

SECTION 14

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single nucleon $s = \pm \frac{1}{2}$

$$j = l + \frac{1}{2} \quad \hat{L} \cdot \hat{S} |\psi\rangle = \frac{1}{2} \left((l + \frac{1}{2})(l + \frac{3}{2}) - l(l+1) - \frac{1}{2}(\frac{1}{2} + 1) \right) |\psi\rangle$$

$$l^2 + \frac{1}{2}l + \frac{3}{2}l + \frac{3}{4} - l^2 - l - \frac{1}{4} - \frac{1}{2}$$

$$= \frac{1}{2} l |\psi\rangle \quad \checkmark \text{ no negative} \Rightarrow \text{lowers } V$$

$$j = l - \frac{1}{2} \quad \hat{L} \cdot \hat{S} |\psi\rangle = \frac{1}{2} \left((l - \frac{1}{2})(l + \frac{1}{2}) - l(l+1) - \frac{1}{2}(\frac{1}{2} + 1) \right) |\psi\rangle$$

$$l^2 - \frac{1}{2}l + \frac{1}{2}l - \frac{1}{4} - l^2 - l - \frac{1}{4} - \frac{1}{2}$$

$$= -\frac{1}{2}(l+1) |\psi\rangle \quad \checkmark \text{ no negative} \Rightarrow \text{raises } V$$

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^{18}O Even-even - no unpaired nucleon $\Rightarrow J^P = 0^+$

^{15}N Unpaired nucleon in $1p_{1/2}$ n in $1p_{1/2}$ $j = \frac{1}{2}$ $l = 1$ $s = \frac{1}{2}$ $P = (-1)^l = -1$ $J^P = \frac{1}{2}^-$

^{10}B Unpaired p in $1p_{3/2}$ $j = \frac{3}{2}$ $l = 1$
 Unpaired n in $1p_{3/2}$ $j = \frac{3}{2}$ $l = 1$

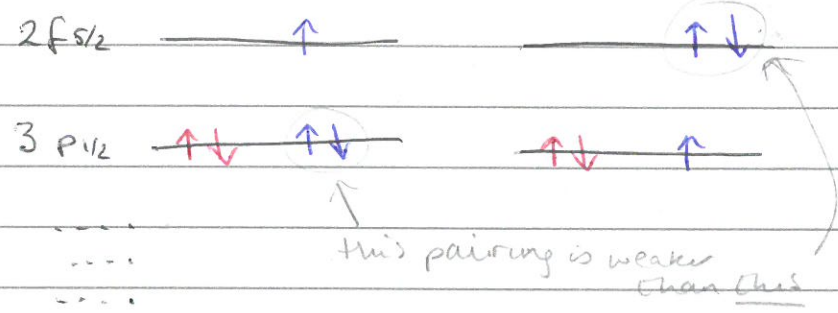
J $|j_1 - j_2| \dots |j_1 + j_2| = 0 \dots 3$ $0, 1, 2, 3$
 $P = (-1)^{l_n} (-1)^{l_p} = (-1)^1 (-1)^1 = +1$
 $\Rightarrow J^P = 0^+, 1^+, 2^+, 3^+$

s p d f
0 1 2 3

82 125
protons neutrons

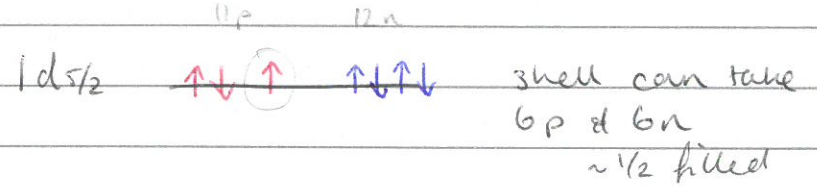
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^{207}Pb
82



slide 14

^{23}Na
11



if one unpaired proton $J = 5/2$ $l = 2$ $J^P = 5/2^+$
 but not observed!
 if all 3p contribute... can have observed $\frac{3}{2}^+$

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Allowed J states for 2 quadrupole phonons $J^P = 2^+$

Allowed J for diphonon state?

Phonons $J^P = 2^+$ $m_j = -2, -1, 0, 1, 2$

	11	12	13
-1	-2	-3	
-2			
2	0		
1	-1		
0	-2		
-1	-3		
-2		-4	

The discarded double-counted states would allow $J = 3$ & $J = 1$, but not allowed if want ψ_{symm} .

Allowed J for diphonon state?

Phonons $J^P = 2^+$ $m_j = -2, -1, 0, 1, 2$

BOSONS

Ψ must be symmetric

i.e. diphonon state formed by $m_1=1, m_2=0$ is indistinguishable from $m_2=1, m_1=0$

→ do NOT double count states! ✗

Can achieve this using $m_2 \leq m_1$.

Phonon 1 m_1	Phonon 2 m_2	Diphonon state $m_j = m_1 + m_2$
2	2	4
	1	3
	0	2
	-1	1
	-2	0
1	2 ✗	3
	1	2
	0	1
	-1	0
	-2	-1
0	2 ✗	2
	1 ✗	1
	0	0
	-1	-1
	-2	-2
-1	2 ✗	1
	1 ✗	0
	0 ✗	-1
	-1	-2
	-2	-3
-2	2 ✗	0
	1 ✗	-1
	0 ✗	-2
	-1 ✗	-3
	-2	-4

$J=0$ (states with $m_j = 0$)
 $J=2$ (states with $m_j = \pm 2$)
 $J=4$ (state with $m_j = 4$)

The discarded double-counted states would allow $J=3$ & $J=1$, but not allowed if want Ψ_{symm} .