

Many authors & significant contributions from Cambridge !
Too long a list to be comprehensive …

[Czakon et al. (2019)]

Combination of state of the art calc.ns

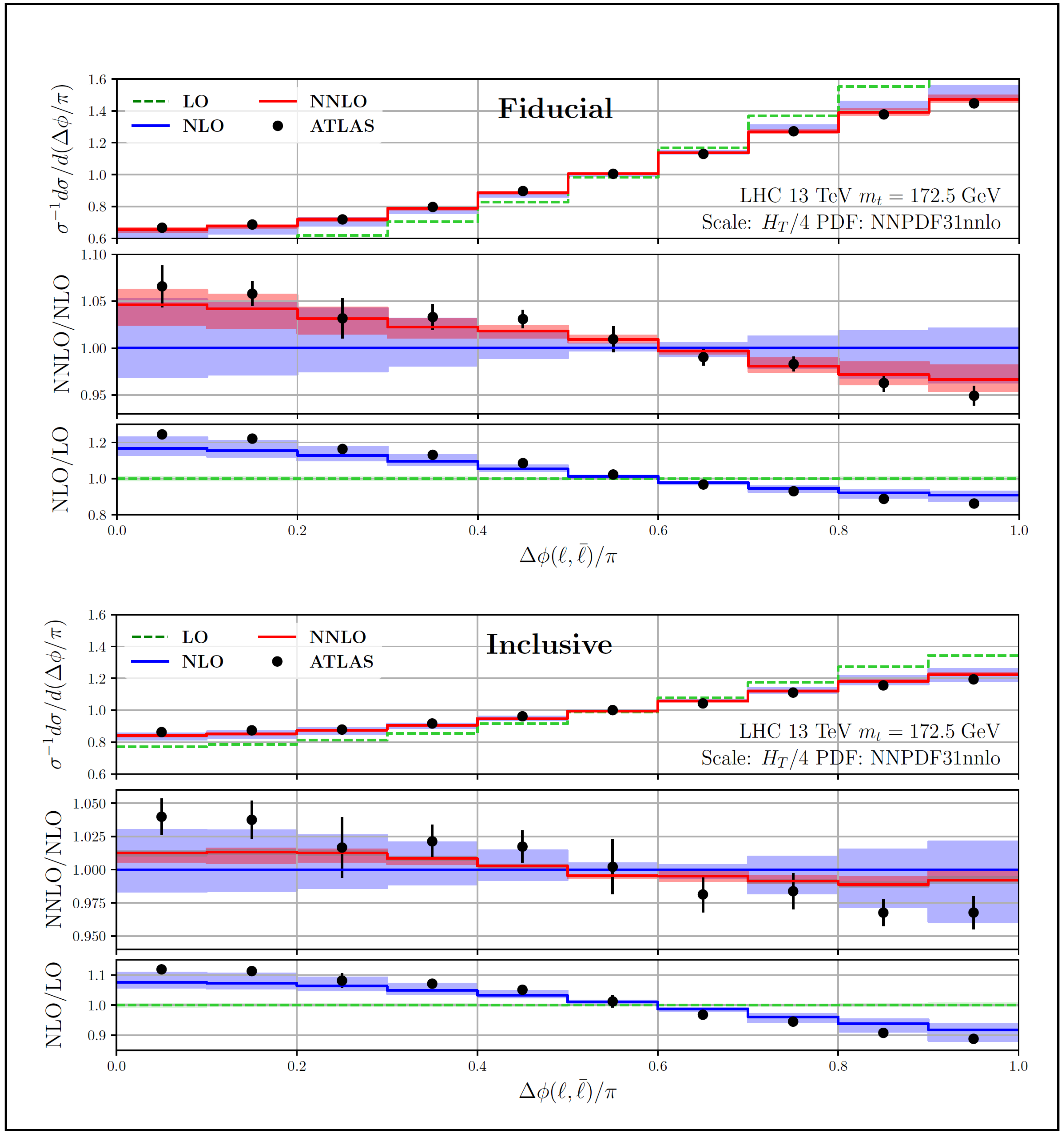


Top pair production at the LHC

- However, very often bridge between theory and experiments relies on Monte Carlo parton showers (PS) … with considerable uncertainties
  - Fiducial measurements: sensitivity to PS dynamics and fragmentation (also non-pert.)
    - e.g. Unfolding to inclusive phase space may hide subtle issues w/ underlying MC accuracy

[Behring, Czakon, Mitov, Papanastasiou, Poncelet (2019)]
see also [Czakon, Mitov, Poncelet (2020)]

e.g. Probing spin correlations in tt final states*



* origin of the discrepancy still unclear, but precedents exist where MC extrapolation was an important factor (e.g. WW)

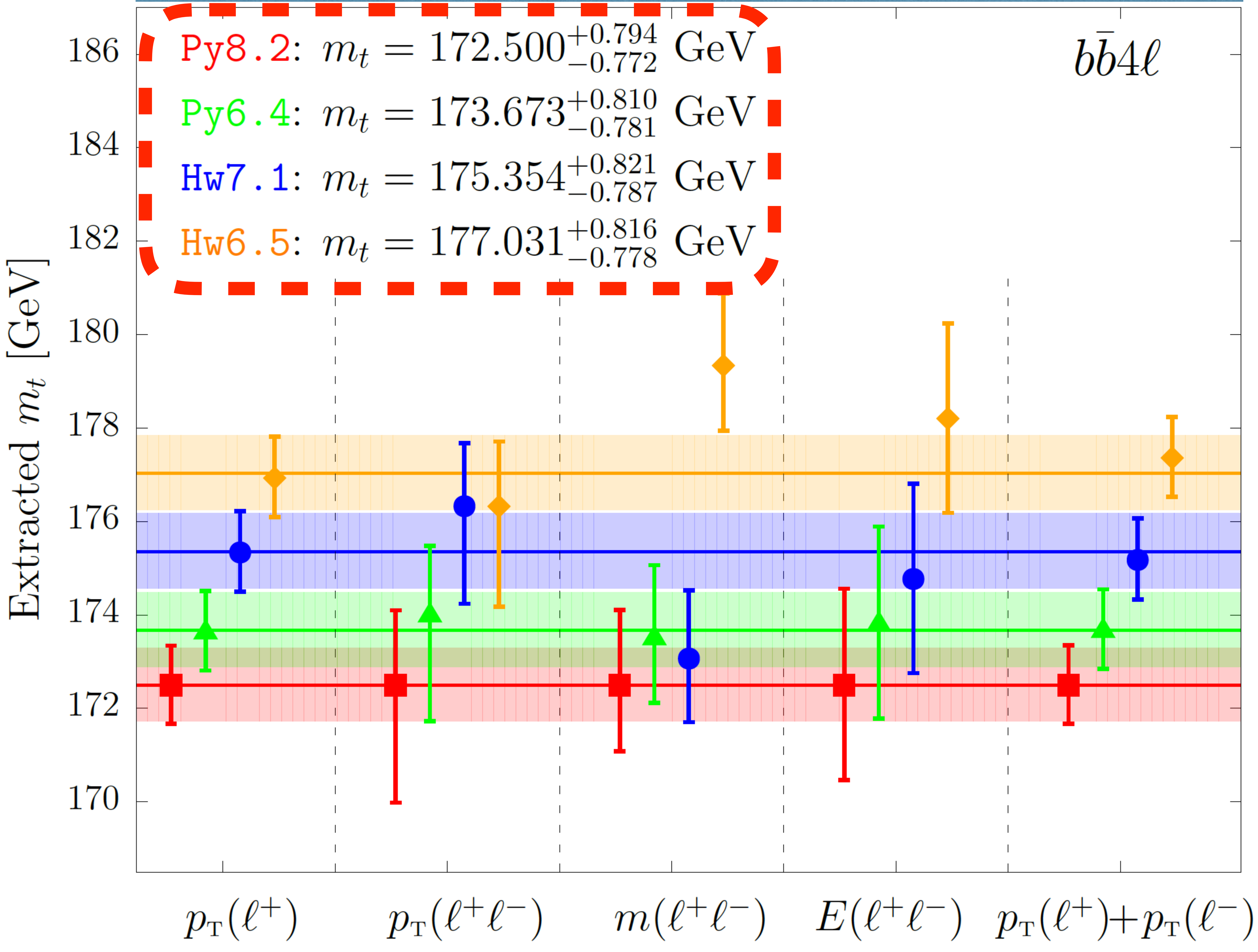
[PM, Zanderighi (2014)]

Top pair production at the LHC

- However, very often bridge between theory and experiments relies on Monte Carlo parton showers (PS) … with considerable uncertainties
  - Fiducial measurements: sensitivity to PS dynamics and fragmentation (also non-pert.)
    - e.g. significant dependence of the extracted pole top mass on MC used in template fits

[Ferrario Ravasio, Jezo, Nason, Oleari (2018-2019)]

m_t dependence on MC (moments of leptonic obs.)



reminder: intrinsic IR ambiguity is $O(\Lambda_{QCD})$!

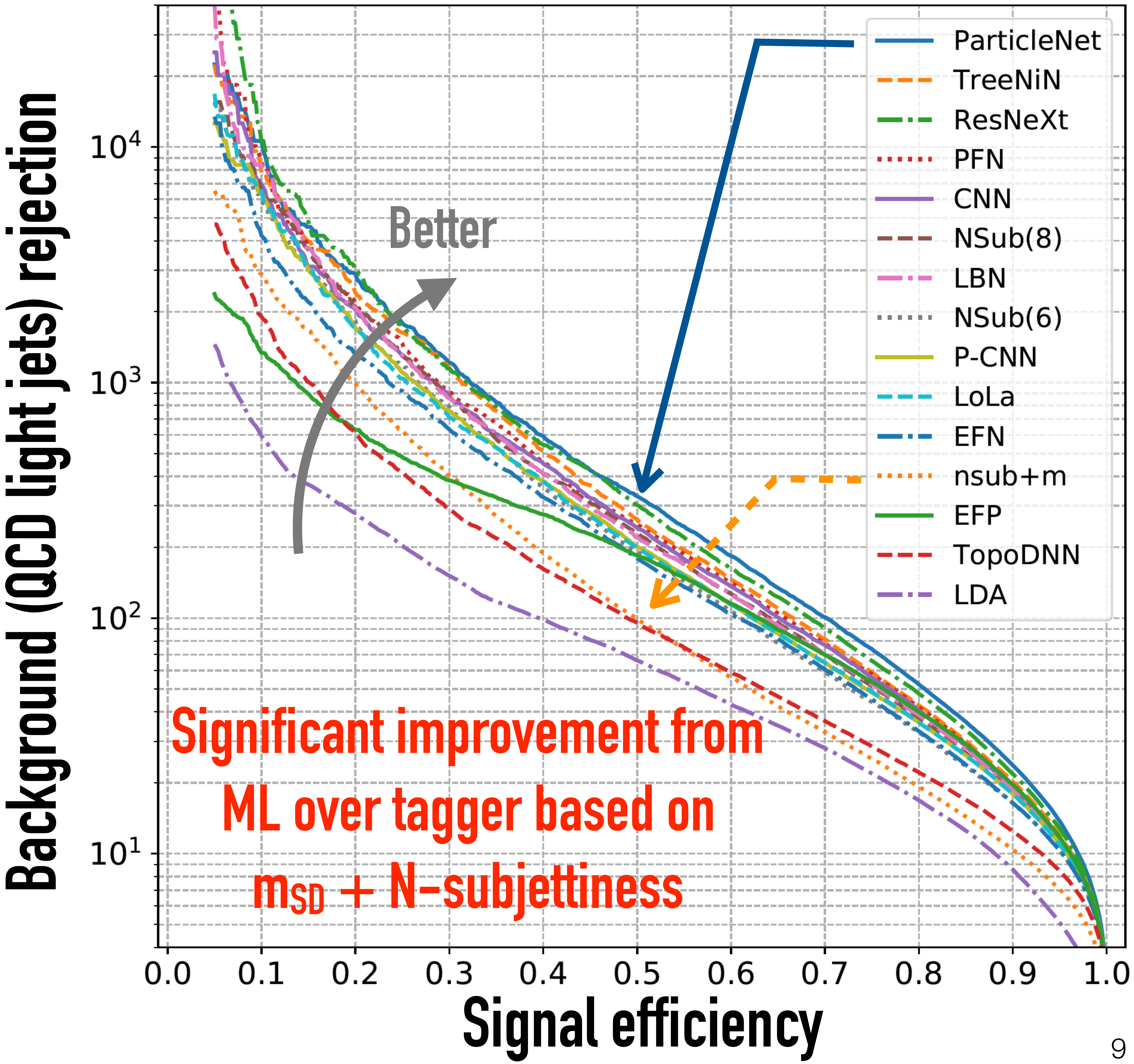
Top pair production at the LHC

[Kasieczka, Plehn et al. (2019)]

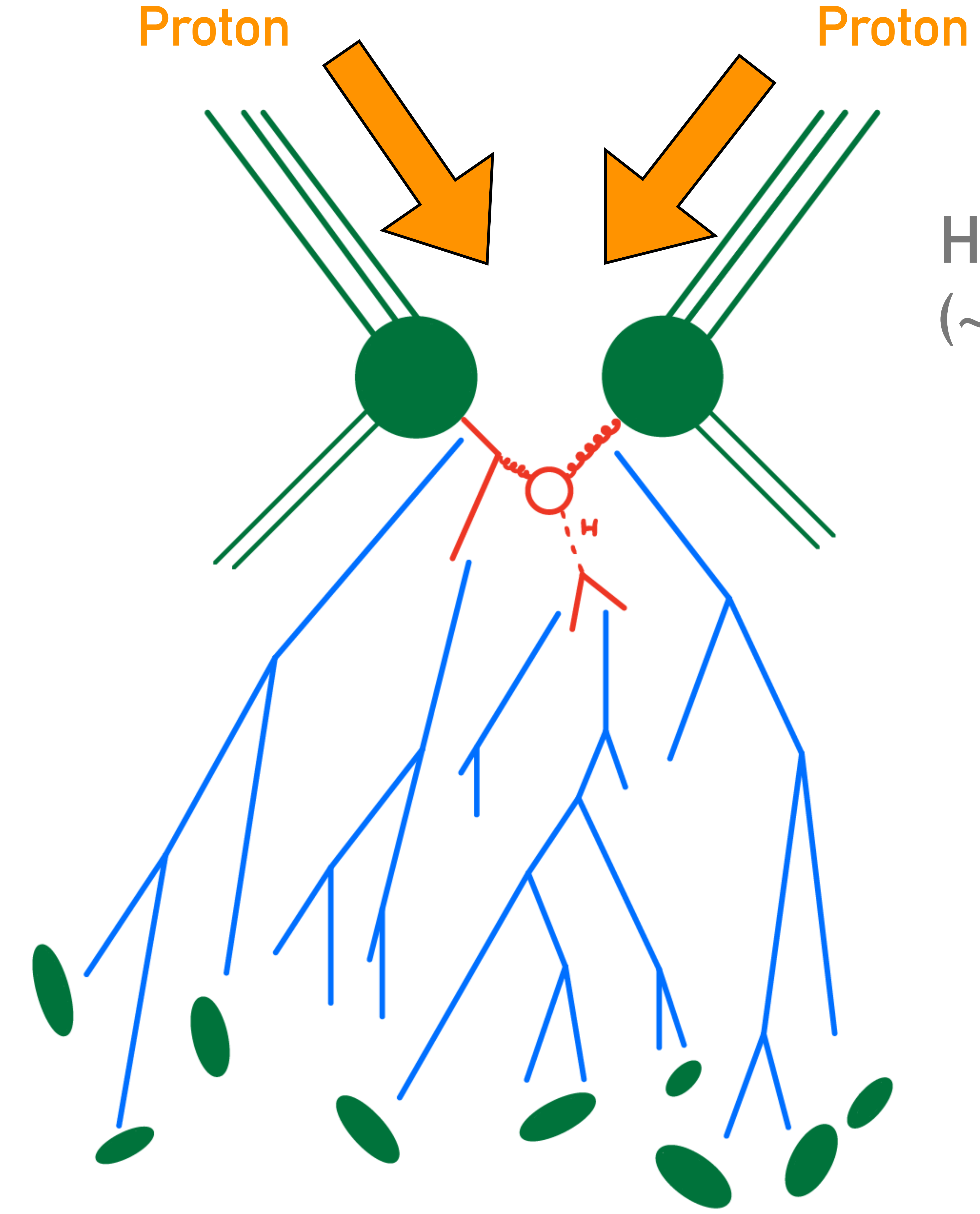
- However, very often bridge between theory and experiments relies on Monte Carlo parton showers (PS) … with considerable uncertainties
  - Fiducial measurements: sensitivity to PS dynamics and fragmentation (also non-pert.)
    - e.g. Assessment of uncertainties in ML tools to study top quarks (e.g. boosted tagging, top mass, …)

