Leading hadronic contribution to the muon magnetic moment from lattice QCD

Laurent Lellouch

CPT & IPhU Marseille
CNRS & Aix-Marseille U.

Budapest-Marseille-Wuppertal collaboration [BMWc]

Borsanyi, Fodor, Guenther, Hoelbling, Katz, LL, Lippert, Miura, Szabo, Parato, Stokes, Toth, Torok, Varnhorst

Nature 593 (2021) 51, online 7 April 2021 $\rightarrow$ BMWc '20
PRL 121 (2018) 022002 (Editors' Selection) $\rightarrow$ BMWc '17
& Aoyama et al., Phys. Rept. 887 (2020) 1-166 $\rightarrow$ WP '20

Lepton magnetic moments and BSM physics

Interaction with an external EM field: QM

Dirac eq. w/ minimal coupling (1928):

$$i\hbar\frac{\partial\psi}{\partial t} = \left[\vec{\alpha}\cdot\left(c\frac{\hbar}{i}\vec{\nabla} - e_\ell\vec{A}\right) + \beta c^2 m_\ell + e_\ell A_0\right]\psi$$

nonrelativistic limit $\downarrow$ (Pauli eq.)

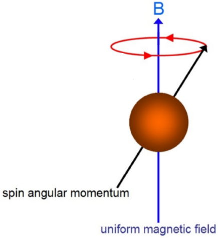
$$i\hbar\frac{\partial\phi}{\partial t} = \left[\frac{\left(\frac{\hbar}{i}\vec{\nabla} - \frac{e_\ell}{c}\vec{A}\right)^2}{2m_\ell} - \underbrace{\frac{e_\ell\hbar}{2m_\ell}\vec{\sigma}\cdot\vec{B}}_{\vec{\mu}_\ell\cdot\vec{B}} + e_\ell A_0\right]\phi$$

with

$$\vec{\mu}_\ell = g_\ell\left(\frac{e_\ell}{2m_\ell}\right)\vec{S}, \qquad \vec{S} = \hbar\frac{\vec{\sigma}}{2}$$

and

$$g_\ell\big|_{\text{Dirac}} = 2$$



"That was really an unexpected bonus for me, completely unexpected." (P.A.M. Dirac)

Interaction with an external EM field: QFT

Assuming Poincaré invariance and current conservation ($q^\mu J_\mu = 0$ with $q \equiv p' - p$):

$$\langle \ell(p')|J_\mu(0)|\ell(p)\rangle = \bar{u}(p')\left[\gamma_\mu F_1(q^2) + \frac{i}{2m_\ell}\sigma_{\mu\nu}q^\nu F_2(q^2) - \gamma_5\sigma_{\mu\nu}q^\nu F_3(q^2)\right.$$

$$\left. +\gamma_5(q^2\gamma_\mu - 2m_\ell q_\mu)F_4(q^2)\right] u(p)$$

$F_1(q^2) \rightarrow$ Dirac form factor: $F_1(0) = 1$

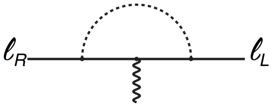
$F_2(q^2) \rightarrow$ Pauli form factor, magnetic dipole moment: $F_2(0) = a_\ell = \dfrac{g_\ell - \overbrace{2}^{g_\ell|_{\rm Dirac}}}{2}$

$F_3(q^2) \rightarrow$ $\not{P}$, $\not{T}$, electric dipole moment: $F_3(0) = d_\ell/e_\ell$

$F_4(q^2) \rightarrow$ $\not{P}$, anapole moment: $\vec{\sigma}\cdot(\vec{\nabla}\times\vec{B})$

- q^2 dependence of $F_1(q^2)$, $F_2(q^2)$ & $F_{3,4}(q^2)$ come from loops but UV finite once charges and masses are renormalized (in a renormalizable theory)
- a_ℓ dimensionless
  - $\Rightarrow$ corrections including only ℓ and γ are mass independent, i.e. universal
  - $\rightarrow$ contributions from particles w/ $M \gg m_\ell$ are $\propto (m_\ell/M)^{2p} \times \ln^q(m_\ell^2/M^2)$
  - $\rightarrow$ contributions from particles w/ $m \ll m_\ell$ are e.g. $\propto \ln(m_\ell^2/m^2)$

Why are a_ℓ special?



$$\longrightarrow \quad \frac{a_\ell}{2m_\ell} eF^{\mu\nu}[\bar{\ell}_L\sigma_{\mu\nu}\ell_R]$$

- $a_{e,\mu}$ are parameter-free predictions of the SM and can be measured directly, very precisely $\rightarrow$ excellent tests of SM
- Loop induced $\Rightarrow$ sensitive to new dofs that may be too heavy or too weakly coupled to be produced directly
- CP and flavor conserving, chirality flipping $\Rightarrow$ complementary to: EDMs, s and b decays, LHC direct searches, . . .
- Chirality flipping $\Rightarrow$ generic contribution of particle w/ $M \gg m_\ell$

$$a_\ell^{\mathrm{M}} = C\left(\frac{\Delta_{LR}}{m_\ell}\right)\left(\frac{m_\ell}{M}\right)^2$$

- In EW theory, $M = M_W$, chirality flipping from Yukawa and weak interactions, i.e.

$$\Delta_{LR} = m_\ell \qquad \text{and} \qquad C \sim \frac{\alpha}{4\pi\sin^2\theta_W}$$

- In BSM, can have chiral enhancement: e.g. SUSY $M = M_{\mathrm{SUSY}}$ and $C \sim \alpha/\left(4\pi\sin^2\theta_W\right)$ & $\Delta_{LR} = (\mu/M_{\mathrm{SUSY}}) \times \tan\beta \times m_\ell$; or radiative m_ℓ model, $\Delta_{LR} \simeq m_\ell$, $C \sim 1$ and $M = M_{\mathrm{N\Phi}}$

Why is a_μ special?

