Mixed QCD-EW corrections to Z and W boson production and their impact on the W mass measurements at the LHC

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based on arxiv:1909.08428 [hep-ph], arxiv:2005.10221 [hep-ph], arxiv:2009.10386 [hep-ph] and arxiv:2103.02671 [hep-ph]

in collaboration with
- Federico Buccioni, Fabrizio Caola (Oxford)
- Maximilian Delto, Matthieu Jaquier, Kirill Melnikov (KIT)
- Raoul Röntsch (CERN)

May 14th, 2021 – HEP phenomenology joint Cavendish-DAMTP seminar

Precision tests of the Standard Model

- Standard Model is a renormalisable QFT
- A finite number of parameters have to be fixed from experiments, e.g.,

$$m_Z, G_F, \alpha_s(M_Z), \alpha_{em}(m_Z), m_H, m_t, m_b, \ldots, V_{CKM}$$

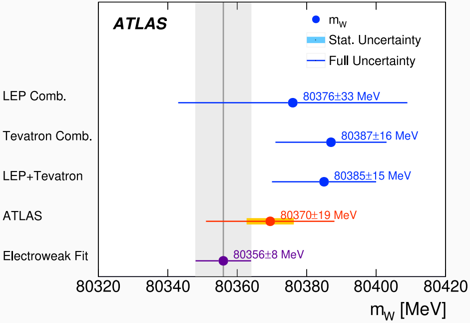
- Every measurement beyond that can be used to cross-check its consistency
- With the choice of input parameters above, we can predict the mass of the W boson

$$m_W^2 = m_Z^2 \left(1 - \frac{\pi\alpha(1 + \Delta r(m_t, m_H, m_Z, \alpha, \ldots))}{\sqrt{2} G_F m_Z^2}\right)$$

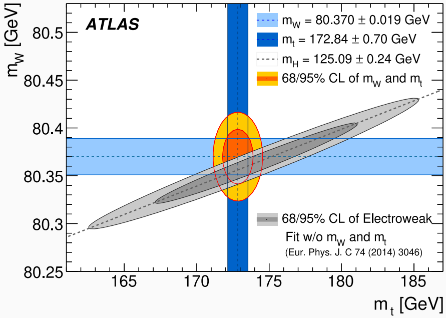
Parameter	Input value	Free in fit	Fit Result
M_H [GeV]	125.1 ± 0.2	yes	125.1 ± 0.2
M_W [GeV]	80.379 ± 0.013	–	80.359 ± 0.006
Γ_W [GeV]	2.085 ± 0.042	–	2.091 ± 0.001
M_Z [GeV]	91.1875 ± 0.0021	yes	91.1882 ± 0.0020
Γ_Z [GeV]	2.4952 ± 0.0023	–	2.4947 ± 0.0014
σ^0_{had} [nb]	41.540 ± 0.037	–	41.484 ± 0.015
R^0_ℓ	20.767 ± 0.025	–	20.742 ± 0.017
$A^{0,\ell}_{\mathrm{FB}}$	0.0171 ± 0.0010	–	0.01620 ± 0.0001
A_ℓ $^{(\star)}$	0.1499 ± 0.0018	–	0.1470 ± 0.0005
$\sin^2\theta^\ell_{\mathrm{eff}}(Q_{\mathrm{FB}})$	0.2324 ± 0.0012	–	0.23153 ± 0.00006
$\sin^2\theta^\ell_{\mathrm{eff}}(\mathrm{Tevt.})$	0.23148 ± 0.00033	–	0.23153 ± 0.00006
A_c	0.670 ± 0.027	–	0.6679 ± 0.00021
A_b	0.923 ± 0.020	–	0.93475 ± 0.00004
$A^{0,c}_{\mathrm{FB}}$	0.0707 ± 0.0035	–	0.0736 ± 0.0003
$A^{0,b}_{\mathrm{FB}}$	0.0992 ± 0.0016	–	0.1030 ± 0.0003
R^0_c	0.1721 ± 0.0030	–	0.17224 ± 0.00008
R^0_b	0.21629 ± 0.00066	–	0.21582 ± 0.00011
$\overline{m}_c$ [GeV]	$1.27^{+0.07}_{-0.11}$	yes	$1.27^{+0.07}_{-0.11}$
$\overline{m}_b$ [GeV]	$4.20^{+0.17}_{-0.07}$	yes	$4.20^{+0.17}_{-0.07}$
m_t [GeV]$^{(\triangledown)}$	172.47 ± 0.68	yes	172.83 ± 0.65
$\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$ $^{(\dagger\triangle)}$	2760 ± 9	yes	2758 ± 9
$\alpha_s(M_Z^2)$	–	yes	0.1194 ± 0.0029

[Gfitter '18]

Precision W mass measurements



[ATLAS '17]



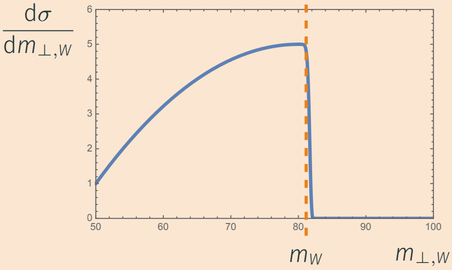
[ATLAS '17]

- Measurements of m_W have come a long way
- ATLAS has measured $m_W = (80\,370 \pm 19)$ MeV [ATLAS '17]
- ATLAS and CMS collaborations aim to reduce uncertainty to $\mathcal{O}(10\,\text{MeV})$
  $\rightarrow$ would rival precision from global electroweak fits
  $\rightarrow$ would mean $\mathcal{O}(0.01\%)$ uncertainty

How to measure m_W at hadron colliders

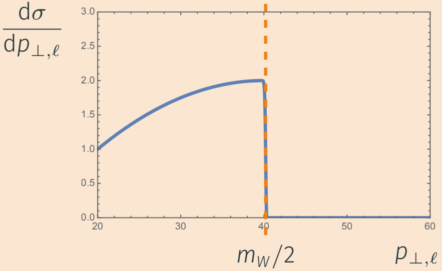
Need observables that are sensitive to m_W:

Transverse mass of W



$$m_{\perp,W} = \sqrt{2p_{\perp,\ell}p_{\perp,\nu}(1-\cos\phi_{\ell\nu})}$$

Transverse momentum of ℓ



At LO and with idealized detectors both observables have sharp kinematic edges.
$\rightarrow$ Very sensitive observables

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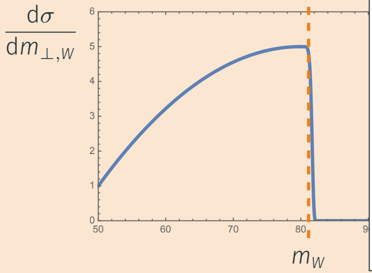