=> training is MC dependent

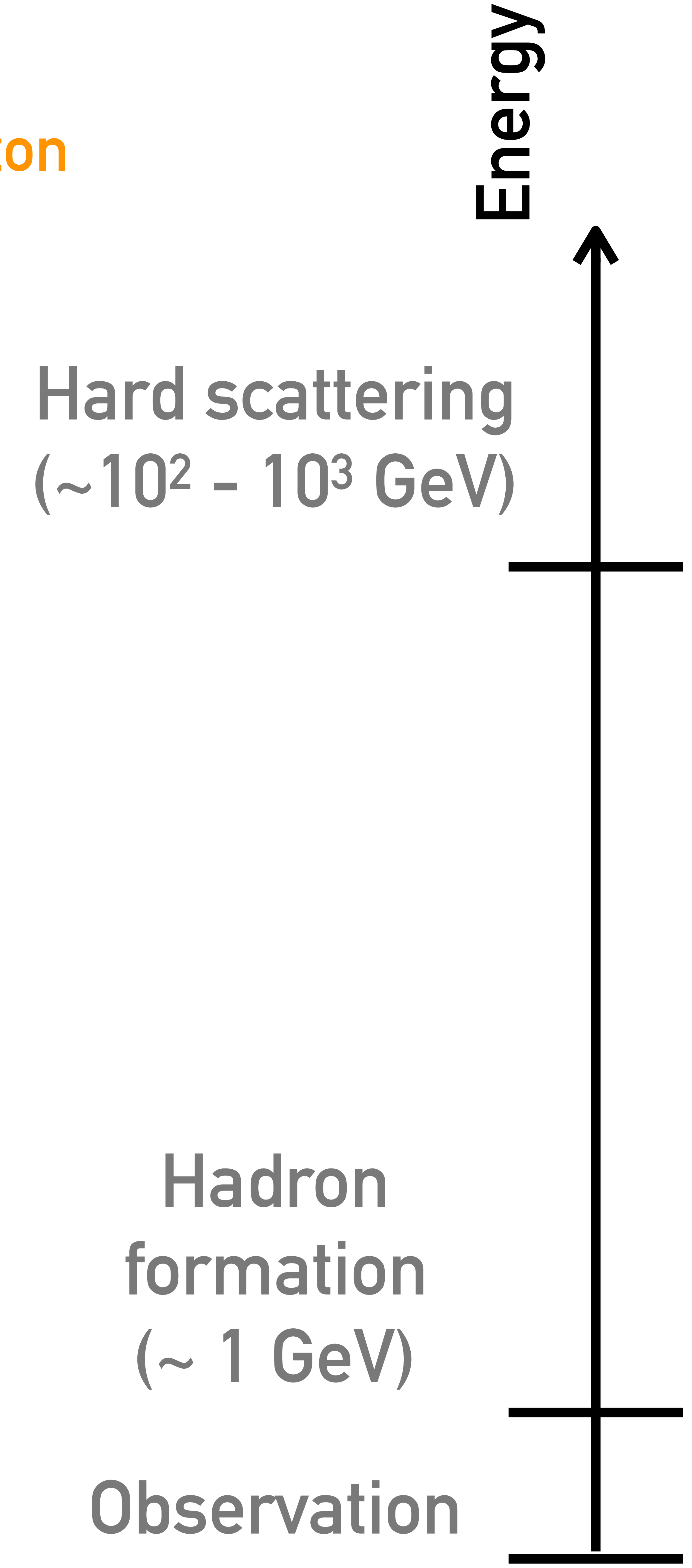
Taggers performance for boosted tops



Tackle event generators



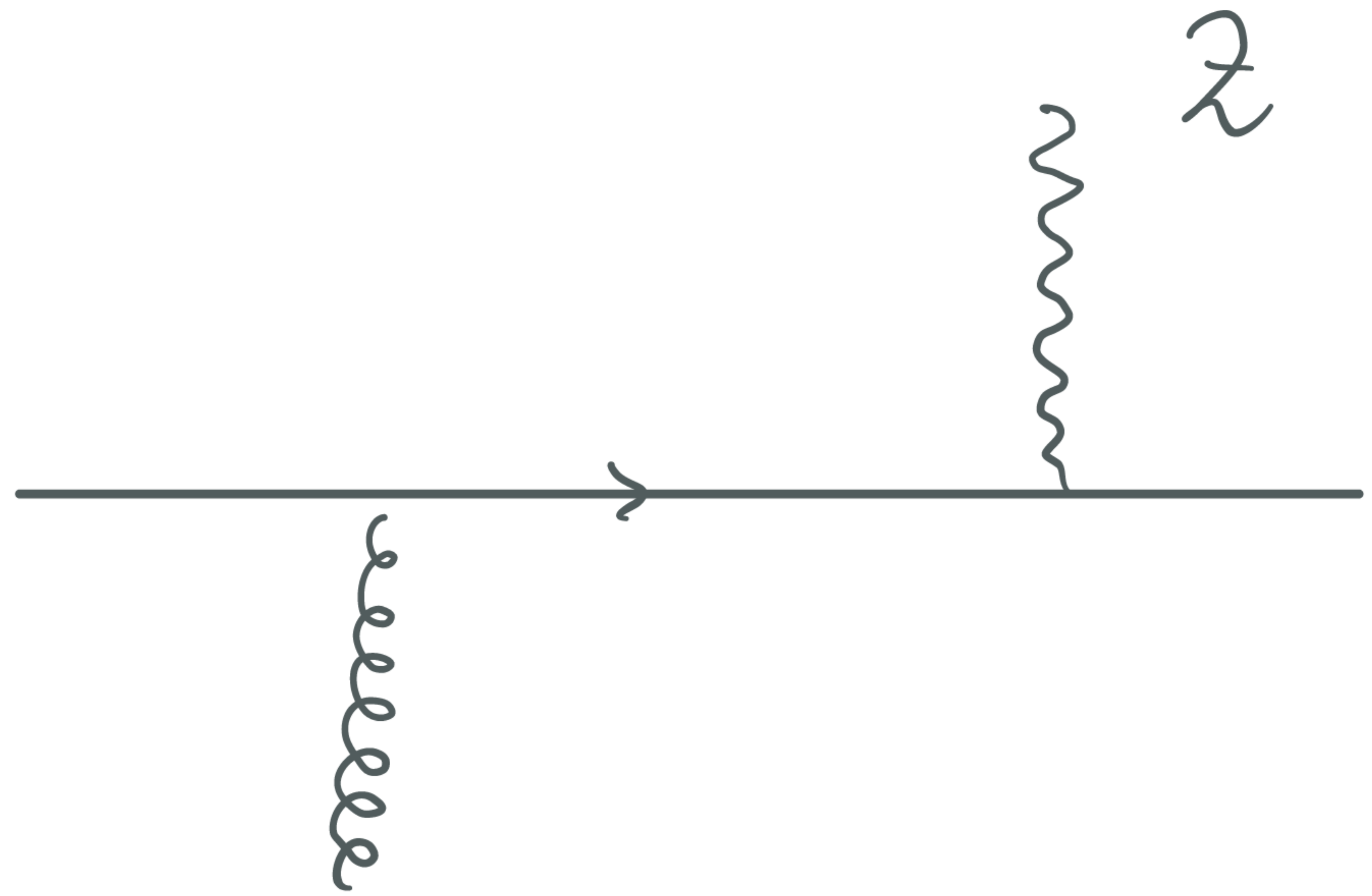
e.g. event generator cartoon in H+jet



This talk:
NNLO QCD corrections to the hard scattering
(+ some considerations on PS)

Factorization of physics at different scales allows one to study each component separately

Matching to Parton Shower: e.g. Z+jet@NLO



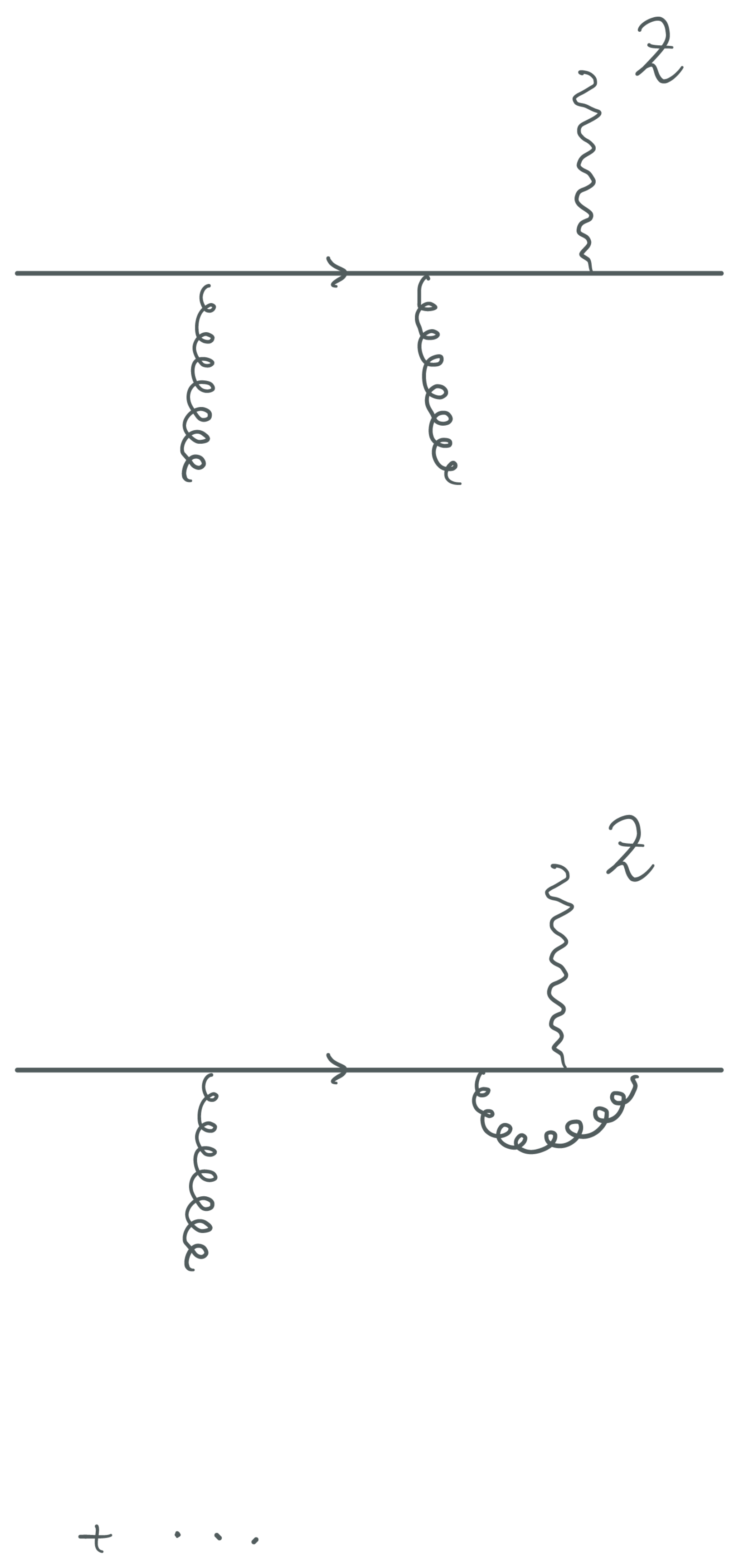
Resolved (e.g. $p_T^{jet} > 30$ GeV) QCD jet

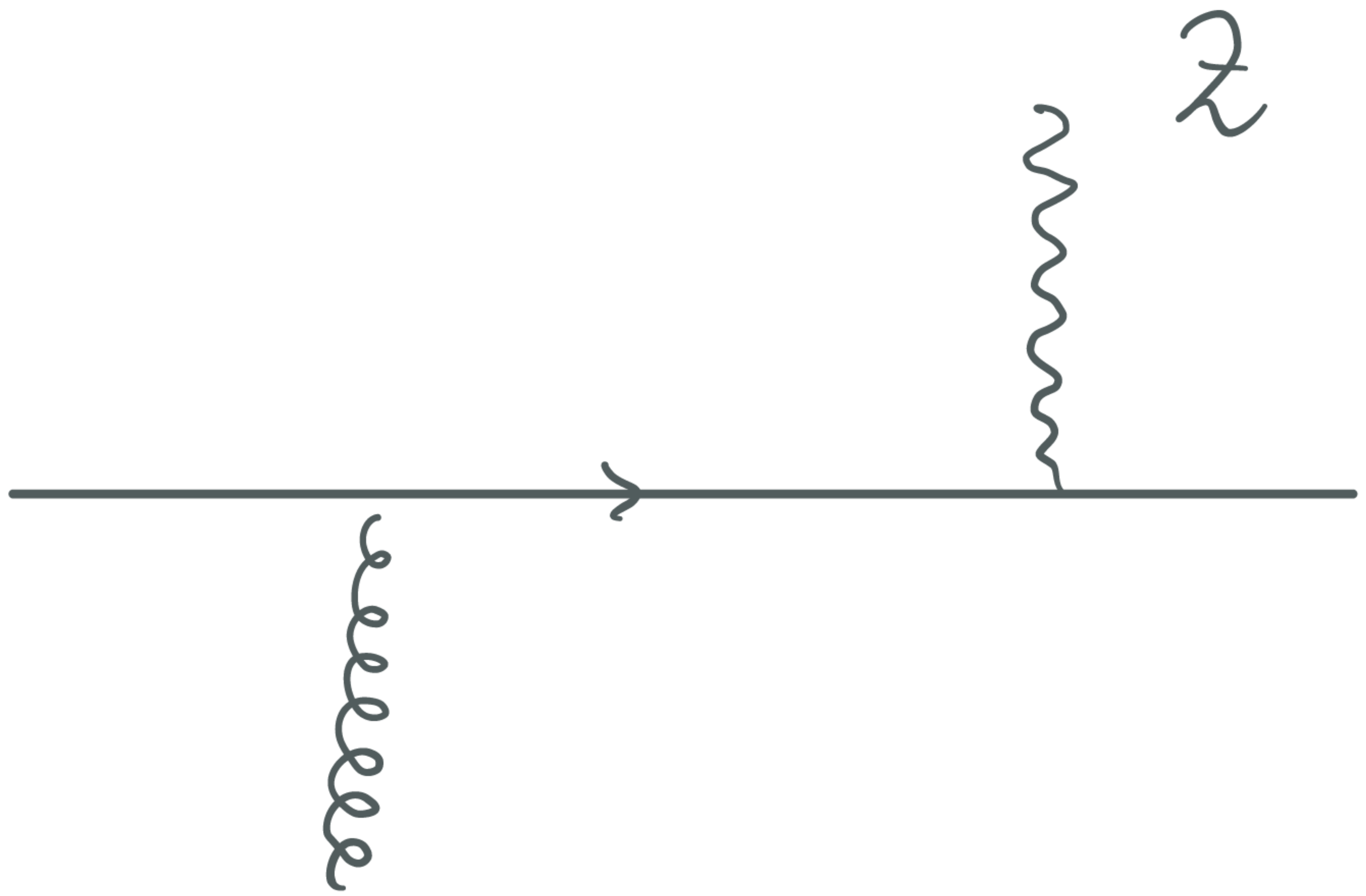
Matching to Parton Shower: e.g. Z+jet@NLO

NLO QCD: fixed order exp.^{n}

$$d\sigma = d\sigma^{(0)}\left(1 + \alpha_s(\mu_R)d\sigma^{(1)} + \mathcal{O}(\alpha_s^2(\mu_R))\right)$$

- Fixed coupling $\alpha_S(\mu_R)$
- Series truncated at FO





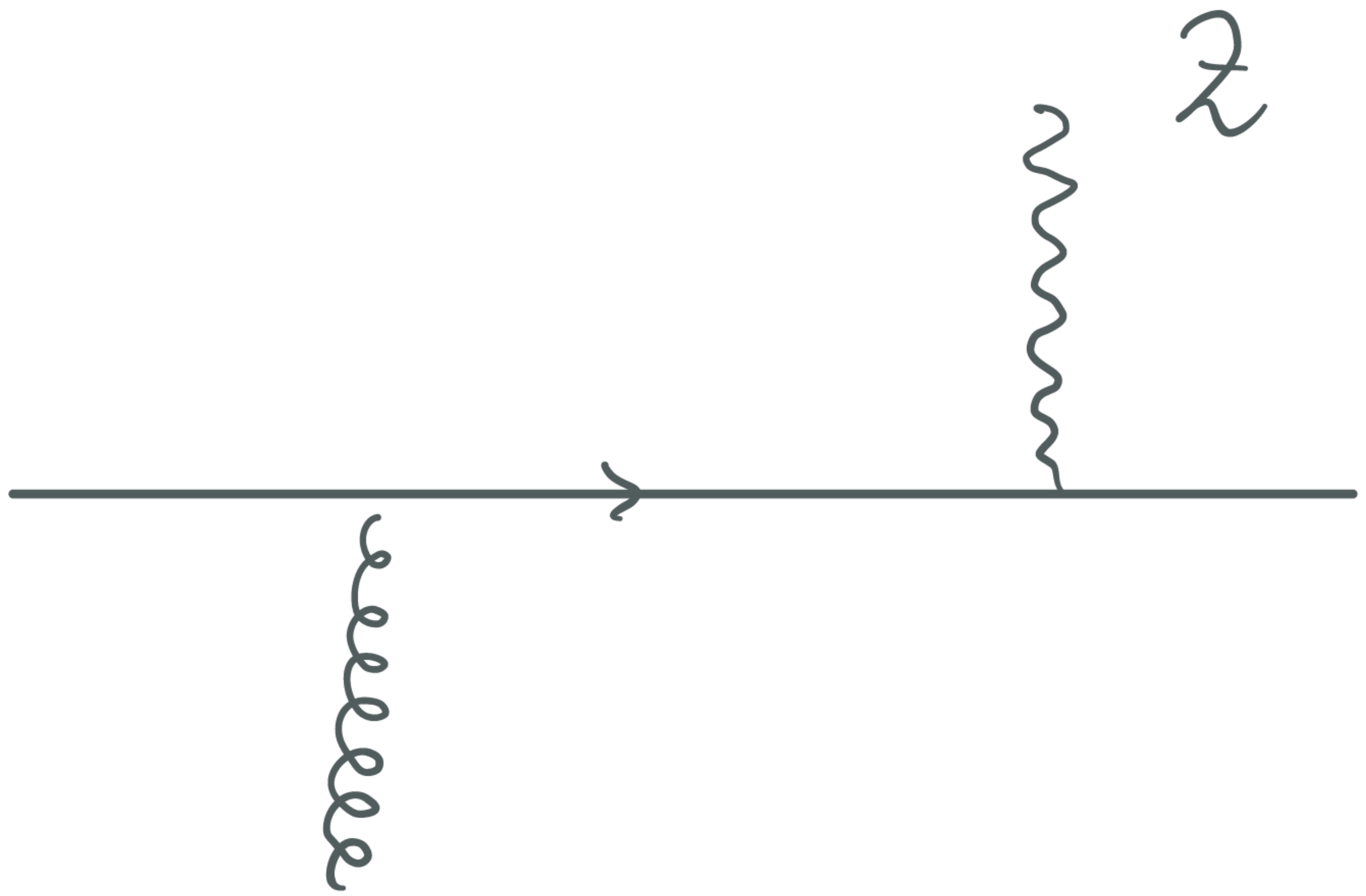
Resolved (e.g. $p_T^{jet} > 30$ GeV) QCD jet