$m_e : m_\mu : m_\tau = 0.0005 : 0.106 : 1.777\,\mathrm{GeV}$ $\tau_e : \tau_\mu : \tau_\tau =$ "∞" $: 2.\cdot 10^{-6} : 3.\cdot 10^{-15}\,\mathrm{s}$

- a_μ is $(m_\mu/m_e)^2 \sim 4. \times 10^4$ times more sensitive to new Φ than a_e
- a_τ is even more sensitive to new Φ, but is too shortly lived
- τ_μ small but manageable

$\rightarrow$ measure & compute a_μ in SM as precisely as possible

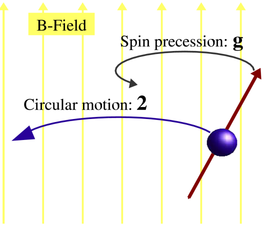
Big question:

$$a_\mu^{\rm exp} = a_\mu^{\rm SM}?$$

If not, there must be new Φ

Experimental measurement of a_μ

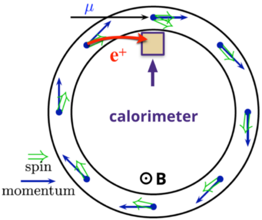
Measurement principle for a_μ



Precession determined by

$$\vec{\mu}_\mu = 2(1+a_\mu)\frac{Qe}{2m_\mu}\vec{S}$$

$$\vec{d}_\mu = \eta_\mu \frac{Qe}{2m_\mu c}\vec{S}$$



$$\vec{\omega}_{a\eta} = \vec{\omega}_a + \vec{\omega}_\eta = -\frac{Qe}{m_\mu}\left[a_\mu \vec{B} + \left(a_\mu - \frac{1}{\gamma^2-1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right] - \eta_\mu \frac{Qe}{2m_\mu}\left[\frac{\vec{E}}{c} + \vec{\beta}\times\vec{B}\right]$$

- Experiment measures very precisely $\vec{B}$ with $|\vec{B}| \gg |\vec{E}|/c$ &

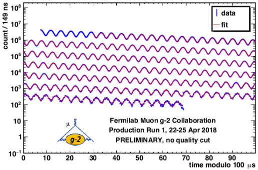
$$\Delta\omega \equiv \omega_S - \omega_C \simeq \sqrt{\omega_a^2 + \omega_\eta^2} \simeq \omega_a$$

since $d_\mu = 0.1(9)\times 10^{-19} e\cdot\mathrm{cm}$ (Benett et al '09)

- Consider either magic $\gamma = 29.3$ (CERN/BNL/Fermilab) or $\vec{E} = 0$ (J-PARC)

$$\to \Delta\omega \simeq -a_\mu B \frac{Qe}{m_\mu}$$

Fermilab E989 @ magic γ: measurement (simplified)



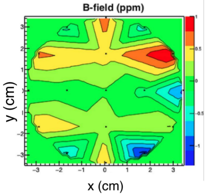
ω_a

Extract from decay positron time spectra

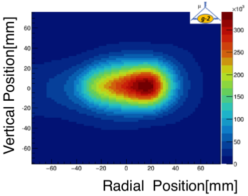
$$N(t) = N_0 e^{-t/\tau_\mu}[1 + A\cos(\omega_a t + \phi)]$$

$$a_\mu = \left(\frac{g_e}{2}\right)\left(\frac{\omega_a}{\langle\omega_p\rangle}\right)\left(\frac{\mu_p}{\mu_e}\right)\left(\frac{m_\mu}{m_e}\right)$$

0.26 ppt — 3 ppb — 22 ppb ⇨ 2017 CODATA



Map the magnetic field



Obtain muon distribution In the storage ring

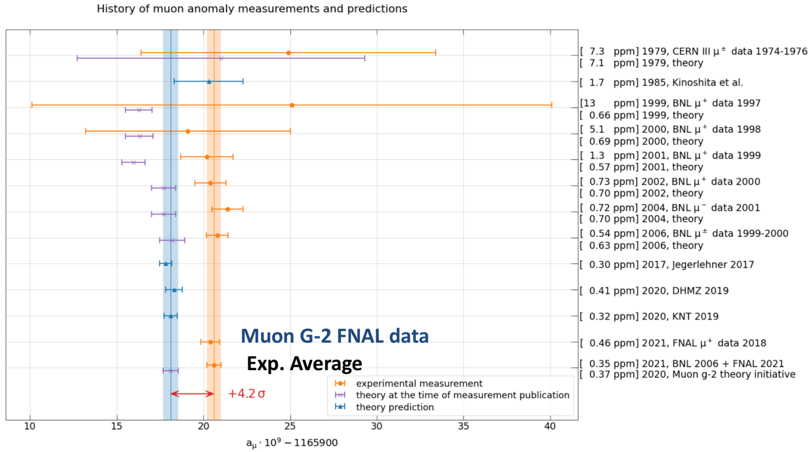
$$\langle\omega_p\rangle \approx \omega_p \otimes \rho(r)$$

Average magnetic field weighted by muon distribution

ω_p: free proton precession frequency

Using proton NMR $\hbar\omega_p = 2\mu_p B$

$g_\mu - 2$ updated history (7 April 2021)



$$a_\mu(\text{AVG}) = 116\,592\,061(41) \times 10^{-11} \quad (0.35\,\text{ppm}).$$

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G. Venanzoni, CERN Seminar, 8 April 2021

Precision of a bathroom scale that could tell you that you that you had just lost a small eyelash !!!

