SECTION 15

slide 21

Density of final states

We have a 3-body final state (Ch.2 looked at 2-body)

Treat 2 particles as independent (3rd given by $E,\vec{p}$ conservation)

$$dN = \frac{p_\nu^2}{(2\pi)^3} d\Omega_\nu dp_\nu \frac{p_e^2}{(2\pi)^3} d\Omega_e dp_e$$

Energy released goes to KE of decay products

$$E_0 = E_\nu + E_e + T_{recoil}$$

$M_{nucleus} >> m_e, m_\nu$ $\quad E_0 \sim E_\nu + E_e$

For a given electron energy $dE_0 = dE_\nu = dp_\nu$ since $m_\nu \sim 0$

$$\frac{dN}{dE_0} = \frac{dN}{dp_\nu} = \frac{E_\nu^2}{(2\pi)^3} d\Omega_\nu \frac{p_e^2}{(2\pi)^3} d\Omega_e dp_e$$

Assume isotropic decay & integrate over $\Omega_\nu \Omega_e$

$$\frac{dN}{dE_0} = \rho(E_0) = (4\pi)^2 \frac{E_\nu^2}{(2\pi)^3} \frac{p_e^2}{(2\pi)^3} dp_e = \frac{E_\nu^2 p_e^2 \, dp_e}{4\pi^4}$$

$$= \frac{E_\nu^2 E_e^2 \, dE_e}{4\pi^4} \quad m_e \sim 0$$

$$= \frac{(E_0 - E_e)^2 E_e^2 \, dE_e}{4\pi^4}$$

Matrix element

4-point interaction $\quad M_{fi} = G_F \int \psi_n \psi_p^* \psi_e^* \psi_{\bar{\nu}_e}^* \, d^3\vec{r}$

e^- & $\bar{\nu}_e$ are free particles

$$\psi_e = e^{i\vec{p}_e \cdot \vec{r}} \qquad \psi_\nu = e^{i\vec{p}_\nu \cdot \vec{r}}$$

$$M_{fi} = G_F \underbrace{\int \psi_p^* e^{-i(\vec{p}_e + \vec{p}_\nu)\cdot\vec{r}} \psi_n \, d^3\vec{r}}_{M_{nuclear}}$$

Decay Rate

$$\Gamma = 2\pi |M_{fi}|^2 \rho(E_f)$$

$$= 2\pi G_F^2 |M_{nuclear}|^2 \int_0^{E_0} \frac{(E_0 - E_e)^2 E_e^2 \, dE}{4\pi^4}$$

$$= \frac{G_F^2 |M_{nuclear}|^2}{2\pi^3} \int_0^{E_0} (E_0 - E_e)^2 E_e^2 \, dE_e$$

slide 28

- $^{34}Cl(0^+) \rightarrow {}^{34}S(0^+)$

  no parity change, ℓ is even
  no J change
  $\Rightarrow S_{ev} = 0$ Fermi allowed
  no GT $S_{ev} = 0$ as $0 \rightarrow 0$ forbidden

- $^{14}C(0^+) \rightarrow {}^{14}N(1^+)$

  no change in parity ℓ is even
  $\Delta J = 1$
  $\Rightarrow S_{ev} = 1$ GT allowed

- $n(\frac{1}{2}^+) \rightarrow p(\frac{1}{2}^+)$

  no change in P
  $\Delta J = 0$
  $\Rightarrow S_{ev} = 0$ Fermi allowed
  $S_{ev} = 1$ GT allowed

  can be super allowed if overlap between ψ is maximal (mirror nuclei)

- $^{39}Ar(\frac{7}{2}^-) \rightarrow {}^{39}K(\frac{3}{2}^+)$

  change in P $\ell > 0$ forbidden
  $\Delta J = 2$
  for $\ell = 1$, 1st forbidden
  $S_{ev} = 1$ GT

- $^{87}Rb(\frac{3}{2}^-) \rightarrow {}^{87}Sr(\frac{9}{2}^+)$

  change in P $\ell > 0$ forbidden
  $\Delta J = 3$
  $\ell = 1$ doesn't work for $S_{ev} = 0, 1$
  $\ell = 3$ $S_{ev} = 0$ or 1
  3rd forbidden, GT, F.

slide 35

^{117}Sn

$7/2^+$
$11/2^-$
$3/2^+$
$1/2^+$

$\frac{3}{2}^+ \rightarrow \frac{1}{2}^+$ no parity change ℓ = even
$\Delta J = 1$ M1

$\frac{11}{2}^- \rightarrow \frac{3}{2}^+$ parity change ℓ = odd
$\Delta J = 4$ M4 or 2x E2 transitions

$\rightarrow \frac{1}{2}^+$ $\Delta J = 5$ E5, but less likely than M4

$\frac{7}{2}^+ \rightarrow \frac{11}{2}^-$ parity change ℓ = odd
$\Delta J = 2$ M2

$\rightarrow \frac{3}{2}^+$ no parity change ℓ = even
$\Delta J = 2$ E2 * most likely

$\rightarrow \frac{1}{2}^+$ ℓ = even
$\Delta J = 3$ M3