Matching to Parton Shower: e.g. Z+jet@NLO

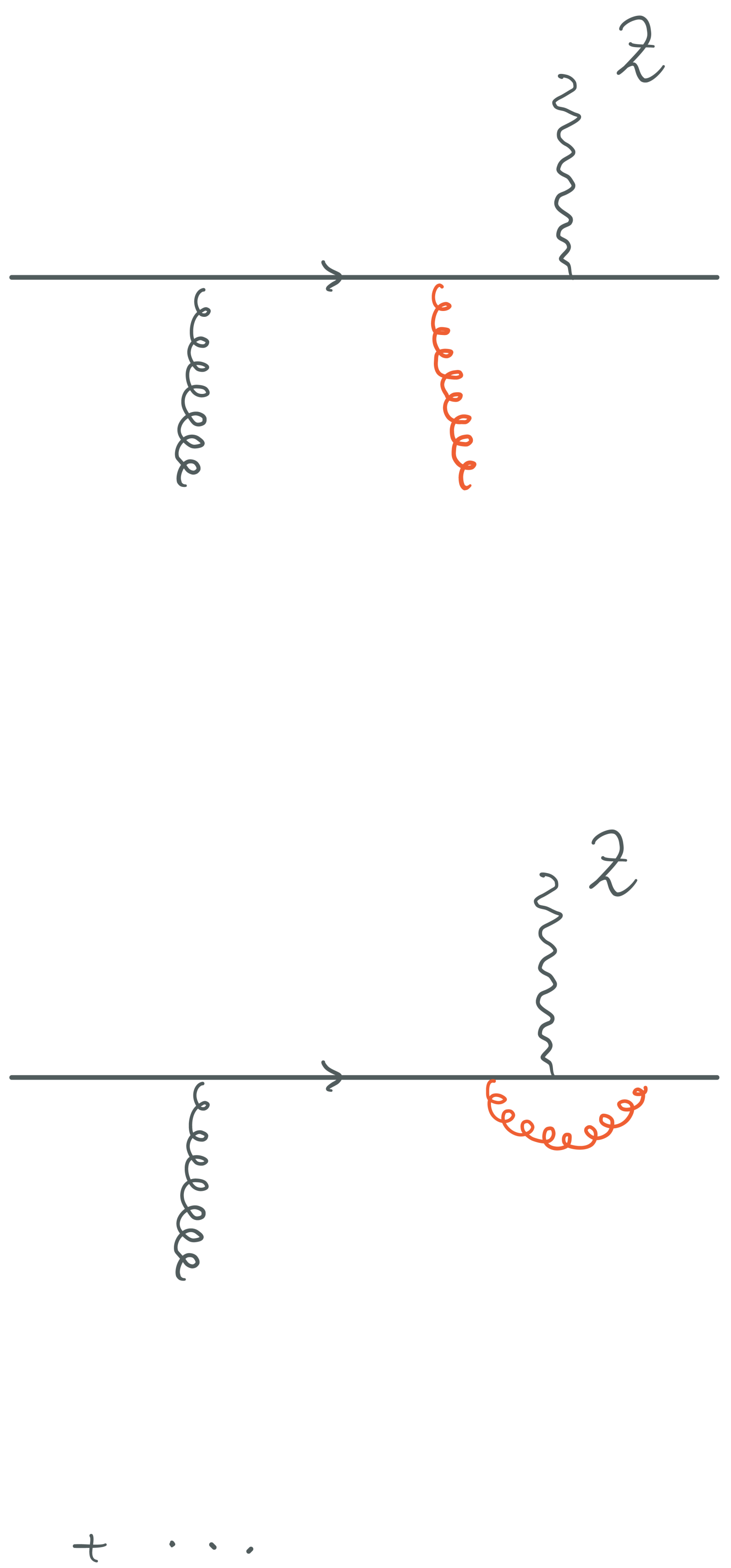
Parton Shower: iterate

$$d\sigma_{\mathrm{n+1}} = d\sigma_{\mathrm{n}} \left(\Delta(v_n) + d\Phi_{\mathrm{rad}} \frac{\Delta(v_n)}{\Delta(v_{n+1})} P(\alpha_s(k_{\perp,\,\mathrm{rad}}), \Phi_{\mathrm{rad}}) \right)$$

- coupling scale of the order of transverse momentum of the radiation
- Virtual corrections encoded in Sudakov FFs (no-emission probability)
- Resummation of radiative corrections at all orders (with some accuracy …)



Resolved (e.g. $p_T^{jet} > 30$ GeV) QCD jet



$$\Delta(v_n) \equiv \exp\left\{ - \int_{v_n > v_{\mathrm{rad}} > \Lambda} d\Phi_{\mathrm{rad}} P(\alpha_s(k_{\perp,\,\mathrm{rad}}), \Phi_{\mathrm{rad}}) \right\}$$

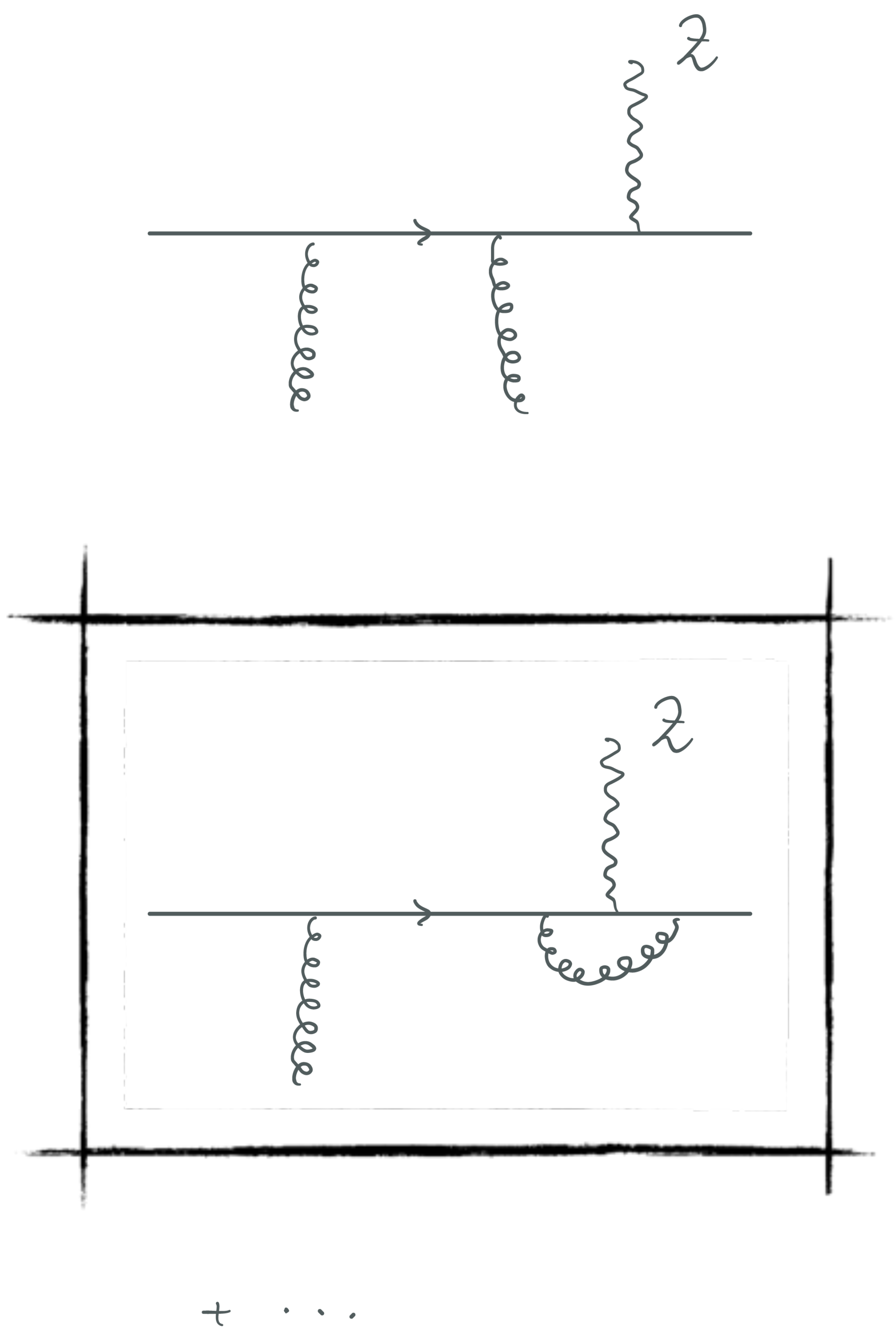
Matching to Parton Shower: e.g. Z+jet@NLO

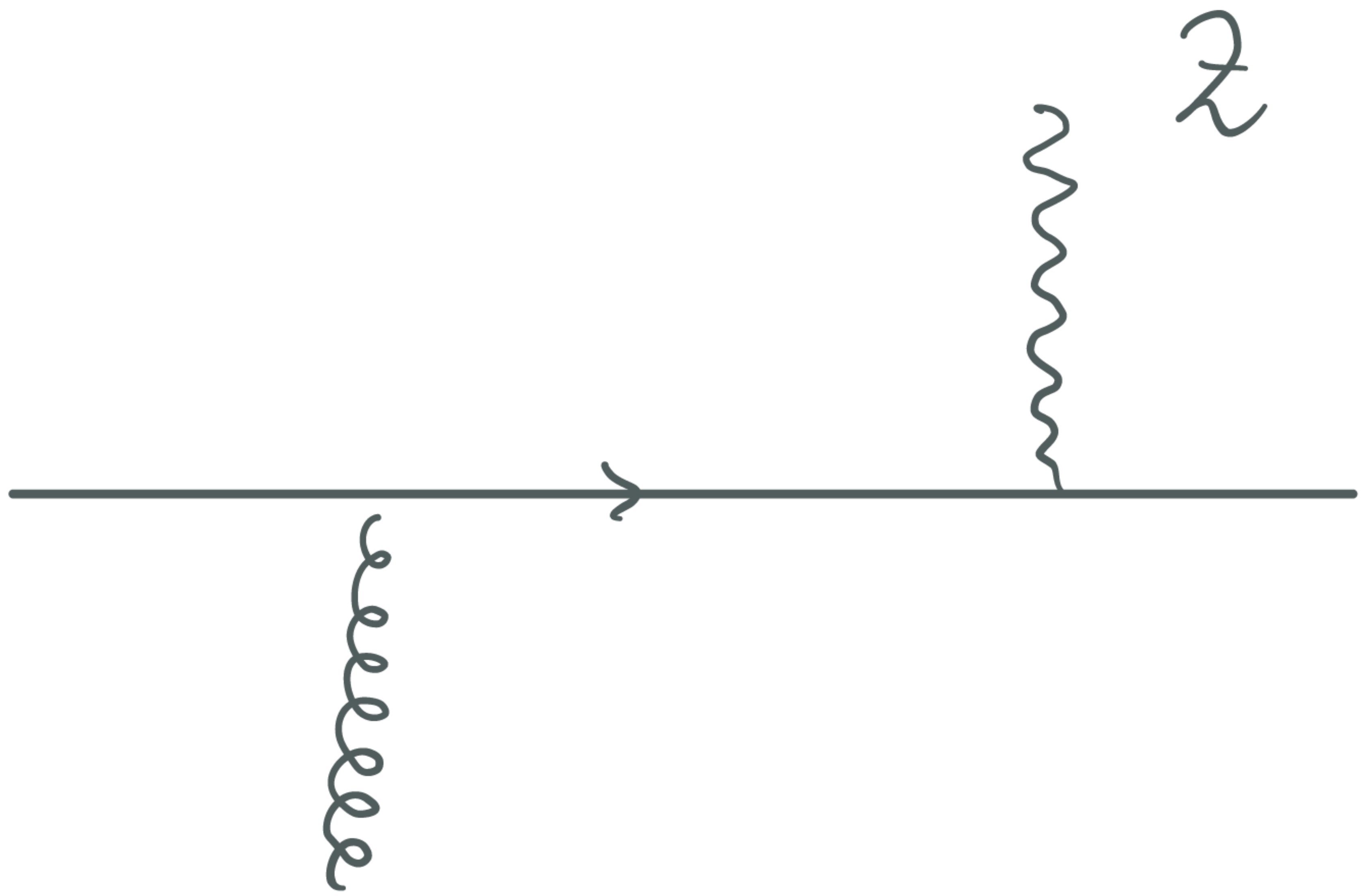
NLO QCD: fixed order exp.^{n}

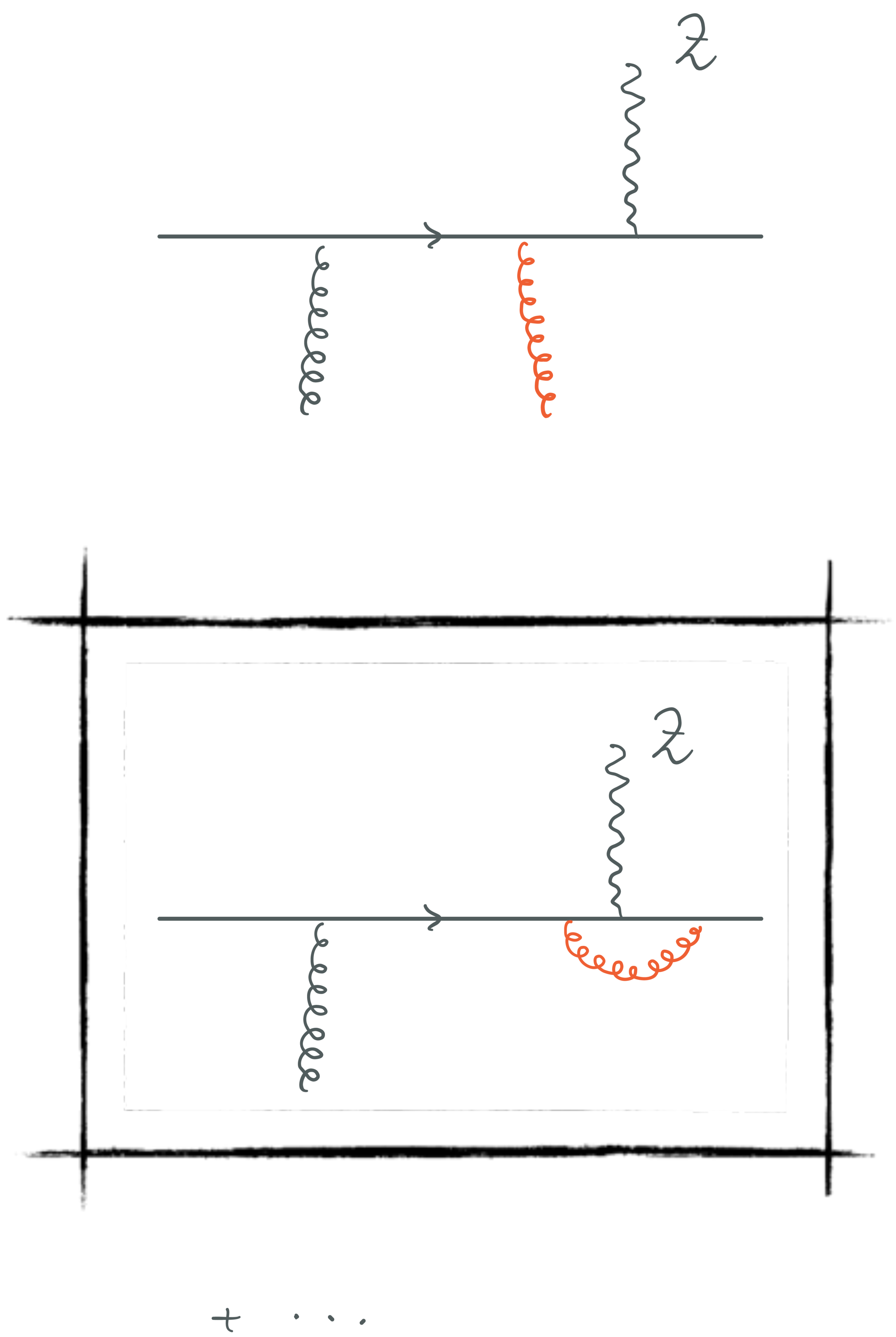
$$d\sigma = d\sigma^{(0)}\left(1 + \alpha_s(\mu_R)d\sigma^{(1)} + \mathcal{O}(\alpha_s^2(\mu_R))\right)$$

Parton Shower: iterate

$$d\sigma_{\mathrm{n+1}} = d\sigma_{\mathrm{n}}\left(\Delta(v_n) + d\Phi_{\mathrm{rad}}\frac{\Delta(v_n)}{\Delta(v_{n+1})}P(\alpha_s(k_{\perp,\mathrm{rad}}), \Phi_{\mathrm{rad}})\right)$$







Double counting of radiative corrections near the singular limits

What do we want from N(N)LO + PS simulations ?

- Simple goal: avoid double counting while
  - a) preserving N(N)LO accuracy of hard scattering process
  - b) preserving the logarithmic accuracy of the parton shower
  - Possible price to pay: inclusion of higher order corrections

- In the following the PS is assumed to have LL accuracy (in the leading colour approximation), i.e. the multi-parton squared amplitude is reproduced correctly in the limit of strongly ordered emissions and $N_c >> 1$
  - This is the case for many modern PS such as Pythia8, though more accurate designs exist for specific observables (e.g. Herwig)
  - Recently new algorithmic ways to reach NLL for broad categories of observable, road to systematic improvement of PS accuracy is being explored [more later on this point]

NLO + PS & merging jet multiplicities: MiNLO

- Problem well understood at NLO, general solutions applicable to virtually any process

[Frixione, Webber (2002); Nason (2004); Frixione, Nason, Oleari (2007); Jadach et al. (2015)]

  - e.g. one option is to recast the hard scattering as if the radiation were generated by a PS …

NLO + PS & merging jet multiplicities: MiNLO

[Hamilton, Nason, Zanderighi (2012) + Oleari (2012)]

- 1) dress the LO with Sudakov FFs, and set the coupling scales to the k_T of the corresponding emission (in a k_T-clustering sense - inspired by CKKW procedure)

[Catani, Krauss, Kuhn, Webber (2001)]

e.g. consider a NLO calculation for Z+jet differential in Φ_{FJ}

$$\bar{B}_{\mathrm{NLO}}^{(\mathrm{FJ})} = \frac{\alpha_s(\mu_R)}{2\pi}\left[B^{(\mathrm{FJ})} + \frac{\alpha_s(\mu_R)}{2\pi}V^{(\mathrm{FJ})} + \frac{\alpha_s(\mu_R)}{2\pi}\int d\Phi_{\mathrm{rad}} R^{(\mathrm{FJ})}\right]$$