Based on only 6% of expected FNAL data! $\rightarrow$ aim $\delta a_\mu = 0.14\,\text{ppm}$

Standard model calculation of a_μ

At needed precision: all three interactions and all SM particles

$$
\begin{aligned}
a_\mu^{\text{SM}} &= a_\mu^{\text{QED}} + a_\mu^{\text{had}} + a_\mu^{\text{EW}} \\
&= O\left(\frac{\alpha}{2\pi}\right) + O\left(\left(\frac{\alpha}{\pi}\right)^2 \left(\frac{m_\mu}{M_\rho}\right)^2\right) + O\left(\left(\frac{e}{4\pi \sin\theta_W}\right)^2 \left(\frac{m_\mu}{M_W}\right)^2\right) \\
&= O\left(10^{-3}\right) + O\left(10^{-7}\right) + O\left(10^{-9}\right)
\end{aligned}
$$

QED contributions to a_ℓ

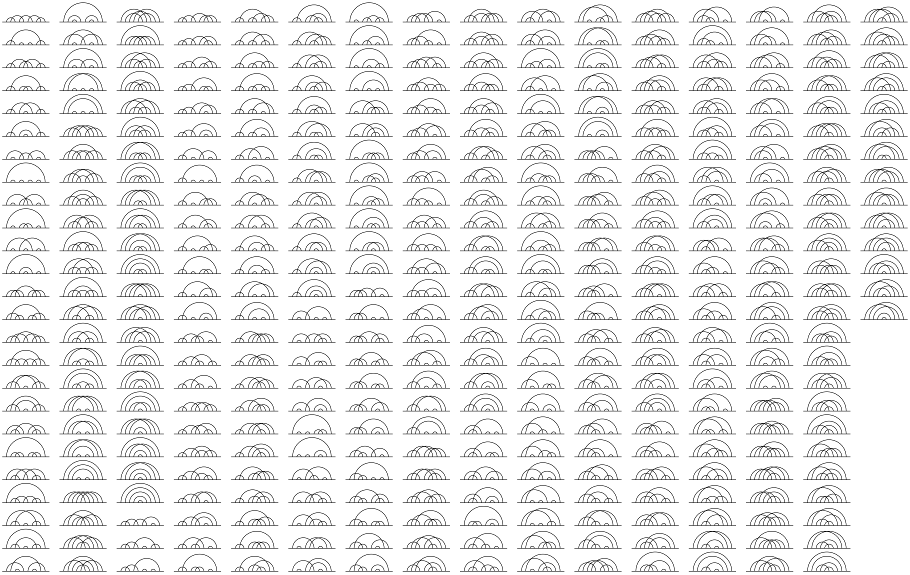
Loops with only photons and leptons

$$a_\ell^{\rm QED} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \cdots$$

$$C_\ell^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_\ell/m_{\ell'}) + A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})$$

- $A_1^{(2)}, A_1^{(4)}, A_1^{(6)}, A_2^{(4)}, A_2^{(6)}, A_3^{(6)}$ known analytically (Schwinger '48; Sommerfield '57, '58; Petermann '57; ...)
- $O((\alpha/\pi)^3)$: 72 diagrams (Laporta et al '91, '93, '95, '96; Kinoshita '95)
- $O((\alpha/\pi)^4; (\alpha/\pi)^5)$: 891;12,672 diagrams (Laporta '95; Aguilar et al '08; Aoyama, Kinoshita, Nio '96-'18)
  - Automated generation of diagrams
  - Numerical evaluation of loop integrals
  - Only some diagrams are known analytically
  - Not all contributions are fully, independently checked

5-loop QED diagrams



(Aoyama et al '15)

QED contribution to a_μ

$$a_\mu^{\text{QED}}(Cs) = 1\,165\,847\,189.31(7)_{m_\tau}(17)_{\alpha^4}(6)_{\alpha^5}(100)_{\alpha^6}(23)_{\alpha(Cs)} \times 10^{-12} \text{ [0.9 ppb]}$$

$$a_\mu^{\text{QED}}(a_e) = 1\,165\,847\,188.42(7)_{m_\tau}(17)_{\alpha^4}(6)_{\alpha^5}(100)_{\alpha^6}(28)_{\alpha(a_e)} \times 10^{-12} \text{[0.9 ppb]}$$

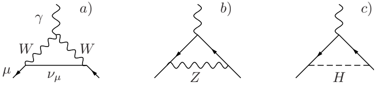
(Aoyama et al '12, '18, '19)

$$a_\mu^{\text{exp}} - a_\mu^{\text{QED}} = 734.2(4.1) \times 10^{-10}$$

$$\stackrel{?}{=} a_\mu^{\text{EW}} + a_\mu^{\text{had}}$$

Electroweak contributions to a_μ: Z, W, H, etc. loops

1-loop

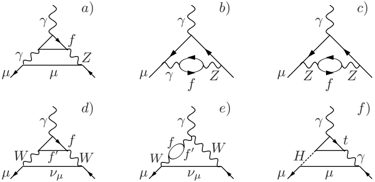


$$a_\mu^{\text{EW},(1)} = O\left(\frac{\sqrt{2}G_F m_\mu^2}{16\pi^2}\right)$$

$$= 19.479(1) \times 10^{-10}$$

(Gnendiger et al '15, Aoyama et al '20 and refs therein)

2-loop



$$a_\mu^{\text{EW},(2)} = O\left(\frac{\sqrt{2}G_F m_\mu^2}{16\pi^2}\frac{\alpha}{\pi}\right)$$

$$= -4.12(10) \times 10^{-10}$$

(Gnendiger et al '15 and refs therein)

$$a_\mu^{\text{EW}} = 15.36(10) \times 10^{-10}$$

Hadronic contributions to a_μ: quark and gluon loops

$$a_\mu^{\text{exp}} - a_\mu^{\text{QED}} - a_\mu^{\text{EW}} = 718.9(4.1) \times 10^{-10} \stackrel{?}{=} a_\mu^{\text{had}}$$

Clearly right order of magnitude:

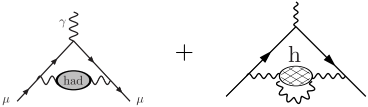
$$a_\mu^{\text{had}} = O\left(\left(\frac{\alpha}{\pi}\right)^2 \left(\frac{m_\mu}{M_\rho}\right)^2\right) = O\left(10^{-7}\right)$$

(already Gourdin & de Rafael '69 found $a_\mu^{\text{had}} = 650(50) \times 10^{-10}$)