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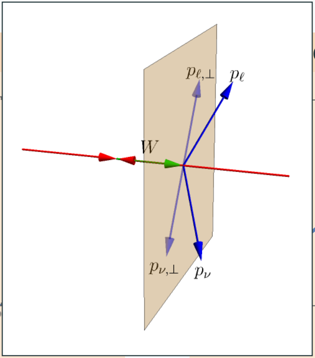

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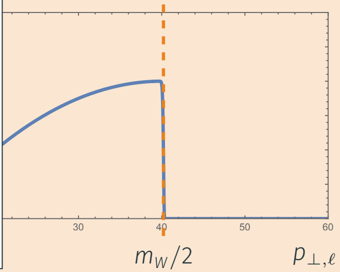

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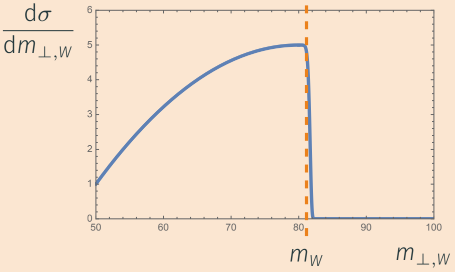

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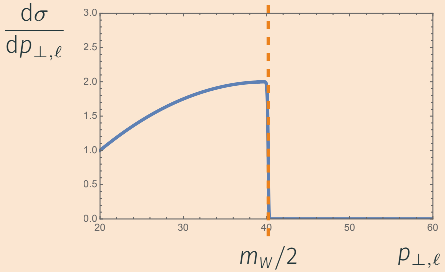

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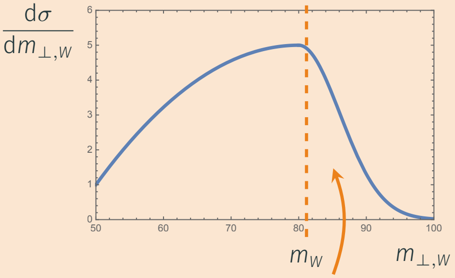
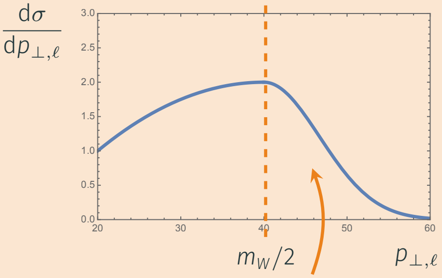


Beyond the edge: Mostly detector effects

Transverse momentum of ℓ



Mostly QCD & QED initial state radiation

Starting from NLO and with realistic detectors the edges are washed out

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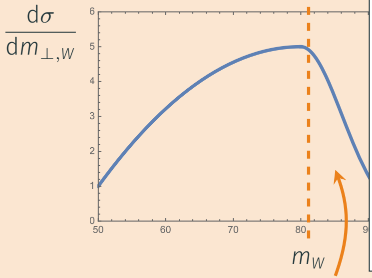

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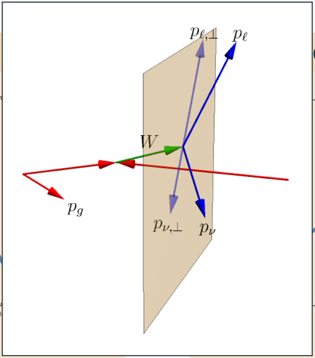

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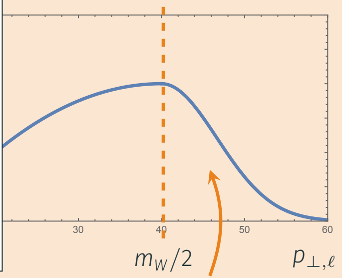


Mostly QCD & QED initial state radiation

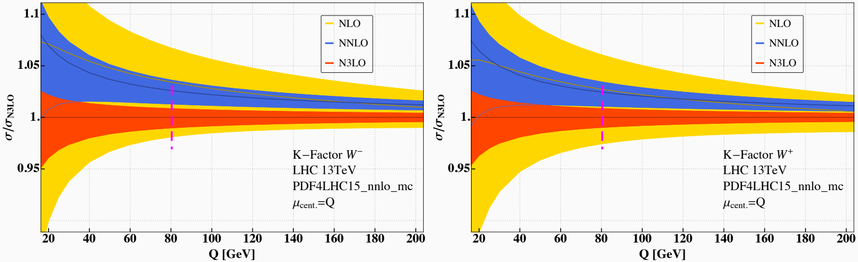
Starting from NLO and with realistic detectors the edges are washed out

Theory predictions for m_W measurements at hadron colliders

- Need very precise predictions for differential distributions for W and Z production
- Standard tools: Collinear factorisation and perturbation theory

$$d\sigma = \sum_{ij} \int dx_1 dx_2 \, f_i(x_1) f_j(x_2) d\sigma_{ij}(x_1, x_2) \qquad d\sigma_{ij} = \sum_{n,m} \alpha_s^n \alpha^m d\sigma_{ij}^{(n,m)}$$

- Typically reaches $\mathcal{O}(1\%)$ or worst uncertainties for inclusive observables



[Duhr, Dulat, Mistlberger '20]

Theory predictions for m_W measurements at hadron colliders (cont.)

To measure m_W to a precision of $\mathcal{O}(10\,\text{MeV})$ we have to control theory uncertainties to a level of about $\mathcal{O}(0.01\%)$.

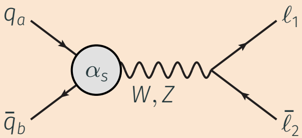
$\rightarrow$ Straightforward application of standard tools falls short of required precision.

Consequences:

1. We cannot hope to predict distributions to this precision from first principles.
   Instead:
   - Measure Z distributions
   - Parametrise them in QCD-motivated way
   - Transfer them to W distributions (bulk of QCD does not distinguish between W and Z)
2. Small effects that distinguish between Z and W bosons may matter.
   $\rightarrow$ Electroweak corrections are obvious examples of such effects.