NLO + PS & merging jet multiplicities: MiNLO

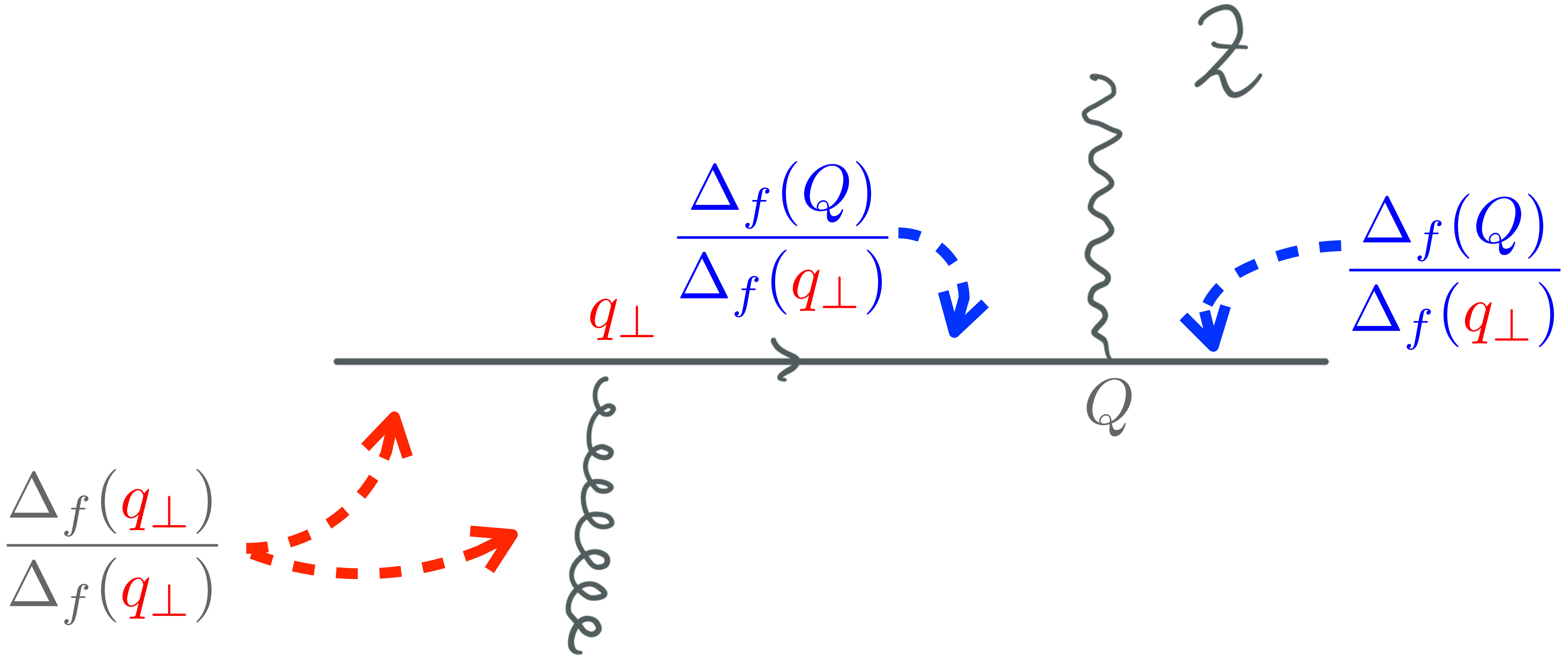
- 1) dress the LO with Sudakov FFs, and set the coupling scales to the k_T of the corresponding emission (in a k_T-clustering sense - inspired by CKKW procedure)

$$\bar{B}^{(\mathrm{FJ})}_{\mathrm{NLO}} = \frac{\alpha_s(\mu_R)}{2\pi}\left[B^{(\mathrm{FJ})} + \frac{\alpha_s(\mu_R)}{2\pi}V^{(\mathrm{FJ})} + \frac{\alpha_s(\mu_R)}{2\pi}\int d\Phi_{\mathrm{rad}} R^{(\mathrm{FJ})}\right]$$

$$\Downarrow$$

$$\bar{B}^{(\mathrm{FJ})}_{\mathrm{MiNLO}} = \frac{\alpha_s(q_\perp)}{2\pi}\left\{\frac{\Delta_f^2(Q)}{\Delta_f^2(q_\perp)}\left[B^{(\mathrm{FJ})}\left(1+\frac{\alpha_s(q_\perp)}{2\pi}S_f^{(1)}(q_\perp)\right) + \frac{\alpha_s(q_\perp)}{2\pi}V^{(\mathrm{FJ})}\right] + \int d\Phi_{\mathrm{rad}}\frac{\alpha_s(q_\perp)}{2\pi}\frac{\Delta_f^2(Q)}{\Delta_f^2(q_\perp)}R^{(\mathrm{FJ})}\right\}$$

Squared = 2 radiating legs
in the unresolved limit

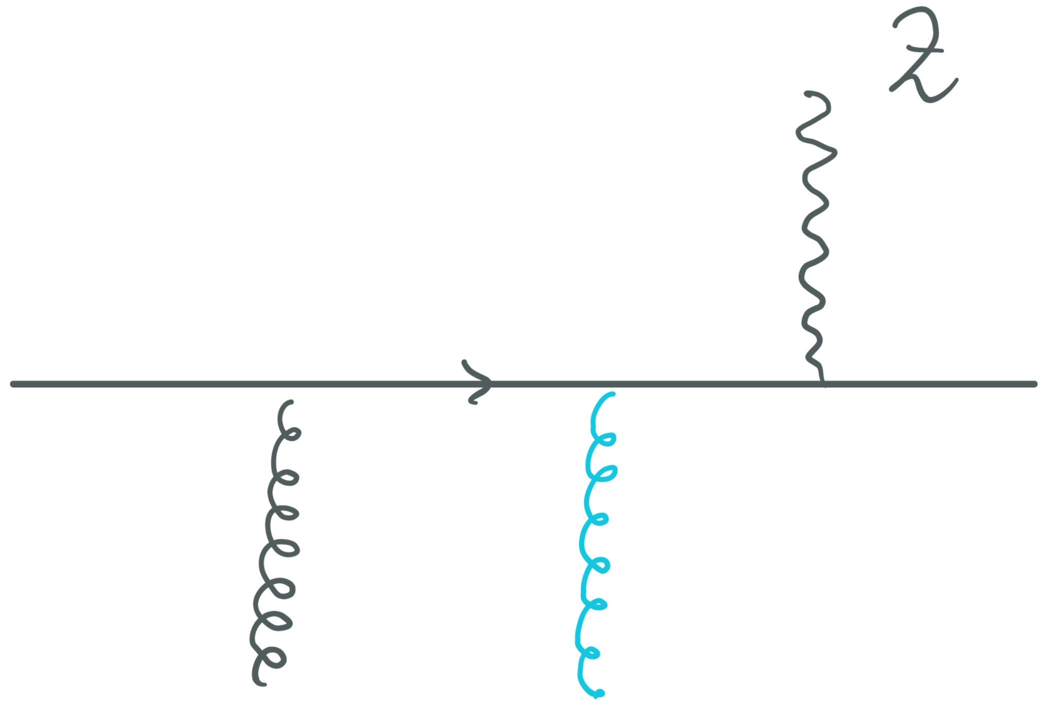


$$\frac{\Delta_f^2(Q)}{\Delta_f^2(q_\perp)} = \left(1 - \frac{\alpha_s(q_\perp)}{2\pi}S_f^{(1)}(q_\perp) + \mathcal{O}(\alpha_s^2(q_\perp))\right)$$

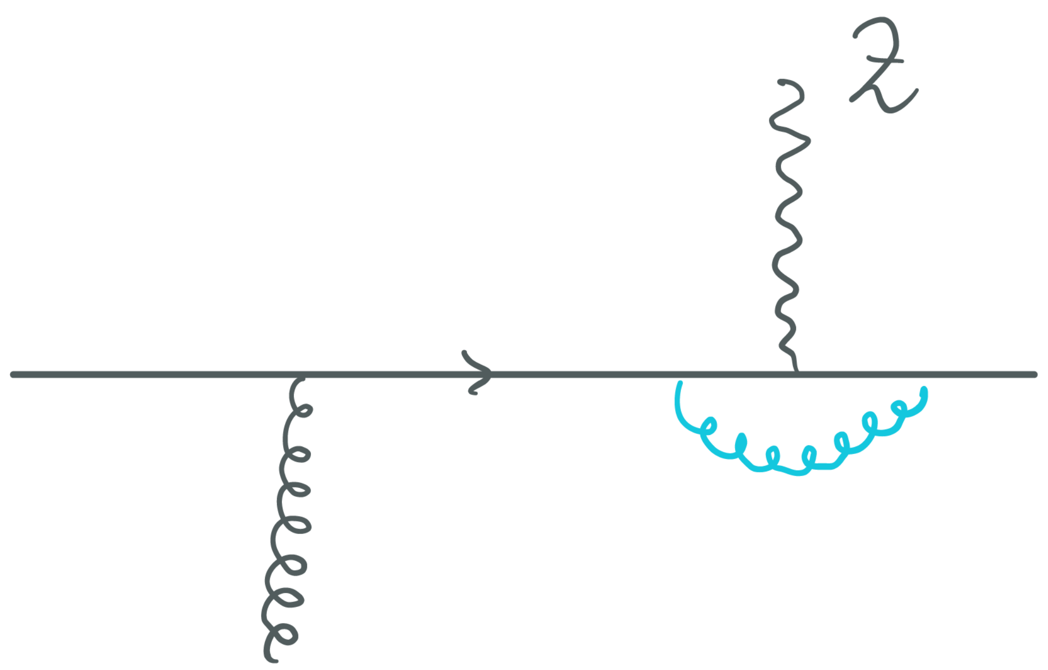
NLO + PS & merging jet multiplicities: MiNLO

- 2) generate NLO correction à la PS, namely (POWHEG) [Nason (2004)]

$$d\sigma^{(\mathrm{FJ})} = \bar{B}^{(\mathrm{FJ})}_{\mathrm{MiNLO}} d\Phi_{\mathrm{FJ}} \left[\Delta_{\mathrm{pwg}}(\Lambda_{\mathrm{gen}}) + d\Phi_{\mathrm{rad}} \Delta_{\mathrm{pwg}}(k_\perp^{\mathrm{rad}}) \frac{R(\Phi_{\mathrm{FJ}}, \Phi_{\mathrm{rad}})}{B(\Phi_{\mathrm{FJ}})} \right]$$



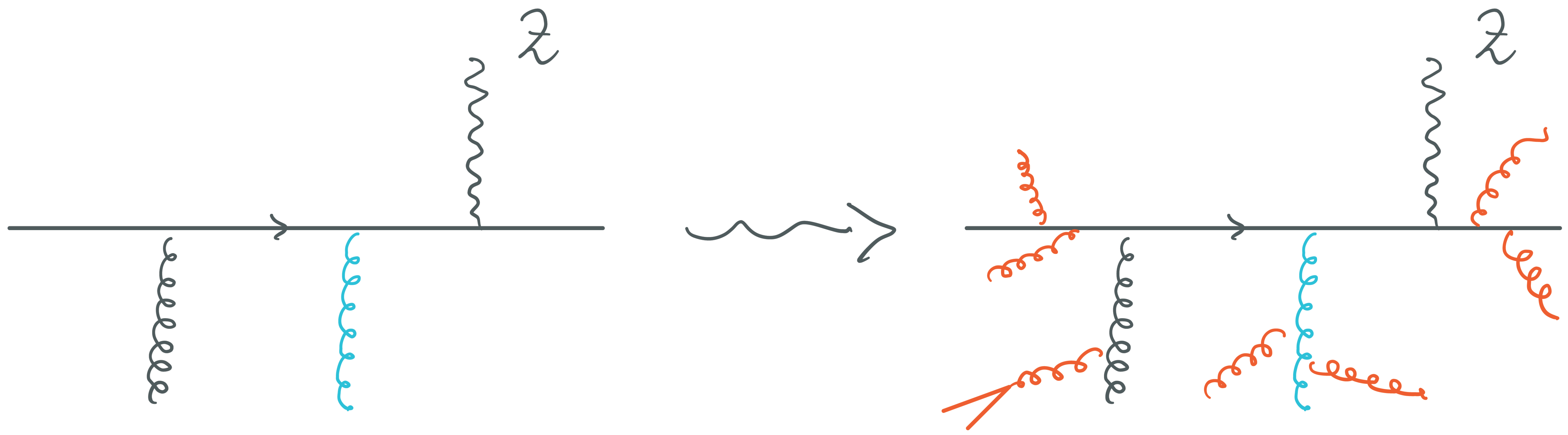
$$\Delta_{\mathrm{pwg}}(q) \equiv \exp\left\{ - \int_{k_\perp^{\mathrm{rad}} > q} d\Phi_{\mathrm{rad}} \frac{R(\Phi_{\mathrm{FJ}}, \Phi_{\mathrm{rad}})}{B(\Phi_{\mathrm{FJ}})} \right\}$$



Mimics a shower step ordered in k_T, with the full real matrix element (virtuals in $\bar{B}^{(\mathrm{FJ})}_{\mathrm{MiNLO}}$)

NLO + PS & merging jet multiplicities: MiNLO

- 3) NLO calculation now mimics the first two steps of a PS, so it is sufficient to let the actual shower (e.g. Pythia8) generate extra radiation requiring it has a transverse momentum smaller than the POWHEG radiation (PS starting scale)



- NB: crucial for the PS ordering to match transverse momentum near singular limit, otherwise extra fixes become necessary (e.g. truncated shower for angular ordering)

- Price to pay: junk beyond NLO in Z+jet (i.e. α_S^3) contaminates the simulation

NLO + PS & merging jet multiplicities: MiNLO

- An important byproduct is that now the jet can go unresolved (i.e. $q_\perp \to 0$)
  - Merging of 1 and 0 jet multiplicities: can one get NLO accuracy for both ?
    - Unresolved (0-jet) limit approached as the leading jet has $p_T^{jet} \to 0$

$$\bar{B}^{(\mathrm{FJ})}_{\mathrm{MiNLO}} = \frac{\alpha_s(q_\perp)}{2\pi}\left\{\frac{\Delta_f^2(Q)}{\Delta_f^2(q_\perp)}\left[B^{(\mathrm{FJ})}\left(1+\frac{\alpha_s(q_\perp)}{2\pi}S_f^{(1)}(q_\perp)\right)+\frac{\alpha_s(q_\perp)}{2\pi}V^{(\mathrm{FJ})}\right]+\int d\Phi_{\mathrm{rad}}\frac{\alpha_s(q_\perp)}{2\pi}\frac{\Delta_f^2(Q)}{\Delta_f^2(q_\perp)}R^{(\mathrm{FJ})}\right\}$$

- With LL accuracy, approximate the p_T^{jet} with the $q_\perp$ of the Z: Sudakov FF must account for the full singularity structure in the limit $q_\perp \to 0$ => Get it from $q_\perp$ resummation !

[Hamilton, Nason, Zanderighi, Oleari (2012)]

Small $q_\perp$ limit for colour singlet systems

- In the limit $q_\perp \to 0$ the differential cross section obeys a simple factorisation theorem*

$$\frac{d\sigma}{d\vec{q}_\perp d\Phi_F} \sim \sum_f |M^{(0)}_{f\bar{f}\to F}|^2 \int \frac{d^2\vec{b}}{(2\pi)^2} e^{i\vec{b}\cdot\vec{q}_\perp} e^{-R_f(b)} H_f \sum_{i,j} (C_{fi} \otimes h^{[i]})(C_{\bar{f}j} \otimes h^{[j]})$$

***Connection with MC manifest in momentum-space formulation (RadISH), not discussed in the following**

[PM, Re, Torrielli (2016); Bizon, PM, Re, Rottoli, Torrielli (2017)]

NLO + PS & merging jet multiplicities: MiNLO

- Simple form when averaged over azimuth of $\vec{q}_\perp$ and LL accuracy

$$\left[\frac{d\sigma}{d\vec{q}_\perp d\Phi_F}\right]_\phi \sim \frac{d}{dq_\perp}\left[\sum_f e^{-S_f(q_\perp)}\mathcal{L}_f(q_\perp)\right] + \mathcal{O}(\alpha_s^3(q_\perp))$$

- Allows us to identify the missing Sudakov FF

$$\bar{B}^{(\mathrm{FJ})}_{\mathrm{MiNLO}} = \frac{\alpha_s(q_\perp)}{2\pi}\left\{\frac{\Delta_f^2(Q)}{\Delta_f^2(q_\perp)}\left[B^{(\mathrm{FJ})}\left(1+\frac{\alpha_s(q_\perp)}{2\pi}S_f^{(1)}(q_\perp)\right)+\frac{\alpha_s(q_\perp)}{2\pi}V^{(\mathrm{FJ})}\right]+\int d\Phi_{\mathrm{rad}}\frac{\alpha_s(q_\perp)}{2\pi}\frac{\Delta_f^2(Q)}{\Delta_f^2(q_\perp)}R^{(\mathrm{FJ})}\right\}$$

- Mind the power counting

$$\int^Q \frac{dq_\perp}{q_\perp}\ln^n\frac{Q}{q_\perp}\alpha_s^m(q_\perp)e^{-S(q_\perp)} \sim \alpha_s^{m-\frac{n+1}{2}}(Q) \quad \Rightarrow$$

Full α_S^2 resummation structure needed to have NLO in both 0 and 1 jet events

[Hamilton, Nason, Zanderighi, Oleari (2012)]

NLO + PS & merging jet multiplicities: NNLOPS*

[Hamilton, Nason, Re, Zanderighi (2013)]

- NNLO for 0-jet events could be achieved by a local reweighing in the phase space of the the Z boson by $d\sigma_{NNLO}/d\sigma_{MiNLO}$: remarkably simple, computationally challenging for final states with many particles, e.g. ZZ, top pair, …

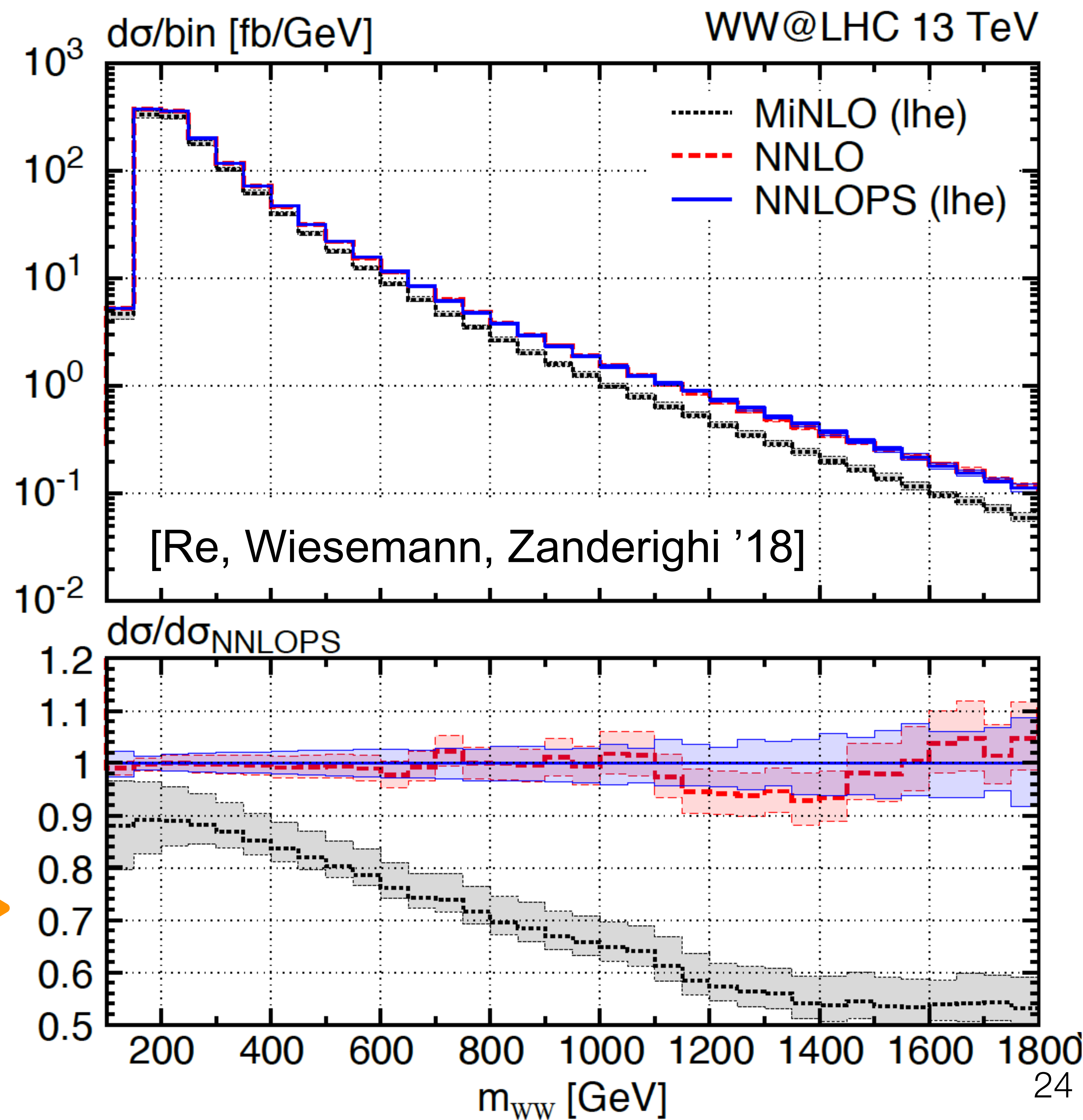
❌ discrete grids, hard to access remote regions

❌ CPU intensive

❌ tough high dimensional reweighing

e.g. W^+W^- production would require a 9-dim. diff. XS (recast as 81 grids using Collins-Soper decomposition) !