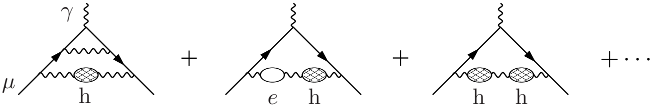
Write

$$a_\mu^{\text{had}} = a_\mu^{\text{LO-HVP}} + a_\mu^{\text{HO-HVP}} + a_\mu^{\text{HLbyL}} + O\left(\left(\frac{\alpha}{\pi}\right)^4\right)$$

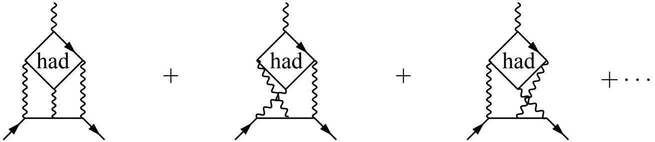
Hadronic contributions to a_μ: diagrams



$\rightarrow a_\mu^{\text{LO-HVP}} = O\left(\left(\frac{\alpha}{\pi}\right)^2\right)$

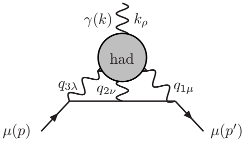


$\rightarrow a_\mu^{\text{NLO-HVP}} = O\left(\left(\frac{\alpha}{\pi}\right)^3\right)$

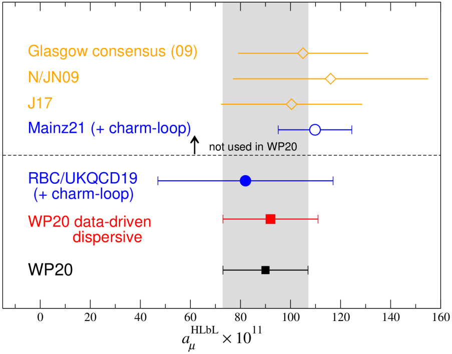


$\rightarrow a_\mu^{\text{HLbL}} = O\left(\left(\frac{\alpha}{\pi}\right)^3\right)$

Hadronic light-by-light

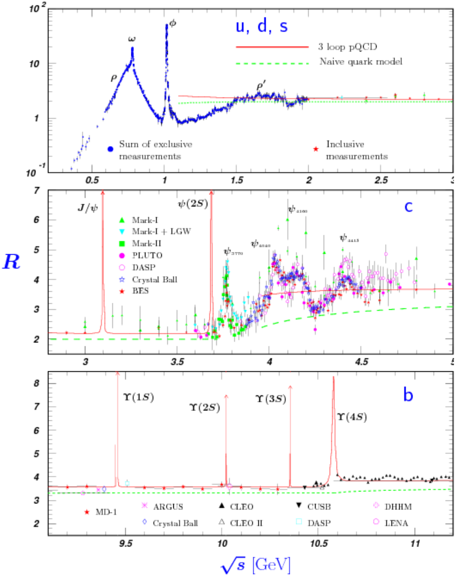


- HLbL much more complicated than HVP, but ultimate precision needed is $\simeq 10\%$ instead of $\simeq 0.2\%$
- For many years, only accessible to models of QCD w/ difficult to estimate systematics (Prades et al '09): $a_\mu^{\rm HLbL} = 10.5(2.6) \times 10^{-10}$
- Also, lattice QCD calculations were exploratory and incomplete
- Tremendous progress in past 5 years:
  - → Phenomenology: rigorous data driven approach [Colangelo, Hoferichter, Kubis, Procura, Stoffer,. . . '15-'20]
  - → Lattice: first two solid lattice calculations
- All agree w/ older model results but error estimate much more solid and will improve
- Agreed upon average w/ NLO HLbL and conservative error estimates [WP '20]
- $a_\mu^{\rm exp} - a_\mu^{\rm QED} - a_\mu^{\rm EW} - a_\mu^{\rm HLbL} = 709.7(4.5) \times 10^{-10} \stackrel{?}{=} a_\mu^{\rm HVP}$



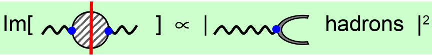
[Colangelo '21]

HVP from $e^+e^- \to$ had (or $\tau \to \nu_\tau +$ had)



(PDG compilation)

Use [Bouchiat et al 61] optical theorem (unitarity)



$$\mathrm{Im}\Pi(s) = -\frac{R(s)}{12\pi}, \quad R(s) \equiv \frac{\sigma(e^+e^- \to \text{had})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

and a once subtracted dispersion relation (analyticity)

$$\hat{\Pi}(Q^2) = \int_0^\infty ds\, \frac{Q^2}{s(s+Q^2)} \frac{1}{\pi} \mathrm{Im}\Pi(s) = \frac{Q^2}{12\pi^2} \int_0^\infty ds\, \frac{1}{s(s+Q^2)} R(s)$$

$\Rightarrow \hat{\Pi}(Q^2)$ & $a_\mu^{\text{LO-HVP}}$ from data: sum of exclusive $\pi^+\pi^-$ etc. channels from CMD-2&3, SND, BES, KLOE '08,'10&'12, BABAR '09, etc.

$$a_\mu^{\text{LO-HVP}} = 694.0(1.0)(3.9) \times 10^{-10}$$

[DHMZ'19] (sys. domin.)