Electroweak and QCD corrections to on-shell W and Z production

NLO QCD



$\rightarrow$ Only corrections to the initial state

Electroweak and QCD corrections to on-shell W and Z production

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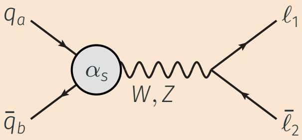
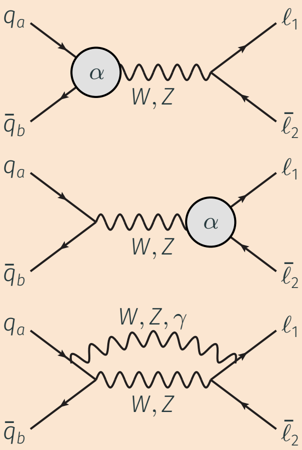


NLO EW



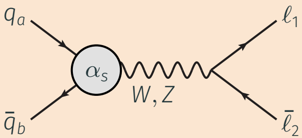
$\rightarrow$ initial state corrections

$\rightarrow$ final state corrections

$\rightarrow$ non-fact. corrections [Dittmaier, Huss, Schwinn '14]:

$$\sim \mathcal{O}\left(\alpha \frac{\Gamma}{m_V}\right) \sim \mathcal{O}\left(\alpha^2\right)$$

NLO QCD



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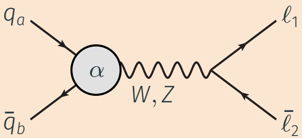

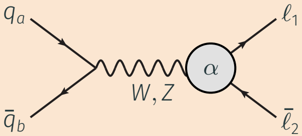

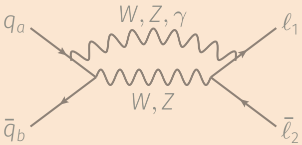

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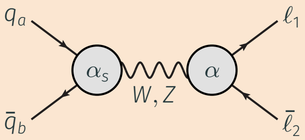
Mixed QCD-EW corrections to on-shell W and Z production

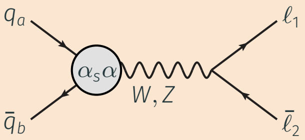
Mixed QCD-EW: Initial-Final



- Correction of NLO $\otimes$ NLO type
- Previously investigated
  [Dittmaier, Huss, Schwinn '15] [Carloni Calame et al. '16]
- Estimated impact on m_W measurement:

$$\delta m_W \sim \mathcal{O}(15\,\text{MeV})$$

Mixed QCD-EW: Initial-Initial



- Correction of NNLO type
- Generated lots of recent activity
  [De Florian, Der, Fabre '18] [Cieri, de Florian, Der, Mazzitelli '20]
  [Bonciani, Buccioni, Rana, Triscari, Vicini '19]
  [Bonciani, Buccioni, Rana, Vicini '20] [Dittmaier, Schmidt, Schwarz '20]
  [Buonocore, Grazzini, Kallweit, Savioni, Tramontano '21]
- Subject of this talk
  [Delto, Jaquier, Melnikov, Röntsch '19]
  [Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch '20]
  [AB, Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch '20]
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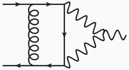
Mixed QCD-EW corrections to on-shell W and Z production

Mixed QCD-EW corrections to $pp \to W/Z$ have been discussed for many years

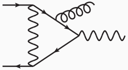
Calculation became possible due to progress on several bottlenecks

- Double Virtual: Complicated integrals with internal and external masses

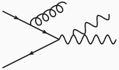
  $\to$ Progress on differential equations, iterated integrals etc.

- Real Virtual: Sufficiently stable numerics close to singular limits

  $\to$ `OpenLoops` can provide this in an automated way

- Double Real: IR singularities require NNLO subtraction scheme

  $\to$ Profit from progress on NNLO QCD subtraction schemes

$\to$ We derive estimates for shifts of W mass due to mixed QCD-EW corrections

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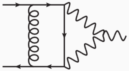

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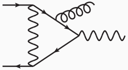

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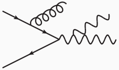

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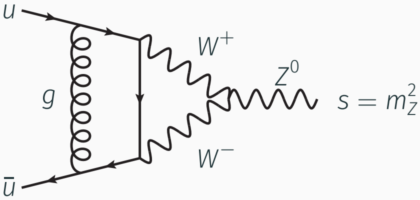
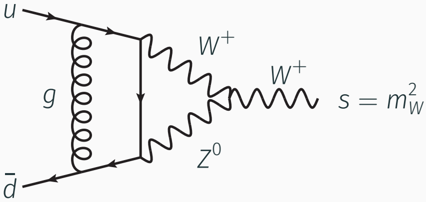
Two-loop amplitudes

Form factors for on-shell W and Z bosons

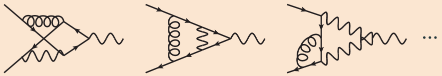




What needs to be calculated? $\rightarrow$ Only on-shell form factors
(Narrow-width approximation simplifies the problem)

- Z: Mixed QCD-EW corrections are known [Kotikov, Kühn, Veretin '07]
- W: Mixed QCD-EW corrections were not yet publicly available
  $\rightarrow$ We calculated the missing integrals and completed the form factor

44 Feynman diagrams



This is a non-trivial, but tractable calculation.

Feynman rules, γ algebra, IBP reductions, ...

35 master integrals

$$I \sim \int \frac{[\mathrm{d}^d k_1][\mathrm{d}^d k_2]}{[k_2^2 - m_W^2] \dots [(k_2 - p_{12})^2 - m_Z^2]}$$

10 MI with internal W and Z

Calculated using differential equations

$$\partial_z I(z,\varepsilon) = A(z,\varepsilon) I(z,\varepsilon) \quad \text{with} \quad z = \frac{m_W^2}{m_Z^2}$$

with the equal mass case ($z = 1$) as boundary conditions

25 MI known in the literature

[Aglietti, Bonciani '03] [Aglietti, Bonciani '04]
[Bonciani, Di Vita, Mastrolia, Schubert '16]

Results can be expressed in terms of well-understood iterated integrals (GPLs)

$$G_{a,\vec{b}}(y) = \int_0^y \frac{G_{\vec{b}}(t)}{t-a} \mathrm{d}t\,, \qquad G_a(y) = \int_0^y \frac{1}{t-a} \mathrm{d}t\,, \qquad G_0(y) = \ln(y)\,, \qquad z = \frac{y}{(1+y)^2}$$

Mixed QCD-EW corrections to the W form factor

The result for the form factor can be brought into a compact form.