W^+W^- invariant mass



*Other NNLO+PS methods developed in

[Alioli, Bauer, Berggren, Tackmann, Walsh, Zuberi (2013)]
[Hoeche, Li, Prestel (2014)] [Hoeche, Kuttimalai, Li (2018)
[Alioli et al. (2019-2021)]

The MiNNLO$_{\rm PS}$ procedure

[PM, Nason, Re, Wiesemann, Zanderighi (2019)]
[PM, Re, Wiesemann (2020)]

- MiNNLO$_{\rm PS}$: compute full NNLO corrections directly in the weight, i.e.

$$\bar{B}^{(\rm FJ)}_{\rm MiNNLO_{PS}} = \frac{\alpha_s(q_\perp)}{2\pi}\left\{ \frac{\Delta_f^2(Q)}{\Delta_f^2(q_\perp)} \left[B^{(\rm FJ)}\left(1+\frac{\alpha_s(q_\perp)}{2\pi}S_f^{(1)}(q_\perp)\right) + \frac{\alpha_s(q_\perp)}{2\pi}V^{(\rm FJ)}\right] + \int d\Phi_{\rm rad}\frac{\alpha_s(q_\perp)}{2\pi}\frac{\Delta_f^2(Q)}{\Delta_f^2(q_\perp)}R^{(\rm FJ)} + D^{(\geq 3)}(\Phi_F, q_\perp)F_\ell^{\rm corr}(\Phi_{FJ}) \right\}$$

New term derived from $q_\perp$ resum., contains all terms required to achieve NNLO according to our power counting (α_S^3 corr.$^{\rm ns}$ needed)

$$D^{(\geq 3)}(\Phi_F, q_\perp) = \underbrace{-\frac{dS_f(q_\perp)}{dq_\perp}\mathcal{L}_f(q_\perp) + \frac{d\mathcal{L}_f}{dq_\perp}}_{D(q_\perp)} - \frac{\alpha_s(q_\perp)}{2\pi}[D(q_\perp)]^{(1)} - \frac{\alpha_s^2(q_\perp)}{(2\pi)^2}[D(q_\perp)]^{(2)}$$

Spreading of new corr.$^{\rm n}$ across $\Phi_{\rm FJ}$ at fixed $\Phi_{\rm F}$ (FKS)

$$F_\ell^{\rm corr}(\Phi_{\rm FJ}) = \frac{J(\Phi_{\rm FJ})}{\int d\Phi'_{\rm FJ} J(\Phi'_{\rm FJ})\delta(q_\perp - q'_\perp)\delta(\Phi_{\rm F} - \Phi'_{\rm F})}$$

$$J(\Phi_{\rm FJ}) = P(\Phi_{\rm rad})(h^{[i]}h^{[j]})$$

The MiNNLO$_{PS}$ procedure

[PM, Nason, Re, Wiesemann, Zanderighi (2019)]
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- MiNNLO$_{PS}$: compute full NNLO corrections directly in the weight, i.e.

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Spreading of new corr.^{n} across Φ_{FJ} at fixed Φ_F (FKS)

$$F_\ell^{\mathrm{corr}}(\Phi_{\mathrm{FJ}}) = \frac{J(\Phi_{\mathrm{FJ}})}{\int d\Phi'_{\mathrm{FJ}} J(\Phi'_{\mathrm{FJ}})\delta(q_\perp - q'_\perp)\delta(\Phi_{\mathrm{F}} - \Phi'_{\mathrm{F}})}$$

$$J(\Phi_{\mathrm{FJ}}) = P(\Phi_{\mathrm{rad}})(h^{[i]}h^{[j]})$$

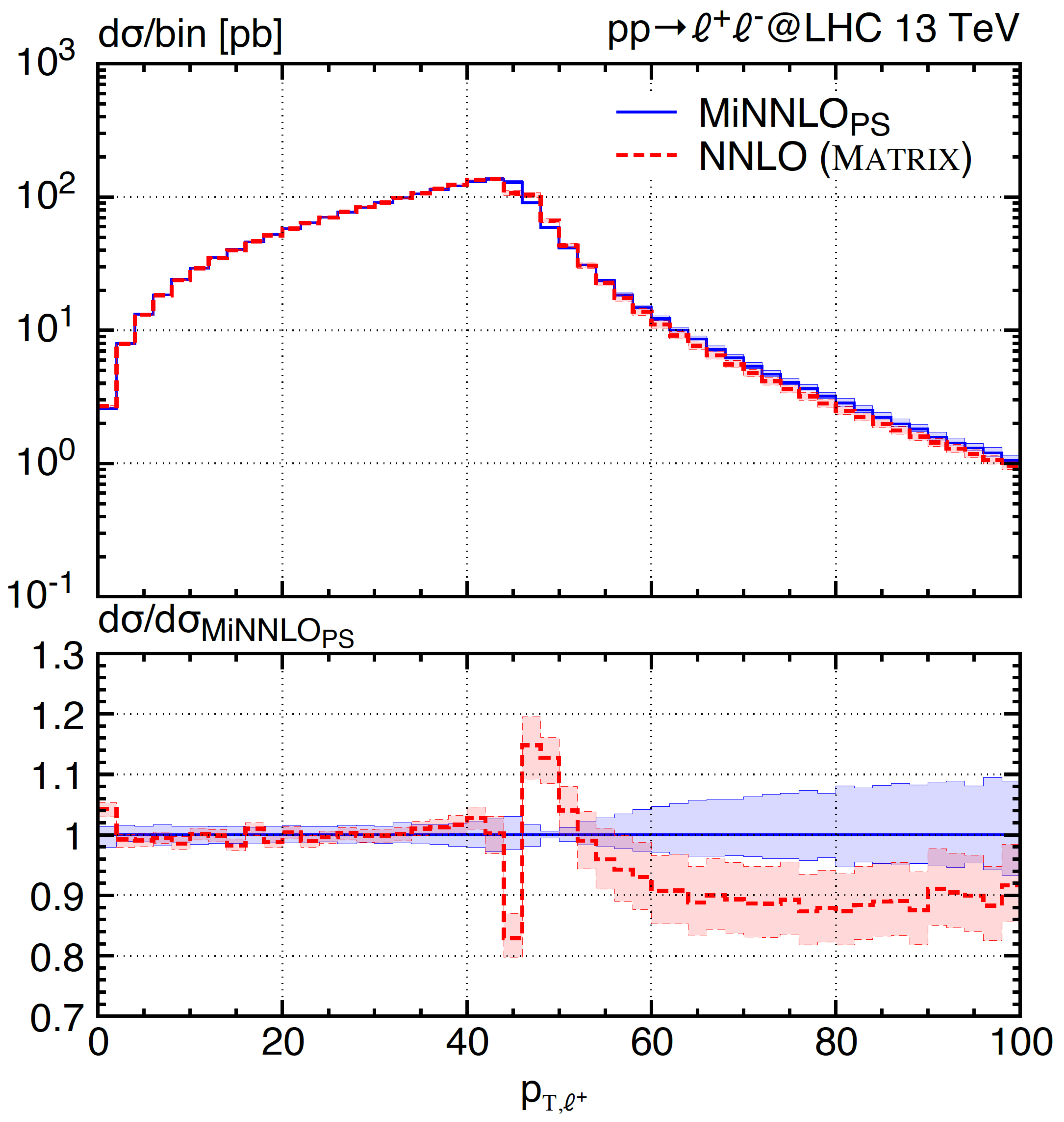
- ✓ Fully differential NNLO upon integration over $q_\perp$
- ✓ Marginal loss in speed w.r.t. NLO calculation
- ✓ Possible to tackle complex processes

MiNNLO$_{PS}$ for colour singlet prod.^{n}

- Higher order difference with fixed order NNLO:
  - Subleading corrections in matching to PS (inaccurate away from singular limits)
  - Scale variation in Sudakov FFs => slightly larger uncertainties than in FO

Total cross section MiNNLO$_{PS}$ vs. NNLO

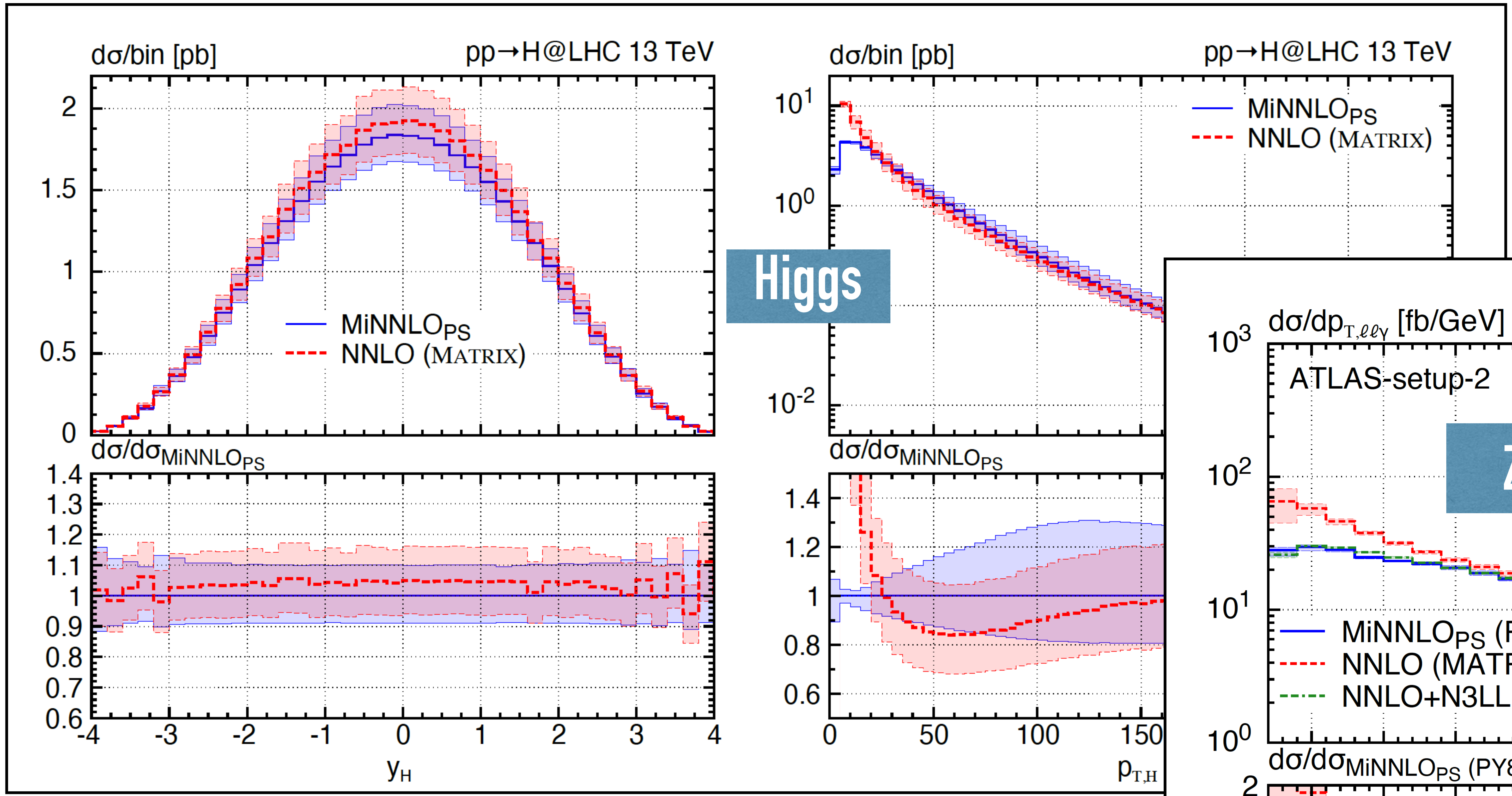
Process	NNLO (MATRIX)	MiNNLO$_{PS}$	Ratio
$pp \to \ell^+\ell^-$	$1919(1)^{+0.8\%}_{-1.1\%}$ pb	$1926(1)^{+1.4\%}_{-1.1\%}$ pb	1.004
$pp \to \ell^-\bar{\nu}_\ell$	$8626(4)^{+1.0\%}_{-1.2\%}$ pb	$8689(4)^{+1.7\%}_{-1.5\%}$ pb	1.007
$pp \to \ell^+\nu_\ell$	$11677(5)^{+0.9\%}_{-1.3\%}$ pb	$11755(5)^{+1.5\%}_{-1.6\%}$ pb	1.007