Can also use $I(J^{PC}) = 1(1^{--})$ part of $\tau \to \nu_\tau +$ had and isospin symmetry + corrections

Standard model prediction and comparison to experiment

SM prediction vs experiment on April 7, 2021 (v1)

SM contribution	$a_\mu^{\text{contrib.}} \times 10^{10}$	Ref.
HVP LO (R-ratio)	692.8 ± 2.4	[KNT '19]
	694.0 ± 4.0	[DHMZ '19]
	692.3 ± 3.3	[CHHKS '19]
HVP LO (R-ratio, avg)	693.1 ± 4.0	[WP '20]
HVP LO (lattice$<$2021)	711.6 ± 18.4	[WP '20]
HVP NLO	-9.83 ± 0.07	
		[Kurz et al '14, Jegerlehner '16, WP '20]
HVP NNLO	1.24 ± 0.01	[Kurz '14, Jeger. '16]
HLbyL LO (pheno)	9.2 ± 1.9	[WP '20]
HLbyL LO (lattice$<$2021)	$7.8 \pm 3.1 \pm 1.8$	[RBC '19]
HLbyL LO (lattice 2021)	$10.7 \pm 1.1 \pm 0.9$	[Mainz '21]
HLbyL LO (avg)	9.0 ± 1.7	[WP '20]
HLbyL NLO (pheno)	0.2 ± 0.1	[WP '20]
QED [5 loops]	11658471.8931 ± 0.0104	[Aoyama '19, WP '20]
EW [2 loops]	15.36 ± 0.10	[Gnendiger '15, WP '20]
HVP Tot. (R-ratio)	684.5 ± 4.0	[WP '20]
HLbL Tot.	9.2 ± 1.8	[WP '20]
SM [0.37 ppm]	11659181.0 ± 4.3	[WP '20]
Exp [0.35 ppm]	11659206.1 ± 4.1	[BNL '06 + FNAL '21]
Exp $-$ SM	25.1 ± 5.9 [4.2σ]	

SM prediction vs experiment on April 7, 2021 (v2)

SM contribution	$a_\mu^{\text{contrib.}} \times 10^{10}$	Ref.
HVP LO (R-ratio)	692.8 ± 2.4	[KNT '19]
	694.0 ± 4.0	[DHMZ '19]
	692.3 ± 3.3	[CHHKS '19]
HVP LO (R-ratio, avg)	693.1 ± 4.0	[WP '20]
HVP LO (lattice)	707.5 ± 5.5	[BMWc '20]
HVP NLO	-9.83 ± 0.07	
		[Kurz et al '14, Jegerlehner '16, WP '20]
HVP NNLO	1.24 ± 0.01	[Kurz '14, Jeger. '16]
HLbyL LO (pheno)	9.2 ± 1.9	[WP '20]
HLbyL LO (lattice$<$2021)	$7.8 \pm 3.1 \pm 1.8$	[RBC '19]
HLbyL LO (lattice 2021)	$10.7 \pm 1.1 \pm 0.9$	[Mainz '21]
HLbyL LO (avg)	9.0 ± 1.7	[WP '20]
HLbyL NLO (pheno)	0.2 ± 0.1	[WP '20]
QED [5 loops]	11658471.8931 ± 0.0104	[Aoyama '19, WP '20]
EW [2 loops]	15.36 ± 0.10	[Gnendiger '15, WP '20]
HVP Tot. (lat. + R-ratio)	698.9 ± 5.5	[WP '20, BMWc '20]
HLbL Tot.	9.2 ± 1.8	[WP '20]
SM [0.49 ppm]	11659195.4 ± 5.7	[WP '20 + BMWc '20]
Exp [0.35 ppm]	11659206.1 ± 4.1	[BNL '06 + FNAL '21]
Exp $-$ SM	10.7 ± 7.0 $[1.5\sigma]$	

Very brief introduction to lattice QCD

What is lattice QCD (LQCD)?

To describe matter w/ sub-% precision, QCD requires $\geq$ 104 numbers at every spacetime point

$\rightarrow \infty$ number of numbers in our continuous spacetime

$\rightarrow$ must temporarily "simplify" the theory to be able to calculate *(regularization)*

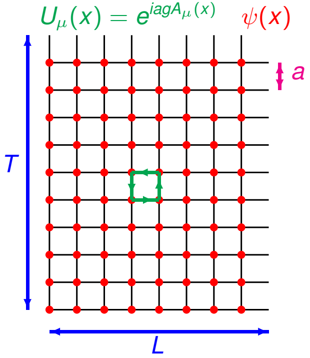
$\Rightarrow$ Lattice gauge theory $\longrightarrow$ mathematically sound definition of NP QCD:

- UV (& IR) cutoff $\rightarrow$ well defined path integral in Euclidean spacetime:

$$\langle O \rangle = \int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi\, e^{-S_G - \int \bar{\psi} D[M]\psi}\, O[U,\psi,\bar{\psi}]$$

$$= \int \mathcal{D}U\, e^{-S_G}\, \det(D[M])\, O[U]_{\rm Wick}$$

- $\mathcal{D}U e^{-S_G} \det(D[M]) \geq 0$ & finite # of dofs $\rightarrow$ evaluate numerically using stochastic methods



LQCD is QCD when $m_q \rightarrow m_q^{\rm ph}$, $a \rightarrow 0$ (after renormalization), $L \rightarrow \infty$ (and stats $\rightarrow \infty$)

HUGE conceptual and numerical ($O(10^9)$ dofs) challenge

Our “accelerators”

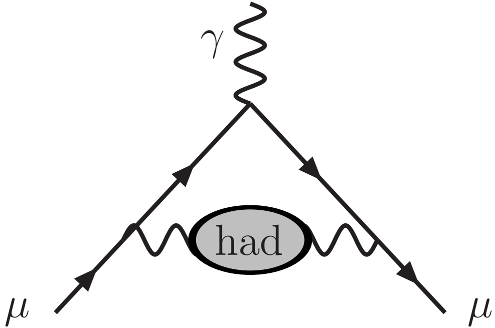
Such computations require some of the world’s most powerful supercomputers

- copyright Photothèque CNRS/Cyril Frésillon

- 1 year on supercomputer
  $\sim$ 100 000 years on laptop

- In Germany, those of the Forschungszentrum Jülich, the Leibniz Supercomputing Centre (Munich), and the High Performance Computing Center (Stuttgart); in France, Turing and Jean Zay at the Institute for Development and Resources in Intensive Scientific Computing (IDRIS) of the CNRS, and Joliot-Curie at the Very Large Computing Centre (TGCC) of the CEA, by way of the French Large-scale Computing Infrastructure (GENCI).