Infrared poles are predicted by a "Catani-like" formula:

$$\left\langle F^{\mathrm{QCD}\otimes\mathrm{EW}}_{\mathrm{LVV+LV}^2} \right\rangle = \left(\frac{\alpha_s(\mu)}{2\pi} \frac{\alpha_{\mathrm{EW}}}{2\pi} \right) \left[I_{12,\mathrm{QCD}} \cdot I_{12,\mathrm{EW}} + \frac{e^{\varepsilon\gamma_E}}{\Gamma(1-\varepsilon)} \frac{H^W_{\mathrm{QCD}\otimes\mathrm{EW}}}{\varepsilon} \right] \langle F_{LM} \rangle$$
$$+ \left(\frac{\alpha_s(\mu)}{2\pi} \right) I_{12,\mathrm{QCD}} \left\langle F^{\mathrm{fin,EW}}_{\mathrm{LV}} \right\rangle + \left(\frac{\alpha_{\mathrm{EW}}}{2\pi} \right) I_{12,\mathrm{EW}} \left\langle F^{\mathrm{fin,QCD}}_{\mathrm{LV}} \right\rangle$$
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$$+\left\langle F^{\mathrm{fin,QCD}\otimes\mathrm{EW}}_{\mathrm{LVV+LV^2}}\right\rangle .$$

Building blocks:

$$I_{12,\mathrm{QCD}} = \left[\frac{e^{\varepsilon\gamma_E}}{\Gamma(1-\varepsilon)}\right]\left(\frac{\mu^2}{M_W^2}\right)^{\varepsilon}\left[-2C_F\cos(\pi\varepsilon)\left(\frac{1}{\varepsilon^2}+\frac{3}{2\varepsilon}\right)\right]$$

$$I_{12,\mathrm{EW}} = \left[\frac{e^{\varepsilon\gamma_E}}{\Gamma(1-\varepsilon)}\right]\left(\frac{\mu^2}{M_W^2}\right)^{\varepsilon}\left[-Q_uQ_d\cos(\pi\varepsilon)\left(\frac{2}{\varepsilon^2}+\frac{3}{\varepsilon}\right)+(Q_d-Q_u)Q_W\left(\frac{1}{\varepsilon^2}+\frac{5}{2\varepsilon}\right)\right]$$

$$H^W_{\mathrm{QCD}\otimes\mathrm{EW}} = C_F\left[Q_u^2+Q_d^2\right]\left(\frac{\pi^2}{2}-6\zeta_3-\frac{3}{8}\right)$$

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$$+ \left\langle F^{\mathrm{fin,QCD}\otimes\mathrm{EW}}_{\mathrm{LVV+LV^2}} \right\rangle .$$

- Pole structure *almost* factorises into NLO QCD $\times$ NLO EW
- Finite remainder $\left\langle F^{\mathrm{fin,QCD}\otimes\mathrm{EW}}_{\mathrm{LVV+LV^2}} \right\rangle$ also consists of a factorising (NLO QCD $\times$ NLO EW) and a non-factorising part

Mixed QCD-EW corrections to the W form factor

$$
\begin{aligned}
&\Re\widetilde{\mathcal{M}}_{\text{mix}} = \\
&\quad (Q_u^2+Q_d^2)C_F\bigg[\frac{1}{\epsilon}\Big(-\frac{3}{16}+\frac{1}{4}\pi^2-3\zeta_3\Big)+\Big(\frac{3}{8}-\frac{1}{2}\pi^2+6\zeta_3\Big)\ln\Big(\frac{M_W^2}{\mu^2}\Big)+\frac{1}{4}\frac{(27z+13)(1-z)^2}{z^3}H_1(z) \\
&\quad +\frac{(1-z)^2(1+z)}{z^3}\Big(\frac{3}{4}H_1(z)\pi^2-\frac{9}{2}H_{1,0,0}(z)-\frac{9}{2}H_{1,0,1}(z)\Big)-\frac{1}{4}\frac{(5z+3)(1-z)(1+z)}{z^3}H_{-1,0}(z) \\
&\quad +\frac{(1-z)(1+z)^2}{z^3}\bigg(-\frac{3}{2}H_{-1,-1,0}(z)+\frac{3}{2}H_{-1,0,0}(z)+3H_{-1,0,1}(z)+2H_{-1,-1,-1,0}(z)-2H_{-1,-1,0,0}(z) \\
&\quad -6H_{-1,-1,0,1}(z)-2H_{-1,0,-1,0}(z)+H_{-1,0,0,1}(z)+H_{0,-1,0,0}(z)+4H_{0,-1,0,1}(z)+\Big(-\frac{1}{4}H_{-1}(z)+\frac{1}{6}H_{-1,-1}(z) \\
&\quad -\frac{1}{6}H_{0,-1}(z)\Big)\pi^2-3H_{-1}(z)\zeta_3\bigg)+\frac{1}{32}\frac{7z^2-72z+64}{z^2}+\frac{1}{24}\frac{50z^2-5z-16}{z^2}\pi^2-\frac{3}{2}\frac{8z^2-z-2}{z^2}\zeta_3-\frac{11}{180}\pi^4 \\
&\quad +\frac{(1-z)}{z^2}\bigg(\frac{1}{2}(9z+11)H_{0,1}(z)-\frac{1}{2}(3z+4)H_{0,0,1}(z)+\frac{1}{4}(23z+16)H_{0,0}(z)+(3z+2)\Big(\frac{1}{2}H_{0,-1,0}(z) \\
&\quad -\frac{17}{8}H_0(z)\Big)\bigg)+\frac{(z^2+3z+1)(1-z)}{z^3}\Big(\frac{1}{3}H_{0,1}(z)\pi^2-2H_{0,1,0,0}(z)-2H_{0,1,0,1}(z)\Big)\bigg]+C_F\bigg[\frac{z+2}{1-z}\Big(-\frac{1}{6}H_{0,0}(z)\pi^2 \\
&\quad +4H_0(z)\zeta_3\Big)+\frac{1}{8}\frac{(5z-2)(2z^2+12z+11)}{(1-z)z^2}H_{0,1}(z)+\frac{1}{8}\frac{43z^2+7z-16}{(1-z)z^2}H_{0,0}(z)-\frac{1}{48}\frac{10z^3+5z^2+20z-16}{(1-z)z^2}\pi^2 \\
&\quad -\frac{1}{16}\frac{8z^3+142z^2+23z-34}{(1-z)z^2}H_0(z)+\frac{1}{120}\frac{5z-36}{1-z}\pi^4-\frac{1}{8}\frac{4z^2-17z+8}{(1-z)z^2}+\frac{2z^2-2z+1}{(1-z)z^2}\bigg(\frac{1}{4}(3z+4)H_{0,0,1}(z) \\
&\quad +(3z+2)\Big(-\frac{3}{4}\zeta_3-\frac{1}{4}H_{0,-1,0}(z)\Big)\bigg)+\frac{(2z^2-6z+3)(1+z)}{z^3}\Big(\frac{3}{4}H_{1,0,0}(z)+\frac{3}{4}H_{1,0,1}(z)-\frac{1}{8}H_1(z)\pi^2\Big) \\
&\quad -\frac{1}{(1-z)z}\Big(\frac{1}{8}H_{0,0,0}(z)+\frac{1}{2}(9z^2-8z-2)\zeta_3+\frac{5}{48}H_0(z)\pi^2\Big)+\frac{(2z^2-2z+1)(1+z)^2}{(1-z)z^3}\bigg(\frac{3}{4}H_{-1,-1,0}(z) \\
&\quad -\frac{3}{4}H_{-1,0,0}(z)-\frac{3}{2}H_{-1,0,1}(z)-H_{-1,-1,-1,0}(z)+H_{-1,-1,0,0}(z)+3H_{-1,-1,0,1}(z)+H_{-1,0,-1,0}(z) \\
&\quad -\frac{1}{2}H_{-1,0,0,1}(z)-\frac{1}{2}H_{0,-1,0,0}(z)-2H_{0,-1,0,1}(z)+\Big(\frac{1}{8}H_{-1}(z)-\frac{1}{12}H_{-1,-1}(z)+\frac{1}{12}H_{0,-1}(z)\Big)\pi^2+\frac{3}{2}H_{-1}(z)\zeta_3\bigg) \\
&\quad +\frac{1}{8}\frac{4z^3+64z^2-z-13}{z^3}H_1(z)+\frac{1}{8}\frac{(5z+3)(2z^2-2z+1)(1+z)}{(1-z)z^3}H_{-1,0}(z)+\frac{z^4-4z^2+z+1}{(1-z)z^3}\bigg(H_{0,1,0,0}(z) \\
&\quad +H_{0,1,0,1}(z)-\frac{1}{6}H_{0,1}(z)\pi^2\bigg)+\bigg[\frac{\sqrt{4z-1}}{8z}\bigg(-\frac{10z+3}{1-z}(H_r(z^{-1})-\pi)-(\pi H_0(z)+H_{0,r}(z^{-1}))+\frac{17z+4}{1-z}H_{r,0}(z^{-1}) \\
&\quad -\frac{6z+1}{1-z}(i\pi^2-3i\pi H_r(z^{-1})-3H_{r,1}(z^{-1}))\bigg)-\frac{1}{8}\frac{3z+2}{(1-z)z}(H_{r,r}(z^{-1})-\pi H_r(z^{-1}))-\frac{1}{8}\frac{30z^2-20z-1}{(1-z)z}H_{r,r,0}(z^{-1}) \\
&\quad +\frac{1}{8}\frac{1}{(1-z)z}(H_{0,r,r}(z^{-1})-\pi H_{0,r}(z^{-1}))-\frac{1}{8}\frac{6z^2-4z+1}{(1-z)z}(H_{r,0,r}(z^{-1})-\pi H_{r,0}(z^{-1}))+\frac{1}{2}\frac{3z-2}{1-z}\bigg(-3H_{r,r,1}(z^{-1}) \\
&\quad -3i\pi H_{r,r}(z^{-1})+i\pi^2H_r(z^{-1})-i\frac{\pi^3}{6}\bigg)+\frac{z+2}{1-z}\bigg(i\frac{\pi^3}{6}H_0(z)+i\pi^2H_{0,r}(z^{-1})-3i\pi H_{0,r,r}(z^{-1})-3H_{0,r,r,0}(z^{-1}) \\
&\quad -3H_{0,r,r,1}(z^{-1})-4i\pi\zeta_3\bigg)\bigg]\bigg]
\end{aligned}
$$