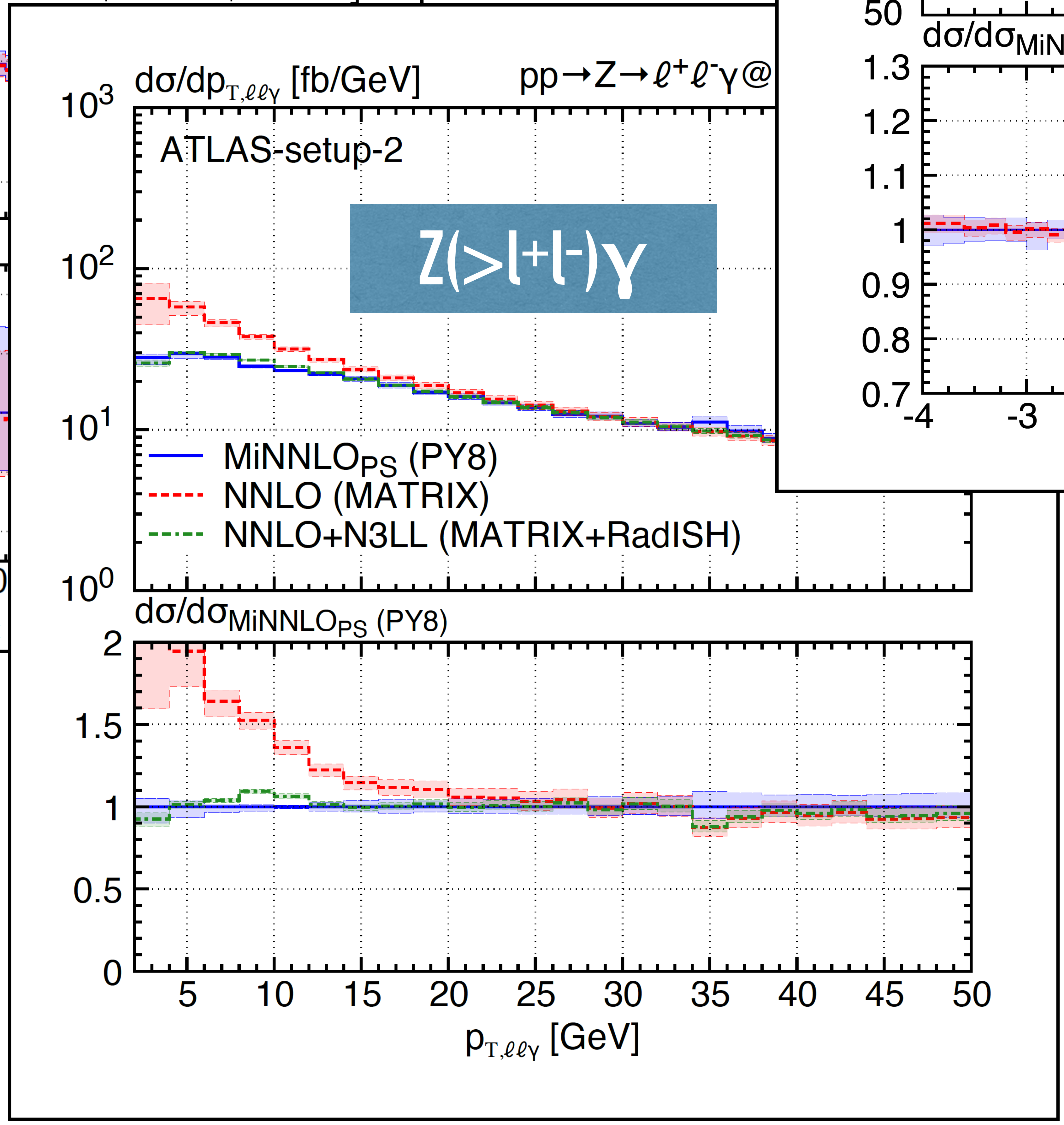


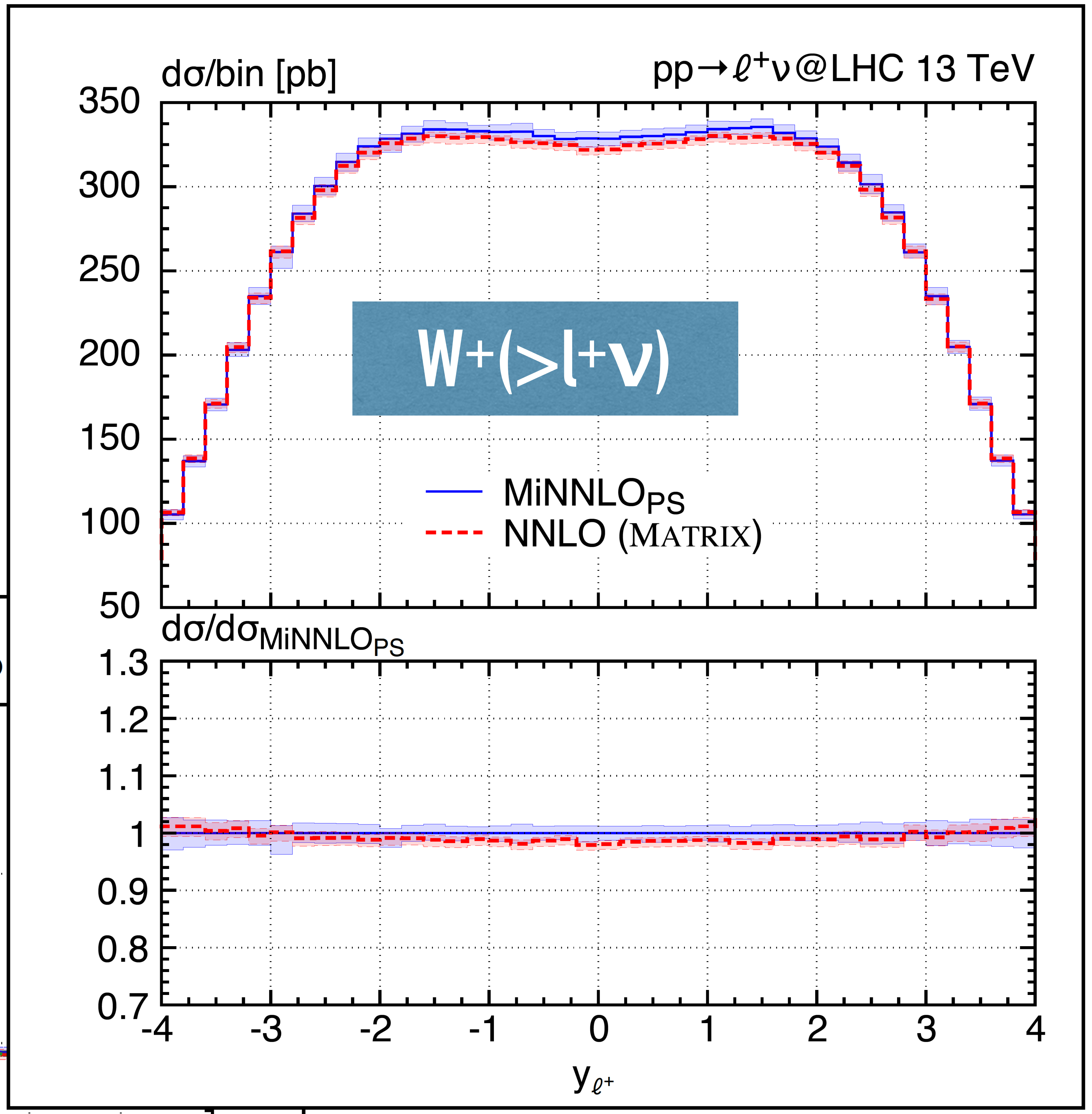
PS cures regions where fixed order description is inaccurate (e.g. Sudakov shoulder in lepton distr.)

MiNNLO$_{PS}$ for colour singlet prod.^{n}

Ongoing applications to colour singlet processes (e.g. Z, W, H, Z γ,. . .)







[PM, Nason, Re, Wiesemann, Zanderighi (2019)]
[PM, Re, Wiesemann (2020)]
[Lombardi, Wiesemann, Zanderighi (2020)]

Colour charges in the final states: top pair production

Reminder:

$$\bar{B}^{(\mathrm{FJ})}_{\mathrm{MiNNLO_{PS}}} \sim \frac{\Delta_f^2(Q)}{\Delta_f^2(q_\perp)} \cdots$$

Squared = 2 radiating legs.
Doesn't account for radiation off tops, notably initial-final & final-final soft interference

Colour charges in the final states: top pair production

[Zhu, Li, Li, Shao, Yang (2013)]
[Catani, Grazzini, Torre (2014)]

$$\frac{d\sigma}{d\vec{q}_\perp d\Phi_F} \sim \sum_f |M^{(0)}_{f\bar{f}\to F}|^2 \int \frac{d^2\vec{b}}{(2\pi)^2} e^{i\vec{b}\cdot\vec{q}_\perp} e^{-R_f(b)} H_f \sum_{i,j} (C_{fi} \otimes h^{[i]})(C_{\bar{f}j} \otimes h^{[j]})$$

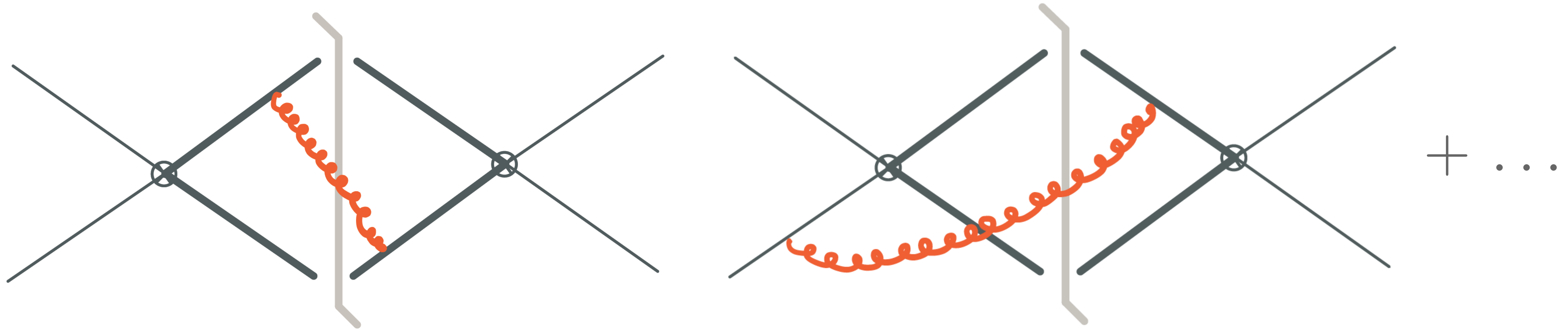
$$\Downarrow$$

$$\frac{d\sigma}{d\vec{q}_\perp d\Phi_F} \sim \sum_f |M^{(0)}_{f\bar{f}\to t\bar{t}}|^2 \int \frac{d^2\vec{b}}{(2\pi)^2} e^{i\vec{b}\cdot\vec{q}_\perp} e^{-R_f(b)} \mathrm{Tr}\left(\mathbf{H}_f \mathbf{\Delta}_{\mathrm{soft}}\right) \sum_{i,j} (C_{fi} \otimes h^{[i]})(C_{\bar{f}j} \otimes h^{[j]})$$

$$\mathrm{Tr}\left(\mathbf{H}_f \mathbf{\Delta}_{\mathrm{soft}}\right) = \frac{\langle M^{(0)}_{f\bar{f}} | \mathbf{\Delta} | M^{(0)}_{f\bar{f}} \rangle}{|M^{(0)}_{f\bar{f}}|^2}, \qquad \mathbf{\Delta} = \mathbf{V}^\dagger \mathbf{D} \mathbf{V}$$

$$\mathbf{V} = \mathcal{P} \exp\left\{ - \int_{\frac{b_0^2}{b^2}}^{M_{t\bar{t}}^2} \frac{dq^2}{q^2} \mathbf{\Gamma}_t(\Phi_{t\bar{t}}, \alpha_s(q)) \right\}$$

V and D encode soft interference up to two loops



Colour charges in the final states: top pair production

- With LL and NNLO accuracy, the azimuthally averaged distribution takes a simpler form

[Mazzitelli, PM, Nason, Re, Wiesemann, Zanderighi (2020)]

Soft interference pattern is split into 3 contributions that can be matched to the MiNNLO$_{PS}$ weight

$$\left[\frac{d\sigma}{d\vec{q}_\perp d\Phi_F}\right]_\phi \sim \frac{d}{dq_\perp}\left[\sum_f e^{-S_f(q_\perp)} \langle M^{(0)}_{f\bar{f}}|(\mathbf{V}_{\rm NLL})^\dagger \mathbf{V}_{\rm NLL}|M^{(0)}_{f\bar{f}}\rangle [\mathrm{Tr}\,(\mathbf{H}_f \mathbf{D}_{\rm soft})\sum_{i,j}(C_{fi}\otimes h^{[i]})(C_{\bar{f}j}\otimes h^{[j]})]_\phi\right] + \mathcal{O}(\alpha_s^5(q_\perp))$$

$$S_f(q_\perp) = \int_{q_\perp^2}^{Q^2} \frac{dq^2}{q^2}\left(A(\alpha_s(q))\ln\frac{Q^2}{q^2} + B(\alpha_s(q))\right)$$

$$A(\alpha_s) = \frac{\alpha_s}{2\pi}A^{(1)} + \frac{\alpha_s^2}{(2\pi)^2}A^{(2)}$$

$$B(\alpha_s) = \frac{\alpha_s}{2\pi}B^{(1)} + \frac{\alpha_s^2}{(2\pi)^2}B^{(2)}$$

Ingredients used in slicing NNLO calculations and derived in:
[Baernreuther, Czakon, Fiedler (2013)] [Czakon (2008)]
[Catani, Grazzini, Torre (2014)]
[Catani, Grazzini, Sargsyan (2018)]
[Catani, Devoto, Grazzini, Kallweit, Mazzitelli (2019) + Sargsyan (2019)]

Scales & modified logs

- Scales setup:
  - 2 Born powers of the coupling @ $\mu_R = K_R\, m_{tt}/2$
  - Everywhere else ($Q = m_{tt}/2$): $\mu_R = K_R\, m_{tt}/2\, e^{-L}$, $\mu_F = K_F\, m_{tt}/2\, e^{-L}$

$$L = \begin{cases} \ln \frac{Q}{q_\perp} & \text{for}\, q_\perp \lesssim \frac{Q}{2} \\ 0 & \text{for}\, q_\perp \geq Q \end{cases}$$

  - Vary scales by a factor of 2 (7 pts), including Sudakov (slightly more conservative than FO)
  - Smooth freezing of PDFs at Q_0=2 GeV
  - Stable top quarks
  - Exp. data from CMS (arXiv:1803.08856) unfolded to inclusive phase space (no fid. cuts)

Rapidity & total cross section

Total cross section slightly (3.5%) smaller than NNLO, with similar scale uncertainties

Inclusive distributions (e.g. tt rapidity) expected to be NNLO accurate (good agreement with NNLO fixed order — small subleading difference)

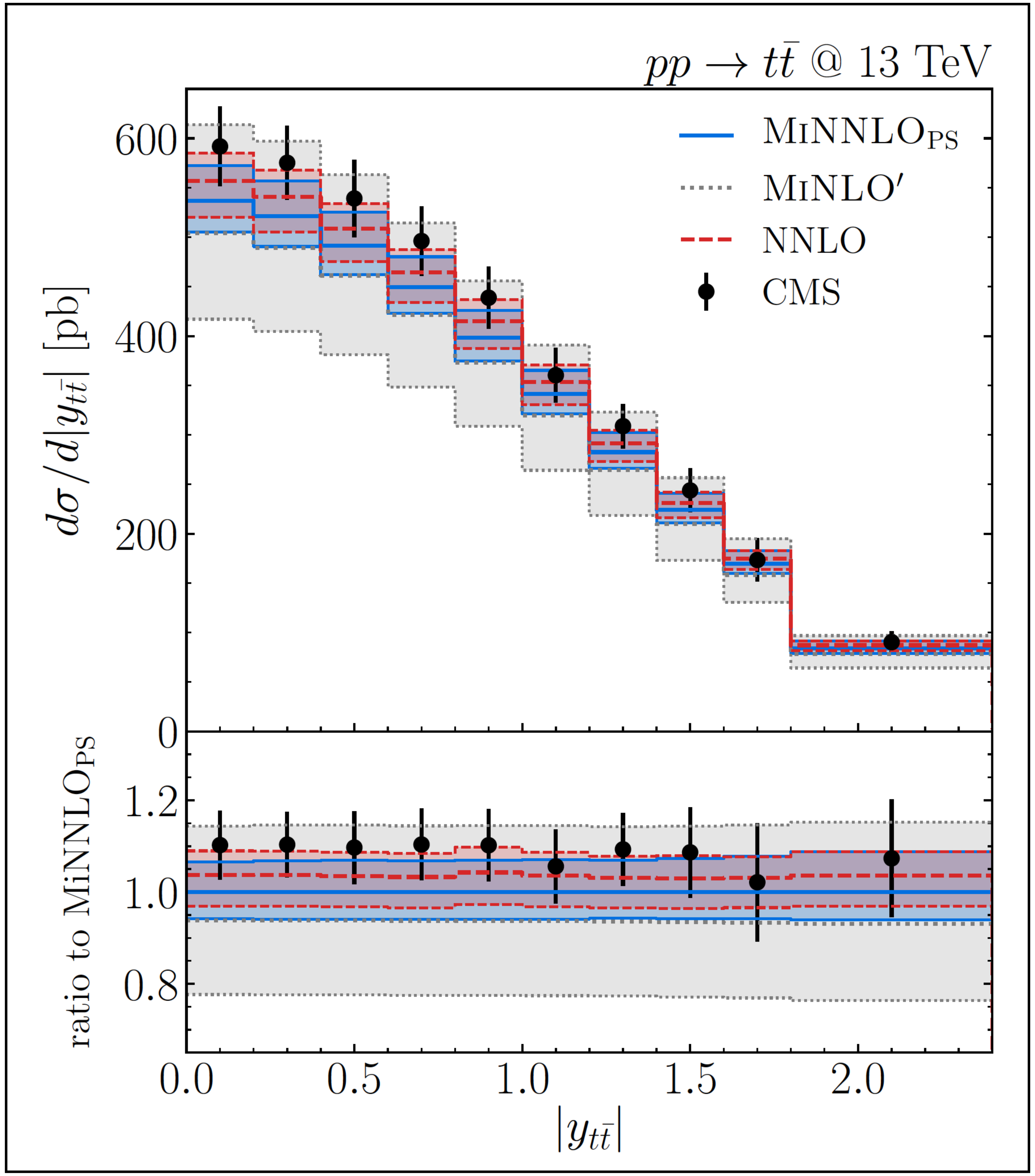
Significant scale reduction w.r.t. MiNLO

Total cross section MiNLO vs. MiNNLO$_{PS}$ vs. NNLO

MiNLO′	NNLO	MiNNLO$_{PS}$
$695.6(3)^{+22\%}_{-17\%}$ pb	$769.8(9)^{+5.0\%}_{-6.5\%}$ pb	$742.6(3)^{+7.2\%}_{-5.9\%}$ pb

NNLO calculation in:
[Baernreuther, Czakon, Mitov (2012); Czakon, Fiedler, Mitov (2013); Czakon, Heymes, Mitov (2015); Behring, Czakon, Mitov, Papanastasiou, Poncelet (2019); Czakon, Mitov, Poncelet (2020), …]
[Catani, Devoto, Grazzini, Kallweit, Mazzitelli (2019) + Sargsyan (2019)]

Rapidity of the top pair



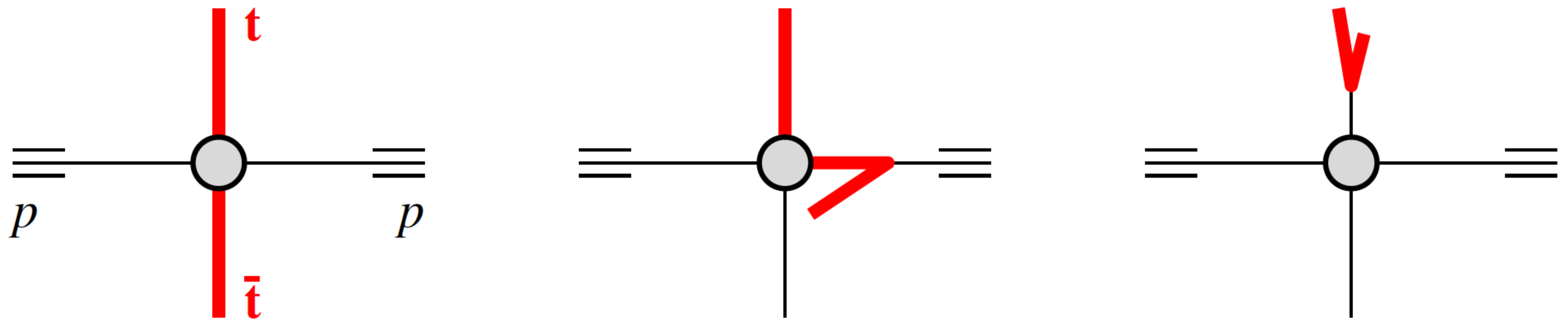
Invariant mass spectrum & scales

Good description of invariant mass spectrum, with the exception of the tt threshold bin (sensitivity to finite width & non-relativistic effects)