Lattice QCD calculation of a_μ^{HVP}



All quantities related to a_μ will be given in units of 10^{-10}

Consider in Euclidean spacetime, i.e. spacelike $q^2 = -Q^2 \leq 0$ [Blum '02]

$$\Pi_{\mu\nu}(Q) =$$

$$= \int d^4x\, e^{iQ\cdot x} \langle J_\mu(x) J_\nu(0)\rangle$$

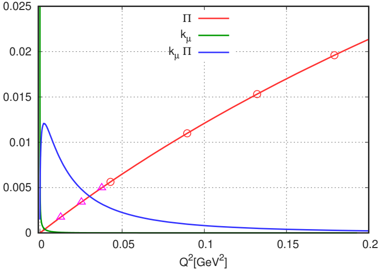
$$= \left(Q_\mu Q_\nu - \delta_{\mu\nu} Q^2\right) \Pi(Q^2)$$

w/ $J_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \frac{2}{3}\bar{c}\gamma_\mu c + \cdots$

Then [Lautrup et al '69, Blum '02]

$$a_\ell^{\text{LO-HVP}} = \alpha^2 \int_0^\infty \frac{dQ^2}{m_\ell^2}\, k(Q^2/m_\ell^2)\hat{\Pi}(Q^2)$$

w/ $\hat{\Pi}(Q^2) \equiv \left[\Pi(Q^2) - \Pi(0)\right]$



$(k_\mu(Q^2) = (\pi/m_\mu)^2 k(Q^2))$

FV & $a \neq 0$: discrete momenta, $\Pi_{\mu\nu}(0) \neq 0$ & $\Pi(0) \sim \ln a$

$\rightarrow$ modify Fourier transform to take care of all three problems and eliminate some noise [Bernecker et al '11, BMWc '13, Feng et al '13, Lehner '14, ...]

Contributions of ud, s, c ... have very different systematics (and statistical errors) on lattice

$\rightarrow$ study each one individually

Key new ingredients

- Noise in *ud* contributions grows exponentially at large distances

  → algorithmic improvements (EigCG, solver truncation [Bali et al '09], all mode averaging [Blum et al '13]) to generate more statistics: $>$25,000 gauge configurations & 10's of millions of measurements

  → exact treatment of IR modes to reduce long-distance noise (low mode averaging [Neff et al '01, Giusti et al '04, ...])

  → rigorous upper/lower bounds on long-distance contribution [Lehner '16, BMWc '17]

  ⇒ statistical error reduced from 1.1% to 0.3%

- Permil determination of QCD scale in our simulations: naively $\frac{\delta a_\mu^{\text{LO-HVP}}}{a_\mu^{\text{LO-HVP}}} \sim 2 \times \frac{\delta a}{a}$

  → Fix w/ Ω^- baryon mass computed w/ 2‰ error

  → Compute Wilson-flow scale [Lüscher '10, BMWc '12] and use to define isospin limit: $w_0 = 0.17236(29)(66)$

  ⇒ lattice spacing error reduced from percent to permil

- Even in our large volumes w/ $L \gtrsim 6.1\,\text{fm}$ & $T \geq 8.7\,\text{fm}$, exponentially suppressed FV effects significant

  - 1-loop SU(2) χPT [Aubin et al '16] suggests ~2%

  → leading 2% systematic in all previous $a_\mu^{\text{LO-HVP}}$ lattice calculations

  → perform dedicated FV study w/ even larger volumes $(\sim 11\,\text{fm})^4$ and use theory for tiny, residual correction

  ⇒ FV error reduced from 2% to 0.35%

Key new ingredients

- Need controlled continuum, $a \to 0$ limit

$\to$ perform all calculations at 6 a's: $0.134 \to 0.064\,\mathrm{fm}$

$\to$ statistical error at finest a reduced from 1.9% to 0.3%!

$\to$ improve approach to continuum limit w/ EFTs and phenomenological models (SRHO) [Sakurai '60, Jegerlehner et al '11, Chakraborty et al '17, BMWc '20] w/ 2-loop SU(2) SχPT for systematic error [Bijnens et al '99, BMWc '20]

$\Rightarrow$ error reduced from 1.1% to 0.6%

- Include all relevant QED and $m_u \neq m_d$ effects

$\to$ compute ALL $O(\alpha)$ and $O(\delta m = m_d - m_u)$ effects on ALL quantities needed

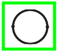
$\Rightarrow$ error reduced from 0.8% to 0.2%

- Thorough and robust determination of statistical and systematic errors
  - Stat. err.: resampling methods
  - Syst. err.: extended frequentist approach [BMWc '08, '14]
    - Hundreds of thousands of different analyses of correlation functions
    - Weighted by AIC weight
    - Use median of distribution for central values & $16 \div 84\%$ confidence interval to get total error

(Nature paper has 95 pp. Supplementary information detailing methods)

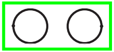
Summary of contributions to $a_\mu^{\text{LO-HVP}}$

Isospin symmetric


connected light
633.7(2.1)(4.2)


connected strange
53.393(89)(68)

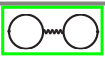
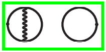

connected charm
14.6(0)(1)


disconnected
-13.36(1.18)(1.36)

QED isospin-breaking: valence

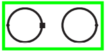


connected -1.23(40)(31)

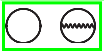


disconnected -0.55(15)(10)

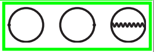
Strong isospin-breaking


connected
6.60(63)(53)


disconnected
-4.67(54)(69)

QED isospin-breaking: sea



connected 0.37(21)(24)

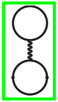


disconnected -0.040(33)(21)

Etc.

bottom; higher order; perturbative

0.11(4)

QED isospin-breaking: mixed


connected -0.0093(86)(95)


disconnected 0.011(24)(14)