The analytic result is now available and even reasonably compact.

Non-factorising part of finite remainder becomes this simple when expressed in terms of iterated integrals over $z = \frac{m_W^2}{m_Z^2}$

$$H_{a,\vec{b}}(z) = \int_0^z f_a(t) H_{\vec{b}}(t)\,\mathrm{d}t$$

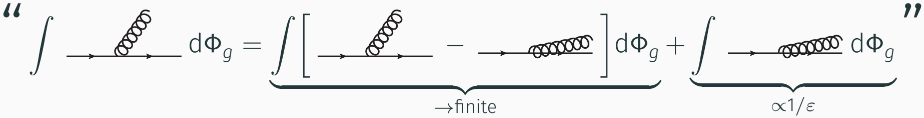
with HPL- and square root letters

$$f_1(t) = \frac{1}{1-t}, \quad f_0(t) = \frac{1}{t},$$
$$f_{-1}(t) = \frac{1}{1+t}, \quad f_r(t) = \frac{1}{\sqrt{t(4-t)}}$$

Subtraction

Cross-sections develop IR singularities in soft and collinear limits of massless particles
$\rightarrow$ cancel between real and virtual corrections

- Use a subtraction scheme to make poles from real radiation explicit



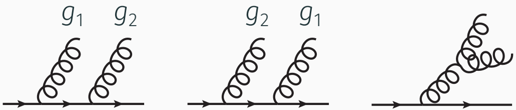
- Build on progress with NNLO QCD subtraction schemes to tackle mixed QCD-EW corrections (here: nested soft-collinear subtraction scheme)
  - Z: Abelianisation of NNLO QCD subtraction is sufficient
  - W: New contributions from radiating W bosons

Subtraction for mixed QCD-EW corrections: triple-collinear limits

We can make use of simplifications compared to NNLO QCD.

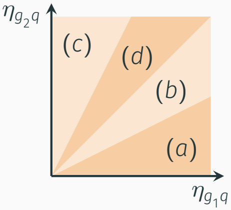
Triple-collinear limits

- NNLO QCD: Overlapping singularities in triple-collinear limits



$\rightarrow$ Needs 4 sectors to disentangle collinear singularities

$\eta_{ij} = \frac{1}{2}(1 - \cos\theta_{ij})$



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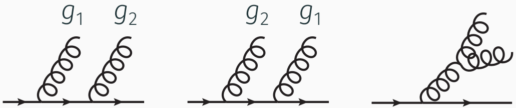
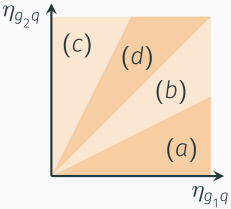
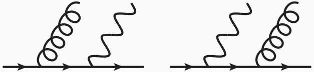
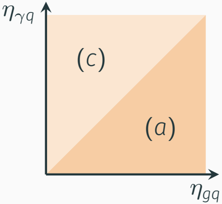




  $\rightarrow$ Needs 4 sectors to disentangle collinear singularities

- Mixed QCD-EW: Collinear limit of photon and gluon is not singular





  $\rightarrow$ 2 sectors can be dropped in $q\bar{q}$ channel

Overall: No new collinear limits arise compared to NNLO QCD

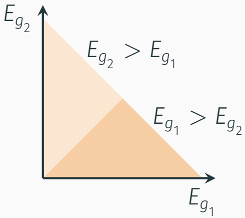
Subtraction for mixed QCD-EW corrections: double-soft limits

We can make use of simplifications compared to NNLO QCD.