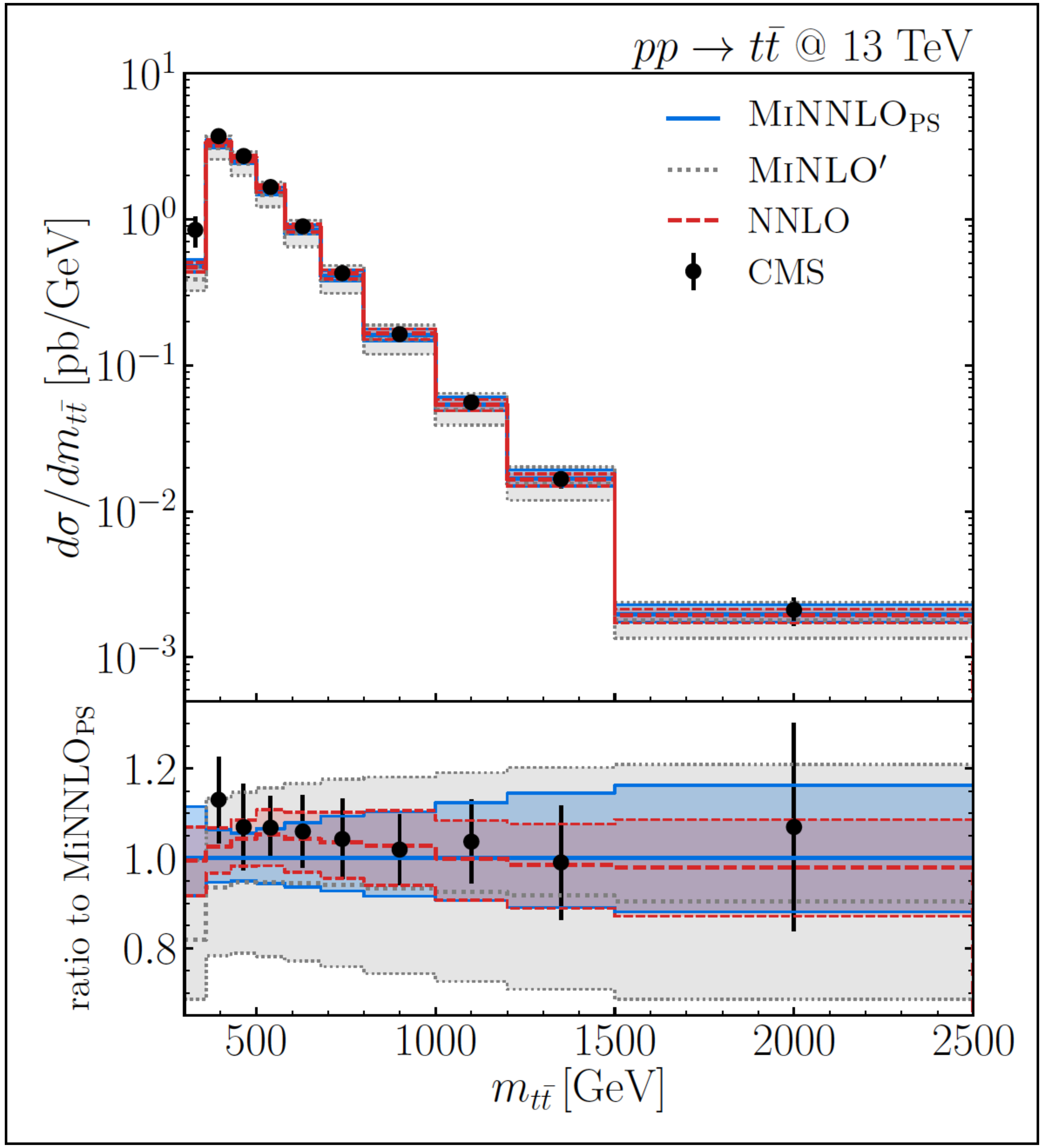
Slightly larger uncertainty in the tail reflects extra sources of scale variation

May be interesting to study scale assignment across tt topologies (including different choices of the hard scale at large $q_{\perp}$)

[Caola, Dreyer, McDonald, Salam (2020)]

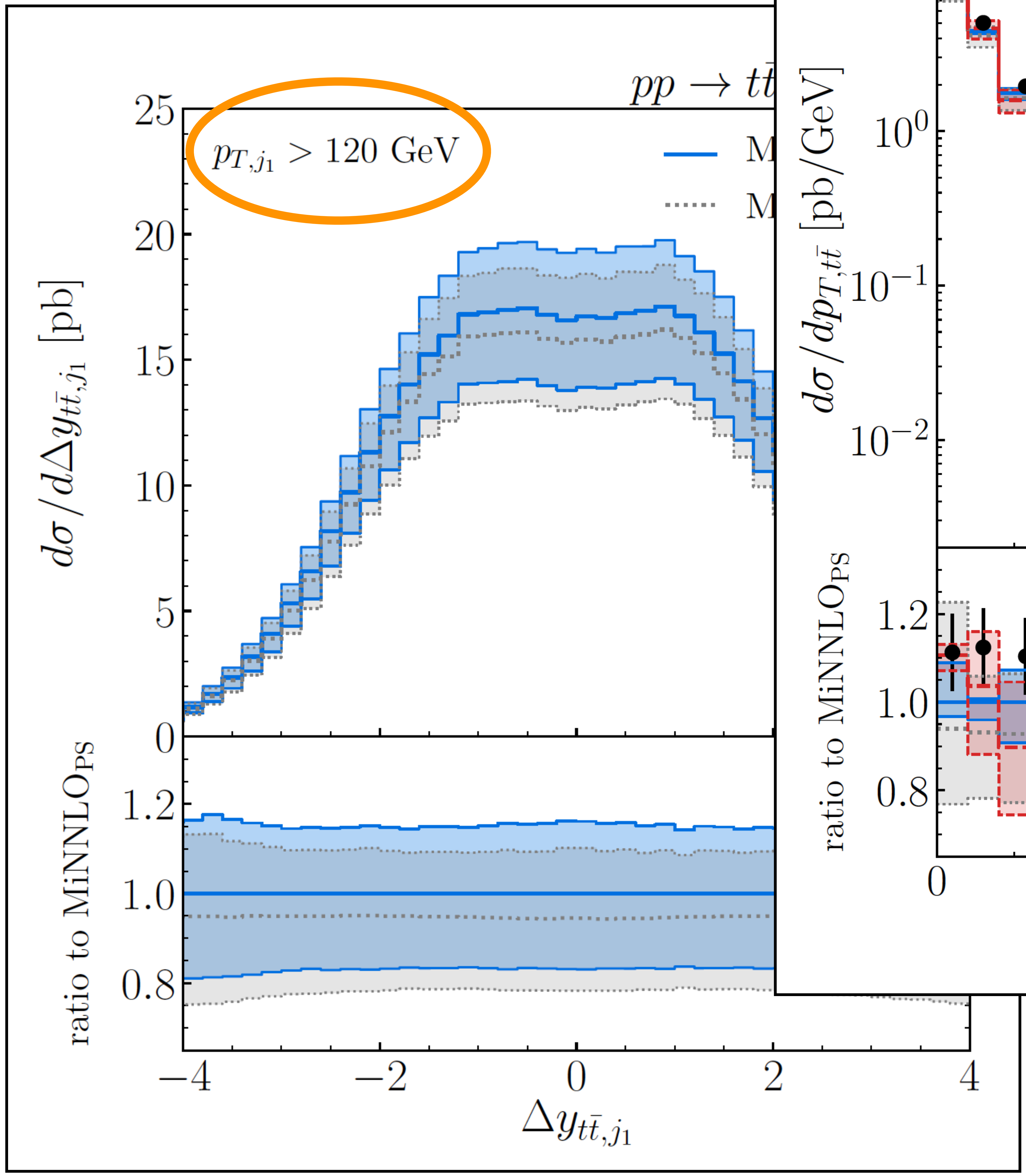


Top pair invariant mass

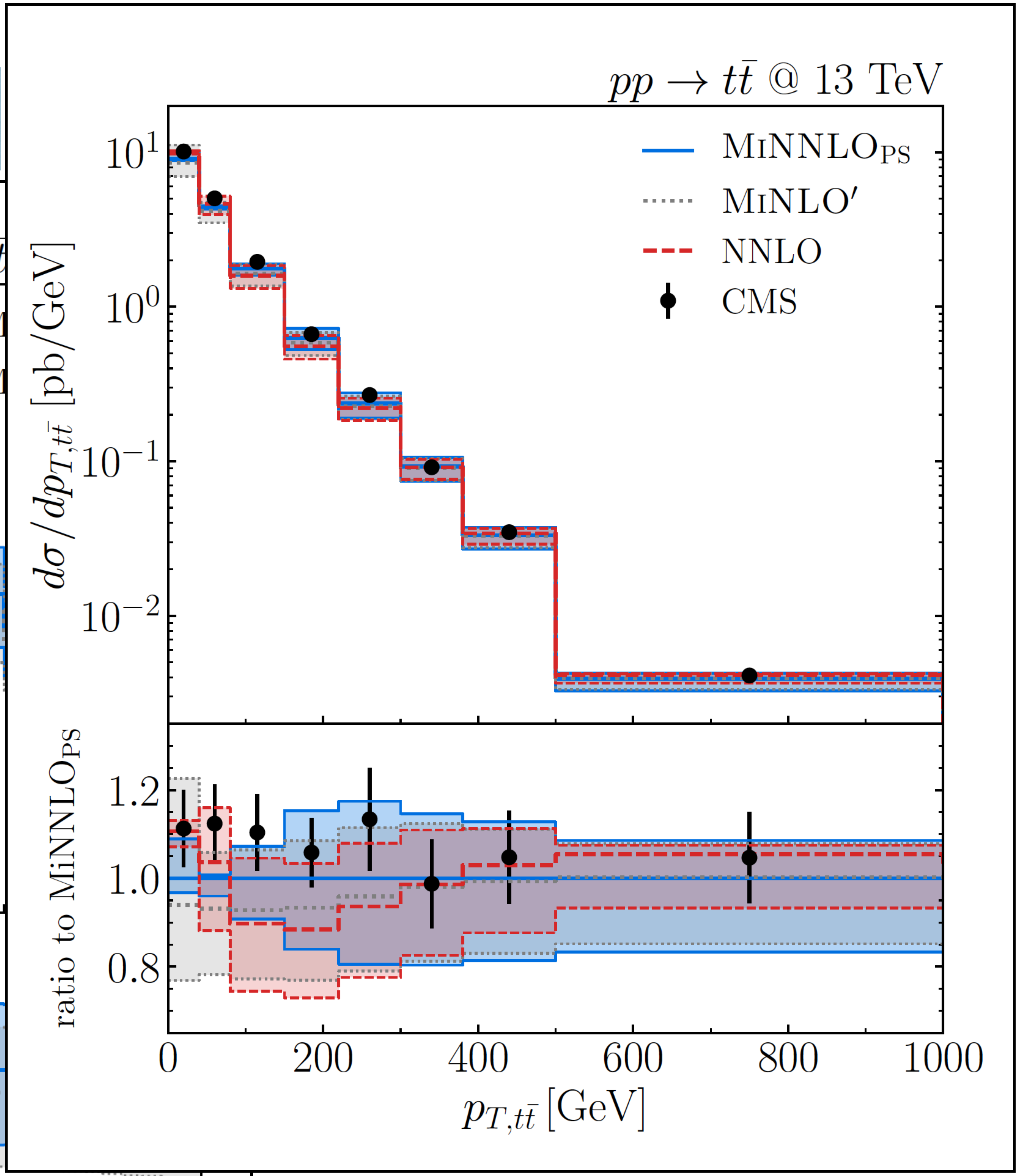


Jet sensitive observables

tt-jet rapidity distance



Top pair's transverse momentum



Obs. which resolve a radiation expected to be NLO, good agreement with MiNLO (except for the small $q_\perp$, unresolved limit)

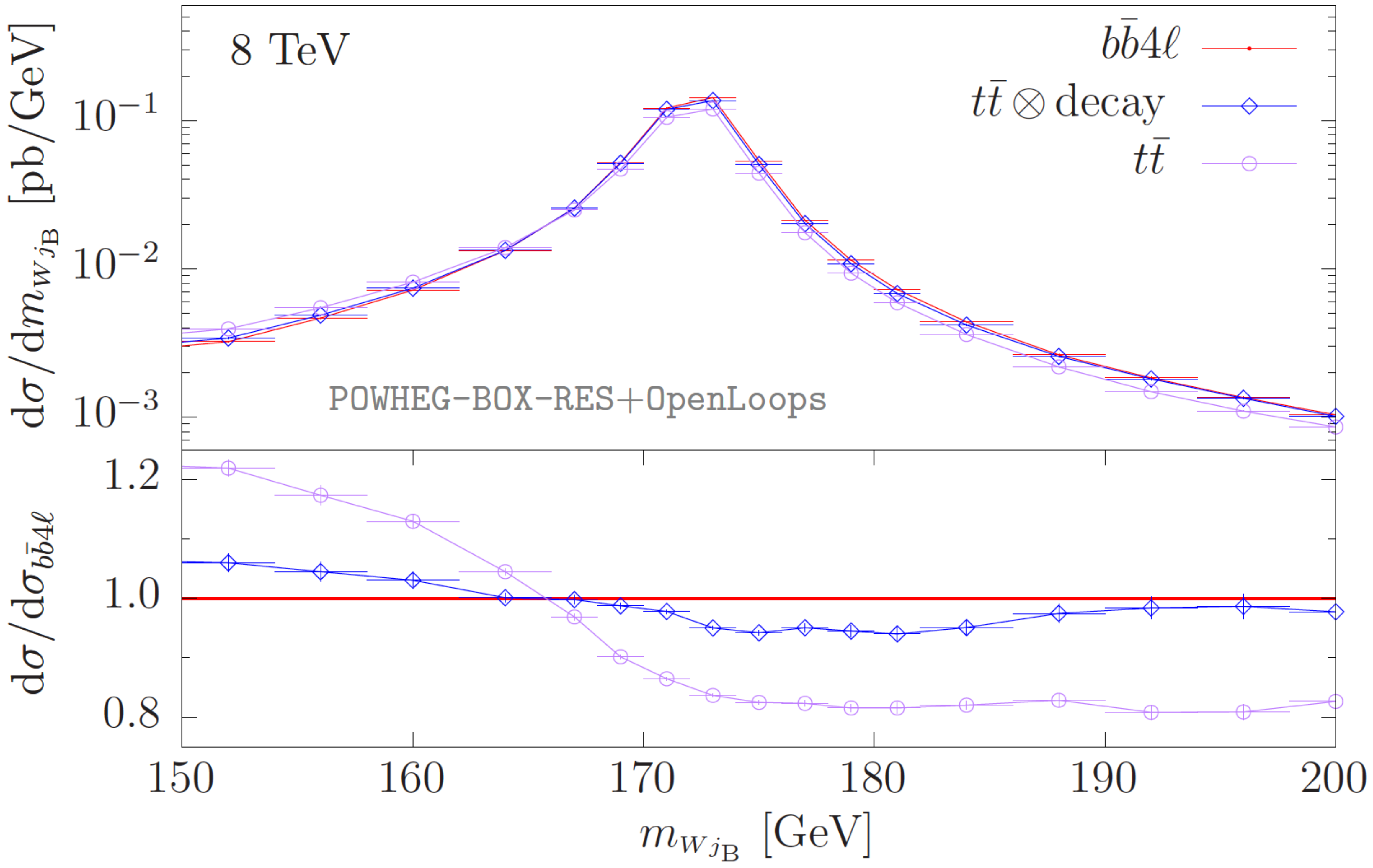
Good agreement with data

Top decays

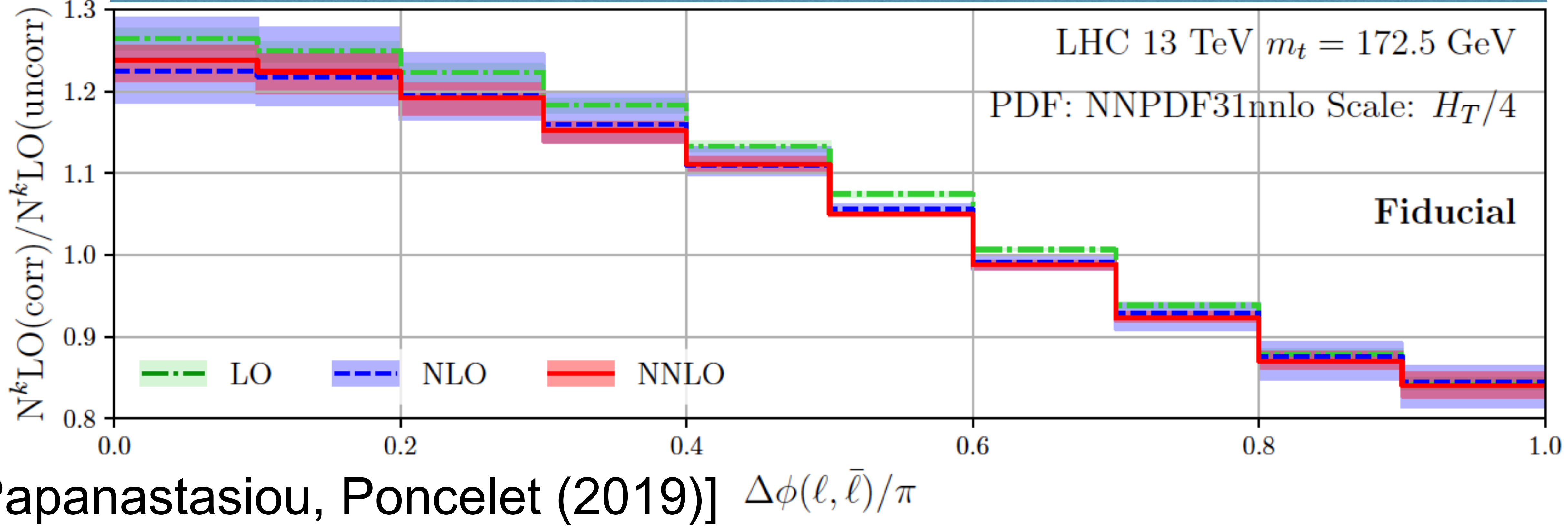
[Jezo, Lindert, Nason, Oleari, Pozzorini (2016)]

- Inclusion of top decays paramount for realistic experimental cuts
  - A cheap option is to let the PS perform the decay, though with very limited pert. accuracy
  - Possible avenue is the inclusion in NWA @ N(N)LO+PS, though significant work is required to retain spin correlations (e.g. density matrix)
- Full NLO (off-shell+spin corr., non-resonant channels) is available @ NLO+PS
  - Interesting to assess effects of spin correlation in leptonic observables @ N(N)LO+PS (possible hints at unfolding/extrapolation issues ?)