Finite-size effects

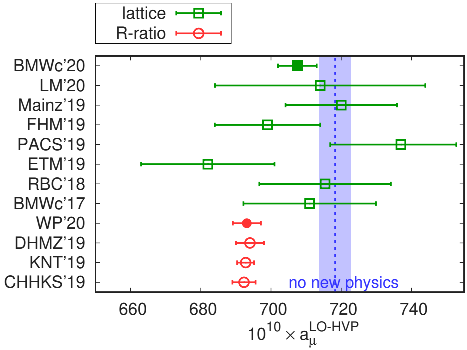
isospin-symmetric
18.7(2.5)

isospin-breaking
0.0(0.1)

$$10^{10}\times a_\mu^{\text{LO-HVP}} = 707.5(2.3)_{\text{stat}}(5.0)_{\text{sys}}[5.5]_{\text{tot}}$$

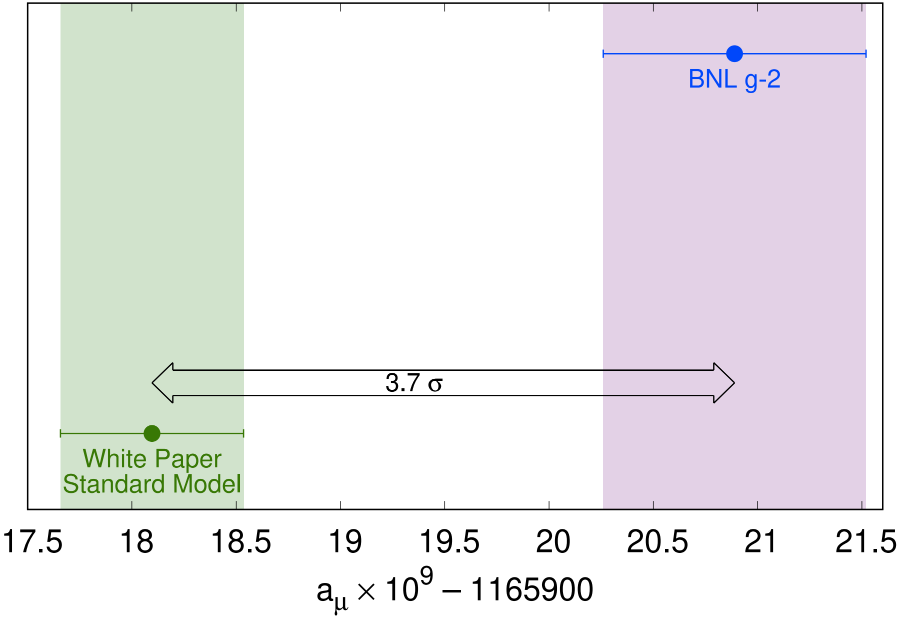
Comparison and outlook

Comparison

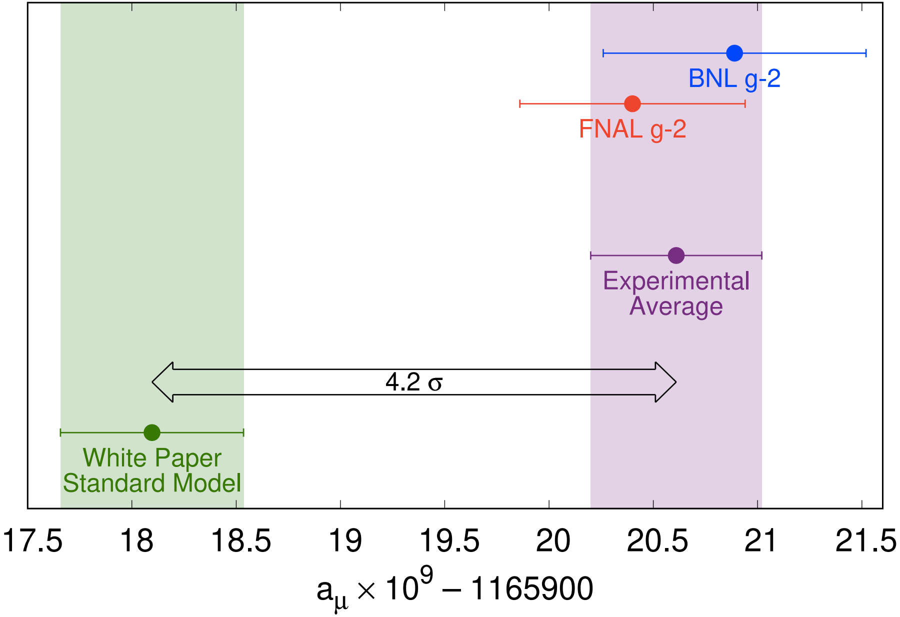


- Consistent with other lattice results
- Total uncertainty is $\sim \div 3$ ...
- ... and comparable to R-ratio and experiment
- Consistent w/ experiment @ 1.5σ ("no new physics" scenario) !
- 2.1σ larger than R-ratio average value [WP '20]