Double-soft limits

- NNLO QCD: Overlapping singularities in the double-soft limit
  - Non-trivial double-soft eikonal function
  - Distinguish rates at which energies of soft particles vanish

$$1 = \theta(E_{g_1} - E_{g_2}) + \theta(E_{g_2} - E_{g_1})$$



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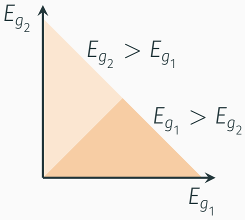


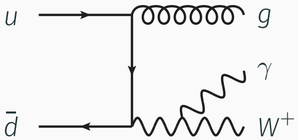
- Mixed QCD-EW: Soft gluons and photons are not entangled
  - Double-soft limit factorises into NLO QCD $\times$ NLO QED

$$\lim_{E_g, E_\gamma \to 0} |\mathcal{M}_{Wg\gamma}|^2 = g_s^2 \mathrm{Eik}_g(p_u, p_{\bar{d}}; p_g)\, e^2 \mathrm{Eik}_\gamma(p_u, p_{\bar{d}}, p_W; p_\gamma) |\mathcal{M}_W|^2$$

$$\mathrm{Eik}_g(p_u, p_{\bar{d}}; p_g) = 2C_F \frac{(p_u \cdot p_{\bar{d}})}{(p_u \cdot p_g)(p_g \cdot p_{\bar{d}})}$$

  - No need to distinguish $E_g > E_\gamma$ vs. $E_\gamma > E_g$

Subtraction for mixed QCD-EW corrections: radiating W bosons



New contribution compared to NNLO QCD: W bosons can radiate photons

- Mass of W boson prevents collinear singularities
- Soft limit of photon is still singular
  - Requires soft eikonal function for massive emitter
  - QCD and QED factorise in soft limit $\rightarrow$ only NLO eikonal functions necessary

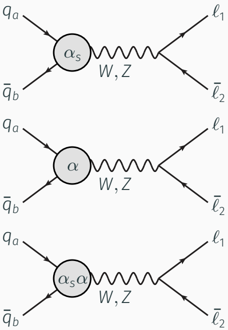
$$
\begin{aligned}
\mathrm{Eik}_\gamma(p_u, p_{\bar{d}}, p_W; p_\gamma) = \Bigg\{ & Q_u Q_d \frac{2(p_u \cdot p_{\bar{d}})}{(p_u \cdot p_\gamma)(p_{\bar{d}} \cdot p_\gamma)} - Q_W^2 \frac{p_W^2}{(p_W \cdot p_\gamma)^2} \\
& + Q_W \left(Q_u \frac{2(p_W \cdot p_u)}{(p_W \cdot p_\gamma)(p_u \cdot p_\gamma)} - Q_d \frac{2(p_W \cdot p_{\bar{d}})}{(p_W \cdot p_\gamma)(p_{\bar{d}} \cdot p_\gamma)} \right) \Bigg\}
\end{aligned}
$$

Estimates for impact on W mass

Results for W production: Cross sections for $pp \to W^+ \to e^+\nu_e$

- Write cross section as $\sigma = \sigma_{\text{LO}} + \delta\sigma^{\text{QCD}}_{\text{NLO}} + \delta\sigma^{\text{EW}}_{\text{NLO}} + \delta\sigma^{\text{QCD-EW}}_{\text{NNLO}} + \ldots$
- We include only initial-initial contributions

σ [pb]	$\mu = m_W$	$\mu = m_W/2$	$\mu = m_W/4$
σ_{LO}	6007.6	5195.0	4325.9
$\delta\sigma^{\text{QCD}}_{\text{NLO}}$	508.8	1137.0	1782.2
$\delta\sigma^{\text{EW}}_{\text{NLO}}$	2.1	-1.0	-2.6
$\delta\sigma^{\text{QCD-EW}}_{\text{NNLO}}$	-2.4	-2.3	-2.8



Results for: 13 TeV LHC, G_μ scheme,
$\mu_R = \mu_F = \mu \in \{m_W, m_W/2, m_W/4\}$,
NNPDF3.1luxQED

Selection criteria: $p_{T,e} > 15$ GeV, $p_{T,\text{miss}} > 15$ GeV, $-2.4 < y_e < 2.4$.

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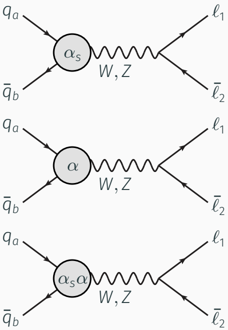

- NLO EW corrections are tiny $O(0.02\%)$ (mostly due to G_μ scheme)

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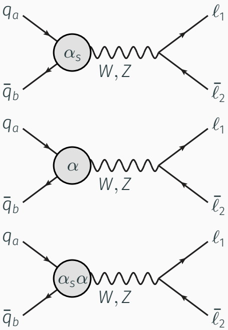
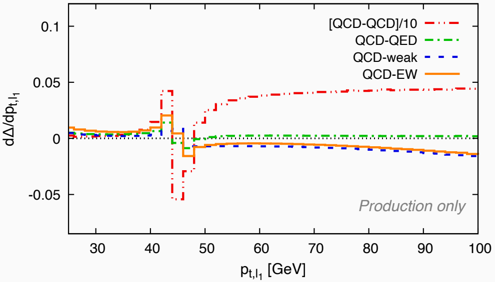


- Mixed QCD-EW corrections are very small, about $\mathcal{O}(0.05\%)$, but not obviously irrelevant for m_W measurements at the LHC

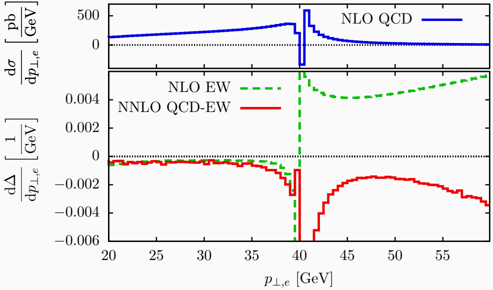
Differential distributions

$pp \to Z^0 \to e^+e^-$



[Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch '20]

$pp \to W^+ \to e^+\nu_e$



[AB, Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch '20]

- Our implementation allows to calculate differential distributions including mixed QCD-EW corrections
- Impact on W-mass measurement is not immediately obvious

Estimate W mass shifts from mixed QCD-EW corrections

Objective: Estimate impact of new corrections on W boson mass

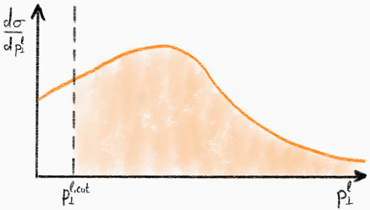
Considerations:

- Should combine W and Z measurements
  - $\rightarrow$ model what is done in experiments
  - $\rightarrow$ make use of available precision for Z mass
- Should be physically and conceptually simple and transparent
- Should be accessible with our calculations

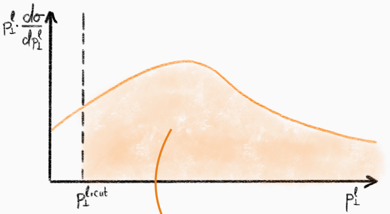
Construction of our observable

We use the average transverse momentum of the charged lepton ($V = W, Z$):

$$\langle p_\perp^{\ell,V} \rangle = \frac{\int \mathrm{d}\sigma_V \times p_\perp^\ell}{\int \mathrm{d}\sigma_V}$$



$$\xrightarrow{\times p_\perp^\ell}$$



$$\langle p_\perp^{\ell,V} \rangle = m_V\, f\left(\frac{p_\perp^{\mathrm{cut}}}{m_V}\right)$$

with $f^{\mathrm{LO}}(0) = \frac{15\pi}{128}$

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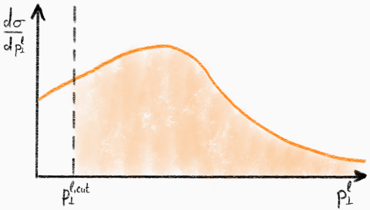

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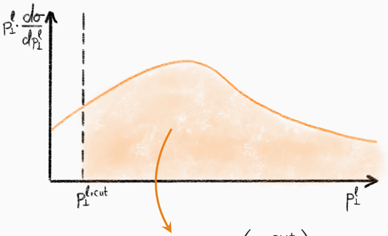

$$\langle p_\perp^{\ell,V} \rangle = \boxed{m_V} f\left(\frac{p_\perp^{\mathrm{cut}}}{m_V}\right)$$

with $f^{\mathrm{LO}}(0) = \frac{15\pi}{128}$

Construction of our observable (cont.)

Use the average lepton $p_\perp$ in W and Z production as well as the Z mass to construct an observable for the W mass:

$$m_W^{\text{meas}} = \frac{\langle p_\perp^{\ell,W} \rangle^{\text{meas}}}{\langle p_\perp^{\ell,Z} \rangle^{\text{meas}}} \, m_Z \, C_{\text{th}}$$

Construction of our observable (cont.)

Use the average lepton $p_\perp$ in W and Z production as well as the Z mass to construct an observable for the W mass:

Measurement from LHC

$\propto m_W$

$$m_W^{\text{meas}} = \frac{\boxed{\langle p_\perp^{\ell,W} \rangle^{\text{meas}}}}{\boxed{\langle p_\perp^{\ell,Z} \rangle^{\text{meas}}}} \, m_Z \, C_{\text{th}}$$

$\propto m_Z$

Measurement from LHC

Construction of our observable (cont.)

Use the average lepton $p_\perp$ in W and Z production as well as the Z mass to construct an observable for the W mass:

Measurement from LEP

$$m_W^{\text{meas}} = \frac{\langle p_\perp^{\ell,W} \rangle^{\text{meas}}}{\langle p_\perp^{\ell,Z} \rangle^{\text{meas}}} \boxed{m_Z} C_{\text{th}}$$

Construction of our observable (cont.)

Use the average lepton $p_\perp$ in W and Z production as well as the Z mass to construct an observable for the W mass:

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Theoretical correction factor

Construction of our observable (cont.)

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Theoretical correction factor

$\rightarrow$ Calculate via $C_{\text{th}} = \dfrac{m_W}{m_Z} \dfrac{\langle p_\perp^{\ell,Z} \rangle^{\text{th}}}{\langle p_\perp^{\ell,W} \rangle^{\text{th}}}$

Construction of our observable (cont.)

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Theoretical correction factor

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Adding a new correction to the theory

$\rightarrow$ changes C_{th}

$\rightarrow$ changes extracted mass m_W^{meas}

$$\frac{\delta m_W^{\text{meas}}}{m_W^{\text{meas}}} = \frac{\delta C_{\text{th}}}{C_{\text{th}}} = \frac{\delta\langle p_\perp^{\ell,Z}\rangle}{\langle p_\perp^{\ell,Z}\rangle} - \frac{\delta\langle p_\perp^{\ell,W}\rangle}{\langle p_\perp^{\ell,W}\rangle}$$

Shifts on W mass (inclusive setup)

Estimated impact of …

… mixed QCD-EW corrections:

$$\delta m_W \approx -7\,\mathrm{MeV}$$

… NLO electroweak corrections:

$$\delta m_W \approx 1\,\mathrm{MeV}$$

$$\delta m_W = \left(\frac{\delta\langle p_\perp^{\ell,Z}\rangle}{\langle p_\perp^{\ell,Z}\rangle} - \frac{\delta\langle p_\perp^{\ell,W}\rangle}{\langle p_\perp^{\ell,W}\rangle} \right) m_W$$

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Mixed QCD-EW corrections appear to have larger impact than NLO EW corrections

- G_μ input parameter scheme reduces size of NLO EW corrections
- Strong cancellation between changes in Z and W

Shifts on W mass (inclusive setup)

Estimated impact of ...

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... NLO electroweak corrections:

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$$\delta m_W = \left(\cancel{\frac{\delta\langle p_\perp^{\ell,Z}\rangle}{\langle p_\perp^{\ell,Z}\rangle}} - \frac{\delta\langle p_\perp^{\ell,W}\rangle}{\langle p_\perp^{\ell,W}\rangle} \right) m_W$$

$\rightarrow \delta m_W \approx 54\,\text{MeV}$ (mixed QCD-EW)

$\rightarrow \delta m_W \approx -31\,\text{MeV}$ (NLO EW)

Mixed QCD-EW corrections appear to have larger impact than NLO EW corrections

- G_μ input parameter scheme reduces size of NLO EW corrections
- Strong cancellation between changes in Z and W

Shifts on W mass (inclusive setup)

Estimated impact of ...

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$\rightarrow$ $\delta m_W \approx -31\,\text{MeV}$ (NLO EW)

$\rightarrow$ Changes are more correlated between Z and W for NLO EW

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Minor influence of PDFs:

- Tested with specialised minimal PDF sets provided by NNPDF collaboration (based on `NNPDF3.1luxQED`)
- Mixed QCD-EW corrections: About $\mathcal{O}(1)$ MeV changes

Scale variation: $\mathcal{O}(\pm 2)$ MeV

The influence of fiducial cuts

Repeat calculation with fiducial cuts (inspired by [ATLAS '17] analysis)

Estimated impact of ...