Full NLO+PS vs. various approximations



Effect of spin correlations on Δφ



[Behring, Czakon, Mitov, Papanastasiou, Poncelet (2019)]

Logarithmic accuracy: bridging PS and resummation

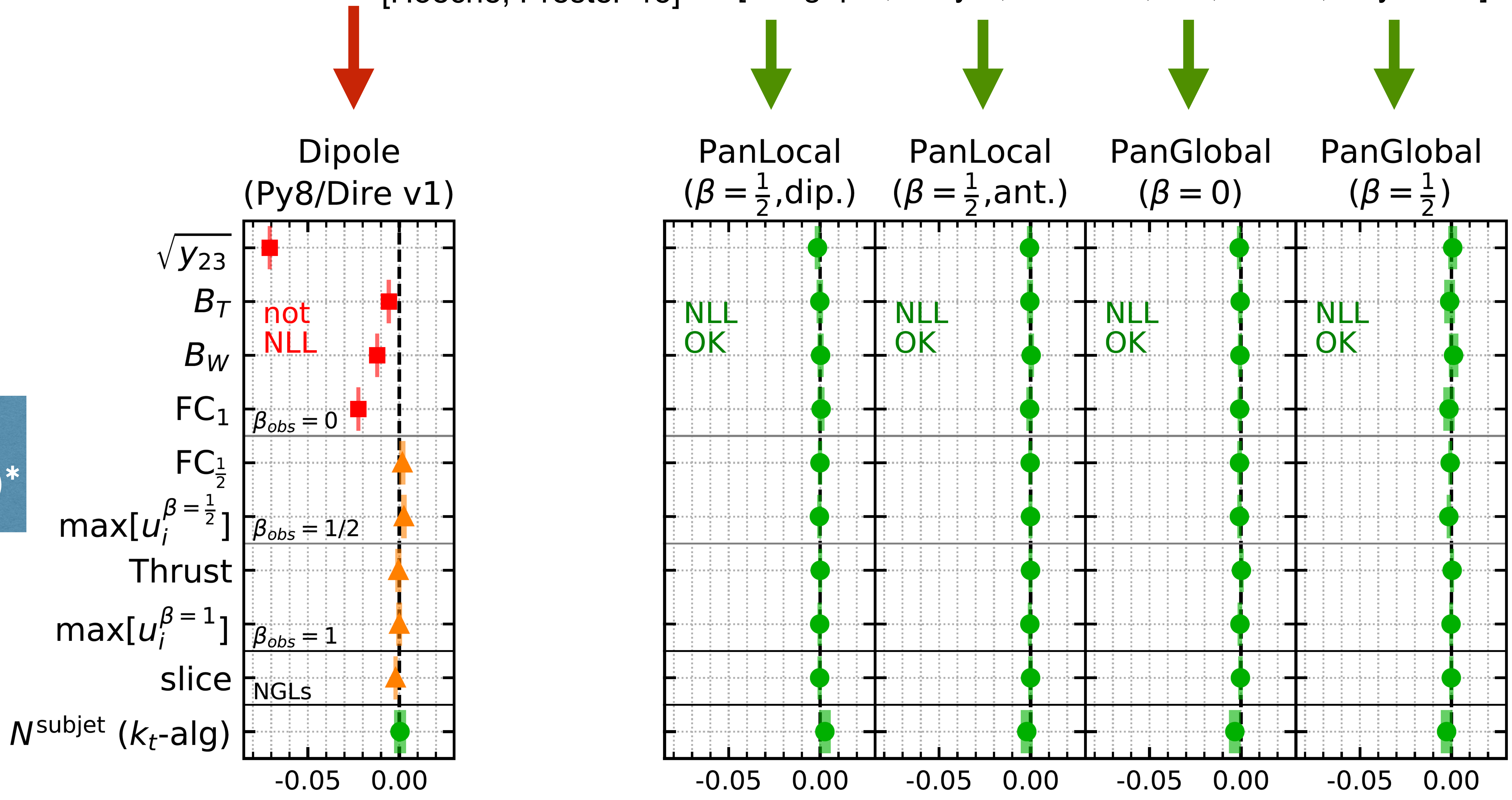
- Parton shower algorithms are being pushed beyond LL … e.g.

New PanScales showers achieve NLL accuracy across many obs.

[Sjostrand et al. ‘15]
[Hoeche, Prestel ’15]

[Dasgupta, Dreyer, Hamilton, PM, Salam, Soyez ’20]

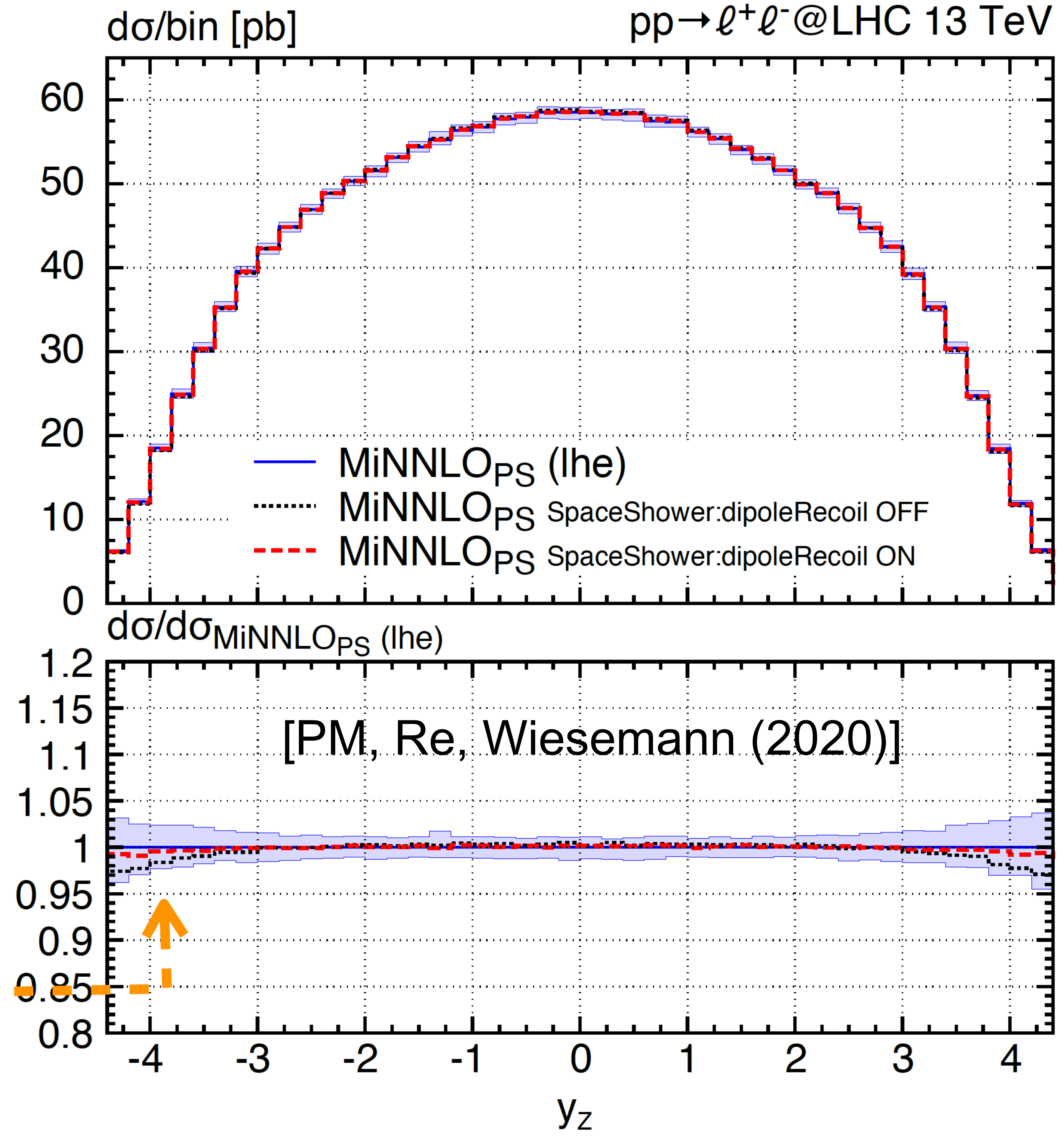
Plots: relative deviation from exact NLL (large N_C)*

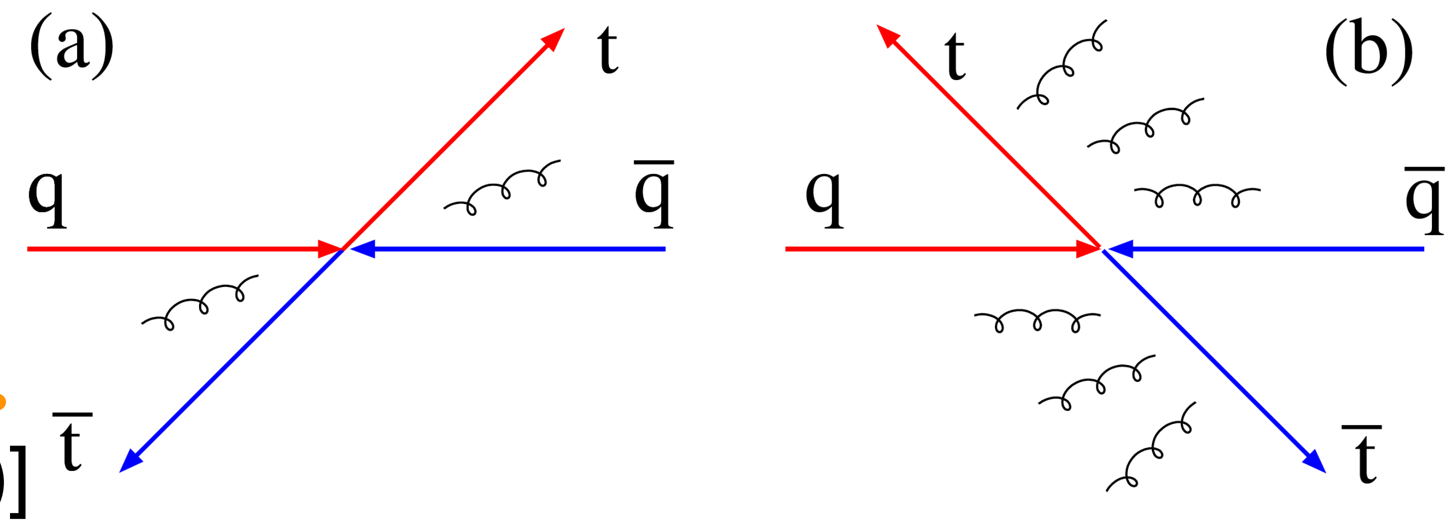


*Full colour can be achieved for global obs. with up to 3 emitters (=>backup)

Logarithmic accuracy: matching to higher order PS

- A N(N)LO+PS generator is at best as accurate as the PS algorithm for given classes of observable
- Crucial to explore consistent matching solutions for NLL (or higher) shower algorithms.
  - Many sources of (logarithmic) problems:
    - resolution variable vs. shower ordering
    - log. accuracy of the weight (pre-shower)
    - kinematic maps & constraint on the shower
    - …
  - Additional questions concern subleading power (regular) effects in distributions (e.g. longitudinal recoil effects @ large y_Z or inclusive A_{FB}'s in tt)





[Skands, Webber, Winter (2012)]

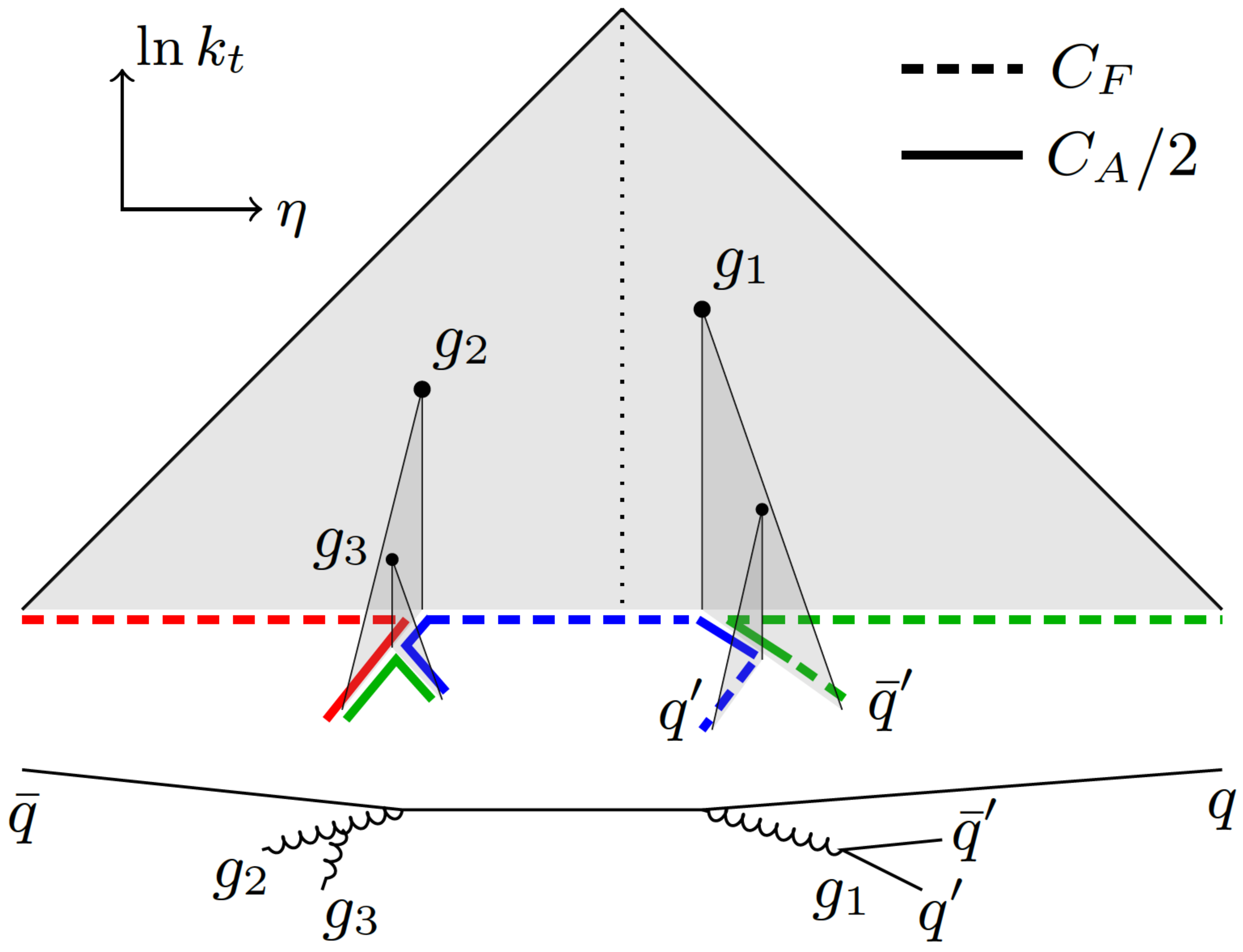
Summary

- MiNNLO$_{PS}$ for tt production:
  - first NNLO+PS simulation for a reaction with colour charges in the final state
  - good description of data, though in depth phenomenological studies yet to be performed across multiple observables, study of scales, etc.
- Avenue towards NNLO+PS for jet processes with appropriately selected resolution variables (full resummation structure up to NNLO needed as an input)
- Future directions necessarily involve study of top decays, as well as developing a solid understanding of the implications of the matching technology for the logarithmic accuracy of MC generators

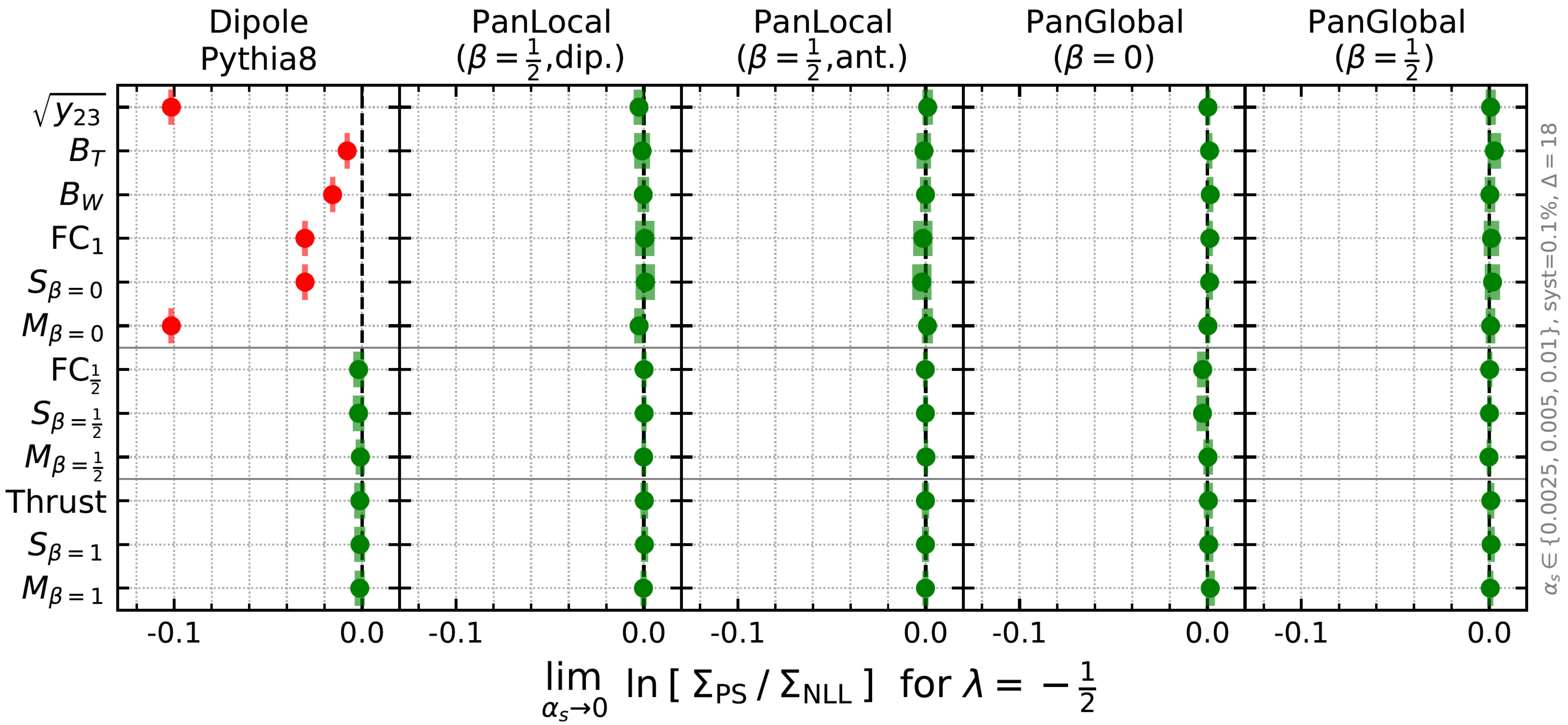
Backup material

Beyond the planar limit: subleading N_c

- Same guiding principles can be used to include some information about subleading colour corrections
- Full colour accuracy can be achieved for global observables in processes with up to three coloured legs



NLL accuracy test – NODS procedure



[Hamilton, Medves, Salam, Scyboz, Soyez ’20]
see also related work by
[Plaetzer, Sjodahl ‘12 + Thoren ’18; Nagy, Soper ’12-’19; Hoeche, Reichelt ’20; De Angelis, Forhsaw, Plaetzer ’20; Forshaw, Holguin, Plaetzer ’20]