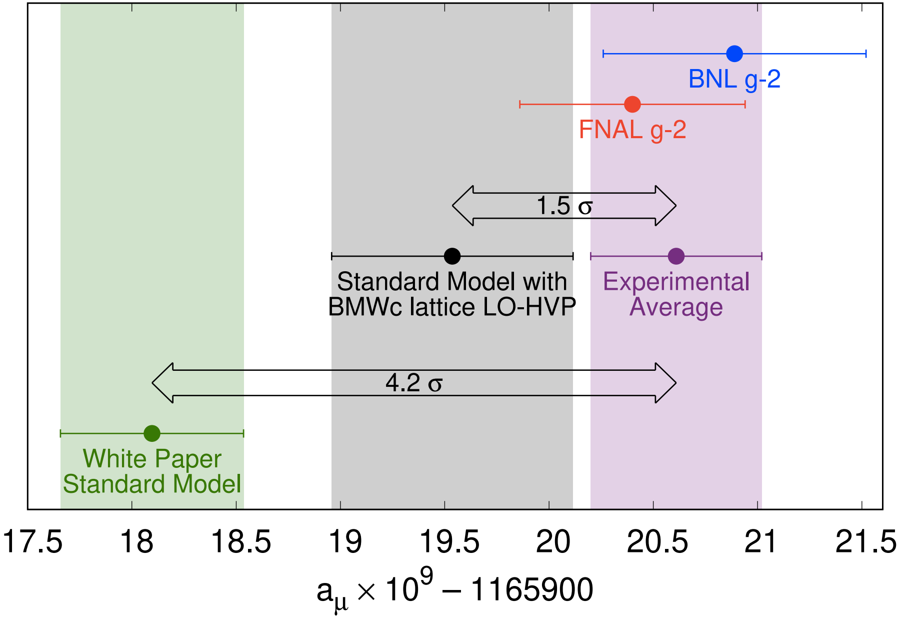
Situation ca. June 2020



Fermilab plot, April 7 2021

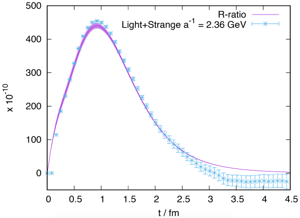


Fermilab plot, April 7 2021, BMWc version

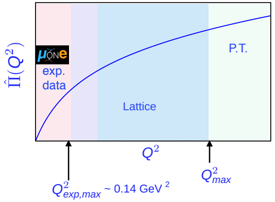


What next?

- FNAL to reduce WA error by factor of 2.5 in coming years
- HLbL error must be reduced by factor of $1.5 \div 2$
- Must reduce ours by factor of 4 !
- Will experiment still agree with our prediction ?
- Must be confirmed by other lattice groups
- If confirmed, must understand why lattice doesn't agree with R-ratio
- If disagreement can be fixed, combine LQCD and phenomenology to improve overall uncertainty [RBC/UKQCD '18]
- Important to pursue $e^+e^- \to$ hadrons measurements [BaBar, CMD-3, Belle III, . . .]
- $\mu e \to \mu e$ experiment MUonE very important for experimental crosscheck and complementarity w/ LQCD
- Important to build J-PARC $g_\mu - 2$ and pursue a_e experiments



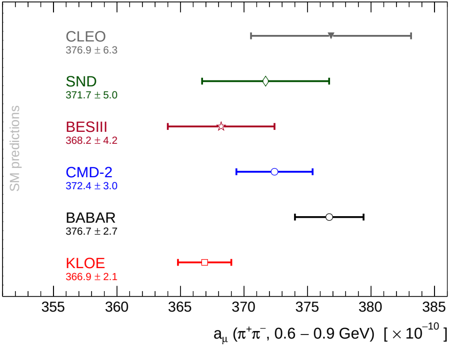
[RBC/UKQCD '18]



[Marinkovic et al '19]

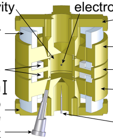
BACKUP

Tensions in R-ratio: $\pi^+\pi^-$ contribution to $a_\mu^{\text{LO-HVP}}$



- BaBar and KLOE contributions for $\sqrt{s} \in [0.6,\ 0.9]\text{GeV}$ disagree by 2.9σ
- $a_\mu^{\text{LO-HVP}}$ from BaBar alone exhibits lesser tension with our result
- Not enough to explain lattice vs R-ratio tension
- New 2020 results by SND between BaBar and KLOE
- New CMD-3 results expected later this year

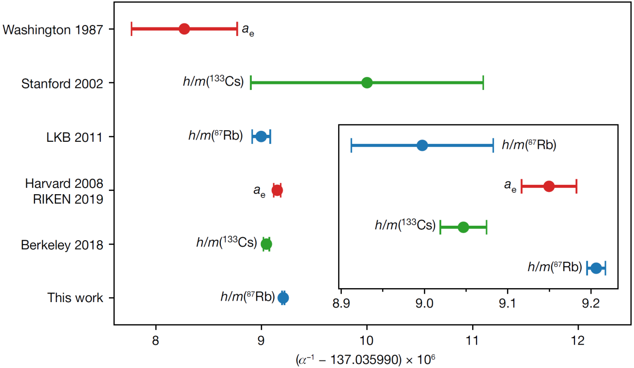
Measurement of a_e and α



$a_e^{exp} = 1\,159\,652\,180.73(28) \times 10^{-12}$ [0.24 ppb]

(Hanneke et al '08)

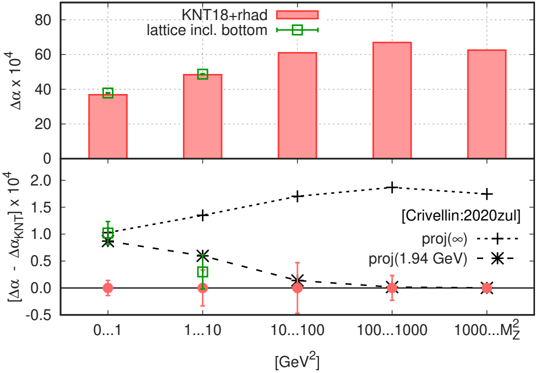
With 5-loop QED $\Rightarrow \sigma_\alpha/\alpha = 2.4 \times 10^{-10}$ vs 0.81×10^{-10} from Rb



(Morel et al '20)

Do our results imply NP @ EW scale?

- Passera et al '08: first exploration of connection $a_\mu^{\text{LO-HVP}} \leftrightarrow \Delta_{\text{had}}^{(5)}\alpha(M_Z^2)$
- Crivellin et al '20, most aggressive scenario (see also Keshavarzi et al '20): our results suggest a 4.2σ overshoot in $\Delta_{\text{had}}^{(5)}\alpha(M_Z^2)$ compared to result of fit to EWPO
- They assume 2.8% relative deviation in R-ratio for all s ($\sim$ excess we found in $a_\mu^{\text{LO-HVP}}$)
- Hypothesis is not consistent w/ BMWc '17 nor new result



- Malaescu et al '20, de Rafael '20 & Colangelo et al '20 also show that values of $a_\mu^{\text{LO-HVP}}$ even as large as needed to explain a_μ^{exp} do not necessarily imply $\Delta_{\text{had}}^{(5)}\alpha(M_Z^2)$ in conflict w/ EWPO