... mixed QCD-EW corrections:

$$\delta m_W \approx -17\,\text{MeV}$$

... NLO electroweak corrections:

$$\delta m_W \approx 3\,\text{MeV}$$

W production:

- $p_\perp^{e^+} > 30\,\text{GeV}$
- $p_\perp^{\text{miss}} > 30\,\text{GeV}$
- $|\eta_{e^+}| < 2.4$
- $m_T^W > 60\,\text{GeV}$

Z production:

- $p_\perp^{e^\pm} > 25\,\text{GeV}$
- $|\eta_{e^\pm}| < 2.4$

$\rightarrow$ Shifts are larger than for inclusive setup

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$\rightarrow$ Shifts are larger than for inclusive setup

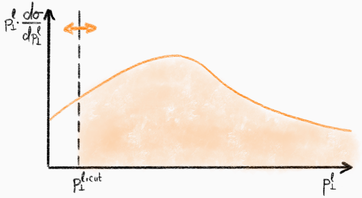
Key reason:

- Relevant transverse momenta: $p_\perp^{e^+}/M_V$
- ATLAS applies larger $p_\perp^{e^+}$ cuts to (lighter) W bosons than to (heavier) Z bosons
- Leads to small decorrelation of corrections to W and Z bosons

Tuning the cuts

Can we “tune” the cuts to reduce the impact of mixed QCD-EW corrections?

- Start from ATLAS-inspired cuts as baseline
- Decrease cuts on $p_\perp^{e^+}$ for W^+ case until $C_{\text{th}} = 1$ at LO
- Leads to $p_\perp^{e^+} > 25.44\,\text{GeV}$



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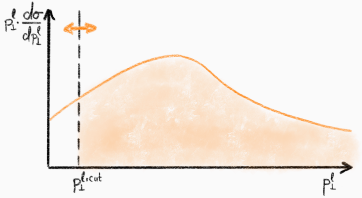

Estimated impact of ...

... mixed QCD-EW corrections:

$$\delta m_W \approx -1\,\text{MeV}$$

... NLO electroweak corrections:

$$\delta m_W \approx -3\,\text{MeV}$$

$\rightarrow$ Strong cut dependence of δm_W allows to “tune away” QCD-EW corrections in our setup

Conclusions

Conclusions

- We calculate mixed QCD-EW corrections to fully-differential on-shell W and Z production at the LHC.
  $\rightarrow$ Possible thanks to progress on amplitude calculations and subtraction schemes.
- Size of mixed QCD-EW corrections to the production part is $\mathcal{O}(0.5)$‰.
  $\rightarrow$ Corrections are small but in line with expectations.
- Experimental measurements of m_W rely on similarity between W and Z distributions. Based on this, we build a transparent and simple model to estimate shifts on m_W via

$$\delta m_W = \left(\frac{\delta \langle p_\perp^{\ell,Z} \rangle}{\langle p_\perp^{\ell,Z} \rangle} - \frac{\delta \langle p_\perp^{\ell,W} \rangle}{\langle p_\perp^{\ell,W} \rangle} \right) m_W \, .$$

- We find that mixed QCD-EW corrections induce shifts on m_W that are comparable or larger than the target precision of $\mathcal{O}(10)$ MeV.
- Further investigations on the impact of mixed QCD-EW corrections on m_W are clearly warranted. They should reflect all relevant details of experimental analyses.

Backup

Input parameters

Input parameters used:

$$
\begin{aligned}
G_F &= 1.166\,39 \times 10^{-5}\,\text{GeV}^{-2} \\
m_Z &= 91.1876\,\text{GeV} \\
m_W &= 80.398\,\text{GeV} \\
m_H &= 125\,\text{GeV} \\
m_t &= 173.2\,\text{GeV}
\end{aligned}
$$

- We use the G_μ input parameter scheme.
- PDFs: NNLO set `NNPDF3.1luxQED` with $\alpha_s(m_Z) = 0.118$
- Simulations for 13 TeV LHC
- Central scale: $\mu_R = \mu_F = m_V/2$

Detailed results for cross-sections and moments

Results for the cross-sections and average transverse momentum of the charged lepton for the inclusive setup of $pp \to Z \to e^+e^-$ and $pp \to W^+ \to e^+\nu_e$ (corrections only to the production part)

$$\mathrm{d}\sigma_{Z,W} = \sum_{i,j=0} \alpha_s^i \alpha_W^j \mathrm{d}\sigma_{Z,W}^{i,j} \qquad F_{Z,W}(i,j,\mathcal{O}) = \alpha_s^i \alpha_W^j \int \mathrm{d}\sigma_{Z,W}^{i,j} \times \mathcal{O}$$

	$V = Z$			$V = W^+$		
	$\mu = m_Z/4$	$\mu = m_Z/2$	$\mu = m_Z$	$\mu = m_W/4$	$\mu = m_W/2$	$\mu = m_W$
$F_V(0,0;1)$, [pb]	1273	1495	1700	7434	8810	10083
$F_V(1,0;1)$, [pb]	570.2	405.4	246.9	3502	2533	1580
$F_V(0,1;1)$, [pb]	$-5810 \cdot 10^{-3}$	$-6146 \cdot 10^{-3}$	$-6073 \cdot 10^{-3}$	$-1908 \cdot 10^{-3}$	$3297 \cdot 10^{-3}$	$10971 \cdot 10^{-3}$
$F_V(1,1;1)$, [pb]	$-2985 \cdot 10^{-3}$	$-2033 \cdot 10^{-3}$	$-1236 \cdot 10^{-3}$	$-8873 \cdot 10^{-3}$	$-7607 \cdot 10^{-3}$	$-7556 \cdot 10^{-3}$
$F_V(0,0;p_\perp^e)$ [GeV pb]	42741	50191	57073	220031	260772	298437
$F_V(1,0;p_\perp^e)$ [GeV pb]	23418	17733	12221	124487	95132	66090
$F_V(0,1;p_\perp^e)$ [GeV pb]	-182.85	-192.77	-189.11	74.53	243.54	484.82
$F_V(1,1;p_\perp^e)$ [GeV pb]	-163.87	-125.22	-92.05	-553.87	-482.0	-448.0

Detailed results for W mass shifts

Detailed results for the shifts δm_W for different setups, orders and scales

δm_W [MeV]		$\mu = m_V/4$	$\mu = m_V/2$	$\mu = m_V$
Inclusive	NLO EW	-0.1	0.3	0.2
	QCD-EW	-5.1	-7.5	-9.3
Fiducial	NLO EW	0.2	2.3	4.2
	QCD-EW	-16	-17	-19
Tuned fiducial	NLO EW	-4.4	-2.5	-0.8
	QCD-EW	3.9	-1.0	-5.7