

SECTION 15

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Density of final states

We have a 3-body final state (Ch-2 looked at 2-body)
Treat 2 particles as independent (3rd given by \vec{E}, \vec{p} conservation)

$$\frac{dN}{d\Omega} = \frac{p_e^2}{(2\pi)^3} d\Omega_\nu d\Omega_e \frac{p_e^2}{(2\pi)^3} d\Omega_e d\Omega_\nu$$

Energy released goes to KE of decay products

$$E_0 = E_\nu + E_e + T_{\text{ recoil}}$$

$$M_{\text{nucleus}} \gg m_e, m_\nu \quad E_0 \sim \bar{E}_\nu + \bar{E}_e$$

For a given electron energy $d\bar{E}_0 = d\bar{E}_\nu = d\Omega_\nu$ since $m_\nu \approx 0$

$$\frac{dN}{d\bar{E}_0} = \frac{dN}{d\Omega_\nu} = \frac{\bar{E}_\nu^2}{(2\pi)^3} d\Omega_\nu \frac{p_e^2}{(2\pi)^3} d\Omega_e d\Omega_\nu$$

Assume isotropic decay & integrate over $d\Omega_\nu d\Omega_e$

$$\frac{dN}{d\bar{E}_0} = \rho(\bar{E}_0) = (4\pi)^2 \frac{\bar{E}_\nu^2}{(2\pi)^3} \frac{p_e^2}{(2\pi)^3} d\Omega_e = \frac{\bar{E}_\nu^2 p_e^2}{4\pi^4} d\Omega_e$$

$$= \frac{\bar{E}_\nu^2 \bar{E}_e^2}{4\pi^4} d\bar{E}_e \quad m_e \approx 0$$

$$= \frac{(\bar{E}_0 - \bar{E}_e)^2 \bar{E}_e^2}{4\pi^4} d\bar{E}_e$$

Matrix element

4-point interaction $M_{fi} = G_F \int \psi_n \psi_p^\dagger \psi_e^* \psi_{\bar{\nu}}^\dagger d^3 r$

e^- & $\bar{\nu}_e$ are free particles

$$\psi_e = e^{i\vec{p}_e \cdot \vec{r}} \quad \psi_{\bar{\nu}} = e^{i\vec{p}_{\bar{\nu}} \cdot \vec{r}}$$

$$M_{fi} = G_F \int \psi_p^\dagger e^{-i(\vec{p}_e + \vec{p}_{\bar{\nu}}) \cdot \vec{r}} \psi_n d^3 r$$

Mnuclear

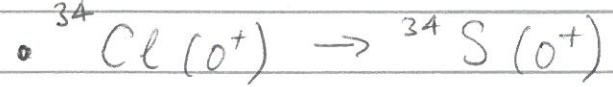
Decay Rate

$$\Gamma = 2\pi |M_{fi}|^2 \rho(\bar{E}_0)$$

$$= 2\pi G_F^2 / |M_{\text{nuclear}}|^2 \int_0^{\bar{E}_0} \frac{(\bar{E}_0 - \bar{E}_e)^2 \bar{E}_e^2}{4\pi^4} d\bar{E}_e$$

$$= \frac{G_F^2 / |M_{\text{nuclear}}|^2}{2\pi^3} \int_0^{\bar{E}_0} (\bar{E}_0 - \bar{E}_e)^2 \bar{E}_e^2 d\bar{E}_e$$

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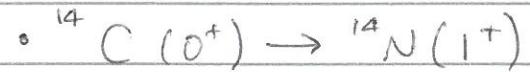


no parity change, ℓ is even

no J change

$\Rightarrow S_{\text{ev}} = 0$ Fermi allowed

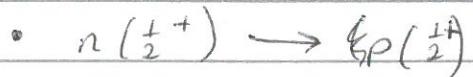
no GT $S_{\text{ev}} = 0$ as $0 \rightarrow 0$ forbidden



no change in parity ℓ is even

$\Delta J = 1$

$\Rightarrow S_{\text{ev}} = 1$ GT allowed

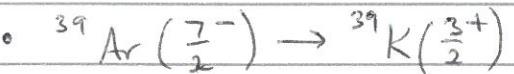


no change in P

$\Delta J = 0$

$\Rightarrow S_{\text{ev}} = 0$ Fermi allowed

$S_{\text{ev}} = 1$ GT allowed

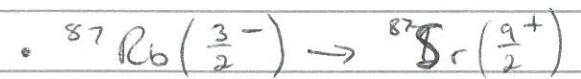


change in P $\ell > 0$ forbidden

$\Delta J = 2$

for $\ell = 1$, 1st forbidden

$S_{\text{ev}} = 1$ GT



change in P $\ell > 0$ forbidden

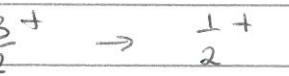
$\Delta J = 3$

$\ell = 1$ doesn't work for $S_{\text{ev}} = 0, 1$

$\ell = 3$ $S_{\text{ev}} = 0$ or 1

3rd forbidden, GT, F.

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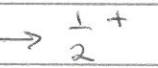
no parity change $\ell = \text{even}$

$\Delta J = 1$ M1

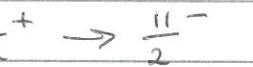


parity change $\ell = \text{odd}$

$\Delta J = 4$ M4 or 2x E2 transitions

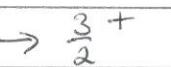


$\Delta J = 5$ E5, but less likely than M4



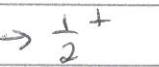
parity change $\ell = \text{odd}$

$\Delta J = 2$ M2



no parity change $\ell = \text{even}$

$\Delta J = 2$ E2 * most likely



$\ell = \text{even}$

$\Delta J = 3$ M3

