SECTION 14

slide 10

single nucleon $s = \pm\frac{1}{2}$

$j = \ell + \frac{1}{2}$ $\quad \hat{L}.\hat{S}|\psi\rangle = \frac{1}{2}\left(\underbrace{(\ell+\tfrac{1}{2})(\ell+\tfrac{3}{2}) - \ell(\ell+1) - \tfrac{1}{2}(\tfrac{1}{2}+1)}_{\ell^2+\frac{1}{2}\ell+\frac{3}{2}\ell+\frac{3}{4}-\ell^2-\ell-\frac{1}{4}-\frac{1}{2} \;=\; \ell}\right)|\psi\rangle$

$= \frac{1}{2}\ell|\psi\rangle$ $\quad V_{so}$ negative $\Rightarrow$ lowers V

$j = \ell - \frac{1}{2}$ $\quad \hat{L}.\hat{S}|\psi\rangle = \frac{1}{2}\left(\underbrace{(\ell-\tfrac{1}{2})(\ell+\tfrac{1}{2}) - \ell(\ell+1) - \tfrac{1}{2}(\tfrac{1}{2}+1)}_{\ell^2-\frac{1}{2}\ell+\frac{1}{2}\ell-\frac{1}{4}-\ell^2-\ell-\frac{3}{4} \;=\; -\ell-1}\right)|\psi\rangle$

$= \cancel{\frac{1}{2}\ell|\psi\rangle}$ $\quad V_{so}$ negative $\Rightarrow$ raises V

$= -\frac{1}{2}(\ell+1)|\psi\rangle$

slide 12

${}^{18}_{8}O$ Even-even – no unpaired nucleon $\Rightarrow J^P = 0^+$

${}^{15}_{7}N$ Unpaired nucleon in $1p_{1/2}$ $\quad n\ell_j \quad j = \frac{1}{2} \quad \ell = 1 \quad s = \frac{1}{2}$

$P = (-1)^{\ell} = -1 \quad \therefore J^P = \frac{1}{2}^-$

${}^{10}_{5}B$ Unpaired p in $1p_{3/2}$ $\quad j_p = \frac{3}{2} \quad \ell = 1$

n in $1p_{3/2}$ $\quad j_n = \frac{3}{2} \quad \ell = 1$

$J \quad |j_p - j_n| \dots |j_p + j_n| = 0, \dots, 3 \qquad 0, 1, 2, 3$

$P = (-1)^{\ell_n}(-1)^{\ell_p} = (-1)^1(-1)^1 = +1$

$\Rightarrow J^P = 0^+ \quad 1^+ \quad 2^+ \quad 3^+$

s p d f
0 1 2 3

82 protons, 125 neutrons

slide 13 $\quad {}^{207}_{82}Pb$

$2f_{5/2}$

$3p_{1/2}$

this pairing is weaker than this

slide 14 $\quad {}^{23}_{11}Na$ $\quad 1d_{5/2}$ $\quad$ 11p $\quad$ 12n

shell can take 6p & 6n
~1/2 filled

if one unpaired proton $\quad j = 5/2 \quad \ell = 2 \quad J^P = \frac{5}{2}^+$

but not observed!

if all 3p contribute... can have observed $\frac{3}{2}^+$
J

slide 21 $\quad$ Allowed J states for 2 quadrupole phonons $\quad J^P_{ph} = 2^+$

Allowed J for diphonon state?

Phonons $J^P = 2^+ \quad m_j = -2, -1, 0, 1, 2$

−1 $\quad \boxed{-2}$
−2 $\quad \boxed{-3}$
−2 $\quad$ 2 ✗ $\quad$ 0
1 ✗ $\quad$ −1
0 ✗ $\quad$ −2
−1 ✗ $\quad$ −3
−2 $\quad \boxed{-4}$

The discarded double-counted states would allow J = 3 & J = 1, but not allowed if want ψ symm.

See the course web page on the TIS, from which there is a link to http://www.sp.phy.cam.ac.uk/teaching/em/

Allowed J for diphonon state?

Phonons $J^P = 2^+$ $m_j = -2, -1, 0, 1, 2$

$\Downarrow$ BOSONS ψ must be symmetric

ie diphonon state formed by $m_1 = 1$, $m_2 = 0$ is indistinguishable from $m_2 = 1$, $m_1 = 0$

$\rightarrow$ do not double count states! ✱

Can acheive this using $m_2 \leq m_1$

Phonon 1 m_1	Phonon 2 m_2	Diphonon state $m_j = m_1 + m_2$
2	2	4
	1	3
	0	2
	-1	1
	-2	0
1	2 ✱	3
	1	2
	0	1
	-1	0
	-2	-1
0	2 ✱	2
	1 ✱	1
	0	0
	-1	-1
	-2	-2
-1	2 ✱	1
	1 ✱	0
	0 ✱	-1
	-1	-2
	-2	-3
-2	2 ✱	0
	1 ✱	-1
	0 ✱	-2
	-1 ✱	-3
	-2	-4

$\rightarrow J=4$

$J=2$

$J=0$

The discarded double-counted states would allow $J=3$ & $J=1$, but not allowed if want ψ_{symm}.